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INVOLUNTARY UNEMPLOYMENT

by

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Abstract

We show that a firm can increase expected profits by undertaking the additional expense of paying unemployment compensation to the workers its lays off, if they are risk averse. When this argument is applied to the implicit contract models it makes the involuntary unemployment derived there disappear, where by involuntary unemployment we mean a situation in which one worker has a job at a wage w and another worker who is known to be productively identical and willing to take on the job at a lower wage h cannot find a job.

We introduce into our model asymmetric information between sectors of the economy. Each agent knows the state of his own firm but not that of others. We suppose also that workers have specific skills which are conformable to some, but not all, firms in the economy. In this model we reestablish the phenomenon of involuntary unemployment in a general implicit contracts equilibrium in which the proportion of layoffs, the "stabilized" wage, and the severance payments are endogenously determined. Moreover, we show that the presence of involuntary unemployment is a signal that there is too little output in the most productive sectors of the economy, thereby restoring the link between underproduction and involuntary unemployment missing in the implicit contracts literature. Finally we ask how large should be the severance payment a profit maximizing firm gives to each worker from a branch it shuts down, when there is uncertainty about what jobs those workers will find. We prove that the rational expected-profit-maximizing firm will offer a contract which provides severance compensation so generous that on average the workers dismissed by the closing of a plant can expect to be better off than if they had been retained, in the case that they have decreasing absolute risk aversion.

Introduction

A fundamental problem in the usual theory of general economic equilibrium, as described, say, by the famous model of Arrow and Debreu, is that it does not allow for involuntary unemployment. Prices, including wages, always adjust in that model to equate supply and demand so that in equilibrium, even though workers may be able to work more and produce more output, they choose, at the going prices and wages, not to. Output is not maximized but welfare is.^{1/}

It seems to us an important empirical fact that workers cannot always find jobs at the going wages, even though they would choose to work at those wages. We therefore take for granted the existence of involuntary unemployment, by which we mean a situation in which one worker has a job at a wage w and another worker who is known to be productively identical and willing to take on the job at a lower wage h cannot find a job. It is the purpose of this paper to show how asymmetries in information together with "implicit contracts" can create such involuntary unemployment. In our explanation, we do not rely on an ad hoc prohibition against severance payments, on fixed coefficients in production, or on the lack of knowledge of aggregate information about the labor market.

An important and plausible explanation for involuntary unemployment, given first in the mid 1970's by Azariadis and Baily, asserts that wage contracts do more than allocate laborers to jobs where they are most productive. By guaranteeing the risk-averse worker a certain wage they protect him from the otherwise uncertain income prospects he would face if he were forced to reoffer his labor services anew every "period" in a changing world. The

consequence, according to this argument, is that wages are not free to adjust continuously to equate demand and supply for labor, and therefore it occasionally happens that a firm finds it profitable to lay off a worker who will not afterwards be able to find another job. Another less attractive, but logically necessary, consequence of this analysis is that the resulting involuntary unemployment is less than the voluntary unemployment in an Arrow-Debreu world and the output greater than the output in an ideal Arrow-Debreu world where wages are free to continuously adjust to equate supply and demand for workers. This last point is disturbing for those who identify the reasons for involuntary unemployment with the reasons for underproduction.

In this paper we extend the "implicit contracts" theory described above to take into account the possibility of private unemployment compensation. We argue that a profit maximizing firm can increase its profits by undertaking the additional responsibility and expense of paying compensation to the workers it lays off. By promising and making such payments the firm can induce potential new employees concerned about the financial risk of job loss to accept lower wages, thereby reducing total expected labor costs. While perhaps appearing to bow to public pressure the firm thus exploits the risk aversion of its workers with severance payments, just as it does with guaranteed stable wages, to reduce its wage bill. Of course the consequence of this argument is that involuntary unemployment again disappears. Yet we acknowledge its existence in the world and moreover we also recognize that some industries make only token contributions directly to the workers their firms lay off, despite the fact that some of them remain unemployed for considerable lengths of time.

In order to reestablish the existence of involuntary unemployment we

modify the implicit contracts theory once more by introducing an asymmetry of information between different sectors of the economy. We show that if there is uncertainty about the job prospects a worker will find once he is laid off, and if the original firm is unable to monitor the wage a laid-off worker finally obtains (so that all laid-off workers must get the same payments) then once again it can happen that a worker is laid off and unable to find another job. Simply put, if there is a high probability that workers laid off by firms in industry A will find good jobs in other industries, then the firm does not gain much by offering to make severance payments since a large part of that money will go to workers who do not value it very highly. On the other hand, we show that in an industry B where workers' skills are so specialized that there is little doubt about the wages they will earn if they are forced to look elsewhere there will be no involuntary unemployment.

If a firm in industry B is aware of which workers will not be able to find other jobs, then they will be compensated so that, taking into account their extra free time, they will be as well off unemployed as the retained workers in that firm.

We prove that when involuntary unemployment does occur in our model, output and employment are lower in the most productive industries than in an idealized Arrow-Debreu world. This argument, which restores the link between underproduction and involuntary unemployment, depends on the inability of the firms to monitor the search efforts of their laid-off workers, as opposed to the idealized Arrow-Debreu world where there are no such moral hazards.

Finally, we ask how big a severance payment the profit maximizing firm should make, if, say, it closes a plant. Will it be so big that on average the laid-off workers end up as well as the retained workers? We show, surprisingly, that if

workers have decreasing absolute risk aversion, then the optimal contract will make their expected utility higher, at the moment of firing, than if they have been retained. Of course those workers who are laid off and do not find other high paying jobs will end up much worse off, but other workers will get the severance compensation and then find other equally good jobs.

In Section 1, we give an overview of previous work on implicit contracts, and in Sections 2 and 3 we present our model. In Section 4 we derive the propositions outlined above and we also analyze an interesting property of incomplete insurance arising, as in our model of layoffs, when there are two stages of uncertainty and only the first can be insured. In Section 5 we calculate a numerical example of our general equilibrium model with involuntary unemployment and severance payments. In Section 6 we make some concluding remarks.

I. Survey of the Literature

There is in the literature a long tradition of fixed price models in which prices are held fixed at nonequilibrium levels, resulting in involuntary unemployment and lower output than is Pareto optimal in the Arrow-Debreu world. The theoretical difficulty with these models is that they often do not explain why prices remain fixed or else how one vector of prices rather than another is chosen.

The implicit contract theory which became popular following the pioneering articles by Baily (1974), D. F. Gordon (1974), and Azariadis (1975) was originally expected to provide a "microfoundation" for the disequilibrium fixed price models (see R. J. Gordon (1976)). In this theory, risk neutral firm owners and risk averse workers negotiate contracts before the productivity (marginal revenue product) of the firm is known. In the partial equilibrium context

described by the original implicit contract papers where workers cannot be bid away by other firms it is easy to see that any optimal "ex ante implicit contract" must specify the same (fixed) real wage in every state of nature. (An ex ante implicit contract (w_s, r_s) specifies for each state of nature, or firm productivity, s , the wage w_s that will be paid and the fraction r_s of the labor pool that will be retained by the firm. It is simple to show that unless w_s is constant for all s there is another contract (\hat{w}_s, \hat{r}_s) which offers a representative worker the same expected utility while increasing the firm's expected profits.) This argument was used to explain why, once the firm's productivity became known, the wage would not move to equal the marginal product of the last member of the labor pool (assuming labor inelastically supplied at any wage above some level of "at home" production h). Thus it seemed at first that the implicit contract theory might provide a basis for fixed wages and involuntary unemployment. By contrast, recall that in a spot market world where prices are determined after the productivity of the firm is known, the real wage will move up and down with the marginal product of labor; either the work force will be fully employed or else the wage will fall to the level of at home production h and part of the work force will be voluntarily unemployed.

Several difficulties, however, have recently emerged with this interpretation of the theory. In the first place many authors, most notably Negishi (1979, Chapter 16) and Akerlof and Miyazaki (1980), have noticed, on the basis of the equation derived by Azariadis, that although in "bad" states the Azariadis model can produce involuntary unemployment ($w_s > h$ and $0 < r_s < 1$) this involuntary unemployment is actually less than the voluntary unemployment which would result in the spot market world with the same productive opportunities. In other words the marginal product of the last worker retained by the firm in a bad state is less than h . Firm production is greater with implicit contracts than without, even though the wage is kept at a higher level. The reason for this curious result is that by negotiating the contract before the state is known the wage can be stabilized across all states,

removing one source of uncertainty to the worker, but at the same time creating another. In the spot market model it made no difference to the worker, given the state s , whether he was employed or not, but in the implicit contracts world the stabilized wage w_s is higher than h . In order to partly insure the worker against this new risk the firm agrees to retain more workers than it would if there were only spot markets (in case every worker is employed in a spot-market world, everybody would be employed in the implicit contracts world, too). Thus Negishi and Akerlof and Miyazaki pointed out that if at home production h is zero, and workers always have positive marginal productivity in the firm (or more generally if full employment marginal product is always greater than h), then there will be no unemployment of any kind in the implicit contracts world (or the spot-market world). Whether the criticism that output fluctuates more in the spot-market model than in the implicit contracts model is terribly damaging, however, is an empirical question. It may well be that if wages were negotiated daily there would be wide swings in voluntary unemployment and that the slight smoothing of such a cycle indicated by implicit contract theory could still yield substantial (involuntary) unemployment in some states.

A second and, we think, more fundamental criticism involves the possibility of private unemployment compensation. If the firm can compensate workers who are laid off, then the risk of job loss can be eliminated and workers will be retained only until the point where marginal product equals h . There will be no involuntary unemployment. The importance of private unemployment compensation to involuntary unemployment has been noted by Sargent (1979, Chapter 8) and Polemarchakis and Weiss (1978). The principal focus of investigation in this paper will be to describe circumstances in which severance payments by the firm are permitted and yet involuntary unemployment still occurs.

A third difficulty with the early implicit contracts literature is that it did not consider the effects of many competing firms. In order for a firm to offer a stabilized wage paying more than a worker's marginal product in

some state of nature, it must be the case that in other states of nature the marginal product exceeds the wage; in those states, however, other identical firms will have an incentive to bid away workers unless they are receiving their full marginal product. We shall see shortly that the problem of negotiating implicit contracts when the workers cannot be compelled to remain with the firm is related to the last difficulty which we mention, namely how the analysis can be extended to more than one time period.

Very recently a number of authors have extended and modified the original implicit contracts framework in order to meet some of the objections raised above.^{2/} In one group of studies, which includes papers by Hall and Lilien (1979), Azariadis (1980), Green (1980), and S. Grossman and O. Hart (1980), asymmetric information between the firm owners and workers is introduced. In these studies a "first best" contract would make the wage and employment conditional on the productivity of the firm, but if that information is available only to the owners, they may have an incentive to misrepresent it, claiming "times are tough and wage cuts are necessary."^{3/} In this framework the best incentive compatible contract yields larger unemployment than the spot-market solution. However, the unemployment is all voluntary in the sense that those laid off by the firm are compensated so as to be no worse off than the workers retained by the firm. We mention that this approach does not rely on or take into account any economy-wide coordination problem such as which sectors of the economy should be allocated more workers — their story can be and is completely told in an economy with only one firm. Indeed in the Grossman-Hart model both the workers and owners of a firm are forbidden to use any information they may have about the aggregate economy (such as the general unemployment rate, etc.) in drawing up their contract. Finally we note one other small point: in the original implicit contracts literature the firm owners were assumed to be risk neutral, whereas in the Grossman-Hart model they must be risk averse.

The additional difficulties connected with adding many firms to the original implicit contract literature have for the most part been solved by Bengt Holmstrom. He made the important observation that if contracts are made for more than one time period, then the firm can pay a wage less than the worker's marginal product in the first period in exchange for providing a stabilized wage in the second period. The workers will not leave in the first period and sell their services at their marginal product wage because they would then forego the second period wage insurance. The firms can thus use the first period surplus to finance the stabilized second period wage. Holmstrom also points out that a worker's marginal product may be so high at another firm in the second period that his employer can retain him only by raising his wage - so wages are downward rigid over time, rather than completely rigid across states of nature. Once again, however, without the ad hoc prohibition against severance payments in Holmstrom's model, involuntary unemployment would disappear and the optimal amount of production would occur. Nevertheless, the Holmstrom model with many firms is a convenient starting point for our analysis.

We show in Section 4 that so long as the firm owners and workers know the worker's best opportunity outside the firm (e.g., the spot market wage in Holmstrom) and so long as that choice is unaffected by the firm's payments it follows as a general proposition that if leisure is a normal good then the optimal ex ante contract, with severance payments allowed, will always make laid-off workers at least as well-off as retained workers. This is true whether the owners are risk neutral or risk averse, whether the workers know the firm's productivity or not, etc. In case leisure and consumption are perfect substitutes, as they are in the Azariadis, Holmstrom, and Grossman-Hart models, the laid-off workers will be exactly compensated so that they are indifferent between working and not working. When workers' utility is separable between consumption and leisure the optimal contract will make laid-off workers better off than retained workers - involuntary employment, as it were. This point has been made independently

The last group of papers, including this one and Arnott, Hosios, and Stiglitz (1980) and Hosios (1980), addresses the problem of involuntary unemployment directly. The crucial idea in these papers is to introduce asymmetric information not between owners of a firm and its workers but between different firms. The paper by Arnott, Hosios and Stiglitz is very comprehensive, considering hours worked as well as number of workers employed, but it relies on behavioral functions (quit rates, etc.) not explicitly derived from optimizing behavior. Agents are not allowed to use information about the aggregate economy. The Hosios paper emphasizes the recruiting efforts of firms. His critical assumption is that the second period market for laid-off workers is much less organized than his first period market for workers seeking implicit contracts.

II. Model

In our paper workers and owners know the conditions affecting their own firms and they are also aware of the general conditions prevailing in the economy, i.e., they know total output, total unemployment, average wage, etc. But neither owners nor workers is precisely sure what job any particular worker will get once he leaves the firm. In order to find out what jobs are available and suitable for his talents, a worker must incur a fixed search cost z . The firm managers are assumed not to have the ability to monitor the opportunity their workers find outside the firm, and since the worker has no incentive to reveal truthfully what job he can get on the outside, private unemployment compensation cannot be made to depend on the eventual salary received. Notice that it is crucial to our story that the worker himself does not find out his opportunities until after the state is revealed. If, on the other hand, the worker knew from the beginning what his wage would be in case he lost his job, he would tell his employer and negotiate to get exactly the difference between his old wage and that alternative salary if he were ever laid off.

Since our story depends so strongly on the uncertain opportunities facing a worker once he is fired, our model must be rich enough to show this uncertainty. In particular, our model must be complex enough so that even with rational expectations it is not possible to infer from the aggregate information on unemployment and wages what the specific markets look like. We have taken the Holmstrom two period, general equilibrium model and added to it differences in information between firms, different types of workers, search costs, and a symmetry between the firms which prevents aggregate information from being "fully revealing" in the sense of Radner (1978) and Beth Allen (1979). Contrary to Hosios, we allow the second period market for laid-off workers to be just as well organized as the first period market. (In our model, when workers arrive in the first period they have already paid their search costs for the first period.)

Our economy is characterized by the following assumptions and notations. There are three industries, indexed by $j = I, II, III$. Each industry has many identical firms. Each industry j is subject to technological uncertainty of a good and bad state of nature, denoted by $\theta^j = G$ or B , respectively. A complete description of states of nature for an economy consists of three industries' states of nature, i.e., $\theta = (\theta^I, \theta^{II}, \theta^{III})$. Let us denote by Ω the set of all possible states of nature: $\theta \in \Omega$. An objective probability for each state of nature is known and denoted by $\phi(\theta)$. Workers have different skills, type a , b , or c . Industry I has a technology under which types a and b workers are perfectly substitutable, but type c workers are not at all productive. Similarly, Industry II's technology permits perfect substitutability between types a and c workers, but not type b ; and types b and c , but not type a workers, are perfectly substitutable in Industry III. Let us call this cyclic technological relation the overlapping conformability of skills.^{4/}

In the following, we refer to the representative firm in the j th industry simply by Firm j .

A firm is able to distinguish conformable workers from nonconformable workers without any costs, once workers apply for a job. Suppose that there are L workers per the representative firm of an industry for each type of labor skill. We consider a two-period economy: In the first period, the current production function is deterministic and known; however, the production function of the second period is stochastic. In the first period, firms bid for workers by offering long-term "implicit contracts," specifying the state contingent wages for both the first and the second periods, thereby guaranteeing some level of expected utility. Firm I is assumed to have an initial labor pool of $L/2$ each of types a and b workers; Firm II $L/2$ each of types a and c workers; and Firm III $L/2$ each of types b and c workers.^{5/} We allow the mobility of workers after the firm specific state of nature is revealed. As Holmstrom pointed out, while a contract wage is binding on the firm, an employee cannot be bound to stay in the same firm because of a ban on slavery. If there is a better opportunity outside of the initially-contracted firm, a worker is free to move without being financially penalized. Specifically, if the marginal product of a firm in a good state of nature is higher than the contracted wage of another firm, which is in either a good or bad state of nature, then the firm in the good state of nature can lure workers away from the other firm by offering a higher wage in the spot market, provided that a worker's skill is known to be conformable.^{6/} Two observations follow from this arbitrage argument. First, the second-period wage becomes state-contingent. Second, it becomes important to define how much information the firm and workers have about the state of nature for an economy. In the following we assume asymmetric information in the following sense. The firm and its contracted workers know the firm's own state of nature but not others' states of nature. In other words, both

the management and workers agree on whether it is a good time or a bad time for the firm to which they belong, while they do not know how the economic condition is affecting other firms. We also allow everyone to have access to aggregate information (total employment, average wage, etc.) which may be obtained for example by reading newspapers.^{7/} An essential noise in information arises when a particular type of worker does not know precisely what his wage would be if he went to the spot market. Formally, the information set, S_j , of the j th firm and its contracted workers in the second period, contains θ^j and any other aggregate information. In our example, the aggregate information is fully summarized by the unique average spot market wage, w_θ^a , of conformable-skill workers. Then we can write the information set of firm j as

$$S_j = \{(\theta^j, w_\theta^a)\} .$$

Given any state of nature $\theta \in \Omega$, define the private signal of firm j about this state of nature: $s^j(\theta) = (\theta^j, w_\theta^a) \in S_j$. For each $s^j \in S_j$, let $\pi_{s^j} = \phi\{\theta | s^j(\theta) = s^j\}$. We can think of each s^j as defining a subset of states of nature θ . We shall often write $w_{s^j}^a$ or even w_s^a when there is no doubt about which firm we are discussing. If information were complete, i.e., $S_j = \{\theta\}$, then the j th firm would know not only its own but also the other firms' states of nature, which we regard as an impossible informational burden. Yet another alternative would be $S_j = \{\theta^j\}$ that is the information of the firm and its workers is restricted to their own state of nature. No aggregate information is observed. Actually, this assumption is used in Arnott, Hosios and Stiglitz (1980). However, we think it more natural to suppose that the agents have some general knowledge of the economy.

Let us assume that workers have identical utility functions which are separable over time and depend only on net income: $u(y_1) + u(y_2)$ where $u' > 0$, $u'' < 0$. We assume that if a worker decides to stay home, he engages in "home production," yielding h . Therefore, there is no leisure in any case.

Individual labor supply is inelastic and normalized as unity. Firms are assumed to be risk-neutral. Even if firm owners are risk-averse, if they own diversified portfolios, then across states of nature θ where aggregate variables are the same, they will direct managers to maximize expected profits. As a result of this observation, our qualitative results are unaffected by assuming that firms maximize expected profits over all states of nature (which from now on we do for simplicity). Technology of production is stochastic in the second period. Let us denote by $f^j(\ell; \theta^j)$ the production function of the j th firm in the θ th state of nature. Let us assume a neoclassical production function for any state of nature. Moreover, assume that the total and marginal products are higher in the good state of nature than in the bad state of nature, given the level of labor input:

$$f^j(0, \theta^j) = 0 \text{ for all } \theta^j, j; f^j(\ell; G) > f^j(\ell; B) \text{ for all } \ell > 0; \text{ and}$$

$$f'^{j\theta} \equiv \partial f^j(\ell; \theta^j) / \partial \ell > 0, f''^{j\theta} \equiv \partial^2 f^j(\ell; \theta^j) / \partial \ell^2 < 0, \text{ and}$$

$$f'^{jG} > f'^{jB} \text{ for all } \ell.$$

In the first period, the production function is deterministic and assumed to be the average of good and bad states of nature: $f_1^j(\ell) = \{f^j(\ell; G) + f^j(\ell; B)\} / 2$.

The wage paid in the first period is denoted by w_1 . Firms and workers are assumed to have no time preference. Later we shall assume that $f'^{jB}(0)$ is sufficiently high, and that $f'^{jG}(L/2)$ and $f'^{jB}(L/2)$ are sufficiently different to assure the existence of involuntary unemployment.

We would like to construct the simplest possible general equilibrium model in which workers and employers are allowed to draw up (albeit "implicitly") optimal contracts with payment schedules conditional on all the information s^j shared between them and yet, because of the limitations of that information, involuntary unemployment persists in equilibrium. Our main idea, as we have

said, is that any severance payment or private unemployment compensation (in our two period model a stream of payments cannot be distinguished from a one-shot payment) must be the same for all laid-off workers. We assume that if a worker searches for a new job, rather than "going home," he incurs a cost $z > 0$. A worker will not search for a new job unless he expects the search to be worthwhile, which normally occurs only after he is fired. If we had $z = 0$ every worker would at every moment be perfectly informed of his job opportunities.

Since we allow severance payments to laid-off workers in our model, the implicit contract $(w_{1j}, w_{sj}, r_{sj}, c_{sj})$ a firm j offers is a bit complicated; to simplify the notation we drop the superscript j when no confusion can arise, it being understood that different firms may offer different contracts and that they observe different events. A worker will choose his employer not simply on the basis of which one offers him the highest first period wage w_1 , but rather according to which contract offers him the highest expected utility. The probability of being retained in the second period if s is observed is denoted by r_s . If a worker is retained, then the wage w_s is paid. However, a worker may be laid off with severance payment $c_s \geq 0$.

Once a worker is laid off he has two alternatives. First, he can go straight home without search and earn utility $u(c_s + h)$. Second, the laid-off worker can participate in the spot market where he will learn whatever wage his particular skill can bring. The worker will take that opportunity, job or going home, whichever offers him the highest payoff. The distribution of possible wages and the probabilities that a worker will get each of those jobs, conditional on the information $s_j = (\theta^j, w_\theta^a)$, will with rational expectations be known to both Firm j and its workers. In our model, which has only two additional sectors, it can easily be shown that s_j always reveals \bar{w}_θ , the wage the highest productivity firm is offering, and \underline{w}_θ , the wage the lowest productivity firm is offering as well as the probabilities of conformability $\bar{\delta}_\theta$ and $\underline{\delta}_\theta$. Notice that since every worker is conformable to precisely two of the three indus-

tries; $\delta_\theta = 1 - \bar{\delta}_\theta$. We shall thus be able to write $\bar{w}_s, \underline{w}_s, \delta_s = \bar{\delta}_\theta$ to simplify our notation looking at the decision problem faced by the firm j and its workers. Since the worker always has the option of going home, the wages \bar{w}_s and \underline{w}_s must be at least h if the other firms are hiring in the spot market. If a firm is not hiring in the spot market, we can assume without loss of generality that it sets its wage, say \underline{w}_s , equal to h , since then every worker is just as happy to go home as to take a job there. Furthermore, we shall show in section IV that if a firm is hiring $l_s^+ > 0$ workers in the spot market, then it will set the wage equal to its marginal product of labor. Therefore it will turn out that

$$(2.1) \quad \underline{w}_s = \max\{h, f'^B(r_s L + l_s^+)\}.$$

In the spot market identical firms in industry j are competing with each other to hire workers for one period. The wage \underline{w}_s is determined by market forces, and will be taken as given by each firm in equilibrium.

Every worker can at any moment choose to quit his firm. When a retained worker decides to quit, he does not qualify for severance payments. The firm is not obligated to consider him if he later reapplies for his old job. Of course no worker will quit his job unless he has already searched and verified that he can get a better one. If the firm intends to keep on its labor force, it must maintain a high enough wage w_s in the second period to discourage workers from looking for other jobs. These decisions in the second period are illustrated in a decision tree in Figure I. A square box represents a workers' decision point. It is important to note that the values of $w_s, r_s, c_s, \underline{w}_s$, and \bar{w}_s are endogenously determined by the equilibrium conditions, though the workers take them as exogenous in making their decisions. For example, \bar{w}_s and \underline{w}_s will turn out to be the marginal products of the high productivity firm and of the low productivity firm, respectively.

FIGURE I about here

Let us briefly return to the informational constraints in our model. First, consider the case in which all firms are in the good state of nature, i.e., $\theta = \{G, G, G\}$. Then the average spot market wage w^a becomes very high, revealing

firm will be forced to raise the second period wage w_s above w_1 , otherwise all workers would quit their firms and offer themselves on the spot market.) Second, just contrary to the first case, suppose that all the firms are in a bad state of nature. This also makes the average spot-market wage able to fully reveal the state of nature. Third, suppose that two of the firms are in the good state of nature and one in the bad. Some workers from the bad (state of nature) firm go to the spot market. Two good (state of nature) firms are demanders in the market. Since a worker from a bad firm knows one of two good firms has technology conformable to his/her skill, the average spot market wage w^a applies to every worker without noise. The competitive mechanism works perfectly efficiently - the resulting wage reveals all the information to the bad firm, and good firms do not need to know the specifics of the rest of the economy since they are not laying off any workers.

The fourth case is our major concern. Suppose that there are two bad firms and one good firm. Without loss of generality, assume that the first firm is in a bad state. Some of its workers go to the spot market. However, it matters for a worker which of the two remaining firms is in a good state of nature, because a worker from the first firm has a skill only conformable to one of the second and third firms.^{8/} On the one hand, not transferring workers from the bad firm to a good firm damages productive efficiency; and on the other hand, laying off workers from the bad firm exposes the workers to uncertainty of the conformability of skills. The optimal contract should be determined considering this trade off. Since the (aggregate) spot-market wage does not have a dimension for a skill differential, it does not reveal all the information needed in the fourth case.

In actual labor contracts, severance payments or any terms are rarely contingent on outside events. This is stated as one of the important features of labor contracts in Hall and Lilien (1979). This may be taken as an evidence that there is substantial noise in aggregate signals in the real economy due to fine skill differentials.

III. Involuntary Unemployment and Incomplete Information

Involuntary unemployment is defined as an ex post state where a laid-off worker who does not find another job has a lower utility than a retained worker, even though it is known that they can both do the retained worker's job equally well.

DEFINITION. Involuntary unemployment is said to exist, if in some state θ , there are unemployed workers who have been laid off by a firm j such that, letting $s^j = s^j(\theta)$, $0 < r_{s^j} < 1$ and

$$(3.1) \quad U(w_{s^j}) > U(c_{s^j} - z + h).$$

As we will prove in the next section, the optimal contract offered by firm j may retain some of the workers, while the others are laid off. Among the laid-off workers, some luckily find their skill conformable with the requirement of a firm which demands additional workers in the spot market, while others are unlucky to find that their best opportunity is to go home. Then twice unlucky workers (once for lay off and twice for uncomformable skill) ex post have a utility worse than a retained workers. These workers we call involuntarily unemployed.

Our argument thus depends on the marginal productivity of the unemployed falling as low as h . (In the spot market, it is obviously in the firm's best interest to offer a new worker precisely his marginal product. If a worker's best opportunity is to go home, then it must be because his marginal product in the other firm to which his skill is conformable is no higher than h). Recall the commonplace observation that any unemployed worker could find a job if he were willing to take a sufficiently low wage. What we show in our model, however, and what justifies the term involuntary unemployment, is that such workers can point to other identical workers who have jobs at wages they

It would be a mistake to conclude from the fact that the involuntarily unemployed workers have a marginal product at or below h in our implicit contracts equilibrium that they would also be unemployed in a first best world free from the monitoring problems which drive our model. Indeed we shall prove in the next section that in an identical world where workers always truthfully report back to their original firms their new wages and search costs there will be higher output and higher employment, at least in the most productive sectors of the economy. In such a world the coordination between sectors of the economy is improved and production is reorganized in a way that could bring the marginal product of some of the workers involuntarily unemployed in the implicit contracts equilibrium above h .

The firm and workers have one other opportunity, besides severance payments, for mutually advantageous trade that we have forbidden. If a laid-off worker says he cannot find another job to his liking, his original firm could rehire him, perhaps at a wage above his marginal product (for example, at the same wage w_s as the retained workers). In our model, we do not allow a worker to return to his original firm once he is laid off. In an earlier version of this paper we allowed for this additional method of insurance and we showed that it did not affect our qualitative results. Here we shall be content to explain why.

The firm, in guaranteeing an expected utility \bar{u} , relies on the fact that when its productivity declines it can lay off workers, some of whom are bound to find high paying (\bar{w}_s) jobs elsewhere. The firm would like to insure the other less fortunate workers who find low-paying jobs. Severance payments are an inadequate remedy, as we have seen, because the firm, unable to distinguish between the fortunate and unfortunate workers, must make the same payment c_s to all. By offering to rehire workers the firm can go one step further to

providing full insurance. What we showed, however, in our earlier version is that the same basic problem still remains. If the firm sets the rehiring wage too high, then none of the workers search - they all simply claim they were unlucky - and the firm is forced to compensate everyone, not just those who would have been unlucky. Even if the firm sets the rehiring wage sufficiently low so as not to discourage search, it cannot be set below h and if the full retention marginal product in a low productive firm falls sufficiently far below h , the optimal implicit contract will specify a percentage less than 1 of workers who return, claiming they cannot find a job, that will be rehired.

IV. Formal Analysis

In this section we examine the conditions which must prevail in an implicit contracts equilibrium. In such an equilibrium firms offer contracts (w_1, w_s, r_s, c_s) with various terms conditional on the information s which will be commonly revealed to the firm and its employees at the beginning of the second period. Workers choose to work for the firms whose contracts maximize their expected utility, which they calculate taking into account the conditional probabilities of a state θ given s and the payoff they will get in state θ if they are laid off. To make this calculation it suffices to know δ_s, \bar{w}_s and \underline{w}_s . The firm, taking $\delta_s, \underline{w}_s$ and \bar{w}_s and the "market" expected utility \bar{u} as given, chooses the contract variables and the number of workers it wants to hire to maximize expected profits, provided that the contract assures a worker of expected utility: at least \bar{u} .^{9/} Equilibrium occurs when each firm is willing to hire one third of the labor force. In this section we assume the existence of equilibrium and examine its properties. In Section V we give an example of a general implicit contracts equilibrium.

We calculate from the decision tree the worker's expected utility if search is undertaken immediately after layoff:

$$(4.1) \quad \delta_s u(c_s - z + \bar{w}_s) + (1 - \delta_s)u(c_s - z + \underline{w}_s)$$

If a worker does not search but instead goes straight home, then he gets a sure utility of

$$(4.2) \quad u(c_s + h).$$

In that case he faces no uncertainty. He will undertake search only if

$$(4.3) \quad \delta_s u(c_s - z + \bar{w}_s) + (1 - \delta_s)u(c_s - z + \underline{w}_s) \geq u(c_s + h).$$

In order to maintain its labor force, the firm must set w_s so that

$$(4.4) \quad u(w_s) \geq \delta_s u(\bar{w}_s - z) + (1 - \delta_s)u(\underline{w}_s - z)$$

otherwise its workers will search for other jobs, and at least half of them will leave. The on the job search decision is illustrated in box 2 in Figure I.

If there is no uncertainty (i.e., if s^j reveals θ) then this reduces to $w_s \geq \bar{w}_s - z$.^{10/}

We are now in a position to calculate from our decision tree the ex ante expected utility of a representative worker who accepts a contract (w_1, w_s, r_s, c_s) and who understands the opportunities $\bar{w}_s, \underline{w}_s, \delta_s$.

$$(4.5) \quad EU = u(w_1) + \sum_{s \in S_j} \pi_s (r_s u(w_s) + (1 - r_s) \max(4.1, 4.2))$$

Letting ℓ_s^+ be the extra labor hired from other firms in the second period at wage w_s^+ , the firm has expected profits;

$$(4.6) \quad E\pi = f(L) - w_1 L + \sum_{s \in S_j} \pi_s (f_s(r_s L + \ell_s^+) - w_s r_s L - w_s^+ \ell_s^+ - c_s (1 - r_s) L)$$

The optimization problem faced by each firm is

$$(4.7) \quad \text{Max}_{\{w_1, w_s, r_s, c_s, l_s^+, L\}} \quad E\Pi$$

$$\text{such that} \quad EU \geq \bar{U}$$

and (4.4), i.e., no quits occur,

$$\text{and } 0 \leq r_s \leq 1, 0 \leq c_s,$$

Observe that w_s^+ is determined in the spot market by competition between firms of the same sector, so the firm takes it as given in equilibrium.

Notice that all these functions are differentiable in all the variables except c_s . When the search constraint (4.3) is not binding, then EU is differentiable in c_s as well. At equilibrium the firm must be optimizing and we can calculate the first order necessary conditions (much as Azariadis did^{11/}) with respect to all the variables except c_s . For c_s we shall have to break our analysis into 3 cases, depending on whether the search constraint 4.3 is not binding, an equality or violated. Let us denote the Lagrangean of problem 4.7 by \mathcal{L} and the Langrange multipliers for $EU \geq \bar{u}$ by λ and for 4.4 by μ .

$$(4.8) \quad \frac{\partial \mathcal{L}}{\partial l_s^+} : f'_s - w_s^+ = 0 \\ \leq 0 \quad \text{if } l_s^+ = 0$$

Hence as we claimed, if any new workers are being hired in the spot market, then they are being paid their marginal product.

$$(4.9) \quad \frac{\partial \mathcal{L}}{\partial L} : f'(L) - w_1 + \sum_{s \in S_1} \pi_s (r_s f'_s (r_s L + l_s^+) - w_s r_s - c_s (1 - r_s)) = 0$$

$$(4.10) \quad \frac{\partial \mathcal{L}}{\partial w_1} : -L + \lambda u'(w_1) = 0$$

$$(4.11) \quad \frac{\partial \mathcal{L}}{\partial w_s} : -r_s L + \lambda r_s u'(w_s) + \mu (u'(w_s) - (1 - \delta_s) u'(w_s - z)) = 0$$

$$(4.12) \quad \frac{\partial f}{\partial r_s} : f'_s(r_s L + \ell_s^+)L - w_s L + c_s L + \lambda(u(w_s) - \max(4.1, 4.2)) \begin{cases} = 0 & \text{if } 0 < r_s < 1 \\ \geq 0 & \text{if } r_s = 1 \\ \leq 0 & \text{if } r_s = 0 \end{cases}$$

In what follows we shall assume the existence of an equilibrium, by which we mean a vector $(w_1^j, w_\theta^j, r_\theta^j, c_\theta^j, \underline{w}_\theta^j, \bar{w}_\theta, \delta_\theta)$, $\theta \in \Omega$, $j \in \{1, 2, 3\}$ solving (4.7) such that \bar{w}_θ , \underline{w}_θ and δ_θ are observable to all and if $s^j(\theta) = s^j(\theta')$ then for any firm j each of the first four variables must also be constant over θ and θ' , and such that the three labor markets clear in the first period and for conformable workers in the second period. The demand for labor by each firm is given by any \hat{L} that is part of a solution to the maximization problem (4.7). We assert, without proof as we have said, that by adjusting \bar{u} we can affect \hat{L}^j so that $\hat{L}^1 + \hat{L}^2 + \hat{L}^3 = 3L \cdot \frac{12}{1}$. We emphasize that markets are no less efficient in the second period than they were in the first period. The only difference is that there is more information in the second period about firm productivity than in the first and the firm must live up to its contractual commitments, established before the firm or workers had access to any information.

We begin by stating three simple propositions, which clarify the role our coordination assumptions (that different sectors of the economy are not fully informed about each other and that the firms cannot monitor the workers once they leave) play in our conclusions.

Proposition 1a: In any state θ in which there is no uncertainty about the wage each laid-off worker will receive, severance payments will be made so that there is no involuntary unemployment. The economy will also be producing efficiently.

Proof: We must show that if the firm has hired L workers of a certain class and retains a nontrivial fraction r_θ of them at wage w while the other $(1-r_\theta)L$ are laid off and subsequently find jobs paying their marginal product m , then the firm must be making severance payments $c_\theta = w_\theta - m$. Notice that the total wage bill of the firm in state θ is $w_\theta r_\theta L + c_\theta (1-r_\theta)L$ while the expected utility of a worker, conditional on state θ , is $r_\theta u(w_\theta) + (1-r_\theta)u(m+c_\theta)$. It follows immediately from the concavity of u that the firm can make the conditional expected utility of the worker higher with the same total wage payment unless $c_\theta = w_\theta - m$. (Since expected utility is separable between states of nature, this would make total utility higher.) Given such a c_θ , it follows at once (see equation(4.12)) that the firm's marginal product is equal to $w_\theta - c_\theta = m$. Q.E.D.

Proposition 1b: Even if a firm is risk averse, if there is no uncertainty about the wage a laid-off worker will receive, then there will be no involuntary unemployment or productive inefficiency in an implicit contracts equilibrium.

Proof: Notice that the proof of Proposition 1a did not use the risk neutrality of the firm, since the firm cares only about its total wage bill in each state θ . Q.E.D

Proposition 1c: Even if the worker faces uncertainty at the moment he is fired, if the firm can count on him to report truthfully his new wage and search cost there will be no involuntary unemployment and no productive inefficiency, even if the firms are risk averse, in an implicit contracts equilibrium.

Proof: The argument for no involuntary unemployment is exactly the same as above. To see that there is productive efficiency, observe that the first order

condition corresponding to equation (4.12), derived on the supposition that the severance payment is made to depend on the laid-off worker's (truthfully) revealed wage, is $f'_\theta L - w_\theta L + E c L = 0$ where $E c = w_\theta - E(m-z)$, E is the expectation operator taken over all the workers laid off and z is the search cost. Notice that even the risk averse firm is indifferent to which particular worker it pays a high or low severance payment. The total severance payment is certain. Hence $f'_\theta = E(m-z)$, which maximizes total output. Q.E.D

One might think that if we allowed the workers' utility function to depend on two arguments, consumption (wealth) and leisure, then there might be involuntary unemployment in an implicit contracts equilibrium even if workers reliably reported their wages. That this is not so is shown by the following proposition:

Proposition 2: If workers have a von Neumann-Morgenstern utility $u(c, \ell)$

$\ell \equiv$ leisure, satisfying $\frac{\partial^2 u}{\partial c \partial \ell} \geq 0$, $\frac{\partial^2 u}{\partial c^2} < 0$, $\frac{\partial^2 u}{\partial \ell^2} < 0$, for example

$u(c, \ell) = u_1(c) + u_2(\ell)$ and if workers can be counted on to report truthfully their rehiring wage, then unemployed workers will be strictly better off than employed workers in an implicit contracts equilibrium: instead of involuntary unemployment, we would get involuntary employment!

Proof: Any equilibrium implicit contract will equalize the marginal utility of consumption to a laid-off worker and a retained worker of the same type. Under the hypothesis of this proposition, the laid-off worker will therefore get more consumption and more leisure. Q.E.D.

We return now to our model of the last two sections in which the firm cannot

monitor the rehiring wage of its laid-off workers and we show that there can be involuntary unemployment and underproduction.

Proposition 3: If the firm cannot monitor the rehiring wage of its laid-off workers, and if there is sufficient variation in the productivity of firms, then there will be involuntary unemployment in some states of nature in an implicit contracts equilibrium.

Proof of Proposition 3: We must show in the states $\theta = (B, G, B)$ and $\theta = (B, B, G)$, both of which produce the same signal $s = s_1(\theta)$ to firm 1, that $0 < r_s < 1$ and that there remain unemployed workers. The reason for the involuntary unemployment, as we said earlier, is that in guaranteeing an expected utility \bar{u} to workers in the first period the firm relies on the fact that if productivity is very bad relative to the rest of the economy in the second period, the firm can discharge the worker knowing that there is some positive probability δ_s that he will get a high paying job elsewhere. Moreover not all the discharged workers will find jobs in the spot market if the marginal product of the low productivity firm falls below h at a small enough quantity of labor.

In what follows we shall assume that $f'_G(2L)$ is sufficiently high and that $f'_B(\frac{1}{2}L)$ is sufficiently low. Recall that by definition the first period marginal product $M \equiv f'(L) = \frac{1}{2}f'_G(L) + \frac{1}{2}f'_B(L)$. In our variation of f'_G and f'_B we shall maintain $M = \frac{1}{2}f'_G(L) + \frac{1}{2}f'_B(L)$ and also $f'_B(0)$ will stay very large (so that always $r_s > 0$). Since $M - w_1$ is the premium the worker pays to get partially insured in the second period, we must have that $M \geq w_1$.

We begin by showing that if there is a large enough variation in f'_G and f'_B , then in equilibrium any worker who is laid off must prefer to search. For we cannot have that $u(c_s + h) = \delta_s u(c_s - z + \bar{w}_s) + (1 - \delta_s)u(c_s - z + h)$ in equilib-

rium with c_s positive unless the derivatives of both sides are the same. In that case, and in the case that workers strictly prefer going straight home to searching, we can replace max (4.1, 4.2) with $u(c_s+h)$

$$(4.13) \quad \frac{\partial f}{\partial c_s} : -(i-r_s)L + \lambda(1-r_s)u'(c_s+h) = 0$$

as a necessary condition for equilibrium. Hence from (4.10) $c_s + h = w_1 \leq M$. But now comparing the search-no search tradeoff (4.3) we see that $u(c_s + h)$ is bounded while the expected utility of search becomes arbitrarily large as $\bar{w}_s = f'_G \geq f'_G(2L)$ goes to infinity, provided that u is unbounded. If u is bounded it may be that the possibility of losing z is so horrible that the workers never undertake any risks, no matter how favorable the odds. Of course in that case we cannot get involuntary unemployment.

For big enough f'_G , we have just seen that if we are in equilibrium, then (4.3) holds strictly (search is preferred to no search). Hence we can write the $\frac{\partial f}{\partial r_s}$ condition as

$$(4.13') \quad \frac{\partial f}{\partial r_s} = f'_s(r_s L + l_s^+)L - w_s L + c_s L + \lambda(u(w_s) - \{\delta_s u(c_s - z + \bar{w}_s) + (1 - \delta_s)u(c_s - z + \underline{w}_s)\}) \geq 0$$

if $r_s = 1$.

We can show this is impossible if $f'_B(L)$ is sufficiently smaller than h . If $r_s = 1$, then c_s doesn't matter: we can assume it is 0. Clearly $w_s \geq h$. Now, as \bar{w}_s gets large, the bracketed term will become negative, implying that the left hand side cannot be greater than or equal to 0, unless w_s is also growing. This is possible, since we must maintain $u(w_s) \geq \delta_s u(\bar{w}_s - z) + (1 - \delta_s)u(w_s - z)$ from the no quit constraint. But then the term $-w_s L$ becomes

dominant, since $\lambda u(w_s)$ is bounded by $\frac{1}{U'(M)} (K + (w_s - (M + \epsilon))u'(M + \epsilon)) \leq K' + \alpha w_s$ with $\alpha < 1$.

Finally, let us show that with sufficient variation in f'_G and f'_B we will find that not only are there workers thrown out of work, all of whom search for other jobs, but some of them remain unemployed. It suffices to assume that $f'_B(L/2) < h$, for the productive sector can absorb at most $2L$ of the worker. Q.E.D

In fact, the good sector is likely to absorb far fewer than $2L$ workers, for if it did that would mean an enormous search cost for society. Notice that in the above argument, taking $f'_G(L)$ large always meant taking $f'_B(L)$ small, since their average was held at M . The argument thus clearly relies on f'_B falling below h (but not below 0).

Proposition 4: Whenever there is involuntary unemployment in an implicit contracts equilibrium, the high productivity firms are hiring too few workers and producing too little output, in the sense that in an identical economy (including the same informational asymmetries between firms) in which workers truthfully reveal their wages and search effort there will be more output and employment in the high productivity firms.

Proof of Proposition 4: The only states in our model in which there can be involuntary unemployment occur when two firms are bad and one is good. Without loss of generality, assume the first and second firms are bad and the third good (of course the first firm is not aware which of the other two is good and which bad). If there is involuntary unemployment, it means by symmetry that the first and second firms are laying workers off and not hiring all the available workers in the spot market. Hence $f'_B(r_s L + \lambda_s^+) \leq h$ for both firms. Now, if the unemployed workers were simply going home without searching, then their wage would be certain and they would be compensated, in an optimal

contract, so as to be exactly as well off as they were before they lost their jobs. Hence if they are involuntarily unemployed, it must be that they searched and were unlucky enough not to find their skills conformable to the high productivity firm. A prerequisite for search is that $\delta_s u(c_s - z + \bar{w}_s) + (1 - \delta_s)u(c_s - z + h) \geq u(c_s + h)$, hence from the concavity of u , it must be that $f'_G = \bar{w}_s > h + \frac{1}{\delta_s} Z$. In a first best world, with the same informational limitation, but where firms could count on laid-off workers to reliably report their new wages and search costs, we would have $f'_G = h + \frac{1}{\delta_s} Z$. Hence the productive industry would be hiring more workers and producing more output. Q.E.D.

As for the unproductive firms, they may retain workers until $f'_B(r_s L)$ is actually less than h , helping to insure those workers that bring the marginal product below h . This is the Azariadis phenomenon. On the other hand, it is possible that $f'_B(r_s L) > h$, in which case the firms will hire additional workers until $f'_B(r_s L + \lambda_s^+) = h$. That is the case in our example, where in a first best world output and employment would go up or remain the same in all sectors of the economy. By assuming that the f'_G curve is sufficiently flat, and the f'_B curve sufficiently steep, we can guarantee that total output and employment would be higher in the first best world than in the implicit contracts equilibrium.

Let us now consider the question whether, including the severance payments, the expected utility of a laid-off worker is higher or lower than that of a retained worker. At the moment of his firing the worker has, on account of the severance payment, more money than he would if he had not lost his job. On the other hand his utility is reduced by the uncertainty with which he views his future income and possibly also by its low expected value. One might expect the latter effects to dominate, otherwise it would seem that dismissal

is a cause for celebration. In fact part of the severance payment is made to alleviate the anxious uncertainty produced by the dismissal: if there were no anxiety that part of the severance compensation would be eliminated. We shall not discuss the psychological question of whether a man worried about finding a job is likely to celebrate the extra money in his pocket, but we do remark that it is perfectly consistent with our model for his expected utility at the moment he is laid off to be higher than if he had been retained so long as without the severance payment he is better off on the job. If the worker quits or attempts to get himself laid off by being incompetent, he will be fired without severance payment. We repeat that our analysis is not meant to focus on the dismissal of individual workers, but on the decision a firm takes to close an entire plant or section due, for example, to lack of demand, not to individual worker incompetence.

The question is only interesting if the workers search after they are laid off. Otherwise we know that the severance payment will exactly compensate them so that they are equally well off before and after their dismissal. Hence we can write

$$(4.14) \quad \frac{\partial f}{\partial c_s} : E_s u'(c_s - z + \tilde{w}_s) - \frac{L}{\lambda} = 0$$

$$\leq \text{if } c_s = 0.$$

where $E_s u'(c_s - z + \tilde{w}_s)$ is the expected marginal utility of income, conditional on the worker being fired and searching for a job, given that at the moment he is fired he and the firm observe s . Consider equation (4.14). If we assume that the no quit conditions are not binding, then $w_s = w_1$ and $\mu = 0$. Hence by substituting $u'(w_s) = L/\lambda$ from equation (4.10) and rearranging terms we get (assuming $r_s < 1$, otherwise the question of severance payments does not arise):

$$(4.15) \quad E_S u'(c_S - z + \tilde{w}_S) = u'(w_S), \leq \text{ if } c_S = 0$$

Notice that \tilde{w}_S is a random variable, depending on how his skill is conformable and that the optimal severance payment c_S equates not the expected utility of the laid-off worker to the retained worker, but the expected marginal utilities. We would like to compare $E_S u(c_S - z + \tilde{w}_S)$ to $u(w_S)$. The following proposition follows immediately from the theorem in Imai, Geanakoplos, and Ito (1981):

Proposition 5: If severance payments are made at all, and if workers prefer not to quit their jobs, then the expected utility of a worker at the moment he is fired is (greater, equal, or less) than the utility of a retained worker according to whether the worker's absolute risk aversion is (decreasing, constant, or increasing).

Thus it follows from Proposition 5 that in the natural case of decreasing absolute risk aversion, an optimizing firm that closes some branch of its operations will pay each of its laid-off workers a sufficiently high severance compensation so that on average they end up strictly better off than they would have been if they were retained. Of course some of the laid-off workers will be unlucky enough not to find another job, so that even with their severance payment they will be worse off than if they hadn't been fired. These are precisely our involuntarily unemployed workers.

We close this section by making one final remark about the rigidity of wages. On account of the no quit constraint, (4.4) wages are not necessarily completely rigid. In fact, in some states the firm may be forced to raise the wage w_S to retain its workers or even lower the wage w_S to retain them!

It is possible (if $(1 - \delta_g)u'(w_g - z) > u'(w_g)$), though not very likely, that the firm will find that it can retain its workers by lowering their wage if by doing so it makes the income that might be wasted, if the worker undertook unsuccessful search, sufficiently dear to him.

V. An Example of Involuntary Unemployment

Now we are ready to illustrate the existence of the involuntary unemployment. For now we assume $\underline{w}_g = h$, that workers never quit, and that they search only after being fired. After calculating the equilibrium we verify that we have indeed found the optimal contract.

Recall the overlapping conformability of skills described in section II. Initially, the representative firm I has a pool of 50 type-a workers and 50 type-b workers; firm II has a pool of 50 type-a and 50 type-c workers; and firm III a pool of 50 type-b and 50 type-c workers. In the "good" state of nature, the marginal productivity of a firm is $f'(L; G) = -4L + 800$, and the "bad" state of nature is $f'(L; B) = -L + 200$, where L is the number of workers. If a worker prefers to stay home, the home production would give the self-employed wage, $h = 140$. Suppose that at the first period, the state is "medium", i.e., $f'(L) = -2.5L + 500$ for all three firms. There are four equally-probable states of nature in the second period,

$$\theta \in \{(G, G, G), (G, B, B), (B, G, B), (B, B, G)\}$$

where G and B denote the good and bad states of nature, respectively, and the entity of (\cdot, \cdot, \cdot) represents the respective firm. Suppose also that the search cost $z = 40$ is incurred at the moment that a worker quits or is laid off from the firm, if he searches.

Now let us restate the first order conditions of the optimal contract.

From (4.9)

$$(5.1) \quad \sum_{s=1}^3 \pi_s (f'_s r_s - (1-r_s)c_s - w_s r_s) + f'_{t=1}(L) - w_1 = 0.$$

Since the four states are assumed to be equally probable, $\pi_{s^1}(\theta) = \frac{1}{4}$, $\theta = (G, G, G)$ or

(G, B, B) , and $\pi_{s^1}(\theta) = \frac{1}{2}$ for $\theta = (B, G, B)$ or (B, B, G) . The information set of the first firm does not differentiate the third and fourth states of nature.

From the first order conditions, (4.15) we have

$$(5.2) \quad u'(w_1) - E_{w_s} u'(c_s - z + \tilde{w}_s) = 0.$$

The first order condition (4.12) in the case of $0 < r_s < 1$, $w_s = h$ and $\delta_s = 1/2$, can be reduced to:

$$(5.3) \quad f'_s + c_s - w_s + \left[u(w_s) - E_{w_s} u(c_s - z + \tilde{w}_s) \right] / u'(w_1) = 0, \quad \forall s.$$

Furthermore, let us assume the quadratic utility function,

$$u = -y^2 + 2000y, \quad y \leq 1000,$$

for every worker. First, in the state of $\theta = (G, G, G)$, the wage will be bid up in the spot market, since the marginal product is surely higher than the stabilized (contract) level. Specifically it is bid up to the marginal product less the search cost (because of quit-no quit decisions):

$$w_{GGG} = f'_{GGG}(100) - z = 400 - 40 = 360.$$

Suppose that each bad-state firm in the state of nature, (G, B, B) , (B, G, B) or (B, B, G) retains r_{GBB} of its labor pool. Then there are $(1 - r_{GBB}) \times 200$ laid-off workers from two bad-state firms. One-half of them, i.e., $(1 - r_{GBB}) \times 100$ workers, on average, are the conformable type

to the good-state firm, so that they are picked up by the good-state firm in the spot market. Therefore the marginal product of the good-state firm is

$$(5.4) \quad f'_{GGB}{}^G (100 + 100 \times (1 - r_{GGB})) = f'{}^G (200 - 100r_{GGB}) = 400r_{GGB}.$$

On the other hand, if the laid-off workers find their skills unconformable to the good-state firm, they go to the other bad-state firms and ask for the spot market wage, which is below the contract stabilized wage. If the spot market wage, which is equal to the marginal product, becomes equal to the home production wage, then no more workers will ask for the return to the firm, but they will go home. Therefore

$$f'_{BGB}{}^B = \max(h, f'_{BGB}{}^B (100 r_{BGB} + 50 (1 - r_{BGB}))) .$$

Specifically, recalling the restriction of $\underline{w}_s \geq h$, the following is true,

$$(5.5) \quad f'_{BGB}{}^B = 140, \quad \text{if } f'_{BGB}{}^B (100 r_{BGB} + 50 (1 - r_{BGB})) < 140 .$$

Since for the first firm, the state of nature $\theta = (B, B, G)$ is indistinguishable from $\theta = (B, G, B)$, we just aggregate these states of nature. The wage rate for all retained workers in the state of nature $\theta = (G, B, B)$, (B, G, B) or (B, B, G) is the stabilized wage, w_1 , provided that $f'{}^G < w_1 + z$.

Now equation (5.1) is

$$0 = f'_{t=1} - w_1 + \frac{1}{4} \{f'_{GGG}{}^G - w_{GGG}\} + \frac{1}{4} \{f'_{GGB}{}^G - w_1\} \\ + \frac{1}{2} \{f'_{BGB}{}^B r_{BGB} - (1 - r_{BGB}) c - w_1 r_{BGB}\} .$$

After substituting the values calculated above, defining $r = r_{BGB} = r_{GGB}$, and arranging terms,

$$(5.1') \quad 0 = 1040 - 5w_1 + 680r - 2c + 2cr - 2w_1 r .$$

The worker who is going to be laid off, does not know whether his skill is conformable with the good-state firm. Therefore the spot market wage he is going to receive is uncertain. If the skill is conformable, he will receive f'_{GGB} determined by (5.4); while if the skill is not conformable, the wage is determined by f'_{BGB} of (5.5). The severance payments less the search cost, $(c_s - z)$, is common for both possibilities. Equation (5.2) becomes,

$$u'(w_1) = \frac{1}{2}u'(c - z + 400r_s) + \frac{1}{2}u'(c - z + 140) .$$

Substituting $z = 40$ and the quadratic utility function into the above equation,

$$(5.2') \quad 0 = -2w_1 + 2c + 60 + 400r .$$

Equation (5.3) becomes nontrivial only when $\theta = (B, G, B)$ or (B, B, G) for the first firm. In either case, equation (5.3) becomes the following,

$$0 = 140 + c - w_1 + \left[u(w_1) - \frac{1}{2}u(c - z + 140) - \frac{1}{2}u(c - z + 400r_s) \right] \div u'(w_1) .$$

After an appropriate substitution, this condition is equivalent to,

$$(5.3') \quad 0 = (140 + c - w_1) \{-2w_1 + 2000\} - (w_1)^2 + 2000w_1 \\ + \frac{1}{2}(c + 100)^2 - 1000(c + 100) + \frac{1}{2}(c + 400r - 40)^2 \\ - 1000(c - 40 + 400r) .$$

Now we have three equations, (5.1'), (5.2') and (5.3') with three unknowns, c , w_1 , r . Solve these equations, we find (w_1, r, c) approximately equal to (220.4468, 0.5554126, 79.36427).

Rounding the numbers, an equilibrium is suggesting the following scenario. In the first period, everyone is employed at the wage rate, 220.45, although the marginal productivity is 250. In the second period, if $s = (G, G, G)$, everyone is still employed at the originally contracted firm and receives

the wage rate 360. Suppose that $s = (B, G, B)$, then all of the workers in the second firm are retained and paid $w_1 = 220.45$. On the other hand, in the first and third firms, only about 56 out of 100 workers of each firm are retained and paid w_1 . The rest, 44 workers from each firm are laid off. Among 88 laid-off workers, 44 workers find their skills conformable with the second firm and become employed. This brings the marginal product of the second firm down to 222.17. So their net wage is $222.17 + 79.36 - 40 = 261.53$. Now there are 44 workers who do not have an opportunity in the high productivity firm and go to another bad-state firm. If all of them were reemployed in the bad-state firms then this would add 22 workers to 56 retained at each firm so that this would drive down the marginal product to 122, far below $140 = h$. Therefore the competitive spot market takes only 4 workers back to each of the bad-state firms at wage $f'_{GBB} = 140$. This implies 36 workers are unemployed. The returning workers and the unemployed laid-off workers receive 140 for their spot market wage. The net income, after adjusting for the severance payment and the search cost, is $140 + 79.36 - 40 = 179.36$.

We have to check whether the constraints which were not explicitly considered in the numerical example are actually satisfied. It is easy to see that no worker will voluntarily quit or search before he has been fired. Furthermore, it can be calculated that the worker prefers searching once fired to going home without searching. Expected utility of search, $\frac{1}{2}\{u(c - z + \bar{w}_s) + u(c - z + \underline{w}_s)\}$ is equal to 390609.02, while no search guarantees $u(c + h) = \underline{390607.86}$. Since a quadratic utility display increasing absolute risk aversion, the expected utility of a worker at the moment he is fired is less than that of the retained workers $u(w_1) = 392296.80$.

TABLE II about here

Table II summarizes the stages of layoff and spot markets.

This shows the nature of involuntary unemployment in this example. A laid-off worker who goes home receives less net income than the fellow worker of the same type, who is luckily retained in the firm.

Finally, it is of interest to note that if the search cost is negligible $z = 0$, then (w_1, r, c) which solve (5.1'), (5.2') and (5.3') are approximately equal to $(202.857, 0.35, 62.857)$. This means that those who are retained and those who are laid off are equally enjoying the net income, 202.857, since $h + c = 140 + 62.857 = 202.857 = w_1$. Therefore the involuntary unemployment disappears in this case.

VI. Conclusion

We have presented a simple general equilibrium model and a numerical example in which asymmetric information creates involuntary unemployment. We noted that in our equilibrium an unemployed worker has job offers, but at salaries equal or below his reservation wage. What makes his unemployment involuntary is that there are also jobs at wages he would take which are held by other workers known to be identical to him in every respect. We also noted that in such times of involuntary unemployment the most productive firms have not optimally expanded their production to take up the slack. The policy implication is not simply to intensify efforts to match unemployed workers to jobs; indeed, the unemployed are fully informed - they know they are not qualified for jobs in the productive sector and there are no jobs in the rest of the economy. What is necessary is to promote the transfer of employed workers from the less productive sectors to the most productive sector, which would have the effect of opening up jobs (by reducing the labor force it raises the marginal product

of potential new workers) for the unemployed where their skills are valuable. Finally, we proved the rather surprising proposition that a firm interested in maximizing expected profits should, if for some reason it closes a section, compensate the workers it lays off so generously (if they show decreasing absolute risk aversion) that on average they do better than the workers it retains.

To keep our analysis tractable, we have ignored several interesting aspects of the labor market, some of which we mention now. First, because we confined our attention to a two-period model, we had to treat all workers equally with regard to a layoff. In a many-period model one might hope to justify the commonly observed fact that layoffs, far from being random, fall nearly always on the most recently employed. Even more importantly, if we had new workers entering the labor force every period we might be able to show that when conditions deteriorate enough, the new workers receive wage offers so low that they choose not to work despite the fact that older workers, productively identical but employed already at a guaranteed high wage, are receiving wages for which they would work.^{13/}

Second, we did not have an explicit model about search behavior. The fixed moving and search costs, z , in our context should be carefully analyzed in the future. The chance of getting an alternative job in the case of lay-off may depend on how much effort workers exercise. Then the unemployment compensation (or severance payments) might cause a more complicated moral hazard in the job-changing process.

Third, one might think that the introduction of the government would improve the performance of our economy. This conjecture depends on whether the government has better information about worker's skill, effort and income than the firm. A worker's net income may be better monitored, say through the IRS,

by the government than by the firm. Indeed the inability of the firm to monitor workers' income caused the lack of stabilization of the wages between the lucky and unlucky laid-off workers. (However, it may also be costly for the government to monitor workers' income, since workers do not have incentives to report correct income, as we all know.) Moreover, the effort in work, which is not explicitly analyzed in our paper, can be better monitored by firms. For example, we could make workers' utility depend also on effort in the first period. The firm could fire at the beginning of the second period those workers who had been lax in their efforts in the first period without severance payments. Thus threatened, the worker might choose to work diligently in the first period. On the other hand, if the government fully insured every worker's income and if it were unable to distinguish the laid-off workers from the fired workers, then no worker would have any incentive to expend any effort in the first period. The more difficult it is for the firm to monitor worker's effort (or the smaller is the range of possible effort), the greater should be the burden of public unemployment compensation, given access to income information. Worker's effort is a question we would like to return to in a future work.

Footnotes

1/ In the sense that in equilibrium resources are used efficiently to produce a Pareto optimum, one can always choose a welfare function which is maximized at that allocation.

2/ Most of these were presented, together with our paper, at the National Bureau of Economic Research symposium on implicit contracts, held in Princeton, in December 1980.

3/ It should be mentioned that this contradicts the very basis of the original implicit contracts literature. The word implicit was used to indicate that while contracts do not normally explicitly specify the contingencies on which the wage and employment depend, these are implicitly understood by all parties and the need for reputation guarantees that the firm honestly fulfills its obligations.

4/ The following example illustrates the overlapping conformability (but not the perfect substitutability) of skills. A symphony orchestra needs violinists and percussionists; a bluegrass band needs violinists (though called fiddlers) and guitarists; a rock'n'roll band needs (electric) guitarists and percussionists (though called drummers). Thus our colleagues have called our model a musical chairs theory of unemployment.

5/ Formally, this is derived as an equilibrium condition of the labor contract market.

6/ Therefore Holmstrom's result of the "downward rigidity" holds true in our model, too.

7/ In an illustration of musicians of an earlier footnote, the aggregate variable is the spot-market wage for a "musician." However, whether it is for a violinist, a guitarist or a drummer is not public information.

8/ One may argue that if there are different numbers of workers for three types, then the spot-market wage would reveal the difference between $\{B, B, G\}$ $\{B, G, B\}$ and $\{G, B, B\}$.

We simplified the number of firms as well as the number of possible states of nature. In the most general formulation, there are continuous states of nature and arbitrary proportions of workers with different skill. Then all the quality demonstrated here will be valid.

9/ The maximization problem with a constraint on the expected level of utility, \bar{u} , is a standard procedure in the literature, such as Azariadis (1978) and Holmstrom (1981). The level of \bar{u} is endogenously determined. See also footnote 12 below.

10/ We assume that even if the firm itself is hiring on the spot market at wage w_s^+ , the contract wage can be constrained only by $w_s \geq w_s^+ - z$: Once a worker s quit a job, he is required to take a skill test in order to reapply for the same job in the same firm.

11/ Although the programming is not concave in w 's, this does not cause a problem (see Holmstrom (1980; Appendix)). At first sight, it seems profitable to depress the wage since it is a choice variable in this programming. However, there is a constraint on the utility of a contract, \bar{u} to be guaranteed. Implicitly, the \bar{u} is adjusted to clear the labor market. In other words, the market clearing, i.e., each firm hired L , is a result of an auction in terms of \bar{u} . (Cf. footnotes 4 and 9.)

12/ This assumption is explicitly stated in Holmstrom (1980, 23): "I view the labor markets in both periods as run by an auctioneer, though in the first period he will announce expected utilities rather than wages." See also Ito (1982) for a numerical example showing how to determine \bar{u} endogenously.

13/ One can imagine that if a period of contract spans more than two periods, then the problem becomes increasingly complicated and the chance of stabilization might increase. Especially having the implicit lifetime employment contract and firm specific fringe benefits would help the complete stabilization of life-cycle incomes. In this respect, the Japanese labor practices are interesting. Their fringe benefits range from firm-owned housing (better apartments for the senior workers), tennis courts, to health clubs in resort areas.

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FIGURE 1: SECOND PERIOD DECISION TREE

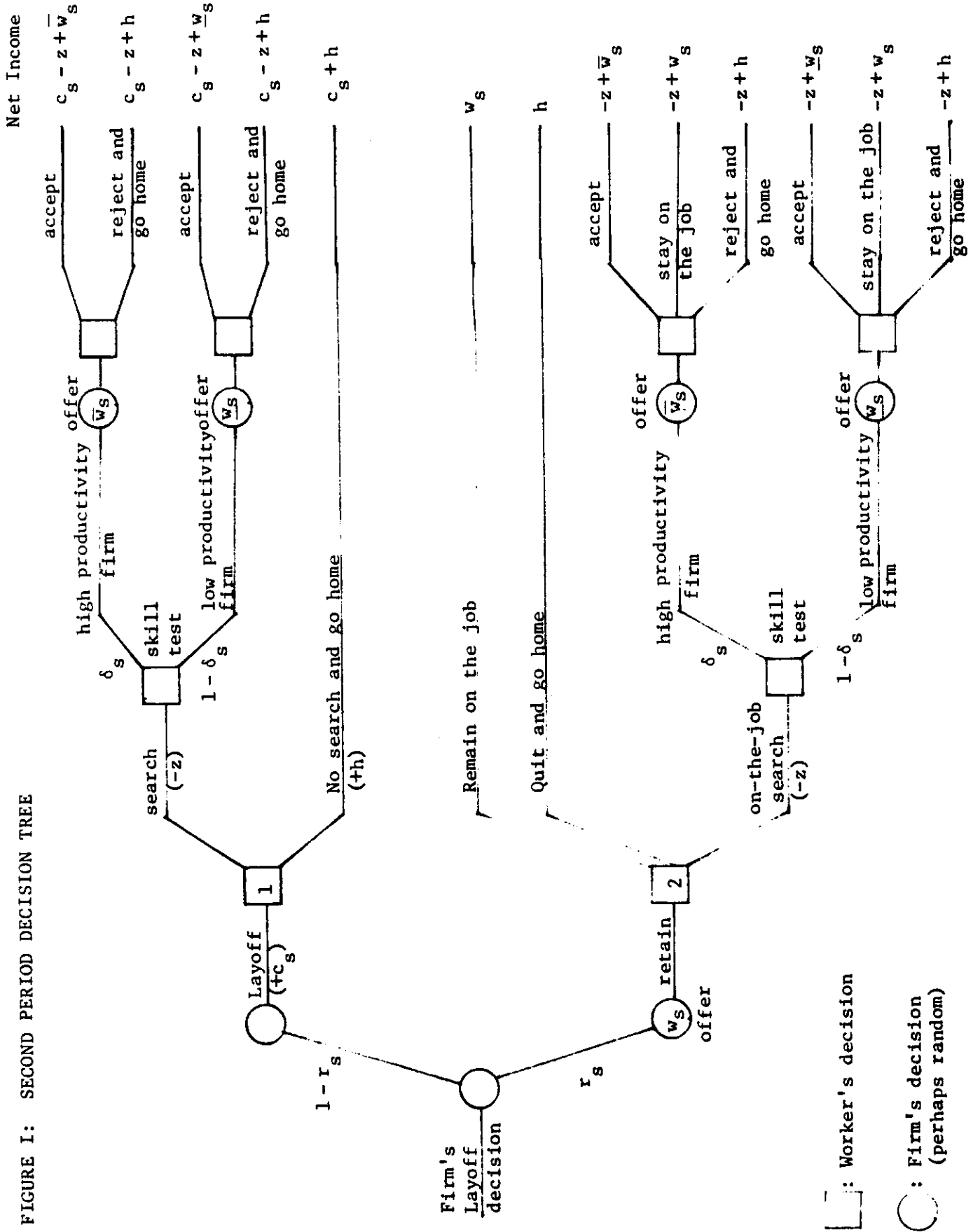


TABLE II

FIRM	I		II		III		home	
	L	w, c	L	w, c	L	w, c	L	w, h
Labor Pool, L	100		100		100			
Retain contract	56		100		56			
Wage w_s		220.45		220.45		220.45		
Layoff, $(1-r)L$	44		→ skill test		44			
sev. pay c		79.364				79.364		
Additional workers, ℓ^+	↓ 4		↓ 44		↓ 4		↓ 36	
spot market wage w^+		140		222.17		140		140

Retained	Laid off and Conformable with II	Laid off, Not Conformable with II
220.45	261.53	179.36
Net Income		