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REVELATION OF INFORMATION IN STRATEGIC MARKET GAMES:
A CRITIQUE OF RATIONAL EXPECTATIONS

by

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1. INTRODUCTION

Consider an economy in which agents have different levels of information concerning exogenous random events. How does the pooled information of the agents get revealed in the process of exchange? In particular what is the role played in this by the price system? At least since Hayek this has been a central problem in economics. "My main contention," Hayek wrote in 1937,

will be that the tautologies, of which formal equilibrium analysis in economics essentially consists, can be turned into propositions which tell us anything about causation in the real world only insofar as we are able to fill those formal propositions with definite statements about how knowledge is acquired and communicated....The really central problem of economics as a social science, which we pretend to solve is how the spontaneous interaction of a number of people, each possessing only bits of knowledge, brings about a state of affairs in which prices correspond to costs, etc., and which could be brought about by deliberate direction only by somebody who possessed the combined knowledge of all those individuals. Experience

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shows us that something of this sort does happen, since the empirical observation that prices do tend to correspond to costs was the beginning of our science. The only trouble is that we are still pretty much in the dark about (a) the conditions under which this tendency is supposed to exist and (b) the nature of the process by which individual knowledge is changed.¹

One approach to this problem has been taken via the notion of a "Rational Expectations Equilibrium" (R.E.E.). Since our paper is juxtaposed to the R.E.E. model, and meant to be a critique of it, let us first briefly recall what the R.E.E. model is.² Let S be the finite set of states of the world. For each agent $n \in N$ let I^n be a partition of S representing the information of n , and denote by I^* the coarsest partition of S which refines each I^n , $n \in N$. Trade takes place in a finite set L of commodities: Thus the space of state-contingent commodities is $\mathbb{R}_+^{L \times S}$. Each agent n is characterized by a utility $u^n : \mathbb{R}_+^{L \times S} \rightarrow \mathbb{R}$ and an endowment $e^n \in \mathbb{R}_+^{L \times S}$. Assume u^n is C^2 , strictly concave and monotonic, and that e^n is measurable with respect to I^n .

Consider a price function $p : S \rightarrow \mathbb{R}_+^L$. Its inverse yields a partition of S which we will denote by $I(p)$. An R.E.E. for this economy is a p , along with allocations $x^n : S \rightarrow \mathbb{R}_+^L$ such that, for $n \in N$,

¹Authors' emphasis.

²We outline here the model described by Radner [14].

- (i) x^n is measurable w.r.t. the coarsest refinement $I^n \vee I(p)$ of I^n and $I(p)$;
- (ii) $x^n = \operatorname{argmax}_{x \in \mathbb{R}_+^{L \ S}} \{u^n(x) : x \text{ is measurable w.r.t. } I^n \vee I(p), \text{ and } p_s \cdot x_s \leq p_s \cdot e_s^n \text{ for each } s \in S\}$;
- (iii) $\sum_{n \in N} x^n = \sum_{n \in N} e^n$.

(Here, for any vector $x \in \mathbb{R}_+^{L \ S}$ and $s \in S$, x_s is the vector in \mathbb{R}_+^L obtained by restricting x .) In words this means that each agent n refines his information I^n by what he can deduce from seeing p , then forms his demand x_s^n (subject to the budget constraint $p_s \cdot x_s^n \leq p_s \cdot e_s^n$), and the ensuing total demand $\sum x^n$ can be met by the supply $\sum e^n$ at hand. Notice that prices play the dual role of simultaneously determining the budget constraint and revealing information. Radner has shown¹ that "generically" (in the space of utilities) an R.E.E. exists and its prices are fully revealing in that $I(p) = I^*$. Prices are thus shown to convey to each agent all the bits of information held originally in separate minds.

An immediate paradoxical upshot of this was first noted by Grossman and Stiglitz. Since prices reveal all of the collective information at a R.E.E., no agent benefits from his initial superior information! And if this information happens to be costly to acquire then no one will gather any, and there will be none for prices to reveal. Recently several authors have attempted to cope with this problem by postulating random exogenous noise and an infinite number of states of nature in order to prevent the R.E.E. prices from being fully revealing.²

¹And this is typical of results obtained in other similar models, e.g., those of Grossman.

²See Grossman [9] and Anderson-Sonnenschein [1].

In our view the paradox stems from, and indeed illustrates, a grievous omission in the R.E.E. model. The R.E.E. model describes how prices can reveal information, but it does not even begin to explain how, in the first place, the diverse bits of information of the agents are pooled and "put into" the prices to be revealed. And, as Hayek emphasized, this step in the market process is the central issue in an understanding of how information is disseminated through the economy. Our essential criticism of the R.E.E. model is that it throws the baby out with the bath-water because it does not represent a process at all.

In this paper we consider a model with an explicit process for the flow of information via prices. Roughly it goes as follows. Economic activity takes place in time periods. Agents initially act on the basis of their privately-held information I^n . This results in observable economic outcomes (e.g. prices) through which their information is "betrayed." The extra information so released to everyone is then available for the next period of activity. Notice that in the initial period agents with superior information can exploit it and make a "killing." The paradox that information is useless is removed by the simple fact that the process that reveals it takes time (as any process must).

This description is, we believe, more realistic than the R.E.E. model (and also more in keeping with what Hayek had in mind). Its very wording invites one to model it as a strategic market game. We shall, for concreteness, choose one such: the Shapley-Shubik model of exchange

¹ Specifically, in the R.E.E. model agents not only must come to understand the price function $p(s)$ but also the particular realization \bar{p} before they can undertake the action which presumably "caused" that realization. This simultaneity problem, which in the special case of complete information disappears, is here seen to be very troublesome.

presented in [15], [16]. But our results seem to be quite robust and not to hinge delicately on this choice (see Remark 2).

Before plunging into the details it might be helpful to describe the contours of our model. S , N , $\{I^n\}_{n \in N}$ are as before. But now there are time periods, for simplicity two. The characteristics of the traders must accordingly be expanded into endowments $e^n, \tilde{e}^n \in \mathbb{R}_+^{L \times S}$ in period 1(2); and utility $u^n : \mathbb{R}_+^{L \times S} \times \mathbb{R}_+^{L \times S} \rightarrow \mathbb{R}$.

The game is best viewed in extensive form. Nature moves first to select a state s in S . At each node s all the players in N move simultaneously with information partitions given by I^n . Let $X^n(s)$ be the set of moves available to n at node s (of course, we must then require that $X^n(s)$ is constant for all $s \in \gamma \in I^n$). Put $X(s) = \prod_{n \in N} X^n(s)$. There are maps $\phi_s^n : X(s) \rightarrow Z^n$, $\psi_s^n : X(s) \rightarrow \mathbb{R}_+^L$. Here Z^n is a space of economic observables for player n (it's best to think of it as prices, and set $Z^n = \mathbb{R}_+^L$). For¹ $q_s = (q_s^1, \dots, q_s^N) \in X(s)$, $\phi_s^n(q_s)$ is what n observes in Z^n as a consequence of the joint choice q_s of moves by the agents; and $\psi_s^n(q_s)$ is his final holding of commodities in time period 1. Thus the maps ψ_s^n satisfy: $\sum_{n \in N} \psi_s^n(q_s) = \sum_{n \in N} e_s^n$. In time period 2 the nodes in the game tree are $q_s \in X(s)$. Let $\tilde{X}^n(s)$ be² the set of moves of n at q_s . The information partition \tilde{I}^n of n on $\bigcup_{s \in S} X(s)$ is given by I^n refined by what he can observe of others' moves through ϕ^n . To make this precise, let \hat{I}^n be the partition of $\bigcup_{s \in S} X(s)$ that is yielded by the equivalence relation: $q_s \sim q'_s$, if $\phi_s^n(q_s) = \phi_s^n(q'_s)$.

¹Without confusion, $N = \{1, \dots, N\}$; $S = \{1, \dots, S\}$ etc.

²In general we could write $\tilde{X}(q_s)$, but in our model we need \tilde{X} to depend on q_s only through the state s .

Also extend I^n to a partition I_{ext}^n on $\bigcup_{s \in S} X(s)$ in the obvious way:

to each $\gamma \in I^n$ corresponds the set $\bigcup_{s \in \gamma} X(s)$. Then define

$\hat{I}^n = I_{\text{ext}}^n \vee \hat{I}^n$. Finally once again there are maps $\hat{\psi}_s^n : \hat{X}(s) \rightarrow \mathbb{R}_+^L$,

where $\hat{X}(s) = \prod_{n \in N} \hat{X}^n(s)$, which specify the transformation of moves to

trades in the 2nd period. (Of course, $\sum_{n \in N} \hat{\psi}_s^n(q_s) = \sum_{n \in N} e_s^n$ for any

$\hat{q}_s \in \hat{X}(s)$.)

A strategy of n is to pick a move at each node in $S \cup \{ \bigcup_{s \in S} X(s) \}$,

subject to the constraint that these be identical at any two nodes that

lie in the same information set. Given a choice of strategies by all agents,

a play $\pi(s)$ is determined in the tree for each $s \in S$ in the standard

manner. Associated with these are moves $q_s = (q_s^1, \dots, q_s^N)$ and

$\hat{q}_s = (\hat{q}_s^1, \dots, \hat{q}_s^N)$ in the two time periods. The final holding that accrues

to n is then $\psi_s^n(q_s)$, $\hat{\psi}_s^n(q_s)$ in periods 1, 2 in state s . His pay-

off is simply the utility of his final holding: $u^n(\{\psi_s^n(q_s)\}_{s \in S}, \{\hat{\psi}_s^n(q_s)\}_{s \in S})$.

We analyze this game for its Nash Equilibria (N.E.) when the X^n ,

\hat{X}^n , ϕ^n , ψ^n , $\hat{\psi}^n$ are according to the Shapley-Shubik model (see the

text section for details). Our results may be summed up as follows. If

N is non-atomic, then for a generic choice of e^n , \hat{e}^n and u^n : N.E.

exist and are finite in number; they fully reveal S in that

$\phi_s^n(q_s) \neq \phi_{s'}^n(q_{s'})$ if $s \neq s'$; and lead to higher utilities for the better-

informed agents. If N is finite then generic revelation fails, and N.E.

exist robustly (i.e. for an open set of e^n , \hat{e}^n and u^n) at which

some agents do not betray all their information in the first period. Thus

generic revelation by prices is a phenomenon that attaches to perfect com-

petition and is seen to break down in an oligopolistic setting. The non-

atomic case is simpler (as is shown in general in [4]) in that the strategies

can be taken to depend on history only insofar as that history reveals something about the state of nature. Fixing q , let \hat{I}^{n2} be the partition of S generated by $\phi_S^n(q_S)$, that is by the equivalence relation $s \sim s'$ if $\phi_S^n(q_S) = \phi_S^n(q_{S'})$. Let $I^{n2} = I^n \vee \hat{I}^{n2}$. In the nonatomic case we may think of the second period strategies q^n as functions of s alone which are measurable with respect to I^{n2} . Threat equilibria, in which strategies depend also on the moves of other agents, disappear. To return to our main point, however, in both the finite and nonatomic cases agents with superior information benefit from it, so that we stay clear of the R.E.E. paradox.

One might wonder if these results are--at bottom--an artifact of the model we have invoked. Could one not concoct an ingenious one-period strategic game whose N.E.'s coincided with the R.E.E.'s of the underlying economy? Such N.E.'s would entail that while strategies are measurable w.r.t. I^n , no one wishes to revise his own even after being informed of the resultant \hat{I}^n . They do not exist in our model, but it is conceivable that in a sufficiently "complex" game they might. Indeed one suggestion is to allow each agent of type n to submit an entire demand function $d_r : R_+^L \rightarrow R^L$ for every $r \in I^n$. The market mechanism then performs a complicated fixed point computation to find prices that clear markets for every $s \in S$. Beja has shown that this will not always work: for at least one economy the R.E.E. cannot be implemented this way. But even if some variant of this game did work, it would be open to the obvious criticism that one cannot imagine agents who have the kind of capacity of computation needed to play it. We take as a dictum--and this is met by our model--that both the strategy sets and the outcome map be simple and "playable."*

*It would be very interesting to be able to state precisely the connection between the implementability of a R.E.E. and the complexity of the game.

what will happen in the market, only upon the information I^n privately held by agents. We believe that a significant proportion of actual trade takes place this way. A farmer offers to the market his crop of wheat, as a matter of prior commitment, no matter what the price is going to be. At the time of planting there is not necessarily much information for prices to reveal: demand decisions will not be made until much later. Even a system of continuous-trading futures markets could not hope to communicate all the relevant information at the appropriate moment in time. It is from the spot prices that the farmer typically learns the information which would have induced him to plant differently had he known it then, and it is these prices which are his most reliable guide to the future.

Our hypothesis is that in many cases these futures markets do not exist anyway. Accordingly we construct a model in which traders learn from past spot prices and undertake simple trading strategies that determine current ones. It is in the spirit of Cournot. And the results we obtain seem to be robust to variations of the model if there are spot markets and uncontingent strategies (Remark 2).

Our model is stripped down to concentrate on the flow of information from period 1 to 2. Agents only buy goods and are forced to put everything up for sale. The commodities in the two periods are completely disjoint and there is no inventorying. All this is for simplicity and could easily be rectified (Remark 2). A more subtle condition is on the space of utilities within which our generic results holds. This consists of all functions $u(x, \tilde{x})$ defined on the joint holdings x, \tilde{x} of periods 1, 2. One could well ask how important the choice of this space is for our results. If we had restricted ourselves, for example, to $u(x, \tilde{x})$ of the form $u(x+\tilde{x})$ then, with inventorying, this would in effect make the two time periods

arbitrarily close to each other.¹ One might expect that the agents would trade very little in the first period and simply wait until the second period when they had more information to do most of their trading. The resulting N.E.'s might then look very much like the R.E.E.'s we have been criticizing. As we show later in an example, however, this intuition is wrong. Agents would trade in both time periods because the prices will in general be different. And the main result of our paper—that information is of value—would not be violated. Moreover, in our example the allocation of the strategic market game Pareto dominates the R.E.E. allocation! Although information is always individually valuable in our strategic market game, it may be socially harmful. Traders may prefer to exchange before they are fully informed, and this is impossible in a fully revealing R.E.E.

One might also consider the special case where $u^n(x, \hat{x}) = w^n(x) + v^n(\hat{x})$ and there is no inventorying. Recalling that when there is a continuum of agents the second period strategies \hat{q}^n can be taken to be functions of the states alone, measurable with respect to I^{n2} , our results imply that generically $I^{n2} = I^*$ and that the second period allocations are precisely the rational expectations equilibrium allocations of the economy $E = (N, v^n, \hat{e}^n, I^n)$. An outside observer seeing the market only in period 2 might conclude that traders were learning from the prices simultaneously as they acted to set them whereas in fact period 2 prices correspond to costs (marginal utilities) because of the information conveyed by period 1 prices.

The paper is organized as follows. In Section 2 the basic strategic

¹It would also have made the proof of our theorem more difficult.

market game is formulated. In Section 3 the main theorem is proved. In Section 4 a series of examples is presented to "round off" the approach. As we mentioned earlier, if N is finite then information may not be revealed at an N.E. We also model the situation in which information may be bought and sold, and in this case it is possible that no N.E. exists. Finally, an excursion is made into a Bertrandian-type of model in which prices can be used as (contingent) strategies. But instead of being functions they are kept very simple, as is the outcome map, in accordance with the dictum stated earlier. We find that again no N.E. may exist; if it does more than one price may prevail for a commodity, and (a) information is not necessarily revealed if N is finite, (b) agents typically benefit from superior information (N finite or continuum).

In our model agents learn from past prices but not from current prices. One might well ask why we could not dispense with our process of price formation and instead simply hypothesize that agents take current prices into account in calculating their budgets but infer information from these same prices with a one period lag. Indeed this is the approach used in Hellwig (13). In our last example, however, we explain that this formulation can lead to grave irrationalities -- an agent who uses prices to calculate income but ignores their informational content can easily be induced to demand a consumption bundle providing strictly less utility than his initial endowment. To put it differently, the agent will make bets with other better informed agents that he can only lose.

2. THE STRATEGIC MARKET GAME

Consider the case when the agent-space is non-atomic.¹ For convenience there is a finite number of types of agents: $1, \dots, N$. Type n consists of the continuum $(n-1, n]$ endowed with the Lebesgue measure for every $n \in N = \{1, \dots, N\}$. (The triple use of n : as the number n , as the set $(n-1, n]$, as the name of the n^{th} type; as well as the additional fourth use of N as the set of types $\{1, \dots, N\}$; should cause no confusion. The usage will always be clear from the context, and it saves enormously on notation.) To recapitulate from Section 1:

$L \equiv \{1, \dots, L\} \equiv$ set of commodities

$S \equiv \{1, \dots, S\} \equiv$ states of nature

$e^n \in \mathbb{R}_+^{L \times S} \equiv$ endowment of $\alpha \in (n-1, n]$ in period 1

$\tilde{e}^n \in \mathbb{R}_+^{L \times S} \equiv$ endowment of $\alpha \in (n-1, n]$ in period 2

$u^n : \mathbb{R}_+^{L \times S} \times \mathbb{R}_+^{L \times S} \equiv$ utility function of $\alpha \in (n-1, n]$

$I^n \equiv$ partition of $S \equiv$ information of $\alpha \in (n-1, n]$

A vector in $\mathbb{R}_+^{L \times S} \times \mathbb{R}_+^{L \times S}$ will be broken into (x, \tilde{x}) where x, \tilde{x} are each in $\mathbb{R}_+^{L \times S}$. Thus $x(\tilde{x})$ is the vector of state-contingent commodities in period 1 (2). Also for $x \in \mathbb{R}_+^{L \times S}$, $x_{\ell s}$ is its component on the axis $(\ell, s) \in L \times S$; and $x_s(x_\ell)$ is the vector in $\mathbb{R}_+^L(\mathbb{R}_+^S)$ obtained by restricting x .

The L^{th} commodity is singled out as a money to be used for bidding. For $s \in S$, let $\underline{e}_{Ls}^n = \min\{e_{Ls}^n : s' \in I^n(s)\}$. (Here $I^n(s)$ is the element of I^n that contains s .) Then $X^\alpha(s)$, the set of moves available to n at s in period 1, is given by:

¹The model for N finite will become clear in the process.

$$X^\alpha(s) = \{b_s \in \mathbb{R}_+^{L-1 \times \{s\}} : \sum_{\ell=1}^{L-1} b_{\ell s}^\alpha \leq e_{Ls}^{\alpha}\} ,$$

where $L-1$ is the set $\{1, \dots, L-1\}$ and $n(\alpha)$ is the type of α . In the interpretation $b_{\ell s}^\alpha$ is the amount of money bid by α in period 1 in state s for the purchase of commodity $\ell \in L-1$. We never allow an agent to bid more than his endowment in the worst possible contingency. A choice of moves $\{b_s^\alpha : \alpha \in [0, N]\}$ determines prices $p_{\ell s}$ and trades $x_s^\alpha \in \mathbb{R}_+^L$ by the rules:¹

$$p_{\ell s} = \frac{\int_0^N b_{\ell s}^\alpha d\alpha}{\int_0^N e_{\ell s}^\alpha d\alpha} ,$$

$$x_{\ell s} = \begin{cases} b_{\ell s}^\alpha / p_{\ell s} & \text{if } p_{\ell s} > 0 \\ 0 & \text{if } p_{\ell s} = 0 \end{cases}$$

for $\alpha \in L-1$;

$$x_{Ls}^\alpha = e_{Ls}^\alpha - \sum_{\ell=1}^{L-1} b_{\ell s}^\alpha + \sum_{\ell=1}^{L-1} p_{\ell s} e_{\ell s}^\alpha .$$

This completely specifies the maps $\{\psi_s^\alpha : \alpha \in [0, N], s \in S\}$. In the interpretation, all of the goods in $L-1$ have to be offered for sale and then the goods (money) are disbursed in proportion to the bids (offers). The sets $X^\alpha(s)$ and the maps ψ_s^α are defined in exactly the same way

¹We consider only the case when the map $\alpha \mapsto b_s^\alpha$ is integrable. See Remark 3, however.

as $X^\alpha(s)$, ψ_s^α with b_{ls}^α , e_{ls}^α replaced by \tilde{b}_{ls}^α , \tilde{e}_{ls}^α . It remains to describe $\hat{\Gamma}^n$, equivalently \hat{I}^n , to complete the definition of the extensive game. Since we are interested in the role played by prices in disseminating information, we shall let prices be the observables, i.e., $Z^n = \mathbb{R}_+^{L-1}$ (the price of the L^{th} commodity being always 1 in our model), and the l^{th} component of $\phi_s^n(b_s) = (\int_0^N b_{ls}^\alpha d\alpha) / (\int_0^N e_{ls}^\alpha d\alpha)$. However, from the technical point of view of the validity of our theorem, much finer observations can be permitted, as explained in Remark 3.

Let us designate the above game by Γ . We will analyze the Nash Equilibria (N.E.) of Γ , i.e., a choice of strategies by the agents in $[0, N]$ at which no one agent can profit by a unilateral deviation. Some readers may now prefer to proceed directly to the examples of Section 4.

3. EXISTENCE OF NASH EQUILIBRIA

Γ has some trivial "inactive" N.E.'s. For instance consider the strategies in which all agents bid zero everywhere. Additional N.E.'s can be constructed which leave any specified subset of the $2 \times L-1 \times S$ trading posts inactive. Our interest is in pinning down conditions which guarantee the existence of active N.E.'s, namely those which produce positive prices in each trading post. Instead, from now on, we shall always mean an active N.E. when we say N.E.

It turns out that N.E.'s do not always exist for Γ . However if we vary Γ then, for a "generic" choice of Γ , it can be shown that N.E.'s do exist. Let us first make the notion of genericity precise. Let A , B , C , D be positive numbers with $A < B$, $C < D$. Consider the

polytope E in $\mathbb{R}_+^{N \times L \times S} \times \mathbb{R}^{N \times L \times S}$ consisting of $e^1, \dots, e^N, \tilde{e}^1, \dots, \tilde{e}^N$

which satisfy:

- (i) $A < e_{ls}^n, \tilde{e}_{ls}^n < B$, for $n \in N$ and $s \in S$
- (ii) $C < \sum_{n \in N} e_{ls}^n < D$ for $l \in L-1$ and $s \in S$.

Each point in E represents a choice of endowments e, \tilde{e} . Clearly we can find an $E > 0$ such that $\max\{\|\sum_{n \in N} e^n\|, \|\sum_{n \in N} \tilde{e}^n\|\} < E$ where $\|\cdot\|$ denotes the maximum norm. Then if $x^1, \dots, x^N, \tilde{x}^1, \dots, \tilde{x}^N$ is any reallocation of $e^1, \dots, e^N, \tilde{e}^1, \dots, \tilde{e}^N \in E$, we automatically have $\|x^n\| < E, \|\tilde{x}^n\| < E$. Thus if endowments are to come from E we can confine ourselves to utilities defined on the cube $C \subset \mathbb{R}_+^{L \times S} \times \mathbb{R}_+^{L \times S}$, whose edges have length E . Let U be the space of all functions defined on a neighborhood N of C which are C^2 , strictly concave, and satisfy (for $0 < \sigma < \sigma'$):

$$\sigma < \frac{\partial u}{\partial x_{ls}}, \frac{\partial u}{\partial \tilde{x}_{ls}} < \sigma'.$$

With the C^2 -topology, U is a Banach manifold. A point in U^N represents a choice of utilities for the N types. We will keep all the other data of the game fixed as in Section 2, and vary only the endowments and utilities. $E \times U^N$ can then be thought of as the space of games. Our existence theorem is now readily stated.

Theorem. There is an open dense set \mathcal{D} in E , whose complement in E has zero Lebesgue measure, such that for $(e, \tilde{e}) \in \mathcal{D}$ there exists an open dense set $\mathcal{D}(e, \tilde{e})$ in U^N with the property:

- (i) N.E.'s exist and are finite in number for $\Gamma \in \{(e, \tilde{e})\} \times \mathcal{D}(e, \tilde{e})$
- (ii) if $b : [0, N] \rightarrow \mathbb{R}^{L \times S}$ is the move at any N.E. in (i), then

b is fully revealing, i.e.,

$$s \neq s' \Rightarrow \phi_s^n(b_s) \neq \phi_{s'}^n(b_{s'}) .$$

The proof consists of three steps. We will define a "potential Nash Equilibrium" (p.N.E.) which exists for every $\Gamma = (e, \tilde{e}, u) \in E \times U^N$ (Section 3.1). Then we define \mathcal{D} and prove that if $(e, \tilde{e}) \in \mathcal{D}$ there is an open dense $\mathcal{D}(e, \tilde{e})$ in U^N such that every p.N.E. of $\Gamma \in (e, \tilde{e}) \times \mathcal{D}(e, \tilde{e})$ is fully revealing (Section 3.2). From this it is deduced that, for such Γ , the set of p.N.E. = the set of N.E. (Section 3.3).

3.1. Potential Nash Equilibria

Fix $\Gamma = (e, \tilde{e}, u)$ in $E \times U^N$. The fictitious game Γ^* is obtained from Γ by the modifications: (i) the information partition I^n of each type in period 2 is replaced by $I^1 \vee \dots \vee I^N$. (W.l.o.g. assume that $I^1 \vee \dots \vee I^N = I^* = (\{1\}, \dots, \{s\})$ from now on.) (ii) Strategies are restricted to be bids contingent only on the information about chance moves and not contingent, beyond this, on others' moves, i.e., $\tilde{b}^\alpha(b(s)) = \tilde{b}^\alpha(b'(s))$ for $b(s), b'(s) \in X(s)$.

For $\Delta > 0$ consider the Δ -modified fictitious game Γ_Δ^* in which (in addition to (i) and (ii)) an external agency is imagined to have placed bids of size Δ in each of the $2(L-1)S$ trading posts. This does not affect the strategy sets of Γ^* but only the strategy-to-outcome map.

A potential Nash Equilibrium (p.N.E.) of Γ is simply an N.E. of Γ^* . If $\eta(\Gamma)$ denotes the set of N.E. of Γ , then clearly p.N.E. of $\Gamma = \eta(\Gamma_0^*)$.

Let \sum^n denote the strategy-set of type n in the game Γ_Δ^* , $\Delta \geq 0$. A typical element of \sum^n consists of a pair of vectors b^n, \tilde{b}^n

in $R_+^{(L-1) \times S}$ measurable w.r.t. I^n , I^* respectively. Since utilities are strictly concave, and the set of agents $[0, N]$ is non-atomic, it is obvious that at any N.E. of Γ_Δ^* agents of a given type use the same strategy. Therefore in our analysis of $\eta(\Gamma_\Delta^*)$ we may restrict ourselves to the set $\Sigma = \Sigma^1 \times \dots \times \Sigma^N$.

For $\mu > 0$ denote by Σ_μ the subset of Σ at which all prices $p_{\ell s}$, $\tilde{p}_{\ell s}$ ($\ell \in L-1$, $s \in S$) in the two periods are at least μ .

Lemma 1. There is a $\mu > 0$ such that if $\Gamma \in E \times U^N$ then $\eta(\Gamma_\Delta^*) \subset \Sigma_\mu$ for $\Delta \geq 0$.

Proof. First let us show that there is a μ_1 such that if the first period moves at some N.E. of Γ_Δ^* are b , then $p_{\ell s}^b > \mu_1$ for all s, ℓ . (By $p_{\ell s}^b$ we mean the first-period prices that accrue from b in the game Γ_Δ^*).

Case 1

$$\sum_{\ell \in L_1} b_{\ell s}^n < \frac{e_{-Ls}^n}{-Ls}$$

for some $n \in N$ and $s \in S$. If an agent of type n increases his bids $b_{\ell r}^n(1)$ ($r \in I_n(s)$) by $\epsilon > 0$ then the increase in his payoff, for small ϵ , is approximately:

$$\epsilon \left[\sum_{r \in I_n(s)} \left(\frac{\partial u^n}{\partial x_{\ell r}} / p_{\ell r}^b - \frac{\partial u^n}{\partial u_{Lr}} \right) \right] \geq \epsilon [\sigma / p_{\ell r}^b - |I_n(s)| \sigma']$$

for any $r \in I_n(s)$. This must be non-positive, therefore

$$p_{\ell r}^b \geq \sigma / (|I_n(s)| \cdot \sigma') \geq \sigma / S \sigma'$$

Case 2

$$\sum_{\ell \in L-1} b_{\ell s}^n = \bar{e}_{-Ls}^n \quad \text{for some } n \in N \text{ and } s \in S.$$

Clearly $p_{\ell s}^b \geq \bar{e}_{-Ls}^n / \bar{e}_{r\ell}$ for any $r \in I_n(s)$, where $\bar{e}_{r\ell}$ abbreviates

$\sum_{n \in N} e_{\ell r}^n$. Consider $q \in I_n(s') \neq I_n(s)$. Put

$$M_\ell = \min\{\bar{e}_{-Ls}^{n'}, \bar{e}_{r\ell} : n' \in N, r \in I_n(s'), s' \in S\}.$$

If an agent of type n reduces $b_{\ell r}^\alpha$ by ε and increases $b_{\ell r'}^\alpha$ by ε ($r \in I_n(s)$, $r' \in I_n(s')$) then his increase in payoff of small ε is approximately:

$$\varepsilon \left[\left(\sum_{r' \in I_n(s')} \frac{\partial u^n}{\partial x_{\ell r'}} / p_{\ell r'}^b \right) - \left(\sum_{r \in I_n(s)} \frac{\partial u^n}{\partial x_{\ell r}} / p_{\ell r}^b \right) \right] \\ \geq \varepsilon [\sigma / p_{\ell r'}^b - |I_n(s)| \sigma' / M\sigma]$$

for any $r' \in I_n(s')$. This must also be non-positive, then

$$p_{\ell r}^b \geq M_\ell \sigma / |I_n(s)| \sigma' \geq M_\ell / S\sigma'.$$

Put $M = \min\{M_\ell : \ell \in L-1\}$ and then $\mu_1 = \min\{\sigma/S\sigma', M, M/S\sigma'\}$. Combining the two cases, we have shown: $b \in \eta(\Gamma_\Delta^*) \Rightarrow p_{\ell s}^b > \mu_1$ for all ℓ and s .

In an exactly analogous manner, one can check that there is a $\mu_2 > 0$ such that if \tilde{b} are the second-period moves at any N.E. of Γ_Δ^* then $\tilde{p}_{\ell s}^b(2) > \mu_2$ for all ℓ and s . Then, with $\mu = \min\{\mu_1, \mu_2\}$, the lemma follows (recall the bounds on endowments in E .)

Q.E.D.

Lemma 2. If $\Delta > 0$, then $\eta(\Gamma_\Delta^*)$ is non-empty for any $\Gamma \in E \times U^N$.

Proof. If $\Delta > 0$ the strategies-to-outcome map is continuous. (It blows up if $\Delta = 0$, i.e. in the unmodified fictitious game Γ_0^* at strategies which produce a zero price in any trading post...hence the importance of Lemma 1.) The proof now involves a straightforward use of Kakutani's fixed point theorem.

Q.E.D.

Lemma 3. $\eta(\Gamma^*)$ is non-empty for any $\Gamma \in E \times U^N$.

Proof. Take a sequence $\{\Delta^m\}$, $\Delta^m \rightarrow 0_+$. Let $m_b, m_b^\nu \in \eta(\Gamma_{\Delta^m}^*)$. (By Lemma 2 such m_b, m_b^ν exist.) Let $*b, *b^\nu$ be a cluster-point of the $\{m_b, m_b^\nu\}$. By Lemma 1 $p_{\ell s}^*b, p_{\ell s}^{\nu *b^\nu} > \mu > 0$ for $\ell \in L-1$ and $s \in S$. Then $*b, *b^\nu$ is a point of continuity of the pay-off functions, from which it easily follows that $*b, *b^\nu \in \eta(\Gamma^*)$.

Q.E.D.

Remark 1. A straightforward fixed-point argument was not possible because of the singularity of the strategies-to-outcome map at places which produced zero prices. This made the Δ -approximation necessary.

3.2. Generic Full Revelation by Prices

Let us first describe the set \mathcal{D} in E . Though it requires somewhat labored notation, the idea is simple. For $\gamma \in I^n$ consider the $L-1$ dimensional simplex of moves $R^n(\gamma)$ available to n in period 1. Suppose (i) there exist $\gamma_1, \gamma_2 \in I^n$ which only n can distinguish, i.e., $\gamma_1 \cup \gamma_2 \in I^j$ for $j \in N \setminus \{n\}$; (ii) e_s^n is constant over $s \in \gamma_1 \cup \gamma_2$. If at an N.E. of Γ^* it happens that n is at the same "vertex" of

$R^n(\gamma_1)$, $R^n(\gamma_2)$, e.g. bidding nothing in both γ_1 and γ_2 , then irrespective of the strategies used by others only $\gamma_1 \cup \gamma_2$ will be revealed at the start of period 2. There is nothing in the model to stop such N.E.'s from existing robustly (in utilities, i.e. for an open set in U^N). Thus we will require that endowments be in "general position," so that if any subset of players is at vertices then their information is still revealed. To make this precise unfortunately calls for quite cumbersome notation.

For $\gamma \in I^n$ let the zero-vertex of R_γ^n be denoted by $v_L^n(\gamma)$ and the remaining $L-1$ (corresponding to putting all bids on some $\ell \in L-1$) by $v_1^n(\gamma), \dots, v_{L-1}^n(\gamma)$. Consider $T_\gamma^n \subset \{v_1^n(\gamma), \dots, v_L^n(\gamma)\}$, $T_\gamma^n \neq \emptyset$, and define:

$$\dot{T}_\gamma^n = \text{relative interior of the convex hull of vertices in } T_\gamma^n.$$

Let $\tau^n = \{T_\gamma^n : \gamma \in I^n\}$ be a collection of subsets of vertices of R_γ^n , $\gamma \in I^n$. A choice of moves $b^n \in \Pi R^n$ by type n is of type τ^n if $b_\gamma^n \in \dot{T}_\gamma^n$ for all $\gamma \in I^n$. Given $\tau = (\tau^1, \dots, \tau^N)$ further define:

(iii) $b = (b^1, \dots, b^N)$ is of type τ if each b^n is of type τ^n .

(iv) $A(\tau) = \text{active players in } \tau = \{n \in N : |T_\gamma^n| > 1 \text{ for some } \gamma \in I^n\}$.

(v) For $n \in A(\tau)$, $R_a^n(\tau) = \text{active strategies of } n \text{ in } \tau = \Pi\{\dot{T}_\gamma^n : |T_\gamma^n| > 1\}$.

(vi) $\dot{R}_a^n(\tau) = \Pi\{\dot{T}_\gamma^n : |T_\gamma^n| > 1, v_L^n(\tau) \in T_\gamma^n\}$.

(vii) $\ddot{R}_a^n(\tau) = \Pi\{\dot{T}_\gamma^n : |T_\gamma^n| > 1, v_L^n(\tau) \notin T_\gamma^n\}$.

(Note: $R_a^n(\tau) = \dot{R}_a^n(\tau) \times \ddot{R}_a^n(\tau)$.)

(viii) $R_\mu(\tau) = \{b = (b^1, \dots, b^N) : b \text{ is a feasible choice of moves in period 1, } b \text{ is of type } \tau, p_{\ell s}^b > \mu \text{ for } \ell \in L-1, s \in S\}$.

By dropping inactive strategies, $R_\mu(\tau)$ can be--and will be--viewed as a subset of $\prod_{n \in A(\tau)} R_a^n(\tau)$.

By Lemma 1 we can confine ourselves to the set $\{(b, \hat{b}) : b \in UR_\mu(\tau)\}$ in the search of N.E. of Γ^* . Observe that U is a finite partition. Also since the moves-to-outcome map ψ is continuous at positive prices, it is uniformly continuous on $UR_\mu(\tau)$. Therefore we can find (sufficiently small) neighborhoods \hat{T}_Y^n of T_Y^n in {Affine hull of T_Y^n } such that (defined by the same formulas) is continuous on $\prod_{n \in A(\tau)} \prod \{\hat{T}_Y^n : |T_Y^n| > 1\}$ and the image of $\prod_{n \in A(\tau)} \prod(\dots)$ under ψ is contained in N .¹ Now define \underline{R}_a^n , $\underline{R}_a^n(\tau)$, $\hat{\underline{R}}_a^n(\tau)$, $R_\mu(\tau)$ exactly as before by using \hat{T}_Y^n in place of T_Y^n . We will consider the τ -subgame defined on the players in $A(\tau)$, each of whom has the strategy-set $\underline{R}_a^n(\tau)$, i.e. all inactive strategies are held fixed and ψ is applied to only the active strategies of the players in $A(\tau)$. These active strategies now vary over \hat{T}_Y^n instead of T_Y^n but this causes no problems. The N.E. of the τ -subgame still lie in $\underline{R}_\mu(\tau) \subset \prod_{n \in A(\tau)} \underline{R}_a^n(\tau)$ by Lemma 1, with μ lowered slightly to allow for the extension of the strategic domain from \hat{T}_Y^n to \underline{T}_Y^n . Define:

$$M(\tau) = \{b \in \underline{R}_\mu(\tau) : p_s^b = p_{s'}^b, \text{ for two distinct } s \text{ and } s' \text{ in } S\}.$$

$M(\tau)$ depends on e . We will say that e is in general position if $M(\tau)$ is a finite union of submanifolds of codimension at least one in $\underline{R}_\mu(\tau)$, for all τ . Then the set

$$\{e : e \text{ is in general position}\}$$

is obtained from $\mathbb{R}_+^{N \times L \times S}$ by removing a finite number of submanifolds of

¹Recall that N is the neighborhood of C on which utilities are defined.

codimension at least one in $\mathbb{R}_+^{N \times L \times S}$. Let

$$\mathcal{D} = \{(e, \tilde{e}) \in E : e \text{ is in general position}\}.$$

Clearly \mathcal{D} satisfies all the properties required by the theorem. To prove (ii) of the theorem it will suffice to show that there is an open dense set $\mathcal{O}(\tau, e, \tilde{e})$ in $U^{A(\tau)}$ (for $(e, \tilde{e}) \in \mathcal{D}$) such that: if $U \in \mathcal{O}(\tau, e, \tilde{e})$ then at any N.E. of the τ -subgame $p_s \neq p_{s'}$, for $s \neq s'$. For then we can simply set $\mathcal{D}(e, \tilde{e}) = \bigcap_{\tau} \{ \mathcal{O}(\tau, e, \tilde{e}) \times \prod_{n \in A(\tau)} U \}$ obtain the conclusion of the theorem. The existence of $\mathcal{O}(\tau, e, \tilde{e})$ in turn is proved by roughly the following argument. The N.E. of the τ -subgame are generically finite in number and vary continuously. On the other strategies b in $M(\tau)$, at which $p_s^b = p_{s'}^b$, for some pair $s \neq s'$, is made up of submanifolds of codimension ≥ 1 . Therefore the N.E. set generically misses $M(\tau)$ and N.E. prices are fully revealing. To change this into a proof requires a routine use of the Transversal Density of Openness Theorems []. Indeed consider the map

$$U^{A(\tau)} \times \underline{R}_\mu(\tau) \xrightarrow{D} \left(\prod_{n \in A(\tau)} (\dot{R}_a^n(\tau) \times \ddot{R}_a^n(\tau)) \right) \times \underline{R}_\mu(\tau)$$

where $b \mapsto b$ for $b \in \underline{R}_\mu(\tau)$, and

$$D_{n,x}(u,b) = \frac{\partial u^n}{\partial x}(b)$$

for $x \in \dot{R}_a^n(\tau) \times \ddot{R}_a^n(\tau)$, i.e., it is the partial derivative of n 's payoff w.r.t. his own active strategy x . Let

$$N = \{y \in \prod_{n \in A(\tau)} (\dot{R}_a^n(\tau) \times \ddot{R}_a^n(\tau)) : y_{n,x} = 0 \text{ if } x \in \dot{R}_a^n(\tau) \\ y_{n,x} = y_{n,z} \text{ if } x, z \text{ are in the same } \dot{T}_\gamma^n \text{ occurring in } \ddot{R}_a^n(\tau)\}$$

For b to be a p.N.E. of $u \in U^A(\tau)$, we must have $D(u,b) \in N \times R_\mu(\tau)$; for b to be a p.N.E. at which prices are not fully revealing we must have $D(u,b) \in N \times M(\tau)$, i.e. $D(u,b) \in N \times M_i(\tau)$ for some $i = 1, \dots, k$ where (since e is in general position) $M(\tau) = M_1(\tau) \cup \dots \cup M_k(\tau)$ breaks $M(\tau)$ into submanifolds of $R_\mu(\tau)$ each of which has $\text{codim} \geq 1$ in $R_\mu(\tau)$. The map D is easily checked to be transverse to every submanifold of its image. The transversal density and openness theorems then show that there is an open dense set $O(\tau, e, \tilde{e})$ such that if $u \in O(\tau, e, \tilde{e})$

$$(a) \text{ codim } D_u^{-1}(N \times M_i(\tau)) \text{ in } R_\mu(\tau) = \text{codim}(N \times M_i(\tau)) \text{ for } i = 1, \dots, k;$$

$$(b) \text{ codim } D_u^{-1}(N \times R_\mu(\tau)) \text{ in } R_\mu(\tau) = \text{codim}(N \times R_\mu(\tau)).$$

Since $\text{codim}(N \times M_i(\tau)) > \dim R_\mu(\tau)$ the sets $D^{-1}(N \times M_i(\tau))$ are empty.

And, since $\text{codim } N \times R_\mu(\tau) = \dim R_\mu(\tau)$, $D^{-1}(N \times R_\mu(\tau))$ has $\dim 0$;

i.e. is a discrete set. But recall that $R_\mu(\tau)$ is a neighborhood of $R_\mu(\tau)$. Hence the intersection of the discrete set with the closure of $R_\mu(\tau)$ is finite, i.e., the number of N.E. of the τ -subgame is finite.

3.3. Completion of the Proof

We have shown that for $\Gamma = (e, \tilde{e}, u)$, if $(e, \tilde{e}) \in \mathcal{D}$ and $u \in \mathcal{D}(e, \tilde{e})$

then:

(i) prices are fully revealing in period 1 at any p.N.E. of Γ

(equivalently N.E. of Γ^*)

(ii) the set of 1st period moves in the p.N.E.'s of Γ is finite.

To strengthen (ii) into finiteness of p.N.E.'s repeat the argument used

for (ii) with D defined not only on the 1st but also the 2nd period moves, i.e. on strategy sets of Γ^* . We avoided doing this in order not to blow up an already cumbersome notation.

It remains to check that the set of p.N.E. of $\Gamma = \text{N.E. of } \Gamma$. This follows from (i) above and Proposition 5 (augmented) with Remark 6) of [1].

Remark 2. We forced the agents to put up all of their commodities for sale. This is not essential. In the more general "bid-offer" model [15] the same theorem would hold (by an identical argument but twice the notation). Adding inventorying also does not affect it. In general, for any smooth strategic-game which is deterministic in spirit,¹ i.e., has a finite number of N.E.'s in Γ^* , the theorem will go through with only one extra stipulation: that the moves of period 1 which are not fully revealing form a submanifold F of codimension ≥ 1 . For then generically the N.E. set would "miss" F . Even when the N.E. set is not finite it is typically a finite union of submanifolds G_1, \dots, G_k each of which has $\text{codim} \geq 1$ ([6]). But then $G_1 \cap F$ will be lower dimensional than G_1 given transversal intersection. Thus "most" N.E.'s (those in $G_1 \setminus F$) will still be fully revealing. The smoothness of the game (i.e. of the moves-to-outcome map) and the condition that $\text{codim } F \geq 1$ both seem likely in any model conceived in the Cournotian spirit. To that extent our results are robust.

Remark 3. We have assumed, in the definition of an N.E., that the strategic choice of the agents lead to jointly measurable moves. This seems to go against the very spirit of a noncooperative game with independent decision-

¹It turns out that a large class of smooth games do yield this when N is non-atomic (see [5]). The argument in [5] is for a simpler setting than Γ^* but we suspect that it could be carried over.

makers. However a model can be described in which measurability is restored after an initial non-measurable choice (see []). This in turn makes the assumption more viable.

Remark 4. If we refined $\hat{\Gamma}^n$ by allowing agents to observe (modulo null sets) the entire measurable function b of 1st period moves, this would still leave the set of p.N.E. of Γ^* unaffected (See (4)).

4. SOME EXAMPLES

In the first three examples there are three types of agents and two goods, a commodity good and a money. There are two time periods and two states of nature. In the second state of nature the commodity has no value to any trader. There is a continuum $(2,3]$ of identical agents $\alpha \in (2,3]$ called sellers who each own 20 units of the good in each period and no money. They have utility only for money. They must always put all their goods on sale in our simple Shapley-Shubik game and, since they have no money to bid, we can suppress their choice of actions from our analysis of the strategic game.

The first two types of traders have the same utility functions

$$(1) \quad u^i = \frac{1}{2}(A \log x_1^i + w_1^i + B \log \frac{v_1^i}{x_1^i} + \frac{v_1^i}{w_1^i}) + \frac{1}{2}(w_2^i + \frac{v_1^i}{w_2^i})$$

$$= \left[\frac{1}{2}(A \log x_1^i + w_1^i) + \frac{1}{2} \frac{v_1^i}{w_1^i} \right] + \left[\frac{1}{2}(B \log \frac{v_1^i}{x_1^i} + \frac{1}{2} \frac{v_1^i}{w_2^i}) \right] = \Pi + \hat{\Pi}$$

where x_1^i (\hat{x}_1^i) is the consumption of the good in state 1 at time one (two) and w_s^i (\hat{w}_s^i) is the holding of money during time period one (two) in state s , $s = 1$ or 2 . Each of these traders has an endowment vector $(0, M)$ in both periods. The trader(s) of type 1 can distinguish between states $s = 1$ and $s = 2$, while the trader(s) of type 2 are uninformed.

Example 1: The Money Quantity Bid Model: Perfect Competition

We assume that there is a continuum of agents $\alpha \in (0,1]$ of the first type and also a continuum of the second type $\alpha \in (1,2]$. The simplest market clearing mechanism model, which may be regarded as "unrealistic," but has the virtue of being well-defined and simple, is where individuals bid a fixed amount of money in each period and obtain whatever quantity allotted by the market price that is formed. For now we shall not allow the inventorying of either good (including the money). In the first period the uninformed traders will each bid a single amount b in both states of nature, while the informed traders will bid state dependent amounts b_1 and b_2 . If we have not made a degenerate choice of utilities, the resulting prices p_1 and p_2 will, according to our theory, be different and so will reveal the state of nature to the uninformed traders in period 2.

Since there is a continuum of traders of each type and the utilities are separable between time periods, our introductory remarks imply that a player α will make his first period move simply to maximize his first period payoff, assuming correctly that he can have no effect on his second period payoff, or on the first period price. As the good is of no value in state 2 we may set $b_2 = 0$. Figure 1 shows market clearance and price formation in states 1 and 2 in period 1.

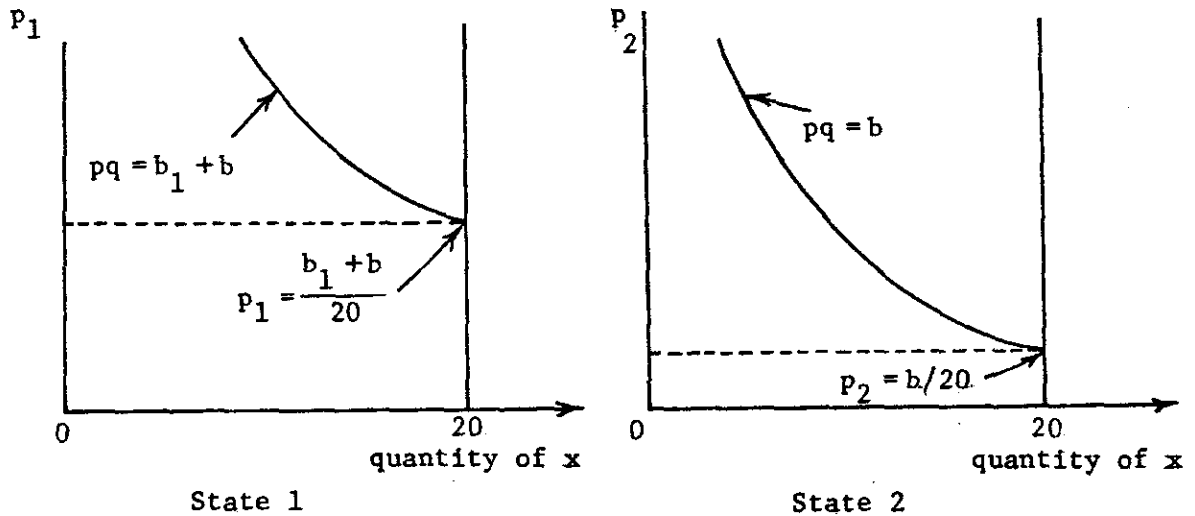


FIGURE 1

The first period payoffs to the informed traders are:

$$(2) \quad \Pi^I = \frac{1}{2} \left[A \log \frac{b_1}{p_1} + M - b_1 \right] + \frac{1}{2} M = \frac{1}{2} \left[A \log \frac{b_1}{p_1} - b_1 \right] + M .$$

The first period payoffs to the uninformed traders are:

$$(3) \quad \Pi^U = \frac{1}{2} \left[A \log \frac{b}{p_1} + M - b \right] + \frac{1}{2} (M - b) = \frac{1}{2} A \log \left[\frac{b}{p_1} - b + M \right]$$

where $p_1 = (b_1 + b)/20$, $p_2 = b/20$ but each infinitesimal player treats p_1 and p_2 are fixed. Agent optimization gives

$$(4) \quad \frac{A}{b_1} = 1$$

and

$$(5) \quad \frac{A}{2b} = 1 .$$

Suppose $A = B = 10$. Then we have $b_1 = 10$, $b = 5$, $p_1 = 3/4$ and $p_2 = 1/4$, $x_1^I = b_1/p_1 = 13-1/3$, $x_1^U = b/p_1 = 6-2/3$, $x_2^I = 0$, $x_2^U = b/p_2 = 20$ and hence,

$$(6) \quad \Pi^I = 5 \log 13\frac{1}{3} - 5 + M$$

$$(7) \quad \Pi^U = 5 \log 6\frac{2}{3} - 5 + M .$$

We may leave off the M to see the gains from trade.

In the second period both traders will be informed since

$p_1 = 3/4 \neq 1/4 = p_2$. In that case their second price payoffs are:

$$(8) \quad \tilde{\Pi}^I = \tilde{\Pi}^U = \frac{1}{2} \left(B \log \frac{\tilde{b}_1}{p_1} - \tilde{b}_1 \right) + M - \frac{1}{2} \tilde{b}_2$$

Of course if state 1 occurs in period 1, then by definition it occurs also in period 2. We find that $\tilde{b}_2 = 0$ for both traders and $\tilde{b}_1 = 10$,

$p_1 = 1$, $p_2 = 0$, and

$$(9) \quad \tilde{\Pi}^I = \tilde{\Pi}^U = 5 \log 10 - 5 + M .$$

Note again that the prices reveal the information, which is already known, anyway.

We have found the equilibrium outcomes and payoffs; we leave to the reader the full specification of the equilibrium strategies. It is clear that

$$(10) \quad \Pi^I + \tilde{\Pi}^I - (\Pi^U + \tilde{\Pi}^U) = 5 \log 2 > 0 .$$

Observation 1. We have shown by example here that it is easy for a whole class of small traders to gain from extra information even though it is revealed by the prices formed.

Before the other examples are presented, we comment briefly upon other market mechanisms. Several have been suggested. In particular

buyers could specify quantities to be bought at any price, or with an upper bound on price; sellers could announce price in advance. Or buyers or sellers could announce whole functions as strategies.

The announcing of whole functions appears to be far less realistic than the money bid we have suggested. But even so it can be considered provided an explicit mechanism for forming price under all circumstances is given.

Figures 2a and 2b illustrate some of the problems in specifying the mechanism for the price-quantity model studied by Dubey and Shubik [] and Dubey [], in the context of differential information. The bid of the informed is p_1^1 , x_1^1 in state 1 and nothing in state 2, the bid of the uninformed is $p^2 x^2$ in either state. The supply is 0 below

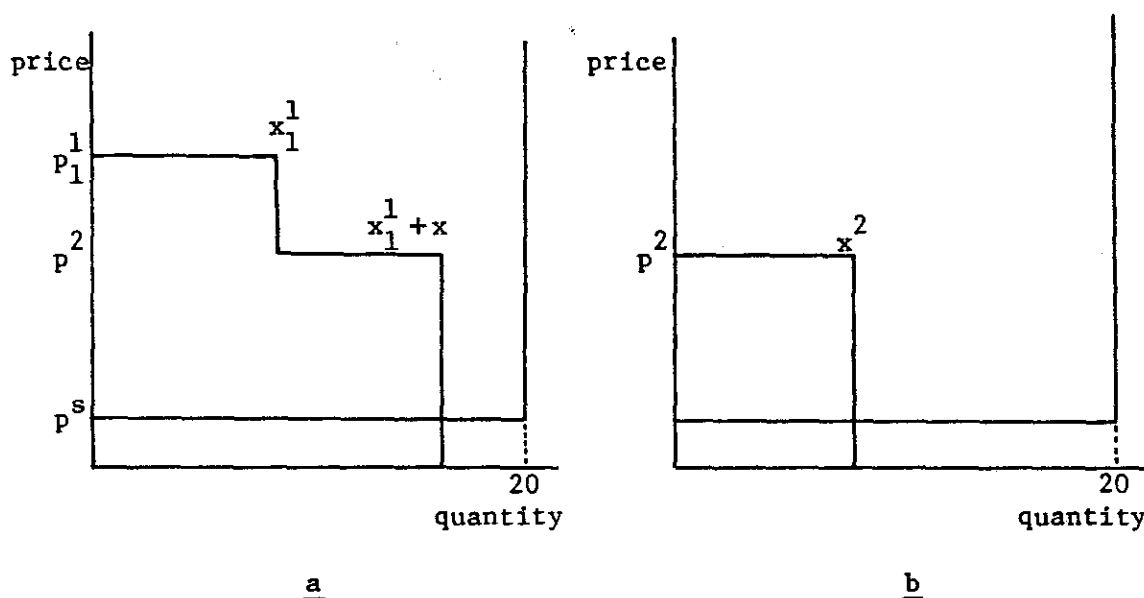


FIGURE 2

p^2 , 0 to 20 at p^s and above. Using the conventions suggested by Dubey and Shubik prices can be formed in each state and qualitatively the same results concerning the value of information can be established. We expect however to encounter robust sets of games with no equilibria and sales with different prices for the same good.

Example 2. The Purchase of Information

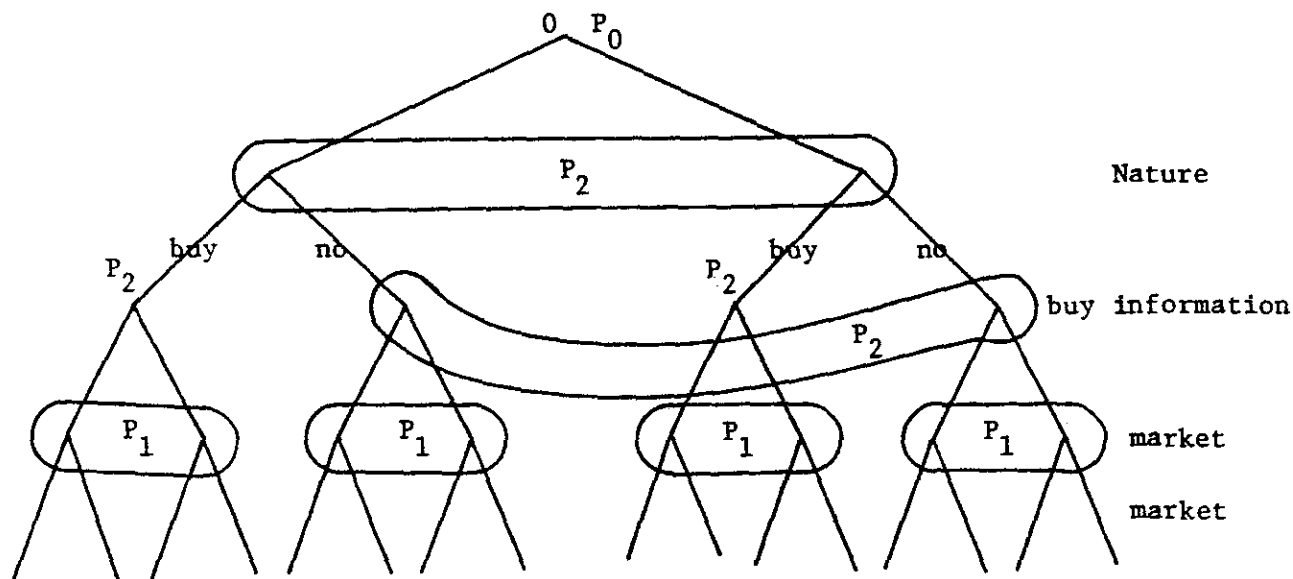


FIGURE 3

In this game tree, after Nature has moved any member of P_2 can choose to pay Δ to buy information about Nature's move. Then P_2 bids. P_1 is not informed of P_2 's bid but he does know if P_2 has bought information or not (we could model this the other way; either case is reasonable).

Let us consider two cases for the cost Δ of information:

$$\Delta < 5 \log 2$$

$$\Delta > 5 \log 2$$

As we have seen, if no trader of type 2 purchases information, then the first period market yields a payoff of $5 \log 13\frac{1}{2} - 5 + M$ to the informed and $5 \log 6\frac{2}{3} - 5 + M$ to the uninformed. Regardless of whether information has been purchased, the second period payoff is $5 \log 10 - 5 + M$. If the cost Δ of information is greater than $5 \log 2$, then this is a

Nash equilibrium--no agent will purchase information. On the other hand, for $\Delta < 5 \log 2$ each agent will be tempted to purchase the information. Now suppose all but one of the continuum type 2 agents has indeed purchased information. Each trader would then be earning $5 \log 10 - 5 + M$ in the first period. If the one remaining trader chose not to purchase the information, he would earn:

$$(11) \quad \text{Max}_b \frac{1}{2} \left(10 \log \frac{b}{1} - b \right) + M - \frac{1}{2}b .$$

Solving his first order condition $5/b = 1$, we get that his expected utility is $5 \log 5 - 5 + M$, hence if $\Delta < 5 \log 2$ he will also purchase the information. If $\Delta < 5 \log 2$ the only (symmetric) equilibrium occurs where everybody purchases information. If $\Delta > 5 \log 2$ the only (symmetric) equilibrium occurs where nobody purchases information.

Let us now consider the game in which neither the type 1 player nor the type 2 player is initially informed, but each one can purchase information at a price $5 \log 2 < \Delta < 10 \log 2$. If nobody purchases information, then the payoffs in both periods will be the same: each player will act to

$$(12) \quad \text{Max}_b \frac{1}{2} \left(10 \log \frac{b}{p} - b \right) + M - \frac{1}{2}b$$

giving the first order condition

$$(13) \quad \frac{5}{b} = 1$$

so that $b = 5$, $p = 1/2$ and the payoff to each player is $5 \log 10 - 5 + M$ in both periods. A player who purchased information could make

$$(14) \quad 2 \operatorname{Max}_{b_1} \frac{1}{2} \left(10 \log \frac{b_1}{1/2} - b_1 \right) + M = 2[5 \log 20 - 5 + M]$$

for a gain of $2[5 \log 2] = 10 \log 2$. Now for $5 \log 2 < \Delta < 10 \log 2$ it is evident that every player will find it profitable to buy information; on the other hand once every player has purchased the information it is no longer so valuable (since it will then be revealed anyway in the second period). For $0 < \Delta < 5 \log 2$ there is a Nash equilibrium in which every player purchases information.

Observation 2. If information is available for sale it may be bought; for sufficiently low but strictly positive prices it always (generically) will be by those agents who don't already know it. Depending on the circumstances, there may be intermediate levels of the price of information for which the symmetric pure strategy equilibrium is destroyed.

Example 3: A Two Buyer Model

Observation 3: If the number of traders is finite it is possible that they will choose to conceal information in early markets if greater profits are to be made later. If all traders are small this will not be so since each, if acting alone could benefit immediately without influencing price, but if all do so they disclose their information and lose future benefits.

Figure 4 shows the game we study in extensive form. Player 1 can pick strategies which will disclose his information, and by doing so may make a higher payoff in the first period. He also has the choice of acting as though he were uninformed. By doing so he earns less in the first period but does not disclose information about the state of Nature.

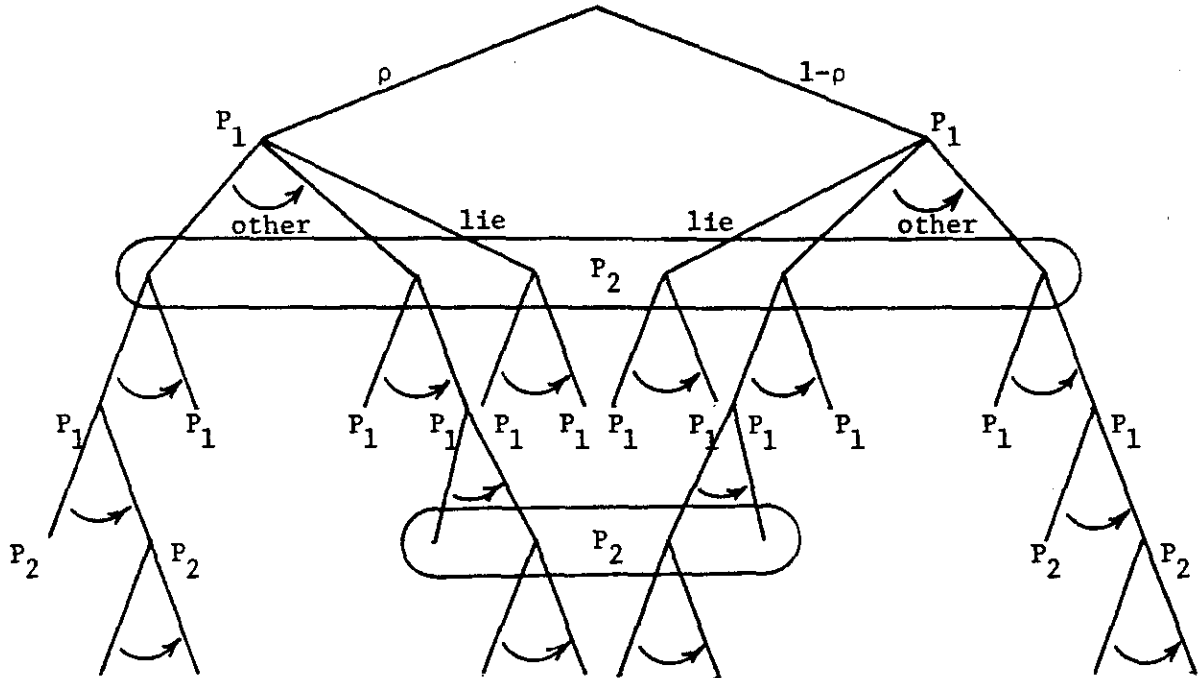


FIGURE 4

Suppose for example we assume that there are two periods and the market structure and preferences in the first period are as in example 1. In the second period the commodity for consumption in state 1 is 10 times as valued as in the first period; $A = 10$, $B = 100$. Furthermore the supply in each period is the same and the good cannot be inventoried by the consumers. The utility function of a traders can be written as:

$$\begin{aligned}
 (15) \quad U^i &= \left[\frac{1}{2}(10 \log x_1^i + w_1^i) + \frac{1}{2}w_2^i \right] + \left[\frac{1}{2}(100 \log \hat{x}_1^i + \hat{w}_1^i) + \frac{1}{2}\hat{w}_2^i \right] \\
 &= \Pi^i + \hat{\Pi}^i .
 \end{aligned}$$

First suppose there are only one period with one trader with information and one without. The payoffs are as follows:

$$(16) \quad \Pi^1 = \frac{1}{2} \left\{ 10 \log 20 \frac{b_1}{b+b_1} - b_1 \right\} + M$$

$$(17) \quad \Pi^2 = 5 \log 20 \frac{b}{b+b_1} - b + M .$$

As there are only two traders they each influence price where

$$p_1 = (b+b_1)/20 \quad \text{and} \quad p_2 = b/20 .$$

A little calculation gives

$$(18) \quad b = \frac{10}{\sqrt{2}(1+\sqrt{2})} = 2.929 , \quad b_1 = \frac{10}{(1+\sqrt{2})} = 4.142$$

$$p_1 = \frac{1}{2\sqrt{2}} = .3536 \quad p_2 = \frac{1}{2\sqrt{2}(1+\sqrt{2})} = .1465$$

$$x_1^1 = \frac{20\sqrt{2}}{1+\sqrt{2}} = 11.716 \quad x_1^2 = \frac{20}{1+\sqrt{2}} = 8.284$$

$$\Pi^1 = 10.234 + M \quad \Pi^2 = 7.643 + M$$

Incomplete-Incomplete Information

If both were uninformed $b = 5/2$ for all $p_1 = p_2 = 1/4$

$$(19) \quad x_1^1 = x_1^2 = x_2^1 = x_2^2 = 10$$

$$\begin{aligned} \Pi^1 &= 5 \log 10 - 5/2 & \Pi^2 &= 5 \log 10 - 5/2 \\ &= 9.013 + M & &= 9.013 + M \end{aligned}$$

Complete-Complete Information

If both were informed $b_1 = 5$, $b_2 = 0$ for all $p_1 = 1/2$

$$(20) \quad p_2 = 0 \quad x_1^1 = x_1^2 = 10$$

$$\begin{aligned} \pi^1 &= 5 \log 10 - 5/2 + M & \pi^2 &= 5 \log 10 - 5/2 + M \\ &= 9.013 + M & &= 9.013 + M \end{aligned}$$

We may construct Table 1 showing the duopsony gains from trade solutions to the one period strategic market game.

		Type 2	
		Informed	Uninformed
Type 1	Informed	9.013, 9.013	10.234, 7.643
	Uninformed	7.643, 10.234	9.013, 9.013

TABLE 1

Four One Stage Games: Duopsony Payoffs

A multiplying of these numbers by 10 yields the payoffs in the second period subgames. It is straightforward to observe that if the informed player chooses to earn 10.234 and thereby reveal his information in the first period, he will earn 90.13 in the second period. If, on the other hand, he plays as if he were uninformed in the first period the totals earned are 9.013 and 102.34. The full payoff with disclosure is 100.364 and without is 111.353. It is important to note that it does not matter that the player of type 2 knows that the other player lies; there is nothing he can do about it. He gets no information on Nature.

This result is robust; we could have had k traders of each type, as long as each had influence on price. With a continuum of traders a single individual who is informed is tempted to save money by not buying worthless goods in state 2. All of them would do this, the price would change and the information would be signalled.

Observation 4. We could easily have introduced uncertainty in the knowledge of traders of type 2 concerning whether or not traders of type 1 were informed. The game tree is similar to that in Figure 3 with the information purchase replaced by a move of Nature.

Observation 5. If the payoffs had been identical in each period then traders of type 1 would have been indifferent between revealing information in period 1 or 2. They could obtain $10.234 + 9.013$ with immediate revelation or $9.013 + 10.234$ with delayed revelation.

Observation 6. If information were for sale in period 1 it would be of considerable value and its availability and not the price mechanism could promote revelation. A reasonable model of this would call for an explicit formulation of the "trustworthiness" of the information and its speed of diffusion, i.e. how fast can people buy it and act upon it. Services such as Disclosure Incorporated indicate the importance of this for obtaining S.E.C. filings.

Example 4: Let us consider an economy in which every agent i has a utility $u^i(x, \tilde{x})$ of the form $u^i = u^i(x + \tilde{x})$. Each agent will be allowed to put up for sale however many goods he wants and to bid money for all the commodities. Inventorying will also be allowed. Since the form of the utility function implies that there is nothing to be gained by having a commodity today rather than waiting until period 2, one might suppose that the agents would wait until the first period prices had revealed all the information before doing nearly all their trading in period 2. In that case the final allocation would apparently be nearly the same as the fully revealing R.E.E. of the one period economy obtained by combining the endowments of the two time periods into one. However, we shall show not only that the Nash equilibrium allocation of the strategic market (bid-sell) game is different, but moreover we shall show that in our example it (ex-ante) pareto dominates the R.E.E. allocation. This of course demonstrates the failure of the first welfare theorem for a R.E.E. allocation. It can be in the interest of all to trade before their information is complete.

Let there be two types of agents $\alpha \in (0, 1]$, $\beta \in (1, 2]$, two commodities (x and money) and 4 states of nature. Let each $\alpha \in (0, 1]$ agent have utility:

$$U^\alpha = \frac{1}{4} \sqrt{x_1^\alpha + \tilde{x}_1^\alpha} + \frac{1}{4} \sqrt{(w_2^\alpha + \tilde{w}_2^\alpha) \frac{1}{2}} + \frac{1}{4} \sqrt{w_3^\alpha + \tilde{w}_3^\alpha} + \frac{1}{4} \sqrt{x_4^\alpha + \tilde{x}_4^\alpha}.$$

For $\beta \in (1, 2]$, let

$$U^\beta = \frac{1}{4} \sqrt{x_1^\beta + \tilde{x}_1^\beta} + \frac{1}{4} \sqrt{(w_2^\beta + \tilde{w}_2^\beta) \frac{3}{2}} + \frac{1}{4} \sqrt{(w_3^\beta + \tilde{w}_3^\beta) \frac{4}{3}} + \frac{1}{4} \sqrt{x_4^\beta + \tilde{x}_4^\beta}$$

In states one and four, only the good has utility, in states two and three only money has utility. Let the agents of type 1 distinguish odd and even

states, and let those of type 2 distinguish $s \in \{1,2\}$ from $s \in \{3,4\}$.

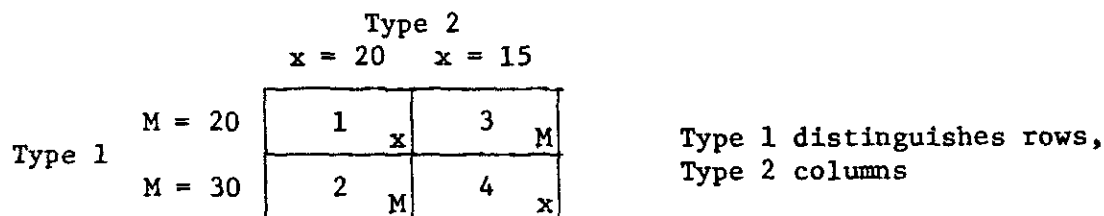


FIGURE 5

Let the endowment of type 1 agents be 20 units of money for $s \in \{1,3\}$ and 30 units of money for $s \in \{2,4\}$ in both periods and nothing else.

Let the endowment of type 2 agents be 20 units of x for $s \in \{1,2\}$ and 15 units of x for $s \in \{3,4\}$ in both periods, and nothing else.

It is easy to see that once the state of nature has been revealed, no further trade will take place. Hence we need only find (b_T, b_B) , the money bids of a trader $\alpha \in (0,1]$ if he sees top or bottom respectively, and s_L, s_R , the amounts of x offered for sale by each $(1,2)$ if he sees left or right, respectively. Thus $\alpha \in (0,1]$ acts to

$$(21) \quad \text{Max}_{b_T, b_B} \frac{1}{4} \sqrt{\frac{b_T}{P_1}} + \frac{1}{4} \sqrt{(60 - b_B) \frac{1}{2}} + \frac{1}{4} \sqrt{40 - b_T} + \frac{1}{4} \sqrt{\frac{b_B}{P_4}}$$

such that $0 \leq b_T \leq 20$

$0 \leq b_B \leq 30$

while each agent $B \in (1,2]$ acts to:

$$(20) \quad \text{Max}_{s_L, s_R} \frac{1}{4} \sqrt{40 - s_L} + \frac{1}{4} \sqrt{p_2 s_L \frac{3}{2}} + \frac{1}{4} \sqrt{p_3 s_R \frac{4}{3}} + \frac{1}{4} \sqrt{30 - s_R}$$

such that $0 \leq s_L \leq 20$

$$0 \leq s_R \leq 15$$

One can easily verify that $b_T = 20$, $b_B = 30$, $s_L = 20$, $s_R = 15$, $p_1 = 1$, $p_2 = 3/2$, $p_3 = 4/3$, $p_4 = 2$ comprise a Nash equilibrium, since for each $s \in \{1, 2, 3, 4\}$, $b_s/s_s = p_s$ and the first order conditions:

$$(23) \quad b_T = \frac{40}{1 + p_1}$$

$$b_B = \frac{60}{1 + \frac{1}{2}p_4}$$

$$s_L = \frac{40p_2}{\frac{3}{2} + p_2}$$

$$s_R = \frac{30p_3}{\frac{4}{3} + p_3}$$

are satisfied. The prices are fully revealing, and in the second period there is no further trade. The expected utility of any type 1 agent is

$$(24) \quad U^\alpha = \frac{1}{4}\sqrt{20} + \frac{1}{4}\sqrt{15} + \frac{1}{4}\sqrt{20} + \frac{1}{4}\sqrt{15} \quad \text{while for } \beta \in (1, 2] \text{ we get}$$

$$(25) \quad U^\beta = \frac{1}{4}\sqrt{20} + \frac{1}{4}\sqrt{45} + \frac{1}{4}\sqrt{\frac{80}{3}} + \frac{1}{4}\sqrt{15}$$

By contrast, in the R.E.E. (where prices together with each agent's information fully reveal the state to him) we get

$$U^\alpha = \frac{1}{4}\sqrt{40} + \frac{1}{4}\sqrt{30}$$

$$U^\beta = \frac{1}{4}\sqrt{40} + \frac{1}{4}\sqrt{30}$$

We see that the Nash allocation pareto dominates the R.E.E. allocation. In our example information is socially harmful, although if any one agent could purchase it before time 1 he would profit from it.

Example 5: In this example we demonstrate that a definition of equilibrium which allowed agents to use prices to calculate their income but not to infer information leads to violations of individual rationality. Agents may simply give away their money.

Let there be two states of nature, two goods, and two agents, both with the same utility functions:

$$U(x_{s_1}, m_{s_1}, x_{s_2}, m_{s_2}) = \frac{1}{2} (m_{s_1} + \alpha x_{s_1}) + \frac{1}{2} (m_{s_2} + 2x_{s_2}).$$

Let agent 1 own 1 unit of the x good in both states and let agent 2 own $\bar{M} \geq 1$ units of the m good in both states. Notice that the sale of the x good is in effect a bet between the agents over whether the state s_1 will occur, or not. Let agent 1 know which state has occurred, and let agent 2 be uninformed. Let the price ratio of x to m be $p_{s_1} = 1$ and $p_{s_2} = 2$. If agent 2 ignores the informational content of prices, then this is a competitive equilibrium: in state 1 agent 2 thinks he is making a fair bet and will purchase 1 unit of x ; agent 1 on the other hand knows that x is worthless and gladly sells his unit holding of x . In state 2 agent 2 demands no x and agent 1 will demand precisely his initial endowment. Clearly agent 2, under this definition of equilibrium ends up betting only when he is sure to lose.

We suggest that our observations pose no paradoxes. They do indicate that further work may require (1) explicit care in formulating the functioning of market mechanisms, (2) the specification of the relative speeds of information diffusion and market reaction, (3) the modeling of the sale of information and its evaluation, (4) the facing up to the possibility that the assumption of the continuum of traders is merely a mathematical convenience which allows us to obtain some insights of interest, but is of limited application as an approximation to many of the phenomenon of interest.

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