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ASSET MARKETS, GENERAL EQUILIBRIUM AND THE NEUTRALITY OF MONEY

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ABSTRACT

When government liabilities (including money) are held in private portfolios only as stores of value, and do not provide additional benefits (as liquidity services), the real variables in an economy with uncertainty are not affected by the government's trading in assets. There are also policies which alter the money supply through taxes or subsidies, and affect the price of money without changing real variables.

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AND THE NEUTRALITY OF MONEY

by

Christophe Chamley* and Heraklis M. Polemarchakis**

1. Introduction

The ability of monetary policy to affect real variables in the economy, is an important issue and has been the subject of numerous theoretical and empirical studies. The most useful theoretical framework to address this problem is a portfolio model of asset markets where the demand for money or other government liabilities can be explained by a model of rational portfolio choice (Tobin, 1958, 1969). These assets may also provide services (transaction, liquidity). However, we will make here the (not very realistic) assumption that these additional services do not exist.

Monetary policy alters the relative supplies of the available assets. As their prices adjust to maintain market equilibrium, the discrepancies which appear between the market prices of physical assets and their replacement costs, induce variations of their supplies, and therefore real effects. In this case, money is not neutral, at least in the short-run.

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It is well known that the effect of monetary policy depends on the method of injection of new money into the economy. Two procedures can be used. When the government issues money it either buys assets (open-market operations), or it distributes subsidies (or decreases taxes).

We show here that the first method of issuing money, i.e., open-market operations, has no effect on real variables in a general equilibrium with market clearing. The argument which can also be applied to the management of the public debt, is presented in a completely general framework in the next section. The result is similar to the Modigliani-Miller theorem in corporate finance.

It is important to emphasize that although government trading in existing assets is neutral, the creation of new types of assets (like indexed bonds) is, in general, not neutral. This is illustrated in the appendix by an example of a model of heterogenous overlapping generations. This example also shows that general equilibrium models where money is held solely for its portfolio properties do exist,and that the theoretical discussions in the paper is not void. Also the introduction of additional markets needs not lead to a Pareto superior contingent allocation. The argument in Hart [1975] indicates that it may even lead to a Pareto inferior one.

In the third section we examine variations of the money supply accompanied by subsidies or taxes. It is well known that in a non-stochastic economy, an unexpected jump of the money stock with subsidies to individuals proportional to their cash balances, induces a proportional jump of the price level and does not affect the real quantity of money or real variables. Also, the only value of an expected jump of the money supply, which is neutral, is equal to zero.[1] In the stochastic case, this result does not apply. There exist contingent anticipated variations of the money supply which do not alter the set of available assets and are neutral. Of course, these policies affect the price of money; in the fourth section, we show how they can be used to offset the effect of open-market operations on the price of money, as Wallace (1981) has already shown in a more restricted framework.

In the last section, we present some concluding comments.

2. Open Market Operations

The government determines the composition of its portfolio by trading between different assets or liabilities on the corresponding markets. In order to analyse the effect of these open-market operations in general equilibrium, we consider a simple model of a one-good economy. This good can either be consumed or used as capital input, in which case it produces a random return. In each period t, the economy is subject to random shocks which are denoted by the elements s_t of sets S_t. These sets have a finite number of elements n_t. Each element s_t determines the distribution of labor endowments of all individuals living at time t, and the total return for each unit of capital r_t.[2]

All agents living at time t know the probabilities of the sequences (s_{t+k}) for the periods in their own lifetime which is finite or infinite, conditional on the state of nature s_t. They determine at time t their own optimal intertemporal program of consumption and portfolio choice for each future period, which is contingent on the value s_{t+k} $(k \geq 1)$ in these periods.

We can assume, without loss of generality, that the government has no expenditure, raises no tax, and has created a fixed amount of fiat money which is held by private agents in their portfolios because of its return properties. The price of money with respect to the unique good is represented by p_t . Denote by K_t , K_t^g , M_t the levels of the total capital in the economy, the capital owned by the government, the (nominal) quantity of money respectively, which are carried from period t to period $t+1$. The budget constraint of the government at time t is given by:

$$(1) \qquad p_t(M_t - M_{t-1}) + r_t K_{t-1}^g = K_t^g$$

The origin of time can be chosen arbitrarily, and is fixed at period 1 . In this period, the government has an open-market policy which determines the level of K_t^g (positive or negative) as a function of s_t for all values of t greater than one. This policy is known by the private agents. The supply of money M_t , is determined in equation (1) for all periods.

We will consider only allocations of resources in equilibrium. Since the purpose of this paper is to show the invariance of an equilibrium under different policies, we do not think that it is necessary to characterize completely an equilibrium, or to prove its existence. However, we present in the appendix, an example of general equilibrium where money is held for its portfolio properties, in order to show that the argument is not void.

The equilibrium in the economy is defined by the functions $c_t(s_t)$, $K_t(s_t)$, $M_t(s_t)$, $p_t(s_t)$ which represent respectively the consumption levels for each of the individuals living at time t , the aggregate capital stock, the quantity and the price of money.[3] These functions satisfy the following restrictions: private consumption programs are supported by portfolio programs which are state contingent; in each period, the sums of the amounts of capital and real quantity of money in all individual's portfolio are equal to $K_t - K_t^g$ and $p_t M_t$, respectively. The equilibrium depends on the government's policy, and will be represented in abbreviated form by the sequence

$$(c_t, K_t, M_t, p_t, K_t^g) \quad t \geq 1$$

which is contingent on s_t.

Let us assume that the economy is in equilibrium under the following government's policy: the quantity of money is fixed for all t and equal to $\overline{M}$, and K_t^g is equal to zero. In the first period the government announces a new policy described by the contingent sequence (K_t^g), for t greater than 1. The following proposition shows that the effect of this policy on the real allocation of resources (c_t, K_t) is nil.

Proposition 1: Assume that the sequence $(\overline{c}_t, \overline{K}_t, \overline{M}, \overline{p}_t, 0)$ describes an equilibrium. Then the sequence $(\overline{c}_t, \overline{K}_t, M_t, p_t, K_t^g)$ describes an equilibrium where M_t is determined by (1) and p_t is given by

$$(2) \qquad \begin{cases} p_1 = \overline{p}_1 \\ \\ \dfrac{p_{t+1}}{p_t} = (1 - \alpha_t)\dfrac{\overline{p}_{t+1}}{\overline{p}_t} + \alpha_t r_{t+1}, \quad \text{for} \quad t \geq 1 \\ \\ \text{with} \quad \alpha_t = \dfrac{K_t^g}{K_t^g + \overline{p}_t \overline{M}} = \dfrac{K_t^g}{p_t M_t} \end{cases}$$

To prove the proposition, we show that individuals "undo" the actions of the government, and that the asset markets are still in equilibrium without change of the consumption levels of any individual. Consider an individual i living both in periods t and $t+1$, and who choses his portfolio for period t. This choice is made in the beginning of his life and is contingent on s_t. Assume that in the first equilibrium $(K_t^g \equiv 0)$, this portfolio is given by $(\bar{K}_t^i, \bar{M}_t^i)$. Construct now the portfolio (K_t^i, M_t^i) as follows:

(3) $$M_t^i = \frac{1}{1-\alpha_t} \frac{\bar{p}_t}{p_t} \bar{M}_t^i$$

(4) $$K_t^i = \bar{K}^i - \frac{\alpha_t}{1-\alpha_t} \bar{p}_t \bar{M}_t^i ,$$ where p_t and α_t are defined in (2).

The two portfolios are related as follows:

(5) $$p_t M_t^i + K_t^i = \bar{p}_t \bar{M}_t^i + \bar{K}_t^i ,$$

and using (2),

(6) $$p_{t+1} M_t^i + r_{t+1} K_t^i = \bar{p}_{t+1} \bar{M}_t^i + r_{t+1} \bar{K}_t^i$$

The first relation shows that the value of the portfolio $(\bar{K}^i, \bar{M}^i)$ at the price of money $\bar{p}$, is identical to the value of (K^i, M^i) at the price p. In the second relation, we see that the real return of these two portfolios (at the prices $\bar{p}$ and p, respectively), are identical. It follows that any intertemporal program of consumption chosen by individual i in the environment described by $\bar{p}(s_t)$, and supported by a program of portfolios $\bar{K}^i(s_t)$, $\bar{M}^i(s_t)$, is also attainable in the environment described by $p(s_t)$, and is sustained in this case by the portfolios $(K^i(s_t), M^i(s_t))$. The reverse is also true. It follows that if $(\overline{K}^i, \overline{M}^i)$ is optimal in the first case, (K^i, M^i) is optimal in the second.

The demand for money in the new equilibrium is determined by summing (3) over all individuals i . Using the definition of α_t in (2) , this demand M_t^D is given by:

$$(7) \qquad p_t\, M_t^D = K^g + \overline{p}_t \overline{M}$$

The supply of money is given by equation (1):

$$(8) \qquad p_t\, M_t^S = p_t\, M_{t-1}^S - r_t K_{t-1}^g + K_t^g$$

At the equilibrium, we must have $M_t^D = M_t^S$, which is equivalent to:

$$(9) \qquad \overline{p}_t \overline{M} = p_t\, M_{t-1}^S - r_t K_{t-1}^g \, .$$

This identity is proven by induction. If $M_{t-1}^D = M_{t-1}^S$, from (7):

$$(10) \qquad \alpha_{t-1} = \frac{K_{t-1}^g}{K_{t-1}^g + \overline{p}_{t-1}\overline{M}} = \frac{K_{t-1}^g}{p_{t-1}M_{t-1}^S}$$

We now use the definition of p_t in (2) to transform the right hand side term in (9):

$$p_t M_{t-1}^S - r_t\, K_{t-1}^g = (1-\alpha_{t-1})\frac{\overline{p}_t}{\overline{p}_{t-1}}\, p_{t-1}M_{t-1}^S + \alpha_{t-1} r_t p_{t-1} M_{t-1}^S$$

$$- r_t\, K_{t-1}^g$$

Replacing α_{t-1} by its value in (10), we see that this expression is equal to the left hand side term in (9).

Adding (4) over all individuals, we find that:

$$\sum_i K_t^i + K_t^g = \overline{K}t ,$$

which concludes the proof.

This result has a simple intuitive interpretation: when the government buys capital with cash, the contingent price of money is altered from p_t to $\overline{p}_t$ where $\overline{p}_t$ satisfies (1) and α_t is equal to $K_t^g / p_t M_t$. The "new" money is a composite asset between "old" money and capital, where the "share" of capital in the determination of the rate of return on money is equal to the ratio between the government's capital and the real quantity of money (in the new equilibrium). When the government buys capital with money, for example, individuals maintain their real contingent consumption plans by reducing their capital and increasing proportionately their holding of money. This increase of the demand for money is equal to the increase of the supply, and the real allocation of resources is unaffected.

The above proposition calls for a few remarks. It should be emphasized that open-market operations do not affect the price of money in the first period. The same result could be extended to operations announced in period 1 and implemented in period t . These policies would not alter the contingent price of money between periods 1 and period t .

We have considered a very simple model with two assets. The proposition would remain valid for an economy with different types of government liabilities. It is intuitive that in this case, open-market operations would only affect the prices of the liabilities traded by the government.

We have assumed that there are no restrictions on individual portfolio choices. If short-sales are bounded or not feasible $(K_t^i \geq b^i$, for some $b^i)$, the above result clearly is not valid. If there are no short-sale restrictions, the proposition is valid whatever the values of K_t^g, which can be greater than K_t.

3. Monetary Policy and Transfers

When the government expenditures are fixed, the revenues generated by the creation of money are used by the government either to purchase capital or other assets, or to lower taxes (or increase subsidies). We saw in the last section that the first type of operation has, in general, no effect on the real variables of the economy. This neutrality surely does not apply in general, for the second type of policy.

Consider, for example, an economy with two period overlapping generations, capital and money. Initially the levels of government expenditures and taxes are equal to zero. In period one, the government announces a one-time increase of the money supply next period, combined with lump-sum subsidies. If this policy has no incidence on individual's consumption, it should not alter the real quantity of money in period one or in period two.[4] A variation of the nominal quantity of money in the second period would affect (inversely) the price of money, and the rate of return on money between the first two periods. In general, we can expect contingent monetary policies to create new assets and not to be neutral.

In order to characterize some neutral policies, we now describe an equilibrium in the economy by the functions of s_t, $(c_t, K_t, M_{t-1}, p_t, w_t)$. The level of K_t^g is assumed without loss of generality, to be identically equal to zero, and w_t represents a vector of transfers from the government to all individuals living in period t . The government budget constraint in period t takes the form:

$$p_t\,(M_t - M_{t-1}) = W_t \ ,$$

where W_t is the sum of all net lump-sum transfers from the government to individuals.

We also introduce the following matrices:

$$A_t(s_t) = [a^t_{si}] \ , \text{ with } \ a^t_{si} = \frac{u'^i_{tt+1}(s)}{u'^i_{tt}(s_t)}$$

the terms $u'^i_{tt}(s_t)$ and $u'^i_{tt+1}(s_{t+1})$ represent the marginal utility of consumption of individual i in period t and $t+1$, considered in period t , in state s_t and s_{t+1} , respectively. The index i covers the set of individuals living in both periods t and $t+1$.[5]

In the determination of neutral policies, we consider only the policies which do not affect the real quantity of money.[6] We call them strongly neutral policies.

Proposition 2: Consider the following properties:

(i) $(\bar{c}_t, \bar{K}_t, \bar{M}_t, \bar{p}_t, \bar{w}_t)$, and $(\bar{c}_t, \bar{K}_t, M_t, p_t, w_t)$ describe two equilibria with $M_1 = \bar{M}_1$, $p_1 = \bar{p}_1$, $w_1 = \bar{w}_1$, $p_t M_t = \bar{p}_t \bar{M}_t$ for $t \geq 2$.

(ii) The columns of the matrices $A_t(s_t)$ generate affine spaces of dimension $n_{t+1} - 2$, respectively.

(iii) Money and capital are not perfect substitutes.

These properties imply that:

$$W_t - \bar{W}_t = \beta_t \bar{p}_{t-1} \bar{M}_{t-1} \left(\frac{\bar{p}_t}{\bar{p}_{t-1}} - r_t \right) \quad , t \geq 2 \tag{11}$$

where (β_t) is a sequence of arbitrary numbers. The numbers β_t are independent of s_t but may depend on s_{t-k}, with $k \geqq 1$.

The proposition is proven by a simple spanning argument. Since the sequences $(\bar{c}_t, \bar{K}_t, \bar{M}_t, \bar{p}_t, \bar{w}_t)$ and $(\bar{c}_t, \bar{K}_t, M_t, p_t, w_t)$ describe two equilibria with the same real allocation of resources, the first order conditions between periods 1 and 2 for an arbitrary individual i, can be written

$$\begin{cases} 1 = \sum_{s \in S_2} \pi_s^i \, r_2(s) \left(\dfrac{u_{12}^{'i}(s)}{u_{11}^{'i}(s_1)} \right) \\ \\ 1 = \sum_{s \in S_2} \pi_s^i \, \dfrac{\bar{p}_2(s)}{\bar{p}_1(s_1)} \left(\dfrac{u_{12}^{'i}(s)}{u_{11}^{'i}(s_1)} \right) \end{cases} \tag{12}$$

and[7]

$$
(13) \qquad \begin{cases} 1 = \sum_{s \varepsilon S_2} \pi_s^i \, r_2(s) \left(\dfrac{u_{12}^{'i}(s)}{u_{11}^{'i}(s_1)} \right) \\[2ex] 1 = \sum_{s \varepsilon S_2} \pi_s^i \, \dfrac{p_2(s)}{p_1(s_1)} \left(\dfrac{u_{12}^{'i}(s)}{u_{11}^{'i}(s_1)} \right) \end{cases}
$$

where π_s^i represents the subjective probability of s_2, at time 1, for individual i. The vector $\pi_s^i p_2(s)/p_1(s_1)$ must be a linear combination of the vector $\pi_s^i r_2(s)$, and $\pi_s^i \overline{p}_2(s) / \overline{p}_1(s_1)$, (otherwise, by condition (iii), the vectors $(u_{22}^{'i}(s) / u_{11}^{'i}(s_1),\ s \varepsilon S_2)$ would be contained in an affine space of dimension $n_2 - 3)$. Therefore, there exists a number β such that[8]

$$
(14) \qquad \frac{p_2(s)}{p_1(s_1)} = (1 - \beta) \frac{\overline{p}_2(s)}{\overline{p}_1(s_1)} + \beta r_2(s) \ , \ s \varepsilon S_2
$$

Using the equality $p_2(s)\, M_2(s) = \overline{p}_2(s)\, \overline{M}_2(s)$, and the government budget constraint in period 2 for each equilibrium, we find that for each s in S_2 :

$$
W_2 - \overline{W}_2 = p_2(M_2 - M_1) - \overline{p}_2(\overline{M}_2 - \overline{M}_1) = \overline{p}_2\overline{M}_1 - p_2 M_1
$$

Replacing p_2 by its value in (14):

$$
W_2 - \overline{W}_2 = \beta \overline{p}_1 \overline{M}_1 \left(\frac{\overline{p}_2}{\overline{p}_1} - r_2 \right)
$$

The proof is concluded by applying the same argument for t greater than two.

In (11), the left hand side is equal to the variation of the nominal quantity of money (measured in real term) in each period. The choice of β_t determines the distribution of returns to the "new" money as a combination of the returns to the "old" money (in the first equilibrium), and the returns of capital.

The spanning argument applies only if generations are sufficiently non-homogenous. In particular, if all individuals living both in periods t and $t+1$ are identical, it is obvious that the condition (11) is much too strong, and can be replaced by a weaker equality obtained from (13).[9]

We now show that there exist strongly neutral policies. For this, we use the same method as in the proof of Proposition 1. Assume that in the equilibrium $(\bar{c}_t, \bar{K}_t, \bar{M}_t, \bar{p}_t, \bar{w}_t)$, the levels of capital and money carried by an individual i from period t to $t+1$ are equal to $\bar{K}_t^i$ and $\bar{M}_t^i$.

Define a new policy as follows: Choose a sequence of values β_t $(t \geq 2)$, where β_t is independent of $s_{t+k} (k \geq 0)$; they define the prices p_t in (14). Set $M_t = \bar{p}_t \bar{M}_t / p_t (t \geq 2)$. The portfolio of individual i in the new equilibrium (K_t^i, M_t^i) is chosen as follows:

$$K_t^i = \bar{K}_t^i$$

$$M_t^i = \bar{M}_t^i \bar{p}_t / p_t$$

This portfolio has with the price p_t, the same value as $(\bar{K}_t^i, \bar{M}_t^i)$ with the price $\bar{p}_t$:

(15) $$\overline{K}_t^i + \overline{p}_t \overline{M}_t^i = K_t^i + p_t M_t^i$$

Choose now a contingent transfer policy w_t^i , such that

(16) $$r_{t+1}\overline{K}_t + \overline{p}_{t+1}\overline{M}_t^i + \overline{w}_{t+1}^i = r_{t+1}K_t^i + p_{t+1}M_t^i + w_{t+1}^i \ , \ t \geq 1 \ .$$

By the same argument as in the previous section, the portfolio (K_t^i, M_t^i) is optimal with the new prices (p_t) and the transfers policy (w_t^i) . It is a trivial exercise to show that the sum of the transfers w_t^i is equal to the variation of the quantity of money in period t (in real terms), and that the policy of transfers defined by (16) is feasible. This discussion proves the following proposition:

Proposition 3: For any equilibrium $(\overline{c}_t, \overline{K}_t, \overline{M}_t, \overline{p}_t, \overline{w}_t)$, and any sequence β_t , where β_t may depend on s_{t-k} $(k \geq 1)$, there exists a strongly neutral monetary policy accompanied by transfers. Aggregate transfers are given by:

$$W_t - \overline{W}_t = \beta_t \overline{p}_{t-1}\overline{M}_{t-1} \left(\frac{\overline{p}_t}{\overline{p}_{t-1}} - r_t\right) \ , \ t \geq 2$$

the path of the price of money under the policy is defined by:

$$\frac{p_t}{p_{t-1}} = (1 - \beta_t) \frac{\overline{p}_t}{\overline{p}_{t-1}} + \beta_t r_t$$

In this and the previous proposition, we have implicitly assumed that there is uncertainty $(n_t \geq 2)$. Without uncertainty, money and capital are perfect substitutes; the identities $p_1 = \bar{p}_1$, $p_{t+1} / p_t = r_{t+1}$, $p_t M_t = \bar{p}_t \bar{M}_t$ imply that $M_t = \bar{M}_t$. No variation of the money supply is strongly neutral.

4. A Special Case

Using the previous results, we now show that there are policies combining open-market operations and transfers, which are neutral and do not alter the price of money. Assume that the economy is initially in the equilibrium described by the contingent sequence $(\bar{c}_t, \bar{K}_t, \bar{K}^g_t, \bar{M}_t, \bar{p}_t, \bar{w}_t)$, with $\bar{K}^g_t \equiv 0$ for all t . Consider the contingent path K^g_t . From proposition 1, we know that the path $(\bar{c}_t, \bar{K}_t, K^g_t, \tilde{M}_t, \tilde{p}_t, \bar{w}_t)$ is in equilibrium when the values of $\tilde{M}_t$ and $\tilde{p}_t$ are determined by the government budget constraint and the relation:

$$(17) \qquad \frac{\tilde{p}_{t+1}}{\tilde{p}_t} = (1 - \alpha_t) \frac{\bar{p}_{t+1}}{\bar{p}_t} + \alpha_t r_{t+1} \text{ , where } \alpha_t \text{ is given in (2) .}$$

Using Proposition 3, there exists a strongly neutral policy of money variation and transfers, such that the total transfers W_t are given by:

$$(18) \qquad W_{t+1} - \bar{W}_{t+1} = \beta_{t+1} \tilde{p}_t \tilde{M}_t \left(\frac{\tilde{p}_{t+1}}{\tilde{p}_t} - r_{t+1} \right) .$$

This policy alters the contingent sequence of money prices to p^*_t which is defined by

$$(19) \qquad \frac{p^*_{t+1}}{p^*_t} = (1 - \beta_{t+1}) \frac{\tilde{p}_{t+1}}{\tilde{p}_t} + \beta_{t+1} r_{t+1}$$

From (17) and (19), p_t^* is identical to $\overline{p}_t$ if and only if β_{t+1} is equal to $-\alpha_t / (1 - \alpha_t)$. In this case, using (17), and the definition of α_t in (2), the expression (18) can be written:

$$(20) \qquad W_{t+1} - \overline{W}_{t+1} = K_t^g \left(r_{t+1} - \frac{\overline{p}_{t+1}}{\overline{p}_t} \right)$$

We have seen that the transfer W_t can be disaggregated into (w_t) , such that the contingent sequence $(\overline{c}_t, \overline{K}_t, K_t^g, M_t, \overline{p}_t, w_t)$ describes an equilibrium.

Policies determined by the sequence (K_t^g, M_t, w_t) , which are neutral and do not alter the price of money have been analysed by Wallace (1981) in the case of models with 2-period overlapping generations and complete markets in contingent claims. Our results have been derived in a completely general framework. We see that these policies can be decomposed in two parts: Open-market operations determine the path (K_t^g) and are always neutral (provided that there is no restriction on short sales). Suitable variations of the money supply with transfers offset the effect of the open-market policies on the price of money without affecting the real allocation of resources.

In our results, condition (20) is identical with the condition (b) proposed by Wallace. However, we do not agree with him that there is a relationship between this condition and the condition of an unchanged path of total taxes minus transfers. This condition is implied by the invariance of the price of money. It is not necessary for the neutrality of a policy, and it is only a special case of the condition (11) (necessary for a policy to be strongly neutral) which is derived from the natural idea: if a policy is neutral it does not, in general, introduce assets of a new type.

5. Conclusion

Recently it has been shown that monetary policv can have real effect in economies with no liquidity constraints in which money is held only as a store of value, and some agents do not have complete information (Weiss 1980).

A monetary policy contingent on a future state of nature s, which is to be applied only when s is revealed to all agents affects the price of money contingent on s and therefore the return of money between the present and the time of the events to the agents informed about s. This affects their demand in the present and therefore the price of money in the present, which conveys some information about s to non-informed agents for their economic decisions. Our results, which do not depend on a perfect foresight assumption, show that in this situation of incomplete information, monetary policy can affect the signalling process of the price level only when the variations of the money supply are accompanied with transfers between the government and individuals.

The results on the neutrality of open-market operations have been derived in a framework where the demand for money or other government liabilities is obtained by rational portfolio choice. Clearly the neutrality does not occur when these liabilities provide additional services, as for transactions. For example, if total output depends on the stocks of capital and money, open-market operations which monetize capital may increase productivity. The assumption of the portfolio choice may be particularly useful to analyse the composition of the government debt. A readjustment of this composition by trading between these assets has no real effect if individuals can vary freely their portfolios under their budget constraint.

FOOTNOTES

[1]This is not the place to summarize the huge literature on the subject. For example of a life-cycle model, see [Weiss 1980b, Drazen 1980]. For a model with infinite horizon, where money is neutral in the long-run, but not in the short-run, see Fisher [1979].

[2]The state of nature at time t, s_t, may also include exogenous events prior to time t. We assume that the factor productivities are independent of the levels of input, and are determined only by the exogenous parameter s_t. This does not restrict the generality of our result since we will see that the contingent path of aggregate capital accumulation is independent of the government's policy.

[3]It is not necessary to introduce the distribution of labor endowments or the rate of return of capital in this sequence, since they are included in the definition of s_t.

[4]The real quantity of money is determined in period one by the consumption of the old (in that period), and in period two by the savings of the young.

[5]The rows are indexed by $s \in S_{t+1}$ which can always be represented by the integers $1, \ldots, n_{t+1}$.

[6]There are many trivial policies which are neutral and change the real quantity of money. An example is given by the combination of a lump-sum grant of money to the young and a contingent lump-sum tax levied on the old.

[7]In (12) and (13) the marginal utilities are identical in both equilibria (because of (i)).

[8]We assume that there exists on i such that $\pi_s^i > 0$ for all $s \in S_2$.

[9]Using condition (9), we have:

$$1 = \sum_{s \in S_{t+1}} \pi_s \frac{\bar{P}_{t+1}(s)\bar{M}_{t+1}(s)}{\bar{P}_t(s_t)\bar{M}_t(s_t)} \frac{M_t(s_t)}{M_{t+1}(s)} \frac{u'^{i}_{tt+1}(s)}{u'^{i}_{tt}(s_t)} , \text{ for } t \geq 1 .$$

his second period. Suppose now that a market is introduced for an asset which pays a unit of the good independently of the state of nature, and has a price q ; its return is equal to $r(s) = 1$, $s = 1, 2$. The total amount of the asset is equal to zero. The unique equilibrium price system is given by

$$(\bar{p}, \bar{q}) = (\bar{p}(1), \bar{p}(2), \bar{q}(1), \bar{q}(2)) = (\tfrac{2}{M}, \tfrac{4}{M}, \tfrac{2}{3}, \tfrac{4}{3}) ,$$

the associated contingent consumption allocation by

$$\{\bar{c}^1(1) = \tfrac{1}{2}, \bar{c}^2(1) = \tfrac{3}{2}, \bar{c}^1(2) = \tfrac{5}{2}, \bar{c}^2(2) = \tfrac{3}{2}\}$$

and the asset holdings which yield the allocation by

$$\{\bar{m}^1(1) = M, \bar{y}_1^1(1) = -\tfrac{3}{2}, \bar{m}^2(1) = 0, \bar{y}_1^2(1) = \tfrac{3}{2}, \bar{m}^1(2) = M, \bar{y}_1^1(2) = -\tfrac{3}{2},$$

$$\bar{m}^2(2) = 0, \bar{y}_1^2(2) = \tfrac{3}{2}\} .$$

The introduction of a riskless asset enables the risk neutral agents to absorb all risk. Furthermore, money not only is held at equilibrium even after the introduction of the riskless asset and even in the absence of a transactions technology, but it is also necessary for the efficient allocation of risk: In the economy describes in the above example, a riskless asset could not enable the risk neutral agents to absorb all risk in the absence of money.

APPENDIX

We show, for a simple model, that an equilibrium with money exists, and that it is affected by the introduction of a new asset: there is one good which is non-durable, in the economy. The population is composed of overlapping generations living two perios. Each generation is divided into two classes of identical individuals represented by the index $h = 1, 2$, who are either risk averse or risk neutral. There are the same (large) numbers of individuals in each class. There is no growth. Individuals derive utility only from their second period consumption. There are two states of nature denoted by $s = 1, 2$. Each individual receives an endowment only during the first period of his life, which depends on the state of nature s :

$$e^h(1) = 1 \ , \ e^h(2) = 2 \ , \ h = 1, 2 \ .$$

If money is the only asset available and its supply M is different from zero, the unique equilibrium price of money is a function of s, and is given by:

$$\bar{p} = (\bar{p}(1), \bar{p}(2)) = (\frac{2}{M}, \frac{4}{M})$$

and the resulting equilibrium contingent consumption allocation is

$$\{\bar{c}^h(s, 1) = 1 \ , \ \bar{c}^h(s, 2) = 2 \ , \ h = 1, 2, \ s = 1, 2\}$$

where $c^h(s, s')$ represents the second-period consumption of individual h when s is the state of nature in his first period, and s' the state in

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