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SCHUMPETERIAN DYNAMICS

II. TECHNOLOGICAL PROGRESS, FIRM GROWTH AND "ECONOMIC SELECTION"

by

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1. Introduction

Business firms strive for survival and growth. They innovate in order to grow; they imitate in order to survive. Firms which fail to innovate or imitate must go out of business or at least forego the opportunity to grow.

In the preceding paper (Part I: Evolutionary Model of Technological Innovation and Imitation), we studied how the dynamic processes of firms' innovation and imitation activities interact with each other and shape up the evolutionary pattern of the state of technology of an industry as a whole. We, however, did not take account of the differential impacts of such diverse technological developments among firms on their growth

the consequent repurcussions on the evolutionary pattern of the state of technology itself. The first purpose of this paper, the second in the series on Schumpeterian dynamics, is to explore the evolution of the industry's state of technology as a dynamic outcome of the interactions between technological developments and growth processes at the micro level of firms.[1]

In his (too) well-known article on the methodology of positive economics, Milton Friedman wrote:

> ...unless the behavior of businessmen in some way or other approximated behavior consistent with the maximization of returns, it seems unlikely that they would remain in business for long. Let the apparent immediate determinant of business behavior be anything at all--habitual reaction, random chance, or whatnot. Whenever this determinant happens to lead to behavior consistent with rational and informed maximization of returns, the business will prosper and acquire resources with which to expand; whenever it does not, the business will tend to lose resources and can be kept in existence only by the addition of resources from outside. The process of "natural selection" thus helps to validate the hypothesis--or, rather, given natural selection, acceptance of the hypothesis can be based largely on the judgment that it summarizes appropriately the conditions for survival. (Friedman [1953], p. 22)

It is thus argued here that the force of competition, in particular, the force of dynamic competition for the acquisition of resources for growth is strong enough to ensure the survival and eventual dominance of the most efficient firms, whether their actions are guided by conscious maximization of returns or resulted from pure chance mechamism. It is the analogy between biological competitive process in natural environment and inter-firm competitive process in economic environment which has always been invoked in this kind of argument.[2] It is easy to claim metaphysically that only the fittest survives in the long-run. It is, however, another matter to examine whether the logic of natural selection in biological evolutionary theory is indeed applicable to the description of the process of inter-firm competition for survival and growth in a genuinely economic environment. The second purpose of the present paper is to put the above "economic selection" argument to scientific scrutiny, on the basis of our explicitly "economic" model of evolutionary processes of firm growth and technological development.

2. State of Technology

Let us begin by representing the "state of technology" of an industry at a given point in time.

Suppose, as in the preceding paper, that there exist n distinct production methods coexisting in the industry at a given point in time. (The number of production methods, n, will of course change, i.e., increase, over time as firms succeed in bringing new production methods into the industry.) Each production method is assumed to be of fixed proportion type, so that the unit cost is constant up to its

productive capacity. If we denote by c the unit cost in terms of numeraire, all the existing production methods can be arrayed in accordance with their unit costs as follows:

$$c_n < c_{n-1} < \ldots < c_i < \ldots < c_1 ,$$

where c_n is the unit cost of the best practice method and c_1 that of the worst production method.[3]

Suppose, further, that the industry consists of M firms, including both active and potential producers. The cost conditions of course vary from firm to firm. Then, let $f_t(c_i)$ represent the relative frequency of firms with unit cost equal to c_i at time t, and let $F_t(c_i) \equiv \sum_{j=i}^{n} f_t(c_j)$ represent the relative frequency of firms with unit costs c_i or less at time t. We call $f_t(c)$ the (discrete) frequency function and $F_t(c)$ the cumulative frequency function of unit costs.

In the preceding paper we could represent the state of technology of an industry by $f_t(c)$ or, equivalently, by $F_t(c)$ alone. As soon as, however, we take into consideration the dynamic interplays between firms' technological innovation and imitation activities and their growth strategies, it is no longer sufficient to look at $f_t(c)$ or $F_t(c)$ alone in evaluating the industry's state of technology. It becomes necessary to study how the industry's total productive capacity is distributed over different production methods with different unit costs. Accordingly, let $k_t(c_i)$ represent the total volume of productive capacities with unit cost equal to c_i at time t, and let $K_t \equiv \sum_{j=1}^{n} k_t(c_j)$ be the total productive capacity of the industry as a whole at time t. (There is no logical difficulty in this calculation of total productive capacity.

See Sato [1975] for a useful discussion on the problem of capital aggregation.) We can then define by $k_t(c_i)/K_t$ the capacity share of unit cost c_i at time t. We denote this by $s_t(c_i)$ and call it the capacity share function. We then denote by $S_t(c_i) \equiv \sum_{j=i}^{n} s_t(c_j)$ the capacity share of unit costs from c_n to c_i at time t, and call it the cumulative capacity share function of unit costs.

Note in passing that the ratio between the capacity share function and the frequency function, i.e., $s_t(c_i)/f_t(c_i)$, represents the average capacity size of the firms with unit cost c at time t (in terms of the average capacity size of all the firms in the industry).[4]

The state of technology of an industry as a whole at a given point in time is now represented by a pair of frequency function $f_t(c)$ and capacity share function $s_t(c)$ or, equivalently, by a pair of their cumulative counterparts, $F_t(c)$ and $S_t(c)$. The task of this paper then is to study how the state of technology, thus represented, evolves over time as a dynamic consequence of the complex interplay among innovation, imitation and growth processes of firms in an industry.

3. Models of Firm Growth

There are basically two causal mechanisms through which successful innovation or imitation leads to the growth of the firm. In the first place, a successful innovation or imitation and the consequent cost reduction allow the firm to lower the price of its product. In fact, the firm may choose to keep the profit margin constant and reduce the price proportionally to the cost reduction. This has the effect of lowering the price of its product relative to the less fortunate ones and directly

promote the growth of its sales volume. Then, the growth of productive capacity follows suit. Alternatively, a successful innovation or imitation may allow the firm to increase its profit margin earned on each sales dollar and raise the rate of profit on the existing productive capacity. Such an increase in profit rate stimulates the firm's investment in productive capacity, either by influencing the expected profitability of investment or, to the extent that capital markets are imperfect, by directly providing internal fund for investment project.[5]

Let us formalize these two causal mechanisms. In fact, we now propose three alternative models of firm growth, which will be later integrated with the evolutionary model of technological innovation and innovation, developed in the preceding paper. The first of these three models formalizes the firm growth process in an industry dealing with a single homogeneous product. Since the firm in such industry is incapable of controlling the price of its own product, and hence the first causal mechanism from successful innovation and imitation to firm growth, explained above, does not work, we only have to consider the second causal mechanism in this case. The remaining two models are concerned with the firm growth process in a monopolistically competitive industry with differentiated products. Since the two causal mechanisms can both work in this case, it is necessary to present two different formulations. They are, however, not necessarily mutually exclusive modellings of the firm growth process. All three models have extremely simplified structures, which are intended to be no more than charicatures of the much more complex firm growth processes in the real market economy.

Consider first the case of a homogeneous product industry which consists of many firms producing exactly the same product. Firms are here

unable to control the price of their own product and take the market price as given. Let $\bar{p}(t)$ be this market price prevailing at time t. (The discussion of how this market price is determined at each point in time is postponed to the following paper (Part III); in the present paper, it is tentatively supposed to be exogenously given.) Then, we can calculate the "profit margin" of the firm whose unit cost equals c at time t as $(\bar{p}(t) - c)/\bar{p}(t)$, which can be approximated as $\ln \bar{p}(t) - \ln c$. If we suppose that the value ratio between output and capacity (or capital stock) varies little from firm to firm as well as from time to time, this index of profit margin can be regarded as a good proxy for the rate of profit for the firm. (This is of course true only for the firm operating at full capacity. The rate of profit of the firm operating below its capacity has to be adjusted by the factor equal to its capacity utilization rate. The discussion on the firm's production decision is postponed to the next paper.)

Let $g_t(c)$ represent the rate of capacity growth of the firm whose unit cost equals c at time t. Our hypothesis concerning the firm's capacity growth policy in this case is that the rate of capacity growth is positively correlated with its current profit margin, either by its effect on the future profitability or as the source of the internal fund for investment. Or, as a first-order approximation, it is assumed that

$$(1) \qquad g_t(c) = g_0' + g' \cdot (\ln \bar{p}(t) - \ln c) \ ;$$

where g_0' and g' (> 0) are given constants. This equation says that the lower the unit cost in relation to the current product price, the larger the rate of profit and hence the greater the rate of capacity growth.

Next, consider the case of a monopolistically competitive industry which consists of many firms competing with each other by producing differentiated products. Unlike the preceding case of homogeneous product industry, the two causal mechanisms from successful innovation or imitation to firm growth, explained at the outset of this section, are now both at work. Let us examine them separately.

In order to formalize the first mechanism, suppose that each firm adopts a mark-up pricing rule and sets the price of its own product p as a constant mark-up on the unit cost c ; that is:

$$p = (1+m)\cdot c \tag{2}$$

where $m > 0$ is a constant mark-up rate, which is assumed to be uniform across firms. Here, we do not analyze how this constant mark-up rate, which is sometimes called (rather tautologically) the degree of monopoly, is determined by the structure of industry. Let $\bar{c}(t)$ be the industry-wide average unit cost and $\bar{p}(t)$ the industry-wide average price, at time t , respectively defined as

$$\ln \bar{c}(t) \equiv \sum_{i=1}^{n} s_t(c_i)\cdot \ln c_i \ , \tag{3}$$

$$\ln \bar{p}(t) \equiv \sum_{i=1}^{n} s_t(c_i)\cdot \ln p_i \ , \tag{4}$$

(where p_i represent the price of the firm whose unit cost equals c_i). In view of the mark-up relation (2), we have the relation between them:

$$\bar{p}(t) = (1+m)\bar{c}(t) \ . \tag{5}$$

Our main hypothesis in this case is that the firm's accumulated stock

of "good-will" of customers and hence its sales volume grow at a rate which is correlated with how low its own price p deviates from the industry-wide average $\bar{p}(t)$. (See, for example, Phelps and Winter [1970] for a model of the dynamics of good-will.) Since the firm expands (or contracts) its productive capacity in accordance with the expansion of its sales volume, we can in fact obtain, as a crude first-order approximation, the following formula for the capacity growth policy of the firm with unit cost c:

$$(6) \qquad g_t(c) = g_0'' - g''(\ell n\ p - \ell n\ \bar{p}(t))$$

$$= g_0'' - g''(\ell n\ c - \ell n\ \bar{c}(t)) \text{ , by (2) and (5);}$$

where g'' is a positive constant representing the responsiveness of the growth rate of sales volume to the relative cheapness of the firm's product and g_0'' is a constant trend growth rate of sales volume. Equation (6) thus says that the growth rate of the firm is governed by the degree of its relative cost advantage, $-(\ell n\ c - \ell n\ \bar{c}(t))$.

The second causal chain from technological change to growth in this monopolistically competitive case is easier to formulate if the innovation is not the process innovation but the product innovation. Accordingly, let us here reinterpret the reciprocal of c, i.e., $1/c$, as the index of the "quality" of the product in question. A product innovation or imitation thus amounts to an event which raises the value of this quality index $1/c$. Our hypothesis here is that the profit margin a firm can enjoy at a given point in time is determined by the relative quality of its product, which may be represented as $\ell n(1/c) - \ell n(1/\bar{c}(t))$. If, furthermore, we suppose that the firm's rate of capital growth is positively correlated with its current profit margin (either by its effect

on the expected profitability or as the source of the internal fund for investment), we can write down, as a very crude first-order approximation, the capacity growth rate of the firm with the quality index $1/c$ as follows:

$$(7) \qquad g_t(c) = g_0''' + g'''\{\ell n(1/c) - \ell n(1/\bar{c}(t))\} \ ;$$

where g''' (> 0) and g_0''' are given constants. This equation says that in this model of product innovation the higher the index of the product quality $1/c$ in relation to its industry-wide average $1/\bar{c}(t)$, the higher the rate of capacity growth of the firm in question.

The study of the process of firm growth has in the past centered around the so-called "Gibrat's law of proportionate effect" (Gibrat [1931]). In its weak form, this is a proposition claiming that the expected growth rate of a firm during a specific period is independent of the initial size of the firm. (Its strong form asserts, in addition, that this probability is independent of time and uniform across firms.) Three examples of the firm's capacity growth policy, given in (1), (6) and (7), are all formulated in such a way that they are consistent with this weak form of Gibrat's law, for they all claim that the growth rate of productive capacity is independent of the size of productive capacity itself. Of course, they at the same time insist that the growth rate varies from firm to firm, depending on the difference in cost conditions or product qualities and that the difference in capacity growth rate persists

over time, to the extent that the difference in cost conditions or product qualities persists across firms.

In what follows, in order not to complicate the exposition, we shall call the real number c the unit cost even if we are dealing with the case of product innovation. No confusion would arise from this convention.

4. The Logic of Economic Selection

We are now in a position of embarking on the detailed analysis of the evolutionary process of the state of technology of an industry, under the joint pressure of firms' capacity growth, technological innovation and imitation. In order to put into relief the effect of the differential growth rates among firms with different cost conditions, however, let us begin our analysis by ignoring the effects of technological innovation and imitation. They will be introduced into the analysis in the sections that will follow.

Now, under the supposition of no technological innovation and imitation, the frequency function, $f_t(c)$, and the cumulative frequency function, $F_t(c)$, of unit costs are both invariant over time. (We have also supposed that there is neither entry into nor exit from the industry in question. An appropriate modelling of the process of entry and exit is an important agenda for future research.) The capacity share function $s_t(c)$ and the cumulative capacity share function $S_t(c)$ do, however, change over time, in response to different growth rates among firms. Their evolutionary pattern must be studied in detail.

For this purpose, differentiate the definition of the capacity share function, $s_t(c_i) \equiv k_t(c_i)/K_t$, with respect to time, and we obtain:

$$(8)\qquad \frac{\dot{s}_t(c_i)}{s_t(c_i)} = \frac{\dot{k}_t(c_i)}{k_t(c_i)} - \frac{\dot{K}_t}{K_t}$$

$$= \frac{\dot{k}_t(c_i)}{k_t(c_i)} - \sum_{j=1}^{n} \frac{\dot{k}_t(c_j)}{k_t(c_j)} \cdot s_t(c_j) \ ;$$

where $\dot{x} \equiv dx/dt$. Since $\dot{k}_t(c_i)/k_t(c_i)$ is nothing but $g_t(c_i)$ introduced in the preceding section, we can substitute into the above equation each of the three formulae for the firm's capacity growth policy, (1), (6) and (7), in the previous section. First, if we substitute (1), we obtain,

$$(9)\qquad \frac{\dot{s}_t(c_i)}{s_t(c_i)} = \{g_0' + g' \cdot (\ell n\ \bar{p}(t) - \ell n\ c_i)\} - \sum_{j=1}^{n} \{g_0' - g' \cdot (\ell n\ \bar{p}(t) - \ell n\ c_j)\} s_t(c_j)$$

$$= -g' \cdot (\ell n\ c_i - \ell n\ \bar{c}(t)) \ .$$

Second, if we substitute (6), we obtain

$$(10)\qquad \frac{\dot{s}_t(c_i)}{s_t(c_i)} = \{g_0'' - g'' \cdot (\ell n\ c_i - \ell n\ \bar{c}(t))\} - \sum_{j=1}^{n} \{g_0'' - g'' \cdot (\ell n\ c_j - \ell n\ \bar{c}(t))\} s_t(c_j)$$

$$= -g'' \cdot (\ell n\ c_i - \ell n\ \bar{c}(t)) \ .$$

Finally, if we substitute (7), we obtain

$$(11)\qquad \frac{\dot{s}_t(c_i)}{s_t(c_i)} = \{g_0''' + g''' \cdot (\ell n(1/c_i) - \ell n(1/\bar{c}(t))\}$$

$$- \sum_{j=1}^{n} \{g_0''' - g''' \cdot (\ell n(1/c_j) - \ell n(1/\bar{c}(t))\}$$

$$= -g''' \cdot (\ell n\ c_i - \ell n\ \bar{c}(t)) \ .$$

Therefore, whether the industry in question produces a homogeneous product or not, and whether, in the case of a differentiated product

industry, innovation is of the process type or of the product type, the growth rate of the capacity share of a given unit cost, $\dot{s}_t(c_i)/s_t(c_i)$, is shown to be proportional to the extent of its relative advantage over the industry average unit cost, $-(\ell n\ c_i - \ell n\ \bar{c}(t))$. In other words, the lower the unit cost relative to the industry average, the more rapidly will the productive capacity with that unit cost gain its share; and the higher the unit cost relative to the industry average, the speedier will the productive capacity with that unit cost lose its share in the industry. For the sake of brevity, let us summarize this result in the form of:

Hypothesis (G): Whether the industry in question produces a homogeneous product or heterogeneous products, the growth rate of the capacity share of the firm with unit cost equal to c_i is determined by the following equation:

$$(12) \qquad \frac{\dot{s}_t(c_i)}{s_t(c_i)} = -g(\ell n\ c_i - \ell n\ \bar{c}(t)) \ ;$$

where g (> 0) is a given positive constant.

The above equation says that the share of productive capacities which have a "cost advantage" in the sense of $c_i < \bar{c}(t)$ grows at a rate which is proportional to the relative size of that cost advantage. It thus appears to grow exponentially at this rate over time and hence exceed any positive number eventually. But, of course, the capacity share can never exceed unity by being a relative fraction! Something was wrong with our reasoning, and it is not difficult to locate where it went wrong. Indeed, it is only necessary to recall the definition (9) of the average

unit cost $\bar{c}(t)$. It is plain from this that $\bar{c}(t)$ is not a given constant, but a weighted geometrical average of existing unit costs with weights being their corresponding capacity shares, which grow or contract according to the very dynamic equation (12) we are analyzing. As time goes by, productive capacities with lower-than-average unit costs grow relatively more than those with higher-than-average unit costs. This has an effect of shifting weights in favor of lower-than-average unit costs, thereby reducing the weighted average unit cost $\bar{c}(t)$. Such a decline of the average unit cost undercuts the existing unit costs one by one and has these capacity shares contract until the productive capacities with the least unit cost dominate the entire industry and the value of the average unit cost is reduced to the level of this least unit cost. (In Appendix 1, we deduce an equation which illustrates this point in a summary form.)

In order to make this point clearer, we find it convenient to transform equation (12) into another differential equation of the cumulative capacity share function in the following manner:

$$(13) \qquad \dot{S}_t(c_i) \equiv \sum_{j=i}^{n} \dot{s}_t(c_j) = \sum_{j=i}^{n} \{-g(\ell n\, c_j - \ell n\, \bar{c}(t)) \cdot s_t(c_j)\}$$

$$= -g\{\sum_{j=i}^{n} \ell n\, c_j \cdot s_t(c_j) - [\sum_{j=1}^{n} \ell n\, c_j \cdot s_t(c_j)] \cdot S_t(c_i)\}$$

$$= -g\{\sum_{j=i}^{n} \ell n\, c_j \cdot s_t(c_j) - [\sum_{j=1}^{i-1} \ell n\, c_j \cdot s_t(c_j) + \sum_{j=i}^{n} \ell n\, c_j \cdot s_t(c_j)] \cdot S_t(c_i)\}$$

$$= [g \cdot \delta_t(c_i)] \cdot S_t(c_i) \cdot [1 - S_t(c_i)] \ ;$$

where $\delta_t(c_i) > 0$ is defined by

$$(14) \qquad \delta_t(c_i) \equiv \sum_{j=1}^{i-1} \ell n\ c_j \cdot s_t(c_j) \Big/ \sum_{j=1}^{i-1} s_t(c_j) - \sum_{j=i}^{n} \ell n\ c_j \cdot s_t(c_j) \Big/ \sum_{j=i}^{n} s_t(c_j) \ ,$$

for $i = 2, 3, \ldots, n$. The function $\delta_t(c_i)$ thus defined represent the difference between the logarithmic average of the subset of unit costs which are at least as high as c_{i-1} and the logarithmic average among those which are at least as low as c_i . When the number (n) of distinct unit costs is two, $\delta_t(c_2)$ equals $\ell n\ c_1 - \ell n\ c_2$ (> 0) , which is constant over time. ($\delta_t(c_1)$ is undefinable in this case.) When the number of distinct unit costs is more than two, $\delta_t(c_i)$ can no longer be treated as a given constant. However, even in this case, its value does not appear to fluctuate much from one unit cost to another, nor from one point in time to another. Indeed, from now on, we proceed our analysis as if the value of $\delta_t(c_i)$ were a given positive constant, and write it simply as δ .[7] Then, the above equation can be rewritten as

$$(15) \qquad \dot{S}_t(c_i) \approx (g\delta) S_t(c_i) \cdot [1 - S_t(c_i)] \ .$$

This is, of course, a logistic differential equation with growth parameter $g\delta$. It thus has an explicit solution of the form:

$$(16) \qquad S_t(c_i) \approx \frac{1}{1 + (1/S_T(c_i) - 1) \cdot \exp[-g\delta \cdot (t-T)]} \ ,$$

where T $(\leq t)$ is a given initial time.

Therefore, if there is neither innovation nor imitation incessantly altering firms' cost conditions, the difference of capacity growth rates between low cost and high cost firms sets in motion the cumulative capacity share function $S_t(c)$ (approximately) along a familiar logistic growth path (16). (See Figure 1.) The logic behind this logistic growth process

is easy to explain. For instance, the capacity share of the least unit cost, $S_t(c_n)$, can grow almost exponentially when it is small. But, as it begins to occupy a non-negligible portion, the weights of the industry average unit cost $\bar{c}(t)$ begins to shift towards the lower unit costs. As a result of this distributional effect, the average unit cost $\bar{c}(t)$ begins to decline as well, and the relative cost advantage of the least cost firms gradually disappears. The capacity share of the least cost firms therefore lags behind the exponential growth path of the initial stage, and decelerates its growth momentum as it becomes larger and larger. It never stops growing, however. With decelerating speed it nonetheless approaches unity asymtotically. The capacity shares of the less efficient production methods, $s_t(c_{n-1})$, $s_t(c_{n-2})$, ..., $s_t(c_1)$, on the other hand, dwindle gradually over time (though some of them may grow temporarily before they start dwindling) and disappear entirely in the long-run.

Other things being equal, the only possible long-run state of technology is the complete monopolization of the industry's productive capacities by the group of firms (or a firm) which are lucky enough to begin with the most efficient, least cost production method. The dynamic competition among firms for capacity growth thus selects out the most efficient ones in the industry. In the long-run only the fittest survives. This model is thus a paradigm of the doctrine of "economic selection" *à la* Milton Friedman *et al.*, in which the analogy to the biological theory of natural selection is perfectly valid.

In the following two sections, however, we shall show that the doctrine of economic selection itself will not survive once the processes of imitation and innovation are explicitly introduced into our evolutionary model of the industry.

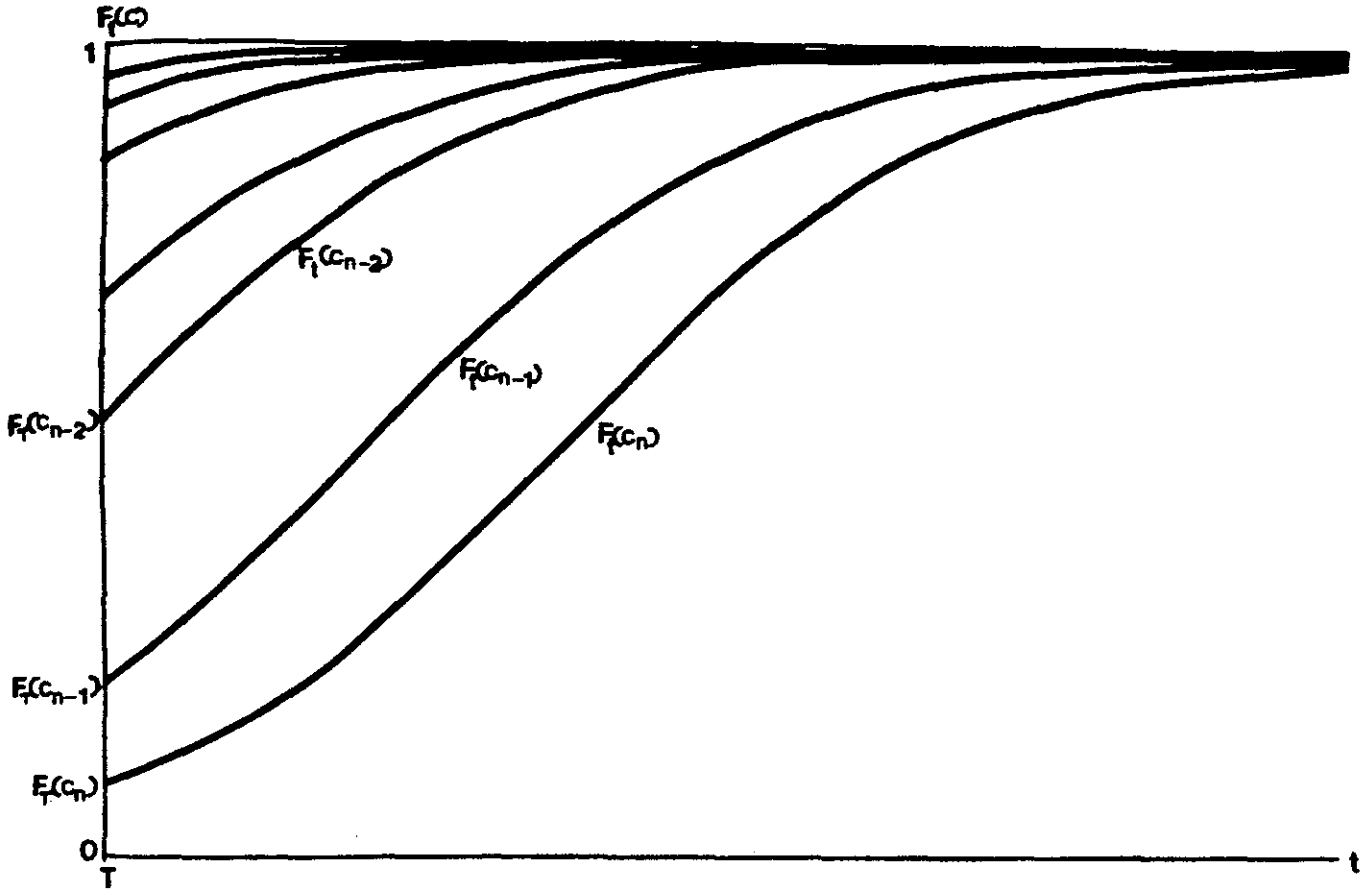

Fig. 1: The evolution of the capacity shares of different unit costs under the sole pressure of economic selection

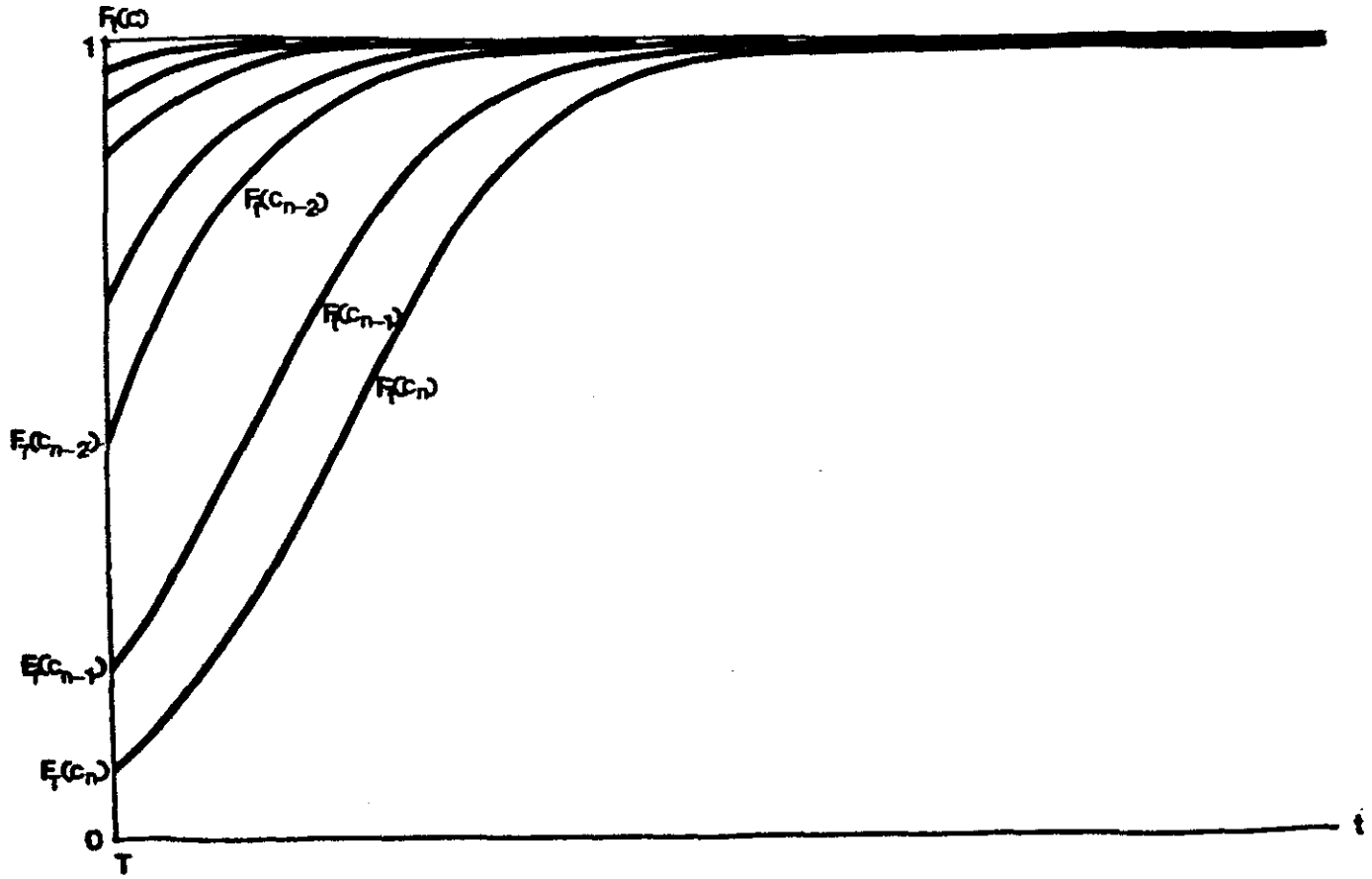

Fig. 2: The evolution of the capacity shares of different unit costs under the joint pressure of economic selection and technological imitation

5. Imitation and Economic "Selection"

Let us now introduce the process of technological imitation into our picture of the industry. To this end, we have to slightly modify the hypothesis (IM') employed in the previous paper concerning the probabilistic law governing the process of imitation. A modified hypothesis is as follows:

Hypothesis (IM): The probability that a firm is able to copy a particular production method is proportional to the share of total productive capacity which employs that method in the period in question. The firm, of course, implements only the method whose unit cost is lower than the one it is currently using.

Formally, this hypothesis says that the probability that a firm with unit cost c_i imitates a production method of unit cost c during a small time interval $[t, t+\Delta t]$ is equal to

$$(17) \qquad \begin{cases} \mu \cdot s_t(c)\Delta t\,, & \text{for } c < c_i\,, \text{ and} \\ 0\,, & \text{for } c \geq c_i\,; \end{cases}$$

where $\mu > 0$ is a parameter summarizing the effectiveness of the firm's imitation activity. Thus, unlike the previous hypothesis (IM'), it is easier for the firm to imitate the production method of a large size firm than that of a small size firm. On the other hand, its own size does not provide the imitator with any particular advantage in the probability of imitation, although, once a better production method is successfully copied, the assumed disembodied nature of technological change confers the proportionally larger fruits on the larger size firm.

Under the modified hypothesis (IM), it is not difficult to apply

the argument similar to the one given in Section 2 of the preceding paper and obtain a differential equation which describes how the firms' imitation activities move the industry's cumulative share of productive capacities with unit costs c_i or less, $S_t(c_i)$. For this purpose, let us first note that the value of $S_t(c_i)$ changes whenever one of the firms whose unit costs are higher than c_i succeeds in imitating one of the production methods with unit cost c_i or less. In fact, the value of $S_t(c_i)$ increases by the magnitude equal to the capacity share of the imitator. Now, let M be the number of firms in the industry. Then, the number of firms whose unit costs are higher than c_i can be counted as $(1 - F_t(c_i))M$. Since the total capacity share of these firms is given by $(1 - S_t(c_i))$, their average capacity share can be calculated as $(1 - S_t(c_i))/(1 - F_t(c_i))M$. On the other hand, since by hypothesis (IM) the probability that each of these firms succeeds in imitating one of the production methods with unit cost c_i or less is equal to $\mu \cdot \{s_t(c_i) + \ldots + s_t(c_n)\} \cdot \Delta t \equiv \mu S_t(c_i)\Delta t$ during a small time interval $[t, t+\Delta t]$, the probability that one of them succeeds in such an imitation during the same time interval is given by $\{\mu S_t(c_i)\Delta t\} \cdot (1 - F_t(c_i)) \cdot M$. We can therefore compute the <u>expected</u> increase in the value of $S_t(c_i)$ during $[t, t+\Delta t]$ as $\{(1 - S_t(c_i))/(1 - F_t(c_i))M\} \cdot \{\mu S_t(c_i)\Delta t(1 - F_t(c_i))M\}$ $= \mu S_t(c_i)(1 - S_t(c_i))\Delta t$. The so-called law of the large numbers then allows us to use this as a good approximation of the <u>actual</u> increase in the value of the cumulative capacity share from time t to time $t+\Delta t$, $S_{t+\Delta t}(c_i) - S_t(c_i)$. If we let Δt approach zero, we finally obtain the following familiar logistic differential equation:

$$\dot{S}_t(c_i) = \mu \cdot S_t(c_i) \cdot [1 - S_t(c_i)] \quad . \tag{18}$$

This equation, however, has not taken account of the effect of the differential growth rates among firms with different cost conditions, which was described by equation (16) of the preceding section. If, therefore, we add (16) and (18), we obtain a new logistic differential equation which describes the combined effect of the processes of capacity growth and technological imitation:

$$(19) \qquad \dot{S}_t(c_i) = (\mu + g\delta)\cdot S_t(c_i)\cdot[1 - S_t(c_i)] \ .$$

Solving this explicitly, we obtain a new logistic growth path of the form:

$$(20) \qquad S_t(c_i) = \frac{1}{1 + (1/S_T(c_i) - 1)\ \exp[(\mu + g\delta)(t-T)]} \ ,$$

for $t \geq T$. Under the combined pressure of capacity growth and technological imitation, the cumulative capacity share function $S_t(c)$ will (approximately) follow a familiar logistic growth path, illustrated by Figure 2. The only formal difference from the preceding case of no technological imitation is that its growth parameter is now the sum of $g\delta$ and μ --the sum of the parameter representing the effect of differential growth rates and the parameter representing the effect of imitation process. In the long-run, therefore, the least cost production method will again completely dominate the industry's total productive capacity. (That is, $S_t(c_n) \to 1$, as $t \to \infty$.)

The process of capacity growth and the process of technological imitation, however, contribute to the logistic growth process of the cumulative capacity share function for entirely opposite reasons. While, as was shown in the preceding section, the former represents the force which tends to amass the industry's productive capacities in the hands

of few technologically advanced firms through their capability of rapid capacity growth, the latter represents the force which dissipates the advantage of the low cost production methods among all firms through their imitation efforts. While the former represents a centralizing tendency, the latter represents a decentralizing tendency of productive capacities.

In order to see in more detail how these two opposite tendencies will interact with each other, let us now turn our attention to the evolutionary pattern of the cumulative frequence function of unit costs, $F_t(c)$. Indeed, a slight modification of the argument employed in deducing the logistic differential equation (18) leads us to the following differential equation:

$$\dot{F}_t(c_i) = \mu \cdot S_t(c_i) \cdot (1 - F_t(c_i)) \ . \tag{21}$$

Here, $(1 - F_t(c_i))$ in the right-hand-side represents the fraction of the firms whose unit costs are higher than c_i , and $\mu \cdot S_t(c_i)$ the probability that one of these firms succeeds in imitating one of the production methods with unit cost c_i or less per unit of time. The expected increase in the value of $F_t(c_i)$ per unit of time, therefore, equals their product, so that an application of the strong law of large numbers gives us the above differential equation. Note that the differential growth rates among firms have no direct impact on the motion of $F_t(c)$, except indirectly through their influence on the probability of the success of imitation.

As is shown in Appendix 2, it is possible to solve the differential equation (21) and deduce the following explicit formula for the growth path of $F_t(c)$:

$$(22) \qquad F_t(c_i) = 1 - (1 - F_T(c_i))\{(1 - S_T(c_i)) + S_T(c_i) \cdot \exp[(g\delta + \mu)(t-T)]\}^{-\mu/(g\delta+\mu)} ,$$

for $t \geq T$ and for all $i = 1, 2, \ldots, n$. It is not difficult to see from this equation that the frequency of the firms employing the best practice method, $F_t(c_n)$, grows slowly at first, but accelerates its speed as the corresponding capacity increases its share logistically. After the corresponding capacity share reaches its midpoint, it loses its growth momentum, but nonetheless approaches unity asymptotically. (That is, $F_t(c_n) \to 1$, as $t \to \infty$.) In the long-run, therefore, all the firms in the industry will come to adopt the best practice method with unit cost c_n.

In the economy with no technological imitation (nor innovation), the firms which are lucky enough to possess the least cost production method and hence able to afford the highest growth rate will in the long-run monopolize the whole productive capacity of the industry. As soon as, however, the possibility of technological imitation by relatively high cost firms is taken into account, this natural-selection-like logic loses much of its effectiveness. True that the lowest cost firms will again monopolize the whole productive capacity in the long-run, but, as we have seen above, the force of technological imitation will eventually allow <u>all</u> the existing firms to join the rank of the lowest cost firms. In fact, it is precisely the very expansion of the productive capacity of the lowest cost firms--their own success--which necessarily invites the vigorous imitation activities of the less fortunate ones and betrays

their own bids for the dominance of the whole industry. The human force of imitation thus has the power to overcome the blind force of economic selection. It is, in other words, the Lamarkian mechanism, not the Darwinean, that assures the survival of the firms in this world of capacity growth and technological imitation.

Still only the fittest survives in this world, and the industry's long-run state of technology is nothing but a neoclassical paradigm of perfect information. The introduction of technological innovation, however, will destroy this last vestige, as we shall see in the section that follows.

6. Innovation and "Economic Selection"

Finally, let us introduce the process of technological innovation into our picture of the industry. To begin with, we follow the notation of the previous paper and denote by $T(c_N)$ the innovation time of a unit cost c_N, i.e., the time at which a production method of unit cost c_N is for the first time put into practice in the industry. Then, at each innovation time $T(c_N)$, the capacity share of the production method of c_N unit cost emerges out of nothingness and starts its long evolutionary journey. Unlike the special model, analyzed in the previous paper, which ignored the difference in firms' productive capacities, we can no longer claim that the initial capacity share, $S_{T(c_N)}(c_N)$, is equal to $1/M$, where M is the number of firms. The initial capacity share may become larger or smaller than $1/M$, depending upon whether the capacity size of the innovator is above or below the industry average. Figures 3 and 4 illustrate, respectively, the evolutionary pattern of the cumulative capacity shares, $S_t(c_i)$, in the special case where only one of

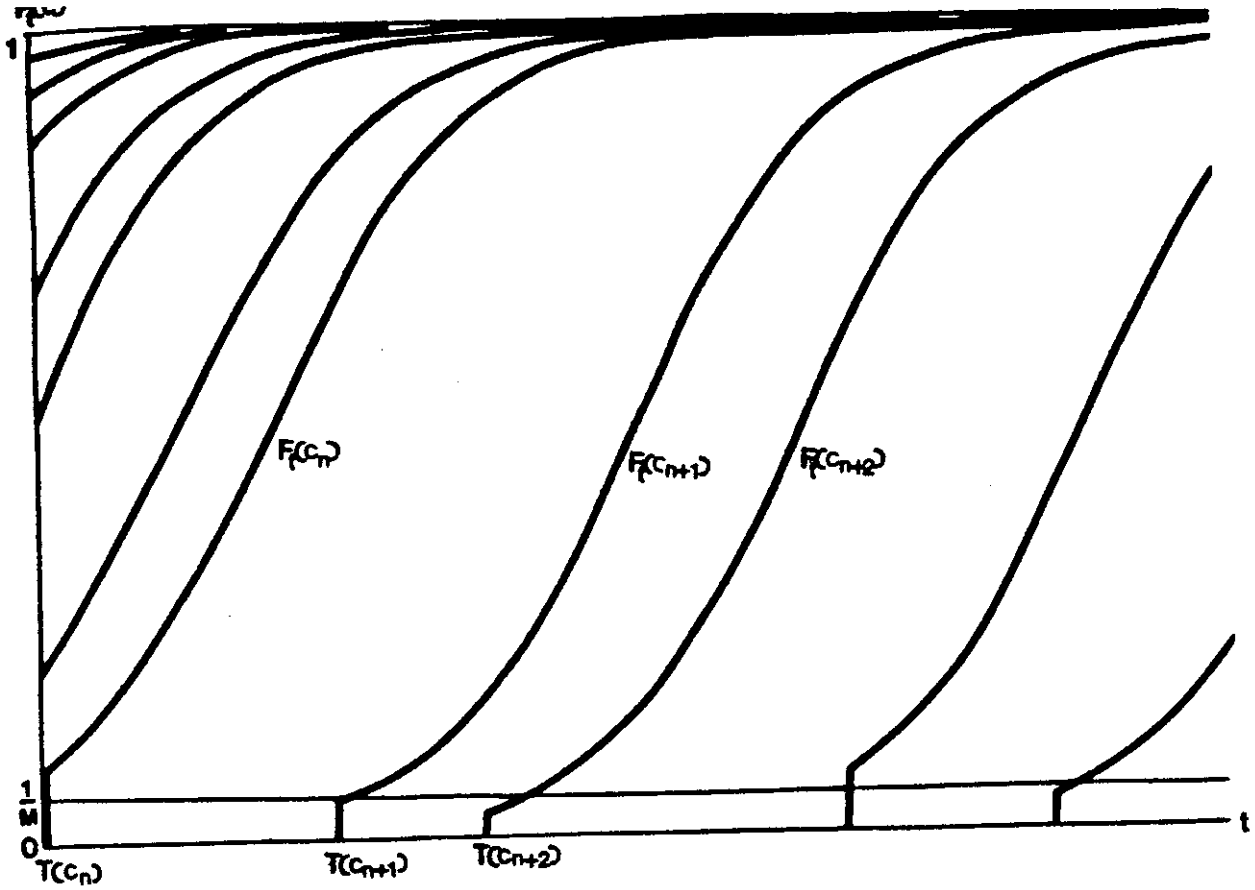

Fig. 3: The evolution of the capacity shares of different unit costs under the joint pressure of economic selection, technological imitation and technological innovation --- the case where only techmologically the most advanced firms can innovate

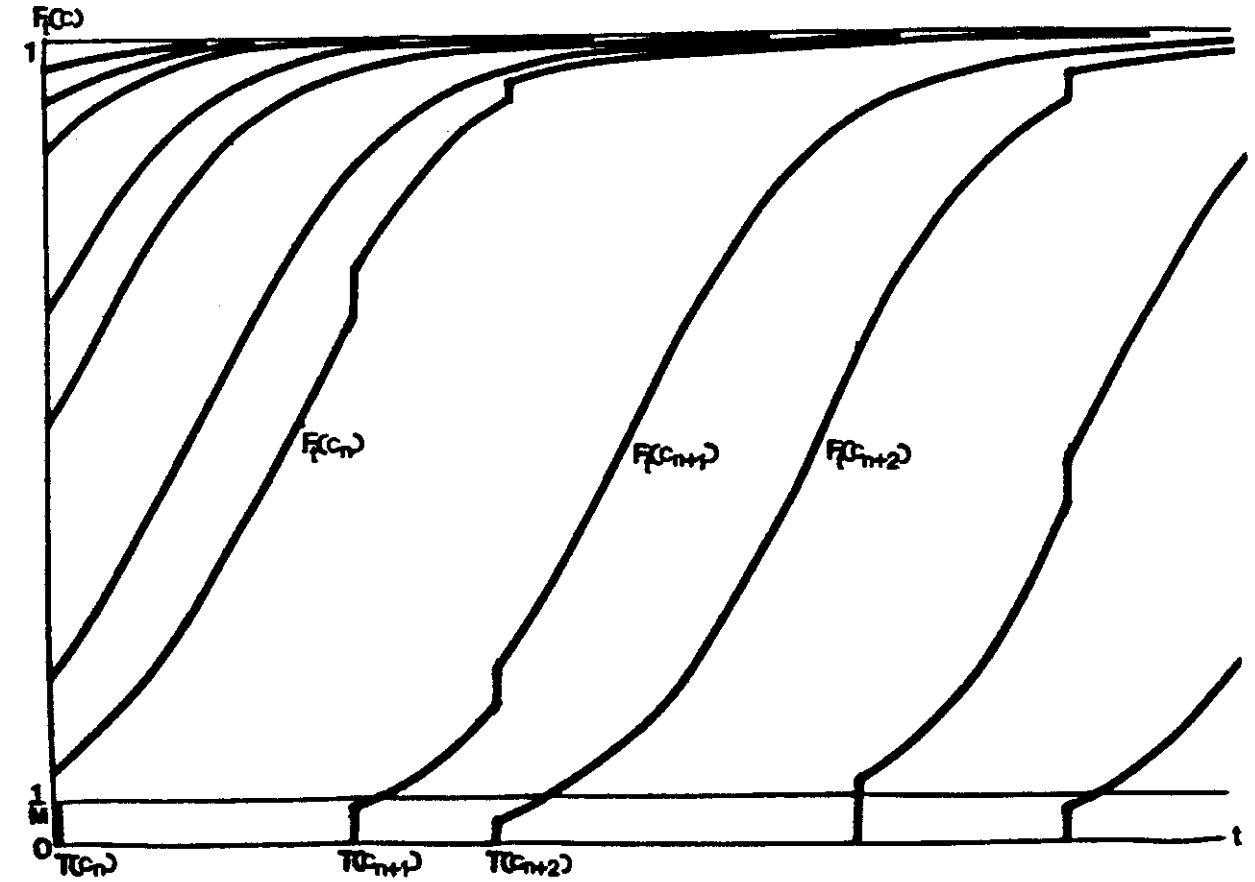

Fig. 4: The evolution of the capacity shares of different unit costs under the joint pressure of economic selection, technological imitation and technological innovation --- the general case

the lowest cost firms can strike an innovation and in the general case where any firm can become an innovator. They are, in fact, revisions of Figures 3 and 4 of the previous paper, respectively.

As soon as we come to explore the long-run consequences of the firms' innovation activities, it is no longer necessary to modify our analysis given in Section 6 of the previous paper in any substantial manner. To see this, let us first restate two hypotheses concerning the process of technological innovation. First, let $C(t)$ be the potential unit cost, i.e., the unit cost of the best production method technologically possible at time t. The movement of this potential unit cost is determined by the basic research activities outside of the industry. Our first hypothesis is:

Hypothesis (PC): The potential unit cost is declining at a constant (positive) rate λ over time; that is:

$$C(t) = \exp(-\lambda t) \; . \tag{23}$$

The second one is concerned with the stochastic nature of innovative activity:

Hypothesis (IN-i): Every firm has a small but equal chance for successful innovation at every point in time. Specifically, the probability that a firm succeeds in carrying out an innovation during a small time interval Δt is equal to

$$\nu \cdot \Delta t \; ; \tag{24}$$

where $\nu > 0$ is a parameter summarizing the effectiveness of the firm's innovation activity.[8]

Hypothesis (PC) amounts to saying that the innovation time of a given production method with c unit cost, $T(c)$, has the explicit functional form of $(1/\lambda)\ell n\, c$. Hypothesis (IN-i), on the other hand, says, within the context of the present paper that the size per se does not provide the firm with any advantage in the probability of its innovative success. At the same time, however, because of the assumed disembodied nature of technological change, the fruits of a successful innovation can be enjoyed by the firm in proportion to its existing capacity size. All in all, hypothesis (IN-i) implies a kind of constant returns to scale with respect to the firm's innovative activity.

Let $S_t^*(c)$ denote the expected value of the cumulative capacity share function at time t. For the purpose of describing the long-run average behavior of the cumulative capacity share function, we only have to concern ourselves with the motion of $S_t^*(c)$.

We know from equation (19) that, if no innovation occurs, $S_t(c)$ increases by $(g\delta+\mu)S_t(c)(1-S_t(c))\Delta t$ during a small time interval $[t, t+\Delta t]$. If, on the other hand, a firm whose unit cost is higher than c succeeds in innovation during this time, $S_t(c)$ increases, in addition to the above magnitude, by the amount equal to the capacity share of the innovator. (Note that an innovation made by a firm with unit cost c or less does not affect the value of $S_t(c)$.) Now, at time t there are $(1-F_t(c))\cdot M$ firms in the industry whose unit costs are higher than c, and the total share of productive capacities of these firms are given by $(1-S_t(c))$. Their average capacity share is therefore equal to $(1-S_t(c))/(1-F_t(c))\cdot M$. Since by hypothesis (IN-i) the probability that one of these firms strikes an innovation during a time interval t is equal to $(\nu\cdot\Delta t)\cdot(1-F_t(c))\cdot M$, the expected increase in the value

of $S_t(c)$ from time t to time $t+\Delta t$, due to innovation, can be calculated as $\{(\nu\cdot\Delta t)\cdot(1-F_t(c))\cdot M\}\cdot\{(1-S_t(c))/(1-F_t(c))\cdot M\} = \nu\cdot(1-S_t(c))\cdot\Delta t$. Hence, the expected increase of $S_t(c)$ during the same time period, as a combined result of growth, imitation and innovation, is given by $\{(g\delta+\mu)S_t(c)(1-S_t(c)) + \nu(1-S_t(c))\}\cdot\Delta t$. In terms of the expected cumulative frequency function $S_t^*(c)$, we can state this result in the form of differential equation as

$$(25) \qquad \dot{S}_t^*(c) \approx (g\delta+\mu)S_t^*(c)(1-S_t^*(c)) + \nu(1-S_t^*(c)) .$$

Solving this explicitly and noting the obvious initial condition: $S^*_{T(c)}(c) = 0$, we obtain

$$(26) \qquad S_t^*(c) \approx \frac{1+\nu/(g\delta+\mu)}{1+\frac{g\delta+\mu}{\nu}\exp[-(g\delta+\mu+\nu)(t-T(c))]} - \frac{\nu}{g\delta+\mu} .$$

Let $z \equiv \ln c - \ln C(t)$ represent the cost gap, i.e., the proportionate difference between a given unit cost c and the potential unit cost $C(t)$. Then, under hypothesis (PC), we can rewrite the above expression as

$$(27) \qquad S_t^*(c) \approx \tilde{S}(z) \equiv \frac{1+\nu/(g\delta+\mu)}{1+\frac{g\delta+\mu}{\nu}\exp\left(-\frac{g\delta+\mu+\nu}{\lambda}z\right)} - \frac{\nu}{g\delta+\mu} .$$

This represents the long-run average cumulative capacity share as a function of the cost gap z, independently of the calendar time t. Its density form can be easily computed as

$$(28) \qquad \tilde{s}(z) \equiv \frac{d\tilde{S}(z)}{dz}$$

$$= \frac{\frac{(g\delta+\mu+\nu)^2}{\lambda(g\delta+\mu)}}{\left\{\sqrt{\frac{\nu}{g\delta+\mu}}\cdot\exp\left[\frac{g\delta+\mu+\nu}{2\lambda}z\right] + \sqrt{\frac{g\delta+\mu}{\nu}}\cdot\exp\left[-\frac{g\delta+\mu+\nu}{2\lambda}z\right]\right\}^2}$$

The long-run average cumulative capacity share function $\tilde{S}(z)$, or equivalently its density form $\tilde{s}(z)$, is a long-run statistical summary of the relative distribution of the industry's total productive capacity over a spectrum of production methods with diverse unit costs. It shows how the dynamic interaction among capacity growth, imitation and innovation will in the long-run generate a statistical regularity out of the seemingly irregular pattern in which the relative shape of the capacity share function reproduces itself over time (without collapsing into a single atom). Indeed, both the long-run average cumulative share function $\tilde{S}(z)$ and its density form $\tilde{s}(z)$ have the same functional forms as the long-run average cumulative frequency function of unit costs and its density form, we deduced in Section 5 of the preceding paper, except that the parameter μ in the latter is now replaced by the sum of μ and $g\delta$. Their general shapes are illustrated in Figures 5 and 6. It is not difficult to see from these capacity functions that an increase in the declining rate of potential unit cost, λ, tends to widen the average of the cost gaps of the industry and at the same time to disperse their distribution across firms, that an increase in the rate of innovation, ν, tends to narrow the average of cost gaps and concentrate their distribution, and finally that an increase in either the rate of imitation, μ, or the growth parameter, $g\delta$, also tends to narrow the average of cost gaps and concentrate their distribution.

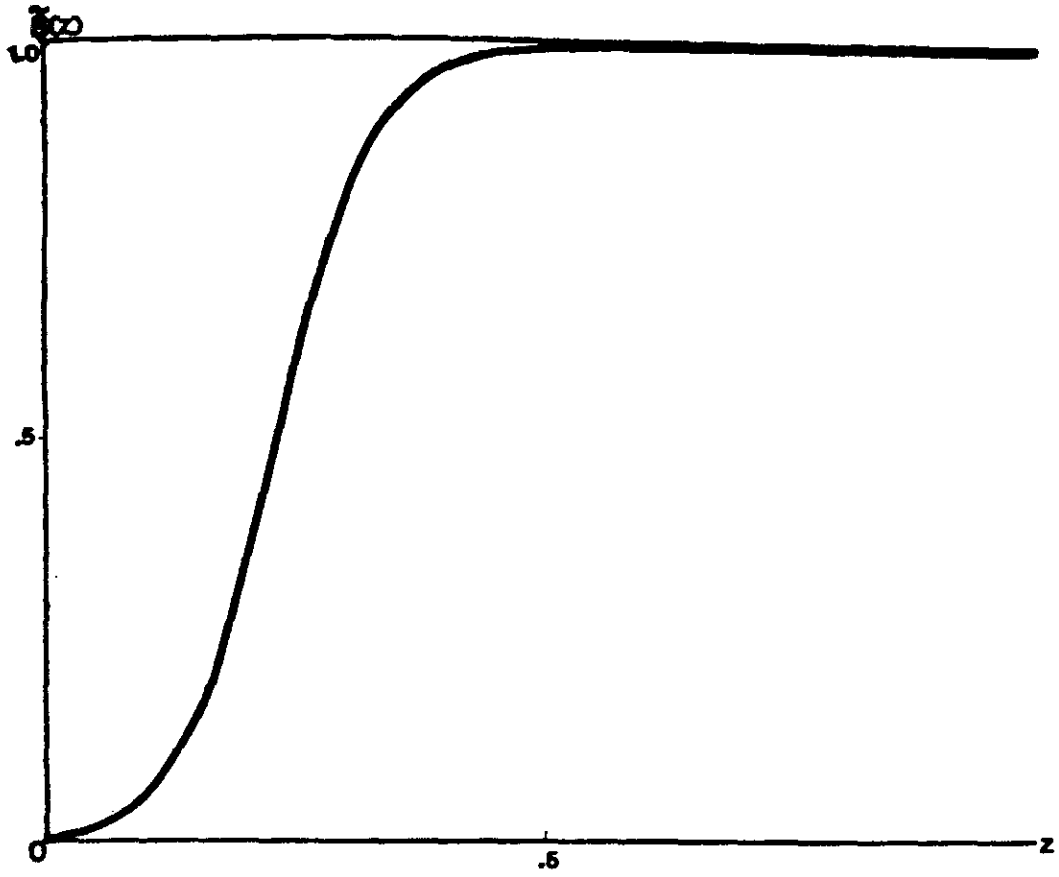

Fig. 5: The long-run average cumulative capacity share function of cost gaps (where $\lambda = .05$, $\nu = .01$, $\mu = .50$ and $g\delta = .50$)

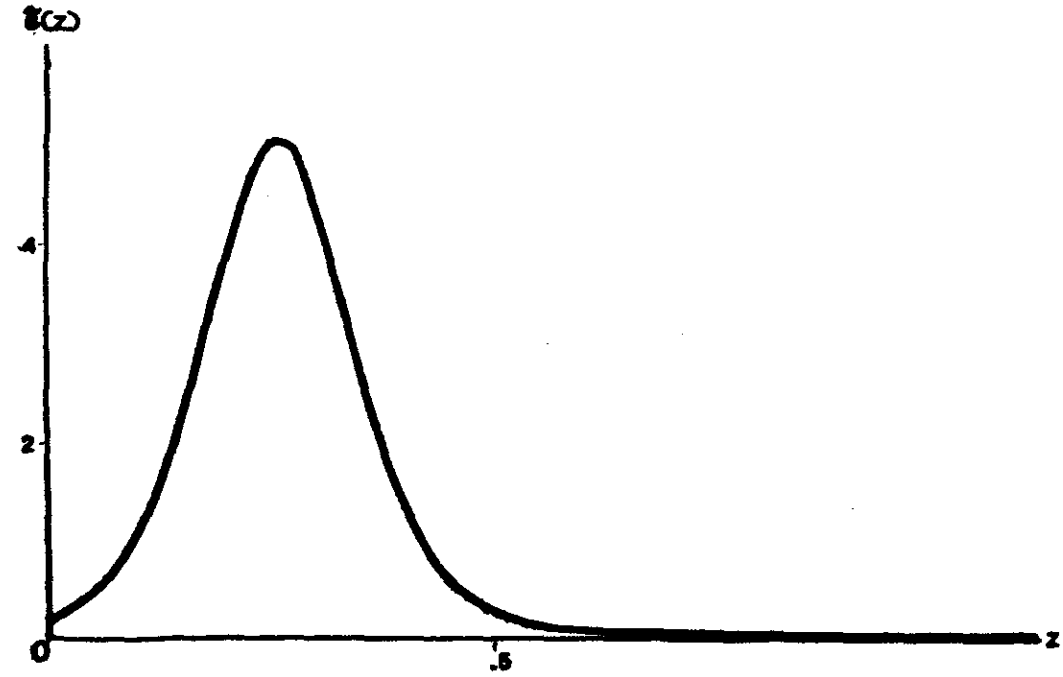

Fig. 6: The long-run average capacity density of cost gaps

Let $\hat{F}(z)$ and $\hat{f}(z)$ denote the long-run average cumulative frequency function of cost gaps and its density form, respectively. In Appendix 3 we are able to deduce their explicit forms as follows:

$$(29) \qquad \hat{F}(z) = 1 - \exp\left[-\frac{\nu g\delta}{\lambda(g\delta+\mu)}z\right]\cdot\left[\frac{g\delta+\mu}{\nu+g\delta+\mu}+\frac{\nu}{\nu+g\delta+\mu}\cdot\exp\left(\frac{\nu+g\delta+\mu}{\lambda}z\right)\right]^{-\mu/(g\delta+\mu)}$$

$$(30) \qquad \hat{f}(z) = \exp\left[-\frac{\nu g\delta}{\lambda(g\delta+\mu)}z\right]\cdot\left[\frac{g\delta+\mu}{\nu+g\delta+\mu}+\frac{\nu}{\nu+g\delta+\mu}\cdot\exp\left(\frac{\nu+g\delta+\mu}{\nu}z\right)\right]^{-\mu/(g\delta+\mu)}$$
$$\cdot\left[\frac{1+\nu/(g\delta+\mu)}{\frac{\lambda}{\mu}+\frac{\lambda(g\delta+\mu)}{\mu\nu}\cdot\exp\left(-\frac{g\delta+\mu+\nu}{\lambda}z\right)}+\frac{\nu g\delta}{(g\delta+\mu)}\right].$$

They represent statistically how the firms in the industry are distributed over production methods with different cost conditions in the long-run. Their general shapes are illustrated in Figures 7 and 8. It is not difficult to show (at least numerically) that these long-run average frequency functions respond to changes in parameters qualitatively in the same manner as the long-run average capacity share functions.

What is of the primal importance is, however, not the specific results of comparative statics concerning these long-run average capacity share and frequency functions, but the general observation that a spectrum of production methods with diverse unit costs will forever coexist in the industry. Not only the fittest but also the second, third, fourth, ..., indeed, the whole range of the less fit will survive in the long-run. The force of economic selection working through the differential growth rates among firms with different unit costs is constantly outwitted by the firms' imitation activities and intermittently disrupted by the firms' innovation activities. Indeed, the processes of growth, imitation and innovation will interact with each other and work only to maintain the

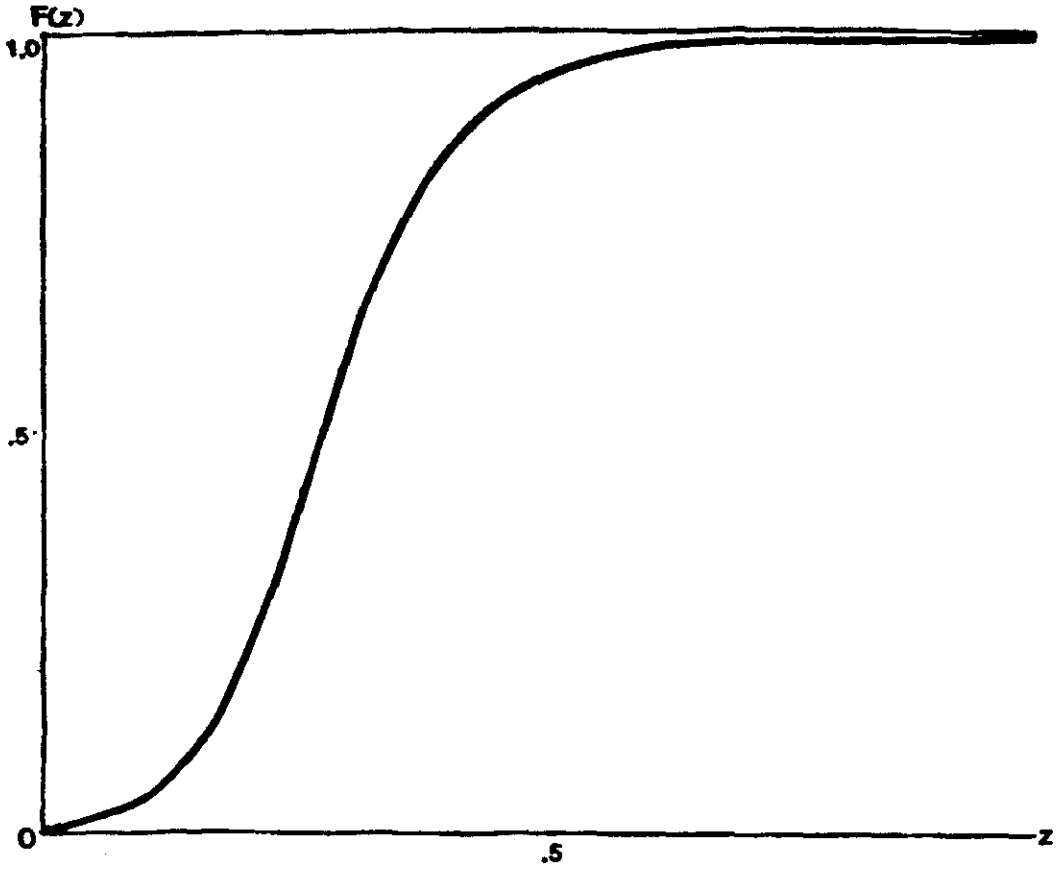

Fig. 7: The long-run average cumulative frequency function of cost gaps

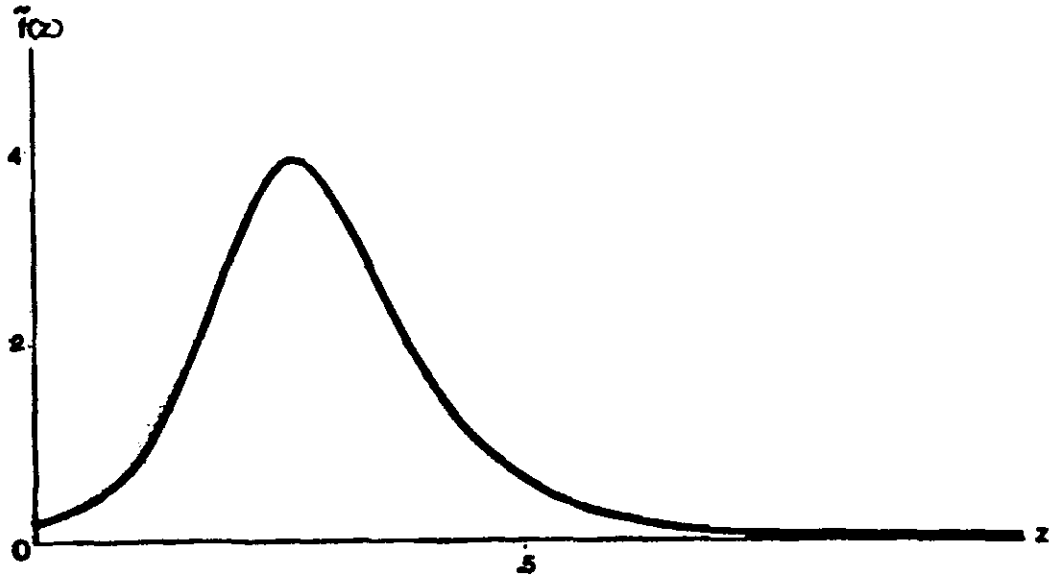

Fig. 8: The long-run average density function of cost gaps

relative structure of the industry's state of technology in a statistically balanced form in the long-run, as is described by the pair of long-run average cumulative capacity share function and frequency function, $\tilde{S}(z)$ and $\tilde{F}(z)$, or of their density forms, $\tilde{s}(z)$ and $\tilde{f}(z)$, given above.

7. Empirical Returns to Scale

The relationship between the sizes of firms and their efficiency (or profitability) has been one of the central issues in the traditional theory of the firm and industrial organization. The question which is usually asked is: "what general effects will the sizes of firms...have on the efficiency attained in production and distribution?" (Bain [1968], p. 165). Behind this question is a static view that the size is an independent variable which functionally explains the degree of efficiency the firm attains in the form of economies or diseconomies of large-scale firms. Numerous empirical studies which try to detect the existence of positive or negative correlations between size and efficiency on the basis of individual firm data have thus been conducted in the hope that these cross-sectional correlations would reveal the underlying functional relationship between firm size and efficiency.

In the present paper, we have started from the premise that the unit cost of production for each firm is constant (up to the position of productive capacity) at each point in time and hence that there exists no systematic relationship between firm size and efficiency at the level of individual firm. The unit cost each firm has attained is the fruit of the firm's innovation and imitation activity in the past, whereas the capacity size of the firm is the cumulative result of its past growth policies whose major determinant is nothing but the profitability or the

relative efficiency. Both size and efficiency have, therefore, the firm's pursuit for technological superiority in the form of innovation and/or imitation as their common cause--the former as its long-run effect and the latter as its more immediate effect.

It is thus expected that this dynamic causal relation in the long-run gives rise to a certain statistical relationship between capacity size and unit cost in our Schumpeterian model, even though any static relationship is by assumption precluded between them. This is indeed the case, and in Figure 9 , we illustrate numerically a typical shape of the ratio between the long-run average density of capacity share and of firm frequency, $\tilde{s}(z)/\tilde{f}(z)$. This ratio represents (approximately) the average capacity size of firm (measured in terms of the average firm size of the industry as a whole) at each value of cost gap z . It normally has a truncated bell-shape, so that there is in the long-run a negative correlation between capacity size and cost gap for relatively efficient firms (i.e., for firms with relatively low values of cost gap) and a positive correlation for relatively inefficient firms. Reason for such non-monotonic correlation is not hard to come by. The firm size is nothing but the legacy of the capacity growth in the past, which was in turn governed by the relative performance of its cost conditions in the past. The most efficient firms at present are those who have recently succeeded in innovation or those which have recently succeeded in imitating the innovator. The fair law of chance then indicates that they were probably not successful in innovation or imitation in the near past, and they are yet to exploit their good luck by rapidly expanding their capacity. The most efficient firms are, therefore, unlikely to be the ones with the large capacity size. The large size firms are, on the other

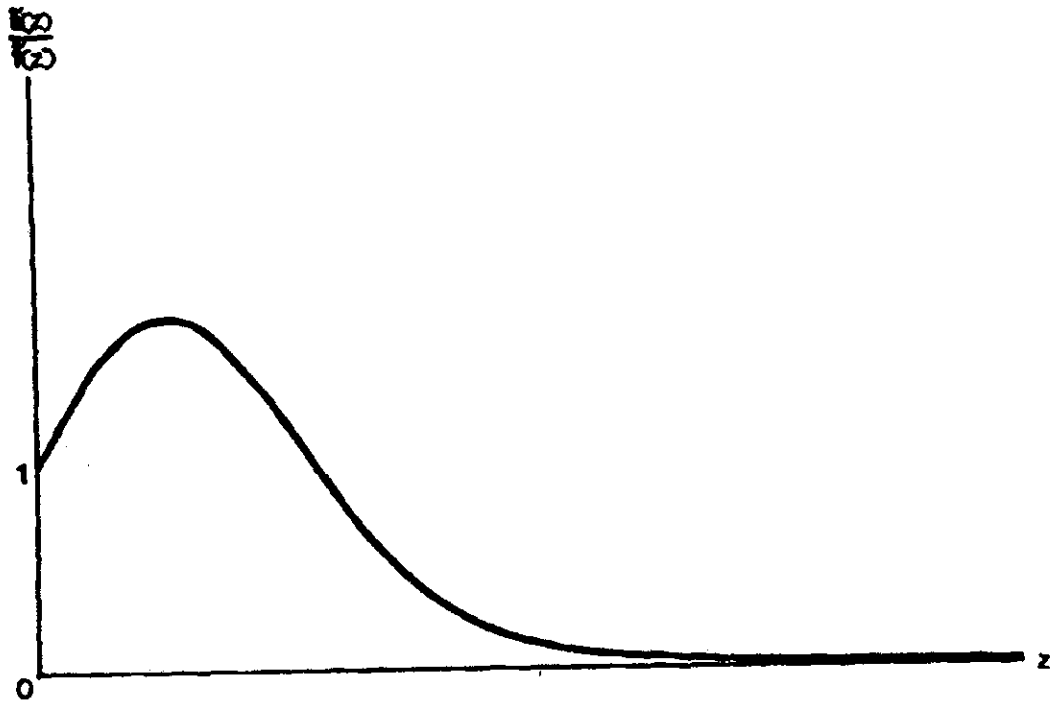

Fig. 9: The long-run average relation between efficiency and firm size (where $\lambda = .05$, $\nu = .01$, $\mu = .50$ and $g\delta = .50$)

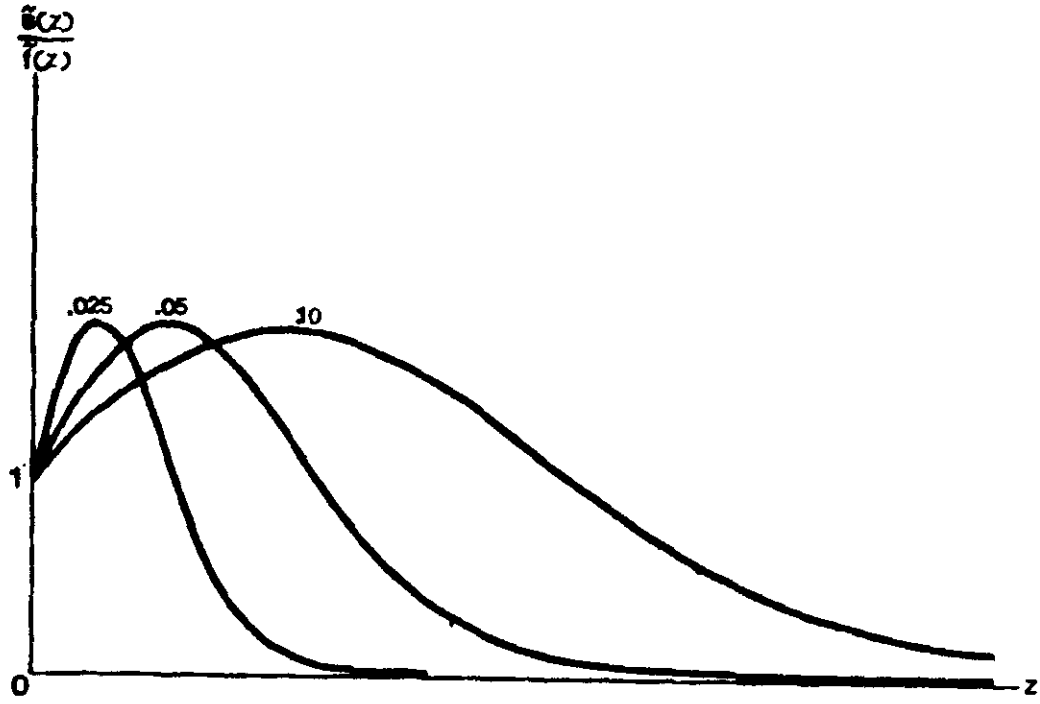

Fig. 10: The long-run average relations between efficiency and firm size for different values of λ (where $\nu = .01$, $\mu = .50$ and $g\delta = .50$)

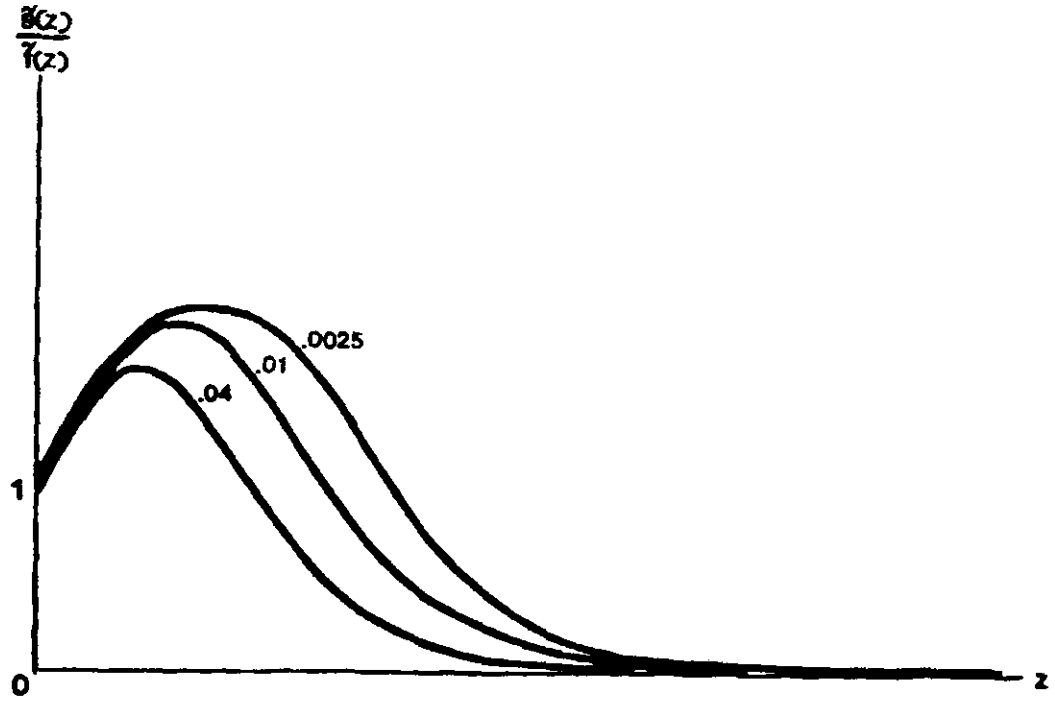

Fig. 11: The long-run average relations between efficiency and firm size for different values of ν (where $\lambda = .05$, $\mu = .50$ and $g\delta = .50$)

hand, probably those which have already passed their prime times and are currently enjoying their past success in innovation or imitation. They therefore tend to dominate in size the class of firms with modest values of cost gap. Finally, the firms with currently poor efficiency are likely to be small because of their relatively lower growth rates in the past.

The above explanation of the spurious relationship between size and efficiency has nothing to do with the conventional explanation based upon the static notion of economies or diseconomies of large-scale firms. If, however, empirical researchers run cross-sectional regressions of unit cost on capacity size or vice versa, they are likely to detect diseconomies of scale if they restrict their data to firms which earn at least some minimum rate of profit and economies of scale if they discard high profit firms as abnormal. If they do not restrict their data set, they are then unlikely to detect any economies or diseconomies, although they are likely to discover in this case that within-the-class-dispersion of unit costs increases as capaicty size decreases. Needless to say, these purely theoretical predictions are very much in conformity with the results of the past empirical analyses of the relationship between size and efficiency or profitability.[9]

Figures 10, 11, 12 and 13 illustrate how this empirical relationship between efficiency and size varies as each of the basic parameters changes its value. Figure 11 shows that an increase in the declining rate of potential unit cost, λ, tends to widen the range of empirical scale diseconomies and moderate the extent of both empirical diseconomies and economies of scale. Figure 12, on the other hand, shows that an increase in the probability of innovation, ν, tends to narrow the range of empirical scale diseconomies but at the same time slightly moderate the

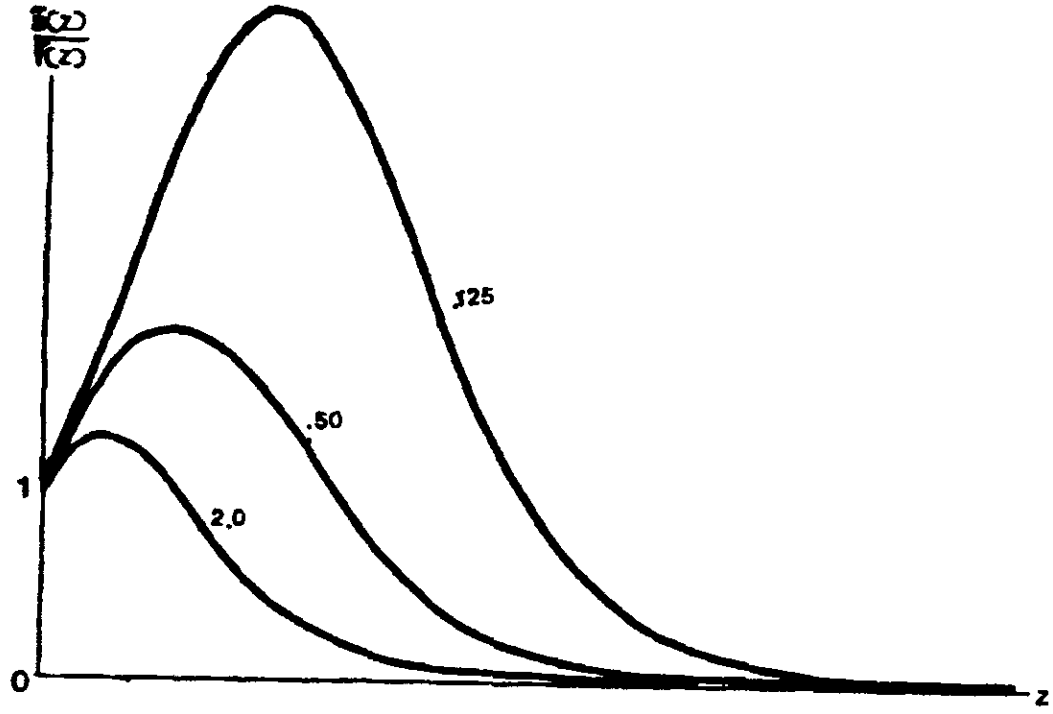

Fig. 12: The long-run average relations between efficiency and firm size for different values of μ (where $\lambda = .05$, $\nu = .01$ and $g\delta = .50$)

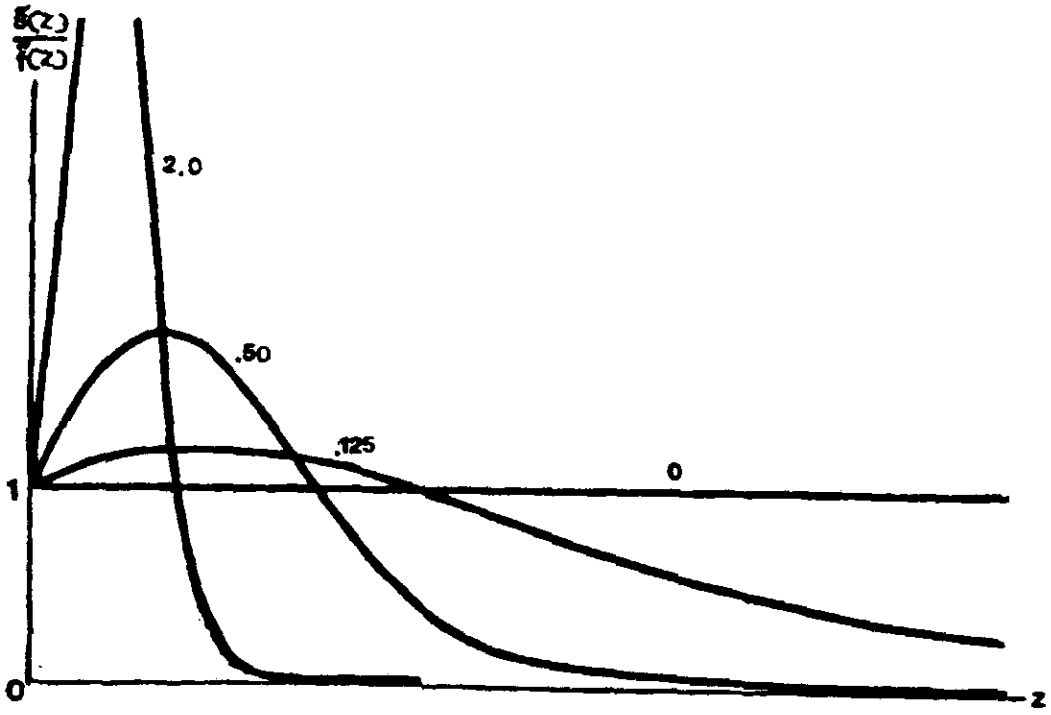

Fig. 13: The long-run average relations between efficiency and firm size for different values of $g\delta$ (where $\lambda = .05$, $\nu = .01$ and $\mu = .50$)

extent of scale economies and diseconomies. Figure 13 also shows that an increase in the likelihood of imitation, μ, tends to narrow the range of empirical scale diseconomies but moderate the extent of both scale economies and diseconomies. Finally, Figure 14 shows that an increase in the force of economic selection, represented by $g\delta$, tends to narrow the range of empirical scale diseconomies and accentuate the extent of both scale economies and diseconomies. In fact, when $g\delta = 0$, that is, when the force of economic selection is completely absent, no statistical relation should be detectable between efficiency and size across firms.

8. Concluding Remarks

The doctrine of economic selection insists, by means of the analogy to the biological theory of natural selection, that only the most efficient firms will survive the long-run competitive struggle for limited resources for capacity growth. It is this doctrine which has served the ultimate foundation of the orthodox belief in the "rationality" of individual economic agents and the "efficiency" of the market system as a whole, but which has seldom been formalized rigorously within the context of economic processes in which self-seeking firms, not biological species or genes, compete with each other for their survival and growth.

In the present paper, we have developed a simple dynamic model of industrial structure in which firms grow or contract (relative to others) in accordance with the success or failure of their innovative and/or imitative activities and the evolution of the state of technology of the industry as a whole is governed by the complex interplay among growth, innovation and imitation of these firms. The "force" of the logic of

economic selection has then been tested within this explicitly evolutionary model of economic process.

In the first place, we have found the paradigm of the economic selection doctrine in an artificially constructed economy in which no possibility of technological innovation or imitation is allowed to the firms. In this special environment it is not difficult to see that a firm or a group of firms which is lucky enough to start with the most efficient production method will outgrow all the other firms and eventually dominate the whole productive capacity of the industry. Only the fittest will survive the competition and the industry will in the long-run find itself in a static-equilibrium of perfect technological knowledge. Once, however, the possibility of technological imitation is brought into our model, the force of the logic of economic selection loses much of its forcefulness. It is true that even in this case only the most efficient firms will survive in the long-run, and the industry will eventually settle down to a static equilibrium with perfect technological knowledge. But such long-run state is brought about, not by the success of the most efficient firms in their striving for the higher growth rate, but by the success of the less efficient firms in their efforts to imitate the most efficient ones. The blind force of economic selection is thus outwitted by the human force of imitation process. Finally, when firms are allowed to innovate in their production methods, the selective forces of market competition is no longer capable of weeding out the less fit even in the long-run. Not only the most efficient but also the whole spectrum of firms with diverse efficiencies will survive forever. Indeed, it has been shown that the dynamic interactions among the processes of growth, imitation and innovation will keep the industry's state of technology from settling down to the static equilibrium and reproduce in the long-run a relative dispersion of efficiencies

across firms in a statistically balanced form. The doctrine of economic selection itself has thus failed the "test of survival."[10]

(As a by-product of our critique of the doctrine of economic selection, we have been able to demonstrate the existence of a statistical relation between efficiency and firm size; indeed, a negative correlation among the firms with relatively high efficiencies and a positive correlation among the firms with relatively low efficiencies. Since each production method is assumed in this model to be of constant-return-to-scale type, this relation is due solely to a statistical effect of the dynamic relations between technological innovation and imitation and capacity growth.)

Our critique of the doctrine of economic selection is, however, still incomplete. Though we have been able to demonstrate the long-run persistence of the difference in efficiencies across firms against the selective force of market competition, this technological diversity is merely an ex post diversity arising from the essentially non-deterministic nature of the laws governing the processes of innovation and imitation. All the firms in our model are, from the ex ante standpoint, identical in their potentiality for success and failure. In a forthcoming part (Part IV) we intend to introduce the ex ante diversity in firms' capabilities to innovate, imitate and grow and to advance our critique of the doctrine of economic selection one more step.

Before we address ourselves to this program, however, we still have to work out some of the important implications of the model developed in the present paper. In the sequel (Part III: "As-If" aggregate production function and evolutionary growth accounting) we shall examine the determination of aggregate supply in our Schumpeterian dynamics and present

a critique of the neoclassical growth accounting which is based upon the notion of aggregate production function.

APPENDIX 1

The purpose of this appendix is to deduce an equation which explains the way in which the average unit cost of an industry move over time under the pressure of differential growth rates among firms.

The average unit cost is defined as $\ln \bar{c}(t) \equiv \sum_{i=1}^{n} \ln c_i \cdot s_t(c_i)$, and so

$$\text{(A-1)} \qquad \frac{d \ln \bar{c}(t)}{dt} = \frac{\dot{\bar{c}}(t)}{\bar{c}(t)} = \sum_{i=1}^{n} \ln c_i \cdot \dot{s}_t(c_i) \ .$$

Substituting equation (12) for the growth rate of capacity share, we obtain

$$\text{(A-2)} \qquad \frac{\dot{\bar{c}}(t)}{\bar{c}(t)} = \sum_{i=1}^{n} \ln c_i \cdot [-g \cdot (\ln c_i - \ln \bar{c}(t))] \cdot s_t(c_i)$$

$$= -g \cdot [\sum (\ln c_i)^2 \cdot s_t(c_i) - \ln \bar{c}(t) \cdot \sum \ln c_i \cdot s_t(c_i)]$$

$$= -g \cdot \sum (\ln c_i - \ln \bar{c}(t))^2 \cdot s_t(c_i) \ .$$

Hence, the rate of <u>decrease</u> in the average unit cost, $-\dot{\bar{c}}(t)/\bar{c}(t)$, is proportional to the cross-section variance of the logarithmic values of unit costs. The average unit cost therefore keeps declining as long as firms which do not have the least cost technique have non-negligible shares. It stops declining only when the share of the least cost firms approaches unity through their most vigorous capacity expansion and the cross-section variance of unit costs becomes zero. The only possible long-run equilibrium in this model without technological progress is the situation in which the least cost firms dominate the entire industry.

The above analysis is a minor variation of the so-called "fundamental theorem of natural selection" in population genetics, due originally to F. A. Fisher [1930]. See, for instance, Crow and Kimura [1969] for its more recent formulation.

APPENDIX 2

The purpose of this appendix is to obtain an explicit solution to the following form of differential equation:

$$\text{(A-3)} \qquad \dot{x}_t = \alpha y_t (1 - x_t) + \beta (1 - x_t) \ ,$$

where

$$\text{(A-4)} \qquad y_t = \frac{\gamma}{1 + \delta \exp[-\varepsilon(t-T)]} - \eta \ , \quad t = T.$$

Let us rewrite (A-3) as

$$-\frac{\dot{x}_t}{1 - x_t} = -\frac{\alpha\gamma}{1 + \delta \exp[-\varepsilon(t-T)]} - (\beta - \alpha\eta) \ .$$

Integrating this, we have

$$\text{(A-5)} \qquad \ell n(1 - x_t) = \int_T^t \frac{-(\alpha\gamma)\,dt}{1 + \delta \exp[-\varepsilon(t-T)]} - (\beta - \alpha\eta)(t-T) + \ell n(1 - x_T) \ .$$

Let $Z(t)$ represent $1 + \delta \exp[-\varepsilon(t-T)]$. Then the integrand in the right-hand-side of the above equation can be rewritten as

$$\text{(A-6)} \qquad -(\alpha\gamma) \cdot \int_{1+\delta}^{Z(t)} \frac{1}{Z(s)} \cdot \frac{dS}{dZ(s)} \cdot dZ(s) = -\alpha\gamma \cdot \int_{1+\delta}^{Z(t)} \frac{1}{Z} \cdot \frac{1}{-\varepsilon(Z-1)} dZ$$

$$= \frac{\alpha\gamma}{\varepsilon} \cdot \int_{1+\delta}^{Z(t)} \left(\frac{1}{Z-1} - \frac{1}{Z} \right) dZ$$

$$= \frac{\alpha\gamma}{\varepsilon} \cdot \left[\ell n \left(\frac{Z-1}{Z} \right) \right]_{1+\delta}^{Z(t)}$$

$$= -\frac{\alpha\gamma}{\varepsilon} \cdot \ell n \left\{ \frac{\delta}{1+\delta} + \frac{1}{1+\delta} \cdot \exp[-\varepsilon(t-T)] \right\}$$

Substituting this back into (A-5), we finally obtain

$$(A\text{-}7) \qquad x_t = 1 - (1 - x_T)\exp[-(\beta-\alpha\eta)(t-T)]\cdot\left\{\frac{\delta}{1+\delta}+\frac{1}{1+\delta}\exp[-\varepsilon(t-T)]\right\}^{-\alpha\gamma/\varepsilon}$$

If we identify x_t with $F_t(c)$ and set α, β, γ, δ, ε and η equal to μ, 0, 1, $1/S_T(c) - 1$, $g\delta+\mu$ and 0, respectively, we obtain (22) of the main text. If, on the other hand, we identify x_t with $F_t^*(c)$ and set α, β, γ, δ, ε and η equal to μ, ν, $1 + \nu/(g\delta+\mu)$, $(g\delta+\mu)/\nu$, $g\delta+\mu+\nu$ and $\nu/(g\delta+\mu)$, respectively, we obtain (A-9) of the next appendix.

APPENDIX 3

Let us denote by $F_t^*(c)$ the expected cumulative frequency functions of unit costs at time t. Under hypotheses (IM) and (IN-i), it is not difficult to deduce the following differential equation concerning its dynamic motion:

$$\text{(A-8)} \qquad \dot{F}_t^*(c) = \mu \cdot S_t^*(c)(1 - F_t^*(c)) + \nu \cdot (1 - F_t^*(c)) \ .$$

In Appendix 2, we are able to solve this explicitly and obtain

$$\text{(A-9)} \qquad F_t^*(c) = 1 - (1 - F_{T(c)}^*(c)) \cdot \exp\left[- \frac{g}{g\delta+\mu}(t - T(c)) \right]$$
$$\cdot \left\{ \frac{g\delta+\mu}{\nu+g\delta+\mu} + \frac{\nu}{\nu+g\delta+\mu} \exp[(\nu+g\delta+\mu)(t - T(c))] \right\}^{-\mu/(g\delta+\mu)} \ .$$

Noting the obvious initial condition: $F_{T(c)}^*(c) = 0$, the hypothesis (PC) enables us to rewrite this as

$$\text{(A-10)} \qquad F_t^*(c) = \hat{F}(z)$$
$$1 - \exp\left[- \frac{g}{\lambda(g\delta+\mu)} z \right] \cdot \left[\frac{g\delta+\mu}{\nu+g\delta+\mu} + \frac{\nu}{\nu+g\delta+\mu} \exp\left(\frac{\nu+g\delta+\mu}{\lambda} z \right) \right]^{-\mu/(g\delta+\mu)} \ .$$

This is the long-run average cumulative distribution of cost gaps by firms. Its density form is then given by

$$\text{(A-11)} \qquad \hat{f}(z) \equiv \frac{d\hat{F}(z)}{dz} = \exp\left[- \frac{\nu g\delta}{\lambda(g\delta+\mu)} z \right] \cdot \left[\frac{g\delta+\mu}{\nu+g\delta+\mu} + \frac{\nu}{\nu+g\delta+\mu} \exp\left(\frac{\nu+g\delta+\mu}{\lambda} z \right) \right]^{-\mu/(g\delta+\mu)}$$
$$\cdot \left[\frac{1 + \nu/(g\delta+\mu)}{\frac{\lambda}{\mu} + \frac{\lambda(g\delta+\mu)}{\mu\nu} \exp - \left(\frac{g\delta+\mu+\nu}{\lambda} z \right)} + \frac{\nu g\delta}{(g\delta+\mu)} \right] \ .$$

FOOTNOTES

1. See Winter [1969], Nelson and Winter [1974], and Nelson, Winter and Schuette [1973], for related theoretical attempts in this direction.

2. Besides Friedman [1953], see also Alchian [1950] for the classical statement of the doctrine of economic selection. See Winter [1975] for its perceptive critique.

3. In an example of product innovation briefly considered in Section 3, we interpret $1/c$ as an index of the product quality.

4. Scherer [1965] has found in his empirical study the prevalence of the first causal mechanism.

5. In Section 7 we shall be concerned with the long-run average performance of this ratio.

6. Gibrat's law has, at least in its weak form, faired quite well empirically. See, for instance, Mansfield [1962], Singh and Wittingenton [1968, 1975], and Ijiri and Simon [1967, 1975].

7. Note, however, that these empirical studies have almost unanimously rejected the strong version of Gibrat's law. This is, of course, a heroic assumption to make. All the qualitative conclusions we shall deduce in this paper are, however, independent of this assumption.

8. Unfortunately, we have not been able to analyze the long-run average performance of the industry's state of technology under the alternative hypothesis (IN-ii), of the previous paper, concerning the stochastic nature of innovation.

9. See Demsetz [1973] for a related discussion.

10. See Stigler [1958] for the characteriztion of the survival test.

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