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SCHUMPTERIAN DYNAMICS

I. AN EVOLUTIONARY MODEL OF INNOVATION AND IMITATION

by

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Introduction

"The essential point to grasp...in dealing with capitalism" is, according to Joseph Schumpeter [1950, p. 82], that "we are dealing with an evolutionary process." The evolutionary character of the capitalist process is due to the fact that "the fundamental impulse that sets and keeps [its] engine in motion comes from the new consumers' goods, the new methods of production or transportation, the new markets, the new forms of industrial organization that capitalist enterprise creates" [p. 83]. Such "innovation" then creates a market power which enables the innovator to earn a monopoly profit or what is called an entrepreneurial profit, and it is this prospect of gaining entrepreneurial profit that in turn supplies the motives for innovative activities. But the innovator's monopoly position is only temporary. As soon as an innovation is made, "the spell is broken" and the way for others to imitate is opened up. The first innovation draws followers, and then successful imitation again makes it easier for more imitators to follow suit, until finally the innovation becomes familiar and the associated entrepreneurial profit is wiped out, or until the appearance of another innovation renders it obsolete (Schumpeter [1961]). This process of "Creative Destruction"--the process that "incessantly revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one"--is what Schumpeter regarded as "the essential fact about capitalism" (Schumpeter [1950], p. 83).

The orthodox theory of competitive equilibrium consists precisely of assuming this "fundamental fact about capitalism" away. The notion of competitive equilibrium in its most basic form is defined to be a state of affairs in which a set of prices, one for each commodity, balance demand and supply of all commodities and coordinates the actions of all market participants who take prices as given and determine demands and supplies accordingly. There is thus no one within the system who has any motivation to change the reached position, not to mention the one who strives for creation or destruction. Indeed, from the perspective of the orthodox analysis, the existence of entrepreneurial profit which arises inevitably from successful innovation must be treated as an example of the "imperfection" of competition; the wave of imitations which relentlessly follows the first success must be classified as an "externality" to markets; and the entire process of creative destruction is merely an "adjustment process" which transfers the economy from one equilibrium to another. What Schumpeter considered to be "the essential fact about capitalism" is regarded here as an aberration from the competitive equilibrium—a slip of the Invisible Hand.

This is the first of a series of papers whose major objective is to develop a coherent theoretical framework which is capable of placing the evolutionary process of creative destruction at its central analytical core.[1/] It is an attempt to analyze the phenomena of innovation, imitation, and growth, not as equilibrium outcomes of the far-sighted choices of optimizing economic agents, but as the dynamic processes moved by complex interactions among individual firms which are constantly striving for survival and growth by their competitive struggle against each other. Indeed, underlying the whole series of papers is a premise that even for the analysis of such "long-run" economic phenomena it is essential to begin with the study of disequilibrium processes working at the micro level of firms and to trace out carefully the manner in which they interact with each other and cause the aggregate economy to move from one position to the next. Such a "disequilibrium" view of technological change and economic development has certainly been foreign to the orthodox economists who tend to identify "long-run" with "equilibrium" and dismiss "disequilibrium" as mere "short-run" problems.

By proposing to construct a disequilibrium theory of technical change and economic development, however, I do not claim that the behaviors of the firms are not economically motivated or that they are outright irrational. Indeed, it was Schumpeter, along with many other social thinkers, who considered the capitalism as the quintessential rationalistic civilization and claimed that "capitalism--and not merely economic activity in general--has after all been the propelling force of the rationalization of human behavior" ([1950], p. 125). By rational behavior or rationalistic civilization, Schumpeter meant a way of social life in which "individuals or groups go about dealing with a given situation, first, by trying to make the best of it more or less--never wholly--according to their own lights; second, by doing so according to those rules of consistency which we call logic; and third, by doing so on assumptions which satisfy two conditions: that their number be a minimum and that every one of them be amenable to expression in terms of potential experience" ([1950], p. 122). The main actors of our scenario--the firms--are all supposed to behave rationally in the sense used by Schumpeter. This notion of rationality should, however, be distinguished from the very limited notion of rationality used in the orthodox theory. While the latter identifies rational behaviors as the optimization of a well-specified objective function over a sharply defined set of alternative actions whose outcomes are (at least probabilistically) fully anticipated, the former assumes a much broader position which takes due account of the limits of human capacities to comprehend and compute in the face of uncertain environment and complex cognitive process. It is, in other words, equivalent to what Herbert Simon called the "procedural rationality" or "bounded rationality" (Simon [1957, 1978]).

In order to describe the behavior of the firm in terms of this broader concept of rationality it is convenient to distinguish the short-run from the long-run decision process.2/ At any point in time the behavior of a firm is governed by a given decision policy, which processes the stimuli from its environment in accordance with a pre-existing schema and routinely transforms them into a set of decisions to be taken. This short-run decision policy is the historical product of the firm's search for the better rules in the past, and not the result of any full-fledged optimizing computations which take account of all the data relevant to the current situation. In the short-run, the firm thus "satisfices" (à la H. A. Simon [1957]) rather than optimizes. In the long-run, however, the decision policy itself is subject to changes. As long as the decision policy on the basis of which the firm makes its day-to-day decisions yields satisfactory outcomes (in terms of profits or some other criterion of evaluation), there is little motivation left on the part of the firm to change that policy. As time goes by, however, the market environment including the rivals' decision rules, changes gradually over time or shifts suddenly over night. The current decision policy then begins to yield unsatisfactory outcomes, which sooner or later induce the firm to search for a better decision policy. Search continues until another policy is discovered, whose performance meets a certain pre-specified aspiration level; and a new round of satisficing behavior on its basis starts from that moment. In the long-run, therefore, the firm's decision policy tends to accommodate to the demands of market reality.

This series of papers will be structured in conformity with the preceding account of the firm's decision process. To be specific, the present paper sets up the basic framework for our Schumpeterian dynamics and deduces certain prototypic results. It will be then followed by papers which enlarge the scope of this basic framework. These papers are to be written under the premise that firms' decision rules are given once and for all and their behaviors can be described by the satisficing principle. This premise will indeed allow us to analyze in detail how the dynamic processes of firms' innovation, imitation and growth will interact with each other and shape up the evolutionary pattern of industry structure. The more complex analysis of how the individual firm comes to choose particular policies concerning innovation, imitation and growth in the long-run is postponed until the evolutionary pattern of industry structure, which each firm has to cope with, is spelled out clearly in these papers.[3/] A truly Schumpeterian picture of the economy will emerge only as a synthesis of these two analyses. The present paper is therefore no more than the introduction to the first half of our Schumpeterian dynamics.

Let us begin by specifying our basic model.

1. The State of Technology

Consider an industry which consists of a large number of firms competing with each other. (We denote by M the total number of firms which participate, either actively or potentially, in the working of the industry.) Each firm may produce an undifferentiated product homogeneous throughout the industry, or a unique product of its own which is differentiated from the products of others, depending upon the structure of a particular industry in question. In fact, we shall present, in this series of papers, a theoretical framework which is capable of dealing with any of these alternative industry structures.

Unless there is a universal access to the same and best technology, production method actually employed are different from firm to firm. Let us identify each production method by a positive real number c. Although we call this number the firm's "unit cost" (in terms of a numeraire) for the sake of concreteness, it is only one of many possible interpretations. All that is needed in most of our subsequent investigations is a convention that the smaller the value of c is, the more profitable is the corresponding production method. (Indeed, in one example dealt in the following paper the inverse of c will be given the interpretation of an index of the "quality" of the product processed by the method in question.) If the number of production methods coexisting in an industry is finite (n) we can represent them by a list of unit costs, $c_n < c_{n-1} < \ldots < c_i < \ldots < c_1$. arranged in ascending order. The first in the list c_n then designates the unit cost of the best practice method and the last one c_1 the unit cost of the worst practice method. To describe the "state of technology" of an industry at a point in time, it is therefore necessary to stipulate how these different production methods are distributed across firms.

Let $f_t(c)$ represent the relative frequency of firms whose unit cost equals c at time t. It is, in other words, the frequency function of unit costs at time t. Since only the production methods with unit costs, $c_1, c_2, \ldots, c_n$ are actually employed at time t, the value of $f_t(c)$ is zero for any other value of unit cost. (By convention we have $f_t(c_1) + \ldots + f_t(c_n) = 1$.)

Let $F_t(c)$ denote the cumulative frequency function of unit costs at time t, defined by

$$(1) \qquad F_t(c) \equiv f_t(c_i) + f_t(c_{i+1}) + \ldots + f_t(c_n) \; ;$$

for $c_{i-1} < c \leqq c_i$. (We set, as convention, $F_t(c) = 0$ for $c < c_n$ and $F_t(c) = 1$ for $c \geqq c_1$.) In words, $F_t(c)$ represents the relative frequency of firms with unit cost c <u>or lower</u> at time t. In terms of this cumulative frequency function, the frequency function $f_t(c_i)$ can now be redefined as

$$(2) \qquad f_t(c_i) = F_t(c_i) - F_t(c_{i-1}) \; ,$$

with an understanding that $f_t(c_N) = F_t(c_N)$. Or, more generally, we have

$$(2') \qquad f_t(c) = F_t(c) - F_t(c-o) \; ,$$

where $c-o$ denotes $\lim_{h \to +o} c-h$. Figure I-1 illustrates the relation between $f_t(c)$ and $F_t(c)$.

The frequency function $f_t(c)$ or alternatively, the cumulative frequency function $F_t(c)$ represents how a variety of production methods from the most profitable one to the least, are distributed across firms.

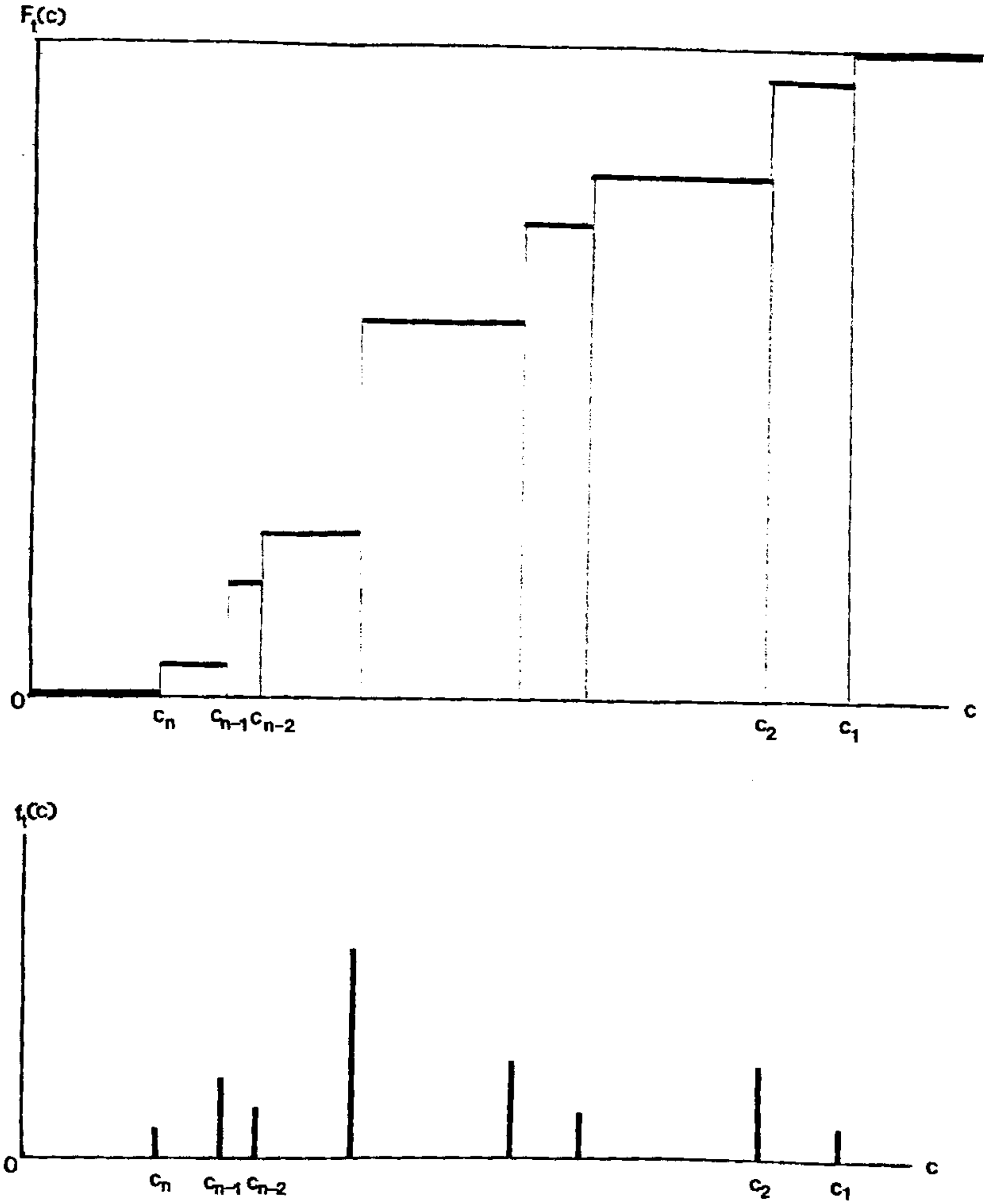

Fig. 1: The relation between $F_t(c)$ and $f_t(c)$

at a given point in time. Either function gives us an equally informative snapshot picture of the industry's "state of technology." Unlike the paradigm of the orthodox economies, however, the state of technology is not a given datum to the industry. As time goes by and future unfolds itself, dynamic competition among firms for technological superiority constantly changes it from one configuration to another. The state of technology is never static and never exogenous in a capitalist economy.

The main aim of the following sections is to develop a theoretical framework which is capable of studying the evolutionary process of the state of technology.

2. Imitation, or Diffusion Process of Technology

In the world of Schumpeterian competition, each firm is constantly striving for a better production method. There are basically two means by which that aim can be achieved. A firm may succeed in putting a new production method into practice by its own R&D effort; i.e., it may succeed in "innovation." The firm can also direct its eyes towards outside; it may indeed "imitate" one of the more profitable methods which are currently employed by other firms. The evolution of the state of technology is therefore determined by the interaction of these two dynamic forces. In order to give an orderly exposition of this complex evolutionary process, however, we shall devote the present section exclusively to the study of the process of imitation, postponing that of the process of innovation until the next section.

Schumpeter wrote:

> [T]he carrying out of new combinations is difficult. ...However, if one or a few have advanced with success, many of the difficulties disappear. Others

> can then follow these pioneers, as they will clearly do under the stimulus of the success now attainable. Their success again makes it easier, through the increasingly complete removal of the obstacles..., for more people to follow suit, until finally the innovation becomes familiar and the acceptance of it a matter of free choice. (Schumpeter [1961], p. 228)

For our purposes it is, however, necessary to translate this somewhat picturesque description of the process of imitation into a much more prosaic mathematical language. Indeed, we shall now introduce an extremely simple hypothesis regarding the probabilistic law which governs the process of imitation among firms in an industry.

Hypothesis (IM'): The probability that a firm is able to copy a particular production method is proportional to the frequency of firms which employ that method in the period in question. The firm, of course, implements only the method whose unit cost is lower than the one currently used by it.[4/]

Formally, it will be assumed that the probability that a firm of unit cost c_i imitates a production method of unit cost c during a small time interval between t and $t+\Delta t$ is equal to

$$(3) \qquad \begin{cases} \mu f_t(c)\Delta t\ , & \text{for} \quad c < c_i\ , \quad \text{and} \\ 0\ , & \text{for} \quad c \geqq c_i\ ; \end{cases}$$

where $\mu > 0$ is a parameter which summarizes the effectiveness of the firm's imitation activity.

The value of the imitation parameter μ should be influenced by the particular imitation policy the firm has come to adopt in its long-run pursuit of survival and growth. In particular, it should be positively correlated with the portion of economic resources the firm is willing to direct towards its imitation activity. The present paper, however, is not concerned with the analysis of how each firm shapes up its imitation policy and chooses (or at least influences) the value of the imitation parameter μ . As was already indicated in the Introduction, the main objective here is rather to work out the mechanism through which a given long-run imitation policy of the firms (along with a given long-run innovation policy, to be discussed in Sections 5 and 6) structures the evolutionary pattern of the industry's state of technology.

We shall, therefore, assume in this paper that the imitation parameter μ is a given constant whose value is a legacy from the past. We shall also assume that the value of μ depends neither on the current unit cost of the firm nor on the unit cost of the production method it wishes to imitate.5/ We shall assume further, for the sake of simplicity, that a new production method once copied can be implemented to the entire productive capacity within a firm without any cost and without any delay. Indeed, throughout this series of papers, all technical changes are supposed to be of the disembodied type. The problem of intra-firm diffusion process of new technical knowledge, as is analyzed, for instance, by Mansfield [1968], Chapter 9, is thus set aside from our investigation.

Now, hypothesis (IM') turns out to be powerful enough to determine (at least approximately) the whole evolutionary pattern of the industry's state of technology. To see this, let us examine the way in which $F_t(c_i)$, the relative frequency of firms with unit cost c_i or less, changes its value from time t to $t+\Delta t$. It is clear that this relative frequency increases whenever one of the firms whose unit cost is higher than c_i succeeds in imitating one of the firms with unit cost c_i or less. Now the relative frequency of firms whose unit costs are higher than c_i equals $f_t(c_{i-1}) + f_t(c_{i-2}) + \ldots + f_t(c_1)$, which can be conveniently rewritten as $1 - F_t(c_i)$ by (1). On the other hand, hypothesis (IM') tells us that the probability that <u>each</u> of these firms succeeds in imitating one of the production methods with unit cost c_i or higher during a time interval between t and $t+\Delta t$ is equal to $\mu f_t(c_i)\Delta t + \mu f_t(c_{i+1})\Delta t + \ldots + \mu f_t(c_n)\Delta t$, which can be conveniently rewritten as $\mu F_t(c_i)\Delta t$ by (1). (Here, we have ignored the very small probability that a firm succeeds in copying two or more production methods simultaneously during a small time interval Δt .) We can therefore compute the <u>expected</u> increase in $F_t(c_i)$ during a time interval between t and $t+\Delta t$ as the product of these two expressions:

$$\{\mu F_t(c_i)\Delta t\} \cdot \{1 - F_t(c_i)\} .$$

In fact, if the total number of firms M is very large, the so-called law of large numbers allows us to use this expression for a good approximation for the <u>actual</u> increase in $F_t(c_i)$. In what follows, we assume this is indeed the case and treat the above expressions as representing the actual increase in $F_t(c_i)$.

Of course, even among firms whose unit costs are lower than c_i , relatively higher cost firms are imitating the production methods of the lower cost firms. It is, however, plain that these infra-marginal imitation activities result only in infra-marginal transfers of frequencies and do not affect the value of $F_t(c_i)$ itself.

We have thus obtained an equation which describes the change in the relative frequency of firms of unit cost c_i or less, from time t to $t+\Delta t$, effected by the firms' imitation activities in an industry:

$$(4) \qquad F_{t+\Delta t}(c_i) - F_t(c_i) = \mu F_t(c_i)(1 - F_t(c_i))\Delta t \ .$$

Furthermore, if we divide the both sides of this equation by Δt and let Δt approach zero, we can transform it in the following differential equation:

$$(4') \qquad \dot{F}_t(c_i) = \mu F_t(c_i)(1 - F_t(c_i)) \ ,$$

where $\dot{F}_t(c_i)$ represents the time derivative of $F_t(c_i)$. Since the same argument can be applied without any modification to any value of unit cost, we have, in fact, obtained the following series of differential equations:

$$(5) \qquad \begin{cases} \dot{F}_t(c_n) = \mu F_t(c_n)(1 - F_t(c_n)) \\ \quad \vdots \\ \dot{F}_t(c_i) = \mu F_t(c_i)(1 - F_t(c_i)) \\ \quad \vdots \\ \dot{F}_t(c_1) = \mu F_t(c_1)(1 - F_t(c_i)) \ . \end{cases}$$

It requires only a moment's reflection to recognize that each of the above series of differential equations is nothing but a well-known "logistic differential equation," which appears frequently in population biology and mathematical ecology. (See, for example, Pearl and Reed [1924], Lotka [1925], or any modern textbook on these subjects. Samuelson [1947], pp. 291-94, also contains a useful discussion on this form of differential equation.) It is very easy to show that this logistic differential equation has the following form of explicit solution, which is called the "logistic growth curve."

$$(6)\qquad \begin{cases} F_t(c_n) = \dfrac{1}{1 + (1/F_T(c_n) - 1)\exp[-\mu(t-T)]} \\ \quad\vdots \\ F_t(c_i) = \dfrac{1}{1 + (1/F_T(c_i) - 1)\exp[-\mu(t-T)]} \\ \quad\vdots \\ F_t(c_1) = \dfrac{1}{1 + (1/F_T(c_1) - 1)\exp[-\mu(t-T)]} , \end{cases}$$

where $\exp(\cdot)$ stands for exponential, and $F_T(c_i)$ represents the cumulative frequency function at a given time T ($\leqq t$) in the past. From this it is also easy to deduce the expression for the growth pattern of the frequency function $f_t(c)$ by invoking the relation (2). But, here, we do not bother to write it down.

Figure 2 illustrates the foregoing result. Each of the S-shaped curves traces a logistic growth curve that represents the growth pattern of the cumulative frequency function of firms. In particular, the one at the lowest layer depicts the growth pattern of the relative frequency of firms with the least unit cost c_n. When only a small number of firms employ this production method, its growth is hesitant and slow. But as this number gradually increases, imitation activities of the less efficient firms become more and more successful. "The spell is broken," and a bandwagon sets in motion. The growth rate then accelerates, until a half of total population comes to adopt this method. Once this median point is passed, the effect of saturation steps in and the growth rate

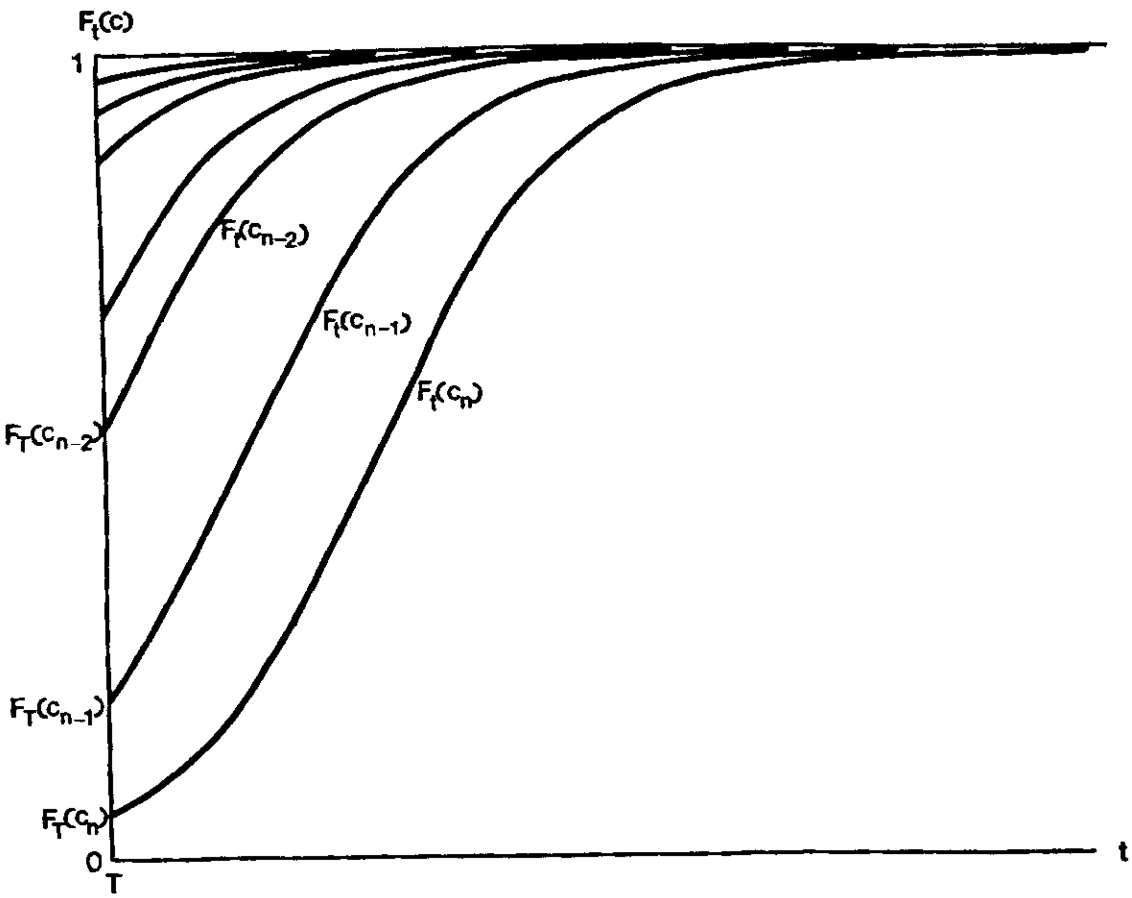

Fig. 2: The evolution of the state of technology under the pressure of imitation process

starts decelerating. But the growth itself continues until the whole population of firms is swamped by this best practice method. The fate of the less efficient production method, on the other hand, can be easily read by tracing out the changing width of a strip formed by two adjacent logistic curves. Initially its number may expand by absorbing the firms with less efficient techniques. But sooner or later it will lose ground to the more efficient techniques, and will find its way to the eventual extinction.

The idea of using the logistic curve to describe a band-wagon phenomenon that can be commonly observed in a variety of diffusion process of a new idea, new technique, new instrument, and so forth is not new. Indeed, there is an abundant literature on this application in economics and other social sciences. (See, for example, Coleman, Katz, and Menzel [1957], Griliches [1957], Ozga [1960], and Mansfield [1968], Ch. 8.) Outside of social sciences the so-called "models of epidemics" deal with mathematically similar problems. (See, e.g., Bailey [1957].) What seems novel about our foregoing analysis is its application of the logistic law to the description of the evolutionary pattern of whole array of production methods coexisting side by side at the same time. And it is this small innovation which allows us to study the dynamic interaction between processes of imitation and innovation in an integrated manner, as we shall soon see.

3. Innovation

As is shown in the preceding section, firms' imitation activities will gradually upgrade their production techniques, and, if other things are equal, all the firms will eventually succeed in adopting the best practice method. This limiting state must be the paradigm of neoclassical economics in which every market participant is supposed to have complete access to the best technical knowledge of the society.

Other things, however, do not forever remain the same. The tendency towards technological uniformity among firms is bound to be upset by a sudden introduction of a new and better production method by one of the firms. Indeed, to destroy the stalemate brought about by imitation process and to create a new industrial structure is the role our capitalist economy has assigned to Schumpeterian entrepreneurs or to innovative firms. It is this "process of creative destruction" that is "what capitalism consists in and what every capitalist concern has got to live in" (Schumpeter [1950], p. 83). Let us now turn to the formal analysis of this perennial gale of creative destruction.

Suppose that at some point in time one of the firms finally succeeds in implementing a new production method whose unit cost equals c_{n+1} ($< c_n$). We denote by $T(c_{n+1})$ the time at which this method is introduced for the first time and call it "the innovation time" for the production method with unit cost c_{n+1}. (This somehwat clumsy notation will make more sense in Section 5.) Since the total number of firms is M and hence each firm's share is $1/M$, this innovation creates a new relative frequency of the magnitude of $1/M$ at the new and lower unit cost c_{n+1}. That is, we have

(7) $$F_{T(c_{n+1})}(c_{n+1}) = \frac{1}{M} .$$

No sooner does this innovation occur than do all the other firms start struggling to imitate it. A firm or two will eventually make a headway, and after that a wave of imitation follows. Under hypothesis (IM), this sets in motion a new logistic growth curve of $F_t(c_{n+1})$ from the initial condition (7) given above. Hence, we have for $t \geqq T(c_{n+1})$.

(8) $$F_t(c_{n+1}) = \frac{1}{1 + (1/F_{T(c_{n+1})}(c_{n+1}) - 1)\exp[-\mu(t - T(c_{n+1}))]}$$

$$= \frac{1}{1 + (M-1)\exp[-\mu(t - T(c_{n+1}))]} .$$

How does this innovation affect the evolutionary pattern of the state of technology of the industry as a whole? The answer to this question depends upon whether or not the innovator has used the best practice production method before innovation. We first examine a special case.

Let us suppose that the innovator of c_{n+1} has employed the then best practice method c_n before the innovation time $T(c_{n+1})$. In this case, the size of $f_t(c_n)$ declines by $1/M$ at the time of $T(c_{n+1})$, but this decline is recouped at the same time by the new creation of an equal magnitude of $f_t(c_{n+1})$, as shown in (7). Obviously, this exchange of an equal mass of frequency leaves unaffected the cumulative frequency $F_t(c_n)$, for it is nothing but the sum of $f_t(c_n)$ and $f_t(c_{n+1})$. It then follows that even after the innovation time $T(c_{n+1})$, the cumulative frequency $F_t(c_n)$ keeps moving along the same old logistic growth curve (6). Indeed, since the innovation in question involves no other production method, all the other cumulative frequencies must follow the same old logistic curves as well. Part of Figure 3 around the innovation time $T(c_{n+1})$ illustrates all this. By comparing it with Figure 2, the reader can immediately see that the only alteration we made to the latter is to superimpose a new logistic growth curve that starts with an initial mass $1/M$ at time $T(c_{n+1})$.

Innovation is not a single-shot phenomenon. No sooner than an innovation occurs, a new round of competition for a better production method begins. And no sooner than a winner of this game is named, another round of competition for a still better production method is set out. And so forth. Innovation is by nature a recurrent process.

Accordingly, let $T(c_{n+2})$, $T(c_{n+3})$, ..., $T(c_N)$, ..., denote times at which production methods with unit costs $c_{n+2} > c_{n+3} > \dots > c_N > \dots$ are introduced for the first time into an industry, respectively. We call $T(c_N)$ the "innovation time" of the production method with unit cost c_N and $T(c_N) - T(c_{N-1})$ the "waiting time" for a new method with c_N . (There is, of course, no reason to believe that these innovation times are evenly distributed over time.) Then, at each innovation time a new frequency emerges

$$(9) \qquad F_{T(c_N)}(c_N) = \frac{1}{M}$$

and a new logistic curve starts its growth path from that instant on,

$$(10) \qquad F_t(c_N) = \frac{1}{1 + (M-1)\exp[-\mu(t - T(c_N))]} , \quad t \geq T(c_N) .$$

If, as in the case of the first innovation, innovations always emerge from the class of firms which have practiced the then best production method, we can repeatedly apply the same argument as was given earlier

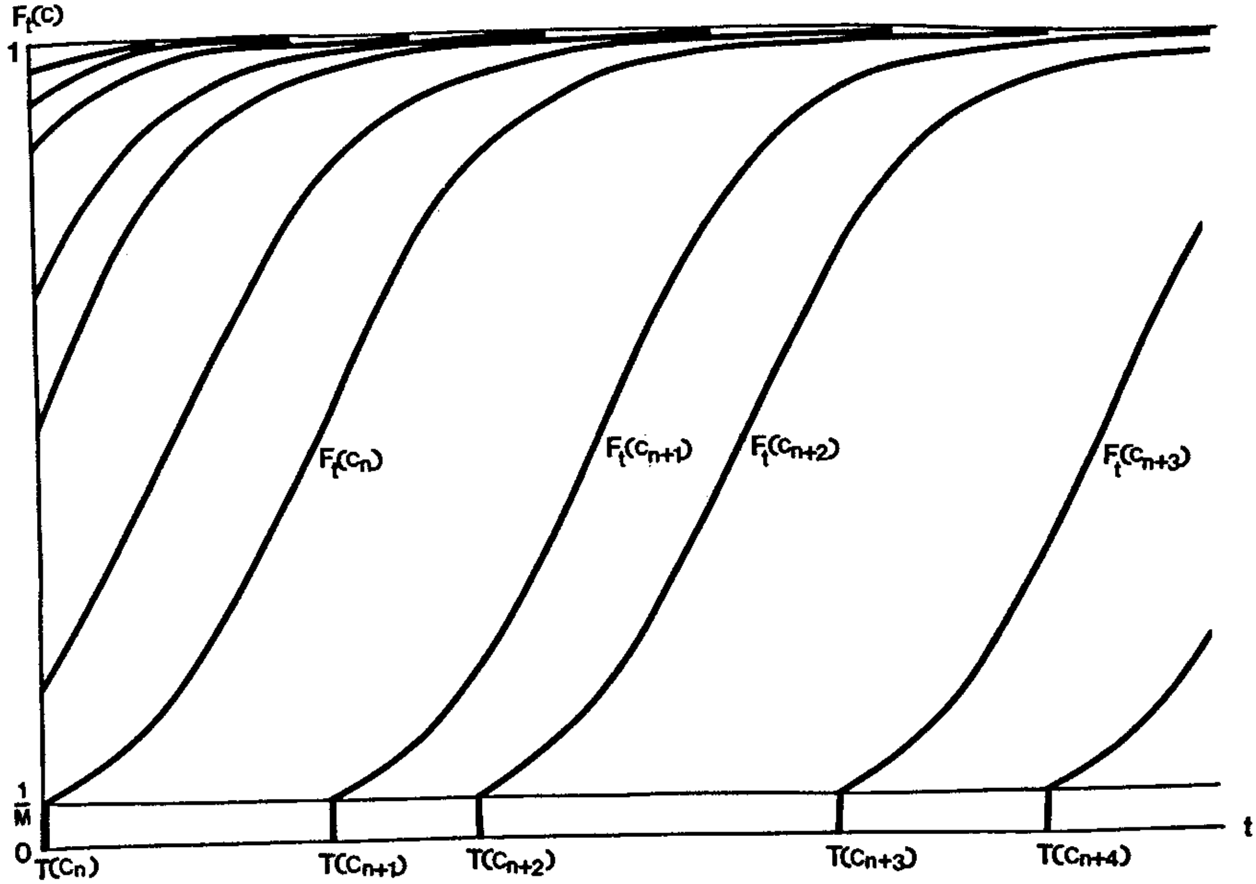

Fig. 3: The evolution of the state of technology under the joint pressure of innovation and imitation -- the case where only the technologically most advanced firms can innovate

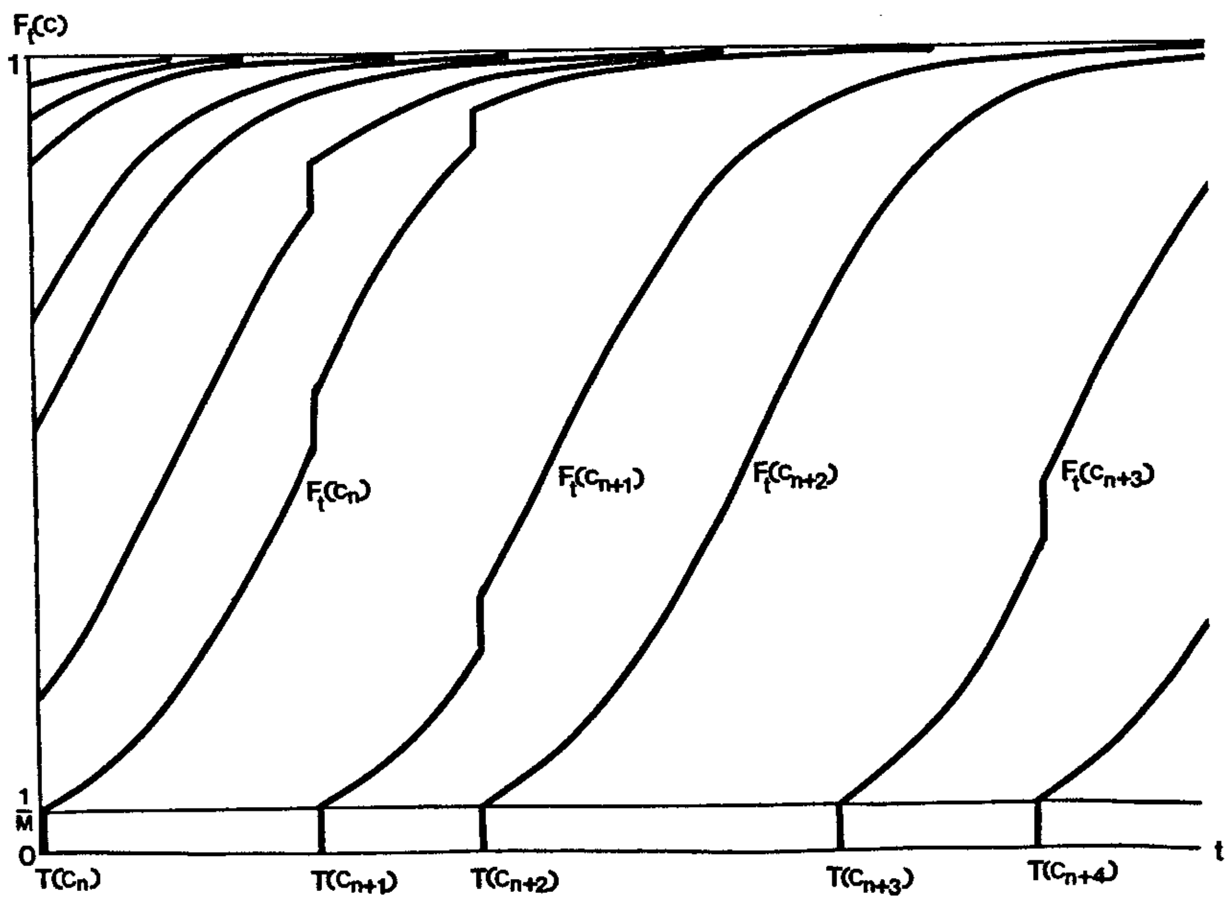

Fig. 4: The evolution of the state of technology under the joint pressure of innovation and imitation -- the general case

and claim that none of these successive innovations perturb the logistic growth patterns of all the cumulative frequencies of the currently practiced production methods. They only add a new logistic growth curve (10) one by one from the bottom at each innovation time. This process is explained in Figure 3.

We are now in a position to examine the more general case by removing the supposition so far made that innovations always emerge from the class of technologically most advanced firms. Then, the evolutionary pattern of the state of technology becomes slightly more complex.

Suppose, for example, that production method with c_{n+1} unit cost is introduced into an industry by a firm whose pre-innovation unit cost c_i is not the most efficient one, that is, $c_i > c_n$. Then, at time $T(c_{n+1})$, the frequency of firms at c_i declines by the magnitude $1/M$, and at the same time a new mass of frequency of firms with the equal size $1/M$ emerges at c_{n+1}. (See (7).) Then, it is not difficult to see that this exchange of a distribution mass also causes an equal discrete jump of all the cumulative frequencies of firms whose unit cost is lower than c_i. Formally, this can be represented as

(11) $$F_{T(c_{n+1})}(c) = F_{T(c_{n+1})-0}(c) + \frac{1}{M}, \quad \text{for} \quad c_i > c \geqq c_{n+1},$$

$$F_{T(c_{n+1})}(c) = F_{T(c_{n+1})-0}(c) \quad , \quad \text{for} \quad c \geqq c_i ;$$

where $T(c_{n+1})-0$ represents a time immediately before the innovation time $T(c_{n+1})$. As soon as the jumps are made at time $T(c_{n+1})$, these cumulative frequencies begin their new logistic growth process from new initial conditions (11). Thus, we have for $t \geqq T(c_{n+1})$

$$(12) \qquad F_t(c) = \frac{1}{1 + (1/F_{T(c_{n+1})}(c) - 1)\exp[-\mu(t - T(c_{n+1}))]} \, .$$

This new logistic growth process will continue until it is again upset by another innovation at the next innovation time $T(c_{n+2})$. At time $T(c_{n+2})$, yet another logistic growth process will be set off only to be upset once again at the next innovation time $T(c_{n+3})$. And so forth. Figure 4 presents an evolutionary pattern of the state of technology in this general case.

4. A Specific Model of Innovation

In the preceding sections we have seen how the process of imitation and the process of innovation interact with each other and mold the evolutionary history of an industry's state of technology. The process of imitation works essentially as an equilibriating force that continually tends the industry towards a static equilibrium, in which all firms employ the same production technique. The function of innovation, on the other hand, lies precisely in upsetting such an equilibriating tendency. It is a discontinuous process which breaks up the existing order of an industry and forces the state of technology to become more progressive but more volatile.

The purpose of this section and the next two is to study the long-run consequence of the interaction of these opposite forces on the development of the state of technology. To this end we have to specify the structure of firms' innovation activities in more detail.

Basic or applied scientific researches in private firms, governmental institutions and academia, weekend experiments of amateur inventors in their backyard garages, and so forth continuously expand the stock of technical knowledge potentially applicable to industrial production. But such a continuous inflow of new technical knowledge or "inventions" does not necessarily lead to a corresponding improvement of production methods actually employed in an industry. "As long as they are not carried into practice, inventions are economically irrelevant" (Schumpeter [1961], p. 88). For the purpose of industrial production, the potentially must be transformed into the actuality; a production method hitherto untried must be put into industrial practice. This is what we mean by the word "innovation," which must be conceptually distinguished from "invention."

Let us denote by $C(t)$ the unit cost of the best production method that is "technologically possible" at time t but has thus far resisted the actual use in the industry. (For the sake of simplicity we ignore all the problems associated with the uncertainty as well as fuzziness inherent in delineating what is technologically feasible from what is not.) We call $C(t)$ the unit cost of the potential production method or, more simply, the "potential unit cost" at time t. It is then reasonable to suppose that the continuous inflow of technological knowledge or continuous supply of inventions constantly reduces the potential unit cost of the industry, so that we have

(13) $\dot{C}(t) < 0$.

This paper, however, does not probe into the mechanism of "inventive" activity itself; it is merely supposed to occur outside of the industry and beyond the control of the individual firms. This is, of course, a heroic assumption to maintain.[6/]

It then becomes possible to characterize the notion of "innovation" formally as the activity by which a firm puts into practice the potential production method and thus succeeds in reducing its unit cost to the level of potential unit cost. Now, let $T(c)$ denote the inverse function of $C(t)$, defined by

(14) $$T(C(t)) \equiv t \quad \text{or} \quad C(T(c)) \equiv c \; .$$

(Because of the monotone decreasingness of $C(t)$ with respect to t, as is assumed in (13), $T(c)$ is also a monotonically decreasing function of c.) We know that if an innovation occurs at time t it introduces a production method with $C(t)$ unit cost for the first time into an industry. It then follows that if a particular production method with unit cost c is presently in use it must have been introduced into the industry at time $T(c)$, for in view of the inverse relation (13) we have $c = C(T(c))$. The function $T(c)$ can then be interpreted as the "innovation time" for a given production method with unit cost c, and this interpretation and the notation are perfectly consistent with the definition of the same concept we introduced in the preceding section.

Later we shall find it useful to introduce the following hypothesis which further specializes the dynamics of the potential unit cost $C(t)$.

Hypothesis (PC): The potential unit cost is declining at a constant rate over time.

More formally this hypothesis supposes that

(13') $$C(t) = \exp(-\lambda t) \; ,$$

where λ is a positive constant. (For convenience, we set $C(0) = 1$.) Under this special hypothesis, the innovation time $T(c)$ --the inverse of $C(t)$ --can be expressed simply as a logarithmic function of c, or

(14') $\quad T(c) = -\frac{1}{\lambda}\ln c$.

This special hypothesis will simplify our later exposition.

We have seen above what innovation consists of. But we have not seen who does innovation. For this purpose we have to specify in more detail the stochastic process that characterizes the way innovation occurs. We shall indeed consider two alternative models, which can be regarded as two polar cases spanning more realistic situations as their convex combinations. Let us explore these two models separately.

5. The State of Technology in the Long-Run (I)

In the first case, we postulate the following hypothesis concerning the stochastic nature of innovative activity.

Hypothesis (IN-i): Every firm has a small but equal chance for successful innovation at every point in time.

Let $\nu \cdot \Delta t$ be the probability that a firm succeeds in carrying out an innovation during a small time interval Δt ; where ν is a positive constant which is supposed to be of the much smaller order of magnitude than the innovation parameter μ . Then, the probability that an innovation is successfully carried out by one of the firms during a time interval Δt becomes equal to

(15) $\quad \nu M \Delta t$.

The probability that two or more firms simultaneously succeed in innovation is extremely small and hence ignored. Hypothesis (IN-i) amounts to saying that the occurrence of innovation is subject to the law of rare events or to the Poisson law which supposes that whether or not an innovation occurs in any time interval is independent of whether or not an innovation occurs in any time interval preceding it. (This is called the lack of memory property of Poisson process.)

The innovation parameter ν represents the effectiveness of each firm's innovation activity. Its value should, therefore, reflect a particular innovation policy the firm has come to choose as a critical pillar of its long-run growth strategy. In the present paper, however, we are concerned only with analyzing how the evolutionary pattern of the industry's state of technology is causally determined by a given innovation policy of the firm, together with its imitation policy. The study of how the firm selects a particular innovation policy in the long-run and how this long-run decision process reflects the evolutionary pattern of the industry's state of technology is left for the future research.

The state of technology of an industry is a historical outcome of the dynamic interactions between the process of imitation and the process of innovation. The process of imitation is a force which moves the entire state of technology along the family of logistic growth curves, whereas the process of innovation is a force which disturbs this smooth journey and restructures the state of technology from time to time. As time goes on, however, innovation takes place over and over again. After a long period of time, it is expected that a certain statistical regularity will emerge out of this random pattern of the occurrency of innovations. (For instance, it is not difficult to show that after a long passage of time the average rate of innovation tends to approach a constant value νM, under the Poisson hypothesis (IN-i).) Indeed, not only the dynamic pattern of innovation but the entire state of technology are also expected in the long-run to exhibit a tendency towards certain statistical regularity as a long-run averaging result of the dynamic balance between the forces of imitation and of innovation.

Let $F_t^*(c)$ represent the expected cumulative frequency function of unit costs at time t. We shall now turn to the study of the behavior of this expected cumulative frequency function. Since we are now concerned only with describing the industry's state of technology "in the long-run," this is all that we have to do.

Now, we know from (5) that the cumulative frequency function $F_t(c)$ increases by $F_t(c)(1-F_t(c))\Delta t$, if no innovation occurs during a time interval $[t, t+\Delta t]$. If, on the other hand, an innovation occurs during the same time interval, it creates a new cumulative frequency $F_t(C(t))$ of the size equal to $1/M$. When the innovator has belonged to the class of firms whose unit costs are higher than c, this innovation increases $F_t(c)$ by the same magnitude $1/M$. When, however, the innovator is from the class of firms whose unit costs are less than or equal to c, the innovation effects only an infra-marginal exchange of an equal mass of frequency and leaves the value of $F_t(c)$ unchanged. Since by hypothesis (IN-i) the probability of an innovation during a time period of Δt is $\nu M \Delta t$ and the fraction of firms whose unit costs are higher than c is $1-F_t(c)$, the expected number of innovators whose unit costs are higher than c can be calculated as $\nu M(1-F_t(c))\Delta t$ during $[t, t+\Delta t]$. We can thus conclude that the cumulative frequency function $F_t(c)$ increases on average by $\{\mu F_t(c)(1-F_t(c)) + \nu M(1-F_t(c))(1/M)\}\Delta t$ from time t to $t+\Delta t$. In terms of expected cumulative frequency function $F_t^*(c)$, we can state this result as

$$(16) \qquad \dot{F}_t^*(c) = \mu F_t^*(c)(1-F_t^*(c)) + \nu(1-F_t^*(c)) ,$$

or

$$(16') \qquad \frac{d}{dt}\left\{F_t^*(c) + \frac{\nu}{\mu}\right\} = (\mu+\nu)\cdot\left\{F_t^*(c) + \frac{\nu}{\mu}\right\}\cdot\left\{1 - \frac{F_t^*(c) + \nu/\mu}{1+\nu/\mu}\right\} .$$

This again is a logistic differential equation of $F_t^*(c) + \nu/\mu$, which has an explicit solution of the form:

$$(17) \qquad F_t^*(c) + \frac{\nu}{\mu} = \frac{1 + \nu/\mu}{1 + \left[\frac{1+\nu/\mu}{F_{T(c)}^*(c) + \nu/\mu}\right]\exp[-(\mu+\nu)(t - T(c))]} ,$$

for $t \geqq T(c)$; where $T(c)$ is the innovation time for a given unit cost c and $F_{T(c)}^*(c)$ is the expected value of the cumulative distribution at that point of time. Although $F_{T(c)}(c)$ equals $1/M$ when an innovation occurs, the probability of an innovation at a particular instant is of course equal to zero. Hence, $F_{T(c)}^*(c) = 0$, and we can simplify (17) as:

$$(17') \qquad F_t^*(c) = \frac{1 + \nu/\mu}{1 + (\mu/\nu)\cdot\exp[-(\mu+\nu)(t - T(c))]} - \frac{\nu}{\mu} .$$

In order to study the structure of this expected cumulative frequency function further, we have to invoke the hypothesis (PC) which assumes that the potential unit cost $C(t)$ declines at a constant rate λ over time. Let

$$(18) \qquad z \equiv \ln c - \ln C(t) \ (\geqq 0)$$

represent the proportionate difference between the unit cost c of a given production method and the potential unit cost $C(t)$. We call this "cost gap," for short. By definition the potential unit cost has zero cost gap, and all the other feasible production methods have positive cost gaps. Then, in view of the logarithmic relation (14') between a given unit cost c and its innovation time $T(c)$, we can rewrite (17') in terms of $z \geqq 0$ as follows:

$$(19) \qquad F_t^*(c) = \tilde{F}(z)$$

$$\equiv \frac{1 + \nu/\mu}{1 + (\mu/\nu)\cdot\exp[-(\mu+\nu)z/\lambda]} - \frac{\nu}{\mu},$$

which is independent of the calendar time! $\tilde{F}(z)$ is the "long-run average cumulative frequency function" of unit costs or cost gaps, we have sought to deduce under the hypothesis (IN-i). It is a function only of the cost gap, $z \equiv \ell n\ c - \ell n\ C(t)$, and not of the value of the potential unit cost $C(t)$ itself. Figure 5 illustrates the structure of this long-run average cumulative frequency function. It has the shape of a truncated logistic growth curve, with growth parameter $(\mu+\nu)/\lambda$ and initial slope ν/λ.

The long-run average cumulative frequency distribution obtained above is a long-run statistical summary of the way in which firms in the industry are distributed over a multitude of diverse production methods with different unit costs. It shows that while the continuous inflow of new technological knowledge constantly reduces the potential unit cost over time, the industry will never be able to enjoy its fruits fully and unit costs of a majority of firms will always lag behind the potential one. The industry's state of technology will thus never approach a neoclassical equilibrium of uniform technological knowledge. Indeed, it is only the relative shape of the distribution of firms over different cost conditions which exhibits any tendency for a statistical regularity over the long-run course of events in the industry.

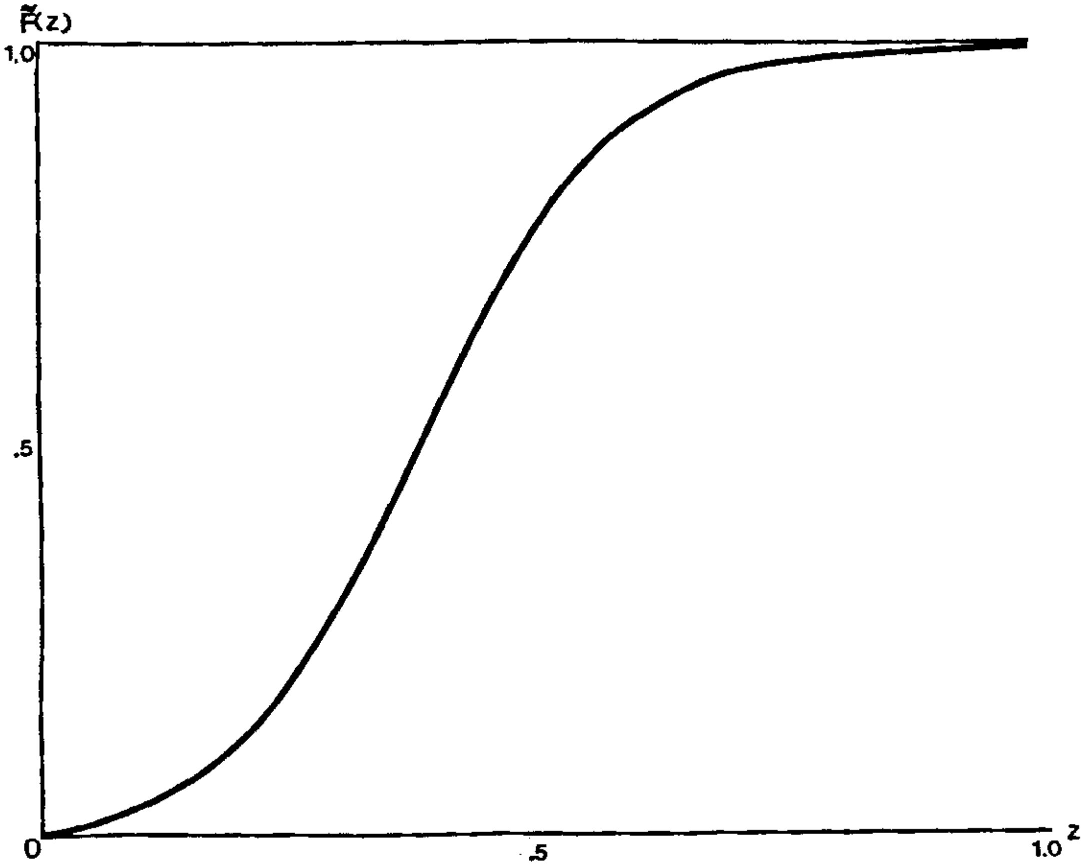

Fig. 5: The long-run average cumulative frequency function of cost gaps under hypothesis (IN-i), when $\lambda = .05$, $\nu = .01$ and $\mu = .50$

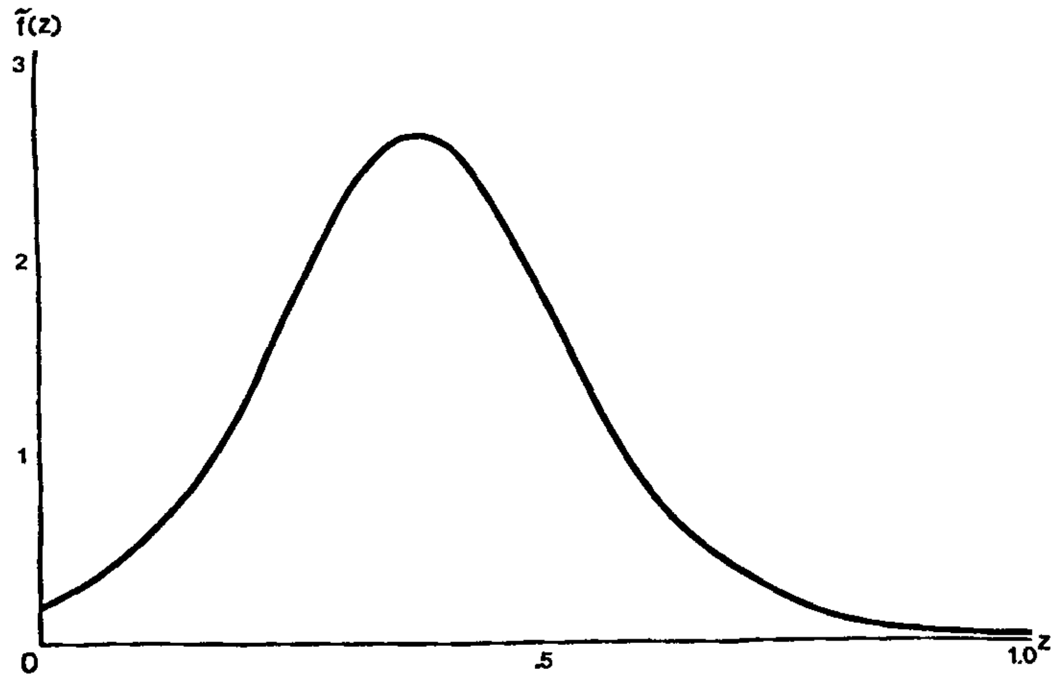

Fig. 6: The long-run average density function of cost gaps under hypothesis (IN-i)

The dynamic interaction between the forces of innovation and imitation, together with the exogenous inflow of new technological knowledge, is what maintains the relative configuration of the state of technology in a statistical equilibrium in the form of (19). In order to study how a change in each of these forces will shift this delicate statistical balance, it is easier to examine the density form of the long-run average frequency function, given by

$$(20) \qquad f(z) \equiv \frac{d\tilde{F}(z)}{df}$$

$$= \frac{\dfrac{(\mu+\nu)^2}{\lambda\mu}}{\left\{\sqrt{\dfrac{\nu}{\mu}}\cdot\exp\left(\dfrac{\mu+\nu}{2\lambda}z\right) + \sqrt{\dfrac{\mu}{\nu}}\cdot\exp\left(-\dfrac{\mu+\nu}{2\lambda}z\right)\right\}^2},$$

for $z \geqq 0$. As is depicted in Figure 6, the long-run average density distribution is a smooth bell-shaped curve, truncated at the left. It has a peak of the height equal to $(\mu+\nu)^2/4\lambda\mu$ at the value of cost gap equal to $[\lambda/(\mu+\nu)]\cdot\ell n(\mu/\nu)$, and the intercept equal to ν/λ at the zero cost gap. It is thus not difficult to see that an increase in the declining rate of potential unit cost, λ, tends to widen on average the cost gaps of the industry and at the same time to disperse their distribution across firms, that an increase in the rate of innovation, ν, tends, albeit weakly, to narrow on average the cost gaps and concentrate their distribution, and that an increase in the rate of imitation, μ, also tends to narrow on average the cost gaps and concentrate their distribution.[7/] Figures 7, 8 and 9, respectively, illustrate these comparative statics results numerically. (The base value of parameters, $\lambda = .05$, $\nu = .01$, $\mu = .50$, means, if the number of firms in the industry is 20, (i) that the potential unit cost declines

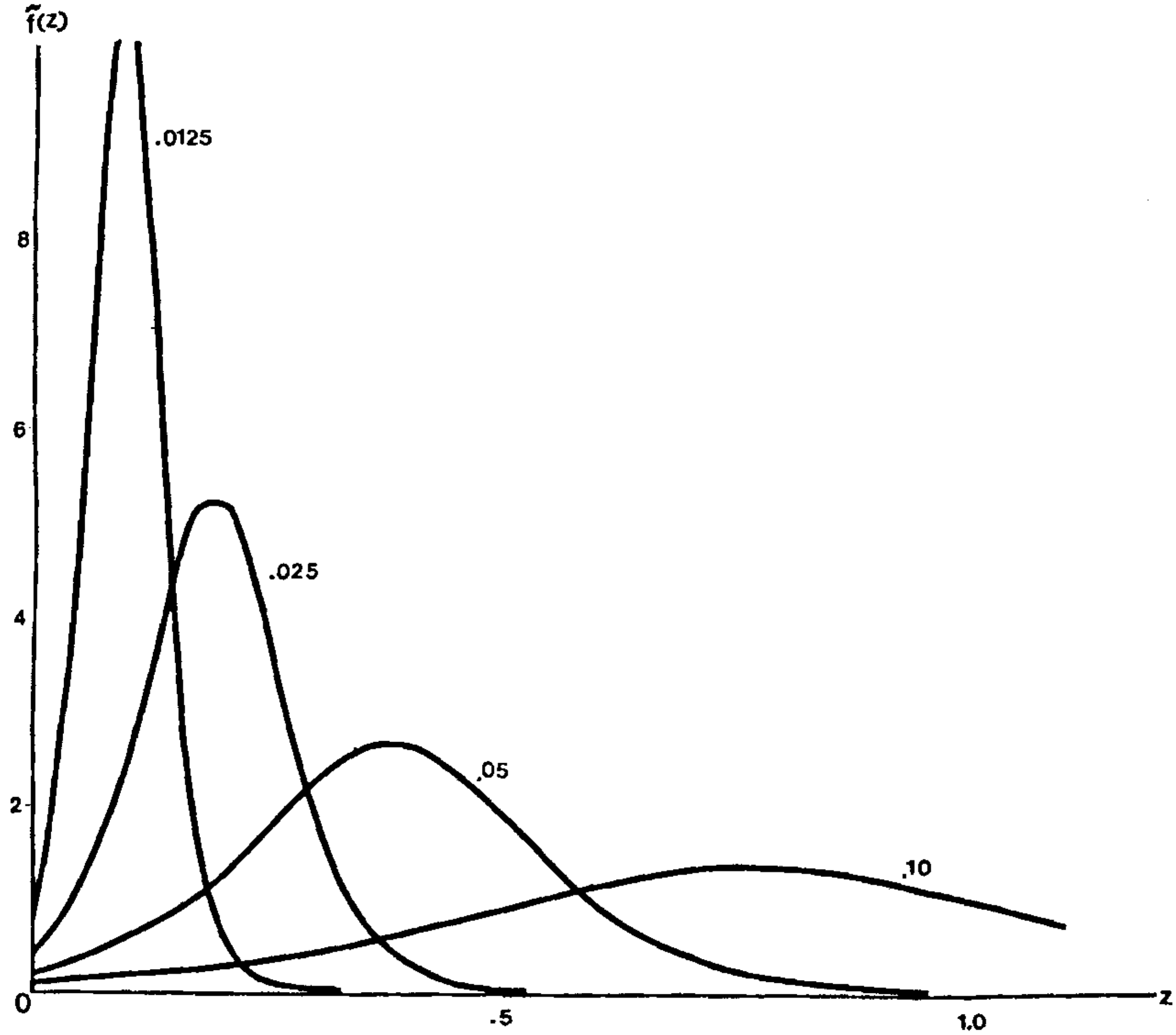

Fig. 7: The long-run average density functions under hypothesis (IN-i) for various values of λ (where ν and μ are fixed at .01 and .50 respectively)

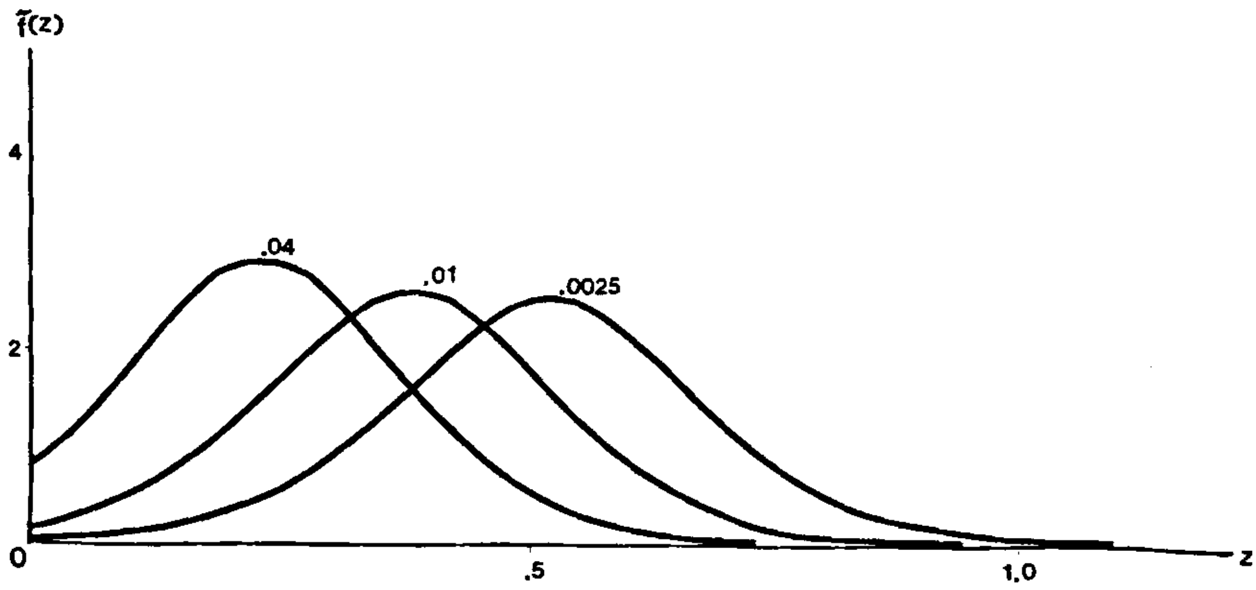

Fig. 8: The long-run average density functions under hypothesis (IN-i) for various values of ν (where λ = .05 and μ = .10)

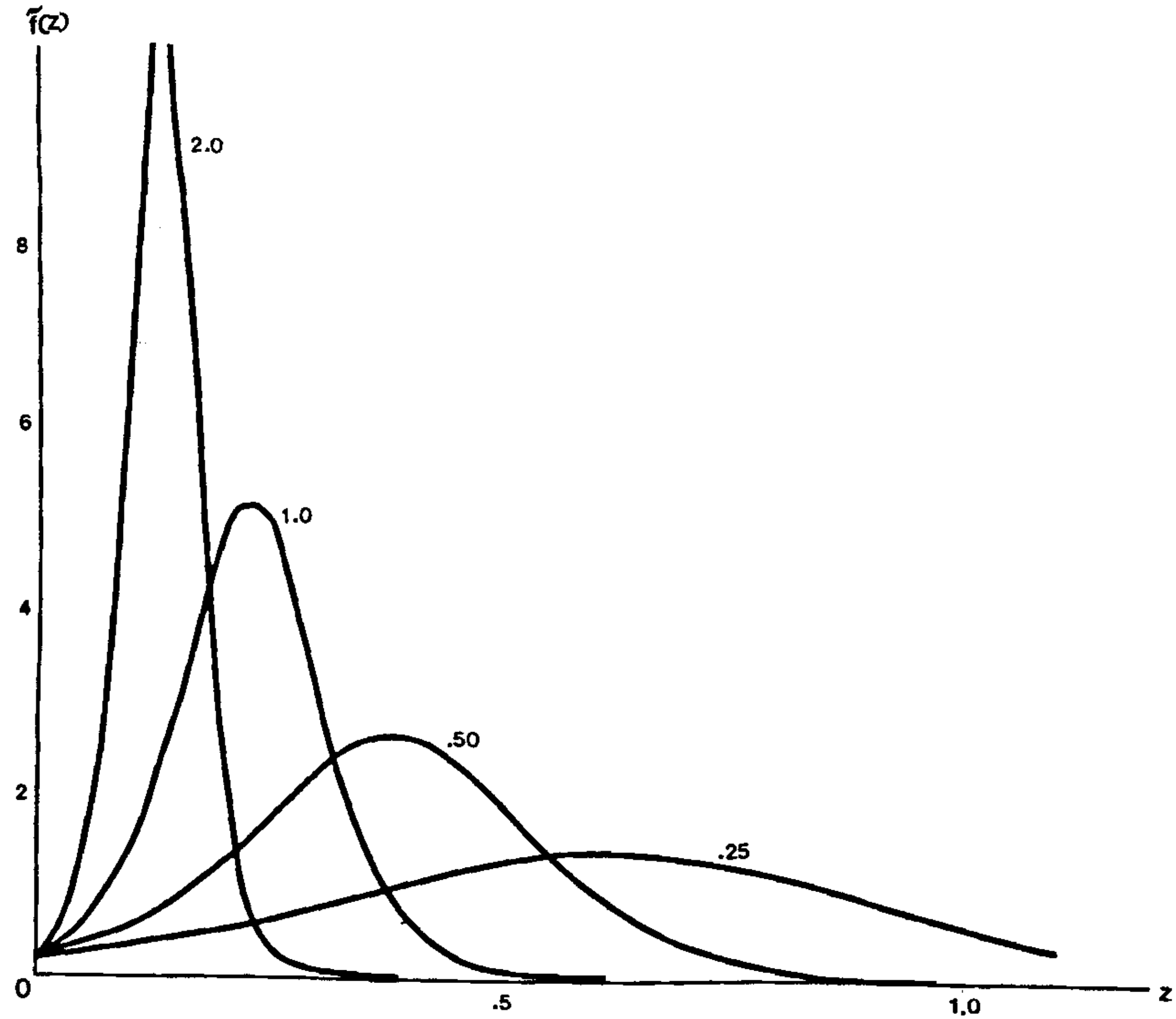

Fig. 9: The long-run average density functions under hypothesis (IN-i) for various values of μ (where $\lambda = .05$ and $\nu = .01$)

5% annually, (ii) that the average lag between invention and innovation is 5 years, and (iii) that it takes on average 5.89 years for a half of the firms to succeed in imitating an innovation.)

It should be borne in mind that the long-run average frequency distribution of cost gaps, $\tilde{F}(z)$ or $\tilde{f}(z)$, is no more than a long-run statistical summary of the evolutionary pattern of the state of technology. It never implies that the industry's state of technology will, in the long-run, converge to a static equilibrium. Far from it, the state of technology is a state of constant flux. As was illustrated in Figures 3 and 4, it is continuously moved by the force of imitation, and discontinuously disrupted by the force of innovation. Its year-to-year or decade-to-decade evolution exhibits no tendency towards equilibrium. All that is claimed here is merely that if the long history of the development of the industry's state of technology is patiently studied, it is possible to detect the existence of certain statistical regularities out of its seemingly irregular evolutionary pattern.

6. The State of Technology in the Long-Run (II)

In the second special case, we introduce the following hypothesis concerning the nature of innovative activities.

Hypothesis (IN-ii): Innovation is always carried out by a firm technologically most advanced at the time of innovation. Among those firms which are potentially capable of carrying out innovation the chance for success is equal at every point in time.

This hypothesis is, of course, an opposite extreme of hypothesis (IN-i) which insisted that every firm, whether technologically advanced or not, is potentially capable of striking innovation. Needless to say, it corresponds to the special case we examined in Section 3. Although we found it easy to illustrate, by means of a diagram, the evolutionary pattern of the state of technology in this case, the analysis of its long-run average performance turns out to be far more involved.

Let $\xi\Delta t$ represent the very small probability that one of the technologically most advanced firms succeeds in carrying out an innovation during a small time interval Δt ; where ξ is a very small positive constant. Then, the foregoing hypothesis can be restated more formally in the following manner. Suppose that the best practice production method at time t has unit cost equal to c_N which was introduced into an industry at time $T(c_N)$ ($\leq t$) . Then the number of firms which employ this production method at time t can be computed as

$$F_t(c_N)\cdot M = \frac{M}{1 + (M-1)\exp[-\mu(t - T(c_N))]} .$$

Since the hypothesis (IN-ii) insists that only those firms whose unit cost is c_N are potentially capable of introducing a new and better production method c_{N+1} and that any of those potential innovators has an equal chance for success, the probability that an innovation occurs during a small time interval between t and $t+\Delta t$ must be equal to $\xi\Delta t$ times the number of those firms given above, or

$$(21) \qquad \frac{M\xi\Delta t}{1 + (M-1)\exp[-\mu(t - T(c_N))]} .$$

Consider the sequence of successive waiting times for innovation, $T(c_2) - T(c_1)$, $T(c_3) - T(c_2)$, ..., $T(c_N) - T(c_{N-1})$, ... Under hypothesis (IN-ii) they can be regarded as random variables which are identically distributed and independent of each other. In fact, that hypothesis enables us to compute explicitly the probability distribution of each waiting time. Let $U(s)$ denote the cumulative probability distribution of the waiting time $T(c_N) - T(c_{N-1})$. Then a calculation whose detail is relegated to Appendix 1, shows that it has the form of

$$(22) \qquad U(s) = 1 - \left[\frac{M-1}{M} + \frac{1}{M}\exp(\mu s)\right]^{-\xi M/\mu},$$

for $s \geqq 0$. From this we can also calculate the expected waiting time for innovation τ as

$$(23) \qquad \tau \equiv E\{T(c_N) - T(c_{N-1})\}$$

$$= \sum_{n=0}^{\infty}\left(\frac{M-1}{M}\right)^n \cdot \frac{1}{\mu n + \xi M},$$

which is a decreasing function of ξ and μ. (See Appendix 2 for the derivation.) The waiting time for innovation is thus expected to shorten as the effectiveness of innovative or imitative activity tends to increase.

In contrast to the first case, the probability of an innovation is uneven under hypothesis (IN-ii). The probability of the next innovation is very small immediately after the occurrence of one innovation (for there is only one firm capable of striking it), but, as more and more firms succeed in imitating the best practice method and become potential innovators, this probability rises accordingly until the whole population of firms become capable of innovation.

As time goes on, however, innovation takes place over and over again. After a sufficient number of years, therefore, a certain statistical regularity is expected to emerge out of the seemingly uneven pattern of the occurrence of innovations. For instance, it is expected that the average number of innovations during a given time interval should depend in the long-run only on the length of that interval, and not on the calendar time itself. This is indeed the case, for we can establish the following limit theorem. Let $N(t)$ be the number of innovations from time zero to time t. (This amounts to saying that the best practice production technique at time t has unit cost $c_{N(t)}$.) Then, as $t \to \infty$, for any time interval $\Delta t > 0$ we have

$$(24) \qquad E\{N(t+\Delta t) - N(t)\} \to \frac{1}{\tau}\Delta t \ .$$

(This is an adaptation of the so-called renewal theorem in probability theory. See, for instance, Feller (1966), Ch. XI, for the proof of the renewal theorem as well as for the related discussions.) This theorem says that after a long passage of time the time rate of innovation, $(N(t+\Delta t) - N(t))/\Delta t$, on average tends to a constant rate whose value is equal to the reciprocal of the expected waiting time τ.

The fact that the dynamic pattern of innovations will in the long-run settle down to statistical uniformity suggests to us that even under the hypothesis (IN-ii) the entire state of technology itself, whose total structure is shaped up by a dynamic interaction between the forces of innovation and imitation, will also exhibit a statistical tendency towards regularity. Let $F_t^*(c)$ be, as before, the expected cumulative frequency function of unit costs at time t. Then, we are indeed able to show that as $t \to \infty$

$$(25) \qquad F_t^*(c) \to \tilde{F}(z)$$

$$= \frac{1}{\tau}\int_0^{z/\lambda} \left[1 + (M-1)\exp\left\{-\frac{\mu}{\lambda}z + \mu y\right\}\right]^{-1} \cdot \left[\frac{M-1}{M} + \frac{1}{M}\exp(\mu y)\right]^{-(\xi M/\mu)} \cdot dy \ ,$$

for $z \equiv \ell n\ c - \ell n\ C(t) \geqq 0$, independently of the calendar time t . Since the proof of this theorem is rather lengthy, it is relegated to Appendix 3. $\tilde{F}(z)$ introduced above is the "long-run average" cumulative frequency function of cost gaps under the hypothesis (IN-ii). It is a function only of the value of cost gap z , the relative measure of the distance between a given unit cost c and the potential unit cost $C(t)$.

Figure 10 illustrates a typical shape of $\tilde{F}(z)$ .8/ As is seen from this diagram, the long-run average cumulative frequency function of cost gaps has also a shape like the logistic growth curve even under hypothesis (IN-ii). But, as is indicated by its density form $\tilde{f}(z) \equiv d\tilde{F}(z)/dz$ illustrated in Figure 11, it is, unlike the true logistic growth curve, skewed to the left. Figures 12, 13, 14 and 15 then illustrate numerically the influence of a change in each parameter value on the shape of the density form of the long-run average frequency function of cost gaps. The first diagram shows that an increase in the declining rate of potential unit cost λ tends to widen the average gap between unit costs and the potential unit cost and at the same time to disperse their distribution across firms. The second one shows that an increase in the rate of innovation (among the technological leaders) ξ has a tendency, albeit weak, to narrow the cost gaps and to make their distribution more concentrated. The third one shows that an increase in the rate of imitation μ also tends to narrow the cost gaps and concentrate their distribution. Finally, the last diagram shows that the shape of the long-run average frequency function is sensitive to a change in the number of firms under hypothesis (IN-ii). An increase in M in fact has a tendency to widen on average the cost gaps and at the same time to make their distribution across firms more concentrated.

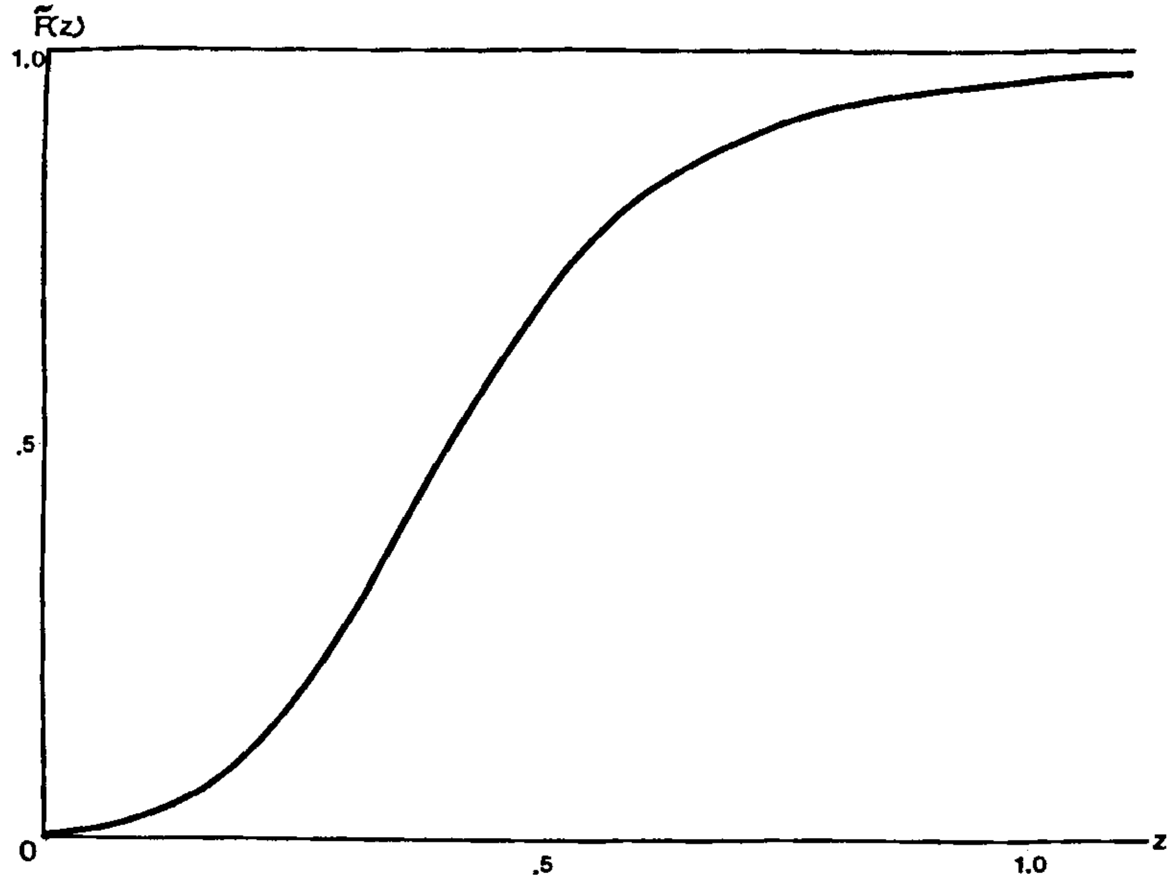

Fig. 10: The long-run average cumulative frequency function of cost gaps under hypothesis (IN-ii), when $\lambda = .05$, $\xi = .01$, $\mu = .50$ and $M = 20$)

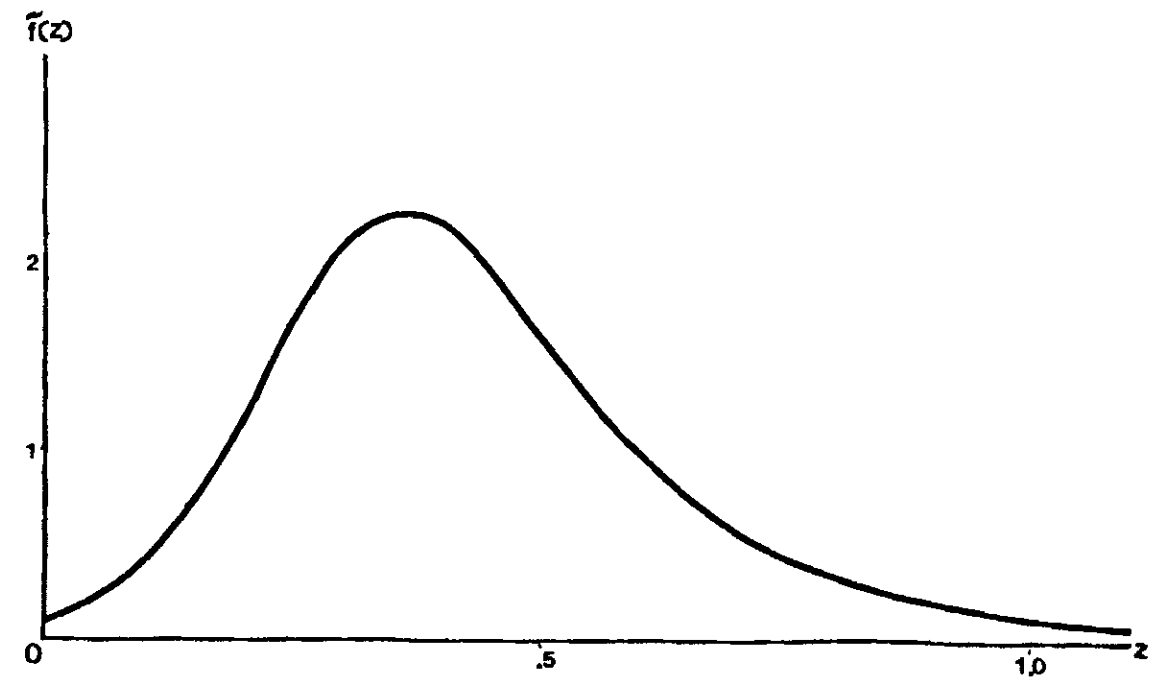

Fig. 11: The long-run average density function of cost gaps under hypothesis (IN-ii)

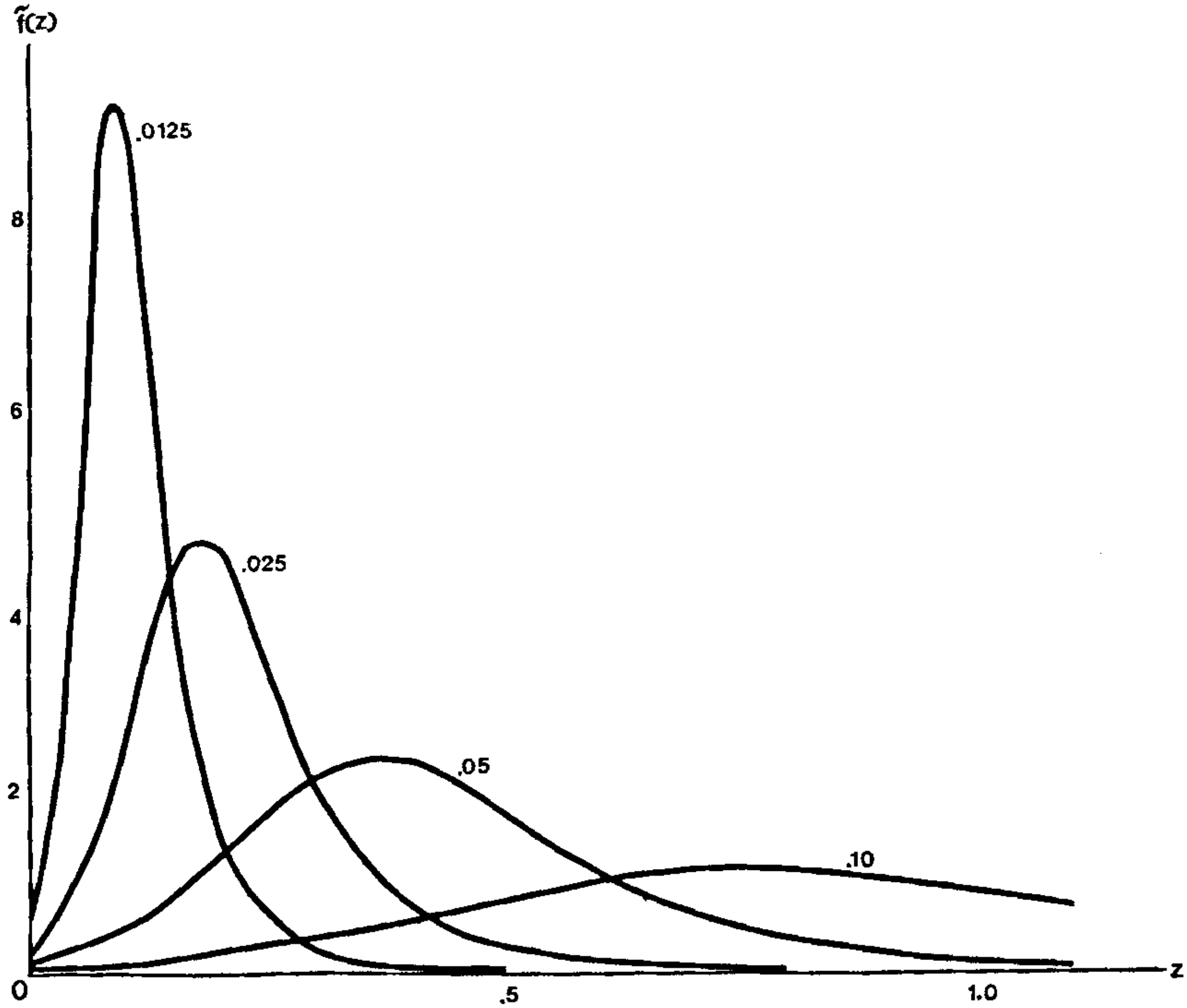

Fig. 12: The long-run average density functions under hypothesis (IN-ii) for various values of λ (where $\xi = .01$, $\mu = .50$ and $M = 20$)

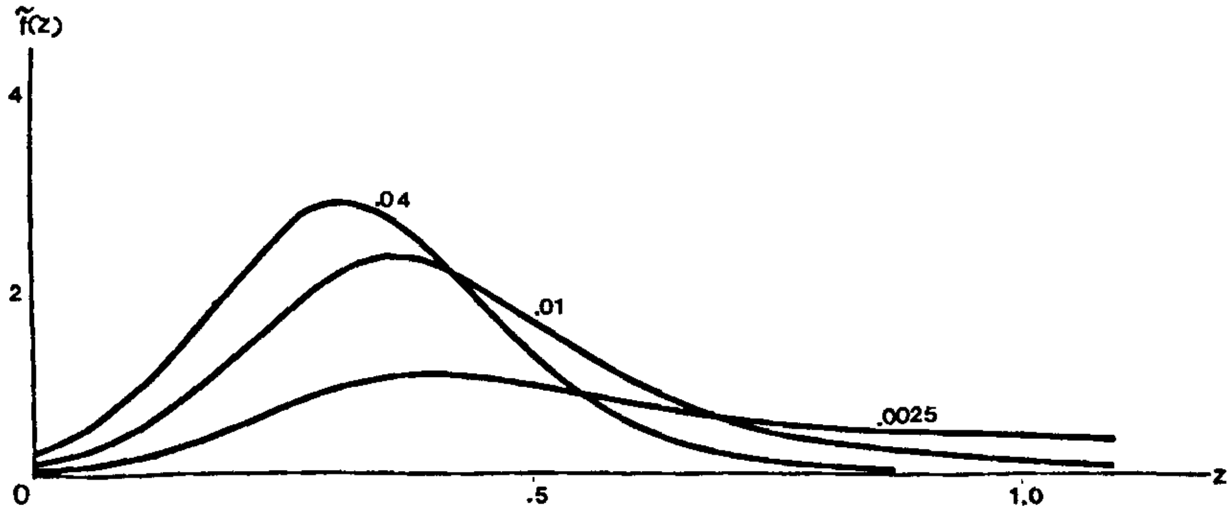

Fig. 13: The long-run average density functions under hypothesis (IN-ii) for various values of ξ (where $\lambda = .05$, $\mu = .50$ and $M = 20$)

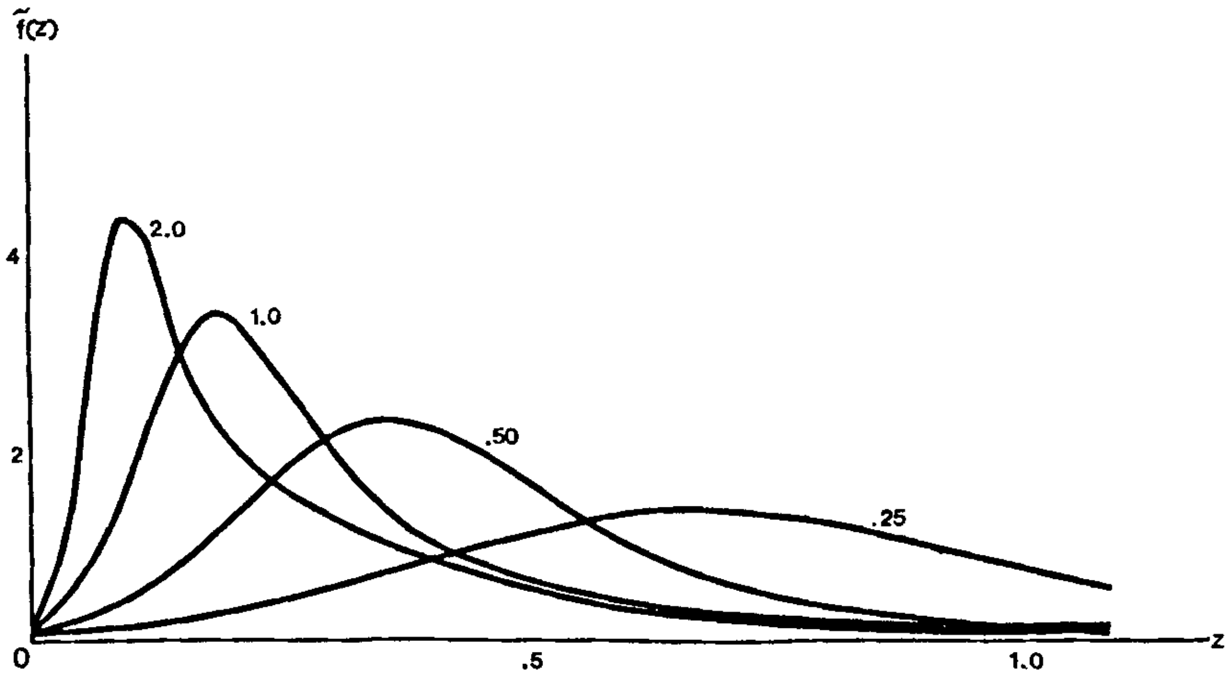

Fig. 14: The long-run average density functions under hypothesis (IN-ii) for various values of μ (where $\lambda = .05$, $\xi = .01$ and $M = 20$)

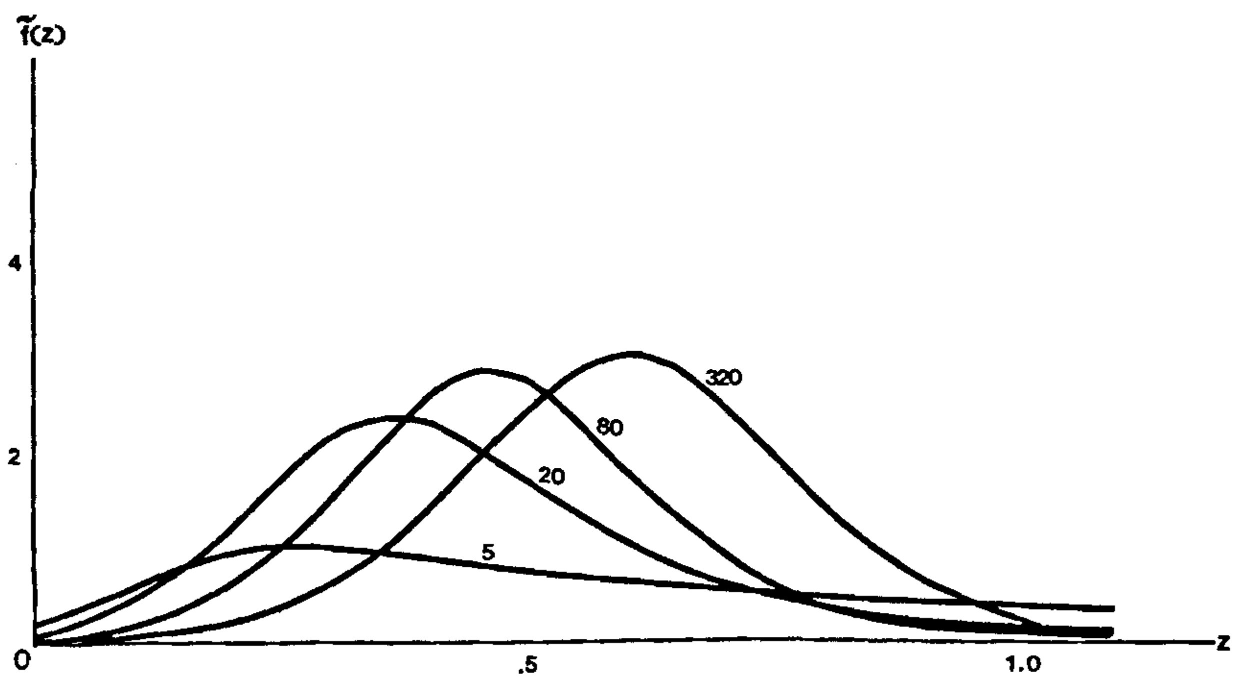

Fig. 15: The long-run average density functions under hypothesis (IN-ii) for various values of M (where $\lambda = .05$, $\xi = .01$ and $\mu = .50$)

7. Concluding Remarks

In the present paper, we have developed a simple evolutionary model of technological innovation and imitation, which captures some of the essential features of what Joseph Schumpeter called the process of creative destruction in a capitalist economy. In the first place, we have been able to describe the dynamic pattern of the diffusion of a new production method through firms' imitation activities as a logistic growth process, in which the rate of diffusion is slow and hesitant initially, but soon accelerates by the so-called bandwagon effect, and then slows down again as the industry is more and more saturated with the production method in question. The firms' imitation activities thus constitute an equilibriating force of technology, which tends the industry's state of technology (not uniformly but logistically) towards a neoclassical equilibrium, in which all the firms have full access to the most efficient production method available. We have then seen that the function of innovation lies precisely in upsetting this equilibriating tendency. Whenever it occurs, it breaks up the existing order of an industry and starts a new round of competitive struggle for imitation. The evolution of the industry's state of technology is therefore governed by the dynamic interaction between the continuous and equilibriating force of imitation and the discontinuous and disequilibriating force of innovation. In fact, we have been able to show how these two opposite forces will work hand in hand to generate a certain statistical regularity in the way in which the relative configuration of the distribution of efficiencies across firms develops itself over time. Under the joint pressure of imitation and innovation, the industry will not reach a neoclassical equilibrium with perfect technological knowledge even in the long-run. While new technological knowledge constantly flows into the industry, actual production methods of a majority of firms always lag behind it, and a multitude of diverse production methods with a wide range of efficiencies will coexist forever. Indeed, it is merely the statistical regularity of the relative pattern of these microscopic disequilibria that characterizes "the long-run" of the industry.

The only economic principle we have employed in the present paper is that of efficiency, namely, that firms always desire to adopt the more efficient or more profitable production method, whenever possible. (That they are not always able to do so is, of course, the starting-point of this paper.) All the results we have obtained here are therefore founded ultimately on this weakest of all economic principles. The task of the sequels is to introduce more specifically economic principles into the basic model and to work out their implications for our Schumpeterian dynamics. In particular, in the sequel (Part II: Technological Progress, Firm Growth and "Economic Selection") we shall introduce another simple economic principle, that firms successful in innovation and imitation grow relatively faster than less successful ones, and study how the process of firm growth and the process of technological innovation and imitation work hand in hand to mold the evolutionary pattern of the industry's state of technology.

APPENDIX 1

Let $U(s)$ be the cumulative probability of the waiting time for innovation $T(c_{N+1}) - T(c_N)$. Suppose that none of the firms have been successful in innovation during a time interval $[0,s)$ after the last innovation time $T(c_N)$. Clearly, the probability of this occurrence is given by $1 - U(s)$. On the other hand, by (21) the probability that one of the firms will introduce a new production method with c_{N+1} unit cost during the next small time interval Δt equals $\xi M\Delta t/\{1+(M-1)\exp[-\mu s]\}$. Since the probability that the production method with c_{N+1} unit cost will be introduced <u>for the first time</u> during the same small interval is the probability that no firms have been successful in $[0,s)$ <u>and</u> one of the firms becomes successful during the next small interval Δt , we have the following equation:

(A1) $$U(s+\Delta t) - U(s) = (1-U(s))\cdot\{\xi M\Delta t/[1+(M-1)\exp(-\mu s)]\} .$$

By letting $\Delta t \to 0$, we obtain:

(A2) $$\frac{dU(s)}{dt} = (1-U(s))\cdot[\xi M/\{1+(M-1)\exp(-\mu s)\}] .$$

We can solve this differential equation as follows. Rewrite (A2) as

(A3) $$\frac{d(1-U(s))}{dt}\Big/(1-U(s)) = -\xi M/\{(1+(M-1)\exp(-\mu s))\} ,$$

and integrate, we obtain

(A4) $$\ell n(1-U(s)) = C - \xi M\int_0^s \frac{1}{1+(M-1)\exp(-\mu t)}dt ;$$

where C is an integration constant. If we note $U(0) = 0$, we can rewrite (A4) as follows

$$\text{(A5)} \qquad U(s) = 1 - \exp\left[-\xi M \int_0^s \frac{1}{1 + (M-1)\exp(-\mu t)} dt\right] .$$

Now, the term inside of the exponential function must be computed. Let $y(s)$ represent $1 + (M-1)\exp(-\mu s)$. Then

$$\int_0^s \frac{1}{1 + (M-1)\exp(-\mu t)} dt = \int_M^{y(s)} \frac{1}{y} \cdot \frac{dt}{dy} \cdot dy$$

$$= \int_M^{y(s)} \frac{1}{y} \cdot \frac{1}{-\mu(y-1)} \cdot dy$$

$$= \frac{1}{\mu} \cdot \int_{y(s)}^M \left(\frac{1}{y-1} - \frac{1}{y}\right) dy = \frac{1}{\mu}\left[\ln\frac{y-1}{y}\right]_{y(s)}^M$$

$$= \frac{1}{\mu} \ln\left[\frac{M-1}{M} + \frac{1}{M}\exp(\mu s)\right] .$$

Substituting of this into (A4), we finally obtain

$$\text{(A6)} \qquad U(s) = 1 - \exp\left\{-\xi M \cdot \frac{1}{\mu} \ln\left[\frac{1}{M}\exp(\mu s) + \frac{M-1}{M}\right]\right\}$$

$$= 1 - \left[\frac{1}{M}\exp(\mu s) + \frac{M-1}{M}\right]^{-\xi M/\mu} .$$

APPENDIX 2

The expected waiting time is given by

$$\text{(A7)} \qquad \tau \equiv E\{T(c_{N+1}) - T(c_N)\} = \int_0^\infty s\cdot dU(s) \ .$$

By integrating the right hand side by parts, we have

$$\int_0^\infty s\cdot dU(s) = \int_0^\infty [1 - U(s)]ds \ .$$

Hence, by (A6), we obtain

$$\text{(A8)} \qquad \tau = \int_0^\infty \left[\frac{M-1}{M} + \frac{1}{M}\exp(\mu s)\right]^{-\xi M/\mu} ds$$

$$= \frac{1}{\mu}\cdot\int_0^1 t^{(\xi M/\mu)-1}\cdot\left(1 - \frac{M-1}{M}t\right)^{-1}\cdot dt \ ,$$

where $t \equiv [(M-1)/M + (1/M)\exp(\mu s)]^{-1}$, which can be further rewritten as

$$= \frac{1}{\mu}\cdot\int_0^1 t^{(\xi M/\mu)-1}\cdot\sum_{n=0}^\infty \left(\frac{M-1}{M}\right)^n t^n\cdot dt$$

$$= \frac{1}{\mu}\cdot\sum_{n=0}^\infty \left[\left(\frac{M-1}{M}\right)^n\cdot\int_0^1 t^{n-1+(\xi M/\mu)} dt\right]$$

$$= \sum_{n=0}^\infty \left(\frac{M-1}{M}\right)^n\cdot\frac{1}{\mu n + \xi M} \ .$$

APPENDIX 3

The purpose of this appendix is to deduce the long-run average cumulative distribution $\tilde{F}(z)$ under hypothesis (IN-ii).

As a preliminary for this, let us introduce several random variables which play useful roles in the analysis that follows. Let $N(t)$ be, as in the main text, the number of innovations from time 0 to t. Then, we can represent by $c_{N(t)}$ the unit cost of the best practice method at time t. We can also represent by $c_{N(t)+1}$ the unit cost associated with the next innovation. Obviously, $c_{N(t)} \geqq C(t) > c_{N(t)+1}$, where $C(t)$ is the potential unit cost at time t. Moreover, since we can represent the innovation times of $c_{N(t)}$ and $c_{N(t)+1}$ by $T(c_{N(t)})$ and $T(c_{N(t)+1})$, this inequality implies by (13) and (14) that $T(c_{N(t)}) \leqq t < T(c_{N(t)+1})$. Then, $T(c_{N(t)+1}) - t$ represents the time period from t to the next innovation time. We call this "the residual waiting time" at time t. (The time elapsed since the last innovation time, $t - T(c_{N(t)})$, may be called "the spent waiting time," but we are not concerned with this in this appendix.) Let us denote by $H_t(y)$ the probability that $T(c_{N(t)+1}) - t \leq y$. In other words, $H_t(y)$ represents the probability that the first innovation time after time t lies within a time-interval $(t, t+y]$. In the theory of renewal process it is well known that as $t \to \infty$ we have

$$(A9) \qquad H_t(y) \to \tilde{H}(y) \equiv \frac{1}{\tau}\cdot\int_0^y [1 - U(s)]ds$$

where $U(s)$ and τ are the distribution function of the waiting time and its expected value, introduced in the preceding two appendices. (See

Feller [1966], p. .) Substituting (A6), we obtain the formula for the limit distribution $\tilde{H}(y)$ as

$$\text{(A9')} \qquad \tilde{H}(y) = \frac{1}{\tau}\cdot\int_0^y \left[\frac{M-1}{M}+\frac{1}{M}\exp(\mu s)\right]^{-(\xi M/\mu)} ds .$$

Consider $F_t(c)$, the relative frequency of firms with unit cost c or less at time t. Let $s \equiv T(c)$ denote the innovation time corresponding to c. Then, $N(s)$ represents the number of innovations from time zero to s, $c_{N(s)}$ the unit cost of the best practice method at time s, and $c_{N(s)+1}$ the unit cost of the first innovation following time s. We have $c \equiv C(s) > c_{N(s)+1}$. By the inverse relation (14), we also have $s < T(c_{N(s)+1})$. (We suppose, for the time being, that $s \neq T(c_{N(s)})$.) Now, when $t < T(c_{N(s)+1})$, an innovation with unit cost $c_{N(s)+1}$ has not occurred as yet, so that $F_t(c_{N(s)+1})$ equals zero. Indeed, since by definition $c < c_{N(s)+1}$, $F_t(c)$ also equals zero. Then, at the innovation time of $c_{N(s)+1}$, i.e., at $t = T(c_{N(s)+1})$, the value of $F_t(c_{N(s)+1})$ jumps to $1/M$. Together with this, the value of $F_t(c)$, for $c_{N(s)+1} \leqq c < c_{N(s)}$, also jumps from zero to $1/M$. After this innovation, $F_t(c)$ starts growing under the pressure of the firms' imitation activities. In fact, it is plain from the discussion of the special example in Section 3 that under the hypothesis of (IN-ii) $F_t(c)$ grows according to a logistic growth curve:

$$\text{(A10)} \qquad F_t(c) = \frac{1}{1 + (M-1)\exp[-\mu(t - T(c_{N(s)+1}))]}, \quad t \geqq T(c_{N(s)+1}),$$

independently of the pattern of the occurrence of innovations that follow.

Let us rewrite (A10) as follows:

$$\text{(A10')}\qquad F_t(c) = \frac{1}{1 + (M-1)\exp\{-\mu(t-s) + \mu[T(c_{N(s)+1}) - s]\}}$$

$$= \frac{1}{1 + (M-1)\exp[-(\mu/\lambda)z + \mu y]} ,$$

for $y \leqq z/\lambda$; where $z \equiv \ell n\ c - \ell n\ C(t) = -\lambda(s-t)$ and $y \equiv T(c_{N(s)+1}) - s$. The variable y in the above expression is nothing but the residual waiting time at time s and has the probability distribution $H_s(y)$. The expected cumulative distribution can be therefore calculated as

$$\text{(A11)}\qquad F_t^*(c) = \int_0^{z/\lambda} \frac{1}{1 + (M-1)\exp[-(\mu/\lambda)z + \mu y]} \cdot H_s(dy) .$$

(In calculating this expected cumulative probablity, we have ignored the possibility of $s = T(c_{N(s)})$, for its probability is zero in any way.) Let t and s go to infinity, while keeping z constant, and take note of the limit theorem on $H_s(y)$, given by (A9'), we finally obtain the limit theorem that

$$\text{(A12)}\qquad F_t^*(c) \to \tilde{F}(z)$$

$$\equiv \int_0^{z/\lambda} \left[1 + (M-1)e^{-(\mu/\lambda)z+\mu y}\right]^{-1} \cdot \frac{1}{\tau} \cdot \left[\frac{M-1}{M} + \frac{1}{M}e^{\mu y}\right]^{-(\xi M/\mu)} \cdot dy .$$

which is independent of the calendar time t .

FOOTNOTES

1. Recent attempts at formalizing the "vision" of Schumpeter · Winter [1969], Nelson and Winter [1973, 1974, 1976, 1977] and Futia [1980]. Our indebtedness to their pioneering works ought to be obvious.

2. See Iwai [1981], Chapter 2 for an application of the similar idea to the theory of expectation-formation.

3. This will link our Schumpeterian dynamics to the so-called neo-Schumpeterian models of Scherer [1967], Kamien and Schwarts [1972, 1975], Loury [1979], Dasgupta and Stiglitz [1980a, 1980b] and others. Their analyses, however, treat the firm's R&D activity as a one-shot game and fail to situate it in a long-run evolutionary process of industrial development.

4. In the next paper which takes an explicit account of the process of capacity growth, this hypothesis will be modified into:

   Hypothesis (IM): The probability that a firm is able to copy a particular production method is proportional to the share of total productive capacity which employs that method in the period in question.

5. It is, however, possible to replace the assumption (3) by another: that the firm imitates only the best practice production method, or $\mu f_t(c)\Delta t$, for $c = c_n$ (the unit cost of the best practice method), and 0 otherwise, and then to reproduce qualitatively most of the results obtained under (3).

6. In the modern economy where many firms are engaged in Research and Development activities, however, the distinction between invention and innovation has become very fuzzy. I am in fact planning to develop an alternative model in which the firm does both inventive and innovative activities.

7. An increase in μ may widen the average cost gap if $1+\nu/\mu < \ell n(\mu/\nu)$. But this somewhat perverse case can be ignored as long as ν is sufficiently small relative to μ .

8. For this illustration, we have chosen the values of parameters as $\lambda = .05$, $\xi = .01$, $\mu = .50$ and $M = 20$.

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