COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 601

THE CONVENTIONALLY STABLE SETS IN NONCOOPERATIVE GAMES WITH LIMITED OBSERVATIONS: DEFINITIONS AND INTRODUCTORY ARGUMENTS

Mamoru Kaneko

THE CONVENTIONALLY STABLE SETS IN NONCOOPERATIVE GAMES

WITH LIMITED OBSERVATIONS: DEFINITIONS AND INTRODUCTORY ARGUMENTS*

by

Mamoru Kaneko**

Abstract

This paper attempts to define a new solution concept for n-person noncooperative games. The idea of the new solution concept is based on that of the von Neumann-Morgenstern stable set, or more precisely, rather on their interpretation of it which they call "standards of behavior." This new approach enables us to consider new interesting problems of information. Further this approach gives us a plausible interpretation of Nash equilibrium. This paper provides the definition and considers the new solution concept for zero-sum two-person games, the prisoner's dilemma, the battle of sexes and games with a continuum of players.

---

*The author wishes to express his thanks to Professors M. Shubik, P. Dubey, L. S. Shapley, R. J. Aumann, K. Iwai and C. Pitchik for valuable discussions and comments on a preliminary draft.

**Mamoru Kaneko, Institute of Socio-Economic Planning, University of Tsukuba, Sakura-mura, Niihari-gun, Ibaraki-ken 305, Japan, Current Address: Cowles Foundation for Research in Economics at Yale University, Box 2125, Yale Station, New Haven, Connecticut 06520, USA.

The research in this paper was partially sponsored by the Office of Naval Research Contract Number N00014-77-C-0518.

1. Introduction

1.1. Noncooperative game theory has developed very rapidly and now has an enormous number of applications.[1] Further, as Nash [1951] holds that noncooperative games are theoretically basic and that cooperative games can and should be subsumed under noncooperative game theory by making communication and bargaining formal moves in a noncooperative extensive game, noncooperative game theory could cover a very large class of game situations including even cooperative aspects. Although Nash's program has not yet been fully accomplished, we are convinced that noncooperative game theory could be the foundation of the whole game theory. In noncooperative game theory, the concept of Nash equilibrium or some modifications of it, e.g., Selten's perfect equilibrium, subgame perfect equilibrium, etc., has played the central role as a solution.

Nonetheless, we could say that there is no convincing foundation nor persuasively consistent interpretation of Nash equilibrium. In other words, we could not answer the question in what situation Nash equilibrium makes sense or whether or not it makes sense in any situation, though Nash equilibrium can be defined mathematically in any games in normal or extensive form. This paper discusses problems of this sort and restricts itself to certain situations which are different from those many game theorists have considered. In doing so, a new solution concept will be proposed, which is applicable to these situations. This approach will provide a plausible interpretation of Nash equilibrium, and in particular, will generate interesting results in games with a small number of players.

---

[1] For a recent survey article, see Schotter and Schwödiauer [1980].

[2] For example, Nash [1953], Aumann [1959], Harsanyi [1974], Selten [1980], Kaneko [1981], etc., discuss cooperative solution concepts in terms of noncooperative games.

1.2. In order to make the problem clear, we should review the existing arguments for the interpretation of Nash equilibrium and their defects.

The following two assumptions have been at times made explicitly or implicitly and in other cases no assumption is discussed:[3]

(i) Each player is fully cognizant of the game in question, i.e., he is fully aware of the rules of the game and the utility function of each of the players.[4,5]

(ii) The game is played exactly once.[6]

The logical relation between assumptions (i) and (ii) is very simple: Assumption (i) is made only if we can consider rational behavior of players in a game which is played exactly once, i.e., for assumption (ii) to make sense assumption (i) is indispensable.[7] Conversely, under assumption (i), any game situation can be regarded as a one-shot game even if a game is repeated.

von Neumann and Morgenstern [1953] argue the validity of the minimax theorem under assumptions (i) and (ii). But it is hard to justify Nash equilibrium in nonzero-sum games in terms of the same argument as

---

[3]As far as the author knows, Basu [1977] only discusses explicitly a different basic assumption from (i) and (ii). The author thinks his assumption itself makes sense only if every player has a dominant strategy in the game in question.

[4]Luce and Raiffa [1957, pp. 49-50].

[5]Harsanyi [1967-8] proposes a theory with incomplete information, but he reduces it to a theory with complete information. We could categorize his theory into the theory with assumptions (i) and (ii).

[6]See Aumann and Maschler [1972, p. 58].

[7]The author does not say that is is impossible to consider the situation where (i) is not true but (ii) is so, but rather says that as far as we consider Nash equilibrium in games with assumption (ii), assumption (i) is necessary.

von Neumann-Morgenstern's, because Nash equilibrium lacks the optimality in nonzero-sum games which is held in von Neumann-Morgenstern's argument in zero-sum games.[8] The purpose of this paper, however, is not to argue that Nash equilibrium can not be well-interpreted under assumptions (i) and (ii), nor to discuss what makes sense under them,[9] but rather to provide an alternative set of assumptions under which Nash equilibrium makes sense. In the following, we call the above interpretation of Nash equilibrium the complete information interpretation.

As far as we regard game theory as a positive science, it would be plausible to say that the complete information interpretation is important only if the number of players is very small, e.g., $n = 2$ or 3 because it is hard for every player to know the other players' utility functions in games with many players. But a recent tendency of game theory is to consider the limit behaviors of solution concepts when the number of players becomes arbitrarily large. It is clear that works of this sort contradict the complete information interpretation. We can, however, provide an opposite and more convincing argument to justify Nash equilibrium in games with a large number of negligible players without assumptions (i) and (ii).

The argument looks like the classical justification of competitive equilibrium in perfect competitive markets.[10] In a game where there are a large number of players and every player has a negligible power to

[8] See Luce and Raiffa [1957, Ch. 5].

[9] In a subsequent paper the author will try to discuss the problem of what makes sense in a situation with assumptions (i) and (ii).

[10] This argument is sometimes but implicitly used to interpret Nash equilibrium. For example, see Shubik [1981, Ch. 9, Sec. 5.1] or Schmeidler [1980].

influence the other players, every player would behave as a player in a game against nature or as a "price-taker," i.e., supposing that the others' behavior is fixed, becuase he could not affect the others. This behavior coincides with that prescribed by Nash equilibrium. Of course, this argument makes sense if the game is repeated a sufficiently large number of times.[11] That is, an important point is that after a sufficiently large number of plays, each player can know the shape of his payoff function to some extent by learning it from the history and can behave rationally even if assumption (i) is not made. This sort of informational problem is very important in repeated situations, which has never been discussed formally.

We now have an interpretation which is diametrically opposed to the complete information interpretation of Nash equilibrium. We call this argument the <u>perfectly competitive interpretation</u> of Nash equilibrium. If this interpretation were natural, the concept of Nash equilibrium might not be persuasive in games with a small number of players, and so, we would like to generalize the interpretation so that it could be compatible with games with a small number of players.

<u>1.3.</u> This paper will attempt to define a new solution concept being based on the idea of the von Neumann-Morgenstern stable set (abbre. vN-M stable set). Strictly speaking, the definition will depend rather upon their interpretation of the vN-M stable set in terms of "standards of behavior." The vN-M stable set and standards of behavior require intrinsically

[11] Aumann and Maschler [1972] say that a situation where a game is repeated can be regarded as a large one-shot game. If we stand at this point, we again need assumption (i). We will consider such a situation to be completely different from a one-shot game.

repeated play.[12] Although von Neumann and Morgenstern emphasize assumptions (i) and (ii) to justify their minimax theorem in two-person zero-sum games, the vN-M stable set is considered in a quite different situation. They do not express this point explicitly. They, however, interpret a vN-M stable set as a standard of behavior (they also use the phrase "an order of society") which is currently accepted by the society and whose acceptance makes each state in the vN-M stable set stable. This concept of a standard of behavior can be reinterpreted as something like convention, moral or tradition, which emerges from a long history.[13, 14]

The vN-M stable set is defined in games in characteristic function form. Hence we can not directly follow their definition, but will define a new solution concept in noncooperative games. The interpretation of vN-M stable set in terms of standards of behavior could be a key stone with which we construct a new mathematical model.

<u>1.4</u>. Now, we have many general questions to which we should give solutions. To make us aware of these problems, we summarize them as follows:

(Q-1): How do we treat the problem of information in repeated situations of noncooperative games?

---

[12] Marschak and Selten [1974, 1977, 1978] define a new solution concept called "convolution" in a repeated situation. The new solution concept of this paper is more similar to their concept than to other game theoretical concepts. The idea of the new concept, however, is based on that of vN-M stable set and is different from Marschak and Selten's.

[13] For more detailed explanations for the vN-M stable set and standards of behavior, see von Neumann and Morgenstern [1953, Sec. 4], Luce and Raiffa [1957, Ch. 9], Harsanyi [1974] and Shubik [1981, Ch. 6].

[14] Lewis [1969] provides a general definition of "convention," though his argument is not fully formal but verbal. The definition of the new solution concept is very similar to his definition of convention. In this paper, we will briefly discuss this.

(Q-2): How do we define a new solution concept based on the vN-M stable set in repeated situations of noncooperative games?

(Q-3): Is the perfectly competitive interpretation of Nash equilibrium consistent with the new theory?

Further if we get satisfactory answers to these questions, we should consider the following questions:

(Q-4): Do any implications of the answers contradict any existing game theoretical studies?

(Q-5): Does the new theory have any intrinsically novel applications to any fields?

(Q-6): What are the limits or defects of the new theory? Is it possible to extend the new theory?

Being conscious of these questions helps us to make the new theory more complete.

1.5. This paper and forthcoming papers attempt to answer these questions. They do not necessarily give a satisfactory answer to every question, but will give a better answer to every question than other existing studies.

In this paper the new solution concept will be defined and considered in several simple games. More precisely, we will consider the new solution concept in two-person zero-sum games, the prisoner's dilemma game and the battle of sexes game, comparing Nash equilibrium. This consideration exhibits that the new solution concept has a great multiplicity like the vN-M stable set. Furthermore, we will also consider the new solution concept in games with a continuum of players in this paper. This shows that the perfectly competitive interpretation is consistent with the new theory. In fact, we get a satisfactory answer to a more precise question than (Q-3). This also teaches us that when an information structure is coarse (low), the multiplicity of the new solution concept disappears.

2. Games in Normal Form with Limited Observations and Conventionally Stable Sets[15]

2.1. A game in normal form with limited observations is given as a quintuple $G = (N, \{S_i\}_{i\varepsilon N}, S, \{h_i\}_{i\varepsilon N}, \{a_i\}_{i\varepsilon N})$. The constituents are as follows: N is the set of all players, which may be finite or infinite. S_i is the set of all pure strategies of player $i \varepsilon N$. S is the set of all outcomes $s = (s_i)_{i\varepsilon N}$ which are functions from N to $\bigcup_{i\varepsilon N} S_i$ with conditions

$$s_i \varepsilon S_i \quad \text{for all} \quad i \varepsilon N , \tag{2.1}$$

$$\text{for all} \quad i \varepsilon N , \quad (s_{-i}, s'_i) \varepsilon S \quad \text{for all} \quad s'_i \varepsilon S_i , \tag{2.2}$$

where s_{-i} is the restriction of s on $N-\{i\}$.[16] h_i is the payoff function of player $i \varepsilon N$, which is a function from S to R (the set of all real numbers). a_i is a partition of S which satisfies

$$\text{for any} \quad s \varepsilon T \varepsilon a_i \quad \text{and} \quad s' \varepsilon T' \varepsilon a_i , \quad \text{if} \quad s_i \neq s'_i \tag{2.3}$$
$$\text{or} \quad h_i(s) \neq h_i(s') , \quad \text{then} \quad T \neq T' .$$

We call $\{a_i\}_{i\varepsilon N}$ an observation structure. Let $a_i(s)$ be the function

[15] We may regard games in normal form as a special class of games in extensive form or alternatively as a general class of games in extensive form by means of strategies. See Selten [1975]. In this paper, to avoid any obscurity, we restrict ourselves to viewing games in normal form in the first way. The author will discuss conventionally stable sets in games in extensive form in a forthcoming paper.

[16] When N is a finite set, conditions (2.1) and (2.2) imply $S = \prod_{i\varepsilon N} S_i$.

on S such that $s \in a_i(s) \in a_i$ for all $s \in S$, which we call the <u>observation function</u> of player $i \in N$.

<u>2.2</u>. Initially we should emphasize that we are considering a situation where the game G is played repeatedly. Due to the repetition of the game we can easily interpret the observation structure $\{a_i\}_{i \in N}$ as follows. When the game is played and an outcome s appears, player i observes the outcome s and obtains information $a_i(s)$. Then he will behave in the next period using this information. The most fundamental assumption we make is that no player can directly know the other players' payoff functions but he can only know what he has observed in the other players' behavior.

We should note two points on the definition of Subsection 2.1. The first note is that we do not consider mixed strategies at all in this paper. If a player plays a mixed strategy and if he does not inform the others, then the others can never know his mixed strategy by observing his play only once. Hence if we permitted the usage of mixed strategies, then we would have to consider the difficult conceptual problem of how to perceive a mixed strategy in a repeated play. In this paper we obviate this problem by allowing only pure strategies. Of course, this does not mean that the author does not think that mixed strategies are important in the situations considered in this paper.

The second note is the interpretation of condition (2.3). If player i plays two different strategies or if he receives different payoffs, then he can distinguish one outcome from the other. This is the minimum requirement for rational players. An important note is that we do not assume that player i can recognize $a_i(s)$ as a set but that player i can distinguish $a_i(s)$ from other $a_i(s')$. This implies that even if a_i were the finest partition, player i could not recognize each outcome itself but could only distinguish it from the others.[17] This assumption is completely different from Aumann's [1976] approach, in which each decision maker knows a priori a_i itself as a partition of S.

2.3. The following concepts will help us in considering the problem of observation and information, though we will not use them directly.

Let A_i be the class of all possible observations a_i of player $i \in N$. We define the partial order $\succsim_i$ on A_i by

$$a_i \succsim_i a_i' \text{ if and only if for every } T \in a_i ,\ T \subset T' \text{ for some } T' \in a_i' . \tag{2.4}$$

Let A be the class of all observation structures $\{a_i\}_{i\in N}$. We define the partial order $\succsim$ by

$$\{a_i\}_{i\in N} \succsim \{a_i'\}_{i\in N} \text{ if and only if } a_i \succsim_i a_i' \text{ for all } i \in N \tag{2.5}$$

We call $\{a_i\}_{i\in N}$ the coarsest observation structure iff for any $\{a_i'\}_{i\in N} \in A$, $\{a_i'\}_{i\in N} \succsim \{a_i\}_{i\in N}$. Similarly we can define the game with the finest observation structure. The finest observation structure consists of the finest partitions of S. The coarsest observation structure also exists and is represented as

---

[17] In fact, the following alternative formulation may be clearer, though it needs one more structure. Let M_i be player i's message space. Let $a_i^*(s)$ be a message function, i.e., $a_i^* : S \to M_i$. The message function $a_i^*(s)$ means that when s occurs, player i obtains the message $a_i^*(s)$. He can know only this message but not s. But we define $a_i(s)$ by

$$a_i(s) = a_i^{*-1}(a_i^*(s)) \quad \text{for all } s \in S .$$

and then, $a_i(s)$ is player i's observation function and has the same meaning as that of $a_i^*(s)$.

$$a_i(s) = \{s' : s_i = s_i' \ \& \ h_i(s) = h_i(s')\}$$

$$\text{for all } i \in N \text{ and all } s \in S .$$

Taking the costs of observation into consideration, coarser observation structures are more important than finer ones. When the number of players is small, the distinction between the finest one and the coarsest one is not so important but we will show in a later section that when the number of players is large, this distinction plays a crucial role.

Example 2.1. Let us consider the following 2-person bimatrix game:

$$\begin{array}{ccc} & & 2 \\ & & \begin{array}{cc} \beta_1 & \beta_2 \end{array} \\ 1 & \begin{array}{c} \alpha_1 \\ \alpha_2 \end{array} & \begin{bmatrix} (5,5) & (-4,6) \\ (6,-4) & (-3,-3) \end{bmatrix} \end{array}$$

This game is usually called "the prisoner's dilemma." In this game, the coarsest observation structure coincides with the finest one. Hence there is a unique observation structure.

Example 2.2. In the following 2-person bimatrix game, there is a difference between the coarsest observation structure and the finest one. The coarsest one is given by $\{a_1, a_2\}$, where $a_1 = \{\{(\alpha_1, \beta_1), (\alpha_1, \beta_2)\},$ $\{(\alpha_2, \beta_1)\},\{(\alpha_2, \beta_2)\}\}$ and $a_2 = \{\{(\alpha_1, \beta_1), (\alpha_2, \beta_1)\}, \{(\alpha_1, \beta_2)\}, \{(\alpha_2, \beta_2)\}\}$. The finest one consists of, of course, the finest partitions of S .

$$\begin{array}{ccc} & & 2 \\ & & \begin{array}{cc} \beta_1 & \beta_2 \end{array} \\ 1 & \begin{array}{c} \alpha_1 \\ \alpha_2 \end{array} & \begin{bmatrix} (5,5) & (5,10) \\ (10,5) & (0,0) \end{bmatrix} \end{array}$$

2.4. Now we are in a position to define the conventionally stable set. A response function of player $i \in N$ is a function from S to S_i which satisfies

$$\text{if} \quad a_i(s) = a_i(s') \ , \quad \text{then} \quad r_i(s) = r_i(s') \ . \tag{2.6}$$

A response configuration r is a function from S to S such that for all $i \in N$, r_i is a response function. r_{-i} denotes the restriction of r on $N-\{i\}$.

Since we are considering a situation where the game G is played repeatedly, we will assume that each player's temporal behavior depends on the temporal state. This does not mean that each player's behavior is independent from his previous history. Rather, we assume that each player has fully learned (with respect to his observations) the society's responses to his actions. Although each player's temporal behavior depends upon the temporal state, his entire behavior depends upon his knowledge on the society's responses to his actions, which has been acquired from the previous history. In particular, condition (2.6) means that his temporal behavior depends upon his temporal observation.

In the following, let r be a fixed response configuration. We call the set $F(r) = \{s \in S \ ; \ r(s) = s\}$ the stationary set of r and call each s in $F(r)$ a stationary state.

Let s and s' be two outcomes, which may be the same. We say that s is absorbed by s' iff

$$\text{for a finite } k \ , \quad r^k(s) = \overbrace{r \cdot r \cdot \ldots \cdot r}^{k \text{ times}}(s) = s' \in F(r) \ . \tag{2.7}$$

We call $\{s, r(s), \ldots, r^{k-1}(s), s'\}$ an absorption of s by s'.

Let $\{s^1, \ldots, s^k\}$ be an absorption of s^1 by s^k , i.e., $s^t = r^{t-1}(s^1)$ for $t = 1, \ldots, k$. After player i observed s^t $(1 \leqq t \leqq k)$, he gets information $a_i(s^t)$. If he notices at s^t that this absorption has started at s^1 , then he can use not only the observation $a_i(s^t)$ but also the accumulation of observations $a_i(s^1), \ldots, a_i(s^t)$. To represent accumulations of observations, we extend $a_i(s)$ as follows: for any absorption $\{s^1, \ldots, s^k\}$, we define $\hat{a}_i$ by

$$\hat{a}_i(\{s^1\}) = a_i(s^1) , \tag{2.8}$$

$$\hat{a}_i(\{s^1, \ldots, s^t\}) = a_i(s^t) \cap r[\hat{a}_i(\{s^1, \ldots, s^{t-1}\})]$$
$$\text{for } t = 2, \ldots, k .$$

See Figure 1. Note that if $\{s^1, \ldots, s^k\}$ is an absorption, then $\{s^t, \ldots, s^k\}$ and $\{s^t, \ldots, s^k, \ldots, s^k\}$ are also absorptions for every t $(1 \leqq t \leqq k)$.

Let s and s' be two outcomes. We say that s' <u>is connected to</u> s by a <u>deviation</u> $\{t_i^1, \ldots, t_i^k\}$ of player i from his response function r_i iff

$$t_i^m \text{ is a response function of player } i \text{ for } m = 1, \ldots, k , \tag{2.9}$$

$$(r_{-i}, t_i^k)\cdot(r_{-i}, t_i^{k-1}) \cdot \ldots \cdot (r_{-i}, t_i^1)[s] = s' . \tag{2.10}$$

We say that player i <u>is effective for</u> $s \in S$ <u>in</u> r iff

$$r_{-i}(s) = s_{-i} . \tag{2.11}$$

If player i is effective for s in r , then he can make s a stationary state, and, in fact, s is a stationary state unless he changes his strategy.

We say that player i <u>can improve upon</u> an absorption $\{s^1, \ldots, s^k\}$ of s^1 by s^k at s^h $(1 \leqq h \leqq k)$ iff

$$\begin{aligned}&\text{player } i \text{ has a deviation } \{t_i^1, \ldots, t_i^m\} \text{ such that} \\ &\text{for any } \bar{s} \in \hat{a}_i(\{s^1, \ldots, s^h\}) \text{, player } i \text{ is} \\ &\text{effective for the outcome } s'' \text{ which is connected to} \\ &\bar{s} \text{ by the deviation } \{t_i^1, \ldots, t_i^m\} \text{ and } h_i(s'') > h_i(s^k) \text{ .}\end{aligned} \tag{2.12}$$

Note the last inequality $h_i(s'') > h_i(s^k)$, i.e., player i compares the new stationary payoff $h_i(s'')$ with the original stationary payoff $h_i(s^k)$, and that s'' is uniquely determined by the deviation $\{t_i^1, \ldots, t_i^m\}$ and $\bar{s}$.

The concept "can improve upon" has the following implication. Let an absorption $\{s^1, \ldots, s^k\}$ be occurring. If a player i notices this absorption with respect to his observations, and if he thinks that it is possible to make this absorption change and converge to another stationary state with a higher payoff than $h_i(s^k)$ by deviating from his response function r_i, then he wants to do so. If some player has such a possibility, then this absorption is not necessarily "stable." Hence a "stable" absorption should be free from such strategic behavior. We call an absorption $\{s^1, \ldots, s^k\}$ of s^1 by s^k <u>stable</u> iff any player can not improve upon this absorption at s^h for any $h = 1, \ldots, k$.

Now it is possible to define the conventionally stable set, and we subsequently interpret the definition.

Definition. A response configuration r is said to be conventionally stable iff

(Stability): every absorption is stable, (2.13)

(Absorptivity): there is a finite k such that for any $s \in S$, there is an absorption $\{s^1, \ldots, s^h\}$ with $s = s^1$ and $h \leqq k$, (2.14)

(Acyclicity): for any $i \in N$, there is a finite h such that for any $s \in S$ and $s_i' \in S_i$, $(r_{-i}, e_i)^h[(s_{-i}, s_i')] = (r_{-i}, e_i)^{h-1}[(s_{-i}, s_i')]$, where e_i is the identity response function, i.e., $e_i(s) = s_i$ for all $s \in S$. (2.15)

We call a subset V of S a conventionally stable set iff there is a conventionally stable response configuration r whose stationary set $F(r)$ coincides with V.

Note that condition (2.14) implies the nonemptiness of the stationary set. Next, let $s \in F(r)$ and $\{s^1, \ldots, s^m\}$ be a sequence such that $m \geqq k$ (k is given in (2.14)) and $s^t = s$ for all $t = 1, \ldots, m$. Of course, $\{s^1, \ldots, s^m\}$ be an absorption. Then the following lemma holds:

Lemma 2.1: Let r satisfy the absorptivity (2.14). Then player i can improve upon $\{s^1, \ldots, s^m\}$ at s^m iff

player i has a deviation such that for any $\bar{s} \in a_i(s) \cap F(r)$, player i is effective for s'' which is connected to $\bar{s}$ by the deviation and $h_i(s'') > h_i(s)$. (2.12')

Proof. We need to show that $\hat{a}_i(\{s^1, \ldots, s^m\}) = a_i(s) \cap F(r)$. Clearly $\hat{a}_i(\{s^1, \ldots, s^m\}) \supset a_i(s) \cap F(r)$. By the absorptivity (2.14), there is a $h \leqq k$ such that for any point $\hat{s} \in a_i(s)$, there is an absorption $\{\hat{s}^1, \ldots, \hat{s}^h\}$ with $\hat{s}^1 = \hat{s}$. Of course, $\hat{s}^h$ is a stationary state. Hence if $\hat{s}$ is in $\hat{a}_i(\{s^1, \ldots, s^m\})$, then $\hat{s}$ is a stationary state. Hence $\hat{\hat{s}} \in a_i(s) \cap F(r)$. Q.E.D.

For simplicity, we say that player i can improve upon a stationary state s iff (2.12') is true. If r is conventionally stable, then any player can not improve upon any stationary state.

2.5. Before we give the interpretation of the definition, we should make its basic assumption clear. This paper restricts itself to considering the possible stationary states after the game has been played a large number of times and the players have learned fully the society's reactions to each player's actions. This restriction could be justified to some extent by introducing strategy changing costs and by limiting the rationality of the players. The justification by limiting the rationality of the players is argued as follows. If there is no regularity in the social movement even after the game is played a large number of times, any player can not behave rationally, because he can expect nothing in the social movement. The regularity which every player can perceive is a periodicity in the social movement, and a special case of periodicity is a stationarity. But if the period is too long and if some player can not perceive the period due to limiting rationality, then he can not behave rationally in our sense and may learn the periodical movement playing more many times or this periodical situation is broken by his nonconformable behavior. Hence a long periodicity is not so

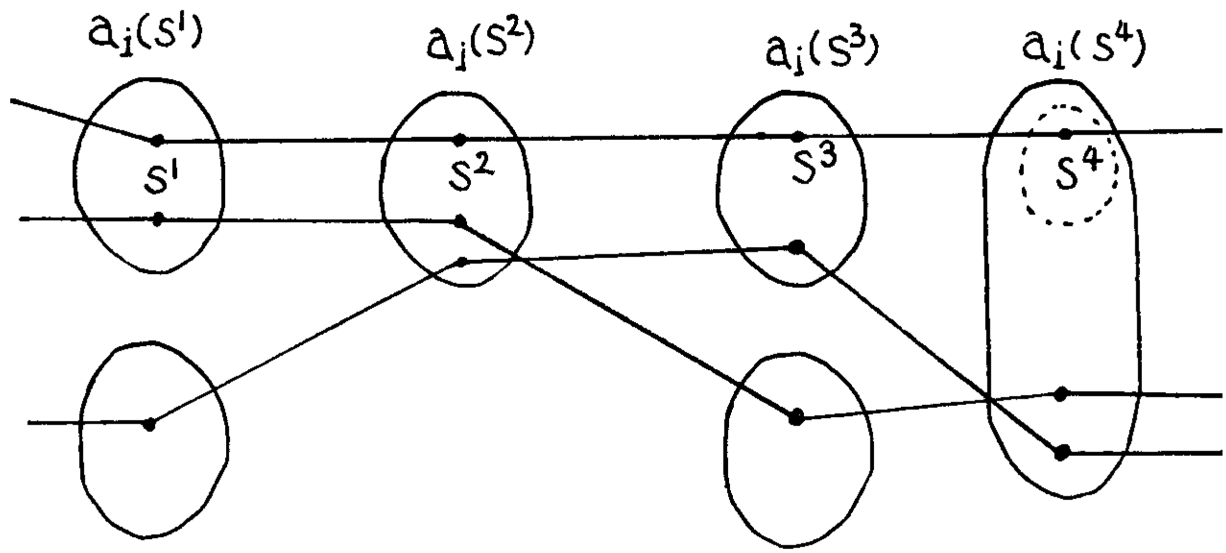


Figure 1. : $\hat{a}_i(\{s^1,s^2,s^3,s^4\})$.

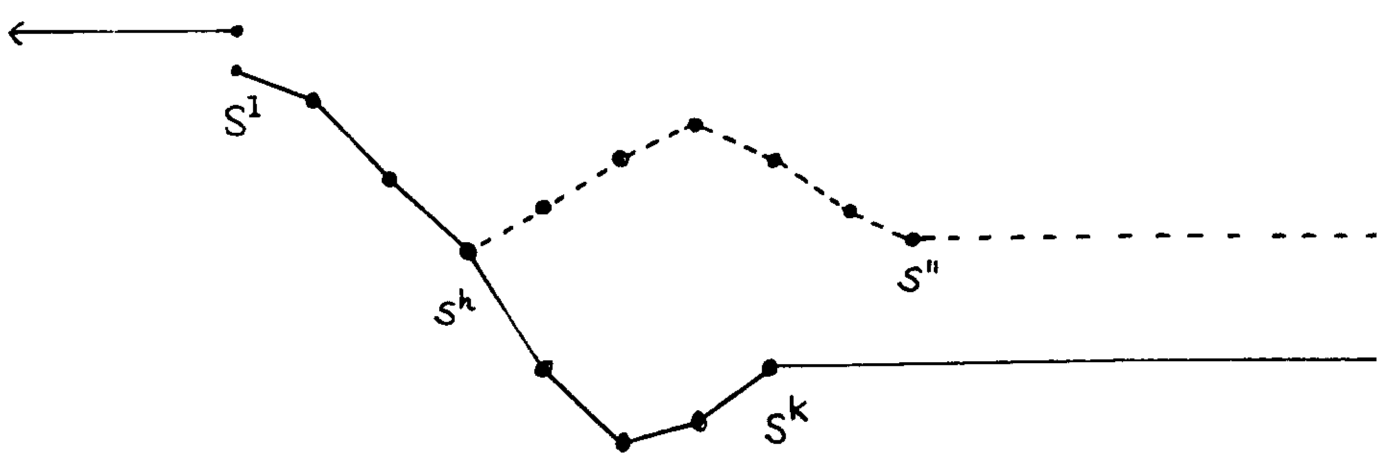


Figure 2.(The finest observation structure cas

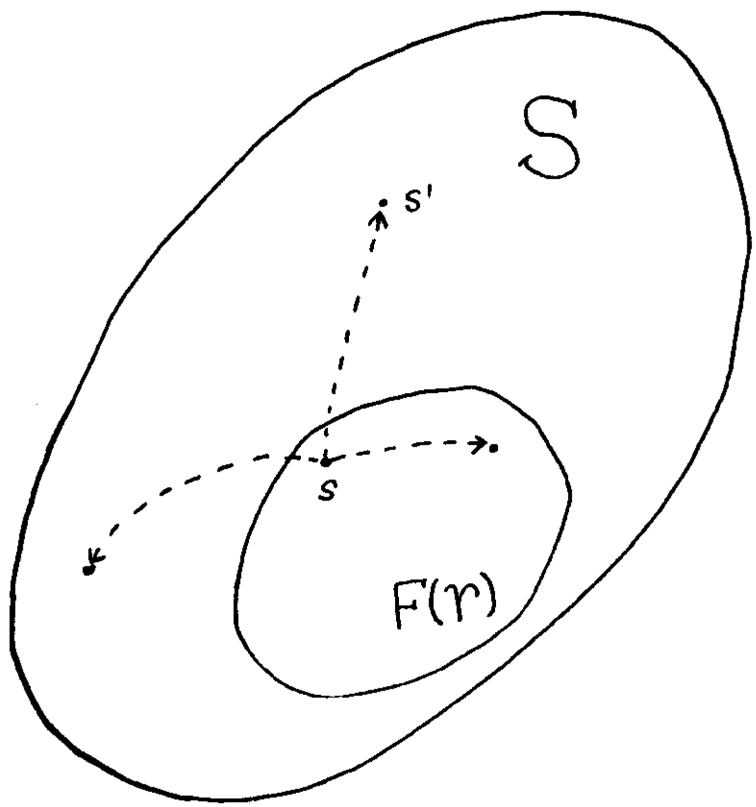


Figure 3.

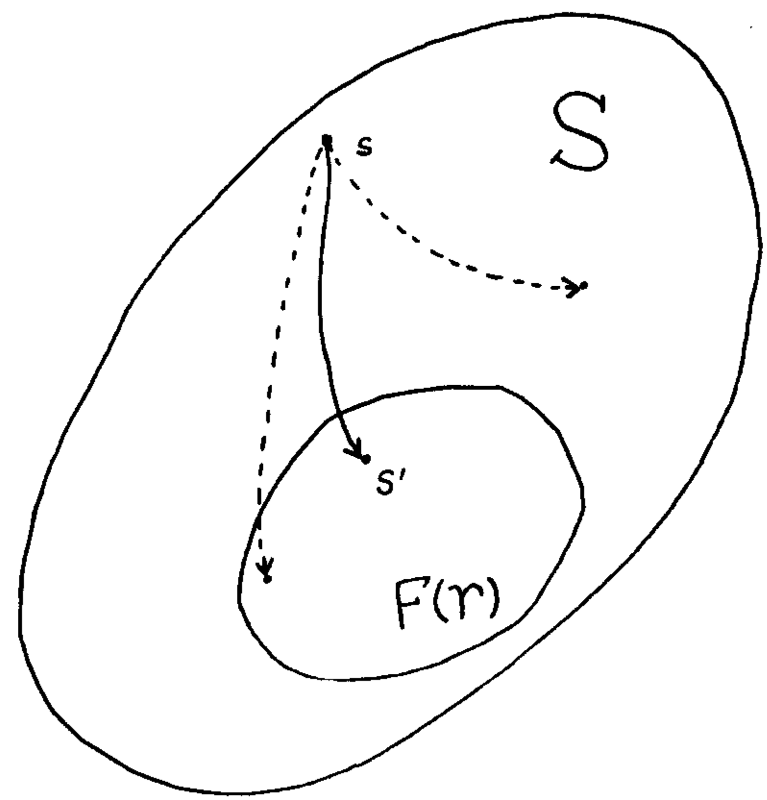


Figure 4.

important in social phenomena. Therefore we assume the stationarity on the possible outcomes.

If we accept the above assumption, then the other assumption is easy to understand. The assumption is that every player's purpose is to maximize his permanent payoff which is given by a stationary state but does not depend upon transitory or temporary payoffs. In other words, this assumption is equivalent to saying that every player's discount rate is zero. This assumption is not a seriously strong condition.

We are now in a position to explain the definition of the conventionally stable set. The absorptivity (2.14) is divided into two cases, and the stability (2.13) requires that every absorption in each case be stable.

First, let us consider a stationary state s. If no player deviates from his response function, then s continues to be a stationary state. If some player i has a deviation $\{t_i^1, \ldots, t_i^m\}$ by which another state s' is connected to s, i.e., $(r_{-i}, t_i^m)\cdot(r_{-i}, t_i^{m-1}) \cdot \ldots \cdot (r_{-i}, t_i^1)[s] = s'$, and if he is effective for s', then the player can make s' a new stationary state, on condition that the others conform to their response functions. See Figure 3. Further if s' gives a higher payoff than s does, then the player wants to deviate from his response function. This is the meaning of "can improve upon." A note is that the player deliberates this possibility of "can improve upon" at the time of s. This deliberation is possible because after the game is played a large number of times, and each player has learned fully the society's reactions to his actions. Therefore a necessary condition for the response configuration to be stable is that no player has such a deviation. The exact definition of "can improve upon" is more complicated because we should take the observation structure into consideration.

Second, let us consider any s which is not in $F(r)$. This s should be absorbed by some stationary state s' because of the basic assumption of stationality. Further we require that this absorption of s by s' itself be stable. In fact, if some player i has a deviation $\{t_i^1, \ldots, t_i^m\}$ such that s'' which is connected to s by the deviation gives a higher payoff to him and if he is effective for s'' , then he wants to deviate from his response function. See Figure 4. Therefore a necessary condition for the absorption itself to be stable is that no player has such a deviation.

The acyclicity (2.15) has the following meaning. In the past each player made trials and errors many times. The simplest trial of a player is to fix a strategy and then to observe the society's reaction to his fixing the strategy. In this case, the acyclicity requires that the social reaction eventually reach a stationary state. This condition is very weak if we accept the stationarity in the social movement mentioned above.

2.6. Here we need to return to the basic assumptions that the game is repeated and that every player has acquired knowledge on the society's responses to his actions by observing outcomes via his observation function. He does not know *a priori* anything. Even after the game has been played a sufficiently large number of times, he can not know certainly anything about the society's responses, e.g., he can not know or infer certainly when an absorption began nor whether or not he is effective for an outcome. Every player, however, has ambiguous knowledge on the society's responses to his actions which he has learned from the history. In our theory, every player behaves strategically being based on this ambiguous knowledge. A conventionally stable response configuration is one which is free from such strategic behavior.

2.7. It is useful to make a comparison between our definition and von Neumann-Morgenstern's definition, though they define their solution in games in characteristic function form. They define their solution V using dominance relation on the set of imputations as follows: (i) (Internal Stability): if x and y are in V, then neither x nor y dominates the other and (ii) (External Stability): for any y not in V, there is an x in V which dominates y. The external stability corresponds to our condition that every state not in $F(r)$ is absorbed by some state in $F(r)$. The internal stability corresponds to that every state in $F(r)$ is stationary. But von Neumann-Morgenstern's definition allows the possibility that some state not in V dominates some state in V. Vickrey [1959] and Harsanyi [1974] criticizes this point as follows: If we allow this possibility, then some players might behave strategically to get higher payoffs and they want to move from a state in V to another state in V, and so, the vN-M stable set is not stable in their motivated sense.[18] Vickrey and more exhaustively Harsanyi modify von Neumann-Morgenstern's definition taking such strategic behavior into consideration. In our case, we take such strategic behavior into consideration by defining "can improve upon absorptions." Thus our definition just corresponds to Harsanyi's version of the vN-M stable set.

It is also valuable to give a small comment to the relation of our definition and Lewis' [1969] definition of "convention," though his argument is not fully rigorous as exhaustively criticized by Gilbert [1981].[19] His general definition is

---

[18] See Harsanyi [1974, Sec. 3].

[19] See also Schotter [1981].

> A regularity R in the behavior of members of a population P when they are agents in a current situation S is a <u>convention</u> if and only if, in any instance of S among members of P ,
> (1) everyone conforms to R ; (2) everyone expects everyone else to conform to R ; (3) everyone prefers to conform to R on condition that the others do, since S is a coordination problem and uniform conformity to R is a coordination equilibrium in S .

Clearly, his regularity corresponds to our response configuration. But, regretfully, as he does not rigorously define several concepts, we can not directly compare our definition with his. Anyway, it is clear that his general idea is the same as ours. Further we can find that Lewis' idea is very similar to von Neumann-Morgenstern's standards of behavior (but not their stable set). In this sense, we might say that our solution concept is a rigorous reformulation of von Neumann-Morgenstern's standards of behavior.

Finally we should give a remark on Marschak and Selten's work. Marschak and Selten [1974, 1977, 1978] define a similar solution concept to ours, which they call a "convolution." Their concept is also defined using response functions, but they assume that every response to some player's deviation is finished after one period. Further they require more local stability than we do, though their stability is "global" to some extent.[20] They consider their solution concept mainly in oligopolistic

---

[20] Except the vN-M stable set, the convolution and the conventionally stable set, other game theoretical solution concepts are defined in terms of local stabilities.

markets. This author will compare their solution concept with ours in oligopolistic markets in a forthcoming paper.

3. The Trivial Conventionally Stable Sets and the Nash Equilibria

3.1. Let G be a game in normal form with limited observations. A Nash equilibrium is an outcome $s \in S$ which satisfies

$$\text{for all } i \in N \text{ , } h_i(s) \geqq h_i(s_{-i}, s_i') \quad \text{for all } s_i' \in S_i \text{ .} \tag{3.1}$$

It should be noted that this definition does not depend upon observation structures. When there exists a Nash equilibrium, the singleton set of it is conventionally stable, as shown in the following proposition. We call r given in the following proposition a trivial response configuration.

Proposition 1. If s is a Nash equilibrium in the game G , then the response configuration r such that $r(s') = s$ for all $s' \in S$ is conventionally stable. This proposition does not depend upon the observation structure.

Proof. Each player i is effective only for (s_{-i}, s_i') for all $s_i' \in S_i$ in r . Since s is a Nash equilibrium, $h_i(s_{-i}, s_i') \leqq h_i(s)$ for any $s_i' \in S_i$. Hence every absorption is stable. It is clear that every s' ($\neq s$) is absorbed by s . The acyclicity is trivial. Q.E.D.

3.2. One might conjecture that if a Nash equilibrium existed, then any conventionally stable set would coincide with some set of Nash equilibria. However, this is not true but in fact, a great variety of conventionally stable sets appears in some games. It is natural for our theory to have a great variety, because it depends not only upon a physical situation described by characteristics of a game (strategies and payoffs) but also upon a history. We will, however, show that the above conjecture is true in some classes of games with some observation structures. Therefore, we could say that the trivial conventionally stable response configurations have a great significance only if there exist no other kind of conventionally stable response configurations which can be interpreted naturally. With this remark in mind, we begin to investigate the conventional stability in various games.

4. Two-Person Zero-Sum Games, the α-Individual Rationality and the "Folk" Theorem

4.1. Let us consider a two-person zero-sum game G, i.e., $N = \{1,2\}$ and $h_1(s) + h_2(s) = 0$ for all $s \in S$. In this game, von Neumann gives a complete solution in the case of complete information. It is the well-known minimax theorem. Although we do not assume complete information, we do assume repeated play and so, can expect a similar result in our new solution concept, because each player could find maximin strategies in repeated play.

Let $s^* = (s_1^*, s_2^*)$ be a Nash equilibrium. Then we call s_i^* player i's maximin strategy and $h_1(s^*)$ the value of G, which is denoted by $v(G)$.

Proposition 2. Assume that G has a Nash equilibrium, i.e., a pair of maximin strategies. Let V be an arbitrary conventionally stable set. Then $h_1(s) = v(G)$ for all $s \in V$. This proposition does not depend upon the observation structure.

Proof. Let $h_1(s) < v(G)$, and let s^* be a pair of maximin strategies. Hence $h_1(s_1^*, s_2') \geqq v(G) > h_1(s)$ for all $s_2' \in S_2$. Let $\bar{s}$ be an arbitrary outcome in $a_i(s) \cap F(r)$. When player 1 fixes his strategy to s_1^* , a stationary state results after a finite repetition of the game (by the acyclicity (2.15)), i.e., there is a h such that $(e_1, r_2)^h[(s_1^*, \bar{s}_2)] = (e_1, r_2)^{h-1}[(s_1^*, \bar{s}_2)]$. Let $(e_1, r_2)^h[(s_1^*, \bar{s}_2)] = (s_1^*, s_2'')$. Thus player 1 is effective for (s_1^*, s_2'') and $h_1(s_1^*, s_2'') > h_1(\bar{s})$. Since $\bar{s}$ is arbitrary in $a_i(s) \cap F(r)$ and h is independent of $\bar{s}$, player 1 can improve upon s , which means that r is not conventionally stable. In the case where $h_1(s) > v(G)$, player 2 can improve upon s by fixing his strategy to s_2^* . Q.E.D.

The following example shows that it is not necessarily true that if r is conventionally stable and $s \in F(r)$, then s is a Nash equilibrium (a pair of maximin strategies), even in two-person zero-sum games.

Example 4.1. Let us consider the following two-person zero-sum game. The components of the matrix are player 1's payoffs.

$$\begin{array}{c c} & \begin{array}{cc} \beta_1 & \beta_2 \end{array} \\ \begin{array}{c} \alpha_1 \\ \alpha_2 \end{array} & \begin{bmatrix} 3 & 5 \\ 5 & 10 \end{bmatrix} \end{array}$$

Table 4.1

$$\begin{bmatrix} (\ \ ) & & (X) \\ \downarrow & & \\ (X) & \longleftarrow & (\ \ ) \end{bmatrix}$$

Table 4.2

It is not difficult to verify that the response configuration by Table 4.2, i.e., $r(\alpha_1, \beta_1) = (\alpha_2, \beta_1)$, $r(\alpha_1, \beta_2) = (\alpha_1, \beta_2)$, $r(\alpha_2, \beta_1) = (\alpha_2, \beta_1)$ and $r(\alpha_2, \beta_2) = (\alpha_2, \beta_1)$, is conventionally stable. X denotes a stationary state. (α_1, β_2) is not a pair of maxmin strategies, but it belongs to $F(r)$.

4.2. Observing Proposition 2 carefully, we can extend it as follows. Let us consider a general game with limited observations. We say that an outcome $s \in S$ is α-individually irrational iff for some $i \in N$, there is a strategy $s_i^* \in S_i$ such that

$$h_i(s'_{-i}, s_i^*) > h_i(s) \quad \text{for all} \quad s'_{-i} \in S_{-i} . \tag{4.1}$$

An outcome s is said to be α-individually rational iff it is not α-individually irrational.

Analogously to the proof of Proposition 2, we can prove the following proposition.

Proposition 3. Let V be an arbitrary conventionally stable set. Then every s in V is α-individually rational. This proposition does not depend upon the observation structure.

4.3. This proposition looks to be similar to the "folk" theorem that every β-individually rational outcome in a game in normal form is attained by a Nash equilibrium in the super game. See Aumann [1959] or Rubinstein [1979]. Although Proposition 3 states only the α-individual rationality of every outcome in every conventionally stable set, we can, in fact, prove a much more similar result to the folk theorem.

Proposition 4. Assume that the observation structure is the finest. Let $\hat{s}$ be an outcome such that for some Nash equilibrium s^N, $h_i(\hat{s}) \geqq h_i(s^N)$ for all $i \in N$. Then there exists a conventionally stable set V such that $\hat{s}$ belongs to V.

Proof. Let r be the response configuration such that

$$r(s) = \begin{cases} s^N & \text{if } s \neq \hat{s} \\ \hat{s} & \text{if } s = \hat{s} . \end{cases}$$

It is easily verified that $F(r) = \{\hat{s}, s^N\}$. Since every player i is effective only for (s^N_{-i}, s_i) for all $s_i \in S_i$ and $\hat{s}$, and s^N is a Nash equilibrium, he can not improve upon any absorptions. Hence r satisfies the stability. The absorptivity and the acyclicity are clear. Hence r is conventionally stable. Q.E.D.

This result may mean the implausibility of our new solution concept, but we did assume the finest observation structure in this proposition. This assumption is not plausible nor persuasive in games with many players. In fact, when the observation structure is coarser, this multiplicity will disappear. We will argue it in subsequent sections.

5. The Prisoner's Dilemma

5.1. Let us consider the prisoner's dilemma given by Example 2.1, i.e.,

$$\begin{array}{c c} & \begin{array}{cc} \beta_1 & \beta_2 \end{array} \\ \begin{array}{c} \alpha_1 \\ \alpha_2 \end{array} & \begin{bmatrix} (5,5) & (-4,6) \\ (6,-4) & (-3,-3) \end{bmatrix} \end{array}$$

Table 5.1

This game has several interesting features. See Luce and Raiffa [1957, Sec. 5.4 and 5.5].

The following proposition lists every conventionally stable response configuration in this game.

Proposition 5. In the prisoner's dilemma, there exist exactly two conventionally stable response configurations, i.e.,

(1) r is a trivial configuration, i.e., $r(s) = (\alpha_2, \beta_2)$ for all $s \in S$.

(2) $r(\alpha_1, \beta_1) = (\alpha_1, \beta_1)$ and $r(s) = (\alpha_2, \beta_2)$ for all $s \neq (\alpha_1, \beta_1)$.

$$\left[\begin{tikzcd} (\ ) \arrow[dr] & (\ ) \arrow[d] \\ (\ ) \arrow[r] & (X) \end{tikzcd}\right]$$

Table 5.2: (1)

$$\left[\begin{tikzcd} (X) & (\ ) \arrow[d] \\ (\ ) \arrow[r] & (X) \end{tikzcd}\right]$$

Table 5.3: (2)

Proof. It is easy to check that these two response configurations are conventionally stable. So, we need to prove that any other response configuration is not conventionally stable.

(1) Case: $r_1(\alpha_1, \beta_k) = \alpha_2$ & $r_1(\alpha_2, \beta_k) = \alpha_1$ for $k = 1$ or 2.

$$\left[\begin{tikzcd} (\ ) \arrow[dr] & (\ ) \\ (\ ) \arrow[u] & (\ ) \end{tikzcd}\right]$$

(1)

(2) Case: $r_2(\alpha_k, \beta_1) = \beta_2$ & $r_2(\alpha_k, \beta_2) = \beta_1$ for $k = 1$ or 2.

$$\left[\begin{tikzcd} (\ ) \arrow[dr] & (\ ) \arrow[l] \\ (\ ) & (\ ) \end{tikzcd}\right]$$

(2)

Let us consider only (2). (α_k, β_1) and (α_k, β_2) are not in $F(r)$. If player 1 fixes his strategy to α_k , the cycle $(\alpha_k, \beta_1) \leftrightarrow (\alpha_k, \beta_2)$ appears. Thus the acyclicity condition is not satisfied.

The next case is also not conventionally stable because of the violation of the absorptivity condition.

(3) Case: $r(s) = s'$ and $r(s') = s$ for some s and s' with $s \neq s'$.

$$\left[\begin{tikzcd} (\ ) \arrow[dr, leftrightarrow] & (\ ) \\ (\ ) & (\ ) \end{tikzcd}\right]$$

(3)

(4) Case: $(\alpha_1, \beta_2) \in F(r)$ or $(\alpha_2, \beta_1) \in F(r)$: Neither of them is α-individually rational. By Proposition 3, r is not conventionally stable.

$$\begin{bmatrix} (\ ) & (X) \\ (\ ) & (\ ) \end{bmatrix}$$

(4)

(5) Case: $(\alpha_1, \beta_1) \in F(r)$ & $(r(\alpha_1, \beta_2) = (\alpha_1, \beta_1)$ or $r(\alpha_2, \beta_1) = (\alpha_1, \beta_1))$: Let $r(\alpha_1, \beta_2) = (\alpha_1, \beta_1)$. Then player 2 can improve upon the absorption $r(\alpha_1, \beta_2) = (\alpha_1, \beta_1)$ by fixing his strategy to β_2 .

$$\begin{bmatrix} (X) \leftarrow (\ ) \\ (\ ) \quad (\ ) \end{bmatrix}$$

(5)

(6) Case: $(\alpha_2, \beta_2) \notin F(r)$: In this case $F(r) = \{(\alpha_1, \beta_1)\}$ by (4). By (5), $r(\alpha_1, \beta_2) \neq (\alpha_1, \beta_1)$ and $r(\alpha_2, \beta_1) \neq (\alpha_1, \beta_1)$. Thus, by the absorptivity, $r(\alpha_2, \beta_2) = (\alpha_1, \beta_1)$ and $r(\alpha_1, \beta_2) = (\alpha_2, \beta_2)$ or $r(\alpha_2, \beta_1) = (\alpha_2, \beta_2)$. Either case contradicts Case (1) or (2), respectively. Here 0 means a nonstationary state.

$$\begin{bmatrix} (X) & (\ ) \\ (\ ) & (0) \end{bmatrix}$$

(6)

(7) Case: $F(r) = \{(\alpha_1, \beta_1), (\alpha_2, \beta_2)\}$ & $(r(\alpha_1, \beta_2) = (\alpha_2, \beta_1)$ or $r(\alpha_2, \beta_1) = (\alpha_1, \beta_2))$: Let $r(\alpha_1, \beta_2) = (\alpha_2, \beta_1)$. Then, by (5), $r(\alpha_2, \beta_1) = (\alpha_2, \beta_2)$. Since player 1 is effective for (α_1, β_1) and $h_1(\alpha_1, \beta_1) > h_1(\alpha_2, \beta_2)$, player 1 can improve upon the absorption $r^2(\alpha_1, \beta_2) = (\alpha_2, \beta_2)$ by fixing his strategy to α_1 .

$$\begin{bmatrix} (X) & & (\ ) \\ & \swarrow & \\ (\ ) & & (X) \end{bmatrix}$$

(7)

(8) Case: $F(r) = \{(\alpha_2, \beta_2)\}$ and $(r(\alpha_1, \beta_1) = (\alpha_1, \beta_2)$ or $r(\alpha_1, \beta_1) = (\alpha_2, \beta_1))$: Let $r(\alpha_1, \beta_1) = (\alpha_1, \beta_2)$. When the current outcome is (α_1, β_1) , player 2 fixes his strategy to β_1 and so, he is effective for (α_1, β_1) and $h_2(\alpha_1, \beta_1) > h_2(\alpha_2, \beta_2)$. Hence player 1 can improve upon the absorption from (α_1, β_1) to (α_2, β_2) .

$$\begin{bmatrix} (\ \ ) & \rightarrow & (\ \ ) \\ (\ \ ) & & (X) \end{bmatrix}$$

(8)

(9) Case: $F(r) = \{(\alpha_2, \beta_2)\}$ and $(r(\alpha_1, \beta_2) = (\alpha_2, \beta_1)$ or $r(\alpha_2, \beta_1) = (\alpha_1, \beta_2))$: Let $r(\alpha_1, \beta_2) = (\alpha_2, \beta_1)$. By (8), $r(\alpha_1, \beta_1) = (\alpha_2, \beta_2)$. Hence r is not conventionally stable by (2).

$$\begin{bmatrix} (\ \ ) & & (\ \ ) \\ & \swarrow & \\ (\ \ ) & & (X) \end{bmatrix}$$

(9)

5.2. Initially we note that we are considering conventionally stable sets in two-person games under the hypothesis (besides our basic assumptions mentioned in Subsection 2.5) that the players can neither communicate nor cooperate with each other. Of course, in an actual situation, these hypothesis are not very realistic.[21] The two-person games, however, are mathematically the most simple, and it is useful to investigate them before we consider more complicated games. We should bear this in mind.

Proposition 5 lists two conventionally stable response configurations. One is the trivial one, which is not so important. The second one deserves a small comment. The second one satisfies the condition of stability as follows. If each player chooses his first strategy, then it is stable since if a player changes his strategy to his second

[21] The author thinks that in a repeated situation of a two-person game, it is natural that some kind of communication and cooperation occurs, because the costs of communication are not so large.

strategy (which raises his temporal payoff), then the other responds to his action by also choosing his second strategy and the resulting stationary state is worse for both of them. That is, due to the mutual deterrence, the pair of the first strategies becomes stable. Of course, then it is also necessary that the pair of the second strategies is also stable. We should not forget that in order for a point to be in a conventionally stable set, it is necessary that the other points in it are also stable. More strictly speaking, the stability of one point in a conventionally stable set depends upon the entire structure of the response configuration.

6. Battle of the Sexes

6.1. A natural question is whether or not configurations of best-response functions are conventionally stable. Here $r_i^* : S \to S_i$ is called a best-response function iff $h_i(s_{-i}, r_i^*(s)) \geqq h_i(s)$ for all $s \in S$. Of course, for r^* to satisfy (2.6), it is necessary to assume some condition on the observation structure. In the prisoner's dilemma, r^* satisfies (2.6), and is conventionally stable. In the battle of the sexes, r^* satisfies (2.6) but is not conventionally stable. The battle of the sexes is given as the following bimatrix game:

$$\begin{array}{c c} & \begin{array}{cc} \beta_1 & \beta_2 \end{array} \\ \begin{array}{c} \alpha_1 \\ \alpha_2 \end{array} & \begin{bmatrix} (2,1) & (-1,-1) \\ (-1,-1) & (1,2) \end{bmatrix} \end{array}$$

Table 6.1

It is easy to see that there is the unique observation structure, i.e., the finest one. The pair of the best-response functions r^* is represented as Table 6.2:

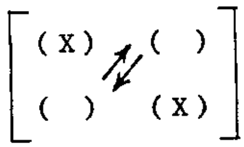


Table 6.2

Here X denotes a stationary state, and so, $F(r) = \{(\alpha_1, \beta_1), (\alpha_2, \beta_2)\}$. But neither of (α_2, β_1) nor (α_1, β_2) is absorbed by any point in $F(r)$. So, r^* is not conventionally stable.

<u>6.2</u>. The following proposition gives the lists of all conventionally stable response configurations in this game.

<u>Proposition 6</u>. In the battle of the sexes, there are exactly eight conventionally stable response configurations:

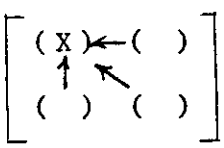


Table 6.3

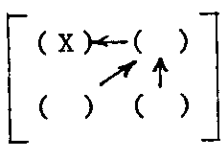


Table 6.4

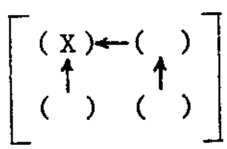


Table 6.5

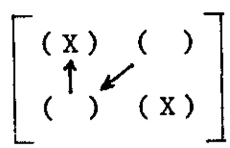


Table 6.6

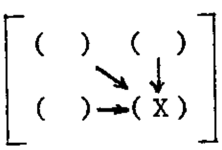


Table 6.7

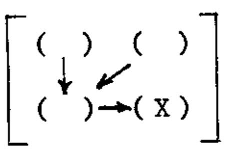


Table 6.8

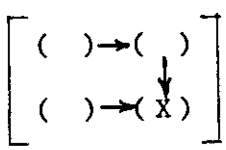


Table 6.9

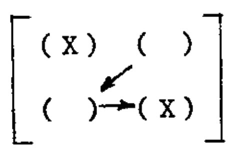


Table 6.10

<u>Proof</u>. In Proposition 1, it was shown that the response configurations given by Tables 6.3 and 6.7 are conventionally stable. Here we prove only that the configuration of Table 6.4 is conventionally stable.

The absorptivity and the acyclicity are clear. Since player 1 is effective only for (α_2, β_2) and (α_1, β_1) but $h_1(\alpha_2, \beta_2) < h_1(\alpha_1, \beta_1)$, player 1 can not improve upon any absorption. Since player 2 is effective only for (α_1, β_2) and (α_1, β_1) but $h_2(\alpha_1, \beta_2) < h_2(\alpha_1, \beta_1)$, player

2 can not improve upon any absorption.

Next we show that any other response configuration is not conventionally stable.

(1) Case: $(\alpha_1, \beta_2) \varepsilon F(r)$ and $r(\alpha_2, \beta_2) = (\alpha_1, \beta_2)$: Since player 1 is effective for (α_2, β_2) and $h_1(\alpha_2, \beta_2) > h_1(\alpha_1, \beta_2)$, player 1 can improve upon the absorption of (α_2, β_2) by (α_1, β_2) .

$$\begin{bmatrix} (\ ) & (X) \\ & \uparrow \\ (\ ) & (\ ) \end{bmatrix}$$
(1)

(2) Case: $(\alpha_1, \beta_2) \varepsilon F(r)$ and $r(\alpha_1, \beta_1) = (\alpha_1, \beta_2)$: Similar to Case (1), player 2 can improve upon the absorption of (α_1, β_1) by (α_1, β_2) .

$$\begin{bmatrix} (\ ) \rightarrow (X) \\ (\ ) \quad (\ ) \end{bmatrix}$$
(2)

(3) Case: $(\alpha_2, \beta_1) \varepsilon F(r)$ and $r(\alpha_1, \beta_1) = (\alpha_2, \beta_1)$.

$$\begin{bmatrix} (\ ) & (\ ) \\ \downarrow & \\ (X) & (\ ) \end{bmatrix}$$
(3)

(4) Case: $(\alpha_2, \beta_1) \varepsilon F(r)$ and $r(\alpha_2, \beta_2) = (\alpha_2, \beta_1)$: In Cases (3) and (4), it is proved similarly to case (1) that r can not be conventionally stable.

$$\begin{bmatrix} (\ ) \quad (\ ) \\ (X) \leftarrow (\ ) \end{bmatrix}$$
(4)

(5) Case: $r(\alpha_1, \beta_1) = (\alpha_2, \beta_1)$: Since player 1 is effective for (α_1, β_1) and $h_1(\alpha_1, \beta_1) > h_1(s_1, s_2)$ for all $(s_1, s_2) \neq (\alpha_1, \beta_1)$, player 1 can improve upon the absorption of (α_1, β_1) .

$$\begin{bmatrix} (\ ) & (\ ) \\ \downarrow & \\ (\ ) & (\ ) \end{bmatrix}$$
(5)

(6) Case: $r(\alpha_2, \beta_2) = (\alpha_2, \beta_1)$: Similar to (5), player 2 can improve upon the absorption of (α_2, β_2) .

$$\begin{bmatrix} (\ ) \quad (\ ) \\ (\ ) \leftarrow (\ ) \end{bmatrix}$$
(6)

(7) Case: $r(s_1, s_2) = (s_1', s_2')$ and $r(s_1', s_2') = (s_1, s_2)$: Neither point is absorbed by any point.

$$\begin{bmatrix} (\ ) \quad (\ ) \\ (\ ) \leftrightarrow (\ ) \end{bmatrix}$$
(7)

(8) Case: $F(r) \supset \{(\alpha_1, \beta_1), (\alpha_2, \beta_1)\}$: Since player 1 is effective for (α_1, β_1) and $h_1(\alpha_1, \beta_1) > h(\alpha_2, \beta_1)$, player 1 can improve upon (α_2, β_1) .

$$\begin{bmatrix} (X) & (\ ) \\ (X) & (\ ) \end{bmatrix}$$
(8)

(9) Case: $F(r) \supset \{(\alpha_2, \beta_1), (\alpha_2, \beta_2)\}$.

$$\begin{bmatrix} (\ ) & (\ ) \\ (X) & (X) \end{bmatrix}$$
(9)

(10) Case: $F(r) \supset \{(\alpha_1, \beta_1), (\alpha_1, \beta_2)\}$.

$$\begin{bmatrix} (X) & (X) \\ (\ ) & (\ ) \end{bmatrix}$$
(10)

(11) Case: $F(r) \supset \{(\alpha_1, \beta_2), (\alpha_2, \beta_2)\}$: Similar to Case (8), these cases can not be conventionally stable.

$$\begin{bmatrix} (\ ) & (X) \\ (\ ) & (X) \end{bmatrix}$$
(11)

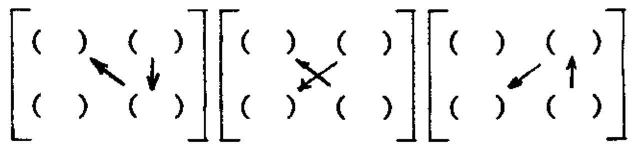
(12) Case: $F(r) = \{(\alpha_1, \beta_1), (\alpha_2, \beta_2)\}$ and $r(\alpha_1, \beta_2) = (\alpha_1, \beta_1)$: Player 1 has the deviation $\{e_1^{\alpha_1}, e_1^{\alpha_1}\}$ by which (α_1, β_1) is connected to (α_2, β_2) , i.e., $(e_1^{\alpha_1}, r_2)\cdot(e_1^{\alpha_1}, r_2)[(\alpha_2, \beta_2)] = (\alpha_1, \beta_1)$, where $e_1^{\alpha_1}$ is the response function such that $e_1^{\alpha_1}(s) = \alpha_1$ for all $s \in S$. Since player 1 is effective for (α_1, β_1) and $h_1(\alpha_1, \beta_1) > h_1(\alpha_2, \beta_2)$, he can improve upon (α_2, β_2) by the above deviation.

$$\begin{bmatrix} (X) \leftarrow & (\ ) \\ (\ ) & (X) \end{bmatrix}$$
(12)

(13) Case: $F(r) = \{(\alpha_1, \beta_1), (\alpha_2, \beta_2)\}$ and $r(\alpha_1, \beta_2) = (\alpha_2, \beta_2)$: Similar to Case (12), player 2 can improve upon (α_1, β_1)

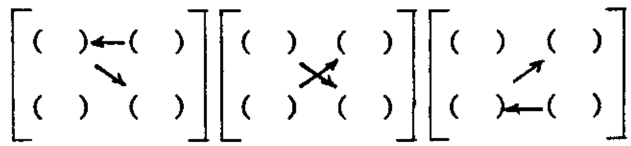
$$\begin{bmatrix} (X) & (\ ) \\ & \downarrow \\ (\ ) & (X) \end{bmatrix}$$
(13)

(14) Case: $r_1(\alpha_1, \beta_k) = \alpha_2$ and $r_1(\alpha_2, \beta_k) = \alpha_1$ for $k = 1, 2$.



(14)

(15) Case: $r_2(\alpha_k, \beta_1) = \beta_2$ and $r_2(\alpha_k, \beta_2) = \beta_1$ for $k = 1, 2$.



(15)

These cases violate the acyclicity condition.

6.3. Every response configuration listed in Proposition 6 can be interpreted naturally. Every stable configuration has a structure of concession and avoids double cross. One player always makes a concession to avoid double cross. Of course, both can not simultaneously make a concession and if so, they meet a double cross. But this concession is not arbitrary. For example, Case (12) of the proof is similar to Tables of Proposition 6 but it can not be conventionally stable.

In this game, (α_2, β_1) and (α_1, β_2) are α-individually rational, but they do not belong to any conventionally stable set. Hence the converse of Proposition 3 is not necessarily true even in games with the finest observation structures.

7. Games with a Continuum of Players--Coarse Observation Structures and Nash Equilibria

7.1. Games with a continuum of players have the following special property about observation structures: Any player can not know other players' strategies precisely by observing his payoff function but can know only a certain tendency of the others' behavior as a whole. Of course, the games of the prisoner's dilemma and battle of the sexes do not enjoy such a property. This property is almost true in games with a large (finite) number of palyers and, of course, games with a continuum of players are regarded as an ideal approximation for ones with a large (finite) number of players. This property makes it possible to treat intrinsically games with coarse observation structures, while in finite cases, this feature appears only approximately. The conclusion of this section is diametrically opposed to the complete information interpretation of Nash equilibrium, that is, Nash equilibrium makes sense in a game only if the observation structure of the game is very coarse. This conclusion would not only support the perfect competitive interpretation of Nash equilibrium but would also give a more precise answer to the question (Q-3).

7.2. Let $(N,\mathcal{B},\mu)$ be a finite measure space which satisfies $\{i\} \in \mathcal{B}$ and $\mu(\{i\}) = 0$ for all $i \in N$ and $0 < \mu(N)$. Of course, N is the set of all players. We assume that S_i is a subset of some fixed set X and S_i is measurable as a correspondence from N to X. Let S be the set of all measurable selections from the correspondence S_i. We assume that S is nonempty. We introduce the equivalence relation $\sim$ into S by $s^1 \sim s^2 \Longleftrightarrow \mu(\{i \in N : s_i^1 = s_i^2\}) = \mu(N)$. We define S^* by $S^* = S/\sim$. We assume that for all $i \in N$, player i's payoff function $h_i(s)$ on S satisfies

$$h_i \text{ is a function on } S_i \times S^* \text{ to } R. \tag{7.1}$$

It is easy to verify that assumption (7.1) does not contradict our general formulation given in Section 2.[22, 23]

[22] We employ exactly the definition of Nash equilibrium given in Subsec. 3.1. even in a game with a continuum of players, i.e., "all $i \in N$" is not replaced by "almost all $i \in N$." Since every one person's behavior is important in noncooperative game theory, we cannot change the definition by nature.

We say that player $i \in N$ has a <u>positive informational influence</u> iff there is a finite sequence $\{s^1, \ldots, s^m\}$ in S such that

$$s_j^1 = s_j^2 \text{ if and only if } j \neq i , \tag{7.2}$$

$$\text{for all } k = 2, \ldots, m-1 , \text{ if } a_j(s^1) = a_j(s^2) = \ldots = a_j(s^k) , \text{ then } s_j^1 = s_j^2 = \ldots = s_j^{k+1} , \tag{7.3}$$

$$\mu(\{j \in N : s_j^m \neq s_j^1\}) > 0 . \tag{7.4}$$

This definition means that if player i changes his strategy but the others do not, then some players observe the change in his strategy and respond to the change and change their strategies, and then some other players observe their changes and change their strategies, and so on, and eventually players with a positive measure change their strategies. On the contrary, if player i has no positive informational influence, then his behavior can not affect the mass of players via the players' observations.

We say that a_i has the <u>price-like information property</u> iff

$$\text{for any } s \text{ and } s' \text{ in } S , \text{ if } a_i(s) = a_i(s') , \text{ then } h_i(s_{-i}, s_i'') = h_i(s'_{-i}, s_i'') \text{ for all } s_i'' \in S_i . \tag{7.5}$$

This condition requires that player i can know his "payoff function" on S_i on condition that the others fix their strategies. For example, noncooperative exchange models with a continuum of players (e.g., Dubey and Shapley [1979]) enjoy this property, because each player can

---

[23] For a sufficient condition for the existence of a Nash equilibrium in a game with a continuum of players, see Schmeidler [1973].

not influence a price vector determined by the rules of the game and he can know his "payoff function" only observing the price vector.

Proposition 7. Assume that for all $i \in N$, player i has no positive informational influence and a_i has the price-like information property. Let r be a conventionally stable response configuration. Then every $s \in F(r)$ is a Nash equilibrium.

Proof. Let $s \in F(r)$ be not a Nash equilibrium. Then there is a s_i' such that $h_i(s_{-i}, s_i') > h_i(s)$. Since a_i has the price-like information property, $h_i(\bar{s}_{-i}, s_i') = h_i(s_{-i}, s_i') > h_i(s) = h_i(\bar{s})$ for all $\bar{s} \in a_i(s) \cap F(r)$. By the acyclicity, there is a k such that $(r_{-i}, e_i)^k[(\bar{s}_{-i}, s_i')] = (r_{-i}, e_i)^{k-1}[(\bar{s}_{-i}, s_i')]$ for all $\bar{s} \in a_i(s) \cap F(r)$. Let $\bar{s}$ be arbitrary in $a_i(s) \cap F(r)$. Let $(r_{-i}, e_i)^m[(r_{-i}, s_i')] = \bar{s}^m$ for $m = 1, \ldots, k$. Since player i has no positive informational influence, $\bar{s} \sim \bar{s}^1 \sim \bar{s}^2 \sim \ldots \sim \bar{s}^k$. Hence $h_i(s) = h_i(\bar{s}) < h_i(\bar{s}_{-i}, s_i')$ $= h_i(\bar{s}^1) = \ldots = h_i(\bar{s}^k)$. It is clear that player i is effective for $\bar{s}^k$. Since $\bar{s}$ is arbitrary in $a_i(s) \cap F(r)$ and k is independent of $\bar{s}$, player i can improve upon s by deviation $\{e_i^{s_i'}, \ldots, e_i^{s_i'}\}$, where $e_i^{s_i'}$ is the constant response function such that $e_i^{s_i'}(s) = s_i'$ for all $s \in S$. This is a contradiction. Hence s is a Nash equilibrium. Q.E.D.

Dubey and Shapley [1979] prove the equivalence of the active Nash equilibria and the competitive equilibria in a noncooperative exchange economy with a continuum of players. Hence it follows from this and Proposition 7 that if a conventionally stable set does not contain any nonactive Nash equilibria, it consists of competitive equilibria in the noncooperative exchange economy with a continuum of players.

We will consider the significance of Proposition 7 in a subsequent subsection.

7.3. Best-response configurations may make sense in nonatomic games with coarse observation structures, because since no player can affect the other players, every player would maximize his payoff on condition that the others fix their strategies. In this subsection we show that this intuition is true to some extent.

We say that player i is the same type as player i' iff S_i and $S_{i'}$ are the same and $h_i(s) = h_{i'}(\bar{s})$ for all s and $\bar{s}$ in S with $s_j = \bar{s}_j$ for all $j \neq i, i'$ and $s_i = \bar{s}_{i'}$. We say that a response configuration r is symmetric iff any two players of the same type have the same response functions. A sequence $\{s^1, \ldots, s^k\}$ is said to be a cycle yielded by r iff $r(s^{t-1}) = s^t$ for $t = 2, \ldots, k$, $s^1 = s^k$ and s^k is not a stationary state.

Proposition 8. Suppose that $(N,\mathcal{B},\mu)$ has a finite number of types and that each player has a finite number of strategies. We assume that every player's observation structure has the price-like information property. Let r be a symmetric best response configuration. Then r is conventionally stable if and only if $F(r)$ is nonempty and r does not yield any cycle.

Proof. The necessity is clear. We show only the sufficiency. Every s in $F(r)$ is a Nash equilibrium because r is a best-response configuration. Since $(N,\mathcal{B},\mu)$ is nonatomic, any player can not influence the others' payoff functions by unilaterally deviating from his best-response function. That is, he can not influence any absorption. Therefore every player i is effective only for s or (s_{-i}, s_i'), where s is a Nash equilibrium and s_i' is an arbitrary strategy in S_i. That is, he can not improve upon any absorption. Since r is symmetric and each player's strategy set and the number of types are finite, any state s in S is absorbed by a state in $F(r)$ because otherwise, a cycle is yielded, which is impossible by the assumption of the sufficiency. The acyclicity of r is clear. Q.E.D.

It should be noted that a best-response configuration makes sense only if every player's observation function has the price-like information property.

7.4. Proposition 7 makes a very strong assertion. We imposed two assumptions on Proposition 7. One is the price-like information property and another one is the zero-informational influence. These conditions are interpreted as lower and upper bounds of informational requirements for the inclusion of the conventionally stable sets by the Nash equilibria. This result is corresponding to the classical interpretation of "perfectly competitive market" (which has never been discussed formally) and rather, it is more precise than the classical one. Of course, these two conditions form only a sufficient condition for the inclusion. Although we might generalize these conditions, the essential part could not be improved.[24] Therefore if we accepted the conventionally stable set as a more adequate solution concept, then we could reach the conclusion that Nash equilibrium makes sense only if the game has a similar structure

---

[24] For example, the price-like information property is a "global" condition but when we consider a well-behaved game (e.g., the payoff functions are concave) we may replace the condition by a "local" version of it.

to "perfectly competitive market." This is our answer to the question (Q-3) of Subsection 1.4.

We should, however, reserve to hold it until we investigate the conventionally stable set in other kinds of games and show that the conventionally stable set exhibits still plausibility in them. The core theory, the value theory and the Noncooperative Nash theory have succeeded in proving the adequacy of competitive equilibrium in large market, but they failed to explain monopolistic and oligopolistic markets. That is, when we apply these concepts to such markets the results are never plausible. For example, see Aumann [1973], Gardner [1976] and also Schotter and Schwödiauer [1980, Sec. 5.1]. Therefore if our theory succeeded in explaining such markets in a plausible way, we have the right to hold the above statement. The author will discuss these problems in a forthcoming paper.

The above implication of Proposition 7 may contradict a recent approach to the social choice theory. One purpose of the social choice theory is a construction of a decision process to satisfy several requirements. The recent approach asserts the informational decentralization and employ Nash equilibrium or variations of it as a necessary solution concept. See Schotter and Schwödiauer [1980, Sec. 3.2]. But if the above implication is true, Nash equilibrium does not necessarily make sense in games with a small number of players and if the number of players is very large, then the price-like information property is necessary. Further the informational assumptions of games constructed by some writers are quite ambiguous. Probably, some of them do not satisfy the informational conditions of Proposition 7 even in the case of a continuum of players. Therefore, the author thinks that this approach needs to be reconsidered from a more fundamental viewpoint.

8. Games with a Continuum of Players--Examples

8.1. In the previous section, we considered a sufficient condition with respect to observation structures for the conventionally stable sets to become Nash equilibria in games with nonatomic players. This sufficient condition requires that observation structures be very coarse in the sense defined by Subsection 2.3. At a first glance, this result may be curious, because the informational influence means that when a player takes some action, other players can only know his action and are not directly affected by it (by assumption of the nonatomic game) and so, if they ignore his action, then no essential change occurs. But if the others respond to his action, each also should conform to this movement. This is the reason why we need the sufficient condition, and is the most important point in our theory. That is, we often find phenomena of this type, i.e., an information does not directly affect any player's payoff but as the society responds to it, each player has to respond to it and so the social response is formed. The other game theories (of course, the other economic theories) can not treat problems of this type. So, we need to investigate whether or not phenomena of this type actually occur.

In this section we provide three examples in which the phenomena mentioned above occur.

8.2. A Dilemma Game with a Watch Machine: Let $N = [0,1]$ and $\mathcal{B}$ be the set of the usual Lebesgue measurable sets. Let μ be a Lebesgue measure with $\mu(N) > 0$. Every player $i \in N$ has two strategies 0 (bad strategy) and 1 (good strategy). His payoff function is given as

$$h_i(s) = 100\int_N s_j d\mu(j) - s_i \quad \text{for all} \quad s \in S . \tag{8.1}$$

It is easily verified that this game satisfies the definition of the previous section. This game is interpreted in several ways. The typical example is the tax-cheat game. That is, each player has two strategies 0 and 1, where 0 means that he cheats on his income tax and 1 means that he pays his income tax legitimately. In this paper, we do not assume that a player will be punished if he cheats on his income tax. Each player's payoff is divided into two parts, $100\int_N s_j d\mu(j)$, the public service he receives from government and $-s_i$, his private consumption.

In this game, there is the unique Nash equilibrium s^0 such that $s_i^0 = 0$ for all $i \in N$.

Assume that this society has a watch machine. The machine observes every player's behavior. The machine is set such that if it finds that some player is playing his bad strategy 0, then it sounds a siren. We assume that each player knows the value $100\int_N s_j d\mu(j)$, his strategy and whether or not the watch machine sounds a siren. Let $w(s) = 0$ or 1 depending on whether or not the watch machine sounds a siren. The observation structure is

$$\text{for any } i \in N , \quad a_i(s) = a_i(s') \iff 100\int_N s_j d\mu(j) = 100\int_N s'_j d\mu(j) , \quad s_i = s'_i \ \& \ w(s) = w(s') . \tag{8.2}$$

In this game, the following proposition holds.

Proposition 9. In the above game, there are exactly two conventionally stable sets (i) $V = \{s^0, s^1\}$ and (ii) $V = \{s^0\}$, where $s_i^0 = 0$ for all $i \in N$ and $s_i^1 = 1$ for all $i \in N$.

Proof. These sets are given as the conventionally stable sets of the following response configurations (i) $r(s) = s^0$ for all $s \in S$ with $s \neq s^1$ and $r(s^1) = s^1$ and (ii) $r(s) = s^0$ for all $s \in S$. It is not difficult to prove that these configurations are conventionally stable. Hence we should prove that if V is an arbitrary conventionally stable set, V coincides with either $\{s^0, s^1\}$ or $\{s^0\}$.

Let r be an arbitrary conventionally stable response configuration. Let $s \in F(r)$ but $s \neq s^0, s^1$. Then there is a player $i \in N$ with $s_i = 1$. Any $\bar{s}$ in $a_i(s) \cap F(r)$ satisfies $\bar{s}_i = 1$, $w(\bar{s}) = 1$ and $h_i(\bar{s}) = h_i(s)$ by (8.2). Since $w(\bar{s}) = 1$, there is a player i' with $\bar{s}_{i'} = 0$. Hence if player i changes his strategy to 0, any other player can not know his change since the watch machine will sound a siren in any case (since $\bar{s}_{i'} = 0$). This means $r_{-i}(\bar{s}_{-i'}, 0) = \bar{s}_{-i}$, i.e., player i is effective for $(\bar{s}_{-i}, 0)$. Of course, $h_i(\bar{s}_{-i}, 0) > h_i(\bar{s})$. Since $\bar{s}$ is arbitrary in $a_i(s) \cap F(r)$, player i can improve upon s by deviation $\{e_i^0\}$, where e_i^0 is the constant response function with $e_i^0(s) = 0$ for all $s \in S$.

In the following we show, in fact, that $\{s^1\}$ is not conventionally stable. Let $F(r) = \{s^1\}$. Let us consider the case where player i changes his strategy to 0. If $(r_{-i}, e_i)[(s_{-i}^1, 0)] = (s_{-i}^1, 0)$, then he is effective for $(s_{-i}^1, 0)$ and $h_i(s_{-i}^1, 0) > h_i(s^1)$. Since $a_i(s^1) = \{s^1\}$ by (8.2), he can improve upon s^1 by deviation $\{e_i^0\}$. Let $(r_{-i}, e_i)[(s_{-i}^1, 0)] \neq (s_{-i}^1, 0)$. Then there is a finite k by the acyclicity condition such that $(r_{-i}, e_i)^k[(s_{-i}^1, 0)] = (r_{-i}, e_i)^{k-1}[(s_{-i}^1, 0)]$. If $(r_{-i}, e_i)^{k-1}[(s_{-i}^1, 0)] = (s_{-i}^1, 0)$, then $(r_{-i}, e_i)[(s_{-i}^1, 0)] = (s_{-i}^1, 0)$, which is a contradiction. Hence $(r_{-i}, e_i)^{k-1}[(s_{-i}^1, 0)] \equiv s' \neq (s_{-i}, 0)$. Of course, $s'_{i'} = 0$ for some $i' \in N$. If $r_i(s') = 0$, then s' is a stationary state, which is a contradiction to that $F(r) = \{s^1\}$. If $r_i(s') = 1$, then $(s'_{-i}, 1)$ is a stationary state because his change can not be known to any other player since the watch machine sounds a siren observing that another player i' plays his bad strategy 0. This is also impossible. Hence if $F(r) = \{s^1\}$, then r is not conventionally stable. Q.E.D.

In this example, any player can not directly observe other players' strategies but he gets an aggregate information of whether or not at least one player plays the bad strategy on condition that he played his good strategy. In the first conventionally stable set, s^1 which is not a Nash equilibrium appears as a stationary state due to the mutual deterrence. That is, if some player changes his strategy to his bad strategy in s^1 , then the watch machine finds it and sounds a siren, and so, the others change their strategies. Eventually the worst state s^0 results. So, no player changes his strategy to avoid this.

Here it is worth to note that in the dilemma game, if we assume the finest observation structure, then Proposition 4 states that every α-individually rational outcome s (i.e., $h_i(s) \geqq 0$ for all $i \in N$) belongs to some conventionally stable set, because s^0 is the unique Nash equilibrium. This unfavorable result is caused by the finest observation structure, which would be inappropriate in this game. There fore we can say that the pathological multiplicity disappears when an appropriate observation structure is assumed.

8.3. A Dilemma Game with Rumor: In this subsection, we provide an example in which players can observe others' strategies more precisely.

Let us consider the dilemma game given in the previous section but with a different observation structure. Let Q be the set of all rational numbers in $[0,1]$. Let $U_c(i)$ be a c-neighborhood of i , i.e., $U_c(i) = \{j \in [0,1] : |i-j| < c\}$. We define the observation structure $\{a_i\}_{i \in N}$ by

$$\text{for all } i \in Q\,,\ a_i(s) \neq a_i(s') \Longleftrightarrow h_i(s) \neq h_i(s') \text{ or } s_i \neq s_i' \text{ or there is a } j \in U_c(i) \text{ such that } s_j \neq s_j'\,, \tag{8.4}$$

$$\text{for all } i \in N-Q\,,\ a_i(s) \neq a_i(s') \Longleftrightarrow h_i(s) \neq h_i(s') \text{ or } s_i \neq s_i' \text{ or } \{j \in U_d(i) : s_j \neq s_j'\} \text{ is dense on } U_d(i)\,, \tag{8.5}$$

where c and d are positive numbers and T is said to be dense in T' iff the closure of T is T' .

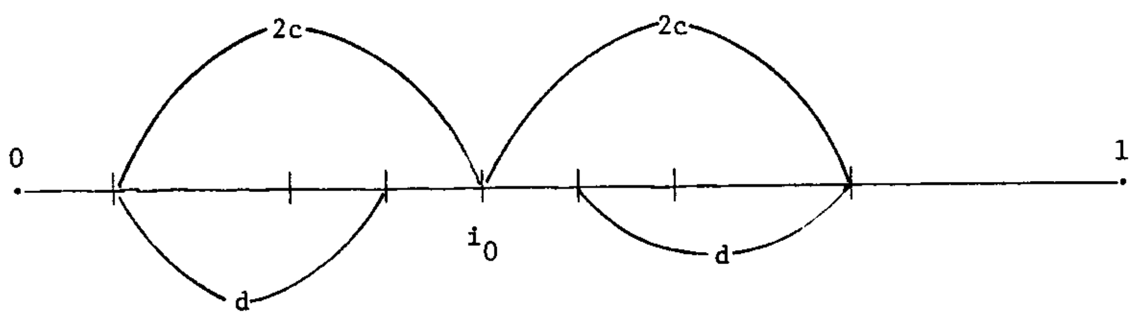


FIGURE 5

Each player i in Q can observe exactly the strategies of players in $U_c(i)$, and each player i in $N-Q$ can make a distinction between s and s' only if the differences between them are dense in $U_d(i)$. If a player i_0 changes his strategy, the players in $U_c(i_0) \cap Q$ know the change, and if they respond to it and change their strategies, then the players in $U_{2c}(i_0) \cap Q$ know the changes. In this case the changes do not influence the whole, because the measure of $U_c(i_0) \cap Q$ is zero.

Let us continue the argument. Let the players in $U_{2c}(i_0) \cap Q$ respond to the changes and change their strategies. If $2c > d$, then the players in $U_{2c-d}(i_0)$ know the changes. They have a positive measure as a group. So, player i_0's change influences on the society after 2 periods. If $kc > d > (k-1)c$, then the change influences the whole society after k periods.

In this game we can not prove Proposition 9's analogue but we can provide a conventionally stable set which does not consist of only the Nash equilibrium.

Proposition 10. Under the observation structure defined by (8.4) and (8.5), $\{s^0, s^1\}$ is a conventionally stable set.

Proof. We construct a response configuration as follows:

$$\text{for all } i \in Q, \quad r_i(s) = \begin{cases} 0 & \text{if } \int_N s_j d\mu(j) < \mu(N) \\ 0 & \text{if } s_j = 0 \text{ for some } j \in U_c(i) \\ 1 & \text{otherwise,} \end{cases} \tag{8.6}$$

$$\text{for all } i \in N-Q, \quad r_i(s) = \begin{cases} 0 & \text{if } \int_N s_j d\mu(j) < \mu(N) \\ 0 & \text{if } \{j \in U_d(i) : s_j = 0 \text{ is dense}\} \\ & \quad \text{in } U_d(i) \\ 1 & \text{otherwise.} \end{cases} \tag{8.7}$$

We prove that this r is conventionally stable and $F(r) = \{s^0, s^1\}$. Clearly $F(r) = \{s^0, s^1\}$. When $\int_N s_j d\mu(j) < \mu(N)$, $r(s) = s^0$. It is also clear that any player can not improve upon this absorption of s by s^0. Let $\int_N s_j d\mu(j) = \mu(N)$ but $s_{j_0} = 0$ for some $j_0 \in N$. Let k is the minimal integer such that $kc \geq 1$. Then the players in

$U_c(j_0) \cap Q$ know that $s_{j_0} = 0$, and so, by (8.6), $r_i(s) = 0$ for all $i \in U_c(j_0) \cap Q$. Let $\mu(\{i \in N : \{j \in U_d(i) : r_j(s) = 0\}$ is dense in $U_d(i)\}) > 0$. Then $r_i^2(s) = 0$ for all $i \in \{i \in N : \{j \in U_d(i) : r_j(s) = 0\}$ is dense in $U_d(i)\}$. Hence $\int_N r_j^2(s)d\mu(j) < \mu(N)$. This implies $r_j^3(s) = 0$ for all $j \in N$ by (8.6) and (8.7). Let $\mu(\{i \in N : \{j \in U_d(i) : r_j(s) \neq 1\}$ is dense in $U_d(i)\}) = 0$. Then $r_i^2(s) = 0$ for all $i \in U_c(s) \cap Q$. If $\mu(\{i \in N : \{j \in U_d(i) : r_j^2(s) = 0\}$ is dense in $U_d(i)\}) > 0$, then the above argument is applicable to this case and so, we get $r_j^4 = 0$ for all $j \in N$. Otherwise we continue the same argument, and if we reach k , $\mu(\{i \in N : \{j \in U_d(i) : r_j^k(s) = 1\}$ is dense in $U_d(i)\}) = \mu(N)$ because $kc \geqq 1$. Therefore $r_j^{k+1}(s) = 0$ for all $j \in N$. That is, s is absorbed by s^0 . This absorption is not influenced by any player's deviation because of (8.6) and (8.7). In this case, every player i is effective only for s^0 and $(s^0_{-i}, 1)$ and $h_i(s^0) > h_i(s^0_{-i}, 1)$, i.e., he can not improve upon this absorption. Similarly, when $s = s^1$, if some player i deviates from 1 to 0, then the outcome goes to s^0 or $(s^0_{-i}, 1)$ and so, his permanent payoff decreases. Hence any player can not improve upon s^1 . Clearly, no player can improve upon s^0 . The acylicity is clear. Q.E.D.

8.4. A Game of Conformists with a Good Leader: Let $(N,\mathcal{B},\mu)$ be the same as that given in Subsection 8.2. Let $S_i = \{1, \ldots, k\}$ for all $i \in N$. Each player i has the following payoff function:

$$h_i(s) = \int_N s_j d\mu(j) - \left(\int_N s_j d\mu(j)/\mu(N) - s_i\right)^2 \text{ for all } s \in S . \quad (8.8)$$

This game is interpreted as follows. Every player has k number of strategies which are ordered with respect to the favor of the society, that is, strategy t gives more social utility that strategy t' if $t > t'$. No player gets any private utility except the disutility which comes from the deviation from the average. Every player hates to deviate from the average.

It is easy to see that in this game

$$\text{an outcome } s \text{ is a Nash equilibrium if and only if } s = s^t \text{ for some } t \in \{1, \ldots, k\}, \tag{8.9}$$

where s^t is the outcome such that $s_i^t = t$ for all $i \in N$. If $t > t'$, s^t is socially preferred to $s^{t'}$. But every s^t is a Nash equilibrium.

Let us consider the case where the society has a leader i_0. The leader is very famous, and so, his strategy is known to every other player after he played it. But let us assume that no other player has any positive informational influence to the other players. So, each player i's observation function $a_i(s)$ is given as

$$a_i(s) = a_i(s') \Longleftrightarrow \int_N s_j d\mu(j) = \int_N s'_j d\mu(j), \quad s_i = s'_i \ \& \ s_{i_0} = s'_{i_0}. \tag{8.10}$$

We assume that the leader also does not have any precise observation. That is, his observation function also satisfies (8.10).

The leader i_0 has only an informational influence on the society but not any direct power to control the society. In this case, an important problem is whether or not the leader can lead the society to the socially optimal outcome s^k. We can answer this question in the affirmative.

Let r_{i_0} be defined by

$$r_{i_0}(s) = \begin{cases} [\int_N s_j d\mu(j)/\mu(N)] + 1 & \text{if } \int_N s_j d\mu(j)/\mu(N) < k-1 \\ k & \text{otherwise} \end{cases} \tag{8.11}$$

where $[\int_N s_j d\mu(j)/\mu(N)]$ is the largest integer which is smaller than or equal to $\int_N s_j d\mu(j)/\mu(N)$. We say that a player i's response function r_i is consistent with a subset V of S iff for some r_{-i}, (r_{-i}, r_i) is conventionally stable and $V = F(r_{-i}, r_i)$.

Proposition 11. r_{i_0} defined by (8.11) is consistent with V if and only if $V = \{s^k\}$.

Proof. Let r_{-i_0} be defined by $r_j(s) = s_{i_0}$ for all $s \in S$ and all $j \neq i_0$. It is not difficult to verify that (r_{-i_0}, r_{i_0}) is conventionally stable and that $V = \{s^k\} = F(r_{-i_0}, r_{i_0})$. The converse is also not difficult. Q.E.D.

Proposition 11 says that if the leader behaves as (8.11) and if the others trust and follow him, then this case is conventionally stable and the socially optimal outcome is reached. But if other players do not trust nor follow him, then this behavior is not conventionally stable. If every player pursues his myopic interest, the following proposition holds.

Proposition 12. In this game, any symmetric best-response configuration is conventionally stable. Further F(r) coincides with the set of all Nash equilibria.

Proof. This proposition follows immediately Proposition 8. Q.E.D.

8.5. These three examples have different kinds of observation structures, but satisfy the condition of positive informational influence. The leaks of information through observation structures subtly affect the conventionally stable set. We can see that these examples automatically satisfy the price-like information property by each player's observation of payoffs. Thus, the price-like information property would be satisfied by many games automatically. This property, however, would not be derived from the observations of payoffs if the rules of game is very complicated. It is easy to construct an example in which every conventionally stable set is not included by the Nash equilibria even under the assumption of no positive informational influence. Therefore we should be always careful for observation structures to consider games of social problems.

9. Conclusion

We defined the new solution concept "conventionally stable set" in games in normal form with limited observations. In our treatment, the meanings of "game," "information" and "solution" have less ambiguities than in other existing theories. This is our answer to the questions (Q-1) and (Q-2). Further we obtained a very precise answer to the question (Q-3). Probably, the examples of Section 8 would be considered as an answer to (Q-5), but they are not enough to say so. Forthcoming papers will provide intrinsically novel applications to monopolistic, oligopolistic markets and other social problems. In Subsection 7.4, we discussed the question (Q-4). We, now, are not in a suitable position to discuss the question (Q-6), and will do it by considering the foundation of our new theory in forthcoming papers.

REFERENCES

Aumann, R. J., Acceptable Points in General Cooperative n-Person Games, Annals of Mathematical Studies, No. 40: Contribution to the Theory of Games, Vol. IV, ed. by A. W. Tucker and R. D. Luce, Princeton University Press, 1959, 287-324.

__________, Agreeing to Disagree, The Annals of Statistics, Vol. 4 (1976), 1236-1239.

__________, Disadvantageous Monopolies, Journal of Economic Theory, Vol. 6 (1973), 1-11.

__________ and M. Maschler, Some Thoughts on the Minimax Principle, Management Science, Vol. 18 (1972), 54-63.

Basu, K., Information and Strategy in Iterated Prisoner's Dilemma, Theory and Decision, Vol. 8 (1977), 293-298.

Dubey, P. and L. S. Shapley, Noncooperative General Exchange with a Continuum of Traders, The Institute for Advanced Studies Report No. 51/79, The Hebrew University of Jerusalem, 1979.

Gardner, R., Shapley Value and Disadvantageous Monopolies, Journal of Economic Theory, Vol. 16 (1977), 513-517.

Gilbert, J. C., Game Theory and Convention, Syntheses, Vol. 46 (1981), 41-93.

Harsanyi, J. C., Games with Incomplete Information Played by 'Bayesian Players,' Parts I, II, III, Management Science, Vol. 14 (1967-8), 159-182, 320-334 and 486-502.

__________, An Equilibrium-Point Interpretation of Stable Sets and a Proposed Alternative Definition, Management Science, Vol. 20 (1974), 1472-1495.

Lewis, D. K., Convention, Harvard University Press, Cambridge, 1969.

Luce, R. D. and H. Raiffa, Games and Decisions, John Wiley, 1957.

Kaneko, M., A Bilateral Monopoly and the Nash Cooperative Solution, Journal of Economic Theory, Vol. 24 (1981), 311-327.

Marschak, T. and R. Selten, General Equilibrium with Price-Making Firms, Springer-Verlag, 1974.

__________, Oligopolistic Economies as Games of Limited Information, Zeitschrift für Gesamte Staats Wissenschaft, Vol. 133 (1977), 385-410.

__________, Restabilizing Responses, Inertia Supergames, and Oligopolistic Equilibria, Quarterly Journal of Economics, Vol. 92 (1978), 71-93.

Nash, J. F., Non-Cooperative Games, Annals of Mathematics, Vol. 54 (1951), 286-295.

__________, Two-Person Cooperative Games, Econometrica, Vol. 23 (1953), 128-140.

Rubinstein, A., Equilibrium in Supergames with the Overtaking Criterion, Journal of Economic Theory, Vol. 21 (1979), 1-9.

Schmeidler, D., Equilibrium Points of Nonatomic Games, Journal of Statistical Physics, Vol. 7 (1973), 295-300.

__________, Walrasian Analysis via Strategic Outcome Functions, Econometrica Vol. 48 (1980), 1585-1593.

Schotter, A., The Economic Theory of Social Institution, Cambridge University Press, Cambridge, 1981.

__________ and G. Schwödiauer, Economics and the Theory of Games: A Survey, Journal of Economic Literature, Vol. 18 (1980), 479-523.

Selten, R., Reexamination of the Perfect Concept for Equilibrium Points in Extensive Games, International Journal of Game Theory, Vol. 4 (1975), 25-55.

__________, A Noncooperative Model of Characteristic-Function Bargaining, D.P. No. 90, Institute of Mathematical Economics, Universitat Bielefeld, 1980.

Shubik, M., Game Theory in the Social Science, Vol. I--An Introduction to Game Theory: Concepts and Solutions, Cambridge, MA: MIT Press, forthcoming (1981).

Vickrey, W., Self-Policing Properties of Certain Imputation Sets, Annals of Mathematical Studies, No. 40: Contributions to the Theory of Games, Vol. IV, ed. by A. W. Tucker and R. D. Luce, Princeton: Princeton University Press, 1959, 213-246.

von Neumann, J. and O. Morgenstern, Theory of Games and Economic Behavior, Princeton: Princeton University Press, 3rd ed., 1953.