COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 598

ON THE INFINITE WELFARE COST OF INFLATION AND OTHER SECOND ORDER EFFECTS

Christophe Chamley

July 30, 1981

ON THE INFINITE WELFARE COST OF INFLATION AND OTHER SECOND ORDER EFFECTS*

by

Christophe Chamley

ABSTRACT

The optimal inflation rate is analyzed in a simple model of intertemporal general equilibrium where agents have an operative bequest motive and taxation is distortionary. Monetary balances are used as a productive input, and agents have perfect foresight. The optimal value of the permanent inflation rate can be approximated by a simple formula.

The case in which the growth of aggregate income exceeds the social discount rate is unlikely to be important, and the optimal value of the permanent inflation rate depends on the existence of a short-run trade-off between unemployment and inflation.

1. Introduction

The benefits and the costs of perfectly anticipated inflation have been the subject of a long debate among economists. To simplify the issues, let us assume, as usual, that anticipated inflation is equivalent to a tax on the monetary base. A permanent inflation has three main effects: First, the reduction of real cash balances implies a welfare cost equal to the reduction of services provided by money. The second effect is substitution of capital for money, which may increase the long-run levels of the capital stock, aggregate output and aggregate consumption. This effect occurs when capital markets are not perfect, or when individuals use intertemporal horizons (for example, their own life-times), shorter than the horizon of society. Finally, since inflation is equivalent to a tax on money holdings (and deflation to a negative tax or a subsidy), it is a substitute for other sources of government revenue, in particular explicit taxes.

The literature on the subject may be confusing because different authors refer, in their arguments, to different models. Three main types of general equilibrium models have been used. In the first, savings are equal to a fraction of disposable income and are equal to the accumulation of capital and money. A portfolio equation between money and capital completes the model. This approach has been useful as a first step (Tobin, 1965), but it relies on an *ad hoc* consumption function.[1]

In the second type of study the framework is provided by life-cycle models of capital accumulation (Weiss, 1979; Drazen, 1980). In these models, individuals save for consumption during their retirement period, planning to leave no bequest. The capital stock is determined only by the individual's taste for retirement consumption. It is not surprising then, that the level of the capital stock is not necessarily socially optimal: the rate of return in the steady state is not equal to the social discount rate (Diamond, 1965).

Two cases should be considered. When the level of the government debt (positive or negative), is exogenously fixed, the tax system can be used as a partial substitute for public savings (or dissavings), because different taxes have different time profiles. For example, when the labor supply is fixed, a combination of a consumption tax and a labor subsidy is equivalent to public savings. Because of this effect, the optimal inflation rate and the optimal explicit tax rates are not determined by the standard rules of optimal taxation.

These rules developed initially by Ramsey do apply in the overlapping generation framework when the public debt can be adjusted by deficits or surpluses (Pestieau, 1974, Atkinson and Sandmo, 1980). On the transition path, the optimal tax rates depend on the initial distribution of endowments between the individuals living at the time of a tax reform. Because we address in this paper only the problem of efficient taxation, we follow the third approach.

All individuals in the same generation are identical and have an operative bequest motive towards their descendants. The private sector can therefore be represented by a large number of identical families which behave competitively, are endowed with perfect foresight, and maximize an intertemporal utility function. When the government can balance its budget by nondistortionary lump-sum taxes, while the inflation tax is distortionary, the optimal inflation rate is equal to the opposite of the real rate of return (Sidrauski, 1967). For the second best, the optimal inflation rate has been analyzed by several authors in the framework of an optimal taxation problem.[2] Since there is no immediate relation between these theoretical results and the small amount of insight we may have on the demand for money, their operational value is limited.

We propose here to consider money as a productive input.[3] In this case, the relation which determines the optimal permanent inflation rate can be approximated by a simple rule which has an immediate interpretation. Furthermore, this rule can be applied in combination with other evidence we have on the demand for money (both through empirical studies or theoretical inventory models).

In the next section an intuitive argument allows us to determine the optimal value of the long-run inflation under the following assumptions: the demand for money depends only on the inflation rate, and the level of money does not affect the gross factor prices.

This result is demonstrated rigorously and generalized in an intertemporal model of general equilibrium in Section 3. We will see that the simple rule in Section 2 gives a good approximation of the optimal inflation rate when the excess-burden of taxation is relatively small with respect to one. This section is more technical and can be omitted without loss of continuity.

The problem of optimal intertemporal taxation and optimal inflation raises well-known issues of time-consistency (Kydland and Prescott, 1977; Calvo, 1978). An answer to this problem has been proposed by Auernheimer (1974). This rule is extended to the model of general equilibrium in Section 5.

The welfare gain of any tax reform including substitution between inflation and other taxes, have traditionally been regarded as small. Recently this view has been challenged by Feldstein (1979):

> In the important case in which the growth of aggregate income exceeds the social discount rate..., if the inflation rate is above its optimal level, the economy should then be deflated to reduce the inflation rate regardless of the temporary consequences for unemployment.

In Section 5, it is argued that the case where the welfare cost of the inflation rate is infinite is not likely to occur.

The problem of the optimal inflation in an economy with a short-run Phillips curve is discussed in Section 6.

2. A Simple Analysis of the Optimal Permanent Inflation Rate

In this section, we determine a simple rule for the optimal inflation rate in a partial equilibrium framework. Money is outside money and is held for the services it provides. These services save resources (time, effort, commodities, etc....) for other productive uses; therefore money can be considered as an input in the production of goods and services.

In equilibrium, the real quantity of money is such that the marginal value of monetary services is equal to the marginal opportunity cost, the difference between the real returns available on other assets and the real interest rate earned on money when money bears zero nominal interest; the marginal value of the services provided by monetary balances is measured by the nominal interest rate on other assets, i . The nominal interest rate i , is equal to the sum of the real rate of return, r , and the inflation rate, π .[4] We assume in this section that r is fixed (or that the demand for money depends only on π).

The economy grows at the rate g (the sum of the growth rates of population and of labor augmenting technological change). The demand for the real quantity of money per efficiency unit of labor, m , depends only on the nominal interest rate.[5]

A benevolent social planner can choose between issuing money and non lump-sum taxes to raise a given total amount of revenues. The distortion created by the taxes is measured by its marginal deadweight loss, i.e. the loss of social income for a marginal unit of tax revenue. Assume that the value of this marginal excess-burden, ν, is known.[6]

To determine the optimal value of the permanent inflation rate, we consider the optimization problem of the government in the steady state at an arbitrary date (fixed at zero). We first determine the social marginal value of the permanent inflation rate, β.

In the steady state the flow of revenues from the issue of money is equal to:

$$(1) \qquad Q = (g+\pi)m \; .$$

A small increase of the permanent inflation rate $d\pi$, has two effects: It reduces the level of cash balances, and the level of social income, by $im'(i)d\pi$, where $m'(i)$ is the derivative of money with respect to the nominal interest rate. The second effect is to increase the government revenues by $(dQ/d\pi)d\pi$. Therefore, the revenues generated by other taxes can be reduced by the same amount. Their excess-burden is then reduced by $\nu(dQ/d\pi)d\pi$. The sum of the two effects is equal to

$$(2) \qquad E = (r+\pi)m'd\pi + \nu[(g+\pi)m' + \nu m]d\pi \; .$$

This flow increases at the same rate as the economy, and the total effect of a permanent increase of inflation on social welfare is equal to the discounted sum of the flow E at each date. Therefore, the social marginal value of the permanent inflation rate is equal to:

$$\beta = \frac{1}{r-g}[(r+\pi+\nu(g+\pi))m'(i)+\nu m] \quad . \tag{3}$$

Following Auernheimer (1974), the government is "honest" and takes the price level at time zero as exogenously given. When the inflation rate increases by $d\pi$, the demand for real balances decreases by $m'd\pi$; the government purchases an equal quantity of money from the private sector, such that the price level is maintained unchanged. This purchase is financed by other explicit taxes, which create an excess-burden equal to $\nu m'd\pi$.

The permanent inflation rate is optimal when this excess-burden is equal to the marginal value of a higher permanent inflation rate:

$$\nu m' + \beta = 0 \ , \tag{4}$$

which after some manipulations, gives the formula

$$\varepsilon = \frac{\nu}{1+\nu} \ , \tag{5}$$

where $\varepsilon = -\frac{r+\pi}{m}m'(r+\pi)$ is the interest elasticity of the demand for money.

This analysis could be extended to the non-steady state case. When the government can adjust the budget deficit or surplus at each instant to minimize the excess-burden of taxation, the term ν is constant over time (see next section). Therefore if the demand for money depends only on the inflation rate, the steady state solution given by (5) is optimal. When the government deficit is exogenously determined, ν and π may vary over time.

In the formula (5) the optimal inflation rate is independent of the growth rate of the economy.[7] Two special cases are well known.

When lump-sum taxation is feasible, the term ν is equal to zero, and the optimal deflation rate is equal to the rate of return r (Sidrauski, 1967; or Friedman, 1969).

When the deadweight loss of other taxes is very large, the government maximizes the inflation tax revenues. When ν is infinite, the optimal inflation rate is obtained when the interest elasticity of money is equal to one (Bailey, 1958).

In general, the variation of the optimal value of the permanent inflation rate as a function of the marginal excess-burden of taxation ν depends on the relation between the interest elasticity of the demand for money and the nominal interest rate. Empirical studies find that ϵ increases with π (Cagan, 1956; Barro, 1972).[8] However, a precise determination of this relation for low inflation rates does not seem to be available currently.

In the previous analysis the demand for money depends only on the inflation rate; the level of cash balances does not affect the productivity of other factors.

Assume now that the demand for real balances increases with the capital stock (or with the level of output). In this case inflation has a "second round" effect on government revenues: an increase of inflation reduces the level of real balances, reduces the level of output (produced by these balances); this reduces further the demand for real balances and the ability of the government to raise revenues by money growth. When the interest elasticity of the demand for money is positively related to the interest rate, this effect tends to reduce the optimal inflation rate. The same argument applies when money is complementary with the other inputs in the production technology.

An analytical discussion of these effects is given in the next section.

The main conclusion of that section is that these effects have the same order of magnitude as ν , the marginal excess-burden of taxation. When ν is small with respect to one, a first order approximation of the optimal inflation rate (in all cases), is given by:

$$(6) \qquad \varepsilon = \nu$$

3. The Intertemporal Model of General Equilibrium

In this section, we analyze the optimal inflation rate in an intertemporal model of general equilibrium where the private sector's intertemporal program of consumption and labor supply is determined by an optimizing behavior.

We consider a one good economy where the private sector is represented by a single household. Its intertemporal utility function is given by:

$$U = \int_0^T e^{-\rho t} e^{gt} u(c_t, \ell_t) dt ,$$

with the following notation:

g : growth rate of the household;

ρ : pure rate of time preference;

c_t : consumption per capita;

ℓ_t : labor supply per capita.

The length of the household's life, T , is assumed to be large and finite, if the growth rate g is greater than the discount rate ρ .[9] It can be infinite if g is smaller than the discount rate ρ .[10]

The household behaves competitively, and takes future prices, which are known with perfect foresight, as given. At time zero, it chooses an intertemporal program which maximizes U subject to the budget constraint. Denote respectively by q_t, $\overline{w}_t$, and $\overline{r}_t$, the private marginal utility of consumption, the net wage rate and the net interest rate at time t. The price of the unique good is normalized to one.

The household's intertemporal program satisfies the following first order conditions:

$$\frac{\partial}{\partial c}u(c_t, \ell_t) = q_t , \tag{7}$$

$$\frac{\partial}{\partial \ell}u(c_t, \ell_t) = -q_t\overline{w}_t , \tag{8}$$

$$\frac{\dot{q}_t}{q_t} + \overline{r}_t = \rho . \tag{9}$$

Using (7) and (8), the levels of consumption and labor at each instant are determined by q and $\overline{w}$:[11]

$$c = c(q,\overline{w}) , \tag{7a}$$

$$\ell = (q,\overline{w}) , \tag{8a}$$

and the level of utility can be also expressed as a function of q_t and $\overline{w}_t$, in an indirect form:

$$v(q,\overline{w}) = u(c(q,\overline{w}), \ell(q,\overline{w})) . \tag{10}$$

The household's portfolio is composed of three types of assets: productive capital, government bonds and money. Capital and money are used in the production process which is controlled by the household. The production technology is represented by the neoclassical function:

$$y = f(k,\ell,m) ,$$

where k and ℓ represent respectively the levels of the capital and the labor supply per capita. When the input levels are optimized,

$$(11) \qquad r = \frac{\partial f}{\partial k} ,$$

$$(12) \qquad w = \frac{\partial f}{\partial \ell} ,$$

$$(13) \qquad r + \pi = \frac{\partial f}{\partial m} ,$$

where r and w denote respectively the growth rate of return and the gross wage rate. For a given nominal quantity of money and an inflation rate π, the price level adjusts such that the equilibrium condition (13) is satisfied. Using (11), this condition can also be written as follows:

$$(14) \qquad m = \phi(k,\ell,\pi) .$$

This is the first portfolio equation. The other equation is expressed by the equality between the rates of return on government bonds and capital, which are perfectly substitutable.

The government finances an exogenous stream of public consumption a_t, by a labor income tax,[12] and issues of bonds and money. To simplify the analysis, we consider the variable h, defined as the opposite of the public debt; h is equal to the level of public assets (per capita). At a given instant, its variation is equal to the government budget surplus:

$$(15) \qquad \dot{h} = (r-g)h + (w-\bar{w})\ell + nm - a ,$$

where n represents the rate of growth of the nominal quantity of money.[13] The intertemporal budget constraint of the government is satisfies when the value of h is equal to zero at date T (when the horizon is finite), or when the present value of public assets tends to zero when t tends to infinity (when the horizon is infinite);

if T is finite, $h_T = 0$,

if T is infinity, $\lim_{t\to\infty} e^{-R(t)} e^{gt} h_t = 0$,

where $R(t) = \int_0^t r_\tau d\tau$, is the gross rate of return between time zero and t .

The problem of the policy maker is then to determine the intertemporal program of net wage rates and nominal money growth rates, which maximizes the utility of the private household, subject to the constraints provided by the aggregate capital accumulation, the government budget, and the private sector's optimizing behavior.

In this model of perfect foresight we can indifferently assume that the government controls the rate of growth of the nominal quantity of money n , or the inflation rate π . The relation between n and π is given by:

$$nm = \dot{m} + (g+\pi)m \ .$$

For technical reasons, we assume that the government controls π ; the level of revenues generated by the creation of money is measured by:

$$Q = \dot{m} + (g+\pi)m \ , \tag{16}$$

where m is expressed as a function of k , ℓ and π in (14).

When no restriction is applied on the policy instruments $\bar{w}$ and π , the optimal solution presents a somewhat pathological behavior at time zero. In order to analyze this problem, the intertemporal program of the policy instruments is defined by the levels of the net wage rate and the inflation rate at time zero, w_0 and π_0 respectively, and by the paths of their time variations, $(\dot{w}_t)_{t\geq 0}$ and $(\dot{\pi}_t)_{t\geq 0}$ respectively. The optimization problem is described as follows:

$$\text{(\underline{P1}):} \qquad \text{Maximize} \quad U = \int_0^\infty e^{-\rho t} e^{gt} v(q_t, \bar{w}_t) dt \ ,$$

where $v(q_t, \bar{w}_t)$ is given by (10),

subject to: $\dot{k} = f(k,\ell,m) - gk - c - a$,

$$\begin{cases} \dot{h} = (r-g)h + (w-\bar{w})\ell + \dot{m} + (g+\pi)m - a \ , \quad \text{with} \\ \dot{m} = \frac{\partial\phi}{\partial k}\dot{k} + \frac{\partial\phi}{\partial\ell}\left(\frac{\partial\ell}{\partial q}\dot{q} + \frac{\partial\ell}{\partial\bar{w}}\dot{\bar{w}}\right) + \frac{\partial\phi}{\partial\pi}\dot{\pi} \ , \end{cases}$$

$$\dot{q} = q(\rho - r) \ ,$$

$$\dot{\bar{w}} = x \ ,$$

$$\dot{\pi} = z \ .$$

At time zero, the level of capital k, is exogenously determined. We also assume that the government cannot raise a capital levy; therefore, the level of public assets h, is fixed at time zero.

At first, we also assume that the values of q, π and of the shadow price of the constraint $\dot{\bar{w}} = x$ (call it α), are exogenously determined at time zero. The solution to the problem (P1) is determined by considering the current value Hamiltonian:

$$\begin{aligned} (17) \qquad H = {} & v(q,\bar{w}) + \lambda(f(k,\ell,m) - gk - c - a) \\ & + \mu((r-g)h + (w-\bar{w})\ell + \dot{m} + (g+\pi)m - a) \\ & + \xi q(\rho - r) \\ & + \alpha x \\ & + \beta z \ , \end{aligned}$$

where m is given as a function of $\dot{k}$, $\dot{q}$, x, and z, in the above description of the problem (P1), and c, ℓ, m are given by (7a), (8a) and (14).

The policy instruments are w_0, x and z. The state variables of the problem are k, h, q, w and π. Their respective shadow prices are denoted by λ, μ, ξ, α and β. The variable λ represents also the <u>social</u> marginal value of the (unique) good in the economy. Because of the second best nature of the problem, λ is in general, different from the <u>private</u> marginal value of the good, q. The variable μ represents the social marginal value of public assets. An increase of public wealth does not increase the real wealth of the economy, but allows the government to lower the revenues raised through distortionary taxes. Therefore μ is equal to the marginal excess-burden of tax revenues.[14] When lump-sum taxation is feasible, the value of μ is equal to zero: since the planner's aim is to maximize the utility function of the household, the maximizing behavior of the latter is not a binding constraint; the shadow prices ξ, α, and β are equal to zero, and the marginal social and private values of consumption, λ and q, are equal.

Let us now consider the optimal inflation rate. It is determined by the equations:[15]

$$\frac{\partial H}{\partial \pi} = (\pi - g)\beta - \dot{\beta}$$

$$\frac{\partial H}{\partial z} = \beta + \mu\frac{\partial \phi}{\partial \pi} = 0 \ ,$$

which give

$$(18) \qquad \frac{\partial H}{\partial \pi} + (\rho - g)\mu\frac{\partial \phi}{\partial \pi} = -\dot{\beta} \ .$$

This equation is the generalization of (4).

In the steady state (assuming a turnpike solution) $\dot{\beta} = 0$, and the rate of return r , is equal to the discount rate ρ ; using (13), (17) and (18), we find:

When μ is equal to zero, $\pi = -r$. When μ is positive,

$$(19) \qquad \boxed{\varepsilon = \frac{\nu}{1 + \nu + A + B}}$$

where $\quad \varepsilon = -\dfrac{(r+\pi)}{\phi}\dfrac{\partial \phi}{\partial \pi}(k, \ell, \pi) \quad , \quad \nu = \mu/\lambda$

$$A = \nu\frac{\partial \phi}{\partial k}(k, \ell, \pi)$$

$$B = \frac{1}{r+\pi}\left(\nu\ell\frac{\partial w}{\partial m} + \left(\nu h - q\left(\frac{\xi}{\lambda} + \nu\frac{\partial \phi}{\partial \ell}\ \frac{\partial \ell}{\partial q}\right)\right)\frac{\partial r}{\partial m}\right) .$$

In the LHS the term ε represents the partial elasticity of the demand for money, with respect to its opportunity cost, keeping capital and labor constant.

The long-run (permanent) inflation rate is determined by (19). It is important to notice that the expression (19) is independent of the initial conditions k_0 , h_0 , q_0 , α_0 and π_0 . However, these initial values determine the long-run values of ν , and therefore the value of the permanent inflation rate.[16]

The relation (19) justifies the intuitive discussion of the previous section: when $\partial\phi/\partial k = 0$ and money does not affect the gross factor prices, $A = B = 0$, and (5) is a special case of (19). Assume that

ε increases with π ; the optimal value of the permanent inflation rate is lower than the value given by (5), when the demand for cash increases with k , or when money is complementary with capital and labor in the production technology.[17] For practical purposes, the term A in (19) is very small and can be neglected.[18] Also the term B is of the same order of magnitude as the marginal excess-burden ν [19] (measured with respect to the social value of the good). When ν is small, the optimal value of the permanent inflation rate is approximated up to the first order by the relation (6), found in the previous section.

$$(6) \qquad \varepsilon = \nu \; .$$

4. The Problem of Time-Consistency

The problem of the time consistency of an optimal policy has been raised in various contexts.[20] For example, assume that there is no restriction on the price of money at time zero. A currency reform is equivalent to a lump-sum capital levy, and introduces no distortion. At time zero, when the excess-burden of taxation is positive, the optimal inflation rate is infinite. Of course, this policy should be unexpected. The optimal policy is time inconsistent.

In the previous section, we have assumed that the government takes the values of k , h , q , π and α at time zero, as exogenously determined. From the budget surplus equation (which determines $\dot{h}$) , we can see that the initial level of the nominal quantity of money M , is assumed to be given at time zero. The values of π and of the other variables in the economy determine the demand for money, and together with M they determine the price level P , at time zero. We could also take the levels of P and π as given, and these values would determine M .

Once the values of k , h , q , π , α and M are fixed at an arbitrary date (fixed at zero), the entire path of all variables can be determined after this date, by solving the problem (P1). Assume that this solution determines the values $\hat{k}_t$, $\hat{h}_t$, $\hat{q}_t$, $\hat{\pi}_t$, $\hat{\alpha}_t$, $\hat{M}_t$ at time t $(t > 0)$. The solution of the problem (P1) computed at time t for the period after t , under the constraint $(k_t = \hat{k}_t, \ldots, M_t = \hat{M}_t)$ is identical with the solution computed at time 0 for the same period, following t . The solution of the problem (P1) is time-consistent.

The constraints on q , π , α and M (or P) at time zero provide a method to solve the time-consistency problem. This method was initially proposed by Auernheimer who showed in a partial equilibrium framework, that when the price level is fixed, optimal inflation policies are time consistent. As we can see, in a general equilibrium model, the constraint on P is necessary but not sufficient.

The constraints which we have introduced prevent the occurrence of time inconsistent jumps of the endogenous variables at the time a new policy can be determined. Currency reforms are excluded. It should also be noticed that restrictions on the other tax rates are necessary (as on $\bar{w}$): the demand for real balances depends on the production inputs, in particular labor. In our model, a large subsidy on labor for an infinitesimal amount of time induces a similar jump of labor and of the demand for real balances. This jump of labor has no effect on welfare because it lasts only for an infinitesimal duration. However the jump of the demand for real balances allows the government to raise revenues by issuing more money, without changing the price level at time zero.[21]

In problems of dynamic optimal taxation, when the constraints on the instruments are not defined carefully, the solutions are not only time inconsistent, but also may have some pathological properties at the beginning of the planning horizon.

5. The Welfare Cost of Taxation and the Choice of a Discount Rate

In a dynamic economy where agents are endowed with perfect foresight, the economic decisions of the private sector at a given date depend on the entire intertemporal program of tax rates. Therefore, the deadweight loss of a tax system should be determined by a single computation for the entire planning horizon, and not period by period. However, when the intertemporal utility function of the private sector is addively separable between periods, as we have assumed in the previous sections, the welfare cost of taxation for a given instant (a flow), can be measured as a fraction of output produced at the same time (the marginal welfare cost was expressed by the variable $\nu = \mu/\lambda$ in the previous sections).

For small tax rates, this fraction is small (it is equal to zero for infinitesimal rates). As it is well known, the deadweight loss of taxation is of the second order with respect to the amount of revenues. In the long-run, the deadweight loss of taxation is a constant fraction of output, and increases at the same rate as the economy. The total welfare cost of the tax system (a stock), is measured by the present discounted value of the flow of the deadweight loss at each instant. We see immediately that the total excess-burden of taxation can be large if the planning horizon is large and if the discount rate is low with respect to the growth rate of the economy. When the horizon tends to infinity, and the discount rate is lower than the growth rate, the welfare cost of taxation tends to infinity.

Feldstein (1979) has argued that this is an "important case," and that in this case the welfare cost of the wrong inflation rate (if it is not optimal) dwarfs any cost associated to the transition towards a new policy. Feldstein is using two arguments. The first is empirical; the average net real rate of return on Aaa bonds for the last 25 years has been less than one percent, and therefore lower than the growth rate. One could remark that bonds represent only one type of asset in the private sector's wealth, and ex post rate on the last 25 years may not be an accurate description of the ex ante real rate. For example, Jorgenson found a higher value for the mean net return on aggregate wealth, around 4%. But let us assume that the observed net risk-free rate is around 1%. What would this indicate about the social discount rate? In the life-cycle model the steady state net rate of return has no a priori relation with the social discount rate (Diamond, 1965). It is somewhat surprising to see that Feldstein after showing in his numerous studies on Social Security that the life-cycle model is the relevant case for the U.S. economy, is using the model with an operative bequest motive for an empirical argument.

In this model, when the horizon is sufficiently large, the economy converges to a turnpike where the net rate of return is equal to the discount rate. Can this discount rate be lower than the growth rate? In this case the level of the capital stock on the balanced growth path is larger than the Golden Rule value. This balanced growth path cannot be sustained ad infinitum by a competitive equilibrium. On this path, the consumption satisfies the equation:[22]

$$c = (\rho-g)k + \bar{w}\ell \ .$$

If the net rate of return ρ (which is equal to the discount rate), is smaller than the growth rate, the present value of consumption is smaller than the present value of labor income $\bar{w}\ell$ (net of taxes), and the intertemporal budget constraint of the private sector cannot be satisfied.

To solve this dilemma, we can choose a finite horizon (a solution also proposed by Feldstein). In this case the economy converges for a certain period to the turnpike where the net rate of return is equal to the discount rate. However capital is accumulated to this apparently inefficient level only because the private sector has a strong taste for consumption in the future. And the future should arrive some day. The private sector intends to benefit from its present thrift. In the case where the discount rate is smaller than the growth rate, the economy diverges sharply from its balanced growth path before the end of the planning horizon, when the large amount of accumulated capital is consumed.[23] It is doubtful that low interest rates in real economies can be explained by the expectations of this period of riotous consumption.

For his second argument, Feldstein relies on introspection: the social discount rate ρ should be equal to the sum of the pure rate of time preference $\tilde{\rho}$, and of the product of the growth rate of consumption per capita g_c of the elasticity of the marginal utility of consumption σ . The historical value of g_c is around 2%. Feldstein chooses somewhat arbitrarily the values $\tilde{\rho} = 0$ (following Ramsey), and $\sigma = 1/2$. Therefore $\rho = 1\%$, again. This argument would apply when the market return is not a good indicator of the social discount rate (as in the life-cycle model). But in this case, intertemporal programs chosen by an enlightened social planner would exhibit the same pathological properties as the programs of the private sector, which were described in the previous paragraph.

6. Short-Run Policy

In the model of perfect foresight all prices adjust costlessly. The model can be easily extended to the case of an economy where variations of the inflation rate affect the level of output (temporarily or permanently). Since we have seen that the partial equilibrium case in Section 2 provides a reasonable approximation of the general equilibrium framework, we will use again for simplicity, the intuitive approach in this section.

Assume that there is a short-run trade-off between inflation and output, i.e., when the variation of the inflation rate is equal to $\dot{\pi}$, output is increased by $F(\dot{\pi})$, where $F'(0) > 0$, $F''(0) \leq 0$ (the short-run supply curve is convex.) Using the same approach as in Section 2, an increase of the inflation rate $d\pi$ in the steady state, increases social welfare by $F'(0)d\pi + \beta d\pi$, where β is the social value of inflation, computed previously (relation (4)). From the discussion in Section 5, we assume that β is finite.

The relation (4) becomes now:

$$\nu m' + F'(0) + \beta = 0 ,$$

and (5) is replaced by

$$\varepsilon = \frac{(r-g)(F'(0)/m) + \nu}{1+\nu} .$$

The existence of a short-run supply curve affects the permanent value of the optimal inflation rate. When r tends to g, the permanent effects dwarf the short-run effects, and we find the same result as in Section 2. Is the effect of the Phillips curve significant? The following values can be considered as upperbound estimates of the different parameters $r-g = 0.05$, $F'(0) = 0.09$,[24] $m = 1/15$ ($F'(0)$ and m are measured with respect to the yearly output). In this case we have:

$$\varepsilon = \frac{0.07 + \nu}{1+\nu} .$$

Current estimates of the marginal excess-burden ν, are in general somewhat larger than seven percent.[25] It does not seem that the existence of a short-run Phillips curve affects too much the permanent optimal inflation rate.

In the frictionless case the adjustment to the optimal permanent inflation rate takes place instantaneously. This is still true when the trade-off between output and the variation of inflation is linear. When the short-term supply curve is strictly concave, the adjustment takes place gradually. The adjustment speed depends in an important way on this convavity.

However, it should be clear that in the model used here, one can easily find conditions under which strong deflationary policies should be recommended, even if the social discount rate is much larger than the growth rate. A sufficient condition for an acceleration of the rate of deflation on the transition path is given by:

$$F(\dot{\pi}) < \frac{m}{r-g}((1+\nu)\varepsilon - \nu)$$

where ε and m are the values of the elasticity and of real balances in the steady state (after the transition period).[26] With the same numerical values as before, this inequality is satisfied when

$$0.07 < (1+\nu)\varepsilon - \nu$$

when ε is equal to 1/4, it is sufficient that $\nu < .24$.

Obviously, the argument can be reversed in other numerical examples, in favor of inflationary policies. Unless the Phillips-curve is strongly concave,[27] the result of the model are very sensitive to the values of the interest elasticity of money and of the marginal excess-burden of taxation, even when the social discount rate exceeds the growth rate.

7. Conclusion

This paper has reexamined the problem of the optimal inflation in a general equilibrium model of optimal taxation. The approach followed here has allowed us to determine conditions under which the optimal value of the permanent inflation rate is well approximated by a simple rule obtained in a partial equilibrium framework. This formula depends on easily identifiable variables, the interest elasticity of the demand for money and the marginal excess-burden of explicit taxation.

The welfare gain of a tax reform depends on the difference between the social discount rate and the growth rate of the economy. It is most likely that this difference is positive. In this case, the existence of a short-run Phillips curve should affect the permanent inflation rate. However, even if the discount rate is significantly larger than the growth rate, the magnitude of this effect seems to be small compared to the present value of the permanent welfare cost of taxation. And indeed, unless the short-run aggregate supply curve is strongly concave, the results in this paper could justify policies such that the rate of inflation converges rapidly to its optimal value, even if this would induce a large temporary variation (positive or negative) of the unemployment rate. But there is also another interpretation. Despite the simplicity of the approximation rule which determines the optimal inflation rate, we still cannot give a precise estimate of this inflation rate because we do not have accurate results about the interest elasticity of the demand for money or about the welfare cost of explicit taxation. The research which could improve our understanding in this matter, and lower permanently the excess-burden of taxation even by a small fraction, would generate social benefits of a very large present value.

FOOTNOTES

1. Also, the definition of a steady state with a growth rate equal to zero is not clear, unless the level of savings is defined as a function of gross income. In this case (with a zero growth rate), inflation has no effect on the steady state capital stock. An empirical study of the optimal inflation rate in this framework is given by Summers (1981).

2. For a partial equilibrium approach, see Siegel (1978) and Feldstein (1979). Phelps (1973), Drazen (1979), and Brock and Turnovsky (1980) present a general equilibrium analysis, where the utility function of the private sector depends on money. The method followed by Drazen is incorrect: the policy maker takes into account the consumer's maximizing choice between different goods at a given date (by using the indirect utility function at this date), but ignores the constraint provided by the private intertemporal maximizing behavior (in expression (32), p. 241).

3. This does not seem to be a worse assumption than to consider money as a consumption good in the utility function. For a discussion, see Fisher (1974).

4. For simplicity, there is no tax on capital income.

5. The services of money depend on the ratio between the nominal quantity of money, and a price deflator. We will assume here that this price deflator is independent of the tax system. This can be justified as follows: in general, the revenues from the inflation tax are relatively small.

Therefore, the variations of the tax rates, which are necessary to keep total tax revenues constant when the inflation rate is altered, are likely to be small, and do not affect significantly the price deflator.

6. Of course, ν should be determined simultaneously with the optimal inflation rate. However, since the inflation tax revenues are small, ν can, in a first order approximation, be considered as exogenous. Also, the level of government expenditures can be optimized by a comparison of their benefits with $1+\nu$.

7. However, the optimal inflation rate depends on the growth rate g , because a higher growth rate allows the government to raise a larger fraction of revenues by issuing money, and this lowers the value of the marginal weight loss ν .

8. Cagan found that a linear relation fits the data well. He also finds that the value of the inflation rate which maximizes revenues $(\varepsilon = 1)$, is large. This result suggests that in our model the permanent inflation rate should be fairly large, even if the marginal welfare cost of other taxes is only moderate. The lowest value for the revenue maximizing inflation rate found by Cagan (p. 81) is 290% per year (792% in the German case). With a linear relation between ε and π , and a marginal excess burden at least equal to 10%, the optimal inflation rate is greater than 29%. On the other hand, if we assume that $\varepsilon = 1/4$ and $\nu = 1/3$, currently observed value of the inflation rate would be about optimal.

9. We assume that the value of T is large enough such that the economy reaches a neighborhood of its turnpike for some interval of time.

When g is greater than ρ, and T is infinite, the integral in the expression of U is not defined. For a further discussion of this problem, see Section 5.

10. When labor-augmenting technological progress takes place at the rate μ, the discount rate ρ should be replaced by $\rho^* = \rho + \sigma\mu$, where σ is the elasticity of the marginal utility of consumption; the level of consumption c_t should then be measured in efficiency units. Except for these alterations, the rest of the analysis remains valid (however, no balanced growth path may exist if the elasticity of substitution between consumption and leisure is different from one).

11. Whenever possible time subscripts will be omitted.

12. Another tax structure (for example, the income tax), could be treated by the same method.

13. "Balanced budget" policies ($\dot{h} \equiv 0$, or h exogenous) could be analyzed by the same method.

14. When the level of the surplus $\dot{h}$, is optimized, the ratio between μ and q is constant over time (use the relations $\partial H/\partial h = (\rho-g)\mu - \dot{\mu}$, and $\dot{q} = q(\rho-r)$).

15. Of course, these equations are only a part of the complete system of dynamic equations to be solved simultaneously.

16. For example, with no restriction on h at time zero, the government can raise a capital levy, $\nu = 0$ and $\pi = -r$.

17. Using the first order condition $\partial H/\partial k = (\rho-g)\lambda - \dot{\lambda}$; in the steady state, $(\mu h - q(\xi + \mu\phi'_\ell \ell'_q))(r'_k + \phi'_k r'_m) = -\mu(\ell w'_k + (r+\pi)\phi'_k) - (r+\pi)\phi'_k(\lambda + \mu\phi'_k)$. It is reasonable to assume that $r'_k + \phi'_k r'_m < 0$ (the total effect of a capital increase on the rate of return is negative). Substituting in B , one finds that $B > 0$.

18. Consider, for example, the following production function: $f(k,\ell,m) = k^\gamma \ell^{1-\gamma} + j\sqrt{m}$, where j is a constant. In this case, $\frac{\partial\phi}{\partial k} = 2(1-\gamma)\frac{r}{r+\pi}\frac{m}{k}$. The ratio m/k is small (smaller than 1/30 in the U.S. economy). If $r/(r+\pi)$ is smaller than two, $\partial\phi/\partial k$ is smaller than 3%. In general if the wealth elasticity of the demand for money is equal to one and m/k is small, $\partial\phi/\partial k \sim m/k$. This ratio is smaller than 2% for the U.S. economy.

19. For example, when the production f is of the Cobb-Douglas type, the terms $\frac{1}{r+\pi}\frac{\partial w}{\partial m}$ and $\frac{1}{r+\pi}\frac{\partial r}{\partial m}$ in the expression B , are bounded. When μ tends to zero, ξ tends to zero, and therefore the same is true for B .

20. See Auernheimer (1974), Calvo (1978), and Kydland and Prescott (1978).

21. More precisely, in (P1), $\frac{\partial H}{\partial \dot{\bar{w}}} = \mu\frac{\partial\phi}{\partial\ell}\frac{\partial\ell}{\partial\bar{w}} + \alpha = 0$, where $\mu > 0$. Where there is no constraint on $\bar{w}$, $\alpha = 0$. Assuming $\frac{\partial\ell}{\partial\bar{w}} > 0$, $\frac{\partial\phi}{\partial\ell} > 0$, we have $\frac{\partial H}{\partial\dot{\bar{w}}} > 0$; the subsidy or labor income should be increased as much as possible to increase the labor supply $(\partial\ell/\partial\bar{w} > 0)$ and the demand for money $(\partial\ell/\partial\bar{w} > 0)$ in order to generate a capital levy which has a social value μ .

22. The notations have already been introduced in Section 3 as follows: consumption c, discount rate ρ, growth rate g, capital stock k, net wage rate $\bar{w}$, labor supply ℓ. All quantities are defined per efficiency unit of labor.

23. See Arrow and Kurz (1970), p. 70.

24. A permanent increase of π equal to one percent induces a decrease of unemployment equal to three percent and an increase of output equal to nine percent.

25. For example, Shoven (1976) finds that the marginal excess-burden of the corporate tax is between 12% and 30%.

26. For the derivation of this sufficiency condition, we assume that ε increases with π. ν is constant over the transition period when the government's deficit is optimized.

27. This concavity could be increased by additional costs not recorded in National Income data. Also it is possible that some of the adjustment costs would not be strictly transitory (in this latter case, the model should be modified accordingly).

REFERENCES

Auernheimer, L. (1974). "The Honest Government's Guide to the Revenue from the Creation of Money," Journal of Political Economy, 82, 598-606.

Arrow, Kenneth and Mordecai Kurz (1970). Public Investment, the Rate of Return and Optimal Fiscal Policy. Baltimore: Johns Hopkins Press.

Atkinson, A. B. and A. Sandmo (1980). "Welfare Implications of the Taxation of Savings," Economic Journal, 90, 529-549.

Bailey, Martin (1956). "The Welfare Cost of Inflationary Finance," Journal of Political Economy, 64, 93-110.

Barro, R. J. (1972). "Inflationary Finance and the Welfare Cost of Inflation," Journal of Political Economy, 80, 978-1001.

Baumol, William (1952). "The Transactions Demand for Cash: An Inventory Theoretic Approach," Quarterly Journal of Economics, 66, 545-556.

Brock, W. and S. Turnovsky (1980). "The Analysis of Macroeconomic Policies in Perfect Foresight Equilibrium," International Economic Review, 22, 179-209.

Cagan, Phillip (1956). "The Monetary Dynamics of Hyperinflation," in Studies in the Quantity Theory of Money, ed. by M. Friedman. Chicago: The University of Chicago Press,

Calvo, Guillermo (1978), "On the Time Consistency of Optimal Policy in a Monetary Economy," Econometrica, 46, 1411-1428.

Chamley, Christophe (1980). "Optimal Intertemporal Taxation and the Public Debt," mimeo, Yale University.

Diamond, Peter (1965). "National Debt in a Neoclassical Growth Model," American Economic Review, 55, 1126-1150.

Drazen, Allan (1979). "The Optimal Rate of Inflation Revisited," Journal of Monetary Economics, 5, 231-248.

_________ (1980). "Inflation and Capital Accumulation under a Finite Horizon," mimeo, University of Chicago.

Feldstein, Martin (1976). "Inflation, Income Taxes, and the Rate of Interest: A Theoretical Analysis," American Economic Review, 66, 809-820.

_________ (1979). "The Welfare Cost of Permanent Inflation and Optimal Short-Run Economic Policy," Journal of Political Economy, 87, 749-768.

Fisher, Stanley (1974). "Money and the Production Function," Economic Inquiry, 12, 517-533.

Friedman, Milton (1969). "The Optimum Quantity of Money," in The Optimum Quantity of Money and Other Essays. Chicago: Aldine Publishing Co.

Goldfeld, Stephen (1973). "The Demand for Money Revisited," Brookings Paper of Economic Activity, 577-646.

Jorgenson, Dale and Laurits Christensen (1973). "Measuring Economic Performance in the Private Sector," in Measurement of Economic and Social Performance, Studies in Income and Wealth, Vol. 38, ed. by M. Ross.

Kydland, F. and E. C. Prescott (1978). "On the Possibility and Desirability of Stabilization Policies," Paper presented at the N.B.E.R. Conference on Rational Expectations and Economic Policy, Bald Peak, N.H., October.

Pestieau, P. M. (1974). "Optimal Taxation and Discount Rate for Public Investment in a Growth Setting," Journal of Public Economies, 3, 217-235.

Phelps, Edmund (1973). "Inflation in the Theory of Public Finance," Swedish Journal of Economics, 75, 67-82.

Shoven, J. B. (1976). "The Incidence and Efficiency Effects of Taxes on Income from Capital," Journal of Political Economy, 84, 1261-1283.

Sidrauski, Miguel (1967). "Rational Choice and Patterns of Growth in a Monetary Economy," American Economic Review, Papers and Proceedings 534-544.

Siegel, Jeremy (1978). "Notes on Optimal Taxation and the Optimal Rate of Inflation," Journal of Monetary Economics, 4, 297-305.

Summers, Lawrence (1981). "Optimal Inflation Policy," Journal of Public Economics, 7, 175-194.

Tobin, James (1956). "The Interest Elasticity of Transactions Demand for Cash," Review of Economics and Statistics, 38, 241-247.

__________ (1968). "Notes on Optimal Monetary Growth," Journal of Political Economy, 76, 833-859.

Weiss, Laurence (1980). "The Effect of Money Supply on Economic Welfare in the Steady State," Econometrica, 48, 565-576.