

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

**Box 2125, Yale Station
New Haven, Connecticut 06520**

COWLES FOUNDATION DISCUSSION PAPER NO. 593

Note: Cowles Foundation Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. Requests for single copies of a Paper will be filled by the Cowles Foundation within the limits of the supply. References in publications to Discussion Papers (other than mere acknowledgment by a writer that he has access to such unpublished material) should be cleared with the author to protect the tentative character of these papers.

MARKET STRUCTURE IN THE "RESEARCH AND DEVELOPMENT" PHASE

John J. Beggs

May 1, 1981

MARKET STRUCTURE IN THE "RESEARCH AND DEVELOPMENT" PHASE

by

John J. Beggs

Introduction

In the competitive struggle to be first to produce a new product or to produce the new product of the highest quality there is an essential economic trade-off of the following form to consider. First, the larger the number of firms, the sooner the new product is likely to become available and the higher the likely quality of the product. Second, a larger number of firms implies greater duplication of the research effort and hence some wastage of resources. This trade-off gives rise to a whole range of policy related issues such as whether anti-trust action is appropriate in certain concentrated high technology industries, whether the government should license only certain numbers of researchers in particular technologies, etc.

The paper proceeds by first developing a model of a competitive market in which firms compete to produce an invention of the highest possible quality. The model produces certain interesting insights, one of which is a natural measure of market concentration. Equilibria are identified for cases where the industry is monopolized, where it is oligopolistic as a result of entry barriers, and where the market is competitive, that is, there are no entry or exit barriers. In the case of physical production of goods the number of firms which can operate competitively in an industry is linked to the minimum optimum scale of

production, so also in the case of new product development the number of firms in a competitive industry is shown to depend on the technological possibilities for producing new products. The final section of the paper examines the question of the optimum market structure from a welfare point of view. The basic result is that the larger the proportion of the total welfare generated by a new product that goes to the consumer the more desirable the competitive solution; that is, the waste of competitive duplication is compensated by surplus accruing to the consumer. The final table of the paper makes some quantitative suggestions on the extent of the trade-off between the two effects.

Model

Consider the following static market in which there are N individual firms. Each firm engages in R&D in an attempt to produce an invention suited to a pre-specified task. The quality, q , of the invention which any one firm can produce is a random variable. The expected quality of the invention is an increasing function of the number of dollars the firm spends on the R&D process. It will be assumed that quality, q , will have an exponential probability density function, so that

$$(1) \quad f(q) = \lambda e^{-\lambda q} .$$

The expected quality is given by $(1/\lambda)$. Let x denote the R&D expenditure of the firm and let

$$(2) \quad \lambda = \lambda(x)$$

where $\frac{\partial \lambda(x)}{\partial x} < 0$.

The objective of competition between firms is to produce the invention of the highest quality. An alternative model could be to produce an invention of pre-specified technical characteristics in the shortest period of time. To retain simplicity it is assumed that the firm producing the invention of the highest quality will collect a prize, and that the value of the prize is greater the higher the quality of the winning invention. (See for comparison, Kamien and Schwartz (1972), Loury (1980).)

In the competitive process it is assumed that each firm is acting with Cournot expectation. When one firm decides to spend more or less on R&D it is done on the assumption that the remaining n firms will not respond. Further all firms are assumed to have identical characteristics, so that when one firm decides it is profitable to change its level of R&D by a given amount, all firms will change their R&D by that amount. Further, actual expenditures only occur at the equilibrium.

Now consider a starting point where each firm is currently contemplating spending \bar{x} dollars on R&D. An individual firm, say the i^{th} firm, might then want to evaluate the expected profits of moving to some alternative level of expenditure, x_i . For notational convenience let us write

$$\lambda_i = \lambda(x_i)$$

$$\bar{\lambda} = \lambda(\bar{x})$$

$$m = n-1$$

and let $b(q)$ be the value in dollars of the prize if the quality of the winning invention is q . The expected profit of moving to expenditure level x_i when all the other firms remain at expenditure level

\bar{x} can be evaluated as in equation (3). The terms in the equations are self-explanatory.

$$(3) \quad \text{Expected Profits from } x_i \text{ given } x_j = \bar{x} \text{ for } j=1, \dots, i-1, i+1, \dots, N = \int_0^{\infty} b(q) \cdot f(q; \lambda_i) \left[\begin{array}{l} \text{Probability that} \\ \text{remaining } m \\ \text{firms will have} \\ \text{inventions of} \\ \text{quality less than} \\ q ; \text{ given } \bar{x} \end{array} \right] dq - x_i .$$

Since there are clearly difficulties in creating a definition of quality of the innovation the function $b(\cdot)$ can be simplified by assuming that quality is one-to-one linear with the value of the invention. Then

$$(4) \quad b(q) = q .$$

Substitution of (4) into equation (3) yields

$$(5) \quad \Pi(x_i | x_j = \bar{x}, \forall j \neq i) = \int_0^{\infty} q \lambda_i e^{-\lambda_i q} [1 - e^{-\bar{\lambda} q}] dq - x_i .$$

By use of the binomial expansion, and integration out under the exponential function it is found that*

$$(6) \quad \Pi(x_i | x_j = \bar{x}, \forall j \neq i) = \lambda_i \sum_{r=0}^m \binom{m}{r} [-1]^{m-r} \frac{1}{(\lambda_i + m\bar{\lambda} - r\bar{\lambda})^2} - x_i .$$

Since all firms are identical, x_i will always be equal to \bar{x} ex post, that is, whenever one firm perceives some advantage in moving to a new level of expenditures each of its competitors will have the same perception. The expected profits to be realized from any level of R&D expenditures can then be calculated from equation (6) by evaluating (6) where

$$* \binom{m}{r} = \frac{m!}{(m-r)! r!} .$$

$x_i = \bar{x}$ for $i = 1, 2, \dots, n$. This is shown below in equations (7) and (8)

$$(7) \quad \Pi(\bar{x}) = \Pi(x_i \equiv \bar{x} | x_j = \bar{x}, \forall j) = \frac{K(m)}{\lambda} - \bar{x}$$

where

$$(8) \quad K(m) = \sum_{r=0}^m \binom{m}{r} [-1]^{m-r} \frac{1}{(m+1-r)^2} .$$

The $K(m)$ term serves as an index of market concentration, which equals 1 when $m = 0$, the monopoly situation, and is a decreasing function of m . An examination of the terms in equation (7) is instructive. The $(1/\lambda)$ term measures the expected value of the winning innovation, \bar{x} is the cost of the R&D effort to the firm, and $K(m)$ measures the extent to which the potential benefits to the firm are frittered away by the presence of competitors.

The graphs of the expected profits functions $\Pi(x_i | x_j = \bar{x}, \forall j \neq i)$ and $\Pi(\bar{x})$ can be plotted but their shapes clearly depend upon the nature of the technology for producing innovations, that is, the functional form of $\lambda(x)$. The form of $\lambda(x)$ is going to depend on the type of innovations one is interested in producing. Substantial quality gains in small mechanized technologies might be available at quite low levels of R&D effort whereas breakthroughs in chemicals or nuclear power industries may require very substantial threshold levels of expenditure before any important innovations can be made. A function is selected below which makes the average quality of an invention a concave function of R&D expenditures. The function in (9) will not suit all empirical cases but the concavity property will facilitate the identification of equilibria.

$$(9) \quad \lambda(x) = \alpha^{-1} x^{-\beta}, \quad 0 < \beta < 1.$$

Now the graph of the expected profit function, $\Pi(\bar{x})$, can be plotted as in Figure 1. The curve will fall as firms enter the industry, and rise as firms leave the industry. This effect is occurring through the market concentration index, $K(m)$, but can be understood as follows. If there is some given number of firms each spending \bar{x} dollars on R&D, and a new firm enters and spends the same \bar{x} dollars on R&D then the expected profits to any one firm must fall.

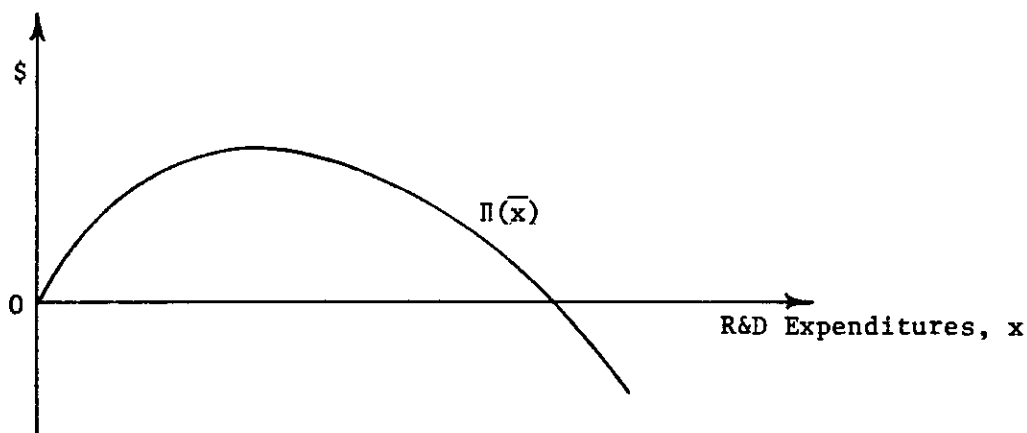


FIGURE 1. Expected Profit Function

Market Structures

The simplest case to investigate is that where $N = 1$, the monopoly case. The profit maximizing monopolist will expand R&D expenditures to the point where the expected marginal profits of an additional dollar of R&D is zero. The expected marginal profit, $M\Pi$, for this case is derived as follows:

$$M\Pi(x) = \frac{\partial \Pi(x)}{\partial x}$$

$$(10) \quad M\Pi(x) = \frac{-\lambda'(x)}{\lambda^2(x)} - 1 .$$

The graphs of $\Pi(x)$ and $M\Pi(x)$ for the case of the monopolist are shown in Figure 2. The profit maximizing level of output is denoted by x_M^* . It is possible to solve for equation (9) back into equation (10) to give the profit maximizing level of output, along with the expected level of profits, in terms of the parameters of the model. These results are shown below in equations (11) and (12)

$$(11) \quad x_M^* = [\beta\alpha]^{\frac{1}{1-\beta}}$$

$$\Pi(x_M^*) = [\alpha]^{\frac{1}{1-\beta}} \left[\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}} \right] .$$

And since in this model the producer is capturing the entire economic surplus, $\Pi(x^*)$ represents the welfare surplus generated by R&D activity in this monopolistic industry.

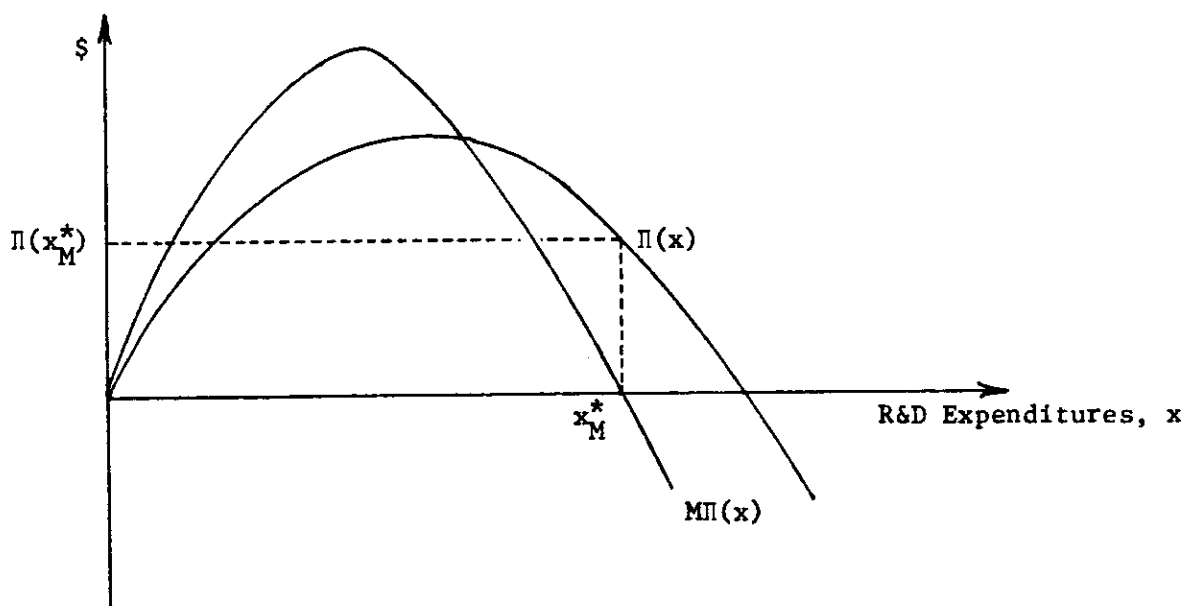


FIGURE 2. R&D Allocations of Monopolist

It is now useful to extend the discussion to the situation where there are a number of firms in competition with one another to produce the best invention. Consider an initial situation in which all firms are currently spending \bar{x} dollars on R&D and one firm, say the i^{th} firm, is deciding whether to spend a little more or a little less on R&D. This is a situation where the firm is limiting itself to considering only "local" adjustments to its expenditure allocations. In such a situation the firm wants to know the expected marginal profit on R&D. This can be evaluated as follows.

$$(12) \quad \text{M}\Pi(x_i | x_j = \bar{x}, \forall j \neq i) = \frac{\partial \Pi(x_i | x_j = \bar{x}, \forall j \neq i)}{\partial x_i} .$$

This gives

$$(13) \quad \text{M}\Pi(x_i | x_j = \bar{x}, \forall j \neq i) = \lambda_i' \sum_{r=0}^m \binom{m}{r} [-1]^{M-r} \frac{(-\lambda_i + m\bar{\lambda} - r\bar{\lambda})}{(\lambda_i + m\bar{\lambda} - r\bar{\lambda})^3} - 1 .$$

Though equation (13) plays a central role in the firm's decision processes it is difficult to interpret in its present form, though it can later be given a simpler representation. To understand something about this function it is necessary to introduce a new notion of marginal profit. The idea we have in mind is the marginal profit which results when the extra dollar a firm spends on R&D is matched by all other firms. This marginal profit function will be denoted $\text{M}\Pi(\bar{x})$ and is formally defined as

$$(14) \quad \text{M}\Pi(\bar{x}) = \frac{\partial \Pi(\bar{x})}{\partial \bar{x}} = \left. \frac{\partial \Pi(x_i | x_j = \bar{x}, \forall j \neq i)}{\partial x_i} \right|_{x_i = \bar{x}} .$$

In the case of $M\Pi(\bar{x})$ any attempt to increase profits is being matched by all firms, whereas in the case of $M\Pi(x_i | x_j = \bar{x}, \forall i \neq j)$ the extra dollar spent on R&D is not being matched by all firms, and since the probability of having the winning invention must be higher in this latter case when $x_i = \bar{x}$, the graph of $M\Pi(x_i = \bar{x} | x_j = \bar{x}, \forall i \neq j)$ must lie everywhere above the graph of $M\Pi(\bar{x})$. The following Figure 3 then shows the relative positions of the graphs. As will shortly be seen, the two figures correspond to the cases of "too many" and "too few" firms in the industry.

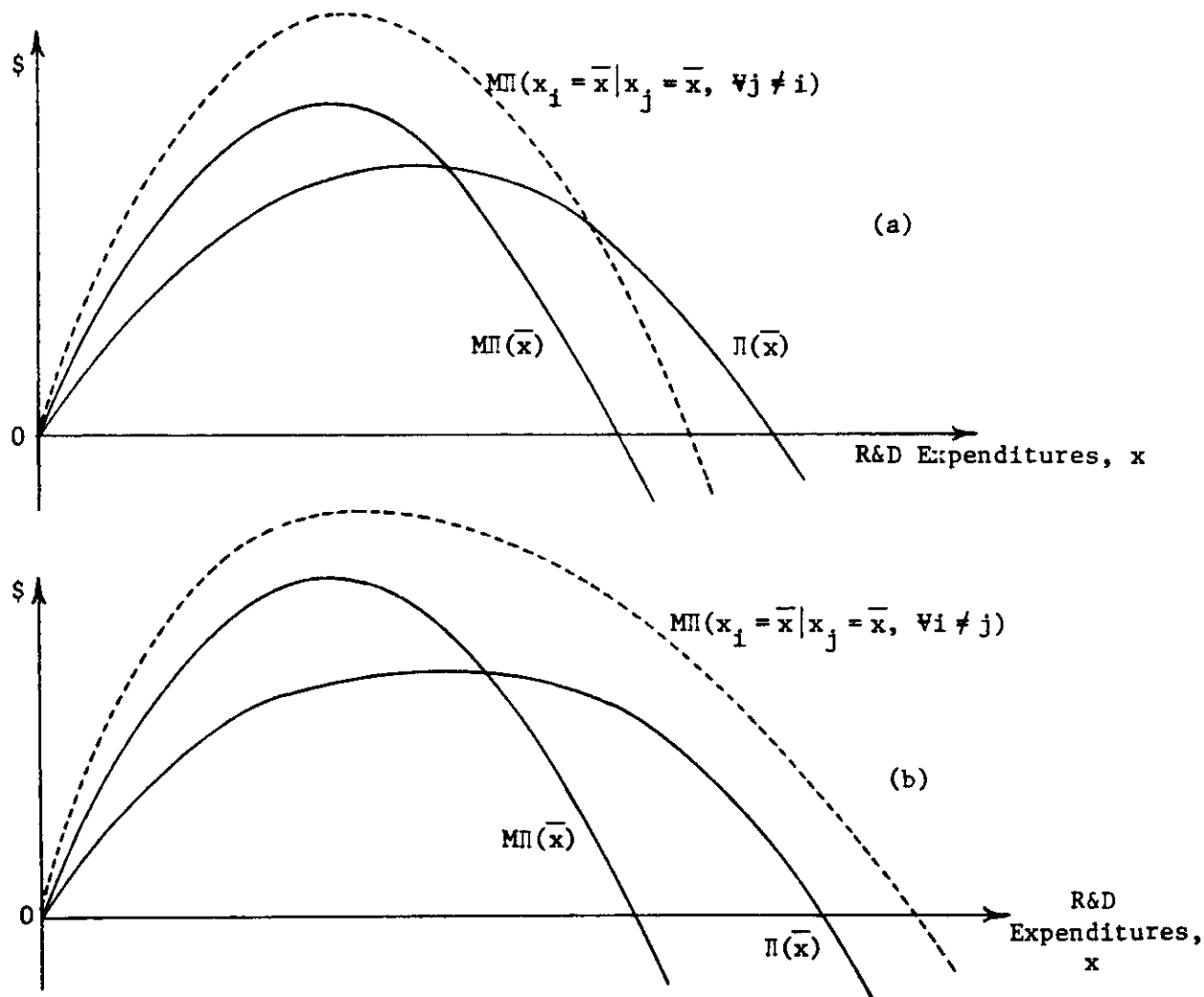


FIGURE 3. Graphs of Marginal Profit Functions

Continue to suppose that the number of firms in the industry is fixed, and let us examine the possible equilibria. An equilibrium will emerge when each firm is in a position such that there are no further benefits to be gained from spending either more or less on R&D. This condition obtains locally when $M\Pi(x_i = \bar{x} | x_j = \bar{x}, \forall j \neq i)$, the broken line in Figure 3, equals zero, that is, where the curve cuts the horizontal axis. The solution is shown in Figure 4 below.

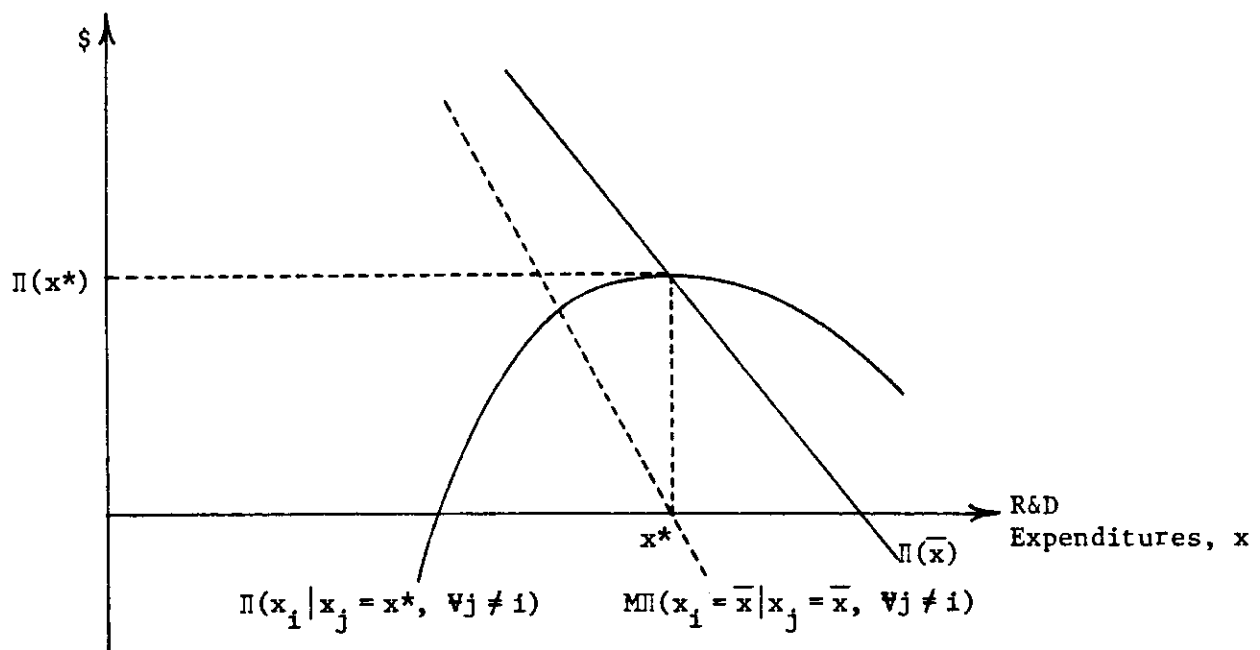


FIGURE 4. N-Firm Equilibrium

In the case shown in Figure 4 there are "too few" firms in the sense that each firm in equilibrium earns excess profits $\Pi(x^*)$ on the equilibrium quantity of R&D expenditures, x^* . A second case will exist corresponding to Figure 3b where there are "too many" firms, so that if all firms persist in participating in the market the rate of return at equilibrium will be negative.

If all entry and exit barriers were removed one would expect firms to enter the market when excess profits are being made, and exit the market in the event of excess losses. For the case described in Figure 3a one would expect entry of new firms, and for the case described in Figure 3b one would expect exit from the industry. A competitive equilibrium will attain when only normal profits are being made, in this case, zero profits. The competitive equilibrium is shown in Figure 5. The conditions for this equilibrium to attain at x^* are that:

$$(15) \quad \Pi(x_1 = x^*) = \Pi(x_1 = x^* | x_j = x^*, \forall j \neq 1) = 0 .$$

These conditions can only be satisfied for the technology production function in (6), only when

$$(16) \quad \beta = \frac{-K(m)}{K_1(m)}$$

where $K(m)$ is defined in (8), the market concentration index, and $K_1(m)$ is defined as

$$(17) \quad K_1(m) = \sum_{r=0}^m \binom{m}{r} [-1]^{m-r} \frac{(m-r-1)}{(m-r+1)} .$$

To illustrate the values which $K(m)/K_1(m)$ can take, consider the sample of values given in Table 1 below.

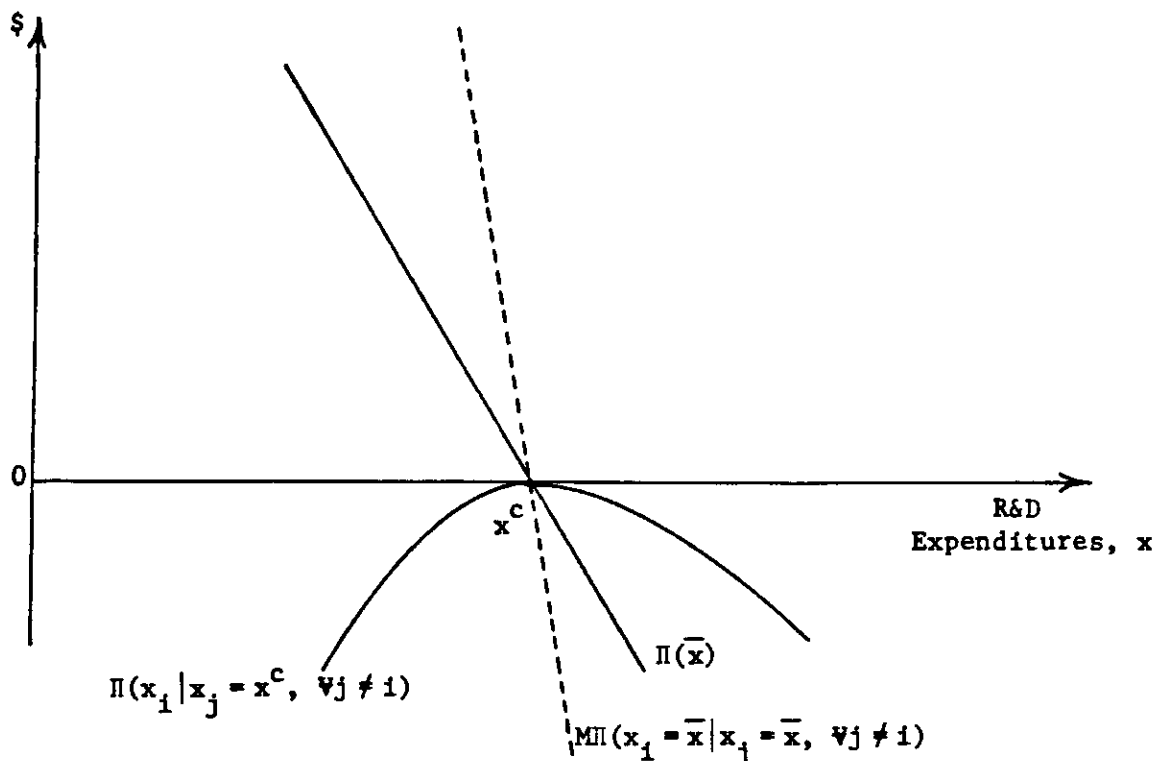


FIGURE 5. Free Entry-Free Exit Competitive Equilibrium

TABLE 1. K-Ratios

	$K(m)$	$K_1(m)$	$-K(m)/K_1(m)$
$M = 1$	0.75	-1.0	0.750
$M = 2$	0.611	-0.963	0.634
$M = 3$	0.521	-0.926	0.562
$M = 4$	0.457	-0.879	0.520
$M = 6$	0.370	-0.806	0.460
$M = 8$	0.314	-0.746	0.421

In defining the technology production function in (6) it was only required that β take on values between zero and one. However, it is clear that a competitive equilibrium, in which there is free entry and exit, will only attain when the β 's in (6) are restricted to certain

values, namely that $\beta \in B$ where B is defined as

$$B = \{-K(m)/K_1(m); m = 1, 2, \dots\}.$$

For a given technology for which $\beta \in B$, the equilibrium number of firms m^* is such that $-K(m^*)/K(m^*) = \beta$. The competitive level of R&D undertaken by each firm will be given by

$$x_c^* = \alpha^{\frac{1}{1-\beta}} [K(m^*)]^{\frac{1}{1-\beta}},$$

where m^* is one defined above for $\beta \in B$. When β is not an element of B an equilibrium will not attain, as there will be some number of firms, say N^0 , at which positive profits are being made, hence stimulating entry, and when $(N^0 + 1)$ are in the market negative profits are realized. This difficulty can be overcome formally by defining N^0 as the equilibrium, this presumes that the $(N^0 + 1)^{\text{th}}$ entrant will not enter the market, realizing that such entry will drive all participants profits to negative levels. The subsequent discussion will be limited to the cases where $\beta \in B$.

Equation (16) and Table 1 together show that the number of firms which will be in a market under competitive equilibrium depends on the technology for producing new products ideas. Consider equation (16) rewritten as:

$$(19) \quad \text{Expected Quality} = E[q(x)] = \alpha x^\beta$$

so that β is seen to measure the expected quality of the invention with respect to R&D expenditures, and is really a measure of the rate at which decreasing returns to R&D expenditures are setting-in. In those

cases where decreasing returns set in slowly, β large, the competitive equilibrium will contain only a small number of firms. Conversely, where decreasing returns set-in rapidly, β small, the large firm is relatively disadvantaged, and an equilibrium attains with a large number of firms.

From the point of view of anti-trust policy the above result is an important one for interpreting market concentration statistics in industries which are very active in the area of new technology. A concentrated market structure may as well be the result of the technology for producing new technology as being the result of collusive or other "nasty" behavior on the part of market participants.

WELFARE IMPLICATIONS OF MARKET STRUCTURE

The basic comparison to be made is between a monopolized industry and a competitive industry with free entry and exit.⁺ The simplest case is where the entire social surplus is appropriated by the producer.⁺⁺ In the competitive equilibrium solution the expected profits of the individual firms is zero (when $\beta \in B$), hence the expected social benefit, the sum of these profits is zero. The profit maximizing monopolist, as was shown, has a positive expected profit, hence there is a welfare sur-

⁺The terms "competitive" and "monopoly" industry refer to the inventive phase. Post-inventive phase the firm with the highest quality invention has monopoly rights in the product market by way of a legal device such as a patent.

⁺⁺This implies a product demand curve of the form

$$Q(\text{price}) = \begin{cases} 0 & \text{price} > P^0 \\ Q^0 & \text{price} \leq P^0 \end{cases}$$

where P^0 is a linear function of the quantity, q , of the product being introduced.

plus equal to this amount. It can be concluded that in the case where all the social benefits are appropriated by the firm, the profit maximizing monopolist will be socially preferred to a competitive market. This is true for the quite simple reason that the competitive market wastes resources by duplicating effort in production of the innovation.

The monopolist not only avoids duplication of effort but also has an incentive to restrict the expected quality of the innovation. The expected quality of the innovation will be higher in the case of the competitive solution as is shown in the relations in equations (20), (21) and (22) below

$$(20) \quad \begin{array}{l} \text{Expected Quality of} \\ \text{Competitive Solution} \\ \text{where } m^* \text{ satisfies} \\ \text{(16) for } \beta \in B \end{array} = (m^* + 1) [K(m^*)]^{-\frac{1}{1-\beta}} \alpha^{-\frac{1}{1-\beta}}$$

$$(21) \quad \begin{array}{l} \text{Expected Quality of} \\ \text{Monopoly Solution} \end{array} = \beta^{-\frac{\beta}{1-\beta}} \alpha^{-\frac{1}{1-\beta}} .$$

The higher expected quality of the competitive solution is ensured by the numerical demonstrating that the inequality (22) holds

$$(22) \quad (m^* + 1) [K(m^*)]^{-\frac{1}{1-\beta}} > \beta^{-\frac{\beta}{1-\beta}} \quad \text{for } \beta \in B .$$

These findings then give rise to the following result: if all the social benefits resulting from an invention are not captured by the monopolist, then there may be circumstances in which the competitive solution is socially preferred. To investigate this question further, suppose that the entire social benefits of the invention are given by $\gamma \cdot q$ where $\gamma \geq 1$. The expected social benefits are given by $E(\gamma q)$. The proportion of the gross benefits going to the producer is, $1/\gamma$ and the proportion going to consumers is given by $(\gamma-1)/\gamma$. The net social benefits for the competitive and monopoly solutions may then be considered as in equations (23) and (24) below.

$$(23) \quad \begin{array}{l} \text{Expected Net} \\ \text{Social Benefit of} \\ \text{Competitive Solution} \\ \text{for } \beta \in B \end{array} = \gamma \left[\begin{array}{l} \text{Expected Quality} \\ \text{of Invention} \end{array} \right] - (m^* + 1)x_C^* \\ = (\gamma-1)(m^* + 1)\alpha^{\frac{1}{1-\beta}} [K(m^*)]^{\frac{1}{1-\beta}}$$

$$(24) \quad \begin{array}{l} \text{Expected Net} \\ \text{Social Benefit of} \\ \text{Monopoly Solution} \end{array} = \gamma \left[\begin{array}{l} \text{Expected Quality} \\ \text{of Invention} \end{array} \right] - x_M^* \\ = \gamma \cdot \alpha^{\frac{1}{1-\beta}} \beta^{\frac{\beta}{1-\beta}} - [\alpha\beta]^{\frac{1}{1-\beta}} .$$

It is true that the competitive solution will be socially preferred to the monopoly solution when the gross surplus of the innovation, relative to that appropriated by the firm satisfies the inequality (25)

$$(25) \quad \gamma > \frac{(m^* + 1) [K(m^*)]^{\frac{1}{1-\beta}} - \beta^{\frac{1}{1-\beta}}}{(m^* + 1) [K(m^*)]^{\frac{1}{1-\beta}} - \beta^{\frac{\beta}{1-\beta}}} .$$

To gain some sense of the importance of this result consider the numerical

evidence in Table 2. The data suggest that when the proportion of the total gross surplus generated by an invention exceeds 40 percent of the total then the competitive solution will always be preferred to the monopoly solution. As one moves to technologies where the diseconomies of scale to innovation set in more rapidly, β smaller, it becomes easier to justify the competitive solution over the monopoly solution.

The quantitative picture drawn in Table 2 is clearly dependent on the model structure that has been developed, but the numbers do offer some insight into the relative importance of the duplication of effort versus the competitive striving for quality that typify a competitive market structure.

TABLE 2. Welfare Comparison of Monopoly and Competitive Solutions⁺

$\beta \in B$	Competitive Equilibrium Number of Firm	Value which consumer benefits as a proportion of gross benefits' invention, $(\gamma-1)/\gamma$, must exceed for competitive solution to be superior
0.611	2	0.38
0.457	3	0.33
0.314	4	0.31
0.245	5	0.29
0.187	6	0.28
0.136	8	0.27

⁺Notice that if the demand function for the product is of the form $Q = [(a - b \cdot \text{price}) \times \text{quality}]$ and there are zero marginal costs of production the monopolist in the product market (by virtue of a winning patent or other such legal devices) would maximize profits by setting prices so that $\gamma = 1.5$ or $\gamma-1/\gamma = 0.33$. This is simply a result of the linearity assumption but it does offer some intuition into relative magnitudes.

Conclusion

Though this paper offers some important modelling and quantitative insights further investigation of related situations would be valuable. In particular it would be useful to have a feel for relative importance of fixed set-up costs in undertaking R&D, or of the effect of threshold levels beyond which the expected quality of the invention may improve rapidly for subsequent dollars of R&D. These modifications, in effect, remove the assumption that average quality of inventions is a concave function of R&D expenditures. In the absence of such concavity the task of describing the competitive equilibria in the market (if any equilibria do exist) becomes a vastly more complex task.

REFERENCES

- [1] Kamien, M. and N. Schwartz (1972). "Uncertain Entry and Excess Capacity," American Economic Review, Vol. 62 No. 5 (1972).

- [2] ~~Lowry~~, G. (1979). "Market Structure and Innovation," Quarterly Journal of Economics, Vol. 93, No. 3.