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EFFICIENT STATIONARY TAXATION AND INTERTEMPORAL GENERAL EQUILIBRIUM

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ABSTRACT

We present a method to analyze the welfare cost of price distortions created by taxes on the incomes of capital and labor and on consumption in an intertemporal model of general equilibrium. This efficiency cost depends in an important way on the production technology. It is not very sensitive to the ratio between the tax rates on capital and labor respectively, when the elasticity of substitution between these factors is small. Our method allows us to determine under the assumptions of the model, the optimal combination of taxes on capital and labor income respectively. The consumption tax is more efficient than the labor income tax.

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1. Introduction

A central issue in the debate on tax reform is the efficiency cost of the price distortions created by the tax rates. In general, this efficiency cost has been analyzed in a partial equilibrium framework with fixed producers' prices (Feldstein 1978, Green and Sheshinski 1979). In these studies the efficiency cost of taxation depends only on the structure of preferences. However, previous work (Chamley 1981) has shown that in an intertemporal model of general equilibrium, this efficiency cost depends in an important way on the production technology.[1] The purpose of this paper is to extend this work in two directions:

We propose a criterion and a computational method when the private sector's utility is not necessarily additive, but satisfies the more general postulates of Koopmans: existence, sensitivity, limited non complementarity between periods and stationarity.

When no lump-sum tax is feasible, the efficiency cost of a tax should be compared to the cost of alternative policies. Also, because of cross effects, the efficiency costs of a given combination of tax rates should be analyzed simultaneously. In this paper, we analyze the welfare cost of policies of constant taxes on capital and labor income and on consumption in a stylized, intertemporal model of general equilibrium.

The model and the general method of measurement of the efficiency

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[1] Consider the efficiency cost of the capital income tax when the production function is of the Cobb-Douglas type, the labor supply is fixed, the intertemporal utility function is additive, with a constant elasticity of the marginal utility α and other parameters (time preference rate, growth rate and capital income share) have plausible values (Chamley 1981). A doubling of the elasticity of the marginal utility α, from one to two has the same effect on the efficiency cost of the tax, as a reduction of the elasticity of substitution between capital and labor from one to 0.92, an 8 percent change.

cost are presented in the next two sections.

Because the consumption tax can be viewed in the dynamic context, as a combination of a lump-sum tax and of a wage tax, we consider in the following sections only taxes on capital and labor.

In order to highlight the importance of the production technology, we analyze in Section 4, the special case where the capital labor and the output labor ratios are fixed. We show that the efficiency cost of a combination of taxes on the incomes of capital and labor depends only on the total amount of revenues, and not on the choice of tax rates.

In Section 5, we apply our method to a specific example of a functional form of the utility function. As expected, the welfare cost of taxation is much less sensitive to the tax policy when the elasticity of substitution between capital and labor takes a plausible value, than in the partial equilibrium case. The welfare cost is approximated by a second order formula. This formula allows us also to determine the optimal ratio between the respective tax rates on capital and labor income, when the tax revenues are relatively small. Because the welfare cost expression is rather complicated, we present various numerical results. A more technical description of the procedure is given in the Appendix.

We analyze briefly the problem of time consistency in Section 6; concluding remarks are presented in the last section.

2. The Model

In a dynamic economy, taxes have a distortionary effect on individual behavior, and also affect the intra- and intergenerational distribution of income. In order to analyze the efficiency cost of distortionary taxation, we assume away the problem of income distribution. We consider an economy where all individuals of the same generation are identical, and where an operational bequest motive takes place (à la Barro 1974). The private sector can then be represented by a single individual who is endowed with an infinite life and behaves competitively.

There is one good in the economy. Call k_t the stock of this good available at the beginning of the period t (per capita). This stock can be partly consumed during the period, or used in the production process. The level of output available at the end of the period y_t is given by the neoclassical production function:

$$y_t = f(k_t - c_t, \ell_t) \tag{1}$$

The capital accumulation constraint is represented by the following equation:

$$k_{t+1} = (k_t - c_t + f(k_t - c_t, \ell_t))/(1+n) , \tag{2}$$

where c_t represents the level of consumption per capita, and n the population growth rate.

An intertemporal program of the private sector is defined by the infinite sequence of consumption and labor supply per capita in each period $(x_t)_{t \geq 1} = (c_t, \ell_t)_{t \geq 1}$. Its utility is valued by the function:

$$U(x_1, x_2, \ldots, x_t, \ldots) .$$

We assume that this function is not identically constant, and is differentiable. Furthermore, following Koopmans (1960), this function satisfies the postulates of limited noncomplementarity and stationarity.[1] It follows

[1] Postulates 3 and 4 pages 292 and 294 in Koopmans (1960).

that this function takes the following form:

(3) $$U(x_1, x_2, x_3, \dots) = F(u(x_1), U(x_2, x_3, \dots)) ,$$

where the function F is an aggregator between the utility of the present period and the utility of all future periods.[1]

The representative individual is endowed with perfect foresight and maximizes the utility function U under the constraints of the initial capital stock k_1, and of the accumulation equation (2). In a given period, the level of U depends only on the level of the capital stock in the same period. This "indirect" utility function is represented by the function $J(k)$. When the private sector's intertemporal program is optimal, (c_t, ℓ_t) maximizes the function $F(u(c_t, \ell_t), J(k_{t+1}))$, subject to the capital accumulation constraint (2).

The first order conditions can be written:

(4) $$F'_1 \frac{\partial u}{\partial c_t} = F'_2 \left(\frac{1 + r_t}{1+n}\right) J'(k_{t+1})$$

(5) $$\frac{1}{1 + r_t} \frac{\partial u}{\partial c_t} = \frac{1}{w_t} \frac{\partial u}{\partial \ell_t} ,$$

where $r_t = f'_1(k_t - c_t, \ell_t)$, and $w_t = f'_2(k_t - c_t, \ell_t)$.

Also, by the envelope theorem, in each period:

(6) $$J'(k) = F'_1 \frac{\partial u}{\partial c_t} .$$

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[1]The existence of an aggregator function follows from the postulate of limited noncomplementarity. This postulate corresponds to an assumption of independence between the ordering of consumption vectors in the present, and the ordering of future programs of consumption. The postulate of stationarity implies that the aggregator F is constant over time, and is necessary (but not sufficient), for the existence of an optimal balanced growth path.

The equations (2), (4), (5) determine the levels of consumption and labor supply as functions of the capital stock in the same period:

(7) $$c_t = c(k_t)$$

(8) $$\ell_t = \ell(k_t) \ .$$

The function $J(k)$ satisfies the relation:

(9) $$J(k_t) = F(u(c_t, \ell_t), J(k_{t+1})) \ .$$

The dynamic path of the economy is defined by the equations (7) and (8) and by the capital accumulation constraint (2). We assume that this path converges to a steady state[1] (where variables are denoted by an asterisk). This steady state is characterized by the stationary equations:

(10) $$c^* = c(k^*)$$

(11) $$\ell^* = \ell(k^*)$$

(12) $$c^* = f(k^* - c^*, \ell^*) - nk^* \ .$$

The utility function $J(k)$ can be approximated around the steady state by a Taylor expansion:

(13) $$J(k) \sim J(k^*) + J'(k^*)(k - k^*) + \frac{J''(k^*)}{2}(k - k^*)^2 \ .$$

The value of $J(k^*)$ is found by solving the equation $J(k^*) = F(u(c^*, \ell^*), J(k^*))$. From (6), the value of $J'(k^*)$ is equal to $F_1' \frac{\partial u}{\partial c}(c^*, \ell^*)$. The value of

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[1] Iwai (1972), provides a complete analysis of a similar dynamic optimization problem.

$J''(k^*)$ is obtained (simultaneously with $c'(k^*)$ and $\ell'(k^*)$) by a differentiation of the equations (2), (4), (5), (6) (see the Appendix).

3. The Efficiency Cost of Taxation

The Wage and Interest Taxes: Assume first that taxes are levied on the income of capital and labor, at the respective rates θ_r and θ_w . These rates are time invariant. The first order conditions of the private sector's optimization can be rewritten as follows:

$$(4a) \qquad F_1' \frac{\partial u}{\partial c_t} = F_2' \frac{(1 + r_t(1 - \theta_r))}{1+n} J'(k_{t+1})$$

$$(5a) \qquad \frac{1}{1 + r_t(1 - \theta_r)} \frac{\partial u}{\partial c_t} = - \frac{1}{w_t(1 - \theta_w)} \frac{\partial u}{\partial \ell_t} .$$

The taxes distort the allocation of resources between the levels of consumption and leisure, and between the levels of consumption in different periods.

Following a standard practice, we assume that the tax revenues are refunded by lump-sum payments to the private sector. The steady state of the economy is now characterized by the equations (4a), (5a) and the following equation:

$$(12a) \qquad \bar{c} = f(\bar{k} - \bar{c}, \bar{\ell}) - n\bar{k} ,$$

where a bar will denote a variable in the steady state with taxation. The values of $\bar{c}$, $\bar{\ell}$, $\bar{k}$ in the steady state with taxation, depend on θ_r and θ_w :

$$(14) \qquad \bar{k} = \bar{k}(\theta_1, \theta_2) .$$

The value of the intertemporal utility in the steady state with taxation $\bar{J}$, is found by solving the equation $\bar{J} = F(u(\bar{c},\bar{\ell}), \bar{J})$. It is a function of the tax rates:

$$(15) \qquad \bar{J} = \bar{J}(\theta_1, \theta_2) \ .$$

Following the method used previously by one of the authors, we measure the efficiency cost of taxation as follows: the taxation system is assumed to have been in place for an infinite amount of time, and the economy is in the steady state with taxation characterized by the equations (4a), (5a), (12a). At the time zero, all taxes are abolished and the economy's variables start to move on a path converging to the new steady state with no tax. The efficient cost of taxation E , is measured by the welfare gain of the tax reform, i.e., the difference between the level of utility measured on the new dynamic path and the level of utility on the stationary path with taxation:[1]

$$(16) \qquad E = J(\bar{k}(\theta_1, \theta_2)) - \bar{J}(\theta_1, \theta_2) \ .$$

When the tax rates are small, this difference is of the second order with respect to the tax rates, and can be approximated by the second order term of its Taylor expansion with respect to the tax rates.

The Consumption Tax: Before applying this method in the next sections, let us consider briefly the properties of a tax on consumption at the rate θ_c . The dynamic equation (4), (5), (6) take now the following form:

[1] Up to the second order, this measure is equal to the welfare cost of introducing the tax system at an original position of no tax.

(4b) $$F_1' \frac{\partial u}{\partial c_t} \frac{1}{(1+\theta_c)} = F_2' \frac{(1+r_t)}{1+n} J'(k_{t+1})$$

(5b) $$\frac{1}{1+r_t} \frac{\partial u}{\partial c_t} \frac{1}{(1+\theta_c)} = -\frac{1}{w_t} \frac{\partial u}{\partial \ell_t}$$

(6b) $$J'(k_t) = F_1' \frac{\partial u}{\partial c_t} \frac{1}{(1+\theta_c)} .$$

Comparing (4a), (5a), (6), and (4b), (5b), (6b), respectively, we see that the only distortion introduced by the consumption tax occurs between consumption and leisure, exactly as for the labor income tax. However, it does not follow that the consumption tax and the labor income tax are equivalent: the labor income tax alters the value of the time in terms of consumption goods. The consumption tax alters the value of time and of the existing capital, in terms of consumption goods; it is equivalent to a combination of a tax on labor income and a lump-sum tax. Consider for simplicity, the steady state with a consumption tax:

$$c(1+\theta_c) = w\ell + (r-n)k .$$

This expression can be rewritten:

$$c = \frac{1}{1+\theta_c} w\ell + \frac{(r-n)k}{1+\theta_c} .$$

The consumption tax is the equivalent of a tax on labor income at the rate θ_w , where $1-\theta_w = 1/(1+\theta_c)$, and of the lump-sum tax (the present discounted value of the latter's revenues is equal to $\theta_c/(1+\theta_c)\cdot k$).

The consumption tax is always more efficient than the labor income tax. Any combination of constant tax rates on capital and labor income is less efficient than a combination of taxes on capital income and on consumption.

4. A Special Case: The Fixed Coefficients Technology

The efficiency cost of taxation depends on the production technology, and on the form of the private sector's utility function. The optimal tax policy minimizes this welfare cost under a tax revenue's constraint. Previous work has shown that the efficiency cost of taxation depends in an important way, on the elasticity of substitution between capital and labor in the production technology. A useful example is provided by the case of a technology with fixed coefficients. We now prove the following proposition:

Proposition. Assume that the private sector's utility is represented by a function of the form (3) and that the capital/output and labor/output ratios are fixed in the production function. The efficiency cost of taxation on the incomes of capital and labor depends only on the ratio between tax revenues and gross output, and is independent of the ratio between the respective tax rates.

Assume that the production technology is represented by the equations:

$$k = ay$$

$$\ell = by \text{ , where } a \text{ and } b \text{ are constants.}$$

In the steady state by the capital accumulation constraint, we have:

$$\bar{c} = \bar{y} - n\bar{k} = \left(\frac{1}{a} - n\right)\bar{k} \ . \tag{17}$$

Also,

$$\bar{\ell} = b\bar{k} \ . \tag{18}$$

Let us prove that the levels of $\bar{c}$, $\bar{\ell}$ and $\bar{k}$ in the steady state depend only on the ratio between tax revenues and output, and not on the respective tax rates on labor and capital income. By (17) and (18), it is sufficient to prove the property for k. In the steady state with taxation, the first order conditions (4a) and (5a) take the following form:

$$\frac{\frac{\partial u}{\partial c_t}}{\frac{\partial u}{\partial \ell_t}} = -\frac{1 + r(1-\theta_r)}{w(1-\theta_w)}$$

$$F_2' = \frac{1+n}{1 + r(1-\theta_r)}$$

where in the left-hand side $\bar{c}$ and $\bar{\ell}$ are expressed as functions of $\bar{k}$ (using (17) and (18)). These equations can be rewritten:

(19) $$G(\bar{k}) = \bar{w}(1-\theta_w)$$

(20) $$H(\bar{k}) = 1 + \bar{r}(1-\theta_r) \ .$$

We also have the equation of the price possibility frontier

(21) $$1 = \bar{r}a + \bar{w}b \ ,$$

and the ratio between tax revenues and gross input R, is equal to:

(22) $$R = ar\theta_r + bw\theta_w \ .$$

Differentiating the system (19), (20), (21), (22) with respect to $\bar{k}$, $\bar{r}$, $\bar{w}$, $\bar{\theta}_r$ as functions of θ_w, keeping R constant, we find that $\bar{k}$ is independent of θ_w (or θ_r).[1]

Q.E.D.

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[1] The simplest method may be to apply Cramer's rule.

The above proposition is not surprising: when capital and labor are used in a fixed ratio in the production of goods, a tax on the capital input is equivalent to a tax on the labor input. (The consumption tax is more efficient than a tax on capital or labor income.)

It should be emphasized that the tax is equivalent to a lump-sum tax in the immediate short-run, but it introduces a distortion in the long-run. The magnitude of this efficiency cost is analyzed numerically in the next section.

5. Application

As an illustration of the method described in Section 3, we analyze the case where the aggregator F in (3) is linear. For simplicity, we consider a time continuous model and we assume that the level of utility function is measured by the following function:

$$U = \int_0^\infty e^{(n-\rho)t} v(c_t,\ \ell_t)dt\ , \tag{23}$$

where $v(c,\ell) = \frac{1}{1-s}[c^b(1-\ell)^{1-b}]^{1-s}$ if $s \neq 1$

$v(c,\ell) = b \log c + (1-b)\log(1-\ell)$ if $s = 1$.

We also assume that labor augmenting technological progress takes place at the constant rate μ . The sum $n+\mu$ is equal to the economy's growth rate g .

In the above expression, we can see that the elasticity of the marginal utility of consumption is equal to $\sigma = 1-b + bs$. When the gross factor prices vary with the capital labor ratio, the economy converges to a balanced growth path which is characterized by a net rate of return $r^* = \rho + \sigma\mu$.

We can express a second approximation of the welfare cost of the taxes on capital and labor income, by a quadratic function of their respective tax rates.[1] This welfare cost can be measured in terms of a permanent reduction of consumption equal to L .[2]

$$(24) \qquad L = a_{11}\theta_r^2 + 2a_{12}\theta_r\theta_w + a_{22}\theta_w^2 \ .$$

For example, when the elasticity of substitution between capital and labor tends to infinity (which corresponds to a partial equilibrium assumption of fixed factor prices), and s is equal to one, L is equal to:[3]

$$(25) \qquad L = \frac{1}{2b}\left[\left(\frac{r^*}{r^*-g}\right)^2 \theta_r^2 + b(1-b)\theta_w^2\right] \ .$$

The optimal values of the tax rates minimize the loss function L under the budget constraint,

$$(26) \qquad \frac{1}{r-g}(\theta_r rk + \theta_w w\ell) = A \ ,$$

where all the variables are measured in the steady state with no tax, and A is the present discounted value of government expenditures.

Therefore, the optimal ratio between the tax rates is equal to:

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[1]A complete derivation for this example is given in the Appendix.

[2]The wealth equivalent of the welfare cost is equal to $Lc^*/(r^* - g)$, where c^* is the level of consumption on the balance growth path.

[3]This expression can also be obtained directly by the Hotelling-Hicks-Harberger formula, and is an extension of the result of Levhari and Sheshinski (1972).

$$(27) \qquad \frac{\theta_r}{\theta_w} = \frac{-\frac{a_{12}}{\alpha} + \frac{a_{22}}{1-\alpha}}{\frac{a_{11}}{\alpha} - \frac{a_{12}}{1-\alpha}} ,$$

where α is the share of capital income. In the special case of fixed gross prices and s equal to one, (27) becomes

$$(28) \qquad \frac{\theta_r}{\theta_w} = \frac{\alpha}{1-\alpha}\left(\frac{r^* - g}{r^*}\right)^2 b(1-b) .$$

We can note that in this last case it is optimal to tax only labor income tax when there is no capital $(\alpha = 0)$, or when the growth rate tends to the rate of return r^* .[1]

In the following table we present some estimates of optimal tax ratios and welfare costs for different values of the parameter b in the utility function, and of the elasticity of substitution between capital and labor ε . The other parameters are chosen such that the growth rate g is equal to 2%, the net rate of return r^* is equal to 4%, the capital income share α is equal to 1/4, and the elasticity parameter s is equal to one.

In each cell, the first number represents the optimal ratio between the tax rates on capital and labor income; the next two numbers represent the welfare costs of the income tax $(\theta_r = \theta_w)$, and of the optimal combination of tax rates, respectively. These numbers are normalized as follows: Call R the ratio between tax revenues and total output. The welfare cost of a tax policy is measured in terms of steady state consumption, and this fraction is equal to the product of the number found in Table 1 and of R^2 .

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[1] Because of our specification of the utility function, this result is true even when the tax rates are not small (see Atkinson and Stiglitz [1976]).

TABLE 1

Optimal Taxation and Welfare Cost

	Elasticity of Substitution between Capital and Labor						
Parameter b	0.00	0.50	0.75	1.00	1.25	1.50	10000.00
0.00	1.083	0.399	0.303	0.245	0.205	0.176	0.000
	0.653	0.807	0.853	0.896	0.936	0.974	1182.393
	0.653	0.784	0.807	0.822	0.832	0.839	0.889
0.25	0.867	0.305	0.240	0.201	0.174	0.155	0.016
	0.517	0.636	0.674	0.708	0.740	0.772	8.301
	0.517	0.605	0.617	0.625	0.630	0.634	0.663
0.50	0.639	0.207	0.167	0.143	0.127	0.115	0.021
	0.366	0.459	0.491	0.521	0.550	0.578	4.232
	0.366	0.416	0.421	0.424	0.426	0.428	0.441
0.75	0.391	0.105	0.086	0.075	0.068	0.062	0.016
	0.197	0.273	0.304	0.333	0.361	0.388	2.784
	0.197	0.215	0.216	0.217	0.217	0.218	0.221
1.00	0.004	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.077	0.110	0.140	0.169	0.196	1.996
	0.000	0.000	0.000	0.000	0.000	0.000	0.000

In each cell the first number represents the optimal ratio between the tax rates on capital and labor respectively; the second and the third numbers represent the normalized welfare cost of the income tax and of the optimal tax respectively. Other parameters: $s = 1$, $r^* = 0.04$, $g = 0.02$.

As an example, consider the case of a Cobb-Douglas technology ($\varepsilon = 1$), and a value of b equal to 1/2: the optimal ratio between θ_r and θ_w is equal to 0.14; the welfare cost of the income tax at the rate 1/4, is equivalent to a permanent reduction of consumption by a fraction equal to $.52 \times (1/4)^2 = 3.25$ percent; this is equal to about 11.5 percent of the government revenues.

From an inspection of this table we can make the following observations:

The welfare cost of taxation increases with the elasticity of substitution between capital and labor ε, and with the elasticity of the labor supply (inversely related to b). The tax on capital income is inversely related to ε, and increases with the elasticity of the labor supply.

For plausible values of ε (around one), the welfare gain of shifting from an income tax to an optimal tax is moderate, especially if the labor supply is elastic. As we have seen in the previous section, the welfare gain of tax reform is nil in the case of fixed coefficients ($\varepsilon = 0$). Also, the values found in the partial equilibrium framework ($\varepsilon = \infty$) are very different from the values obtained in the case of a Cobb-Douglas technology.

The optimal tax ratio is somewhat sensitive to the other parameters of the model. When the intertemporal substitutability in the utility function increases, the parameter s decreases, and we can expect a lower tax on capital income (and vice-versa). Table 2 presents the optimal tax ratios for a case of a high substitutability ($s = 0$, first number in each cell), and low substitutability ($s = 4$, second number).

TABLE 2

Optimal Capital Labor Tax Ratios

	Elasticity of Substitution between Capital and Labor						
Parameter b	0.00	0.50	0.75	1.00	1.25	1.50	10000.00
0.00	1.083 1.084	0.399 0.399	0.303 0.303	0.245 0.245	0.205 0.205	0.176 0.176	-0.000 0.000
0.25	0.699 1.236	0.214 0.503	0.158 0.417	0.125 0.366	0.102 0.332	0.086 0.307	0.001 0.001
0.50	0.479 0.929	0.131 0.331	0.101 0.273	0.084 0.238	0.073 0.215	0.065 0.197	0.002 -0.001
0.75	0.254 0.561	0.051 0.150	0.042 0.120	0.037 0.101	0.033 0.087	0.031 0.077	0.001 0.001
1.00	0.000 0.008	0.000 0.000	0.000 0.000	0.000 0.000	0.000 0.000	0.000 0.000	0.000 -0.000

In each cell the first number corresponds to a case of high intertemporal substitutability $(s = 0)$, and the second to a case of low substitutability $(s = 3)$.

Another important parameter in the model is the growth rate. The welfare cost and the optimal tax rates depend on the ratio between the growth rate and the steady state rate of return. For a given rate of return,[1] the welfare cost of taxation increases with the growth rate, and the capital income tax decreases with the growth rate.

When the growth rate tends to the steady state net rate of return, the optimal tax rate on capital income tends to zero, independent of the parameters of the utility function.[2]

Since there is considerable uncertainty about the parameters of the model, this sensitivity of the optimal policy may appear to be disturbing. However, we have seen previously that when the elasticity of substitution between capital and labor is not too large, the loss function L may be relatively "flat" with respect to the ratio between the tax rates. In order to illustrate this point, we consider a ratio between the tax rates on capital and labor respectively, equal to .25 (an arbitrary average of different values). We then determine the difference between the welfare cost of this tax policy and of the optimal policy. We then compute an upper bound of this difference for all values of the parameters b , s , and the growth rate g . This upper bound is presented in Table 3 as a function of the elasticity ε .[3]

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[1]The welfare cost of taxation (measured in consumption equivalent) depends on r^* and g only through the ratio g/r^* .

[2]Of course this result depends on the functional specification of the utility function.

[3]As in Table 1, this upper bound is expressed in consumption equivalent and for a given policy should be multiplied by the square of the ratio between tax revenues and income.

TABLE 3

ε	0.00	0.50	0.75	1.00	1.25	1.50	20.00
	0.000	0.0105	0.0158	0.0210	0.0263	0.0316	0.4208

Upper bound of the difference between the normalized welfare cost of a policy $(\theta_r/\theta_w = 1/4)$ and of the optimal tax respectively (for all values of b, $g \leq r$, and $\varepsilon \leq 3$).

Assume for example that the technology is of the Cobb-Douglas type, and that the level of government revenues is about one quarter of income. The difference between the welfare costs of a policy with a tax ratio of 1/4 and the cost of the optimal policy, is for all values of b, $s \leq 3$, g, r^*, less than one tenth of one percent of income $(0.1 \sim 2.1/16)$.

6. The Problem of Time-Consistency

A tax on capital has two effects: in the beginning of the planning period, it is equivalent to a lump-sum tax; however the expectation of a capital income tax has a distortionary effect on savings and on the capital stock in the long-run. When the tax rate of capital can be altered over time, and individuals have perfect foresight, the optimal policy may be to tax capital in the short-run to accumulate a government surplus, and to impose no capital income tax in the long-run (Chamley 1980b).

It is immediate that this policy cannot be time-consistent since what is the long-run at some instant will become the short-run in the future. The social planner may have an incentive to depart in the future from an announced policy. When the tax rate can vary freely there is, in general, no time-consistent policy.[1] In our study, we impose the restriction that the tax rates on capital and labor respectively, are constant over time. We find, in general, a positive tax rate on capital income because the lump-sum advantages of the tax in the short-run compensate for the distortions in the long-run. Is it possible that in this framework, there are

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[1] For a discussion of this problem, see Kydland and Prescott (1980) and Fisher (1980). A similar problem occurs in the definition of an optimal monetary policy (Calvo (1978), Chamley (1980)).

time-consistent policies? The answer is no, except for a set of probability zero.

Imagine that tax rates have been consistent for a long time, and that the economy is in a steady state. Assume that at time zero, the planner can revise the ratio between the tax rates on capital and labor, λ_0, under the constraint that the tax rates should be constant thereafter. The optimal value of the new ratio depends on the capital stock at time zero; it is not impossible that this optimal ratio is equal to the past ratio λ_0. In this case there is no incentive to alter it, and the policy _is_ time-consistent.[1] However, the probability of this event is equal to zero. Given the choice, the social planner alters, in general, this ratio to a new value λ_1; after time zero, the capital stock converges to a new value k_1. At a later date when k is near k_1, the "old" ratio λ_1, is no longer optimal and a social planner would, if possible, alter it again. The optimal policy is not time-consistent.

We should remark that the optimal policies described in the previous section are independent of the initial state of the economy. This is so because the values in Tables 1 and 2, are only asymptotic values which are accurate for relatively small tax revenues.

An important conclusion of Section 5 was that when the elasticity of substitution between capital and labor is finite, the welfare cost of taxation is not sensitive to the capital labor tax ratio, provided that this ratio is not too far from the optimum. It follows then that although in a strict sense, a social planner would like to revise at each instant a preannounced policy, the incentive for these time inconsistent revisions

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[1]If we relax the constraint to time invariant tax rates, the past policy of a tax ratio λ is no longer time consistent.

is very small. In this approximate sense, optimal policies restricted to time invariant tax rates are time-consistent.[1]

7. Conclusion

The generality of the results of this study depend on the assumptions of the model of general equilibrium we have used. This model should be considered as a stylized version of more disaggregated models (Hudson and Jorgenson (1976), Fullerton et al. (1979)). Also, as Green and Sheshinski (1979) have shown, a complete study of the welfare cost of large tax rates should not be restricted to a second order approximation, but should rely on a more exact measurement of the path of the economy (in this dynamic case, numerical computation seems to be the only satisfactory solution).

The advantage of a stylized model is to provide an easy test of the sensitivity of the results to various assumptions about the important parameters of the model. It seems that if the elasticity of substitution between capital and labor is not much greater than one, the welfare cost of taxes or capital and labor is not very sensitive to the ratio between their respective tax rates. The shift from a general income tax to an optimal balance between these two taxes reduces the welfare cost by a value smaller than one percent of output in most cases[2] (for a ratio between tax revenues and aggregate income equal to one quarter). Because of the uncertainty about the parameters of the model the lack of sensitivity of the welfare

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[1] Also, a change of the tax policy creates an additional distortion if individuals do not have perfect foresight about the future dynamic path of the economy. An example of this cost of imperfect foresight is described in Chamley (1981).

[2] This value is about of the same order of magnitude as the cost of misallocation of the corporate tax (Shoven 1976).

cost to the optimal ratio between the tax rates on capital and labor income respectively, is somewhat comforting. In a specific example we find that when this ratio is equal to one quarter, the difference between the welfare cost of taxation and its minimum optimal value is for most parameteric values, very small, less than one tenth of one percent of total output.

APPENDIX

The Second Order Approximation of the Welfare Cost of Taxation

We describe here the approximation method of computation of the welfare cost for the discrete model. This method can be applied to the discrete model as well.

In a continuous model, we assume that the utility function of the private sector is given by:

$$U = \int_0^\infty e^{-\rho t} e^{nt} u(c_t,\ 1-\ell_t)dt \ ,$$

where $$u(c,\ 1-\ell) = \frac{1}{1-s}\left(c^b(1-\ell)^{1-b}\right)^{1-s} \ ,$$

and c_t is the level of consumption per capita. For simplicity, we will measure c_t per unit of effective individual (ℓ_t is measured per capita); the function U can then be written:

$$U = \int_0^\infty e^{-(r^*-g)t} u(c_t,\ 1-\ell_t)dt \ ,$$

where $r^* = \rho + \mu\sigma$; $\sigma = -(u''_{11}/u'_1)c$.

The optimal path is characterized by the first order conditions

(A-1) $$\frac{b}{c} = \frac{1-b}{w(1-\ell)} \ ,$$

(A-2) $$u''_{11}\dot{c} + u''_{12}\dot{\ell} = u'_1(r - r^*) \ ,$$

and by the capital accumulation equation

(A-3) $\dot{k} = f(k,\ell) - gk - c$.

This path converges to the steady state determined by the equations:

(A-4) $r = r^*$

(A-5) $c = \frac{b}{1-b} w(1-\ell)$

(A-6) $c = f(k,\ell) - gk$.

At each instant, the levels of c and ℓ are functions only of the capital stock. We now determine the derivative of these functions $c(k)$ and $\ell(k)$ respectively, at the steady state determined by (A-4)-(A-6). Taking the ratio of (A-2) and (A-3) when k tends to k^*, we find:

$$u''_{11}c'(k^*) + u''_{12}\ell'(k^*) = \lim_{k \to k^*}\left(\frac{r - r^*}{f(k,\ell) - gk - c}\right) .$$

The value of $\ell'(k^*)$ can be expressed as a function of $c'(k^*)$ by differentiating (A-1) with respect to k :

$$\ell'(k^*) = B_1 + B_2 c'(k^*)$$

with $$B_1 = \frac{\frac{\alpha}{\varepsilon k}}{\frac{\alpha}{\varepsilon \ell} + \frac{1}{1-\ell}} , \quad B_2 = \frac{-1}{c\left(\frac{\alpha}{\varepsilon\ell} + \frac{1}{1-\ell}\right)} .$$

We find that $c'(k^*)$ is the positive root of the following second order equation

$$A_0 x^2 + A_1 x + A_2 = 0 ,$$

with

$$A_1 = (1 - B_2 w)H_2 B_1 - (B_1 w + r - g)(H_1 + B_2 H_2) + (1-\alpha)\frac{r}{\varepsilon \ell} B_2$$

$$A_2 = -(B_1 w + r - g)H_2 B_1 - (1-\alpha)\frac{r}{\varepsilon k} + (1-\alpha)\frac{r}{\varepsilon \ell} B_1$$

$$H_1 = -(u''_{11}/u_1) \ , \ \ H_2 = -(u''_{12}/u'_1) \ .$$

On the optimal path the level of utility depends only on the capital stock

$$U = J(k) \ .$$

By optimization,

$$J'(k) = u'_1(c(k), \ \ell(k)) \ .$$

Differentiating this expression at k^* :

$$J''(k^*) = u''_{11} c'(k^*) + u''_{12} \ell'(k^*) \ .$$

The values of the variables in the steady state with taxation (denoted by a bar), are determined by the equations:

(A-7) $$\bar{r}(1 - \theta_r) = r^*$$

(A-8) $$\bar{c} = \frac{b}{1-b}(1 - \theta_w)(1-\bar{\ell})$$

(A-9) $$\bar{c} = f(\bar{k}, \bar{\ell}) - g\bar{k} \ .$$

The welfare cost of taxation is measured by the difference E :

$$E = J(\bar{k}) - \frac{1}{r^* - g} u(\bar{c}, \bar{\ell}) \ .$$

Taking a second order approximation, and using the fact that (A-9) is verified exactly, we find:

$$\text{(A-10)} \qquad E = \frac{1}{2}\left[J''(k^*)d\bar{k}^2 - \frac{u_1'}{r^* - g}(f_{11}''d\bar{k}^2 + 2f_{12}''d\bar{k}d\bar{\ell} + f_{22}''d\bar{\ell}^2 + u_{11}''d\bar{c}^2 + 2u_{12}''d\bar{c}d\bar{\ell} + u_{22}''d\bar{\ell}^2)\right] .$$

The values of $d\bar{c}$, $d\bar{\ell}$ and $d\bar{k}$ are found by differentiating the system (A-7), (A-8), (A-9), and are linear functions of these rates (if they are small). Replacing in (A-10), the welfare cost E can be written under the form:

$$E = \frac{c}{r^* - n}L$$

with $L = a_{11}\theta_r^2 + 2a_{12}\theta_r\theta_w + a_{22}\theta_w^2$

The last number in each cell of Table 1 in the text is equal to the sum $a_{11} + 2a_{12} + a_{22}$.

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