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SOCIETY, LAND, LOVE OR MONEY

(A STRATEGIC MODEL OF HOW TO GLUE THE GENERATIONS TOGETHER)

by

Martin Shubik

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Martin Shubik³

There are nine and sixty ways
Of constructing tribal lays
And every single one of them is right!

Rudyard Kipling

1. INTRODUCTION

In 1958, Paul Samuelson (1958) published a seminal article on the problem of linking the generations together, if the individuals consisted of economic men with finite lives, only concerned with their own welfare. He noted, but did not develop in detail, that the social contrivance of money as a device for intergenerational transfer might link the generations together.

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²This supercedes a previous preliminary paper on intergenerational political economy.

³Many of the ideas in this paper were developed jointly in close collaboration with Brian Arthur over the last few years.

The purpose of this paper is to show that if reasonable care is taken in well-defining the types of games of strategy which arise naturally from considerations of the multigenerational aspects of any society: then many different plausible explanations of intergenerational links will all be consistent with an ongoing economy with intergenerational transfers of wealth. Which of the models (or blends of the models) is the best, appears to be both an empirical and a theoretical question.

This suggests that a specific program of interdisciplinary work involving both model building based upon empirical investigation and the development of the mathematical methods of intergenerational or multistage variable person games is called for. In particular, it is argued that biology, anthropology, sociology, demography, political science, law and economics all are concerned with behavior with highly different time spans and different degrees of conscious behavior. The emphasis here is not only economic, the stress is that for the successful development of an economic dynamics the correct links with the other disciplines must be forged.

It is not possible to do justice to the many relevant aspects of the demographic, political and social conditions which constrain economic life in an ongoing society in a single paper. No attempt to do so is made here. Instead the approach is to sketch what we perceive to be the central problem in the study of multigenerational economics. Having stated the central problem we need to specify why it is important.

Given that the problem is stated and deemed to be important what can we do about it? In this paper we proceed by developing (in highly simplified form) the apparatus needed to consider a *multistage variable person game of strategy*. This is applied to several relatively simple

examples involving conscious altruism, coded or instinctive behavior, the threat of social custom and the role of ownership. We note, but defer to a further projected paper several more directly economic models involving markets, taxes, money and traded capital goods.¹

In conclusion it is argued that although the examples analyzed are simple they are examples of general phenomena. The level of mathematical analysis itself is kept at a minimum. Further development calls for a considerable enlargement of notation and further discussion of items such as information, games with infinite horizons and the many problems associated with the development of adequate solution theories for games played in extensive form. As the stress is upon the modeling problems of glueing the generations together, the game theory is presented only as a useful method for furthering this purpose. Because it stresses the general concepts of strategic analysis it is our belief that it is particularly suited to the analysis of situations in which economic, social, political and other decisions and motivations must be considered together.

1.1. The Problem

If man were individualistic, utilitarian and primarily self concerned, then without an extraeconomic structure to provide the connections there is no guarantee that the generations would hold together in a stable way.

Among animals the problem of glueing the generations together appears to have been taken care of genetically. Care of offspring appears to be programmed in, if for no other reason that a species lacking such a gene would rapidly disappear. The care for offspring needs to be forthcoming until the last of the new generation is selfsufficient and able to reproduce.

¹For a partial treatment of these in this manner, see Shubik (1980).

After the last reproduction and rearing of offspring the parent is of no more "use" and can die. If we were to accept this argument in full we would see no reason for the care of the aged among animals. Thus we would expect to find no old-folks homes or retirement centers for kangaroos, fish or seagulls. But we do expect to find nests, dens and other locals for rearing the young.

It would appear that among the animals, at least some of the human species exhibit some form of built-in perversion which might be most easily explained by a desire for living and the quality of life. This desire may even influence the way the young are looked after or abandoned. It also provides a justification for considering the economics of family planning.

The intergenerational linkage problem, for the reasons given above, seems to be a distinctly human problem, and furthermore it appears to be primarily an old age support problem--although with humans, the genetic programming for parenthood can be overruled by a host of other factors ranging from cultural tradition to individual economic interest.

In the remainder of this paper the society is modeled as though there is a single individual per generation, although it could be extended to a mass of homogeneous nuclear families. In the course of human history the structure of the family has varied; depending upon time and culture there have been nuclear families, extended families, tribal groups, monogamy, polygamy, polyandry and so forth. With the variation in family structure comes varying degrees of internalization of insurance, health care and other forms of mutual support. Depending upon this structure the needs for savings banks, loan societies or joint stock ventures will vary as the essential functions of insurance and support are provided by family, private economic, social or political mechanisms.

1.2. Why the Problem Is Important

There are at least four reasons why we believe that the intergenerational connection problem is important. An adequate analysis of the stability of and mechanisms behind generational linkages should be of the following use.

(i) It should help us to better understand the origin and persistence of certain institutions; for example, peasant proprietorships.

(ii) It helps to better understand and more rigorously define welfare questions between generations. The economic growth theorists have frequently used an infinite utility function with a real discount factor in order to provide a bounded optimization model of economic society as a single entity undifferentiated by generations. This appears to be more of a mathematical convenience than an adequate portrayal of reality. We believe that the type of formulation presented here offers several alternatives to this type of model, which are of interest.

(iii) It should help us to better understand the processes of social change and economic development.

(iv) It helps to embed the general equilibrium model of the price system in a richer institutional context yielding an explicit set of mixed socio-economic and politico-economic models which highlight the tradeoffs between strictly economic and other solutions to distribution.¹

¹There is now a large literature on economic intergenerational models involving the price system and various extensions of the pure consumption loan model. As this paper does not deal with the price system we defer a discussion of this literature until a subsequent publication.

2. MULTISTAGE VARIABLE PERSON GAMES

Population

Consider a game with individuals who live only at most M time periods. Each individual is strategically active only at most from age m_1 to m_2 . From birth to the end of $m_1 - 1$ the individual is a child and must be supported if it is to survive. From age $m_2 + 1$ to M the individual is deemed to be non-productive in an economic sense and beyond the zone of childbearing and raising.

Suppose that the individual reproduces at m_3 where $m_1 \leq m_3 \leq m_2$. The individual has one offspring at precisely the age m_3 and this is endogenously determined. The offspring is a precise replication and lives for at most M periods. Many new more realistic phenomena appear when we introduce uncertainty concerning length of life, birth of children, health and length and quality of productivity. These are suppressed at this time. Furthermore it would be far more realistic to introduce two sexes and the size of family as an endogenous variable. For the prime purpose of this paper, the first look at generational interlinkage can be achieved without these extra realistic but complicating features.

As can be seen from the above, population will at best remain stationary, but there is a chance of early death if not enough sustenance is obtained, and if a sufficient number of individuals die before producing offspring the population will die out.

A digression on "one or many"

In the subsequent mathematical models in this paper the most rigorous way of proceeding without considerable elaboration is to consider that there is one representative individual in each generation. Thus a game involving a set of N generations is represented by n players (where

n is the number of generations). It is asserted but not rigorously formulated here that all the results noted would equally well apply to a formal model where the single individual per generation is replaced by many identical individuals (i.e., a continuum). This statement will not hold true when formal economic markets are introduced. (A justification for using a price system depends explicitly upon the existence of many competitors in a decentralized economy.)

Production and consumption

We consider one general aggregate consumption good (food) and one capital good (land). At time t an individual of age m and generation g obtains an amount of food:

$$(1) \quad g_{mt}^a(g_{c_1}^c, g_{c_2}^c, \dots, g_{c_m}^c, g_{k_{mt}}^k) > 0 \quad \text{if } m_1 \leq m \leq m_2 \\ = 0 \quad \text{otherwise.}$$

Let $g_{c_j}^c$ = the amount consumed by an individual of age j and generation g .
 $g_{k_{mt}}^k$ = the amount of land held at time t by an individual of age m and generation g .

This form indicates that current "earnings" or productivity are a function of previous nurture and current ownership of land (or capital).

We assume that food can be transferred between individuals but cannot be stored beyond a single period.

Survival

We assume that there is a minimum positive subsistence level below which an individual dies within a period. Let this level be D .

Ownership and inheritance

The only durable considered here is "land." The meaning of ownership of land is deeply rooted in the interpretations of the law and the customs and power of society and the polity. Here we follow a relatively straightforward simplification which is that society is organized so that the young may own but have no strategic freedom to utilize or dispose of land until of age. The old have strategic freedom to dispose of their land as they see fit.

As soon as a durable is introduced into a model in which individuals die, a strategic game cannot be defined unless *inheritance rules* are specified. The simplest are that the closest surviving offspring inherits all. If an individual dies without heirs in this rudimentary society we might assume that the state or the king inherits all--but this requires that we either let land disappear from the system or model the activities of the state. As a simple way to get around this we could alternatively assume that any estate without heirs is split up evenly among all.

2.1. Moves and Strategies

As our concern is with intergenerational transfers the scope of the strategy of an individual is as follows:

From birth until age m_1 the individual has no strategic freedom.

From m_1 until age m_2 the individual decides upon intergenerational transfers to offspring and parent.

From m_2 until M the individual "earns" or obtains no food directly but may possibly control land and may transfer it.

At age m where $m_1 \leq m \leq m_2$ an individual of generation g has a move which consists of at most four nonnegative numbers they are g_m^x , g_m^{x-1} and g_m^y , g_m^{y-1} which stand respectively for the amount

of food given by an individual of age m and generation g to generations $g+1$ and $g-1$ (denoted by superscript 1 and -1), and the amount of land given to generations $g+1$ and $g-1$.

At age m where $m_2 < m \leq M$ an individual has a move which consists of at most two numbers y_m^1 , y_m^2 . Land can be given directly to children or grandchildren.

Strategy and information

A strategy is a complete policy or plan. It depends upon the amount of information available to the individual, thus it will be a function of the contingencies perceived by the individual. The simplest strategy possible for an individual who lives the full life span of M will contain $4(m_2 - m_1 + 1) + 2(M - m_2 + 1)$ numbers.

The general strategy set of an individual of generation g is denoted by S_g and a specific strategy by $s_g \in S_g$.

2.2. Outcomes, Utility Functions and Payoffs

The *outcome* from the interaction of the strategies of all individuals will be a set of consumption levels for each period and a terminal amount of assets. The *utility* or preferences of an individual will depend upon his own outcomes if we adopt a strictly individualistic view; or it may depend upon outcomes to others as well if we consider a more altruistic concern. The *payoff function* for each individual is a function which relates strategies of all to outcomes and hence to final utility.

2.3. Starting and Ending Conditions

When trying to model the flow of generations it hardly seems reasonable to have a first and a last generation especially if we are trying to portray conditions for the existence of a stationary state.

One may ask why should one even be bothered with the stationary state. The answer provided here is that it provides the simplest step towards full dynamics and that as such it is a reasonable starting point for analysis.

The method adopted here is that a satisfactory way to approach problems of the infinite horizon is to be able to set up a finite model which can then be examined for a lengthening horizon with the goal being to establish that the solutions to the series of limiting models approach some smooth limit.

In order to set up the multigenerational model described here as a finite n person game we must provide some fairly complex initial and final conditions. In particular the set of individuals who are alive at the start of time may include some middleaged and old who were alive "before the start of time." The "prehistorical" outcomes to them must be specified as parameters of the model. If we decide to "end the game" after the death of the youngest member of the n^{th} generation; at his time of death there will be many members of the next generation or two alive. Their strategies "beyond the end of time," as long as they were alive before the end of time might be relevant to the equilibrium conditions. These somewhat cryptic statements will be made precise in the specific games with $M = 3$ which are presented below.

3. MULTISTAGE GAMES WITH THREE PERIOD LIVES

We formulate in full detail several models with three period lives. In particular we set the following values:

- $M = 3$ maximum length of life
- $m_1 = 1$, $m_2 = 2$ the range of productivity, i.e., the second period offspring are born at the start of the age of productivity.

The three period life with offspring at period two enables us to be particularly parsimonious on notation as a new generation appears every period.

Samuelson's model was implicitly a three period life model where he dropped the first period by a quick assumption that child support was to be purely instinctive and hence not in the analysis.

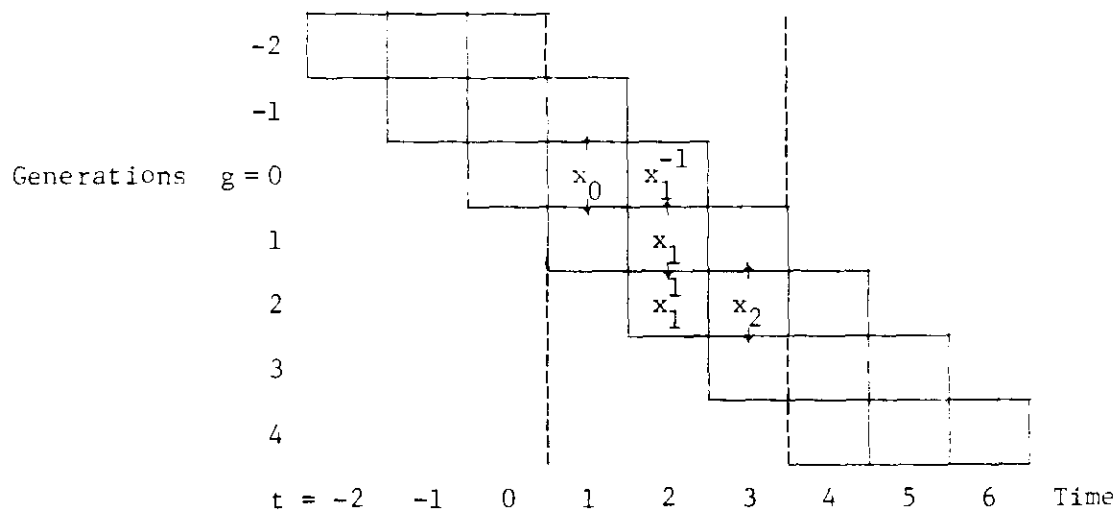


FIGURE 1

In Samuelson's model there was only one nondurable consumer good supplied exogenously to individuals aged two periods old. Here we have introduced a durable good as well because we believe that an important essence of socio and politico-economic relationships is that capital goods

plus law provide the complex of escrow, threat and hostage relationships needed to enforce cooperation in an environment that is not intrinsically cooperative. However both for comparison and contrast we consider the cases where the supply of the capital good is 0 or 1.

Starting with the case where the amount of capital is 0 we simplify along the lines of Samuelson and assume that 1 unit of the consumer good is earned by any individual who survives to age two. This obliterates the effects of nutrition, education, etc. upon productivity.

$$x_{gt}^a = a = 1 .$$

The move of generation g can be described as a distribution x_g^1, x_g, x_g^{-1} where

$$(2) \quad x_g^1 + x_g + x_g^{-1} = 1 .$$

Subsistence level D is set at some number considerably less than 1.

In Figure 1 we illustrate the game for periods $t = 1, 2, 3$, this encompasses the full life of generation 1 whose parent's (generation 0) strategy may be contingent upon his parent's move (generation -1) in period -1. If we assume that the formulation of an individual's strategy is at most dependent upon the actions of those who were alive in the life time of his parent we need go only two periods back before $t = 1$. Initial conditions require that the moves of generations -1 and -2 be specified in advance.

Similarly we will require conditions to be specified for $t = 4$ and 5; but the terminal conditions may be somewhat more complicated than the initial conditions because the strategy of generation g at equilibrium may be contingent upon the strategy of generation $g+1$. Thus for terminal

conditions we need the strategies of generations 3 and 4 as data.

Given the moves of generations -1 and -2 and the full strategies of generations 3 and 4 the test for a stationary state is, using these as data will there exist a noncooperative equilibrium in the 3 person game played by generations 0, 1 and 2 which has the same strategies as those assigned to 3 and 4 and produces moves that are the same as those assigned to -1 and -2.

The consumption of generation g of age i is given by ${}_g c_1, {}_g c_2, {}_g c_3$ where in this simple example

$$(3) \quad {}_g c_1 = x_{g-1}^1, \quad {}_g c_2 = x_g \quad \text{and} \quad {}_g c_3 = x_{g+1}^{-1}.$$

3.1. Utility Functions

The utility function of an individual of generation g is given by a concave function

$$(4) \quad U({}_g c_1, {}_g c_2, {}_g c_3) \quad (\text{individualistic})$$

if he is modeled as a purely self centered individual.

We may argue that there is some index of love, altruism or concern between the generations. The simplest way to introduce this is to introduce a coefficient of concern or sympathy between the generations. This type of link was suggested by Edgeworth (1881).

Several problems in modeling must be faced:

- (i) How many generations should be linked?
- (ii) Should the linkage go in one direction, down the generational tree, or should it go both ways?

- (iii) Should the measure of the linkage be in terms of direct transfer of resources, or in terms of the donor's perception of the satisfaction or welfare of the donee?

Even casual empiricism indicates that personal concern for "the generations unborn" while a splendid political phrase has little if any operational content beyond the level of grandchildren. An important key distinction for direct concern is existence. Humans appear to be the only species¹ that has the concept of grandchild and although there may have been five or even six generations alive at the same time, direct interaction is at most considerable only between parent and child; even in the extended family it is far less between grandparent and child, and beyond that hardly of significance. The approximation suggested here is that only proximate generations need to be linked. For a finer relationship we might wish to include the concern of grandparents for grandchildren.

The linkage of concern appears to go heavily in one direction, as considerations for the survival of the species would indicate. The support of elderly and weak parents or grandparents calls for claims on resources which could be used on the young and vigorous.

"You will eat your spinach because it is good for you," and "my happiness is in seeing you happy" present two different views of interpersonal concern, the first involves concern with what the young get, not particularly how they like it. The second is with the preferences of others.

We now sketch some forms for the utility functions reflecting these various conditions.

¹Possibly baboons or other primates?

Proximate generation linked utility functions

The individual's utility function contains the consumption of young offspring and elderly parents as arguments

$$(5a) \quad U(g_{-1}^c, g^c, g^c, g^c, g_{+1}^c)$$

We might wish to specialize this structure by introducing two coefficients of concern or sympathy modifying (5a) to:

$$(5b) \quad U(g^c, g^c, g^c) + \eta g_{-1}^c + \theta g_{+1}^c$$

where η = coefficient of concern of offspring for parent

θ = coefficient of concern of parent for offspring.

Forms (5a) and (5b) portray the "eat your spinach," or "make sure they are fed" type of concern. Utility functions expressing preferences for the preferences of others are difficult to operationalize and can give rise to infinite regresses. One way of defining such a function using the separability suggested in (5b) is as follows:

$$(5c) \quad \phi_g = U(g^c, g^c, g^c) + \eta U_{g-1}(g_{-1}^c, g_{-1}^c, g_{-1}^c) \\ + \theta U_{g+1}(g_{+1}^c, g_{+1}^c, g_{+1}^c) .$$

In (5c) the utility function of g is broken into three separable parts, the first depends only upon his lifetime consumption; the next upon his parent's preference for his lifetime's consumption; here the first two arguments are data. The third component is the offspring's preference for his lifetime consumption. The last two components have to be estimated as they will be evaluated after the decisions of generation g .

If we assume that all generations have the same preferences then $\phi_g = \phi$ and if furthermore $\tau = \theta = 1$ then

$$(5d) \quad \phi = U(g^c_1, g^c_2, g^c_3) + U(g^{-1}\bar{c}_1, g^{-1}\bar{c}_2, g^{-1}c_3) \\ + U(g^{+1}c_1, g^{+1}\hat{c}_2, g^{+1}\hat{c}_3)$$

and the successive generations act as if they were a single individual.

3.2. Extensive Form and Strategies

In order to draw a full game tree for illustration we limit the moves of each generation to three alternatives 1, (0,1,0), 2, (1/2, 1/2, 0), and 3, (1/3, 1/3, 1/3) rather than the continuum. We assume $D < 1/3$.

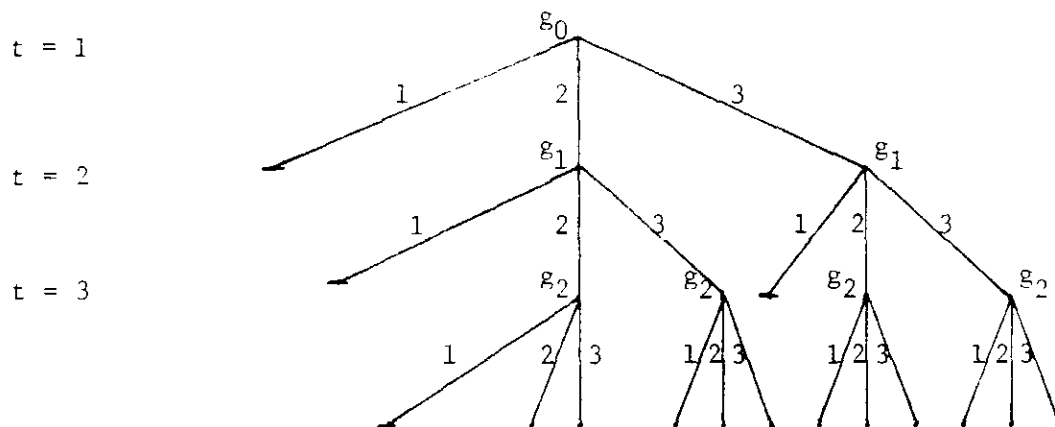


FIGURE 2

In Figure 2 no information sets are indicated implying that this is viewed as a game with perfect information, i.e. at any stage all previous history is known to all. Thus in this simple example viewed as a finite three stage three player game g_0 has 3 strategies; g_1 has $3^2 = 9$ strategies and g_3 has $3^4 = 81$ strategies. g_3 can recognize 4 positions at which he has 3 choices hence he has $3^4 = 81$ strategies.

In order to capture the effect of continuing generations we need

the strategic interlinkage to look the same from any generation, thus we need at least the moves of generations -1 and -2 as data if generation 0 is to have a strategy set of 81 elements.

Similarly in order to be able to calculate the full payoffs to 1 and 2 we must be given the strategies for 3 and 4 as data. The example calculated in Section 4.3 will fully specify why this extra information is used and how it is used in calculating the payoffs which are not indicated at the bottom of the game tree in Figure 2.

4. SOLUTIONS TO MULTISTAGE GAMES WITH THREE PERIOD LIVES

4.1. Historical Strategies, Threats, Aggregation and Information

An important contribution of the formal theory of games to an understanding of the logical possibilities of strategic behavior is made in the treatment of information and strategy. Even for a simple matrix game played more than once, as information is made available the domain of the strategy sets expands enormously. A simple example of a 3×3 matrix game played twice serves to illustrate both the expansion of strategies and the concept of threat.

	1	2	3
1	5,5	-5,10	-10,-5
2	10,-5	0,0	-10,-5
3	-5,-10	-5,-10	-20,-20

TABLE 1

The game shown in Table 1 has three strategies for each of two players. A strategy (which here is the same as a move) for each player is to pick a number $i = 1, 2, 3$ or $j = 1, 2, 3$. Suppose that the game is played

twice and after the first play both players are informed of each other's choice in the play. Each player can now recognize 9 positions $(1,1), (1,2), \dots, (3,3)$ at each of these positions he has 3 alternatives thus the number of different strategies available to each is $3^9 = 19,583$. A strategy will contain 9 contingent clauses; more completely it is a number (the first move) followed by a function on the 9 information sets. In total there are $3 \cdot 3^9$ strategies but many are redundant--for example if Player 1 chooses 1 for his first move he really does not need to plan for a position of $(2,2)$ for his second move, as his action has ruled it out.

Two examples illustrate qualitatively different strategies.

The first is the *stationary* or *Markovian* strategy. Whenever you are in the same subgame do the same thing. There are only 3 strategies of this type:

Select i , $i = 1, 2, 3$ then regardless of (i,j)
 select $k = i$.

The second is a historical strategy which in certain contexts can be reasonably well interpreted as a threat. Consider the following strategy:

Select $i = 1$, if he selects $j = 1$ select $k = 2$
 " " " $j \neq 1$ " $k = 3$

A glance at Table 1 shows that $k = 3$ is the punishment for not cooperating by setting $j = 1$.

In the example above there are only two players and a threat strategy by one is clearly aimed at the other. In large societies it merits distinguishing between personal and impersonal threats. In a large society,

at a personal level involving possibly high information and communication the individual recognizes family and acquaintances and "the rest of the world" where the rest of the world's behavior and identity is aggregated in some manner. It is possible to distinguish strategies where whole sets of individuals are willing to "punish" any individual, not necessarily identified by name, but only be action if that individual "steps out of line" (Shubik, 1959).

4.2. The Noncooperative Equilibrium

An n-person game has an equilibrium point in pure strategies denoted by a set of n strategies $(s_1^*, s_2^*, \dots, s_n^*)$ if the following holds true. Let the strategy set for player i be denoted by S_i and an individual strategy by $s_i \in S_i$ then for all i

$$(6) \quad \max_{s_i \in S_i} f_i(s_1^*, \dots, s_{i-1}^*, s_i, s_{i+1}^*, \dots, s_n^*) \implies s_i = s_i^* .$$

In the game shown in Table 1 it should be clear that the strategies for each player "play i = 2 each time, regardless" form an equilibrium point, with payoffs of 0 to each. Less obvious is the fact that the threat strategies, if used by both players also form an equilibrium point. That is "play i = 1, if competitor replies with 1 then use 2; otherwise 3." This yields 5 for each as the payoffs.

Although the second equilibrium yields a higher payoff than the first, it is more complicated.

4.3. Selfish Individuals and Threat Strategies (Model 1)

In this section an example is fully specified and solved to show the possibility of a stationary state existing via threat. This should also serve to complete the discussion of Figure 2 in Section 3.2.

We consider that an individual of generation g has an individualistic utility function of the type indicated in (4). The conditions for positive marginal utility for each period's consumption are assumed together with decreasing marginal utility for additional consumption. We add the condition that consumption at less than survival level¹ $D \leq 1/3$ is highly undesirable. In particular the only further conditions imposed are that:

$$(7) \quad U(1,1,0) < \max U(c_1, c_2, c_3)$$

where $c_1 + c_2 + c_3 = 1$,

and that the maximum on the righthand of (7) is achieved at three numbers c_1^* , c_2^* , c_3^* all greater than or equal to D .

A strategy by an individual of generation g is a function of the information he has on the moves of the two previous generations. In particular we display a specific strategy below, interpret it and show that if all individuals adopt this strategy, the resultant set of strategies form an equilibrium.

A strategy by an individual of generation g is as follows:

¹We need the condition $D \leq 1/3$ for our result as otherwise $3D > 1$ would imply it would not be possible for the society to survive keeping the old.

$$(8) \quad \text{Set } x_g^1 = c_1^*, \quad x_g = c_2^*, \quad x_g^{-1} = c_3^* \quad \text{if information reveals that:}$$

$$x_{g-1}^1 \geq c_1^* \quad \text{and} \quad x_{g-1}^{-1} \geq c_3^* .$$

If however $x_{g-1}^1 < c_1^*$ or $x_{g-1}^{-1} < c_3^*$ then set

$$x_g^1 = c_1^*, \quad x_g = c_2^* + c_3^*, \quad x_g^{-1} = 0 .$$

In words this states that g will feed his parent and offspring in the manner noted on the right side of equation (7) if he observes that his parent did that to him and his grandparent. If his parent failed to do so he lets him starve.

We are given the following initial conditions in this game shown in Figures 1 and 2.

$$x_{-2}^1 = c_1^*, \quad x_{-1}^1 = c_1^*, \quad x_{-1}^{-1} = c_3^* .$$

We are also given as data the strategies to be employed by generations $g = 3$ and 4 . We assume they both employ the strategy set forth in (8). Without this information it would not be possible for generations 1 and 2, who are active players in this game, to calculate their payoffs.

Denote a strategy employed by an individual of generation g by s_g then we may write the payoffs in this particular game with three strategic players as follows:

$$(9) \quad \begin{aligned} \Pi_0(s_0, s_1) &= U(0^{c_1}, 0^{c_2}, 0^{c_3}) \\ \Pi_1(s_0, s_1, s_2) &= U(1^{c_1}, 1^{c_2}, 1^{c_3}) \\ \Pi_2(s_1, s_2) &= U(2^{c_1}, 2^{c_2}, 2^{c_3}) . \end{aligned}$$

We may now check to see that if all use the strategy given by (8) the system is in a stationary equilibrium.

Generation 0 has already been sustained in childhood by -1 at the level ${}_0c_1 = c_1^*$. He faces a strategy of the type (8) played by generation 1. If he fails to conform in setting $x \geq c_1^*$ and $x \geq c_3^*$ then the most he can obtain is $(c_1^*, 1, 0)$ but

$$U(c_1^*, 1, 0) < U(c_1^*, c_2^*, c_3^*)$$

hence he is in equilibrium if he sticks to the strategy in (8).

We may now apply the same argument to generation 1 and then generation 2.

It is straightforward to extend this argument for any arbitrary number of generations.

4.4. Selfish Individuals and Simplistic Strategies: Not Enough Glue (Model 2)

The simplest of strategies are those which ignore all information thus we could restrict the strategies to be equivalent to moves. A strategy by generation g is a triad of numbers

$$(x_g^1, x_g, x_g^{-1}) \text{ such that } x_g^1 + x_g + x_g^{-1} = 1.$$

The only information needed from the past to initialize any game in which these types of strategies are employed is that $x_{-1}^1 \geq D$, i.e. generation 0 survived childhood so that it is alive and strategically active at $t = 1$. Suppose that $x_{-1}^1 = k_1 \geq D$, i.e. some number above the survival level, then it is simple to show that no stationary equilibrium exists. We need only check generation 0. Suppose that there were a stationary equilibrium which would require that $x_1^{-1} = k_2 \geq D = x_0^{-1}$ if the old were to survive. This cannot be because

$$(10) \quad U(k_1, 1 - k_1, k_2) > U(k_1, 1 - k_1 - k_2, k_2) .$$

In words, if generation 0 has been raised by -1 and knows that if it raises generation 1 it will obtain k_2 regardless of its strategy it can gain by failing to support its parent as is shown in (10). Thus if a stationary strategy were to exist it would not include support of the old. Suppose $x_1^{-1} = x_0^{-1} = 0$. If generation 0 is selfish and knows that generation 1 does not intend to provide support in its old age then the optimal strategy will be to set $x_0^1 = 0$ as

$$(11) \quad U(k_1, 1, 0) > U(k_1, 1 - k_1, 0) .$$

This means that generation 1 will not survive childhood and the species will end if noncontingent strategies are employed.

4.5. Linkage by Coding or Instinctive Behavior (Model 3)

When there is enough food around most animals instinctively feed their young. This can be modeled here by removing the selection of x_g^1 as part of the strategy set and setting it to some number $k_1 \geq D$. If we do this it is simple to check that there will be a stationary non-cooperative equilibrium with $x_g = 1 - k_1$ and $x_g^{-1} = 0$, each individual will obtain

$$(12) \quad U(k_1, 1 - k_1, 0) < U(c_1^*, c_2^*, c_3^*) .$$

This is clearly not Pareto optimal, thus instinct to support the young does not necessarily help the survival of the old.

4.6. Linkage by Love: The Altruism Finesse (Model 4)

Suppose that we assume that an individual has a utility function of the shape indicated in (5d), i.e. generation g 's concern for its parent and offspring is as for itself. Equations (9) now become

$$\begin{aligned}
 \Pi_0(s_1, s_2) &= U(-1^{c_1}, -1^{c_2}, -1^{c_3}) + U(0^{c_1}, 0^{c_2}, 0^{c_3}) \\
 &\quad + U(1^{c_1}, 1^{c_2}, 1^{c_3}) \\
 (13) \quad \Pi_1(s_0, s_1, s_2) &= U(0^{c_1}, 0^{c_2}, 0^{c_3}) + U(1^{c_1}, 1^{c_2}, 1^{c_3}) \\
 &\quad + U(2^{c_1}, 2^{c_2}, 2^{c_3}) \\
 \Pi_2(s_1, s_2) &= U(1^{c_1}, 1^{c_2}, 1^{c_3}) + U(2^{c_1}, 2^{c_2}, 2^{c_3}) \\
 &\quad + U(3^{c_1}, 3^{c_2}, 3^{c_3}) .
 \end{aligned}$$

Because of the overlap in the arguments of the utility functions it is easy to check that there will be a Pareto optimal stationary non-cooperative equilibrium with simplistic strategies. We check by assuming that all employ $x_g^1 = c_1^*$, $x_g = c_2^*$, $x_g^{-1} = c_3^*$.

Consider generation 0 given that $x_{-2}^1 = c_1^*$, $x_{-1} = c_2^*$, $x_{-1}^1 = c_1^*$ then:

$$(14) \quad \Pi_0(s_0, s_1^*) = U(c_1^*, c_2^*, x_0^{-1}) + U(c_1^*, x_0, c_3^*) + U(x_0^1, c_2^*, c_3^*) .$$

An optimal strategy for generation 0, given information concerning what -1 has done and knowing the strategy s_1^* of generation 1 is one that maximizes $\Pi_0(s_0, s_1^*)$. A strategy here is to select three numbers

$$(x_g^1, x_g, x_g^{-1}) \text{ subject to } x_g^1 + x_g + x_g^{-1} = 1 .$$

From the fact that U is concave and has a maximum at (c_1^*, c_2^*, c_3^*) when

the consumption is constrained to sum to 1, it follows that $\Pi_0(s_0, s_1^*)$ has a maximum at:

$$x_g^1 = c_1^* , \quad x_g = c_2^* , \quad x_g^{-1} = c_3^* .$$

We may now consider $\Pi_1(s_0^*, s_1, s_2^*)$ and show $s_1 = s_1^*$ is in equilibrium, and then consider $\Pi_2(s_1^*, s_2)$.

It should be noted that the assumption of enough love turns a "game into a team." Furthermore if love were only of parent for child this would be enough to achieve a stationary state, but not one with the Pareto optimal survival of the old enforced by simplistic strategies. More complex strategies of the variety in Model 1 (Section 4.3) will do.

An important reason for the "love or altruism" model to be attractive to economists with a noninstitutional viewpoint is that enough love minimizes the need for institutions, laws, rules, or any social or political paraphernalia. The need for a systems design of any complexity to produce optimal behavior from individuals with self interest and different goals vanishes if each cares for the other as himself.

It is suggested here that the coefficients of concern or sympathy are not symmetric. The prime concern of the parent is for the child; the secondary concern is with parents and grandchildren in a Western society with stress on the nuclear family. How much this concern is biologically, sociologically or economically determined appears to be an open question as great variety is seen among different nations, tribes and other social groupings.

5. THE ROLE OF ASSETS

Explicitly modern economic and political models to account for inter-generational linkages are not dealt with in this paper. Fiat money, taxation, social security and the financial and political infrastructure of the modern state attest to the sophistication of the rules of the game and the importance of law and behind law power to enforce the rules.

The belief expressed here is that the existence of assets and power to enforce laws concerning property are more elementally central to the process of transfer than is the existence of commodity or fiat money. The world of paper provides a more convenient way of manifesting and executing transactions involving property and power. Money and markets come after, not before the state, law and property.

The last model considered here introduces a single asset, "land" into our economy in which markets and money have not been described. It must be noted that the market transfer of resources among nonrelated (or even related) individuals is not inconsistent with nonmarket methods for intergenerational transfer. Here we are ignoring the former.

5.1. Instinct, Simplistic Strategies and Owned Land (Model 5)

In (1) and Section 2 a general relation was given linking the amount of food directly obtained by an individual of age m and generation g to previous consumption and ownership of land. Implicit in the description was a production function depending on the amount of land used and the quality of unspecified labor determined by previous consumption.

Rather than fully develop the production function model here we select the simplest condition on production involving land. We assume that there is 1 unit of land which is of unlimited durability and is initially owned by generation -1 who is the first old person alive at time $t = 1$ (see

Figure 1). The conditions on production of food are, for the three period life model

$$(15) \quad g_{mt}^a = 1 \quad \text{if } m = 2 \quad \text{and } k \geq 1 \\ = 0 \quad \text{otherwise.}$$

The difference between (15) and the condition in the previous models is that here only the middleaged are productive, but they need the use of land before they can realize their productivity.

As in Model 1 the utility function is assumed to be

$$U(g^c_1, g^c_2, g^c_3) .$$

Consideration is limited to simplistic strategies, but before a strategy can even be specified involving the transfer of land, some further specification concerning ownership must be given as the game of strategy is not yet fully specified. In particular as has already been noted in Section 3 inheritance conditions must be specified, but so do guardianship use and transfer conditions.

Inheritance. The possessions of an individual on death pass to the nearest surviving relative. If none survive they pass to the state, which we may assume divides and distributes them equally to all. Inheritances cannot be left to the unborn.¹

Ownership. An individual of age who owns land has during his life full protection of his ownership. The land or its use cannot be forcibly taken from him. He may give away or sell his land, or give away or rent its use.

¹This poses a question in well defining the rules of the game as to whether age should be counted from point of conception or birth.

Trusteeship. Land owned by a minor is administered in trusteeship by the parent for the good of the minor and passes to his control on coming of age. In this simple three period life model no further elaboration is needed as the parent is bound to be alive (starving himself to death is feasible but not individually rational).

We must give an operational interpretation to the phrase "for the good of the minor." The simplest is that law or custom or both require that the parent must at least provide the minor with sustenance, i.e., $x_g^1 \geq D$. We assume A is provided by instinctive behavior (Model 3), where $A \geq D$; hence instinct and trusteeship cannot be distinguished at this level.

Transfer and Use. It is assumed that if g gives $g+1$ land, $g+1$ is required to accept it, but can give it to the state (a form of the assumption of free disposal). g may permit $g-1$ the use of land that g owns; but in general this is done by means of a contract or bilateral agreement which does not easily lend itself to a simple formulation as a noncooperative game. It is more reasonable to consider contract in the terms of Edgeworth (1881) or a cooperative solution. Our problem modeling this as a noncooperative game will not be the existence of noncooperative equilibria, but with too many of them. In fact any point on the contract curve between generation g offering¹ D to $1-D$ units of food in exchange for the use of land will be in equilibrium and any point between D and $1-2D$ will be consistent with a stationary equilibrium. Although

¹The selection of D rather than 0 as the lower bound requires an extra but somewhat "fussy" assumption. We must assume that if generation g offers $g-1$ less than D for the use of his land even though $g-1$ dies, g does not inherit the land until one period later and cannot use it unless $g-1$ accepted a contract for less than D . If we further assume that $g-1$ is not indifferent to letting g use his land for less than D , but would prefer to deny the use if he were going to starve anyhow, then the range of equilibria would start at D not 0.

this leaves a great amount of indeterminacy it is noted here that in a more complicated and more realistic model with many individuals of each age group then we could consider a market for land rentals and this would narrow the range of contract.

If we attempted to write down the notation for a simplistic strategy for even this three period life model with the rental, lease¹ or gift of land this would require 13 components:

3 components for distribution	x_g^1, x_g, x_g^{-1}	at time	$g+1$
2 components for gift of land to	$g-1$ or $g+1$	at time	$g+1$
2 components for lease of land to	$g-1$ or $g+1$	at time	$g+1$
2 components for rent of land from	$g-1$ or $g+1$	at time	$g+1$
2 components for gift of land to	$g+1$ or $g+2$	at time	$g+2$
2 components for lease of land to	$g+1$ or $g+2$	at time	$g+2$
<hr/>			
13			

We confine our notation to the case where an individual of age 3 owns the land and show an equilibrium with rental by the individual of age 2 with inheritance and no gift.

Let y_{g-1}^1 stand for the contract offered by $g-1$ to g for a one period rental of one unit of land in return for an amount of food y_{g-1}^1 or more.

Let z_g^{-1} stand for the amount of food offered by g to $g-1$ for a one period rental of one unit of land.

If $z_g^{-1} \geq y_{g-1}^1$ the land is rented, otherwise there is no contract.

¹Another minor fussy detail is that a lease given by someone of age 3 to his offspring of age 2 is constructively a sale if 2 lives, as he will inherit the land at the end of the lease.

A simplistic strategy by generation g is a collection of 5 numbers

$$(x_g^1, x_g, x_g^{-1}, z_g^{-1}, y_g^1)$$

where $x_g^1 + x_g + x_g^{-1} + z_g^{-1} = 1$.

The first 4 numbers symbolize moves at age 2, the last at age 3.

It is straightforward to show that there are stationary equilibria anywhere from:

$$(16) \quad (D, 1-2D, 0, D, D) \text{ to } (D, D, 0, 1-2D, 1-2D) .$$

Instinct keeps the young alive with D ; and property protects the old within contract limits of D up to $1-2D$. The presence of a mass market for rented land would narrow the range, but even with all the apparatus erected there is no mechanism here which gets the young the amount c_1^* if $c_1^* > D$.

A threat model of the variety of Model 1 could easily be constructed with a Pareto optimal noncooperative equilibrium. But as the young have no strategies or markets to operate in perhaps a better measure of optimality should exclude their preferences until they are strategically active thus instead of considering:

$$U(c_1, c_2, c_3) \text{ we consider } \hat{U}(c_2, c_3) = U(A, c_2, c_3) ,$$

where $A \geq D$ is the customary or instinctive level of child support.

If we accepted this and defined the relevant optimality on \hat{U} then all of the equilibrium points in (16) are Pareto optimal.

6. CONCLUDING REMARKS

Long run economics must take into account the multigenerational problem. At least to the level of healthy survival the bringing of the young to maturity can be assumed to be genetically coded or instinctive. The support of the old does not appear to be such. If society is rich and has law, to some extent the old can protect themselves if they control assets (Model 5). If society is sophisticated and strategies dependent on history are utilized then a historical strategy can be interpreted as a threat or a custom which can enforce optimal behavior (Model 1).

Markets (which have not been modeled in this paper) appear to be better suited to the distribution of goods and services among more or less strategically equally active agents than among the generations.

It is at the intergenerational interfaces where genetic, social, legal and political considerations vie with the economic. The long term economic goals cannot even be defined unless the generations are connected together. Yet these connections may not be primarily economic.

Paradoxically as was observed in model 4, the economist can claim the whole of long term social process for himself by postulating enough love. Love seals the links and leaves only the economics of optimization unsullied by institutional arrangements. Unfortunately even casual empiricism would seem to indicate that there is not enough love available in actuality.

Having suggested that there are many ways to glue the generations together the next step is to explore which ways are the most likely and what features of society, polity and the economy are involved in their selection. The mechanisms appear to be of mixed origin. In particular, genetic coding appears to cover the survival of the young; love and custom their extra nurture; economics to some extent their education and use of

child labor. The genetic basis for the support of the old seems to be flimsy at best (Dawkins, 1974). Love, culture and politics appear to dominate. The mechanisms, but not the driving forces appear to be manifested considerably in public and private financial institutions and instruments.

The emphasis has been upon the stationary state, merely as a reasonably simple starting point. However the type of game proposed in Section 2 can be examined more generally. In particular it must be stressed that the type of equilibria we examine depend on the amount of history included in the strategies. This has nothing to do logically with either temporary equilibrium analysis or limited rationality. For finite games in extensive form Dubey and Shubik (1981) have proved that as the amount of information utilized by strategies is increased the set of pure strategy equilibria may grow including all equilibria using less information. The attraction of noncooperative equilibria using little information is that they are simple and can be interpreted as though they are a manifestation of limited rationality although this is not necessarily the case.

Full justice to the interpretation of the threat possibilities of historical strategies cannot be done without the careful modeling of large numbers of individuals in each generation. The specific mathematical models presented here have had only one member in each generation.

Exogenous uncertainty has not been considered; paradoxically the presence of exogenous uncertainty provides extra incentives for cementing the generations together inasmuch as the needs for insurance call for the carrying of extra capital stock.

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