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ABSTRACT

DUOPOLY WITH DIFFERENTIATED PRODUCTS AND ENTRY BARRIERS*

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Product differentiated duopoly with a potential entrant facing a single period fixed cost entry barriers is modeled as a noncooperative game. In addition to characterizing the equilibrium solutions and relating them to entry costs and product differentiation, a comparison of price and quantity competition shows that entry conditions are qualitatively sensitive to the strategic variables used in a given industry. Quantity competition appears to be more favorable for entry than price competition. The use of threats and other exclusionary tactics, such as limit pricing, decisively determine the outcome when entry costs are moderate.

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I. Introduction

It is well-known since Bain [1], that established firms with product differentiation, scale economic, or cost difference advantages over potential entrants may impede entry by limit pricing. Motivated by the limit pricing hypothesis, Kamien and Schwartz [9], Gaskins [8], and others, under various assumptions, have determined how a group of colluding oligopolists may optimally control a fringe of entrants using dynamically generated limit prices. Although these models have contributed to our understanding of some aspects of the entry problem, they have, unfortunately, tended to avoid its truly oligopolistic features: in oligopolistic competition, all firms, potential and established, jointly and strategically control the outcome. Thus, an important question is: given the interlinkages between the strategic and conscious actions of all the firms, what varieties of market structures and behavior may be expected when strategic potential entrants are disadvantaged in one or more of the ways mentioned above?

Recent research on strategic oligopoly with entry are increasingly addressing these issues and providing many useful insights into the problem. Spence [13] has demonstrated that investments and capacity enhance the ability of an established firm to transmit credible threats to a potential entrant; Nti and Shubik [11] have characterized equilibrium solutions under scale-economic entry barriers; Dixit's [5] results on duopoly also discusses the possibility of multiple equilibria and some aspects of product differentiation; and Friedman [7] has initiated dynamic entry and exit super-game models. A unifying theme runs through all these models: they explicitly consider strategic interlinkages between potential entrants and established firms, and they are game-like.

In this paper, we contribute to the game theoretic or game-like analytic framework for studying the strategic entry problem. We examine two models of product differentiated duopoly with a potential entrant which faces a single period fixed entry cost disadvantages. In the first model, the firms compete with quantity and entry decisions as strategic variables, while in the second, price and entry decisions are the strategic variables.

For each model, noncooperative equilibrium solutions are identified and characterized as functions of entry cost and product differentiation. It is shown that strategic factors alone work to exclude the potential entrant when entry costs are high, and that the industry is routinely penetrated when entry costs are low. However, for intermediate levels of entry costs, there is a multiplicity of equilibrium solutions and the outcome depends decisively on the use of threats and other exclusionary tactics. Limit pricing appears as a special kind of threat strategy. Although several "limit prices" are shown to exist, we only compute symmetric limit prices for both models.

A comparative analysis of the price and quantity models is particularly insightful. The quantity model appears to be more favorable for entry than the price model. Thus, the ease or difficulty or entry depends not only on entry barriers, but also on the strategic variables being employed. That the use of different strategic variables leads to qualitatively different features is well-known from Cournot [4] and Bertrand [2]. What is new is that these characteristics carry over into the entry problem, with profound regulatory and market structure implications. Also, the sensitivity of the models to the parameter of product differentiation are qualitatively different.

The quantity model is formulated and analyzed in Section 2. In Section 3, the price model is analyzed after a brief discussion of some modeling difficulties. A comparison of the two models, as well as their sensitivities to several parameters of the model is presented in Section 4. A discussion on limit prices appears in Section 5. And finally, Section 6 sums up the results and indicates directions for future research.

2. The Quantity Model

Consider two established firms selling differentiated products in an industry with one potential entrant. Upon entry, the entrant also offers a quantity of the differentiated product for sale. The variable costs of all firms are assumed to be linear and they sell their products in a market where the market-clearing price for each firm's product depends on output levels and the level of product differentiation, in a manner to be specified later. Established firms have no fixed costs, but the potential entrant incurs a one-period fixed cost if it enters into competition. Thus, the potential entrant faces a differential cost disadvantage or scale economic barriers arising from, perhaps, a set-up cost associated with activating production or marketing the product. The firms are assumed symmetric in all other respects.

Let the variable cost of the i^{th} firm be $C_i(q_i) = Cq_i$, where C is positive and q_i is the non-negative output of the i^{th} firm, $i = 1, 2, 3$. In the sequel, firms 1 and 2 are the established duopolists, and firm 3 is the potential entrant. Also, let D be the non-negative entry cost of the entrant.

Let the relationship $\phi_i(\cdot)$ determining the price of the product of the i^{th} firm be

$$P_i = \phi(q_i, q_j) = \alpha/\beta - nq_i/(1+\gamma)\beta - \gamma(\sum_{j=1}^{n} q_j)/(1+\gamma)\beta \tag{1}$$

where q_j is the output of the j^{th} firm, n(= 2 or 3) is the number of active firms in the industry, γ is a measure of the substitutability of the products, α is a measure of the absolute size of the market, and β is a positive parameter related to the own price elasticity of demand.

The above demand function is the inverse of the piecewise linear (kinked) demand function used for the price model, and justified in section 3. Using a single demand function permits meaningful comparison of the price and quantity models. It should be noted, however, that as $\gamma \to \infty$ and the products become identical, price simply becomes a linear function of total output.

Now consider the following noncooperative game. Each established firm must select an output level q_i, $i = 1, 2$, while the potential entrant selects a probability of entry δ and a corresponding output level q_3, if it enters. All decisions are made simultaneously and noncooperatively. We wish to identify and interpret the noncooperative equilibrium solutions.

A noncooperative (Nash) equilibrium for the model is a set of strategies q_1, q_2, (δ, q_3) with the property that no active firm has a profit incentive to change production or exit and no firm outside the industry has an incentive to enter.

If the potential entrant selects an entry-production strategy (δ, q_3) and the established firms produce q_1 and q_2, then the expected profits of the firms are given by

$$\pi_1 = (1-\delta)[\alpha/\beta - 2q_1/\beta + \gamma(q_1-q_2)/\beta(1+\gamma) - C]q_1$$

$$+ \delta[\alpha/\beta - 3q_1/\beta + \gamma(2q_1-q_2-q_3)/\beta(1+\gamma) - C]q_1 , \tag{2a}$$

$$\pi_2 = (1-\delta)[\alpha/\beta - 2q_2/\beta + \gamma(q_2-q_1)/\beta(1+\gamma) - C]q_2$$

$$+ \delta[\alpha/\beta - 3q_2/\beta + \gamma(2q_2-q_1-q_3)/\beta(1+\gamma) - C]q_2 , \qquad (2b)$$

$$\pi_3 = \delta[\alpha/\beta - 3q_3/\beta + \gamma(2q_3-q_1-q_2)/\beta(1+\gamma) - C]q_3-D\delta . \qquad (2c)$$

If $\delta = 0$ is part of an equilibrium solution to (2a), (2b), and (2c) then the first order conditions for maximum,[1] $\partial\pi_1/\partial q_1 = \partial\pi_2/\partial q_2 = 0$, imply

$$[\alpha/\beta - 2q_1/\beta + \gamma(q_1-q_2)/\beta(1+\gamma) - C]$$

$$+ [\gamma/\beta(1+\gamma) - 2/\beta]q_1 = 0 \qquad (3a)$$

$$[\alpha/\beta - 2q_2/\beta + \gamma(q_2-q_1)/\beta(1+\gamma) - C]$$

$$+ [\gamma/\beta(1+\gamma) - 2/\beta]q_2 = 0 \qquad (3b)$$

But $\delta = 0$ can be part of an equilibrium solution only if the potential entrant cannot make a positive profit if it enters. With firms 1 and 2 producing q_1 and q_2 respectively, the maximum post-entry profit attainable by the entrant is

$$\pi_3 = [(1+\gamma)(\alpha-C\beta) - \gamma(q_1+q_2)]^2/[2\beta(1+\gamma)(6+2\gamma)] - D$$

Thus, the potential entrant has no positive profit incentive to enter if $\pi_3 \leq 0$; that is, if

$$\gamma(q_1+q_2) \geq (1+\gamma)(\alpha-C\beta) - \sqrt{[2D\beta(1+\gamma)(6+2\gamma)]} . \qquad (3c)$$

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[1] Second order conditions for maximum hold in this and subsequent optimizations.

The unique solution to (3a) and (3b) is

$$q_1 = q_2 = (1+\gamma)(\alpha-\beta C)/(4+3\gamma) \tag{4}$$

which is the classical Cournot duopoly solution for differentiated products. Combining (4) and the non-positive profit incentive condition (3c), we obtain a critical number D_1 of entry cost above which the entrant can be excluded from competition.

$$D \geq (1+\gamma)(4+\gamma)^2(\alpha-\beta C)^2/[2\beta(4+3\gamma)^2(6+2\gamma)^2] = D_1$$

D_1 is related to the total profit in the market.
Hence,

$$\delta = 0,\ q_1 = q_2 = q_{duopoly} \text{ is an equilibrium solution if } D \geq D_1.$$

Similarly, if $\delta = 1$ is part of an equilibrium solution to (2a), (2b) and (2c) then

$$[\alpha/\beta - 3q_1/\beta + \gamma(2q_1-q_2-q_3)/\beta(1+\gamma) - C]$$
$$+ [2\gamma/\beta(1+\gamma) - 3/\beta]q_1 = 0 \tag{5a}$$

$$[\alpha/\beta - 3q_2/\beta + \gamma(2q_2-q_1-q_3)/\beta(1+\gamma) - C]$$
$$+ [2\gamma/\beta(1+\gamma) - 3/\beta]q_2 = 0 \tag{5b}$$

$$[\alpha/\beta - 3q_3/\beta + \gamma(2q_3-q_1-q_2)/\beta(1+\gamma) - C]$$
$$+ [2\gamma/\beta(1+\gamma) - 3/\beta]q_3 = 0 \tag{5c}$$

And the corresponding non-negative profit incentive condition, which ensures that the potential entrant actually enters, is given by

$$\gamma(q_1+q_2) \leq (1+\gamma)(\alpha-C\beta) - \sqrt{[2D\beta(1+\gamma)(6+2\gamma)]} \qquad (5d)$$

The unique solution to (5a), (5b), and (5c) is

$$q_1 = q_2 = q_3 = (1+\gamma)(\alpha-C\beta)/(4\gamma+6) \qquad (6)$$

which is the Cournot triopoly solution. Substitution (6) into (5d) we obtain another critical number D_2 of entry cost below which the entrant can penetrate the industry.

$$D \leq (1+\gamma)(2\gamma+6)(\alpha-\beta C)^2/[2\beta(4\gamma+6)^2] = D_2$$

Hence,

$$\delta = 1,\ q_1 = q_2 = q_3 = q_{triopoly} \text{ is an equilibrium solution}$$

if $D \leq D_2$.

Figure 1 summarizes the results schematically and shows the variation of equilibrium solutions with entry cost.

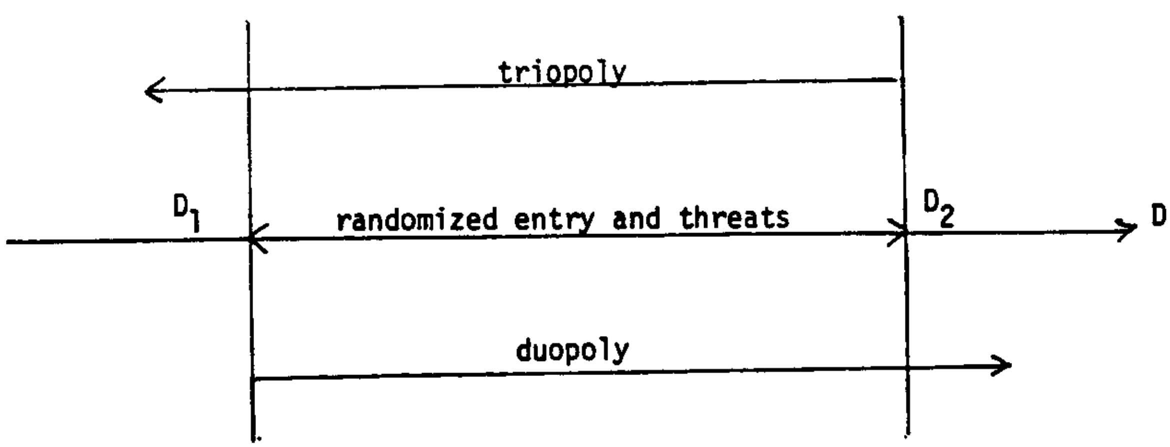

Figure 1. Variation of Equilibrium Solutions with Entry Cost

The critical entry costs D_1 and D_2 satisfy $D_1 < D_2$. And they demarcate the regions of low and high entry barriers. If entry cost is lower than D_1, then the entrant routinely penetrates the industry. But entry is blockaded when entry cost exceeds D_2. However, when entry cost lies between D_1 and D_2 (the intermediate zone), the outcome is uncertain and extremely dependent on the use of threats and other exclusionary tactics.

In the intermediate zone, where entry cost is moderate, both the triopoly and duopoly solutions are enforceable as threat strategies. For example, there is a cowardly duopolists' solutions where the potential entrant makes known its intention to enter at the triopoly level and the established firms, believing that entry is a foregone conclusion, accommodate the entrant. There is also a cowardly entrant's solution where the established firms make known their intentions to maintain a duopoly output and the potential entrant, believing that entry will be fought, stays out. The established firms may also exclude entry by limit pricing, a special kind of threat strategy discussed in section 5.

Finally, there is another equilibrium solution that does not involve the use of threats; all firms fight it out symmetrically. The potential entrant randomizes its entry decision at positive probability $\delta < 1$. That solution satisfies

$$\alpha - \beta C - 4q_1 + \gamma(2q_1-q_2)/(1+\gamma) + \delta[-2q_1 + \gamma(2q_1-q_3)/(1+\gamma)] = 0 \tag{7a}$$

$$\alpha - \beta C - 4q_2 + \gamma(2q_2-q_1)/(1+\gamma) + \delta[-2q_2 + \gamma(2q_2-q_3)/(1+\gamma)] = 0 \tag{7b}$$

$$\alpha - \beta C - 6q_3 + \gamma(4q_3-q_1-q_2)/(1+\gamma) = 0 \tag{7c}$$

$$q_3[\alpha-\beta C - 3q_3 + \gamma(2q_3-q_1-q_2)/(1+\gamma)] - D\beta = 0 \tag{7d}$$

The unique solution to the above system is

$$q_1 = q_2 = [(1+\gamma)(\alpha-\beta C) - (2\gamma+6)q_3]/2\gamma$$

$$q_3 = \sqrt{[(\gamma+1)D\beta/(3+\gamma)]}$$

$$\delta = [(1+\gamma)(\alpha-\beta C)/(4+3\gamma) - q_1]/[(\gamma q_3+2q_1)/(1+\gamma)].$$

The solution satisfies the following inequalities:

$$q_3 < q_{triopoly} ; \tag{8a}$$

$$q_{triopoly} < q_1, q_2 < q_{duopoly} , \tag{8b}$$

$$0 < \delta < 1 . \tag{8c}$$

Hence, if entry is truly randomized then the outputs of the established duopolists lie between the Cournot duopoly and triopoly solutions, and the potential entrant produces less than the Cournot triopoly level if it enters. The asymmetry between the potential entrant and established firms is revealed in the distribution of outputs, with the established firms holding a large market share even if entry occurs. But if entry does not occur, and one views the problem quasi-dynamically, then inventories would be accumulated to enhance the capacity of the established firms to transmit credible threats to future potential entrants (Spence [13]).

3. The Price Model

We now let the firms above compete with price and entry decisions as strategic variables. But before specifying the model, it is useful to make two historical comments on price duopoly.

First, there is the issue of demand conditions under price duopoly: which firm gets which segment of the market if their prices are different? Bertrand [2] assumed, in the case of identical products, that the firm charging the lower price gets the entire market and deduced that an equilibrium would be attained with all firms charging the unit production cost. But upon introducing the possibility of capacity limitations, Edgeworth [6] showed that prices could fluctuate within a range - the Edgeworth cycle - giving rise to a quasi-dynamic price cutting phenomenon and instabilities.

Secondly, there is the issue of the realism of Bertrand's model: there is virtually no market where a price cut (infinitesimal) would lead to a discontinuous market swing in favor of the firm charging the lower price. Allowing for product differentiation eliminates this discontinuity. Shubik [12] has indicated how families of demand functions for price duopoly (or oligopoly) with product differentiation, which includes Sweezy's [14] and Chamberlin's [3] demand curves, may be constructed. Within such a class of demand curves Bertrand-type price noncooperative equilibria exist. Unfortunately, the equilibria may be destroyed if the firms have limited capacity (Levitan and Shubik [10]).

For our model, we will use the simplest price duopoly demand function, which is piecewise linear when the price of the other firm is fixed.

Let the demand, q_i, for the product of the i^{th} firm when it charges a price P_i be

$$q_i = [\alpha - \beta(P_i+\gamma(P_i - \sum_{j=1}^{n} P_j/n))]/n \qquad (9)$$

where P_j is the price charged by the j^{th} firm, and β,γ, $n(= 2 \text{ or } 3)$ are as defined in the previous section. This demand function was inverted for the quantity model (equation (1)). It has the property that as one firm undercuts the other, its share of market increases, not abruptly but at a decreasing rate.

We also eliminate Edgeworth-type instabilities by assuming that each firm has enough capacity to support its market.

As in the quantity model, let D be the non-negative entry cost and C, the unit variable cost.

If the potential entrant, firm 3, selects an entry-price strategy (δ, P_3) while the established duopolists, firms 1 and 2, select prices P_1 and P_2 respectively, then the expected profits of the firms are given by

$$\pi_1 = (1-\delta)(P_1-C)[\alpha-\beta(1+\gamma)P_1+\beta\gamma P_2/2]/2$$

$$+\delta(P_1-C)[\alpha-\beta(1+2\gamma/3)P_1+\beta\gamma(P_2+P_3)/3]/3 \ , \tag{10a}$$

$$\pi_2 = (1-\delta)(P_2-C)[\alpha-\beta(1+\gamma)P_2+\beta\gamma P_1/2]/2$$

$$+ \delta(P_2-C)[\alpha-\beta(1+2\gamma/3)P_2+\beta\gamma(P_1+P_3)/3]3 \ , \tag{10b}$$

$$\pi_3 = \delta(P_3-C)[\alpha-\beta(1+2\gamma/3)P_3+\beta\gamma(P_1+P_2)/3]/3 - D\delta \ . \tag{10c}$$

Following the method of analysis employed in section 2, we identify the noncooperative equilibria as:

$$\delta = 0, \ P_1 = P_2 = [\alpha/\beta + C(1+\gamma/2)]/(2+\gamma/2) = P_{duopoly}$$

is a noncooperative equilibrium if

$$D \geq [(2+7\gamma/6)(\alpha-\beta C)]/[12\beta(1+2\gamma/3)(2+\gamma/2)^2] = D_1 \ .$$

And

$$\delta = 1,\ P_1 = P_2 = P_3 = [\alpha/\beta + C(1+2\gamma/3)]/(2+2\gamma/3) = P_{triopoly}$$

is a noncooperative equilibrium if

$$D \leq (1+2\gamma/3)(\alpha-\beta C)^2/3\beta(2+2\gamma/3)^2 = D_2 .$$

Thus, a product-differentiated price duopoly equilibrium results if entry costs are less than D_1, and triopoly results if entry costs exceed D_2. Again, the intermediate range $D_1 \leq D \leq D_2$ is characterized by uncertainties and possible use of exclusionary tactics. The price model has the same qualitative structure as the quantity model. (See Figure 1).

The randomized entry solution in the intermediate region satisfies

$$P_3 = C + \sqrt{[3D/(1+2\gamma/3)\beta]}$$

$$P_1 = P_2 = 3[\sqrt{(12D\beta(1+2\gamma/3))} - \alpha + \beta C(1+2\gamma/3)]/2\beta\gamma$$

$$\delta = \frac{3(\alpha-\beta P_1(2+\gamma/2) + \beta C(1+\gamma/2))}{\alpha-\beta P_1(2-\gamma/2) = 2\gamma P_3/2 + \beta\gamma(1+\gamma/9)}$$

Furthermore, P_1, P_2, P_3 and δ satisfy

$$0 < \delta < 1 \text{ (for } \delta \text{ not too big)}$$

$$c < P_3 < P_{triopoly} < P_1,\ P_2 < P_{duopoly}$$

4. Comparison of the Price and Quantity Models

The critical entry cost levels D_1 and D_2 computed for the models permit a comparison or sensitivity of the entry problem with respect to strategic variables and levels of product differentiation. As noted earlier, D_1 is the maximum level of entry cost below which noncooperative triopoly solution exists, and D_2 is the minimum level of entry cost above which entry is blockaded.

Superscripting the results of the price and quantity models with P and Q respectively, we write

$$D_1^Q = [(1+\gamma)(4+\gamma)^2(\alpha-\beta C)^2]/[2\beta(4+3\gamma)^2(6+2\gamma)^2]$$

$$D_2^Q = [(1+\gamma)(2\gamma+6)(\alpha-\beta C)^2]/[2\beta(4\gamma+6)^2$$

$$D_1^P = [(2+7\gamma/6)^2(\alpha-\beta C)^2]/[2\beta(1+2\gamma/3)(2+\gamma/3)^2]$$

$$D_2^P = [1+2\gamma/3)(\alpha-\beta C)^2]/[3\beta(2+2\gamma/3)^2]$$

And it is easily verified that

$$D_1^P < D_2^P < D_1^Q < D_2^Q \tag{11}$$

for γ positive.

The inequality $D_2^P < D_2^Q$ implies that, for a fixed level of product differentiation, price competition blockades entry sooner than quantity competition. Likewise, the inequality $D_1^P < D_1^Q$ implies that a three firm solution is more likely under quantity competition than under price competition. Thus, price competition should be less favorable for entry than quantity competition. In general, we would expect that the use of other strategic variables, singly or in combination with others, would affect entry in diverse, and, perhaps, surprising ways.

The variation of the critical numbers with γ, the coefficient of product differentiation, is also important. D_1 and D_2 decrease with γ for both models. That is, the critical numbers D_1 and D_2 shift to the right (See Figure 1) as the products become more differentiated (γ decreasing). Yet, quantity competition retains its edge over price competition. In the extreme case when $\gamma \to \infty$, and the products become identical, entry is completely blockaded in the price model (D_1^P, $D_2^P \to 0$), but quantity competition still supports entry, retaining the basic equilibrium structure in Figure 1. A more subtle result may be obtained if the level of product differentiation becomes a strategic variable, subject to the control of the firm.

5. Limit Pricing

Possibilities for limit pricing is recognized within the framework of our analysis, and limit prices can be computed endogenously for each of the models. Following Bain [1], we roughly define limit price as the highest price established firms can charge without inducing entry. The corresponding concept, limit production, for the quantity model would be the minimum quantity established firms can produce without inducing entry. We say "roughly" because, as we will soon show, limit price (or quantity) is not a single number but a relationship between the prices (or outputs) of the established firms with a multiplicity of solutions.

Suppose, for the price model, that the established duopolists are charging P_1 and P_2, then the non-positive profit incentive condition, corresponding to (3c), is

$$\gamma\beta(P_1+P_2) < 3(\sqrt{[12D\beta(1+2\gamma/3)]} - \alpha + \beta C(1+2\gamma/3)) \tag{12}$$

If we also add the condition

$$P_1, P_2 \geq C \tag{13}$$

to ensure that none of the duopolists makes a loss, then <u>any</u> solution to (12) and (13) is a limit price. It is clear that, in a duopolistic or oligopolistic setting, several limit prices may exist, depending on the bargaining abilities of the established firms. Furthermore, limit prices are threat solutions that must be communicated to and believed by potential entrants. Limit pricing necessitates a binding agreement between established firms against potential entrants.

Under the symmetric assumptions of our model, we may compute a symmetric limit price $P_1 = P_2 = P_{lim}$ by solving (12).

$$P_{lim} = C + 3(\sqrt{[12D\beta(1+2\gamma/3)]} - (\alpha-C\beta))/2\gamma\beta \quad .$$

The condition $P_{lim} \geq C$ yields another critical number D_{lim}, of entry cost, above which limit pricing is feasible. P_{lim} obtained above is feasible if

$$D \geq (\alpha-\beta C)^2/(2\beta(1+2\gamma/3)) = D^P_{lim} \; .$$

The following inequalities also hold;

$$D^P_{lim} < D_1^{\;P} < D_2^{\;P} \; .$$

Thus, the range of entry costs for which the potential entrant may be limit priced out of competition extends into the triopoly range. Certainly, the established duopolists have no need to limit price in the duopoly range. Starting from a minimum value of C at D_{lim}, the symmetric limit price increases with entry cost until it coincides with the duopoly equilibrium price at $D = D_2^P$, beyond which limit pricing is unnecessary and counterproductive. Also, D^P_{lim} decreases with γ, the coefficient of product differentiation, implying that the more identical the products (γ increasing) the more extensive the prospects, and perhaps the attractiveness, for limit pricing.

The conditions for limit producing in the quantity model are

$$\gamma(q_1+q_2) > (1+\gamma)(\alpha - C\beta) - \surd[2D\beta(1+\gamma)(6+\gamma)] \tag{14}$$

$$P_1 = \alpha/\beta - 2q_1/(1+\gamma)\beta - \gamma(q_1+q_2)/(1+\gamma)\beta \geq C \tag{15}$$

$$P_2 = \alpha/\beta - 2q_2/(1+\gamma)\beta - \gamma(q_1+q_2)/(1+\gamma)\beta \geq C \tag{16}$$

$$q_1,\ q_2 \geq 0 \tag{17}$$

Inequality (14) is the non-positive profit incentive condition (3a) for the potential entrant, (15) and (16) ensure that none of the duopolists makes a loss, and (17) restricts outputs to be non-negative.

As before, there are multiple solutions to the limit production problem, but the symmetric limiting outputs, satisfying $q_1 = q_2 = q_{1im}$, is given by

$$q_{1im} = ((1+\gamma)(\alpha - C\beta) - \surd[2D\beta(1+\gamma)(6+2\gamma)])/2\gamma$$

In addition

$$0 \leq q_{1im} \leq (\alpha-\beta C)/2$$

yielding two critical numbers, D^1_{1im} and D^2_{1im}, delimiting the range of entry costs within which limit producing is feasible.

$$D^1_{1im} = (\alpha-\beta C)^2/(2\beta(1+\gamma)(6+2\gamma))$$

$$D^2_{1im} = (1+\gamma)(\alpha-\beta C)^2/(2\beta(6+2\gamma)$$

Also, depending on the size of γ, we have the following inequalities.

$$D^Q_1 < D^1_{1im} < D^Q_2 < D^2_{1im} \qquad \text{(for } \gamma \text{ not too big)} \qquad (18a)$$

And

$$D^1_{1im} < D^Q_1 < D^Q_2 < D^2_{1im} \qquad \text{(for } \gamma \text{ large)} \qquad (18b)$$

According to inequalities (18a) and (18b), the range for limit producing extends into the triopoly range only when γ is large; that is, when the products become more identical. Otherwise, the possibility for limit producing does not destroy the triopoly solution. This is another sense in which quantity competition is more favorable for entry.

6. Conclusion

The two models of duopoly with entry analyzed and compared in this paper help shed some insights into several important structural and strategic issues connected with the entry problem.

For both the price and quantity models, we showed that the potential entrant penetrates the industry if entry costs are low, and that entry is blockaded when entry costs are high. The critical numbers delimiting high and low entry costs are related to the total industry profits and computed endogenously. However, for moderate entry costs complex and subtle possibilities involving the use of threats, limit pricing, and randomized strategies arise.

A comparison of the two models demonstrates that the outcome in the entry problem depends decisively on strategic variables being used. Although the price and quantity models have similar structural characteristics, they also exhibit important qualitative differences. In many ways, quantity competition appears more favorable to entry than price competition. And we expect other strategic variables to influence entry in other ways. Therefore, economists or regulators studying entry conditions in industries ought to pay careful attention to the strategic variables being employed and, above all, ascertain their influences on the outcome.

Our investigation contributes to an understanding of the entry problem, but we expect that if the single-stage, one variable models treated here are extended to include dynamics, capital structure, advertising or other institutional and behavioral factors, other and more detailed insights into competition with potential entry may be obtained.

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