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PERFECT OR ROBUST NONCOOPERATIVE EQUILIBRIUM:

A SEARCH FOR THE PHILOSOPHER'S STONE?

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by

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1. PRELIMINARY REMARKS

The Nash noncooperative equilibrium point offered an important and appealing solution concept to the study of n-person games. Nash proved the existence of a noncooperative equilibrium point in mixed or pure strategies for any n-person game,¹ where the individuals have finite sets of strategies. The chances that an n-person game will have at least one pure strategy noncooperative equilibrium point increase considerably with the number of players.² In general, unless a game in strategic form has special structure associated with it the modeler of a game of strategy may find himself with a surfeit of pure strategy equilibrium points to choose from rather than one or more.

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¹Nash (1950).

²Dresher (1968).

In essence the conditions imposed upon the players in a game in strategic form by the noncooperative equilibrium amount to a set of consistent expectations. There is a circular stability in the reasoning. If all players' conjectures about the behavior of the rest are consistent, then each player can maximize his payoff individually with the resultant outcome being stable, as ex ante conjectures of behavior will match ex post observations of behavior.

Much of the literature in current economic theory going under the horrendous misnomer of rational expectations in actuality utilizes consistent expectations but utilizes the word rational instead of consistent as though consistency and rationality in a multiperson game of strategy were equivalent. This writer can find nothing particularly rational about the highly suboptimal but stable equilibrium in the prisoner's dilemma game as shown below in Table la, or in the suboptimal equilibrium point given by strategies (2,2) in the game shown in Table lb (where all four outcomes are obtained as equilibria).

	1	2
1	5,5	-3,10
2	10,-3	0,0

a

	1	2
1	10,6	5,6
2	10,1	5,11

b

Table 1

One might wish to use the phrase rational expectations to characterize noncooperative equilibria which furthermore have the property that the outcome from play is Pareto optimal. Thus in the game shown in

Table 2a, the single equilibrium might be called a rational expectations equilibrium and the two equilibria (1,1) and (2,2) in the game in Table 2b would also have this property.

	1	2
1	5,5	-3,-1
2	-1,-3	-4,-4

a

	1	2
1	4,1	0,0
2	0,0	1,400

b

Table 2

The two games shown in Tables 3a and 3b have respectively a single mixed strategy equilibrium and no pure strategy equilibrium and a single pure strategy equilibrium and a continuum of mixed strategy equilibria.

	1	2
1	1,-1	-1,1
2	-1,1	1,-1

a

	1	2	3
1	1,-1	0,0	-1,1
2	0,0	0,0	0,0
3	-1,1	0,0	1,-1

b

Table 3

We denote a mixed strategy over m pure strategies by an array

$$\vec{p} = (p_1, p_2, \dots, p_n) \text{ where } \sum_{i=1}^n p_i = 1 \text{ and } p_i \geq 0 \text{ for } i = 1, \dots, n.$$

Thus the equilibrium point for the game in Table 3a is given by the pair of mixed strategies $((1/2, 1/2), (1/2, 1/2))$. The mixed strategies equilibria for the game in Table 3b are of the form

$((p/2, 1-p, p/2), (q/2, 1-q, q/2))$ where $0 \leq p \leq 1$ and $0 \leq q \leq 1$. When $p = q = 0$ we have a pure strategy equilibrium which can be described as a special case of a mixed strategy equilibrium.

The mixed strategy equilibrium in the game in Table 3a has the consistent and even rational expectations property as weakly defined above. But why should a "rational player" use a mixed strategy in a game he is going to play only once? One may evoke the argument of safety level. He protects himself against being found out in case there is an accidental leak of his intentions. But this is a matter of complete and careful specification of the model. If we have specified that in our definition of the model the players move simultaneously, then the information leakage is irrelevant. We may argue a principle of insufficient reason or that the consistency property of the expectations should be used to define rational behavior. This point is an old point and brought up again here merely to stress that there appears to be no particular reason why the normative advice "do anything you want" should be worse than "play your mixed strategy $(1/2, 1/2)$." In the example in Table 3b the "do anything" advice still holds. But one could also argue that the pure strategy equilibrium $((0,1,0), (0,1,0))$ is preferred as it has the saddle-point property.

When a game such as that in Table 3a and 3b is played many times with perfect information available after each play, then a frequency record of actual previous behavior can be built up and the argument concerning the use of mixed strategies becomes more complicated, because now there is the possibility of trying to mislead one's opponent.

When Cournot first applied the concept of a noncooperative equilibrium, he did so to a simple model where a strategy had a natural interpretation in terms of production.¹ In his example there was a unique equilibrium point with an immediate interpretation. Furthermore as the

¹Cournot (1838).

numbers of competitors are increased (in an appropriate manner) in the Cournot model, a relationship appears between the noncooperative equilibrium and the competitive equilibrium. Wald established that neither existence nor uniqueness of a noncooperative equilibrium in a relatively simple economic model were guaranteed except under relatively strong conditions,¹ which were satisfied by Cournot's simple model.

Recently Dubey has shown² under extremely general conditions that mixed strategy equilibria in games in strategic form are almost always inefficient, i.e. are not Pareto optimal. Yet Dubey and Shubik have a series of economic games³ which have the property that noncooperative equilibria which are Pareto optimal exist.

The thrust of the last two remarks in specific, and the discussion in general is that a property deemed to be desirable for a particular solution (in this case the noncooperative equilibrium) may be present for a class of games with special structure and interest, but may easily not be present in general.

The search for appropriate solution concepts may proceed along several different lines. The main distinction is between behavioral and normative approaches. Most of the cooperative solution theories are usually defended normatively. In contrast, for the most part a behavioral rather than normative argument is made out for the noncooperative equilibrium. Harsanyi and Selten⁴ argue otherwise.

¹Wald (1951).

²Dubey (1979).

³Dubey and Shubik (1978).

⁴Harsanyi and Selten (1980).

2. SOCIAL ENGINEERING: NORMATIVE AND BEHAVIORAL APPROACHES

Whether one adopts a normative or descriptive approach to the non-cooperative equilibrium, there is still a choice to be made concerning the domain of applicability of the solution. Does the normative or behavioral theorist wish to assert that the noncooperative solution is going to give a "reasonable or desirable" answer as to what individuals will do or should do in all circumstances?

There is a third point of view, which can be adopted, which contains an intermix of both behavioral and normative components. One may adopt the view that in some circumstances noncooperative behavior is reasonable. However, the task of the social engineer is to design games or institutions such that the noncooperative equilibria have normatively desirable properties. These properties may differ; they may involve the design of self-policing systems¹ or of cheatproof systems, for example.

3. A LIST OF POSSIBLY DESIRABLE PROPERTIES?

3.1. Strategic or Extensive Form

Our examples and comments so far have been addressed to games in strategic form. There is a not completely innocent modeling assumption that any finite game in extensive form can be reduced to a game in strategic form which is equivalent to the original description of the game from the viewpoint of the application of a solution theory. There are enough difficulties with this, as has been shown by Selten² and others,

¹Shubik (1964).

²Selten (1975).

that extra considerations must be taken into account when the noncooperative equilibrium solution is applied to a game in extensive form. We return to this point in Section 5.

3.2. The Modeler's Tool Kit

The end all of the normative theorist would be to find the unique perfect equilibrium which is normatively the solution to games in strategic or extensive form. The goal of the normative-behaviorist social engineer is to find robust equilibria, i.e. equilibria which have desirable properties in a given context. And in general, an important set of properties is robustness or smooth behavior under various perturbations of the model being investigated.

A contrast has been made here between the normative search for a single unifying solution concept, which to some may be aesthetically satisfying, and a much more mundane approach which calls for the modeling and mathematical handtailoring of solutions to cover only special domains of problems. If the first approach were to be truly successful, then clearly it would encompass, as a set of special cases, the second approach. Thus this more limited approach being advocated here is not opposed to the search for the final perfect equilibrium point. It is sceptical that this may turn into a search for the philosopher's stone. At best, both the interea and measurements of successful application of mathematical methods in the behavioral sciences are shaky. Thus, as a first cautious step, the procedure advocated here is to start with the Nash equilibrium and devise a shopping list of desirable properties which it may or may not have in general; together with a set of alternative assumptions concerning the goals, and abilities of individuals and the environment in which they operate.

The shopping list of solution conditions is by no means meant to be complete. It is divided into three broad sets: (I) Aesthetic properties; attractive norms; sensitivity analysis and limit properties; (II) goals of the individuals; limitations on analytical abilities; levels of ignorance and (III) communication, enforcement and special structures.

Group I

1. Aesthetic properties

a. Uniqueness: A single point prediction or recommendation is clearly the most powerful and is aesthetically satisfying, but beyond aesthetic considerations there is no overriding reason to require this condition and there is little evidence that the outcomes from socio-biological systems are or should be unique.

b. Symmetry: If the actors have completely symmetric roles, then the solution (however it is described) will reflect those symmetries. Unfortunately it is easy to describe games in strategic form which have symmetric sets of nonsymmetric equilibria with an unsatisfactory symmetric equilibrium. The game shown in Table 4 has two pure strategy equilibria

	1	2
1	4,10	0,0
2	0,0	10,4

Table 4

(1,1) and (2,2) with payoffs (4,10) and (10,4). It has a symmetric mixed strategy equilibrium $((4/14, 10/14), (10/14, 4/14))$ giving payoffs of $(40/7, 40/7)$, but this is clearly worse than the average payoff to each

in the two pure strategy equilibria (which would be (7,7)).

c. Value: Even if a game did not have a unique equilibrium it would be comforting if all the equilibria yielded the same payoffs to all players. The class of two person constant sum games has the property but the games shown in Tables 1 and 2 do not.

2. Attractive Norms

a. Pareto Optimality: We may take as an axiom that whatever the outcome is, it is Pareto optimal. There should be no way to make any party better off without harming another. This assumption is a basic axiom in many cooperative theories. The matrix in Table 1a provides an example where this property is not true for the noncooperative equilibrium.

b. Independence of Irrelevant Alternatives: Suppose that an individual goes in to a restaurant which has steak or trout on the menu and he chooses steak. Suppose instead that he enters the restaurant and finds that it has not only steak and trout on the menu, but also whale meat. Given that we know that the individual under no circumstances will ever eat whale meat, we might argue that the addition of the choice of whale meat presents him with an irrelevant alternative and can be removed from consideration. It must be stressed that extreme care must be taken in going between the verbal description and mathematical formulation of any assumption of irrelevant alternatives.

3. Sensitivity Analysis

Can we describe two games as being in some sense close to each other? When we consider complex information and move structures this is difficult, but it is an intuitively meaningful question. Two perturbations can be defined which can be applied to games in strategic or in extensive form. Two further perturbations are suggested which at best

apply only two games in extensive form; and for perturbation d below even further specification is required.

a. Perturbation of Payoffs: If we were to add the same number ϵ to every entry in a payoff matrix we should expect that this should have no influence on the way a game is played. The actual experimental evidence on this does not appear to bear this out. Leaving the question of evidence aside and accepting for the purpose of this discussion, that the games shown in Tables 5a and 5b are strategically equivalent, the

5,5	0,0	5+ ϵ , 5+ ϵ	ϵ , ϵ	5,5	ϵ , ϵ
0,0	3,3	ϵ , ϵ	3+ ϵ , 3+ ϵ	ϵ , ϵ	3,3
<u>a</u>		<u>b</u>		<u>c</u>	

Table 5

game in Table 5c is clearly strategically different from Table 5a. For $-\infty < \epsilon \leq 3$ the game has two noncooperative equilibria at (1,1) and (2,2). For $3 < \epsilon < 5$ the game has one equilibrium point. For $\epsilon = 5$ there are three equilibrium points. For $\epsilon > 5$ there are three equilibrium points; two pure strategies at (1,2) and (2,1) and a mixed strategy with probabilities $\left(\left(\frac{\epsilon-3}{2\epsilon-8}, \frac{\epsilon-5}{2\epsilon-8} \right), \left(\frac{\epsilon-3}{2\epsilon-8}, \frac{\epsilon-5}{2\epsilon-8} \right) \right)$.

In this instance we observe that the equilibria are completely stable in zones, but change going over critical boundaries.

When this is a natural metric over the strategy sets, and this is translated into a direct way of generating payoffs, the ϵ perturbation of the payoffs may be more naturally defined than when one is confronted with an arbitrary payoff function.

b. The "Trembling Hand": The trembling hand perturbation has been utilized by Selten¹ in his exploration of perfect equilibria. The basic idea is that there may be a failure of "rationality" or of ability to transmit a strategy with precision. Thus if an individual has m pure strategies and wishes to play strategy i with probability 1, he can only play i with probability $1-\epsilon$ with $\frac{\epsilon}{m-1}$ on the remaining $m-1$ strategies. Selten replaces a game Γ with a game Γ_ϵ then he studies the behavior of equilibrium points as $\epsilon \rightarrow 0$.

c. Random Information Leaks: We might argue that for games in existence form there is always the possibility of an information leak. In Figure 1a, a simple two move game is drawn in extensive form. Figure 1b shows the same game with the modification that after Player 1 has moved, with a probability p the information concerning his move is leaked. With probability $(1-p)$ secrecy is maintained.

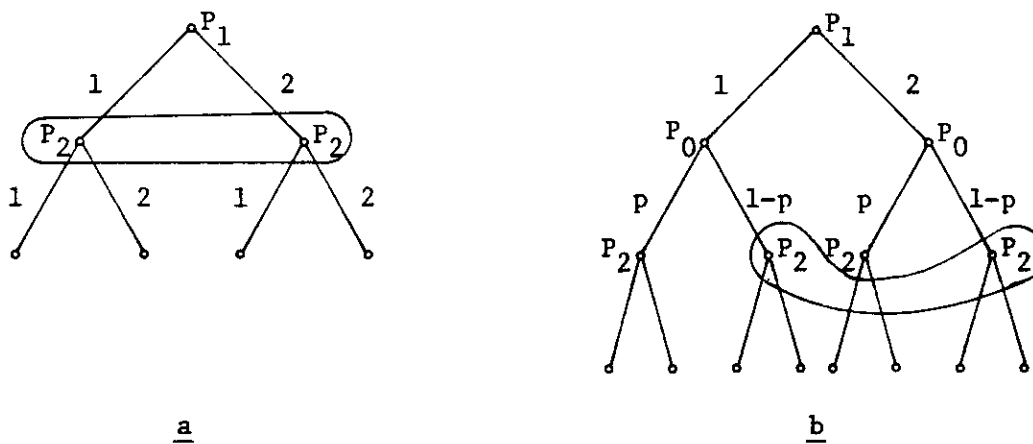


Figure 1

If nothing else, this type of perturbation would enable us to consider the value of a slight leak in information if the equilibria in the two games can be related to each other.

¹Selten (1975).

d. Random Order of Moves: In some situations, such as in trading or voting, individuals appear at random. Thus there is a possibility p that Player 1 moves before Player 2 or $(1-p)$ that Player 2 moves before Player 1. In an extremely complicated game arising from some underlying physical process, it may be unreasonable to try to permute moves as the meaning and function of the process might be destroyed. In mass economic and other phenomena, however, randomness in order is a natural possibility. For the game shown in Figure 1a, this can be modelled as is shown in Figure 2. In this form the sequence of moves makes no strategic difference. With leaks in information it may easily make a difference.

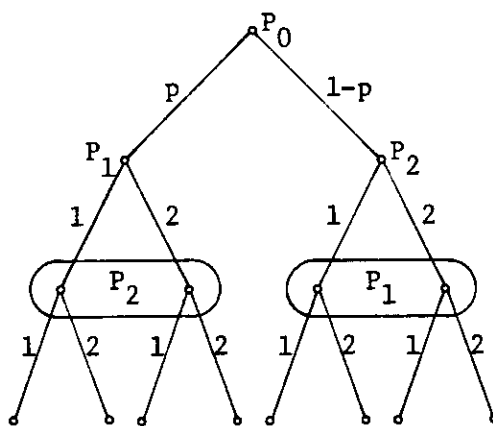


Figure 2

4. Limit Properties

a. Large Numbers of Players: It may well be that the noncooperative equilibria of a game defined for few players do not show particularly attractive properties, yet there is a natural interpretation for a game with "many" or even a continuum of players. Work by Shapley and Shubik, Dubey and Shubik, Postlewaite and Schmeidler and Dubey and Shapley¹ has

¹Dubey and Shapley (1979).

been addressed to this phenomenon for market economies represented in strategic form.

In general, it is not easy to define and compare in a natural and satisfactory manner games with different numbers of players unless he has a natural structure on the underlying game.

b. Games with Infinite Horizons: In many of the most important multiperson decision problems, the end of the game is indeterminate. Although we all die, if someone is alive at t there is a finite chance he is alive at $t+\Delta t$. We might, for individual life, introduce an arbitrary bound of, say, 300 years. For the end of the human race as a whole, in any attempt to study multigenerational models, the selection of an end point appears to be even more arbitrary. We can avoid this approach by arguing that the probability of termination approaches certainty, or that total payoffs are bounded or that there is a bound on the flow of payoffs.

For games with a finite termination, an attractive property is that of backward induction; i.e. starting from the bottom of the game tree, any subgame regarded as an independent game will have an equilibrium point defined by the relevant sections of strategies applicable to the information sets in the subgame. Selten discusses this in detail,¹ and the simple example in Figure 3 illustrates this. This game tree could represent a game where a 2x2 matrix is played twice. After the first play the two players are completely informed. They then play again; hence there are in essence four subgames, one strategy from each of the four outcomes of the first play of the 2x2 matrix. The backward induction requirement calls for considering equilibrium points which are in equilibrium in each of the four subgames and in the game considered as a

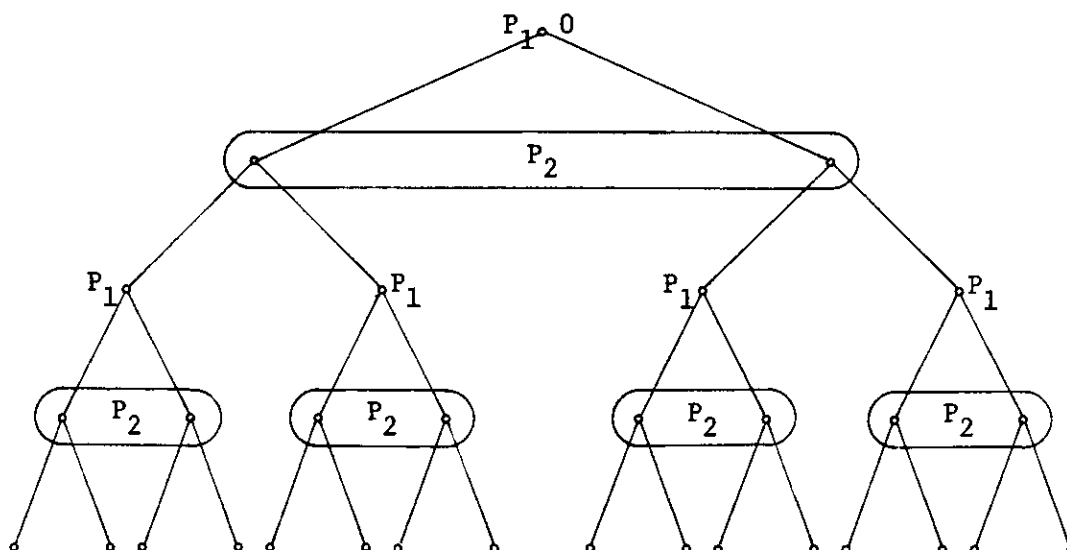
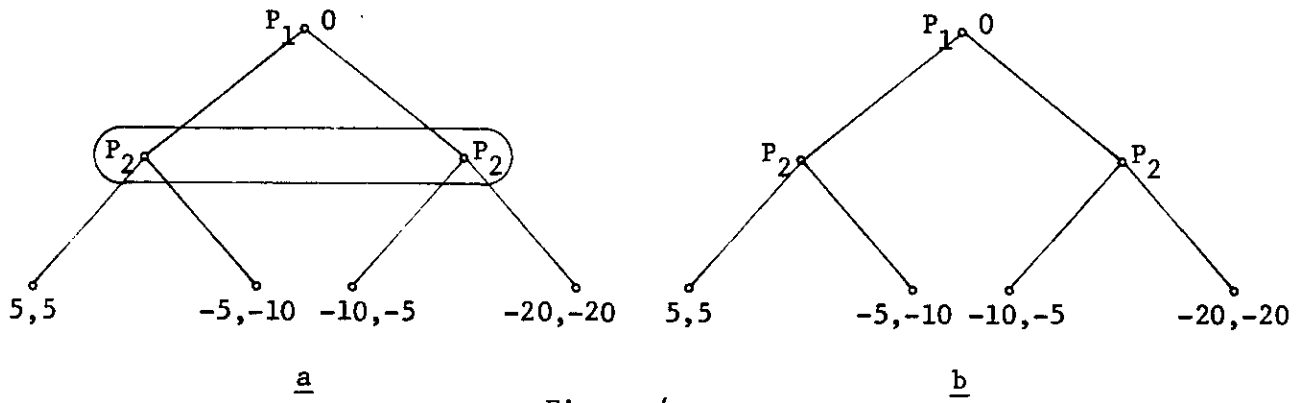


Figure 3

whole. This condition cuts down considerably on the number of equilibria by ruling out threats, in essence. Another way of considering this is that the backward induction condition heavily limits the type of strategies permitted.

c. Limit Properties of Information in Games: A finite game in extensive form without information conditions specified can be described by a game tree without information sets. We may consider the set of all games with the game tree but different information conditions. This set can be structured into a partial ordering of games with more or less information. We may define a game with the most information (perfect information) and a game with least information.

A criterion for robustness of an equilibrium point might be that payoffs (although not necessarily the number of equilibria) are uninfluenced at equilibrium by the amount of information. An example is given by the games in Figures 4a and 4b and Tables 6a and 6b. In these tables the N.E.s are denoted by a .



	1	2
1	5,5	-5,-10
2	-10,-5	-20,-20

a

	1	2	3	4
1	5,5	5,5	-5,-10	-5,-10
2	-10,-5	-20,-20	-10,-5	-20,-20

b

Table 6

Dubey and Shubik¹ have noted that as information is increased pure strategy equilibria never diminishes, but may proliferate. In contrast mixed strategy equilibria are diminished thus for a game with no pure strategy equilibria, by increasing information eventually pure strategy equilibria will appear and the mixed strategy equilibria will disappear.

The injection of exogenous uncertainty into a game without uncertainty may offer pure strategy equilibrium alternatives to a mixed strategy. This is fairly obvious as a mixed strategy uses the mixture to equalize expectations from active strategies; hence if outside uncertainty equalizes the expectations, a pure strategy equilibrium will appear. Figures 5a and 5b show matching pennies without and with exogenous uncertainty.

¹Dubey and Shubik (1980).

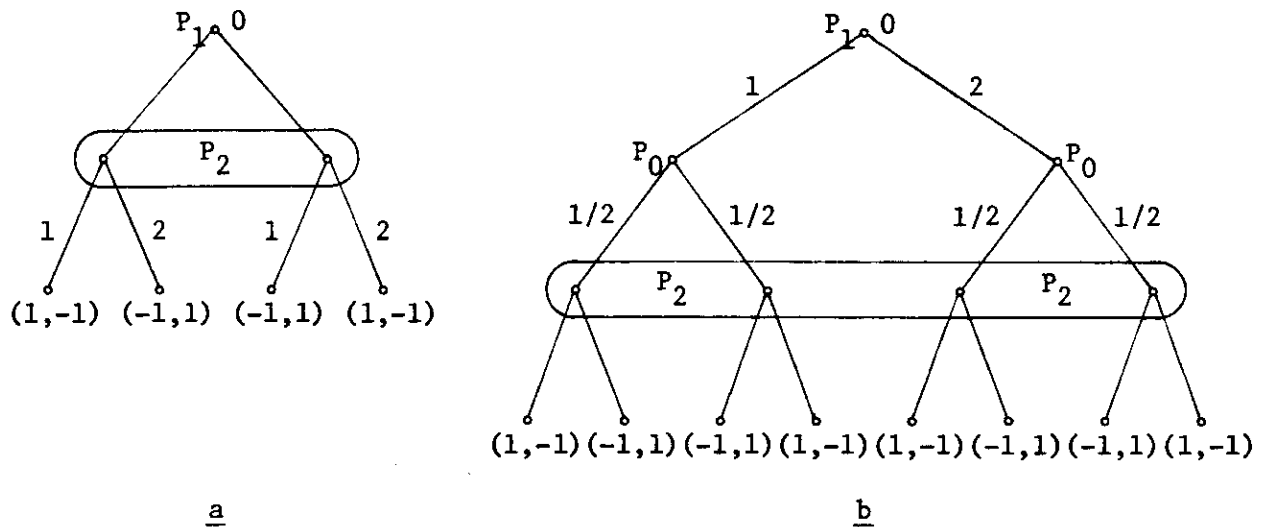


Figure 5

d. Aggregation of Players: In general lumping players together may have a radical effect on the strategic structure and hence the equilibrium points may change radically. However when mass player games representing political, economic or social processes are considered we may expect a regular variation in the noncooperative equilibria as players are aggregated.

5. Goals

What are the underlying preference ordering or utility function assumptions made about the players in a game in strategic or extensive form? Several alternatives can be selected with reason.

a. Ordinal Payoffs: Leasing aside partially ordered preferences, lexicographic preferences and a host of other relatively specialized assumptions concerning preferences; the first somewhat general assumption made is that a preference ordering over the outcomes is given. This assumption wipes out the raison d'etre for mixed strategies. The matrices in Tables 7a, 7b, and 7c illustrate the problem. The matrix in Table 7b is a form of the classical Prisoner's dilemma game which has only a

	1	2
1	b_1, b_2	d_1, a_2
2	a_1, d_2	c_1, c_2

a

	1	2
1	5,5	-5,10
2	10,-5	0,0

b

	1	2
1	5,5	-5,200
2	200,-5	0,0

c

Table 7

pure strategy equilibrium. The game in Table 7c still has a pure strategy equilibrium at (2,2) but also not has the opportunity for the use of a correlated mixed strategy of 1/2 on (1,2) and 1/2 on (2,1). This joint strategy is not self policing hence is not in equilibrium although it might be attractive as a solution. The correlated mixed strategy of $\frac{1}{2}(1,1)$ and $\frac{1}{2}(2,2)$ for the game in Table 8 however is self policing. Both in the games in Tables 7c and 8, if they were presented only in ordinal form there would be little meaning to mixed strategies.

	1	2
1	0,100	-1,-1
2	-1,-1	100,0

Table 8

b. Utility Functions Defined Up to a Linear Transformation: If we accept von Neumann's axioms on the measurability of utility then the numbers on a payoff matrix take on specific meaning and the expected worth of a mixed strategy can be usefully defined. Given the proposed axiomatization of measurable utility the behavior of individuals should not be influenced by any linear transformation of their scales of preference. In actuality this does not appear to be the case. But the purist can immediately point out that in general payoff matrices used in experiments usually contain outcomes not payoffs. E.g. they indicate that a certain

amount of money may be won or lost, but not the value of the money to the player.

c. Measurable and Comparable Utility: It is sometimes argued that as noncooperative games do not involve direct transfers of resources among the player the concept of interpersonal comparison is irrelevant. This is not so. In some sense the title "noncooperative equilibrium" is a misnomer. At best all it implies is that no assumption of Pareto optimality of the outcome is made. But especially in games in extensive form where threats are possible, a so called noncooperative outcome may be Pareto optimal.

Games in strategic or extensive form are played strategically or behavioristically. This has little to do with cooperation or no cooperation. When we talk of a cooperative game we usually mean that (i) Pareto optimality is an axiom; (ii) sidepayments outside of the structure of the game may be permitted and (iii) communication and bargaining outside of the formal structure of the game may be permitted. There is nothing intrinsically cooperative about the last two conditions. They both can be accounted for in the extensive form by introducing extra detailed moves.

Let us consider the two games shown in Tables 9a and 9b. Clearly

	1	2
1	1,2	0,0
2	0,0	2,1

a

1, 2×10^6	0,0
0,0	2, 10^6

b

Table 9

if we take an absolutely purist view of our models then both of these games should be played the same way and have pure strategy equilibria at (1,1) and (2,2). What little evidence there is suggests that individuals

would not play these games the same way. In the game in Table 9, player 1 would compare player 2's 1 or 2 million with his own 1 or 2. If this were a one shot game to be played among strangers I suspect that the results would challenge an assumption of lack of interpersonal comparison. Even leaving this aside, if such a game is intrinsic in some social, political or economic process the chances are that if interpersonal comparisons are likely, the society will modify the game so that the equivalent of side-payments can be made via the mechanism of the game. Contracts and escrow agreements are more or less enforceable agreements which can be added to a game in extensive form which enable sidepayment possibilities to be introduced and strategically (i.e. noncooperatively) enforced.

d. Fiduciary Evaluation: Much of strategic decisionmaking in a society is made by individuals acting as fiduciary for other individuals. This poses a modeling problem. Are those for whom the agent acts to be regarded as strategic dummies or active players? Frequently the trustee is an organization such as a corporation rather than a single individual. Should more than one member of the corporation be regarded as a player? In some game theoretic discussions of games in extensive form an individual player may have many information sets. One way of dealing with this is to imagine that each information set is operated upon by a single individual who is an agent for his overall palyer. Thus we may look at a player as being a corporation and the agents as members of the corporation. If they are totally dedicated to the corporation then all should have the same utility function. This reflects the concept of a "team" as noted by Marshak and Radner.¹

¹Marshak and Radner

e. Minmax Bureaucratic Blame: When we look at a player as a set of agents in a formal game structure where the agents are fiduciaries then there emerges the possibility of applying special measures to the agents' goals. In particular in keeping with the idea of minimizing regret suggested by Savage¹ as a way for an individual to handle unknown exogenous risk we might wish to try to model the feature apparently present in some bureaucracies that the true goals of agents may be best served by minimizing their exposure to bureaucratic blame. This poses a problem in modeling games; and it is not suggested here that the analysis of such games would necessarily provide general results. It is suggested however that this consideration provides an interesting and possible useful extra specification or structure for a class of games.

6. Limited Ability

a. Limit to the Complexity of Strategies: The number of feasible strategies in a game as simple as chess is hyperastronomical. Neither people nor institutions appear to plan with anywhere near the number of contingencies available in the strategies available in a game in extensive form of more than two or three moves. How can we cut down the set of strategies which can be used? And what is our justification for doing so? A fairly natural way is to limit the amount of history which can be used explicitly. In games in which the same physical state can be reached in many ways we may assume that behavior at any state is limited to depend only upon position not how the position was reached. We could restrict ourselves to a limited amount of history as is sometimes done in forecasting procedures based upon a geometric averaging over a few previous periods.

¹Savage (1954).

Rather than limit strategies in an a priori manner one can add conditions that one requires of an equilibrium point which in effect leaves the strategy set the same, but cuts down the set of strategies eligible to be part of a solution. The requirements for a perfect, proper or strong equilibrium (see Section 5) all do this.

b. Limits or Costs to Memory: An explicit way to limit both the size of individual strategies and the number of strategies considered is to put a bound on memory. This requires essentially micromodeling as the way players code their information about the game and their moves must be specified. For example in repeated plays of a matrix game we might limit a player's memory to the extent that all that he can keep track of is where he is and what the average of his and his competitor's payoffs have been. In particular limitations on memory may easily result in less than perfect recall.

c. Limits or Costs to Data Processing: Highly related with but different from limits to memory are limits to data processing. In a world without exogenous uncertainty where everyone could, at a price obtain any information and process anything aggregating procedures would be called for by cost considerations and with these procedures a form of uncertainty must appear.

d. Other Limits on Intelligence or Ability: The trembling hand phenomenon already noted in 3 may be regarded as a limitation on ability. The limits on data processing and memory appear to be at least a clear type of limitation one can place on player ability which is consistent with the view of a consciously optimizing player. Unfortunately these do not cover phenomena such as lack of perception which may be an important lack of ability in many conflict situations.

7. Levels of Ignorance

In the formal modeling of a game of strategy it is assumed that the individuals know the rules of the game. In actuality there are many situations in which this is not a good approximation. A brief listing of some of the ways in which this can fail is as follows.

a. Lack of Knowledge of Number of Other Players: In many mass participation phenomena be they markets, voting situations or otherwise the "rest of the world" is treated as an aggregate and the precise number of other players does not matter. In games with few highly interrelated players the presence of an unknown player could clearly make a considerable difference.

b. Lack of Knowledge of Payoffs of Others: In many situations such as voting or being assessed for contribution to a joint product it is frequently the case that others do not know the true preferences of the individual and it may well pay him to lie.

c. Lack of Knowledge of One's Own Preferences: It is by no means clear that individuals clearly know their preferences. Much of bargaining and of long term investment involves a gradual formation or clarification of preference. Learning and changes in aspirations may be present.

d. Lack of Knowledge of Moves or Strategic Choices: It is a common occurrence that after an event an individual discovers that there was an alternative he should have taken that he did not consider. The alternative was overlooked either because of actual lack of knowledge, lack of recognition of its implications or because it was buried in a mass of incompletely analyzed possibilities.

Many game theorists and economists in general have noted all of the above problems in the modeling of the lack of knowledge. What can

be done about them? A simple position to adopt which covers all of them is to argue that at least for a normative theory we can ignore ignorance due to sloppy decisionmaking as that could be trained away. However for the rest we can represent the initial ignorance of all varieties by allowing our players all to commence with prior probabilities which represent their different degrees of ignorance. This can give a mathematically closed well defined model with Bayesian players. Harsanyi¹ has developed this approach to games without knowledge of the rules.

For what circumstances is this treatment of uncertainty a good guide? This appears to raise several deep empirical questions concerning the "quality of uncertainty" in human decisionmaking. Frequently it seems that individuals are not merely ignorant of some probability distribution but are not even in a position to conceptualize about the nature of their uncertainty.

8. The Nature of Enforcement Mechanisms

Underlying a social-engineering approach to the application of games in strategic or extensive form is the idea that the structure of the society and its institutions provide the rules of the game for most of economic conflict or cooperation. In the short run these are given and fixed. In the longer run if enough individuals are dissatisfied with the laws and institutions they will be modified.

If the above view is correct then it may well be that some game structures have, by some criterion, unsatisfactory noncooperative equilibria. But the way societies resolve this unfortunate fact is not by changing a definition of solution but by changing the structure of the game. Two examples of game modifications are noted below.

¹Harsanyi (1967).

a. Positional Payoffs: In a game in extensive form, as usually defined, all payoffs are made at the end of the game. This can be modified as can be seen easily where a matrix game is played several times, but with payments being made after each play of the matrix game. When there are positional payoffs the players may not need to trust each other as much as when there are no positional payoffs, as the frequent payoffs offer the opportunity for giving "hostages."

b. Voluntary Escrow Arrangements: When A does not trust B it may be in the interest of both parties to find a third to hold "earnest money" or to run an escrow account for them. Thus for example the paradox of the jointly nonoptimal behavior found in experiments with the Prisoner's dilemma is no paradox at all. If a society finds its members constantly trapped in a dilemma game to the expense of society as a whole, the game is changed by the introduction of the appropriate escrow mechanism.

9. Communication Conditions

In our discussion of goals we have already observed that the distinction between cooperative and noncooperative games is not necessarily a good distinction for many purposes. In particular the distinction was made to finesse problems with communication and bargaining. The only game that is played experimentally without any communication whatsoever is the one-shot matrix game. The players simultaneously and anonymously select a strategy and that is it. Even in a matrix game played two or more times, if any results of moves are transmitted the players have, at least within the definition of the rules of the game, a method to signal to each other. Thus in the definition of what is the distinction between communication in a game in cooperative or extensive form, in the former communication outside of the regular channels specified by the game tree

is permitted and is timeless and costless; in the latter if it is not in the tree it is not permitted.

a. Costly Communication, Process and Institutions: It is relatively easy, somewhat ad hoc and undoubtedly tedious to model costly communication directly into the extensive (and hence into the strategic) form of a game. This is tantamount to providing an institutional specification for how communication takes place. The many models of money and financial institutions by Shubik, Shapley and Shubik, Dubey and Shubik and others,¹ all call for a description of the specific mechanisms of trade. Hence they are one manifestation of this type of modeling.

Setting aside the linguistic problem of what sentences mean it is possible to run games where there is a choice for players to send each other items from a menu of messages (like greetings, telegrams) at a cost. For example in the game shown in Figure 6a we could add the option that Player 1, at a cost of c units may transmit one of two messages: "I am going to select 1" or "I am going to select 2." He also has the option of sending no message at no cost. The game tree for this game is shown in Figure 6b. In 1962 Greisner and Shubik ran some pilot studies of this variety. Although no formal statistic analysis was made the play in the games with messages appeared to be considerably different from those without.

¹Shubik (1972), Shapley and Shubik (1972), Dubey and Shubik (1977).

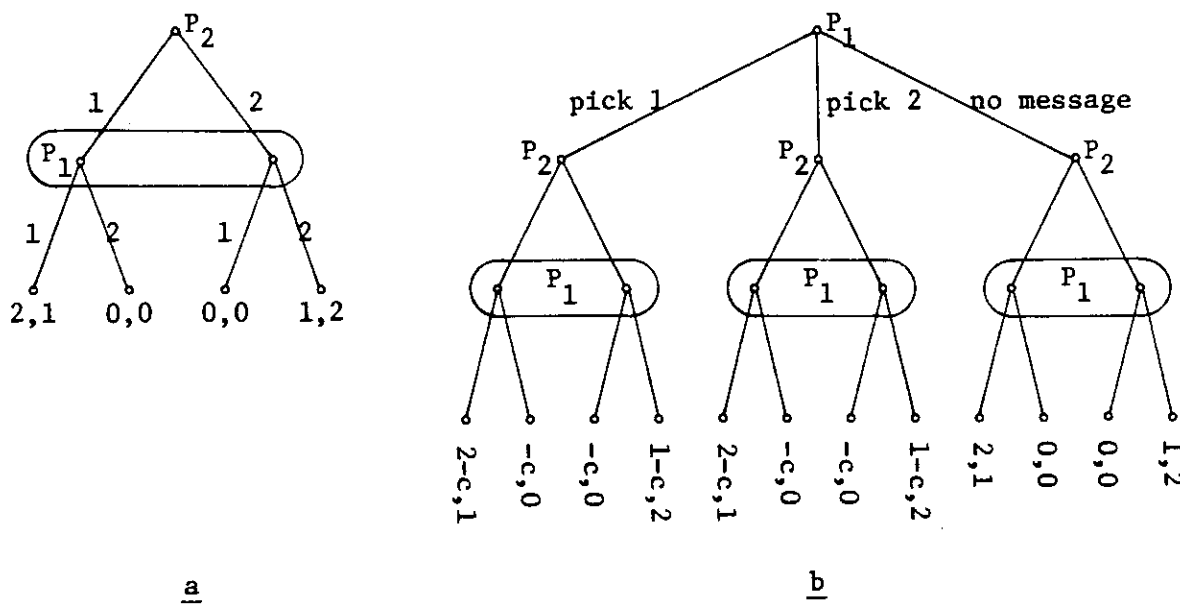


Figure 6

Some individuals may feel that the introduction of costly (unenforceable) messages as moves calls for an intolerable level of detail. The job of the theorist, however, is not to ignore detail but to appropriately generalize for classes of mechanisms and to offer proof that slight variations in the arrangements for message sending do in fact make no substantial difference.

b. Coding: The key difficulty in the meaningful modeling of communication is the coding of the meaning of verbal and other messages such as gestures. It is possible that a normative theory requires an assumption of external symmetry¹ of such strength that all language means precisely the same to every listener. This is clearly not the view of the social psychologist, the diplomat or the labor negotiator.

Mass markets or voting procedures are to a great extent quasi-anonymous mechanisms which try (but clearly do not fully succeed) to minimize the personality and other sociopsychological aspects of economic

¹Shapley and Shubik (1972).

and political intercourse. Arbitration tries to do the same for some bargaining procedures. But especially where numbers are few and communication face to face the game theorist does not have adequate models of people or process. The ad hoc models of labor-management, or international trade may possibly provide more insight into bargaining processes and problems than the models which purport to be more general. This is because the descriptive models present a closer picture of man than do the normative models. The normative models have the attraction that they provide a finesse around empirical work. We are not concerned with man as he is, but how he should be.

10. Special Structures and Other Considerations

Prior to accepting or rejecting the usefulness of the various modifications to the noncooperative equilibrium which have been suggested, a few more special features are suggested which may produce limit classes of games for which the noncooperative equilibria appear to be better behaved than if the domain of definition of the games were broader.

a. Assets and Hostages: In finance as well as in ancient and medieval warfare the availability of assets or hostages of rank offer relatively direct ways of achieving compliance without trust. A rich society should be in a better position to achieve a higher level of enforceable cooperation without trust than a poor society because hostages can be provided. The banker can sleep easier over the highly secured loan than one based on trust alone ceteris paribus.

b. A Metric on Moves: Many situations do not have a natural structure which makes aggregation easy or provides for the construction of related classes of games. Three player chess is really no longer chess;¹

¹Although there is some indication that chataranga the predecessor of chess may have been a four person game.

an eleven person wheat market is closely related to a twelve person wheat market. In many economic situations moves involve the naming of quantities or prices and quantities and there appear to be several natural ways of adding or otherwise aggregating these moves. This possibility tends to produce games with considerable structure.

4. A COMMENT ON EXPERIMENTAL GAMING

There have by now been several thousand experiments performed using matrix games played once or repeatedly under a host of control conditions. Possibly the most exhaustive and concentrated report has been that of Rapoport and his associates. Although the largest number of experiments appears to be socio-psychologists.

The purpose of the comments here are to present a quick and possibly biased view of what a theorist might glean from looking at the little experimental evidence around. The more "nice" properties (uniqueness, Pareto optimality, symmetry, for example) that are loaded on a noncooperative equilibrium the better a predictor it becomes. The experimental subjects are influenced by changes in scale and also appear to make interpersonal comparisons in games played in strategic form. In multistage games even relatively sophisticated players do not appear to use full strategies in the game theoretic sense.

There is not too much evidence but my guess is that irrelevant alternatives appear to be relevant. Computational limitations and memory limitations appear to be important. Unless there is considerably regularity a 3x3 matrix is about as high as one can experiment with without great confusion. The play of a game is influenced by the verbal briefing.

Different scenarios for the same mathematical structure get different results. The noncooperative equilibrium point in the one shot Prisoner's dilemma is a reasonably good predictor. In the multistage battle of the sexes (Table 9a) a small percentage of players who do not even know each other will alternate (1,1) (2,2). Both altruism and spite exist. Unless the briefing and task are specifically economic, homo oeconomicus appears to be a relatively poor approximation of homo ludens.

5. PERFECT, STRONG AND OTHER EQUILIBRIA

Let there be a set N of n players, each player i with a strategy set S_i . Each player i has a payoff function $P_i(s_1, s_2, \dots, s_n)$ defined over all strategies.

A set of strategies $(s_1^*, s_2^*, \dots, s_n^*)$ forms a noncooperative equilibrium if for all $i \in N$

$$\max_{s_i \in S_i} P_i(s^*/s_i) \Rightarrow P_i(s^*)$$

where (s^*/s_i) stands for $(s_1^*, s_2^*, \dots, s_n^*)$ with s_i^* replaced by s_i .

If we stick only to this definition then no distinction is made between games in extensive or strategic form as one reduces any game in extensive form to its strategic form and applies the solution. As some of our examples have already shown the number of equilibrium points can become extremely large, thus the resolution offered by the solution is low.

One way of grouping equilibria is into classes which give the same outcome. For example consider the class of all games formed by varying the information conditions on the same game tree. Figure 7 illustrates this:

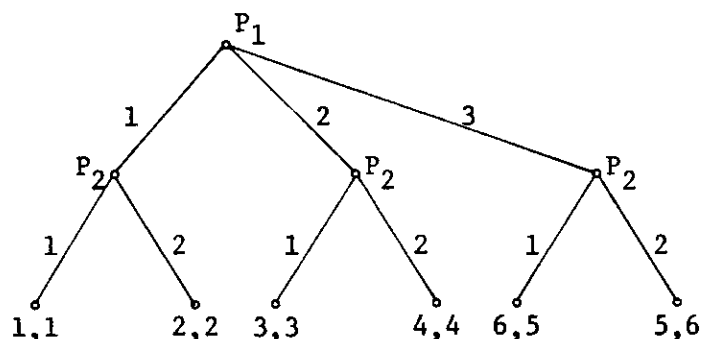


Figure 7

There is one game with perfect information giving a 3×8 matrix. There are three games with two information sets for player 2 giving 3×4 matrices. There is one game with one information set for player 2 giving a 3×2 matrix. As can be seen from the game tree there are only at most 6 distinct outcomes even though the matrix can have as many as 24 entries.

As we increase information the number of pure strategy equilibria which give the same outcome may pile up considerably. Thus in a certain sense we are told that an outcome which can result from many strategies is insensitive to certain variations of information.

The saddlepoint in a two-person constant sum game have the properties of a single value and interchangeability; i.e. if there is more than one pair of strategies forming a saddlepoint say (a_1, b_1) and (a_2, b_2) then the strategies (a_1, b_2) and (a_2, b_1) will also form saddlepoints; furthermore all will have the same value. These properties have been suggested for nonconstant sum games. Unfortunately they do not hold in general.

We briefly review and comment on some of the various specialized equilibria modifications suggested. They are:

Pareto optimal equilibria
 Twisted equilibria
 Information insensitive equilibria
 Perfect equilibria
 Strong equilibria

Pareto optimal equilibria: In general, as we have already noted noncooperative equilibria are rarely Pareto optimal, however for some important special classes of games such as strategic market games with a continuum of traders this property may appear.

Twisted equilibria: Aumann¹ has noted that there is a class of nonconstant sum games for which at an equilibrium neither player can decrease the other's payoff unilaterally. When this property is present Aumann calls the game almost strictly competitive and observes that it has a unique equilibrium payoff.

Information insensitive equilibria: Following the result on information of Dubey and Shubik² the pure strategy equilibria of a game in extensive form with some given configuration of information remain as equilibria as the information is refined (leaving information on exogenous uncertainty unchanged). Thus although many new equilibria may appear, the original equilibria are not destroyed by the change in information. The games in Figure 7 provide an example. For the game with no information (3,2) forms an equilibrium point with payoffs (5,6). For all other games the pair (3, "play 2 regardless of information") gives an equilibrium point with payoffs (5,6).

¹Aumann (1961).

²Dubey and Shubik (1980).

Perfect equilibria: Selten originally suggested the idea of a perfect equilibrium as an equilibrium point which defines an equilibrium in every subgame of the overall game. Essentially the perfectness condition rules out the possibility of most threats, as known history has no influence on what individuals do. In essence it only matters where you are, not how you arrived there. Selten now calls this equilibrium subgame perfect.

As the example in Figure 8 below shows.¹ There are some games which do not have subgames, for which the definition of perfect equilibrium appears to give unreasonable results. In particular as can be

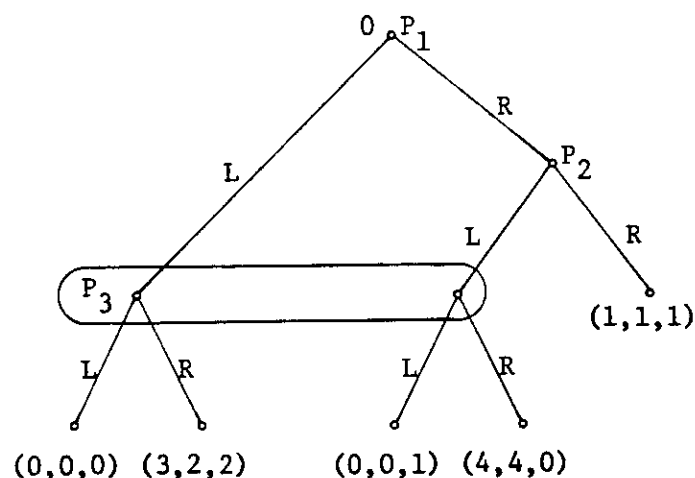


Figure 8

seen in Figure 8 this game has the property that player 1 can give the move directly to 2 or 3 and 2, if called upon can end the game or give the move to 3. There are two sets of equilibrium points for this game. Denoting the mixed strategy for i by p_i for play R with probability p_i and L with probability $1-p_i$ then the two sets of noncooperative

¹Selten (1975).

equilibria are:

$$\text{Type 1: } p_1 = 1, \quad p_2 = 1 \quad \text{and} \quad 0 \leq p_3 \leq 1/4$$

or

$$\text{Type 2: } p_1 = 0, \quad 1/3 \leq p_2 \leq 1 \quad \text{and} \quad p_3 = 1.$$

In the second set of equilibria player 2 is never called upon to play, yet the equilibrium depends upon what he does. Selten argues that this is unreasonable and proposed the mathematical device of a perturbed game to distinguish among perfect equilibria. The perturbed game $\hat{\Gamma}$ is related to the original game Γ in the sense that it is the same game with the modification that slight errors are made in the selection of strategies. For each player i there is the restriction that $1-\epsilon \leq p_i \leq \epsilon$. Thus no matter how the game is played, all information sets will be reached. An equilibrium in the perturbed game will be a best reply set of strategies. A limit equilibrium is an equilibrium point in a sequence of perturbed games $\hat{\Gamma}_1, \hat{\Gamma}_2, \dots, \hat{\Gamma}_k$ where as $k \rightarrow \infty$, $\epsilon \rightarrow 0$.

Examining the example for $\epsilon_1 < 1/4$ it can be shown that the perturbed game has only one equilibrium point of the form

$$p_1^k = 1 - \epsilon_k; \quad p_2^k = 1 - 2\epsilon_k/(1 - \epsilon_k) \quad \text{and} \quad p_3^k = p_3^*.$$

Thus in the sense of Selten all equilibria of Type 1 are perfect. Selten shows that equilibria of Type 2 are not perfect.

Some Criticism

Kohlberg argues that an acceptable equilibrium point should have three properties:

- (1) if the game has perfect information it should be given by backward induction;
- (2) if the game is given in normal form the equilibria should be just the Nash equilibria;
- and (3) the equilibria should be insensitive to the addition of dominated strategies.

Consider the two games shown in Figures 9a and 9b. In the first

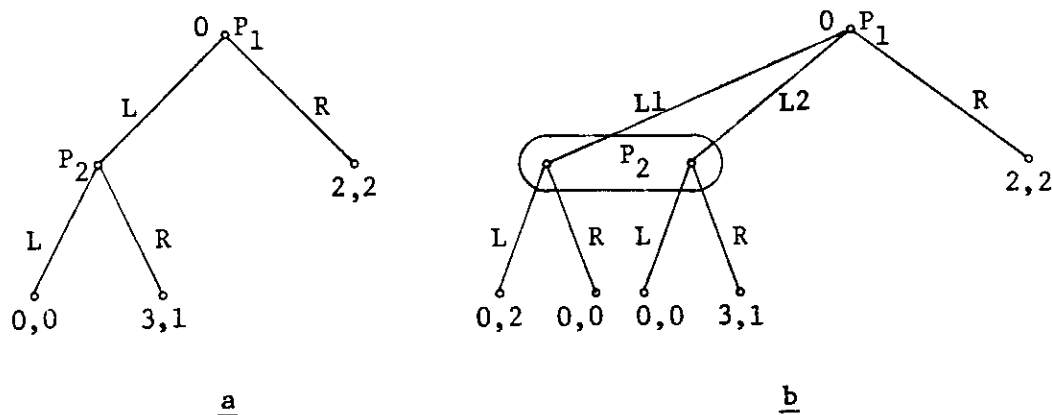


Figure 9

game the only perfect equilibrium is (1,1), in the game in Figure 9b both L2 and R and R and L are perfect equilibria, but the additional strategies for player 1 are completely dominated.

Returning to Selten's example if we consider a sensitivity analysis involving (a) A leak of information; (b) the construction of an escrow scheme, all of these point to the outcome of (3,2,2) as being far more reasonable than (1,1,1) (leaving aside the Pareto optimality criterion).

Figure 10 shows the Selten example with an ϵ information leak.

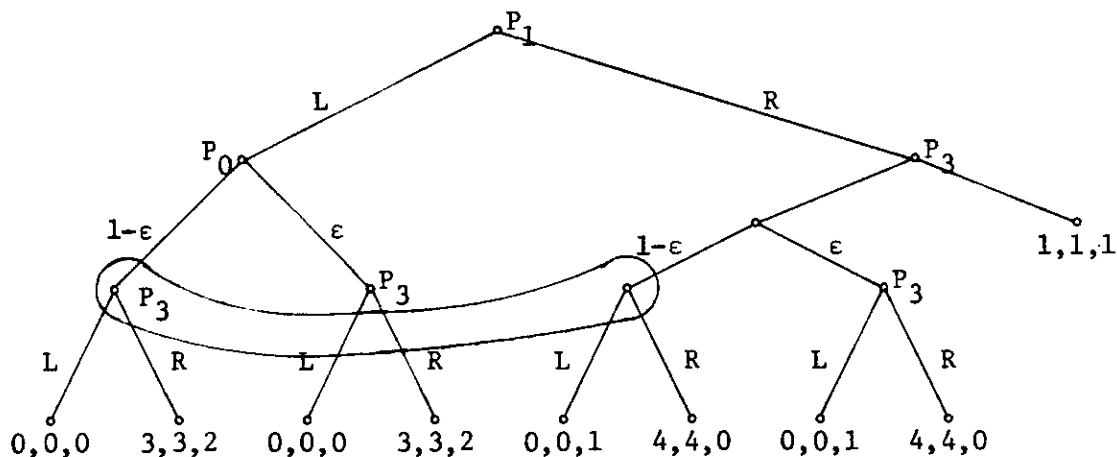


Figure 10

For $\epsilon > 1/4$ the strategy where players 1 and 3 play L and R is impervious to the behavior of player 2. In his second and third information sets, by backward induction player 3 always selects R and L. For $\epsilon > 1/3$ the strategies of Type 2 are never stable.

The Possibility of Communication

How is a one-shot game such as that in Figure 8 actually played if we were to use it experimentally? All players submit their moves or strategies to the referee who then announces the results. In general moves not strategies are handed in hence even mixtures are not disclosed. We could however insist that players hand in the probabilities if they employ mixed strategies. The way we test for an equilibrium point is by imagining that one player has been informed of the moves of all others and optimizes given this information. If we introduce the test as part of the game the equilibrium property will remain. Thus we may consider a game in which the initial moves are statements of intention which are not enforceable. Considering the example of Selten, player 1 has considerable motivation to signal to player 3 that he will select L, even if communication were costly up to a point.

The argument being made here is one concerning modeling and social science not pure mathematics. If we merely take the Nash definition then there are undoubtedly two classes of noncooperative equilibria for the Selten example. Looking at them Selten selects an intuitively appealing model of mistakes of one type which gets rid of the equilibrium points which are the most plausible outcome from other considerations such as Pareto optimality or other perturbations such as leaks of information or the chances for communication. If we take the attitude that much of the normative component to our theorizing should take place at the level of the design of institutions, i.e. the rules of the game; then the existence of the two types of equilibria in the Selten example should not bother us. Our attitude should be that a society confronted with a game such as this in an important context will redesign the rules by having communication, information leaks, escros arrangements, etc. The game design should be able to take care of paradoxical equilibria.

Strong Equilibria

A strong equilibrium has been defined by Aumann¹ as one which is in equilibrium against any subset of players. This implies that the outcome from it must be Pareto optimal and in the case of the associated cooperative game there are $2^n - n - 1$ more conditions imposed upon the strong N.E. than upon an ordinary N.E. hence such an equilibrium exists only under relatively special circumstances.

¹Aumann (1959).

6. CONCLUDING REMARKS

Around thirty considerations in modeling games in strategic or extensive form and solving them for noncooperative equilibria have been suggested.

It is suggested that the noncooperative equilibrium solution is not a good candidate as a normative solution but appears to have some plausibility as a behavioral solution. There are many properties that are normatively attractive which equilibrium points may possess in specially designed games. Thus the approach advocated here is a combination of behavioristic and normative, where the norms are manifested in game design.

At least from the point of view of the application of game theory to the social sciences this writer is highly sceptical that a search for general normative refinements of the noncooperative equilibrium will prove to be as fruitful as the identification of special structures where the noncooperative equilibria display desirable properties not otherwise generally held.

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