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Box 2125, Yale Station
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INCENTIVE ARBITRATION AND TIME-RELATED BARGAINING COSTS

Zvi A. Livne

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By Zvi A. Livne[1]

In the context of the Nash Bargaining Problem an arbitrator is supposed to choose a solution point on the basis of 'fairness' requirements. By introducing time-related costs to the parties' utility functions, a new class of arbitration procedures emerges. Instead of choosing a point, the arbitrator changes the rules of the negotiations, and particularly the parties' payoffs, and lets the parties negotiate. The changes are such that one or both parties are reimbursed by the arbitrator for their bargaining costs. A procedure in which only the party who makes the final concession is compensated, leads (in the case of complete information) to single dominating equilibrium strategies and to a unique solution. This solution can also serve as an arbitration point in the traditional sense.

1. INTRODUCTION

In this paper we propose a class of arbitration procedures that may be applied to negotiations in which the bargainers are adversely affected by costs related to the duration of the negotiations. The main component of such a procedure is an incentive mechanism in the form of a compensation awarded to the parties by the arbitrator after the negotiations are successfully completed. Under conditions of complete information this compensation is such that the final outcome is efficient from the bargainers' viewpoint. The arbitrator in effect guarantees to subsidize the duration-related costs of the negotiators.

A prominent arbitration procedure in this class is the one which assigns positive compensation only to the party who was the first to accept his

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opponent's proposal. (The terms "first" and "proposal" may seem intuitively clear, but this is misleading. We will be careful to give them precise meaning.) This procedure has some desirable properties, among them that it uniquely satisfies some "weak" requirements on arbitration procedures of the type we consider. It also affects the bargainers' behavior in the negotiations. New equilibrium negotiation strategies emerge, and the final negotiations outcome becomes highly predictable (if the players are indeed utility maximizers). This outcome can then serve as an arbitration point to the original problem, viewed as a cooperative game. The arbitration procedure can therefore be viewed as a generator of a solution function for a problem which is an extension of the Nash Bargaining Problem [5]. This extension, characterized by the introduction of time-related costs, has been formulated recently in Livne [2].

The arbitration procedures we propose transform the original two-party cooperative game into a non-cooperative game, and supply mappings from the outcomes of the latter to the outcomes of the original game. This is exactly what Kalai and Rosenthal [1] suggest is the task of an arbitrator, although in their case the non-cooperative game is an 'announcements' game, and not actual negotiations. This view of arbitration is a generalization, to the case of incomplete information, of the usual view (Luce and Raiffa [4]) that in a cooperative game with complete information the role of the arbitrator is to compute a solution point on the basis of a set of fairness criteria. In this paper we will treat only the complete information case, and show that the arbitration point yielded by the procedure mentioned before, although the result of incentive-motivated non-cooperative game, satisfies uniquely a set of fairness requirements.

In a separate paper (Livne [3]) we will study these arbitration procedures under conditions of incomplete information on the part of both the arbitrator and the bargainers. The key issue there is of course how to design the compensation so that it is acceptable to the parties but is still not too expensive for the arbitrator. Unlike in the complete information case, the arbitrator might find himself paying heavily for subsidizing the parties' duration-related costs. This might still be satisfactory to him if he is significantly affected by the duration of the negotiations.

Before concluding this section we describe in some detail the arbitration procedures we have in mind. The arbitrator starts by announcing his estimates of the parties' utility functions including the duration-related costs. In the case of complete information these are of course common knowledge. Next, the arbitrator announces a new process of negotiations. In this process all communications between the parties are forbidden, and they are required to submit their new proposals, whenever they are made, to the arbitrator. The latter passes each proposal to the other party immediately. The next step of the arbitrator is to commit himself to compensate the first party whose proposal is identical to the most recent proposal of his opponent. The amount of this compensation is such that the utilities corresponding to the final outcome constitute a point on the Pareto frontier of the set (on the utility plane) representing all the feasible agreements at the <u>beginning</u> of the negotiations. Under the condition of complete information these rules are enough to guarantee a unique outcome. The link to the traditional arbitration schemes is now clear. By offering the parties infinitesimal rewards if they accept the negotiations outcome right away, the arbitrator saves himself the subsidy without hurting the parties. His commitment has therefore helped in the identification of a 'fair' arbitration solution, in the traditional sense.

This paper is organized as follows. In Section 2 we describe the model and list necessary definitions. In Section 3 we state our main results and in Section 4 we discuss the fairness criteria which one of our arbitration-incentive mechanisms uniquely satisfies. All proofs are relegated to the Appendix.

2. THE MODEL AND DEFINITIONS

As in Livne [3] we define a <u>bargaining domain</u> as a set $B \subseteq R^2$ satisfying:

1. B is convex and compact.
2. There exists a unique point $w(B) \varepsilon B$ such that:
   (i) every $x \varepsilon B$ satisfies $x \geq w(B)$, and
   (ii) for all $y \geq w(B)$, if there exists an $x \varepsilon B$ such that $y \leq x$, then $y \varepsilon B$.

The points $\max\{y_1 | (y_1, w_2(B)) \varepsilon B\}$ and $\max\{y_2 | (w_1(B), y_2) \varepsilon B\}$ are denoted by $\bar{x}_1(B)$ and $\bar{x}_2(B)$ respectively. The Pareto frontier and the strong Pareto frontier of B are denoted by P(B) and SP(B) respectively. The <u>extension</u> of B, denoted B^e, is the set of all point $x \varepsilon R^2$ such that there exists $y \varepsilon P(B)$, $y \geq x$. The set of all bargaining domains is denoted by $\bar{B}$. A partial ordering d on $\bar{B}$ is defined as follows. For all $B_1, B_2 \varepsilon \bar{B}$, $B_1 d B_2$ if: (1) $w(B_1) > w(B_2)$, and (2) for every $y \varepsilon B_2$ there exists an $x \varepsilon B_1$, such that $x \geq y$. A subset C of $\bar{B}$ is a <u>bargaining chain</u> if: (1) it is totally ordered by d; (2) it is a closed set with respect to the Hausdorff metric on $\bar{B}$; (3) it contains both maximal and minimal elements, the latter consisting of one point.

A bargaining chain C can be represented by $\{B(t)|0\leq t\leq T\}$ where $B(t)\supseteq B(s)$ for all $t,s\varepsilon[0,T]$ such that $t<s$. A bargaining chain is <u>end-point continuous</u> if the curves $A_1 = \{(\bar{x}_1(B(t)),w_2(B(t)))|0\leq t<T\}, A_2=\{(w_1(B(t)),\bar{x}_2(B(t))|0\leq t<T\}$ are continuous in R^2. The set of continuous chains, as defined in Livne [2], is a subset of the set of end-point continuous chains. We denote the latter by $\bar{C}_{ec}$. For mathematical convenience, we will restrict the analysis throughout this paper to end-point continuous chains, although it is possible to extend it to more sets in $\bar{C}$.

In $C = \{B(t)|0\leq t\leq T\}$, a bargaining domain $B(s)$, $0\leq s\leq T$, represents the set on the utility plane of all possible outcomes after negotiations of length s. The <u>conflict point</u> $w(B(s))$ corresponds to the parties' utilities in the case that negotiations terminate unsuccessfully at time s. The definition of a bargaining chain reflects the fact that both parties have non-negative time-related bargaining costs. End-point continuity means that the maximal utility each bargainer can achieve, as a function of the negotiations duration, is continuous (and non-increasing).

For each $C = \{B(t)|0\leq t\leq T\}$ in $\bar{C}_{ec}$, a <u>play on C</u> is a pair of curves $L = (L_1,L_2)$ in $R^2 x R^2$ (not necessarily continuous curves) which can be parameterized as $L_1 = \{z^1(t)|0\leq t\leq T\}$, $L_2 = \{z^2(t)|0\leq t\leq T\}$, so that they satisfy the following conditions:

(1) For all $0\leq t\leq T$, $z_1^1(t)\geq z_1^2(t)$, $z_2^2(t)\geq z_2^1(t)$.

(2) There exists a minimal t in [0,T], such that $z^1(t) = z^2(t)$. (Denote this t by $t^*(L)$, and $z^1(t^*(L))$ by $z^*(L)$.)

(3) For all $0\leq t<t^*(L)$, $z^i(t)\varepsilon P(B(t))$, for i = 1,2.

(4) The point $z^*(L)$ is either on $P(B(t^*(L)))$, or it is $w(B(t^*(L)))$.

(5) For all $T\geq t\geq t^*$, $z^i(t) = z^*(L)$, i = 1,2.

The set of all plays on C will be denoted by $\lambda(C)$. A play on C corresponds to the sequences of proposals made by the parties non-cooperatively during negotiations in which the parties behave according to the rules designed by the arbitrator (see Section 1 for a description of these rules). The time $t^*(L)$ is the time in which an agreement has been achieved (if $z^*(L)\varepsilon P(B(t^*(L)))$, or the negotiations terminated with no agreement (if $z^*(L) = W(B(t^*(L)))$.

A bargainer i (i = 1 or 2) will be called a <u>conceding party</u> if $z^*(L)$ is on $P(B(t^*(L)))$ and if at least one of the following two conditions holds: (1) There exists some $0\leq t^0 < t^*$ such that $z_j^j(t) = z_j^*(L)$ for all $t^0 \leq t \leq t^*$ ($j = 1$ or 2, $j \neq i$); (2) $z_i^*(L) = w_i(B(t^*(L)))$. Obviously a conceding party is one who is the first to accept his opponent's last proposal, in the sense that the utility his opponent derives from the proposal $z^*(L)$ is the same as the utility the opponent has demanded for himself just before $t^*(L)$, or it is the maximum utility the opponent can expect at time $t^*(L)$. When $z^*(L)$ is on $P(B(t^*(L)))$ but both conditions are not satisfied, it means that both bargainers yielded simultaneously. We assume that in such a case the arbitrator chooses one of the parties, by tossing a fair coin, and designates him the conceding party.

We are now ready for the definition of an <u>incentive arbitration function</u>. For each triplet (C,L,i) such that $C\varepsilon\overline{C}_{ec}$, $L\varepsilon\lambda(C)$, $i\varepsilon\{1,2\}$ (indicating the conceding party), $g(.,.,.)$ is such a function if:

(1) $g(C,L,i)\varepsilon P(B(0))$ whenever $z^*(L)\varepsilon P(B(t^*(L)))$; $g(C,L,i) = z^*(L)$ otherwise.

(2) $g(C,L,i) \geq z^*(L)$.

The point $g(C,L,i)$ represents the utilities of the bargainers after the arbitrator gives them their compensations. These are non-negative and have the effect of moving the final outcome to the Pareto frontier at time 0, provided that some agreement has been achieved. Otherwise, no compensation is received.

Denote the set of incentive arbitration functions by G. One particular function in G, denoted g^o, will be proven to be of particular interest. This is the function that assigns positive compensation only to the conceding party. To define it formally, let us denote the point $\min\{a|a \geq y_1, (a,y_2) \in P(B)\}$ by $b_1[y,B]$, where B is any bargaining domain and y is a point in B^e. Then $g^o(C,L,1) = (b_1[z^*(L),B(0)], z_2^*(L))$. The definition of $b_2[y,B]$ and $g^o(C,L,2)$ are analogous.

The incentive arbitration function g^o depends only on the final result of the negotiations, on the identity of the conceding party and on the Pareto frontier at time 0. It does not depend on other aspects of the play L. This might seem extreme, since it can be argued that fairness requires that whenever both parties make about equal concessions during the negotiations, they should be compensated about equally, even if one of them happened to make the last concession. After all, the last concession might be very small with respect to other concessions. This argument ignores the fact that g^o is an <u>incentive</u>-arbitration function, namely it provides incentives to change negotiations behavior, with the specific intention of bounding the duration of the negotiations. By making about equal concessions during <u>long</u> negotiations, the parties may deserve equal compensation on fairness grounds, but they should be punished on efficiency grounds, the punishment being an extremely 'unfair' compensation scheme. It can be shown easily that if the parties make about equal concessions during very <u>short</u> negotiations, then g^o will have only a small discriminating effect, and the final outcome will indeed be about equal. Moreover, we will show in Section 3 that g^o is the only function satisfying a set of fairly weak requirements on incentive-arbitration functions, hence it is more than merely an arbitrary scheme. We will also show that when faced with g^o, the parties have 'stron' Nash equilibrium strategies in the non-cooperative game (the negotiations) imposed on them by the arbitrator. (We mean that these strategies constitute a Nash equilibrium and by using his strategy, even unilaterally, a party can guarantee a certain minimal reward.). The existence of such strategies, and the resultant predictability of the final outcome, justify its use as an arbitration point in the usual sense (Luce and Raiffa [4]).

3. MAIN RESULTS

Throughout this section we will make the convenient assumption that whenever there exists a conceding party, it is party 1. All the definitions and results in the other case will be analogous. Let us denote the set of all triplets (C,L,i), where $C\varepsilon\overline{C}_{ec}$, $L\varepsilon\lambda(C)$ and $i\varepsilon\{1,2\}$, by K. An incentive-arbitration function $g:K\rightarrow R^2$ will be called <u>penalizing</u> if wherever $g(C,L,i)$ is on $P(B(t^*(L)))$ the following conditions hold:

1. If $z^2(t) = (w_1(B(t)), \overline{x}_2(B(t)))$ for all t, $0\leq t\leq t^*(L)$, then $g_2(C,L,1) = z_2^*(L)$
2. If $z_2^2(t) = z_2^*(L)$ for all C, $0\leq t\leq t^*(L)$, then $g_2(C,L,1) = z_2^*(L)$
3. Let $L = (L_1,L_2)$, $M = (M_1,M_2)$ be two plays in $\lambda(C)$ such that $t^*(L) = t^*(M)$. Let $L_2 = \{z^2(t)\,|\,0\leq t\leq t^*(L)\}$, $M_2 = \{y^2(t)\,|\,0\leq t\leq t^*(M)\}$. If $y_2^2(t)\leq z_2^2(t)$ for all t, $0\leq t\leq t^*(L)$, then $g_2(C,M,1)\leq g_2(C,L,1)$.
4. Let $L = (L_1,L_2)$, $M = (M_1,M_2)$, be two plays in $\lambda(C)$ such that $t^*(L)\leq t^*(M)$, $L_2\subseteq M_2$, and every $(y_1,y_2)\varepsilon M_2 - L_2$ satisfies $y_2 = z_2^*(L)$. Then $g_2(C,L,1)\leq g_2(C,M,1)$

To see why the functions which satisfy conditions 1-4 are called 'penalizing', examine our interpretation of these conditions. The first states that if party 2 makes no concession from his most extreme demand throughout the negotiations, then he receives no compensation. This is the penalty for making no concession.

The second condition states that if party 2 asks for a certain constant level of utility for himself throughout the negotiations (except at t=0), then he again receives no compensation. One rationale is that party 2 had never demanded more than what he finally received, hence he is not entitled to any compensation. Another explanation would be that this is a penalty for insisting on a particular level of utility, irrespective of the other party's behavior.

The third condition guarantees that if party 2 asks for less throughout the negotiations but still does not make the final concession, then his final reward is lower. More generous behavior is interpreted as weakness and is penalized when it is not followed by a yielding concession.

The fourth condition states that by insisting on a certain utility for a longer period, and still letting party 1 make the final concession, party 2 cannot increase his final reward.

Theorem 1: The only incentive arbitration function in G satisfying 1-4 is g^0.

For each bargaining chain $\overline{C}\varepsilon C_{ec}$, let us define the t-subplays of $\lambda(C)$. For each $L = (L_1,L_2)$, where $L_i = \{z^i(t),|0\leq t\leq T\}$, for i=1,2, L^t is a t-subplay if $L^t = (L_1^t,L_2^t)$, where $L_i^t = \{z^i(t)|z^i(s)\varepsilon L_i, 0\leq s\leq t\}$, for i = 1,2. A strategy for party i assigns a point z to each t-subplay of $\lambda(C)$, which means that at time t, when the t-subplay becomes known to party i, he chooses to make his next proposal z at time t. There is no need for more formality here, because the strategy which will interest us is particularly simple. Let this strategy be called the 'one-concession' strategy. It consists of a simple rule. For each time t, let $z_i^i(t-)$ denote the point $\lim_{\varepsilon\to 0} \sup \{z_i^i(s) | t-\varepsilon<s<t\}$. The rule for party 1 is that if $(z_1^1(t-), z_2^2(t-))$ is in B(0), he chooses $z^1(t)$ to be the point of P(B(t)) such that $z_2^1(t) = z_2^2(t-)$. Otherwise, he chooses $z^1(t) = (\bar{x}_1(B(t)), w_2(B(t)))$.

Theorem 2: The 'one-concession' strategies constitute a strong Nash Equilibrium.

When both parties adopt the 'one-concession' strategy, the resultant play is given by $L = (L_1,L_2)$ with $L_1 = \{(\bar{x}_1(B(t)), w_2(B(t))) | 0\leq t\leq t^*\}$, $L_2 = \{(w_1(B(t)), \bar{x}_2(B(t))) | 0\leq t\leq t^*\}$; t^* is the smallest t satisfying $(\bar{x}_1(B(t)), \bar{x}_2(B(t))) \in P(B(0))$. The incentive arbitration function g^0 has obviously a very strong effect on the bargainer behavior. Each strategy degenerates into a 'one-concession' strategy. Less degenerate strategies, involving probability distributions on the concession time, are expected in the case of incomplete information.

4. FAIRNESS REQUIREMENTS

Consider a subset $\bar{\bar{C}}$ of $\bar{C}_{ec}$, which contains all the bargaining chains $C = \{B(t) | 0\leq t\leq T\}$ such that there exists t, $0\leq t\leq T$, for which the point $(\bar{x}_1(B(t)), \bar{x}_2(B(t)))$ is in B(o). A solution function on $\bar{\bar{C}}$ is any function $h:\bar{\bar{C}}\to R^2$ satisfying the following requirements:

1. Monotonicity. Let $C = \{B(t) | 0\leq t\leq T\}$, $C' = \{B'(t) | 0\leq t\leq T\}$ be two bargaining chains in $\bar{\bar{C}}$. If $B'(t)\subseteq B(t)$, and $\bar{x}_i(B'(t)) = \bar{x}_i(B(t))$ (for i = 1 and 2) for all t, then $h(C')\leq h(C)$.
2. Individual Rationality. Let $C = \{B(t) | 0\leq t\leq T\}\in\bar{\bar{C}}$. If there exists an s, $0\leq s\leq T$, and $x \in SP(B(0))$ such that $SP(B(s))=\{x\}$, then $h(C) = x$.

The first requirement guarantees that if there is a change either in the utility functions of the parties or in the set of feasible agreements, which can be interpreted as beneficial to both parties, then their utility from the arbitrated solution does not decrease.

The second requirement guarantees that the bargainers do not receive from the arbitrator less than what they would give themselves in unarbitrated negotiations. If there is no agreement at time s, then it is certain, assuming the parties are 'rational', that they settle on the point x, because at time s they cannot gain by further negotiations.

Theorem 3: The only function $h:\overline{\overline{C}}\to R^2$ satisfying 1 and 2 is given by $h^0(C) = SP(B(0)) \cap \{(\bar{x}_1(B(t)), \bar{x}_2(B(t))) | 0 \le t \le T\}$.

It is easy to see that h^0 satisfies several 'natural' requirements like Strong Pareto Optimality ($h^0(C) \in SP(B(0))$), Independence of Affine Transformations (if $F:R^2 \to R^2$ is affine, then $h^0(F(C)) = F(h^0(C))$), and Symmetry (if B(t) is symmetrical for all t, then $h_1^0(C) = h_2^0(C)$). These requirements were used in Livne [2], where they, and several additional requirements, lead to an entirely different solution (the Extended Raiffa Solution). The difference is explained by the introduction of the Monotonicity assumption which was avoided there. (Individual Rationality is fairly innocuous. It is satisfied by the Extended Raiffa Solution also.)

The solution function h^0 leads of course to the same point to which the incentive-arbitration function g^0 leads. For each utility function of the form

$$\begin{aligned} U_1(a,t) &= \exp(-r_1 t)a \\ U_2(a,t) &= \exp(-r_2 t)(1-a) \end{aligned} \qquad a \in [0,1],\ 0 \le t < \infty ,$$

$h^o(C)$ is the point $(x,1-x)$ where x satisfies $x + x^{r_2/r_1} = 1$. For $r_1/r_2 = 2$ we get $h^o(C) = (.382, .618)$. For $r_1/r_2 = 3$ we get $h^o(C) = (.318, .682)$. In comparison, the Extended Raiffa Solution (Livne [2]) would be the points (.333, .666), (.25, .75) respectively. The incentive-arbitration solution penalizes the 'weak' player less than the Extended Raiffa Solution, basically because the former limits the time horizon which affects the solution, while the latter does not.

APPENDIX

Proof of Theorem 1: It is easy to see that g^0 satisfies 1-4. Condition 4 is satisfied with an equality (i.e., $g_1^0(C,L,1) = g_1^0(C,M,1)$). Assume that g satisfies 1-4. Let $L = (L_1,L_2)$, where $L_1 = \{z^1(t) \mid 0 \leq t \leq T\}$, $L_2 = \{z^2(t) \mid 0 \leq t \leq T\}$. We assume 1 is the conceding party. Let $L' = (L_1', L_2')$ be a play in C that satisfies the following: (1) $t^*(L) \leq t^*(L')$; (2) $L_2 \subseteq L_2'$; (3) every $(y_1,y_2) \varepsilon L_2' - L_2$ satisfies $y_2 = z_2^*(L)$; (4) $z_1^*(L') = w_1(B(t^*(L)))$. Such L' always exists. Let L'', represented by $\{(z^{1}{}''(t), z^{2}{}''(t)) \mid 0 \leq t \leq T\}$, be such that (1) $z_2''(t) = z_2^*(L)$, for all $0 < t \leq T$; (2) $z^*(L'') = z^*(L')$. Let L''' satisfy (1) $L_2''' = \{(\bar{x}_2(B(t)), w_1(B(t))) \mid 0 \leq t \leq t^*(L')\}$; (2) $z^*(L''') = z^*(L')$. Then from condition 4, $g_2(C,L,1) \leq g_2(C,L',1)$. From condition 3, $g_2(C,L',1) \leq g_2(C,L'',1)$ and $g_2(C,L',1) \leq g_2(C,L''',1)$. From condition 2, $g_2(C,L'',1) = g_2(C,L''',1) = g_2^0(C,L'',1)$. Since $g_2(C,L,1) \geq g_2^0(C,L,1)$ we get $g_2(C,L,1) = g_2^0(C,L,1)$.

Proof of Theorem 2: The theorem is an immediate result of the conditions satisfied by bargaining chains in $\bar{\bar{C}}$. It is easv to see that using the 'one-concession' strategy each party can guarantee a reward which is smaller than his reward of the point $P(B(0)) \cap \{(\bar{x}_1(B(t)), \bar{x}_2(B(t))) \mid 0 \leq t \leq T\}$. The strategies are in equilibrium, although this is not the only equilibrium.

Proof of Theorem 3. First, h^0 is well defined. The curve $\{(\overline{x_1},(B(t)), \overline{x_2}(B(t)))|0\le t\le T\}$ is a continuous curve in the plane which intersects the Pareto frontier of B(0), also a continuous curve. If they intersect at SP(B(0)), then due to the slopes of both curves, they do not have another intersection point on SP(B(0)). Suppose they intersect at the point y on the Pareto frontier of B(0). Let t be such that $(\bar{x}_1 B(t)), \bar{x}_2(B(t))) = y$. Assume the Pareto frontier is horizontal at y, i.e., $\bar{x}_2(B(s)) = \bar{x}_2(B(t))$ for all $s\ge t$. Define y^1 by $y_2^1 = y_2$, $y_1^1 = \max\{\bar{x}_1(B(S))|0\le s\le t, (\overline{x_1}(B(t), y_2^1)$ is on the weak Pareto frontier$\}$. The point y^1 is on the strong Pareto frontier, and the curves intersect there.

It is easy to check that h^0 satisfies 1 and 2. We show now that it does it uniquely. Let $C = \{B(t)|0\le t\le T\}$ be in $\bar{\bar{C}}$, and let s be the smallest time such that $(\bar{x}_1(B(s)), \bar{x}_2(B(s)))\varepsilon SP(B(0))$. Define $C^s = \{B(t)|0\le t\le s\}\cup\{w(B(s))\}$. For all $0\le t\le s$ let $B^1(t)$ be the convex hull of B(t) and the point $(\overline{x_1}(B(s)), \overline{x_2}(B(s)))$. It is easy to see that the set $D = \{B^1(t)|0\le t\le s\}\cup\{w(B(s))\}$ is in $\bar{\bar{C}}$. From requirements 1 and 2 we get $h(C) = h(C^s) \le h(D)$. But $h(D) = (\bar{x}_1(B(s)), \bar{x}_2(B(s)))$, and this point is on SP(B(0)). Therefore $h(C) = h^0(C)$.

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