COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 525

STOCHASTIC GAMES, OLIGOPOLY THEORY

AND COMPETITIVE RESOURCE ALLOCATION

Martin Shubik and Matthew J. Sobel

May 30, 1979

PROJECT CENTER FOR COMPETITIVE AND CONFLICT SYSTEMS RESEARCH

STOCHASTIC GAMES, OLIGOPOLY THEORY AND COMPETITIVE RESOURCE ALLOCATION*

by

Martin Shubik and Matthew J. Sobel

ABSTRACT

We define discrete time sequential games which are multiperson Markov decision processes. The extant theory is sketched and compared with our assessment of research needs in dynamic models of oligopoly and other competitive resource allocation problems. A special class of economic survival games is noted.

1. INTRODUCTION

An oligopolistic market is one with only a few firms who supply the commodity being purchased. Oligopoly theory, until recently, evolved without regard to the institutional details encountered in specific markets and without addressing the role played by time. Oligopoly models were treated statically, or at best, conversationally dynamically. However, dynamic oligopoly models have been analyzed with increasing frequency

---

*This work relates to Department of the Navy Contract N00014-77-C-0518 issued by the Office of Naval Research under Contract Authority NR 047-006. However, the content does not necessarily reflect the position or the policy of the Department of the Navy or the Government, and no official endorsement should be inferred.

The research was partially sponsored by National Science Foundation Grant SOC 78-05770 and Office of Naval Research Grant ENG 76-46156.

in recent years and some of these analyses are responsive to institutional details.

Here we compare the literature on dynamic models of oligopoly with our construal of the objectives of oligopoly theory. We use discrete time sequential games, sometimes called "stochastic games," as a canonical form in which to discuss the issues. The stochastic game model encompasses many interesting oligopoly models and it seems to offer an appropriate level of generality to address research needs. Incidentally, we do not believe that there is any importance to economic theory associated with the distinction here between continuous and discrete time models, i.e., between stochastic games and differential games. In principle, the discussion could be couched in terms of continuous time models instead of stochastic games.

We believe that the primary objective of oligopoly theory is to provide an understanding of pricing and resource allocation over time in large firms and the consequent market behavior of such firms. We begin by enumerating some issues involving dynamics that are inherent in this goal. We sketch a general stochastic game model and use the model to address the task of constructing satisfactory dynamic oligopoly models. This effort in part becomes a specification of research needs and opportunities in stochastic game theory and oligopoly theory. At this stage in the development of both subjects, it is useful to identify problems rather than only describe past accomplishments. Section 5 cites specific recent results, and Section 6 presents a blending of problems in optimization and survival.

2. DYNAMICAL ISSUES IN OLIGOPOLY

What are some characteristics of the dynamics of pricing and resource allocation in large firms and their consequent market behavior? Here, we mention three kinds of problems. Firstly, there are the dynamics of the composition of the set of firms in the market. This "entry and exit" problem in oligopoly is the subject of current research but most analyses have either been static or ignored the multiplicity of "players" in such games. A closely related problem is how to distinguish "competition among the few" from "competition among the many." The modeling issue is how large must a market become in order for game-like individual behavior to become unimportant. Secondly, in a given oligopolistic market, why do prices fluctuate as they do? In most markets, the prices fluctuate more slowly than the prices of the factors comprising the inputs in the production process. This phenomenon of "sticky prices" is widely recognized but has hardly been analyzed in a dynamic oligopoly model. Lastly, in some oligopolistic markets, there is one firm that acts as a leader in changing the price level. Why? Why is there price leadership behavior in some markets but not in others? Why might a firm act passively as a follower under some conditions but bolt the pack under other conditions?

Another collection of dynamical issues concerns the role of information in market behavior. How do firms tacitly communicate their objectives, strategies, and threats to one another? How do divisions of a large firm communicate with one another so that their decentralized actions are mutually supportive of the overall goals of the firm. This is the general problem of managerial control. Furthermore, how do accounting conventions affect firm and market behavior? Technically, this question can be posed in terms of alternative aggregations of information. Lastly, what are the effects of imperfections in information, particularly those due to delays in transmission of information? Little progress has been made on a general treatment of this last issue and the prior one has been analyzed in some detail only in static models (Team Theory).

What are the effects of market size? Most facets of this issue are not particularly dynamical in nature but we should know how to analyze them in dynamic models. As one example, what is the effect on product quality of the number and size of firms that are competing? An issue that is primarily dynamical is the dependence of the number of firms in the market upon the time rate at which information spreads, and vice versa.

Preference structures have been treated somewhat incidentally in oligopoly theory. Important research on intertemporal preference orderings is currently occurring (Kreps & Porteus, forthcoming) for models of individual decision making over time. Comparable investigations of dynamic multi-person decision models have not yet begun. The situation becomes even more complicated if we construct "behavioral models of the firm" (Cyert & March, 1963, Williamson, 1975) which discard the notion of a single monolithic "decision maker" making all the decisions in each firm. The models in "Team Theory," for example, can be construed as noncooperative games amongst players having the same preference ordering over outcomes but differing in the information and the actions available to each. We have yet to see an investigation of sequential models of this kind. Lastly, the dynamic oligopoly models analyzed thus far are predicated on a scalar objective such as each firm's discounted operating profit. However, various economists argue that managers in firms behave as if they were maximizing vector objectives. Components other than profit might include rate of growth in sales, number of employees, share of the market and survival. An important first step has been taken in the analysis of sequential games with vector payoffs (Henig, 1978) but this general theory has yet to be applied to a dynamic oligopoly model.

We now turn to some issues of constructing satisfactory dynamic oligopoly models. The canonical form of a general stochastic game will be useful for that purpose. The next section briefly defines a stochastic game and enumerates some notions of the "solution" of such a model.

3. STOCHASTIC GAMES

Let I be a set of players, S a set of states, and A_s^i a set of actions available to player $i \in I$ when the process is in state $s \in S$. These sets are assumed to be nonempty. The composite action of all the players, when the process is in state s, must be an element of $C_s = \times_{i \in I} A_s^i$. We write $a = (a^i) \in C_s$. An outcome of a stochastic game is a sequence $s_1, a_1, s_2, a_2, \ldots$ where $a_t \in C_{s_t}$ for all t. Let $W = ((s,a) : a \in C_s, s \in S)$.

The dynamics are determined by the decision rules used by the players to choose their actions and by a collection $\{q(\cdot|s, a) : (s,a) \in W\}$ of probability measures on B_s, the Borel subsets of S. For any period t and $H \in B_s$, if $s_t = s$ and $a_t = a$ then $q(H|s, a)$ is the probability that $s_{t+1} \in H$.

A two-person zero-sum matrix game is a special case of a stochastic game where S is a singleton. It is easy to see that in such a game, in general, one may wish to admit randomized strategies. This complicates the measurability and integrability issues which are already imbedded in the one person stochastic game, namely the Markov decision process.

Our exposition suppresses these issues which are the subject of some current research on stochastic games. The interested reader should read the fine survey by Parthasarathy and Stern (1977) and the neat recent paper by Whitt (1977).

With the preceding caveat, let π^i denote the set of player i's nonanticipative decision rules (including rules that are history dependent and randomized) for choosing a_t^i, for each t, on the basis of the outcome to date, namely $s_1, a_1, \ldots, s_{t-1}, a_{t-1}, s_t$. The stationary policies are particularly interesting decision rules. Let D_s^i be the set of probability measures on the Borel subsets of A_s^i. An element of D_s^i can be used to choose a randomized action when the game is in state s. Let $\Delta^i = \times_{s \in S} D_s^i$, $\Delta = \times_{i \in I} \Delta^i$, and $\pi = \times_{i \in I} \pi^i$. An element of π is a policy. A policy $\gamma \in \pi$ is stationary if there exists $\delta \in \Delta$ such that $\gamma = (\delta, \delta, \delta, \ldots)$ so $a_t = \delta(s_t)$ for all t. We write $\gamma = \delta^\infty$ in this case. Let Δ^∞ denote the subset of stationary policies in π. Finally, it is convenient to represent any $\gamma \in \pi$ as (γ^i, γ^{-i}) where $\gamma^{-i} \in \prod_{j \neq i} \pi^j$.

There has been some research on the ergodic properties of $\{(s_t, a_t)\}$ induced by stationary policies (cf. Sanghvi & Sobel, 1976, Sanghvi, 1978) but most literature concerns real-valued payoff functions. Let $r^i(s,a)$ denote the (expected) immediate reward to player i in any period t if the state s_t is s and the composite action a_t is $a \in C_{s_t}$. Let β_i be player i's single period discount factor and let

$$(1)\qquad \begin{cases} V^i(\gamma|s) = \sum_{t=1}^{\infty} \beta_i^{t-1} r^i(s_t, a_t) \\ \\ v^i(\gamma|s) = EV^i(\gamma|s) \end{cases}$$

denote the total discounted payoff and its expectation when $\gamma \varepsilon \pi$ is the policy and $s_1 = s$ is the initial state. Some literature concerns the average payoff per period rather than the discounted payoff but the latter is more appropriate for oligopoly and other open (or partial equilibrium) economic models.

A policy $\gamma \varepsilon \pi$ is said to be an <u>equilibrium point relative to</u> $H \subseteq S$ iff

$$(2)\qquad v^i(\gamma|s) = \sup\{v^i(\rho, \gamma^{-i}) : \rho \varepsilon \pi^i\}\ , \quad s \varepsilon H\ , \quad i \varepsilon I\ .$$

We say simply that γ is an <u>equilibrium point</u> if it is an equilibrium point relative to S. An equilibrium point relative to H is noncollusively optimal for every initial state in H and every player. Shapley (1953), in a magnificent early paper, established existence of an equilibrium point amongst stationary policies in a two-player model with $\bigcup_{s\varepsilon S} A_s^i$ a finite set for each player and $r^1(\cdot,\cdot) + r^2(\cdot,\cdot) \equiv 0$. More general existence results concern nonzero sum games played by more than two players (Rogers, 1969, Sobel, 1971, Parthasarathy & Stern, 1977, Whitt, 1977). Also, Henig (1978) has recently established existence of an equilibrium point for games where $r^i(\cdot,\cdot)$ is vector-valued.

4. MODELING ISSUES

We have observed that the extent theory fails to explain why price leadership occurs and which firms are likely to be followers while others are leaders. More generally, there is no satisfactory sequential analogue to "cooperative" theory for static games. The primitive element in most of the cooperative static theory is the coalition, namely a subset of the players who join together for mutual benefit. However, a satisfactory dymamic theory must admit changes in coalition composition as time passes but the present theory does not include this feature.

Another modeling issue stems from the embarrassment of riches provided by the size of the set of equilibrium points. It is known that the size of this set increases as information conditions in a game proliferate (cf, Dubey and Shubik, 1979). Therefore, oligopoly models that strive to include the design of information systems and managerial control may induce distressingly many equilibrium points. The problem is to decide which one, more generally which subset, is the appropriate object for analysis. We believe that the solution to this problem should vary depending upon the context which motivates the model. In other words, behavioral considerations and institutional details should direct our definition of "the appropriate object for analysis."

It has already been mentioned that no satisfactory canonical model exists to analyze the effects of imperfections in information due to delays in transmission. Indeed, the intricacy of the analysis in a relatively simple case analyzed by Scarf and Shapley (1957) is alarming. We doubt that a Bayesian approach is appropriate here although one of us has explored this elsewhere (Sanghvi and Sobel, 1976). The extant theory of stochastic games would oblige us to assume that each firm knows the prior distribution held by every other firm.

Careful modeling of many industries leads to the explicit inclusion of bankruptcy conditions in a model. Such conditions exemplify "exit fees" in the class of Markov decision processes called stopping problems. There are several interesting qualitative results concerning the structure of optimal policies in stopping problems. As yet, there is no comparable theory for "stopped sequential games." The payoff to oligopoly theory from developing such a theory might include a deeper understanding of the effects of alternative bankruptcy laws and the dynamics associated with the entry and exit of firms from an industry. One of us has suggested a class of "Games of Economic Survival" to pick up the ruin possibilities (cf. Section 6, Shubik, 1958, Shubik & Thompson, 1959).

This list of modeling issues is necessarily brief and we have not discussed some pertinent material. Aumann (1959) has developed results for "supergames." A supergame is a sequence of static games in which the nature of the static games is not contingent on players' past actions. The case of the same static game at each point (in the sequence) has been investigated more than any other. This case is a stochastic game with $|S| = 1$. Friedman (1977) analyzes this case in oligopoly models. He focuses on "reaction function" strategies; each player's present decision is contingent on the opponent's preceding decision. Such decision rules induce a stochastic game in which S is the set of players' possible single game decisions. Recently, Rosenthal (forthcoming) has investigated sequences of games with varying opponents. His point of view may be useful for construction of dynamic models of entry and exit in oligopoly. Shefrin (1978) has interesting results for dynamic market games with incomplete information.

5. SPECIFIC RESULTS

Stochastic game models of oligopoly, even with the limitations enumerated above, are forbiddingly complex. Nevertheless, some progress has been made either by reducing the potential complexity or by building a model for a particular kind of industry and then posing correspondingly special questions. Firstly, we discuss the reduction of complexity.

Stochastic game models are difficult to analyze because the number of players is greater than one, so the players interact with one another, and because the game process extends over time and each player indulges in a variety of intertemporal tradeoffs. Several writers (Lippman, 1977 and its references) have suppressed the complexity due to the interaction of firms by analyzing models of leader-follower behavior where the identities of the leader and followers are known at the outset. The problem is then the selection of an optimal dynamic policy by the leader and this latter problem is a (one person) Markov decision process which is much less complex than a stochastic game.

Another suppression of complexity has been obtained by preserving a multiplicity of players (firms) but reducing the original dynamic game to a static game. Specifically, an equilibrium point of a stochastic game is said to be myopic if it consists of the ad infinitum repetition of an equilibrium point of a static game. The principal sufficient conditions (Sobel, 1978), satisfied by various dynamic oligopoly models, are:

(3a) for each $i \in I$ and $(s,a) \in W$, $r^i(s,a)$ depends additively on the state s and action a, i.e., there are functions K^i and L^i such that

$$r^i(s,a) = K^i(a) + L^i(s) ;$$

(3b) transition probabilities depend on the actions taken but not on the state from which transition occurs, i.e.,

$$q(H|s, a) = p(H|a) \quad \text{for all} \quad H \in B_S \quad \text{and} \quad (s,a) \in W ;$$

(3c) suppose the static game Γ , defined below, has an equilibrium point a_* in pure strategies and let $S^* = \{s : s \in S, a^* \in C_s\}$. Then $p(H|a^*) = 1$ for all $H \in B_s$ having $S^* \subseteq H$, i.e. if $s_1 \in S^*$ then a^* is repeatable <u>ad infinitum</u> (with probability one).

Let $\xi(a)$ be a random variable with the measure $p(\cdot|a)$ in (3b). Then I is the set of players in the static Nash game Γ, player i's payoff function is $K^i(a) + \beta_i E[L^i(\xi(a))]$, $a \in \bigcup_{i \in I} A^i$, and player i has available the set of moves $A^i = \bigcup_{s \in S} A^i_s$. If a_* is randomized then there is an assumption comparable to (3c). It follows from (3) that $a_t = a_*$ for all t is an equilibrium point [in the stochastic game sense of (2)] relative to S^* .

Numerous Markov decision processes in the literature satisfy (3) but the myopia of their optimal policies was either overlooked or deduced by special and sometimes intricate arguments. Also, various oligopoly models satisfy (3). Kirman and Sobel (1974) assume that firms make production and pricing decisions each period and they have linear production costs and arbitrary single-period inventory-related costs. The oligopoly model in Sobel (1977) focuses on advertising decisions. It is assumed there that each firm's demand each period is a random variable whose distribution depends on all firms' "goodwill." The goodwill is an exponentially weighted moving average of past amounts spent on advertising. Myopia has been applied to other oligopoly models where the competition involves expenditures on research and development, expansion of capacity, and the harvesting of interacting fish species in a coastal fishing industry.

6. GAMES OF ECONOMIC OR SOCIAL SURVIVAL

The goals of profit maximization or cost minimization are present in many economic models. Games of survival, as characterized by Shapley (1953), stress the binary outcomes of survival or not. Yet in many social, ecological and economic processes the goals include both survival and optimization of the quality of life for the survivor.

Below we describe a general class of games of social or economic survival. With notational changes, they can be recast as stochastic games.

An n person game of economic survival is described as follows by:

$$\alpha^i(t) \ ; \ W_1^i \ ; \ B_1^i \ ; \ S_1^i \ ; \ L_1^i \ ; \ \Psi_1^i \ ; \ \text{and} \ V^i \ ,$$

where:

$\alpha^i(t)$, $i = 1, \ldots, n$ are the single period payoffs faced by the players at time t . They depend on actions described below and will in general depend upon time.

W_t^i , $i = 1, \ldots, n$ and $t = 1, 2, \ldots$ are the wealths of the players at the start of time t . Initial assets W_1^1 and W_1^2 are given as parameters.

B^i are the ruin conditions, bankruptcy levels or "absorbing barriers" for the player; i.e., if the assets (or strength or viability) of a player i drop to below B^i , that individual is out of the game.

S^i are the survival values; i.e., if individual i is the sole survivor in the game, S^i is the present value of the remaining one person game.

L^i are the liquidation values. If an individual i is ruined, he may still have residual assets at the point of ruin. The value of these assets is given by L^i.

Ψ^i are discount factors. We assume that each individual has a discount factor on future consumption. The Ψ^i could be dependent upon the age of the individual, thus reflecting life cycle considerations.

T is the time at which bankruptcy (if any) occurs: $T = \inf\{t : (W_t^1, W_t^2) \not> (B^1, B^2)\}$.

V^i is the payoff function to player i. It cannot be fully specified until the strategies and the relationship among income, consumption, and survival are specified.

Two versions are given, the game where pure survival is the goal and the game where the maximization of expected discounted consumption (or utility $\phi^i(\cdot)$ of consumption) is optimized.

At the start of any time t an individual i has W_t^i. A Markov strategy by an individual i is a plan for the selection of an investment amount x_t^i and a consumption amount b_t^i dependent upon $W_t = (W_t^1, \ldots, W_t^n)$.

$$W_{i+1}^i = \xi_t[W_t^i + \alpha^i(x_t^1, \ldots, x_t^n) - b_t^i - x_t^i]$$

where $0 < x_t^i + b_t^i < W_t^i$ and $\xi_1, \xi_2, \ldots$ are independent and identically distributed random variables.

Pure Survival

$$V^i = 1 \text{ if } W_t^i > B^i \text{ for } t = 1, 2, \ldots,$$
$$= 0 \text{ otherwise.}$$

Pure Consumption Optimization

$$V^i = \sum_{t=1}^{\infty} (\Psi^i)^{t-1} \phi^i(b_t^i) \qquad W_t^i > B^i ,\quad t = 1, \ldots, \infty \text{ and game continues indefinitely.}$$

$$= \sum_{t=0}^{T} (\Psi^i)^{t-1} \phi^i(b_t^i) + (\Psi^i)^T S^i , \qquad W_t^i > B^i ,\quad t = 1, \ldots, T \text{ and game continues until } T.$$

$$= \sum_{t=1}^{T} (\Psi^i)^{t-1} \phi^i(b_t^i) + (\Psi^i)^T L^i \qquad W_t^i > B^i ,\quad t = 1, \ldots, T-1 , \; W_T^i \le B^k \text{ and game continues until } T-1.$$

For either criterion, let I indicate which player (if either) is ruined:

$$I = \begin{cases} 0 & \text{if } T = \infty \\ j & \text{if } W_T^j \le B^j . \end{cases}$$

Then, for either criterion, consider the vector (V^1, V^2, T, I) . The adoption of a policy by the players induces a (joint) probability distribution of this vector and we may compare the distributions induced by alternative policies. An obvious comparison is according to $v^i = E(V^i)$. For example, in the pure survival criterion, $v^i = P\{V^i = 1\} = P\{I \neq i\}$. Although we shall not pursue the matter here, the two criteria can be treated in a unified manner by first defining an appropriate stochastic game.

6.1. A "Guns or Butter" Example

A two player example illustrates the tradeoff between consumption and survival. Let b_t and x_t denote player 1's consumption and investment in period t and let d_t and y_t denote the same amounts for player 2.

Let

$$\alpha^1(t) = A\frac{x_t}{x_t + y_t} \quad \text{if } x_t + y_t > 0$$

$$= 0 \quad \text{if } x_t = y_t = 0$$

$$\alpha^2(t) = A\frac{y_t}{x_t + y_t} \quad \text{if } x_t + y_t > 0$$

$$= 0 \quad \text{if } x_t = y_t = 0 \ .$$

Let $W_1^1 = M$; $W_1^2 = M$; $B^1 = B^2 = L^1 = L^2 = 0$, $S^1 = S^2 = A/(1-\Psi)$; $\Psi^i(c) = c$; $\Psi^1 = \Psi^2 = \Psi$ where $0 < \Psi < 1$. Say $b_t^1 = b_t$ and $b_t^2 = d_t$.

If the goals are pure survival then $x_t = y_t = b_t = d_t = 0$ gives $M_{t+1} = \xi_t M_t > 0$, $m_{t+1} = \xi_t m_t > 0$ where $\xi_t > 0$ is a random variable.

If the goals are maximization of expected consumption then if a solution with joint survival is feasible, player 1 wishes to

$$\max_{b_t, x_t} E \sum_{t=1}^{\infty} (\Psi^i)^{t-1} b_t \ , \quad b_t \geq 0 \ , \quad x_t \geq 0$$

subject to $0 \leq b_t$, $0 \leq x_t$,

$$b_t + x_t \leq M_t$$

and

$$M_{t+1} = \xi_t \left[M_t - b_t + A\left(\frac{x_t}{x_t + y_t}\right) - x_t \right]$$

and player 2 wishes to

$$\max_{d_t, y_t} E \sum_{t=1}^{\infty} (\Psi^i)^{t-1} d_t$$

subject to $0 \leq d_t$, $0 \leq y_t$,

$$d_t + y_t < m_t$$

and

$$m_{t+1} = \xi_t \left[m_t - d_t + A\left(\frac{y_t}{x_t + y_t}\right) - y_t \right] .$$

Let $\mu = E[\xi_t]$. If $\Psi\mu \leq 1$ and $P\{\xi \geq \Psi\mu/2\} = 1$ (for which $P\{\xi \geq 1/2\} = 1$ is sufficient) then we can show that there is a myopic equilibrium point with respect to $\{(M_1,m_1); (M_1,m_1) \geq \Psi\mu A/4(1,1)\}$ given by

$$x_t = y_t = \mu A\Psi/4, \quad \text{for} \quad t = 1, 2, \ldots$$

$$b_t = d_t = A(2\xi_{t-1} - \Psi\mu)/4, \quad \text{for } t = 2, 3, \ldots$$

$$b_1 = M_1 - \mu A\Psi/4, \quad d_1 = m_1 - \mu a\Psi/\ 4$$

and expected payoffs are:

$$v^1 = M_1 - A\mu\Psi/4 + \sum_{t=2}^{\infty} \Psi^{t-1}(2\xi_{t-1} - \Psi\mu)A/4$$

$$= M_1 + A\mu\Psi/[4(1-\Psi)]$$

$$v^2 = m_1 + A\mu\Psi/[4(1-\Psi)].$$

If $P\{\xi_1 < \Psi\mu/2\} > 0$, or $(M_1,m_1) \not\geq \Psi\mu A/4(1,1)$, or $\Psi\mu > 1$ then the analysis and solution become complicated.

The "net earnings" or gains from competition are given by the terms

$$A\frac{x_t}{x_t + y_t} - x_t .$$

These portray the resource struggle or the "battle conditions." The payoffs involve only the b_t, i.e., the resources drawn off for consumption.

REFERENCES

Aumann, R. J. (1959). "Acceptable points in general cooperative *n*-person games," in R. D. Luce and A. W. Tucker, editors, *Contributions to the Theory of Games IV*, Ann. of Math. Study 40, Princeton, New Jersey, Princeton University Press, pp. 287-324.

Cyert, R. M. and J. G. March (1963). *A Behavioral Theory of the Firm*. Englewood Cliffs, N.J.: Prentice Hall.

Dubey, P. and M. Shubik (forthcoming, 1979). "Information Conditions, Communications, and General Equilibrium," *Math of Oper. Res.*

Friedman, J. W. (1977). *Oligopoly and the Theory of Games*. Amsterdam: North-Holland.

Henig, M. (1978). *Multicriteria Dynamic Programming*, Ph.D. Thesis, Yale University.

Kirman, A. P. and M. J. Sobel (1974). "Dynamic Oligopoly with Inventories," *Econometrica*, Vol. 42, 279-287.

Kreps, D. M. and E. L. Porteus (forthcoming). "Temporal Resolution of Uncertainty and Dynamic Choice Theory," *Econometrica*.

Lippman, S. A. (1977). "Optimal Pricing to Retard Entry," *Review of Economic Studies* (December).

Marschak, J. and R. Radner (1972). *Economic Theory of Teams*. New Haven: Yale University Press.

Parthasarathy, T. and M. Stern (1977). "On Markov Games," *Lecture Notes in Pure and Applied Mathematics*, Vol. 30. Marcel Dekker Publishing Company, pp. 1-46.

Rogers, P. D. (1969). "Nonzero-Sum Stochastic Games," Report ORC 69-8, Operations Research Center, University of California, Berkeley.

Rosenthal, R. W. (forthcoming). "Sequences of Games with Varying Opponents," *Econometrica*.

Sanghvi, A. P. (1978). "Sequential Games as Stochastic Processes," *Stochastic Processes and Their Applications*, Vol. 6, pp. 323-336.

__________ and M. J. Sobel (1976). "Bayesian Games as Stochastic Processes," *International Journal of Game Theory*, Vol. 5, pp. 1-22.

Scarf, H. E. and L. S. Shapley (1957). "Games with Partial Information," in M. Dresher, A. W. Tucker, and P. Wolfe (eds.), *Contributions to the Theory of Games, Volume III*. Princeton, N.J.: Princeton University Press, pp. 213-230.

Shapley, L. S. (1953). "Stochastic Games," *Proceedings of the National Academy of Sciences, USA*, Vol. 39, pp. 1095-1100.

Shefrin, H. M. (1978). "Bayesian Convergence in Market Games with Incomplete Information," presented at the Third Kingston Conference on Differential Games, June.

Shubik, M. (1958). Strategy and Market Structure. New York: Wiley.

_________ and G. L. Thompson (1959). "Games of Economic Survival," Naval Research Logistics Quarterly, Vol. 6, pp. 111-123.

Sobel, M. J. (1971). "Noncooperative Stochastic Games," Annals of Mathematical Statistics, Vol. 42, pp. 1930-1935.

__________ (1977). "Sequential Marketing Games," unpublished manuscript.

__________ (1978). "Myopic Equilibrium Points in Stochastic Games," unpublished manuscript.

Whitt, W. (1977). "Representation and Approximation of Noncooperative Stochastic Games," unpublished manuscript.

Williamson, O. E. (1975). Markets and Hierarchies. New York: Free Press.