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THE OPTIMAL PAYMENT OF UNEMPLOYMENT INSURANCE BENEFITS OVER TIME

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1. Introduction

The primary purpose of unemployment insurance (U.I.) is no doubt to insure individuals against the loss of wage income. However, U.I. is commonly believed to lengthen the duration of unemployment because of its effect on the effort devoted to job search and on the minimum acceptable wage offer, the "reservation wage." Of course, if the government monitored job search behavior, no such problem would exist: U.I. benefits could be withheld if effort or the reservation wage was unsatisfactory. But this is not easy to do, so that, in fact, monitoring of job search behavior is limited.

With these issues in mind, this paper asks how U.I. benefits ought to be paid over time.² Specifically, the paper asks how to pay benefits over time so as to maximize the expected utility of the unemployed, subject to two constraints. The first is that the unemployed are assumed to act in a personally optimal way given the U.I. program. And the second is that the total size of the U.I. budget is taken as fixed (see Baily [1975] for a discussion of the determination of the correct size of the budget). The paper does not inquire into how individuals become unemployed and consequently is not concerned with the effect of U.I. on layoff and quit behavior (see Feldstein [1974, 1976]).

The problem is initially studied under the assumption that unemployed individuals have no wealth and cannot borrow. This seems to be a reasonable approximation of reality for some but certainly not for all of the unemployed. Under this assumption (i) if it is supposed that individuals have no influence over the probability of getting a job (a case which we think it informative to consider), then the optimal time sequence of benefits is a constant sequence. This is as expected, since a constant sequence equates the marginal utility of benefits in the different periods. However, (ii) if it is assumed that individuals do have an influence over the probability of getting a job -- and that the government does not monitor individual behavior -- then the optimal time sequence declines and, although always remaining positive, tends to zero. A declining sequence is desirable (individuals are induced to get jobs sooner, at least on average) even though it reduces the role of benefits as insurance (individuals who have the bad luck to remain unemployed a long time collect lower benefits). This result is illustrated by computing the optimal time path of benefits using estimates from other studies of the effect of unemployment insurance on the duration of unemployment. If it is instead assumed that the government monitors individual behavior, then because there is no problem of adverse incentives, the optimal time sequence of benefits is a constant sequence.

The problem is then examined under the assumption that unemployed individuals begin their spell of unemployment with positive wealth -- or, equivalently, that they may borrow -- and may save or dissave. Under this assumption, (iii) if it is supposed that individuals have no influence over the probability of getting a job, then the optimal time sequence of benefits starts at zero and then jumps to a positive and constant level. This is because benefits ought to be given when the marginal utility of wealth is sufficiently high, which is when individuals have been unemployed long enough to have sufficiently depleted their wealth. However (iv) if it is assumed that individuals do have an influence on the probability of getting a job, it is suggested that the optimal time sequence may rise and then fall, reflecting results (ii) and (iii). Although this case proved analytically intractable, the optimal solution to a two period version of the model was calculated. The complicated nature of that solution indicates that no simple general qualitative results are possible -- the solution depends on the magnitudes of conflicting (but intuitively plausible) forces.

2. The model

The formal problem is to pay benefits over time so as to maximize the expected utility of newly unemployed individuals subject to the constraints that individuals act in a self-interested way given the presence of U.I. and that expected discounted per capita benefits are fixed. As noted in the introduction, it will not be asked why individuals become unemployed. However individuals become unemployed, and whatever the size of the U.I. budget, certainly the budget should be used in a manner which maximizes expected utility.³

The individuals in the model are assumed to be identical and risk averse. They are assumed to be unable to borrow in the case when they begin their spells of unemployment without wealth⁴ but to be able to borrow, save and dissave in the other case.

During each period, it is assumed that an unemployed individual first collects U.I. benefits and then either finds a job or does not. If he finds a job, it is assumed for convenience that he works there forever.⁵

The probability of finding a job in each period is either taken to be exogenous or to depend on individual behavior. When the former assumption is made, it is not because we think it necessarily describes an empirically important situation, but rather because we think it is informative to determine how benefits ought to be paid if their only role is as insurance, thus ignoring any adverse incentive effects. When the latter assumption is made,

individuals are assumed to look over job offers and to accept any one with a wage as high as the reservation wage. The choice of the reservation wage is made each period and depends on the time sequence of future U.I. benefits and the probability distribution of wage offers. This distribution is assumed to be influenced by the effort devoted to job search. Such effort (which may be interpreted as a subtraction from leisure) is assumed to involve disutility. The probability distribution of wage offers as a function of effort is assumed to be known and, for simplicity, fixed from period to period.

Let us define the following notation before beginning the analysis in the next section.

$U(\cdot)$	strictly concave increasing function giving the utility of consumption each period
b_t	unemployment insurance benefit paid at the beginning of the t^{th} successive period of unemployment
p_t	probability of finding a job in period t
e_t	effort (measured in utility cost) devoted to job search in period t
w_t	wage offer in period t
w_t^*	reservation wage in period t
$f(w_t, e_t)$	probability density of a wage offer given effort
r	one period discount rate (assumed to be equal to the interest rate)

Let B_1 be the expected discounted amount that the government has in the U.I. fund per unemployed individual, so B_1 , which is assumed to be positive, satisfies

$$(1) \quad B_1 = b_1 + (1-p_1)b_2/(1+r) + (1-p_1)(1-p_2)b_3/(1+r)^2 + \dots$$

$$= \sum_{t=1}^{\infty} b_t \prod_{j=1}^{t-1} (1-p_j)/(1+r)^{t-1}.$$

(Note that $\prod_{j=1}^{t-1} (1-p_j)$ is the probability of being unemployed at

the beginning of period t .) More generally, let

$$(2) \quad B_t = \sum_{k=t}^{\infty} b_k \prod_{j=t}^{k-1} (1-p_j)/(1+r)^{k-t},$$

so that B_t is expected benefits, discounted to t , which are paid to an individual from t onward, given that he is unemployed at t .

Observe that

$$(3) \quad B_t = b_t + (1-p_t)[b_{t+1}/(1+r) + (1-p_{t+1})B_{t+2}/(1+r)^2].$$

3. The optimal time sequence of benefits assuming that the unemployed have no wealth and cannot borrow.⁶

In this case, an individual who is unemployed at the beginning of period t receives and consumes b_t , enjoying utility from consumption of $U(b_t)$. If he finds and takes a job during the period, it is assumed that he will begin to be paid for work at the job in period $t + 1$.

Let us first suppose that the probabilities are fixed. Thus, we will not consider the role of effort and we will assume that the wage of any job which might be found is known with certainty to be w -- so that the value, discounted to t , of being paid for a job beginning in t is $U(w) (1 + 1/(1+r) + 1/(1+r)^2 + \dots) = U(w)/r$. Consequently, discounted expected utility of a newly unemployed individual is

$$\begin{aligned}
 (4) \quad EU_1 &= p_1 [U(b_1) + U(w)/(r(1+r))] + \\
 &\quad p_2 (1-p_1) [U(b_1) + U(b_2)/(1+r) + U(w)/(r(1+r)^2)] + \\
 &\quad p_3 (1-p_1)(1-p_2) [U(b_1) + U(b_2)/(1+r) + U(b_3)/(1+r)^2 + \\
 &\quad U(w)/(r(1+r)^3)] + \dots \\
 &= \sum_{t=1}^{\infty} \{ [p_t \prod_{j=1}^{t-1} (1-p_j)] [\sum_{j=1}^t U(b_j)/(1+r)^{j-1} + U(w)/ \\
 &\quad (r(1+r)^t)] \}
 \end{aligned}$$

The t 'th term in EU_1 gives the contribution to discounted expected utility if a job is found in period t . The first factor in brackets is the probability of finding a job in period t . The second is the discounted utility if that happens, which is comprised of the utility from t periods of U.I. benefits plus that from wages earned later. The probability of never finding a job is zero.⁷

More generally, define

$$(5) \quad EU_t = \sum_{k=t}^{\infty} \{ [p_k \prod_{j=t}^{k-1} (1-p_j)] [\sum_{j=t}^k U(b_j)/(1+r)^{j-t} + U(w)/(r(1+r)^{k-t+1})] \}$$

so that EU_t is expected utility, discounted to t , given that an individual is unemployed at t . Note that

$$(6) \quad EU_t = U(b_t) + 1/(1+r) (p_t U(w)/r + (1-p_t) (U(b_{t+1}) + 1/(1+r) (p_{t+1} U(w)/r + (1-p_{t+1}) EU_{t+2}))).$$

The problem is to maximize EU_1 over sequences $\{b_t\}_1^{\infty}$ subject to the constraint that B_1 is a constant. It will, however, be convenient to treat the equivalent problem, minimize B_1 over sequences $\{b_t\}_1^{\infty}$, subject to the constraint that EU_1 is a constant. It is clear that a necessary condition for optimality of $\{b_t\}_1^{\infty}$ for this equivalent problem is that for any t , EU_t should be attained at least cost, that is, B_t should be minimized given that EU_t equals a constant, say \overline{EU}_t .⁸ Thus, it must be true in particular that B_t is minimized over just b_t and b_{t+1} subject to the constraint $EU_t = \overline{EU}_t$.

PROPOSITION 1. Suppose that unemployed individuals

- (a) have no wealth, cannot borrow and
- (b) cannot influence the probability of getting a job each period.

Then

- (c) U.I. benefits should be the same from one period to the next (i.e., $b_1 = b_2 = \dots$)

This is true for familiar reasons: Suppose that $b_t < b_{t+1}$ for some

t (the case $b_t > b_{t+1}$ is analogous) and consider a small reduction in b_{t+1} and a small increase in b_t which are actuarially fair (i.e., calculated to keep $b_t + (1-p_t)b_{t+1}/(1+r)$ constant). Since $U'(b_t) > U'(b_{t+1})$, the change will increase expected utility.

Proof: The Lagrangean for the problem of maximizing B_t over b_t

and b_{t+1} subject to $EU_t = \overline{EU}_t$ is (making use of (3) and (6))

$$(7) \quad L = b_t + (1-p_t)(b_{t+1}/(1+r) + (1-p_{t+1})B_{t+2}/(1+r)^2) \\ - \lambda (U(b_t) + \frac{1}{1+r}(p_t U(w)/r + (1-p_t) U(b_{t+1}) + \dots) \\ - \overline{EU}_t).$$

The first order conditions are

$$(8) \quad dL/db_t = 0 = 1 - \lambda U'(b_t)$$

$$(9) \quad dL/db_{t+1} = 0 = (1-p_t)/(1+r) - \lambda U'(b_{t+1})(1-p_t)/(1+r).$$

Since (9) reduces to $0 = 1 - \lambda U'(b_{t+1})$, these conditions imply $U'(b_t) = U'(b_{t+1})$, so that $b_t = b_{t+1}$.

Let us now consider the situation when unemployed individuals do influence the probability of getting a job by their choice of effort and the reservation wage. In this situation an individual who is unemployed at the beginning of period t and receives b_t enjoys $U(b_t) - e_t$, as effort in t involves disutility e_t . The marginal utilities of effort and consumption are taken as independent; this assumption is necessary to our results. The probability of getting a job as a function of effort and the reservation wage is

$$(10) \quad p_t = p(w_t^*, e_t) = \int_{w_t^*}^{\infty} f(w_t, e_t) dw_t.$$

Let u_t be the value of expected utility, discounted to t , given

that an individual gets a job in period t ,

$$(11) \quad u_t = u(w_t^*, e_t) = \int_{w_t^*}^{\infty} \frac{U(w_t)}{r} \frac{f(w_t, e_t)}{p_t} dw_t.$$

Suppose that at time t an unemployed individual has selected $\{e_j\}_t^{\infty}$ and $\{w_j^*\}_t^{\infty}$, implying $\{p_j\}_t^{\infty}$ and $\{u_j\}_t^{\infty}$. Conditional upon being unemployed at this time, his expected utility, discounted to t , is

$$(12) \quad E_t = \sum_{k=t}^{\infty} \{ [p_k \prod_{j=t}^{k-1} (1-p_j)] [\sum_{j=t}^k (U(b_j) - e_j) / (1+r)^{j-t} + u_k / (1+r)^{k-t+1}] \}$$

The interpretation of the terms in E_t is similar to that of the terms in EU_t .

Assume that the government does not observe effort or the reservation wage, so that individuals treat the b_t as fixed. Then E_{t+1} must satisfy the following equation (the "principle of optimality")

$$(13) \quad E_t = \text{Max}_{w_t^*, e_t} [U(b_t) - e_t + 1/(1+r) [p(w_t^*, e_t) u(w_t^*, e_t) + (1-p(w_t^*, e_t)) E_{t+1}]] .$$

The first order conditions for selection of w_t^* and e_t are

$$(14) \quad \frac{dE_t}{de_t} = -1 + 1/(1+r) [p_e (u - E_{t+1}) + p u_e] = 0$$

$$\frac{dE_t}{dw_t^*} = 1/(1+r) [p w^* (u - E_{t+1}) + p u_{w^*}] = 0 .$$

Thus,

$$(15) \quad \frac{dE_t}{dE_{t+1}} = \frac{dE_t}{de_t} \frac{de_t}{dE_{t+1}} + \frac{dE_t}{dw_t^*} \frac{dw_t^*}{dE_{t+1}} + \frac{dE_t}{dE_{t+1}}$$

$$= \frac{dE_t}{dE_{t+1}} = (1-p_t)/(1+r).$$

This is, of course, explained by the envelope theorem. Also, differentiation of (14) and use of the second order conditions give ⁹

$$(16) \quad \frac{dp_t}{dE_{t+1}} < 0.$$

In other words, anything which increases the utility of being unemployed in $t + 1$ increases the probability of that event. Now by the logic used in regard to Proposition 1, a necessary condition for optimality of $\{b_t\}_1^\infty$, is that for any t , B_t should be minimized over just b_t and b_{t+1} , subject to the constraint that E_t equals a constant \bar{E}_t . This necessary condition and (13)-(16) are used to prove the first part of the next proposition.

PROPOSITION 2. Suppose that unemployed individuals

- (a) have no wealth, cannot borrow and
- (b) can influence the probability of getting a job each period -- by their choice of a reservation wage and a level of effort devoted to job search.

Then

- (c) if the government does not monitor this choice on an individual basis, U.I. benefits should decline from period to period and, although remaining positive, approach zero in the limit (i.e., $b_1 > b_2 > \dots > b_t > 0$ for all t , and $\lim_{t \rightarrow \infty} b_t = 0$); and

(d) if the government does monitor this choice, U.I. benefits should be constant from period to period.

The idea behind the proof of (c) is straightforward. Suppose that $b_t = b_{t+1}$ and consider a small actuarially fair reduction in b_{t+1} and increase in b_t . Since $U'(b_t) = U'(b_{t+1})$, the first order approximation of the direct effect of this change on expected utility will be zero. And, by the envelope theorem, the indirect effect of this change through an altered level of effort or the reservation wage can be ignored. But E_{t+1} will be lowered, and thus by (16), e_t and w_t^* will change so as to raise the probability of getting a job in period t . Since the probability of getting a job in periods other than the t^{th} will be unaffected, the increased probability of success in period t will lower the expected cost of providing benefits. A similar argument rules out the case $b_t < b_{t+1}$.

The result (d) is analogous to the well-known fact that for single period models of insurance and moral hazard, if the insurer can monitor the behavior of insureds, then the optimal insurance policy is of the same character as that when the probability distribution of loss is exogenous.

Proof: To prove (c), first note that i) $dE_{t+1}/db_t = 0$, ii) $dw_t^*/db_t = 0$, iii) $de_t/db_t = 0$, and iv) $dE_{t+1}/db_{t+1} = U'(b_{t+1})$. Item (i) is obvious since E_{t+1} can hardly depend on benefits consumed prior to period $t+1$. Items (ii) and (iii) are also obvious, since the conditions (14) are independent of b_t . Item (iv) follows from (13), as applied to E_{t+1} and E_{t+2} .

The Lagrangean for the problem of minimizing B_t over p_t and b_{t+1} subject to $E_t = \bar{E}_t$ is

$$(17) \quad L = b_t + (1-p_t)(b_{t+1}/(1+r) + h_t) \\ - \lambda (U(b_t) - e_t + \frac{p_t u_t}{1+r} + (1-p_t) \frac{E_{t+1}}{1+r} - \bar{E}_t)$$

where by (3) $h_t = (1-p_{t+1}) B_{t+2}/(1+r)^2$ and where e_t and w_t^* (and therefore p_t and u_t) are implicitly determined as functions of b_{t+1} by (14). The first order conditions are, using (i) - (iii),

$$(18) \quad dL/db_t = 0 = 1 - \lambda U'(b_t)$$

$$(19) \quad dL/db_{t+1} = 0 = -(dp_t/db_{t+1})(b_{t+1}/(1+r) + h_t) \\ + (1-p_t)/(1+r) \\ - \lambda d(-e_t + \frac{p_t u_t}{1+r} + (1-p_t) \frac{E_{t+1}}{1+r})/db_{t+1}$$

But, using (iv) and (15),

$$(20) \quad d(-e_t + \frac{p_t u_t}{1+r} + (1-p_t) \frac{E_{t+1}}{1+r})/db_{t+1} = \\ d(-e_t + \frac{p_t u_t}{1+r} + (1-p_t) \frac{E_{t+1}}{1+r})/dE_{t+1} \cdot dE_{t+1}/db_{t+1} = \\ \frac{(1-p_t)}{1+r} U'(b_{t+1}).$$

Using (18) to eliminate λ , we get

$$(21) \quad 0 = -(dp_t/db_{t+1})(b_{t+1}/(1+r) + h_t) + (1-p_t)/(1+r) \\ - (U'(b_{t+1})/U'(b_t)) ((1-p_t)/1+r)$$

or

$$(22) \quad \frac{U'(b_{t+1})}{U(b_t)} = 1 - \left(\frac{dp_t}{db_{t+1}}\right) \frac{(1+r)}{(1-p_t)} (b_{t+1}/(1+r) + h_t).$$

Note that

$$(23) \quad \frac{dp_t}{db_{t+1}} = \frac{dp_t}{dE_{t+1}} \frac{dE_{t+1}}{db_{t+1}} = \frac{dp_t}{dE_{t+1}} U'(b_{t+1}) < 0$$

by (16) and (iv).

First, it will be shown that both $b_{t+1} \leq 0$ and $b_{t+1}/(1+r) + h_t \leq 0$ cannot hold: If $b_{t+1}/(1+r) + h_t \leq 0$, as $dp_t/db_{t+1} < 0$, (22) implies $U'(b_{t+1})/U'(b_t) \leq 1$ so that $b_t \leq b_{t+1}$. Hence $b_t \leq 0$ and $b_t/(1+r) + h_{t-1} \leq 0$. Repeating the argument, $b_{t-1} \leq 0$ and $b_{t-1}/(1+r) + h_{t-2} \leq 0$. By induction, $b_{t-2} \leq 0, \dots, b_1 \leq 0$. Thus $B_1 \leq 0$, a contradiction.

It follows that $h_t \geq 0$: if $h_t < 0$, select the first b_j with $j \geq t+1$ and which is negative (this element, say b_{j_0} , must exist since $h_t < 0$). But then $b_{j_0}/(1+r) + h_{j_0-1} < 0$ must hold since $h_t < 0$. This contradicts the result of the previous paragraph.

Thus $b_{t+1}/(1+r) + h_t > 0$: if not, it must be that $b_{t+1}/(1+r) + h_t \leq 0$ and thus, by the result of the paragraph before the last, $b_{t+1} > 0$. But this implies that $h_t < 0$, which contradicts the result of the previous paragraph.

As $b_{t+1}/(1+r) + h_t > 0$, (22) gives $U'(b_{t+1})/U'(b_t) > 1$, or $b_t > b_{t+1}$. Thus, benefits are strictly decreasing. Therefore, if for any j , $b_j \leq 0$, $b_{j+k} < 0$ for all $k \geq 1$ so that $h_j < 0$, a contradiction. Hence, benefits are always positive.

Let $\ell = \lim_{t \rightarrow \infty} b_t$ (ℓ exists since b_t is a decreasing and bounded sequence). As $b_t \geq 0$, $\ell \geq 0$. From (22)

$$(24) \quad \lim_t U'(b_{t+1})/U'(b_t) = 1 - \left(\lim_t dp_t/db_{t+1} \right) \left(\lim_t (1+r)/(1-p_t) \right) \cdot \left(\lim_t (b_{t+1}/(1+r) + h_t) \right).$$

The first limit, say ℓ_1 , is positive and it will be assumed that the probability of getting a job is bounded away from 1, so that the second limit, ℓ_2 , is also positive. If $\ell > 0$, then

$$\lim_t (b_{t+1})/(1+r) + h_t \geq \ell/(1+r) \text{ so that}$$

$$(25) \quad \lim_t U'(b_{t+1})/U'(b_t) \geq 1 + \ell_1 \ell_2 \ell_3 \ell / (1+r) > 1.$$

But this implies that $\lim_{t \rightarrow \infty} U'(b_t) = \infty$, a contradiction, since

$$\lim_{t \rightarrow \infty} U'(b_t) = U'(\ell) < \infty. \text{ Hence } \ell = 0, \text{ as claimed; } \underline{\text{the level of}}$$

benefits tends to zero.¹⁰

To prove (d), consider the problem of maximizing E_1 over $\{b_t\}_1^\infty$, $\{e_t\}_1^\infty$ and $\{w_t^*\}_1^\infty$ subject to B_1 equals a constant \bar{B}_1 .

Denote the optimal values by a " $\hat{\cdot}$ ".

Suppose now that the government tells individuals that $b_t = 0$ unless $e_1 = \hat{e}_1, \dots, e_t = \hat{e}_t$ and $w_1^* = \hat{w}_1^*, \dots, w_t^* = \hat{w}_t^*$, in which case $b_t = \hat{b}_t$. That is, if in any period t an individual ever fails to choose \hat{e}_t and \hat{w}_t^* , he cannot collect \hat{b}_t nor any benefits in the future. It then follows that an individual will always choose \hat{e}_t and \hat{w}_t^* (otherwise he would be better off not getting any benefits after some point, which can easily be shown to contradict the optimality of $\{\hat{b}_t\}_1^\infty$). Thus \hat{E}_1 will in fact be achieved if the time sequence of benefits is $\{\hat{b}_t\}_1^\infty$. But a necessary condition for selection of $\{\hat{b}_t\}_1^\infty$ was that it solved the problem maximize E_1 subject to $B_1 = \bar{B}_1$ where the probabilities are fixed at the optimal levels $\{\hat{p}_t\}_1^\infty$ (determined by $\hat{p}_t = p(\hat{w}_t^*, \hat{e}_t)$).

And by the proof to Proposition 1, the solution to this problem involves a constant time sequence of benefits.

Example: Let us illustrate the first part of Proposition 2 by finding the least cost time sequence of benefits which achieves the same expected utility that a newly unemployed individual enjoys under the current U.I. system. (This sequence is, of course, the one which maximizes expected utility given the least cost level of benefits.) In the calculations it is assumed that U is logarithmic. It is also assumed that there is a reduced form equation summarizing (14) which gives the probability of getting a job, p_t , as a function of the difference between the value u_t of getting a job and the value E_{t+1} of continuing optimally if one is not found. Let

$$(26) \quad p_t = 1 - a \exp \{-\lambda (u_t - C_{t+1})\}.$$

Here, a and λ are parameters which are to be determined from data. For simplicity, several additional assumptions are made: i) all jobs pay a wage normalized at one in each period, so that $u_t = 1/r \log 1 = 0$. ii) The discount rate is set at zero, as the period is taken to be one week and computations are made only for 50 weeks. iii) The current time sequence of benefits is characterized as remaining constant -- at a level b_0 -- rather than as terminating after some time. Under this assumption, the decision that an individual has to make is independent of how long he has been unemployed. Therefore, his level of effort e^* and probability of finding a job p^* will be the same each period. It may be assumed without loss of generality that the scale on which effort is measured is such that $e^* = 0$. Then, by (12),

$$\begin{aligned}
(27) \quad E_1 &= \sum_{t=1}^{\infty} p^*(1-p^*)^{t-1} t \log b_0 \\
&= (p^* \log b_0) \sum_{t=1}^{\infty} \left(-\frac{d(1-p^*)^t}{dp^*} \right) \\
&= (-p^* \log b_0) \frac{d}{dp^*} \sum_{t=1}^{\infty} (1-p^*)^t \\
&= (-p^* \log b_0) \frac{d}{dp^*} \left(\frac{1}{p^*} - 1 \right) = \frac{1}{p^*} \log b_0
\end{aligned}$$

and similarly, $E_T = \frac{1}{p^*} \log b_0$ for any T . Hence, by (25),

$$(28) \quad p^* = 1 - a \exp \left(\frac{\lambda}{p^*} \log b_0 \right).$$

If the average duration of unemployment is 5.6 weeks and U.I. benefits amount to about 60% of the wage (this is in rough accord with Feldstein [1974] and Marston [1975]), then $p^* = 1/5.6$ when $b_0 = .6$ and, from (28),

$$(29) \quad 1/5.6 = 1 - a \exp (5.6 \lambda \log .6).$$

Most reported estimates of the elasticity of expected duration of unemployment with respect to U.I. benefits lie in the range from .105 to .29 (Marston [1975]); to be conservative, it is assumed here to be .1. As

$$(30) \quad \frac{dp}{db_0} = -\frac{a}{p} \frac{\lambda}{b_0} e^{\lambda((1/p) \log b_0)},$$

we have

$$(31) \quad \frac{b_0}{p} \frac{dp}{db_0} = .1 = -a\lambda(5.6)^2 \exp \{ \lambda(5.6 \log .6) \}.$$

Equations (29) and (31) imply that a is .826 and λ is -.00385.

Using these values, the optimal sequence of benefits may be computed recursively. Rewrite equation (22) (recall that $r = 0$ is assumed here),

$$(32) \quad U'(b_{t+1})/U'(b_t) = 1 - dp_t/db_{t+1} (b_{t+1} + (1-p_{t+1})B_{t+2})/(1-p_t),$$

approximate the left hand side of (32) by $1 + (U''/U')db_{t+1}$ (where $db_{t+1} = b_{t+1} - b_t$) and denote by R the coefficient of relative risk aversion, $-bU''/U'$. Then (32) becomes

$$(33) \quad db_{t+1}/b_{t+1} = dp/db_{t+1} (b_{t+1} + (1-p_{t+1})B_{t+2})/R(1-p_t).$$

Now dp/db_{t+1} equals $a\lambda \exp\{\lambda C_{t+1}\} (dC_{t+1}/db_{t+1})$, which for the logarithm is $-a\lambda \exp(\lambda C_{t+1})/b_{t+1}$. Since R is identically equal to one and $1-p_t = a \exp \lambda C_{t+1}$,

$$(34) \quad db_{t+1} = -\lambda (b_{t+1} + (1-p_{t+1}) B_{t+2})$$

and

$$(35) \quad b_t = b_{t+1} (1+\lambda) + \lambda(1-p_{t+1}) B_{t+2}$$

Here b_t is expressed as a function of all future b_j . Since it is necessary to start somewhere to compute B_{t+2} , it is assumed that when benefits reach 1% of the wage, they remain there forever. Then, using the estimates of a and λ , it is easy to calculate the optimal time sequence of benefits. In particular, initial benefits may be chosen so that expected utility under the optimally declining sequence is exactly that under the current scheme (namely -- see

(28) -- $1/5.6 \log .6$). This optimally declining sequence is shown in the following table.

The Optimal Time Sequence of Unemployment Insurance Benefits

Week	% of Wage	Week	% of Wage
1	66.0	13	52.9
2	64.8	14	52.0
3	63.7	15	51.0
4	62.5	16	50.1
5	61.3	17	49.2
6	60.2	18	48.3
7	59.1	19	47.4
8	58.0	20	46.6
9	56.9	25	42.6
10	55.9	30	38.9
11	54.9	35	35.6
12	53.9	40	32.5

The average cost to the government under the optimal sequence equals 2.8 weeks salary and represents a 16 percent savings over the 3.36 weeks (5.6 weeks average duration times .6 weeks benefit level) salary currently paid to unemployed workers. The savings comes about because beyond week 6, unemployment benefits are less than 60 percent of take home pay. Workers are, nonetheless, just as well off because they are induced to get jobs somewhat earlier -- average duration falls about 1 percent -- and, if they find a job anytime before week 11, receive more benefits in total than under the present scheme.

4. The optimal time sequence of benefits assuming that the unemployed begin with positive wealth or can borrow.

In this case, individuals are assumed to begin their spell of unemployment with positive wealth; or, equivalently, they are assumed to be able to borrow up to some positive amount against future income. Additionally, individuals are assumed to be able to save or dissave. Let $c_t^u \geq 0$ and $c_t^e \geq 0$ be consumption during period t if the individual is, respectively, unemployed or employed at the beginning of the period. Let z_t be wealth exclusive of any benefits or wages at the beginning of the period. Then if the individual was not employed at the beginning of period t , clearly $z_{t+1} = (z_t + b_t - c_t^u)(1+r)$. Consumption is assumed to be feasible, that is, $c_t^u \leq z_t + b_t$ and $c_t^e \leq z_t + w$.

Let us first suppose that the probabilities p_t are exogeneously determined and, thus (as in the previous case) that effort is ignored and that the wage if a job is found is known to be w with certainty. Let $J(z)$ be the discounted utility of an individual given that he has just found a job and has wealth z . Suppose that at time t an unemployed individual has optimally selected $\{c_k^u\}_t^\infty$. Conditional upon being unemployed at this time, his expected utility, discounted to t , is

$$(36) \quad V_t = \sum_{k=t}^{\infty} \left\{ [p_t \prod_{j=t}^{k-1} (1-p_j)] \left[\sum_{j=t}^k U(c_k^u) / (1+r)^{j-t} + J(z_{k+1}) / (1+r)^{k-t+1} \right] \right\}.$$

The quantity V_t depends on wealth at the beginning of period t , so that it will usually be written $V_t(z_t)$. Note that $V_t(\cdot)$ and $V_{t+1}(\cdot)$ must satisfy

$$(37) \quad V_t(z_t) = \text{Max}_{c_t^u} U(c_t^u) + \frac{1}{1+r} [(1-p_t)V_{t+1}(z_{t+1}) + p_t J(z_{t+1})].$$

The problem to be solved is to maximize discounted expected utility, $V_1(z_1)$, subject to the constraint $B_1 = \bar{B}_1$.

PROPOSITION 3. Suppose that unemployed individuals

- (a) have wealth or may borrow and may save or dissave,
- (b) cannot influence the probability of getting a job.

Then,

- (c) U.I. benefits should at first be zero and then should rise to a constant value (i.e., $0 = b_1 = \dots = b_T < b_{T+1} = \bar{b} = b_{T+2} = b_{T+3} = \dots$).

The intuition behind the result is that when an individual is first unemployed, he has relatively high wealth and consumes relatively much, meaning that the marginal utility of U.I. benefits is low compared to what it is later, when he has reduced his savings. This suggests that U.I. benefits should not be given until wealth has fallen to a critical level -- until the marginal utility of wealth has risen to a critical level -- and then that positive benefits should be given. And once positive benefits are given, they should be constant by much the same argument given for Proposition 1. It should also be mentioned that if z_1 is sufficiently low, positive U.I. benefits should be given at the outset ($T = 0$) and that $\bar{b} < w$. Otherwise $\bar{b} = w$.

Proof: The argument, which is similar to that used in Propositions 1 and 2, will only be sketched here. Assume that the optimal consumption c_t^u does not exhaust wealth z_t ; that is, assume that

c_t^u is determined by setting equal to zero the derivative of (37) with respect to c_t^u :

$$(38) \quad U'(c_t^u) = (1-p_t) V'_{t+1}(z_{t+1}) + p_t J'(z_{t+1}).$$

Because $V'(z_{t+1}) \geq J'(z_{t+1})$ (i.e., the marginal utility of wealth if an individual is unemployed is at least as high as that when he is employed), (38) implies

$$(39) \quad U'(c_t^u) \leq V'_{t+1}(z_{t+1}).$$

But by the envelope theorem, $V'_{t+1}(z_{t+1}) = U'(c_{t+1}^u)$ and $J'(z_{t+1}) = U'(c_{t+1}^e)$, so that (38) and (39) may be rewritten as

$$(40) \quad U'(c_t^u) = (1-p_t) U'(c_{t+1}^u) + p_t U'(c_{t+1}^e)$$

$$(41) \quad U'(c_t^u) \leq U'(c_{t+1}^u) \text{ or } c_t^u \geq c_{t+1}^u.$$

In other words, consumption cannot rise with the duration of unemployment. Now we claim that (i) if $b_t > 0$, then $c_t^u = c_{t+1}^u = \dots$: if not, by (41) there must be a positive integer j such that $c_t^u > c_{t+j}^u$. But by the envelope theorem, a small decrease in b_t and increase in b_{t+j} (calculated so as to continue to satisfy $B_1 = \bar{B}_1$) will raise expected utility since $U'(c_t^u) < U'(c_{t+j}^u)$. Furthermore (ii) if $b_t > 0$, then $w = b_{t+1} = b_{t+2} = \dots$: By (i) $c_t^u = c_{t+1}^u$ so (40) implies $c_{t+1}^u = c_{t+1}^e$. It is, however, easy to show that $c_{t+1}^e = c_{t+2}^e = \dots$. Hence, the optimal future consumption stream is independent of whether an individual ever finds a job. It is, again, easy to show that this can be true only if benefits equal the wage, that is, $\bar{b} = w$.

If initial wealth is sufficiently small, it might be the case that it would be optimal to exhaust wealth. If so, the equality sign in (38) is in general replaced by ">" and an argument similar to that of the preceding paragraphs establishes the claim of the proposition, the only difference being that $\bar{b} < w$.

We have not been able to characterize the optimal time sequence of U.I. benefits when unemployed individuals do influence the probability of getting a job by their choice of effort and the reservation wage, and when the government does not monitor these variables. But the relevant considerations seem clear. There is an advantage (illustrated by Proposition 2(c)) to having benefits decline -- due to the creation of an additional incentive to get a job. But having benefits decline reduces the role of benefits as insurance for those who are unemployed for long periods. This disadvantage is especially apparent when the unemployed have significant wealth (as was explained in regard to Proposition 3). Thus, one might expect the optimal time sequence of benefits to decline when the wealth of unemployed is sufficiently low. This is, in fact, the case in the example presented below, but there are complicating factors.

On the other hand, when the government monitors individual behavior, the optimal time sequence of benefits is (paralleling the case in section 3) as described in Proposition 3. The proof is analogous to that given for Proposition 2(d).

Example: The following table shows for a two period model how the optimal time sequence of U.I. benefits changes as initial wealth changes.

The Optimal Time Sequence of Unemployment Insurance Benefits in a Two Period Model

Initial Wealth	Benefits in Week 1	Benefits in Week 2
0	58	46
10	56	51
20	52	60
30	50	64
40	49	66
70	53	56
90	54	54
120	57	48
200	65	27

The example was numerically solved for each level of initial wealth. In the example, the wage was 100, per capita expected discounted benefits (B_1) was 75, $p_t(e_t) = 1 - \frac{10}{10.3 + e_t}$, and $U(c) = 100(c/100)^3$.

Note that, as expected, for very low levels of wealth benefits decline, for higher levels of wealth they rise, but for even higher levels of wealth they again decline. This last fact is no doubt due to the following effect. When wealth is very high, risk aversion is very low (assuming of course that absolute risk aversion decreases with wealth). When risk aversion

is very low, the optimal time sequence should tend toward what it would be for the risk neutral case. But for the risk neutral case, it is easy to show that it is optimal to give all the benefits in the first period. (This maximizes the incentive to find a job, and the risk that this imposes on those who are unemployed is of no concern since they are risk neutral.)

5. Concluding remarks

This paper abstracted from a variety of considerations, two of which we will comment on here. First, the probability distribution of wage offers was assumed to be given. As is well recognized in models of job search, this assumption is not always realistic. To take the extreme Keynesian view, the number of jobs is fixed at a level determined by aggregate demand. Thus, for example, an attempt to reduce the duration of unemployment by a change in the time sequence of benefits would lead to increased competition for jobs and would probably be offset to some extent by an adverse shift in the schedule giving the probability of getting jobs as a function of effort and the reservation wage. On the other hand, the assumption about the probability distribution of wage offers is more nearly descriptive of a situation in which unemployment is in large part "search unemployment", when vacancies are high relative to unemployment.

Second, it was assumed that newly unemployed individuals are identical. If the government cannot easily detect individual differences (in order to subsequently adjust benefits), then two approaches can be adopted. A single time sequence of benefits can be used for all unemployed individuals. Presumably, the design of such a time sequence would reflect the changing composition of the unemployed by duration of unemployment -- that individuals who have been unemployed longer are more likely to have a lower probability of finding a job, other things equal. Alternatively, several time sequences of benefits could be offered, and the newly unemployed allowed to select their most preferred sequence. But

for this approach to be used to advantage, different groups must be induced to select different sequences of benefits.¹¹

Footnotes

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2. A recent paper by Mortensen [1977] is also of interest but he does not determine the optimal time sequence of benefits. Rather, he assumes that benefits are paid out in the usual way -- at a constant rate over the benefit period -- and he asks how job search behavior of the unemployed responds to changes in the benefit rate and the length of the benefit period.
3. This statement is correct assuming that quit and layoff behavior are taken as given, but that may not be appropriate: Suppose, for example, that a firm's contribution to the U.I. fund does not fully reflect the benefits paid to the employees it lays off. Then, since an expected utility maximizing time sequence of benefits may provide a "high" initial level of benefits, the firm may be induced to increase the number of temporary layoffs.
4. Needless to say, this assumption is often realistic; for reasons of moral hazard, unemployed individuals frequently find it difficult to borrow.
5. Allowing for the possibility that individuals who get jobs might subsequently become unemployed would not change either the results or the proofs (since the only effect would be to

change by a constant the "value of the optimal continuation" given that a job is found), but it would increase the notational burden on the reader.

6. It can be shown that the results in this case hold as well when unemployed individuals have some constant, exogenous source of income, a working spouse for example.
7. This is true since it is assumed that the probability of getting a job is bounded away from zero.
8. Suppose that this were not true. That is, suppose that $\alpha = \{b_t\}_1^\infty$ is optimal and yet for some t , say T , EU_T could be achieved by $\beta = \{b_j^*\}_T^\infty$, where the discounted cost of this sequence is less than B_T . Then it is easy to check that the sequence $\gamma = \{b_1, b_2, \dots, b_{T-1}, b_T^*, b_{T+1}^*, \dots\}$ has a discounted cost less than B_1 and that discounted expected utility is the same as with α .
9. Differentiating (14) with respect to E_{t+1} and solving for

$\frac{dw^*}{dC_{t+1}}$ and $\frac{de}{dC_{t+1}}$, we obtain

$$\begin{bmatrix} \frac{dw^*}{dE_{t+1}} \\ \frac{de}{dE_{t+1}} \end{bmatrix} = H^{-1} \begin{bmatrix} p_{w^*} \\ p_e \end{bmatrix}, \text{ where } H \text{ is the Hessian from (13) and}$$

is negative definite (the second order condition for a regular maximum). Thus

$$\frac{dp}{dE}_{t+1} = [p_{w^*} \ p_e] H^{-1} \begin{bmatrix} p_{w^*} \\ p_e \end{bmatrix} < 0 \quad \text{since } H^{-1} \text{ is negative}$$

definite, being the inverse of a negative definite matrix.

10. Note that benefits tend to zero even though the possibility of infinite marginal utility of wealth at zero was not ruled out. However, benefits do remain positive, which agrees with the result in a single-period model of insurance that an optimal policy under moral hazard always involves positive coverage (Shavell [1977]).
11. For example, suppose that there are two different groups of unemployed, each beginning with no wealth and being unable to borrow. Assume that the duration of unemployment in the first group is short and is very sensitive to changes in the time sequence of benefits, while the opposite is true in the second. Assume that a choice is offered between two time sequences of benefits: the first begins at a high level and declines rapidly, the second at a lower level but declines slowly. Then the first group might choose the first sequence and the second group the second sequence. This might be more desirable than offering one sequence, for the groups are induced to select the sequence which best meets their needs.

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