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ON "ON THE FOUNDATIONS OF THE THEORY
OF MONOPOLISTIC COMPETITION"

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by

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1. INTRODUCTION

In a brief stimulating paper John Roberts and Hugo Sonnenschien survey various attempts to embed monopolistic competition into a general equilibrium model [1]. These include Arrow and Hahn [2], Fitzroy [3], Gabszewicz and Vial [4], Laffont and Laroque [5], Negeshi [6], and Marschak and Selten [7].

The work noted by Roberts and Sonnenschien is criticized by them because "the properties thus assumed are not derived from hypotheses on the fundamental data of preferences, endowments and technology." They in particular criticize as an *ad hoc* assumption that the optimal choices by each firm should define an upper semi-continuous convex-valued correspondence.

They offer a model of oligopolistic competition with quantity setting firms for which they demonstrate that no equilibrium exists. In the be-

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ginning of Section 2, we reexamine their model. In Section 3 of this paper we reconsider this model and offer an alternative model based upon the same economic data. We construct a game in strategic form for which a pure strategy noncooperative equilibrium always exists.

Shubik [8], Shapley and Shubik [9], Dubey and Shubik [10, 11], Dubey and Shapley [12] and Postlewaite and Schmeidler [13] have all considered models of a closed trading or trading and production economy viewed as a game in strategic form and solved for its noncooperative equilibria. Remarkably in all of this work a close relationship among the roles of markets, money and oligopolistic competition appears. It seems as if a natural way to model a closed economy as a formal game of strategy is to distinguish some commodity as a money or to specify some form of fiat money or credit mechanism.

Most of the work on oligopoly theory has been done in the context of open or "partial equilibrium." In such a context the role of money and markets is natural. Our game theoretic formulations pick up this aspect of oligopolistic competition. In a series of papers referred to above [8-13] and elsewhere [14] distinctions between fiat money, credit and commodity money conditions are discussed in detail. Here in keeping with the Roberts and Sonnenschien models we merely reinterpret the holdings of the consumer as though there were a commodity money.

In essence our results differ from those of Roberts and Sonnenschien because the nature of our modelling in terms of a fully formally defined game in strategic form forces us to pay attention to certain details concerning the specification of information conditions, the nature of strategies by individuals and different types of noncooperative equilibrium points which may or may not exist.

Roberts and Sonnenschein suggest that "to provide the proper foundations for the theory of imperfect competition one must answer the question of 'what functions can be reaction functions?'" We believe that the priority is more basic. Reaction functions, if that construct is to be used at all must be specified in such a way that information conditions and the definition of the strategic possibilities of all players are made completely explicit.

2. THE ROBERTS SONNENSCHN EIN MODEL

A model is presented consisting of two monopolists each costlessly producing up to one unit of a good. A strategy for a producer i is to offer a quantity x_i to the market.

There is a single consumer (presumably representative of a continuum of nonatomic consumers?). "The consumer is assumed to act competitively, maximizing his utility subject to the budget constraint he faces, taking prices and profits as given."

There are two alternative ways in which we can model this simple extension of Cournot duopoly as a game of strategy. In the first we assume that the duopolists move simultaneously, after which the customers are completely informed.* In the second all players (firms and customers) move simultaneously without further information. These distinctions are shown in the extensive forms in Figures 1a and 1b. The arrows indicate

*There is a third case which we actually use in 3.1 below. This is where the firms move first but the customers are informed only of the aggregate outcome of their moves. In this section we define what is meant by a perfect equilibrium point which is associated with games which have points of complete information. In 3.1 we show that we can generalize this idea to instances where moves can be aggregated.

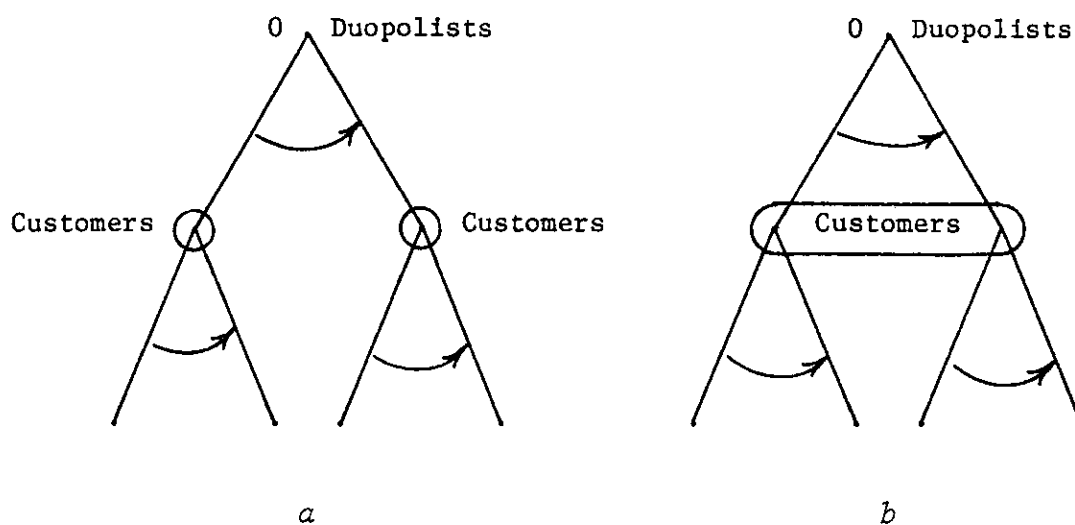


FIGURE 1

that the moves may be selected from a continuous set. In Figure 1a customers are informed hence their moves and strategies must be distinguished as the strategies can utilize the information. In Figure 1b the strategies and moves of customers must coincide. Which of these two is more realistic poses an empirical question. Both can be well defined. Furthermore a result by Dubey and Shubik [15] shows that if a game has any pure strategy noncooperative equilibrium points then any associated game which differs from the first only in a refinement of its information sets will have at least the pure strategy equilibria of the original game as equilibrium points. Thus if the game in Figure 1b has any equilibria so does the game shown in Figure 1a. In Section 3 we show that this is generally the case for quantity variation or Cournot models.

If we interpret the Roberts Sonnenschein model as a game in strategic form then it most closely fits the game illustrated in Figure 1a. Rather than assume that the customer is by definition a price taker (especially since there is no indication how price is formed) we prefer to deduce

this possibility by specifying precisely what his strategies are and to consider, asymptotically or directly, a continuum of traders.

The counterexamples supplied by the authors are undoubtedly correct in establishing that no perfect noncooperative equilibrium points may exist in a noncooperative game with information conditions of the variety shown in Figure 1a. By the term "perfect noncooperative equilibrium point" we mean a set of strategies such that at any point of complete information in the game tree the remaining part of the strategies form an equilibrium point in the subgame remaining. Furthermore the strategies are in equilibrium in the overall game [16].

A simple example illustrates a game with both a perfect equilibrium point and an equilibrium composed of historical strategies; i.e. strategies whose components depend upon the past path to the current state rather than merely upon the current state.

Consider the following 3x3 matrix game played twice by two individuals A and B . After the first play there is complete information and then the players play again. The 3x3 matrix game shown in Figure 2.

	1	2	3
1	5, 5	-5, 10	-10, -5
2	10, -5	0, 0	-12, -7
3	-5, -10	-7, -12	-20, -20

FIGURE 2

(The payoffs can be interpreted as having been generated from a duopoly model, but this is incidental to the point.) The extensive form of the game is shown in Figure 3. A strategy for each player involves planning

for 9 contingencies. We consider two symmetric equilibria. The strategies for player A are specified; the strategies for B are of the same form.

Strategy for a Perfect Equilibrium

"I play move 2 on each occasion regardless of information."

Historical Strategy

"I play move 1 then if B has played his move 1, I play move 2; if he has played anything else I play 3."

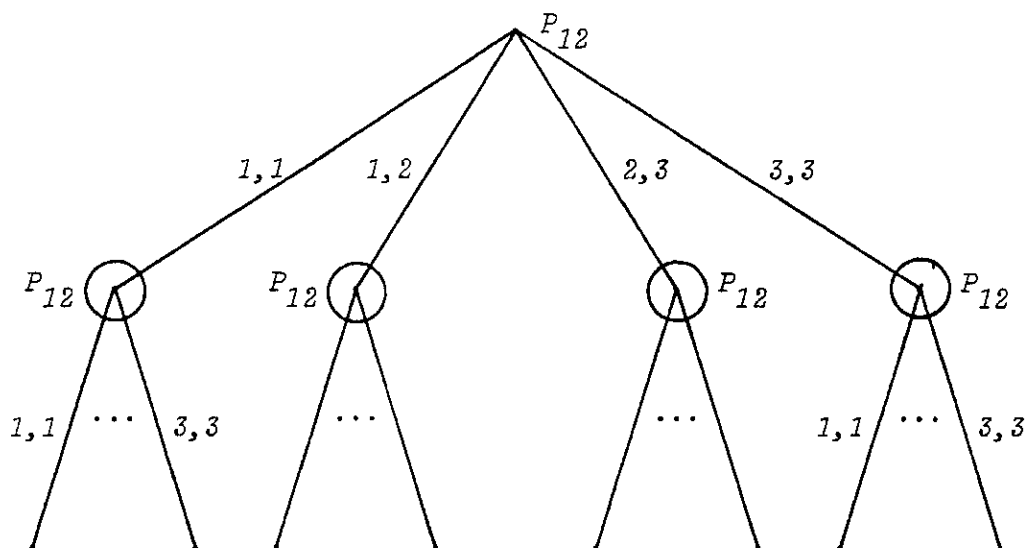


FIGURE 3

It is easy to check that a pair of strategies of the first type are in equilibrium in all subgames. A pair of strategies of the second type are not in equilibrium in the subgames.

The negative results of the Roberts Sonnenschein model, we believe are real. Without even having to be as elaborate as they have been the lack of pure strategy equilibria in price variation duopoly open models

is well known [17, 18]. The simple noncooperative solutions for competition among few firms even when they have pure strategy equilibria do not appear to be particularly instructive.

A formally interesting mathematical result that ties these models into general equilibrium theory is that if we were to assume that the firms were nonatomic then the perfect equilibria and the "ignorance equilibria" of the game in Figure 1b can be shown to coincide even though for few firms the perfect equilibria may not even exist.

It is conjectured that the mixed strategy equilibria might even approach the pure strategy equilibria under replication in a manner suggested in Chapter 6 of reference [17]. An example which has this property may be constructed with relative ease.

3. THE GAME IN STRATEGIC FORM

3.1. The Market \mathcal{E}

Let $\{I, \mathcal{C}, \mu\}$ be a measure space where I the set of agents in the market, $\mathcal{C} \equiv$ the σ -algebra of coalitions (subsets) of I , $\mu \equiv$ a measure on $\{I, \mathcal{C}\}$. Let $I_1 \in \mathcal{C}$ and $I_2 \in \mathcal{C}$ denote the set of firms and consumers respectively. (We have, of course, that $\mu(I_1) > 0$, $\mu(I_2) > 0$, $I_1 \cap I_2 = \emptyset$.) When I_1 (or I_2) is a finite set, μ restricted to I_1 (or I_2), will always be assumed to attach the same weight to every element of I_1 (or I_2).

For any positive integer ℓ , let Ω^ℓ be the nonnegative orthant of the Euclidean space of dimension ℓ . For any $x \in \Omega^\ell$, let x_j stand for the j^{th} component of x .

The initial data of the market is described by the following set of measurable mappings:

$$Y : I_1 \rightarrow 2^{\Omega^{m+1}}$$

$$a : I_2 \rightarrow \Omega^{m+1}$$

$$u : I_2 \times \Omega^{m+1} \rightarrow \Omega^1$$

$$\eta : I_2 \times I_1 \rightarrow \Omega^1, \quad \int_{I_2} \eta(i, j) d\mu(i) = 1 \quad \text{for all } j.$$

We will abbreviate $a(i)$ by a^i , $Y(i)$ by Y^i , $u(i, x)$ by $u^i(x)$.

Let us now explain our symbols:

Ω^{m+1} = the space of commodity bundles

Y^i = the production set of firm i

a^i = the initial endowment of consumer i

$u^i(x)$ = the utility of consumer i for the bundle $x \in \Omega^{m+1}$

$\eta(i, j)$ = share of consumer i in firm j .

3.2. The Market Game $\Gamma_C(\mathcal{E})$ with Aggregate Information

To recast the market as a game in extensive form, we single out the $(m+1)^{\text{st}}$ commodity as a money, and set up m trading-posts for the remaining m commodities. First all firms move *simultaneously* and decide which bundle they will produce and put up for sale in the trading-posts. Thus if $i \in I_1$ selects $y^i \in Y^i$, he is required to send y_j^i for sale to trading-post j ($1 \leq j \leq m$), and to hold back y_{m+1}^i . The aggregate amount of j put up for sale is $\bar{y}_j = \int_{I_1} y_j^i d\mu(i)$.* The consumers are now informed about this aggregate output \bar{y} . Their move is then to bid money for the purchase of the commodities and/or supply these for sale.

*Let \bar{y} denote the vector $(\bar{y}_1, \dots, \bar{y}_m)$.

Let us denote by b^i (q^i) the bid (offer) vector of $i \in I_2$. Thus $b^i \in \Omega^m$ ($q^i \in \Omega^m$), $b_j^i = i$'s bid on commodity j ($q_j^i = i$'s offer of j for sale), $\sum_{j=1}^m b_j^i \leq a_{m+1}^i$ ($0 \leq q_j^i \leq a_j^i$). See Figure 1 for the extensive form. The information set shown is over all nodes at which the moves by the firms lead to the same aggregate output.

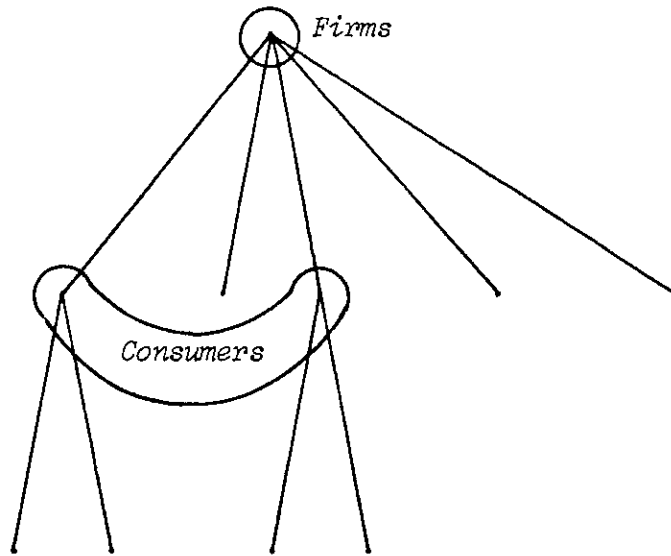


FIGURE 4

Let us now view this in strategic form. Suppose $Z \subset \Omega^m$ denotes the set of all aggregate output vectors that could possibly result from the firms' choices. Recalling that a strategy of any agent is a specification of a move at every information set that belongs to him, we have that the strategy sets S_c^i of $i \in I$ are:

$$S_c^i = Y^i \quad \text{for } i \in I_1,$$

$$S_c^i = \{Z \rightarrow \Omega^m, Z \rightarrow \Omega^m : \text{for any } z \in Z, \sum_{j=1}^m b_j^i(z) \leq a_{m+1}^i, q_j^i(z) \leq a_j^i\}$$

Given a choice of strategies s , $s = [\{y^i\}_{i \in I_1}, \{(b^i, q^i)\}_{i \in I_2}]$ by all the agents, prices $p(s) \in \Omega^m$ are determined in the m trading-posts as follows:

$$p_j(s) = \frac{\int_{I_2} b_j^i(\bar{y}) d\mu(i)}{\int_{I_2} q_j^i(\bar{y}) d\mu(i) + \bar{y}_j}$$

for $1 \leq j \leq m$; the revenue earned by $i \in I_1$ is

$$P^i(s) = y_{m+1}^i + \sum_{j=1}^m y_j^i p_j(s);$$

the final holding of $i \in I_2$ is $x^i(s) \in \Omega^{m+1}$ where:

$$x_j^i(s) = a_j^i - q_j^i(\bar{y}) + \frac{b_j^i(\bar{y})}{p_j(s)}, \quad 1 \leq j \leq m,$$

$$x_{m+1}^i(s) = a_{m+1}^i - \sum_{j=1}^m b_j^i(\bar{y}) + \sum_{j=1}^m q_j^i p_j(s) + \int_{I_2} \eta(i, j) P^j(s) d\mu(j).$$

The payoff to $i \in I_2$ is therefore the utility of this final bundle, i.e. $P^i(s) = u^i(x^i(s))$.

This defines the game $\Gamma_C(\mathcal{E})$.

A *noncooperative equilibrium* in pure strategies (N.E.) of this game is a choice of strategies $*s$ such that

$$P^i(*s | s^i) \leq P^i(*s), \quad s^i \in S_C^i,$$

for all $i \in I$, where $(*s | s^i)$ is the same as $*s$ but with s^i substituted for $*s^i$.

An *active* N.E. of this game is one which produces positive prices in every trading-post.

A modified *perfect* N.E. (denoted "m-perfect N.E.") is an N.E. with the property that for every $z \in Z$, the strategies $\{b^i(z), q^i(z)\}_{i \in I_2}$ comprise a N.E. for the subgame $\Gamma_C(\mathcal{E}, z)$ obtained by looking at the forward portion of the tree starting from the (coincident) information sets of the consumers that correspond to z . (Note that the subgames following from any two nodes in such an information set are isomorphic, and give rise to the same strategic form, hence our definition makes sense.)

3.3. The Game $\Gamma_I(\mathcal{E})$ with No Information

This is the same as $\Gamma_C(\mathcal{E})$ except that consumers move *without any information* on the aggregate output of the firms. (See Figure 5.)

Thus a strategy by $i \in I_2$ must consist of *constant* functions

$b^i : Q \rightarrow \Omega^m$, $q^i : Q \rightarrow \Omega^m$, and we may collapse the strategy-set S_I^i

to $\{(b^i, q^i) = \sum_{j=1}^m b_j^i \leq a_{m+1}^i, q_j^i \leq a_j^i\}$. Note that $S_I^i \not\subset S_C^i$ if $i \in I_2$,

and $S_I^i = S_C^i$ if $i \in I_1$.

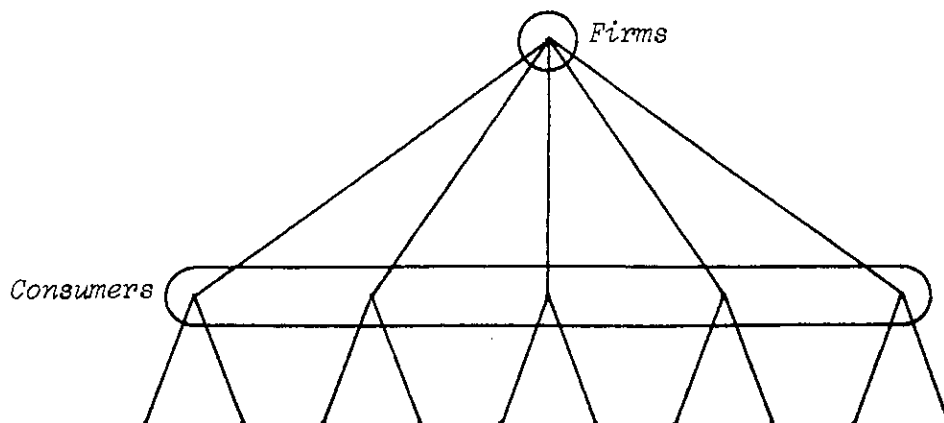


FIGURE 5

3.4. Equilibrium Points

A trivial N.E. exists for both $\Gamma_I(\mathcal{E})$ and $\Gamma_C(\mathcal{E})$, for instance the selection of strategies which involve no bids and offers by the consumers and firms in all the trading-posts. Other N.E. occur which leave some subset of the trading-posts inactive. Our interest is really in active N.E.

For the case where I_2 is nonatomic, introduce the following assumption: for every commodity j , $1 \leq j \leq m$, there is (1) a non-null set of traders who desire j and are moneyed, (2) a non-null set of j -furnished traders who desire money.*

If I_2 is finite, then we make the same assumption* replacing "a non-null set of traders" by "at least two traders."

Given this assumption*** we can state:

Theorem 1. An active N.E. exists for $\Gamma_I(\mathcal{E})$.

Theorem 2. An active N.E. exists for $\Gamma_C(\mathcal{E})$.

Theorem 1 is proved** as Theorem 1 in [19]. Theorem 2 follows from Theorem 1 and the theorem in [15]. Indeed from [15] we know that any N.E. of $\Gamma_I(\mathcal{E})$ also constitutes a N.E. of $\Gamma_C(\mathcal{E})$.

While (as shown by Roberts and Sonnenschein) active m -perfect N.E. of $\Gamma_C(\mathcal{E})$ need not exist when I_1 is a finite set, they do if I_1 is

*We can replace (2) by (3): "there is a nonnull set of (or 'at least two') firms and a positive constant K who can produce more than K units of the j^{th} commodity." (A trader is said to desire commodity j if his utility function is strictly increasing in the variable x_j).

**The proof in [19] is for the "sell-all" model, but similar techniques work for the "bid-offer" model described here.

***The other assumptions are the standard ones, i.e. the traders' characteristics vary measurably, and their utilities are concave, continuous and nondecreasing in each variable.

nonatomic. Indeed if we agree to call an N.E. of $\Gamma_C(\mathcal{E})$ equivalent to a N.E. of $\Gamma_I(\mathcal{E})$ when they select the same path in the game tree, then we have:

Theorem 3. Suppose I_1 is nonatomic. Then any active N.E. of $\Gamma_I(\mathcal{E})$ is equivalent to an active m -perfect N.E. of $\Gamma_C(\mathcal{E})$, and vice-versa.

This is obvious. First consider an N.E. *s of $\Gamma_I(\mathcal{E})$, and suppose $\hat{z} \in Z$ lies on the path selected by *s . Denote the strategies used (in *s) by $[\{^*y^i\}_{i \in I_1}, \{^*b^i, ^*q^i\}_{i \in I_2}]$. For any $z \in Z \setminus \{^*\bar{y}\}$, let $\{b^i(z), q^i(z)\}_{i \in I_2}$ be an N.E. of the game among I_2 defined by the forward portion of the tree starting from z . (These exist, given our assumption, as shown in [19].) Define

$$\{Z \xrightarrow{b^i} \Omega^m, Z \xrightarrow{q^i} \Omega^m\}_{i \in I_2} \text{ by}$$

$$(b^i(z), q^i(z)) = \begin{cases} (^*b^i, ^*q^i) & \text{if } z = ^*\bar{y} \\ (b^i(z), q^i(z)) & \text{otherwise.} \end{cases}$$

Then $[\{y^i\}_{i \in I_1}, \{b^i, q^i\}_{i \in I_2}]$ is an active m -perfect N.E. of $\Gamma_C(s)$ equivalent to *s . Conversely suppose $s = [\{y^i\}_{i \in I_1}, \{b^i, q^i\}_{i \in I_2}]$ is an active m -perfect N.E. of $\Gamma_C(s)$. Then $[\{y^i\}_{i \in I_1}, \{b^i(\bar{y}), q^i(\bar{y})\}_{i \in I_2}]$ is an active N.E. of $\Gamma_I(\mathcal{E})$ equivalent to s .

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