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ON MODELING THE ECONOMIC LINKAGES AMONG COUNTRIES

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ON MODELING THE ECONOMIC LINKAGES AMONG COUNTRIES*

by

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I. Introduction

This paper is concerned with modeling the economic linkages among countries. Although there are by now a number of multi-country macroeconometric models in existence, it seems safe to say with respect to the treatment of capital flows and exchange rates that econometric work has not kept pace with theoretical developments. Since Mundell's pioneering theoretical work [17] in the 1960's, the potential empirical importance of capital flows among countries has been known, and yet in most multi-country econometric models capital flows are either ignored completely or else taken to be exogenous. This usually means that exchange rates are also taken to be exogenous, which in the present regime of floating exchange rates is clearly an important limitation. Econometric model builders are not, of course, unaware of these limitations. For a number of reasons, econometric work in this area is difficult, and these difficulties have undoubtedly impeded progress. One difficulty is the lack of good data for a number of countries. Another

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is the sheer size of the task of linking a number of single-country models together. Dealing with hundreds or thousands of equations is painstaking, and there is a natural tendency in this type of work to be less concerned with theoretical purity than with the practical issue of getting the model running.

Theoretical work in this area has, on the other hand, ignored a number of important economic linkages among countries that are accounted for in multi-country econometric models. The two-country theoretical models of the type surveyed by Myhrman [19] and Mussa [18], for example, are too small to incorporate all the main features and links in the international economy, particularly with respect to price and wage behavior. There is thus currently a fairly wide gap in international economics between theoretical and econometric work, the former emphasizing capital flows and exchange rates at the expense of other features of the economy and the latter emphasizing some of the other features of the economy at the expense of capital flows and exchange rates.

This paper has three main purposes. The first is to present a comparison of the quantitative properties of seven multi-country econometric models; the second is to discuss briefly a quasi-empirical model of the author's that has the detail of large-scale econometric models and yet also accounts for all capital flows and allows for the endogenous determination of the exchange rate; and the third is to suggest an approach for the future construction of multi-country econometric models.

The comparison of the quantitative properties of the seven models is presented in Section II. The evidence presented in this section should give one a general idea of the current range of estimated effects of U.S. actions on the economies of other countries. Given the diversity of the seven models, their quantitative properties are actually closer than one might have expected, although there are still some very large differences. With one exception, however, the results from the models are based on the assumption of exogenous capital flows and exchange rates, and this should be kept in mind in interpreting the results.

The quasi-empirical model is discussed in Section III. This model, which will be called Model A, is a 180-equation two-country model. It was constructed by linking the 84-equation econometric model of the U.S. economy in Fair [7] to itself. Model A is "quasi-empirical" in that half of it is an actual empirical model of the U.S. and half is completely made up. This model accounts for all flows of funds between the two countries and allows for the endogenous determination of the exchange rate. It also has, of course, much more detail and many more links between the two countries than do the standard two-country theoretical models in the literature. Model A is an attempt to bridge in part the gap between theoretical and econometric work mentioned above. As will be seen, the properties of this model are quite sensitive to the treatment of capital flows and the exchange rate. This evidence, along with what is already known from the theoretical literature, rather strongly indicates that further work on making capital flows and exchange rates endogenous in multi-country econometric models is needed before much confidence can be placed on their properties.

The suggested approach for the future construction of multi-country econometric models is presented in Section IV. At the risk of some over-simplification, it will be useful to distinguish between two approaches to making capital flows and exchange rates endogenous in a multi-country econometric model: a "large" approach and a "small" approach. The large approach is to take a model like LINK and modify the single-country models in it to account for all flows of funds among the domestic and foreign sectors.[1] The problem with this approach is again the size of the task. It is a tedious job to account for all flows of funds in a large single-country model,[2] and the amount of effort involved in doing this for all the single-country models in LINK, some of which currently have fairly weak monetary sectors, is enormous.

The small approach, which is the approach discussed in Section IV of this paper, is to specify and estimate a relatively small, highly aggregated multi-country model, but yet a model in which all flows of funds among the countries are accounted for. The emphasis in this approach is on the determination of the key aggregate macroeconomic variables in the system (e.g., prices, interest rates, and exchange rates) and on accounting for all the aggregate flows of funds and goods among the countries. The aim of this approach is to end up with an econometric model that, within its aggregate framework, accounts for all the adding-up constraints and is relatively easy to estimate and analyze. The aim is also to end up with a model that can, if desired, be fairly easily disaggregated later without changing its basic structure. In short, then, the small approach is to start with a small model that accounts for all the aggregate flows of funds and get larger later, rather than, as with the large approach, to start with a large model that does not account for all the flows of funds and work later on accounting for them.[3]

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[1]Hickman [13], p. 203, has stated that work is currently in progress on making capital movements endogenous in the LINK model.

[2]See the 84-equation model in Fair [7] for an example of a single-country model in which all flows of funds are accounted for. See in particular Section 1.3 for a description of the linking (by sector) of the U.S. national income accounts with the flow-of-funds accounts.

II. A Comparison of the Quantitative Properties of Seven Multi-Country Models

There is by now a considerable amount of evidence on the quantitative properties of various multi-country models. The purpose of this section is not, however, to review this evidence in detail, since a fairly extensive review is already contained in Deardorff and Stern [4]. The purpose of this section is rather to take from this evidence results for a common experiment for each model and compare these results across models. The common experiment is an autonomous increase in U.S. income of one percent. Some adjustment of the results for some models had to be made in order to make them comparable, and even given these adjustments, it should be stressed that the results are only approximately comparable.[4] The present comparison should give one a general idea of the different properties of the models, but it is by no means a rigorous evaluation of the differences.

The seven models are: (1) the Morishima-Murata (MM) trade-multiplier model [16], (2) the LINK model [1], (3) the OECD model [21], (4) one of the multiplier models in De Rosa and Smeal (DS) [5], (5) the METEOR model of

[3] The approach of Berner *et al.* [2], who are concerned with the specification and estimation of a five-country model, is perhaps somewhere in between the small and large approaches. There is an attempt in this approach to account for capital flows, although the proposed treatment of exchange rate determination as described in [2] is suspect. The proposed single-country models in [2] are also much larger than the proposed single-country models in Section III of this paper.

[4] For example, the properties of nonlinear models are different for different starting points, and the starting points were not all the same for the results presented in this section. The results also may be sensitive to what is assumed about monetary policy, although most of the models considered here have either no or a weak monetary sector. Finally, the properties of nonlinear models are different in absolute value for positive and negative changes, and for some of the results presented in this section the U.S. policy change was negative rather than positive. For present purposes, the signs of the effects were merely reversed when the U.S. policy change was negative.

the Netherlands Central Planning Bureau [3], (6) the price-linkage model Kwack (KWACK) [15], and (7) the RDX2-MPS model of Canada and the United States [11]. The results presented in this section are taken from the following seven sources: Morishima and Murata [16] for the MM model, Hickman [13] for the LINK model, OECD [20] for the OECD model, De Rosa and Smeal [5] for the DS model, Deardorff and Stern [4] for the METEOR model, Kwack [15] for the KWACK model, and Helliwell [11] for the RDX2-MPS model. The income effects from the autonomous increase in U.S. income are presented in Table 1, and the price effects are presented in Table 2. Table 3 contains a description of how the numbers in Tables 1 and 2 were obtained. The results in the two tables are fairly self explanatory, and so only a brief discussion of them will be presented here.

Except for some of the results for RDX2-MPS, the income effects in Table 1 are all positive. For the one-year results, the DS effects are larger than the LINK effects, which is due in large part to the use in the DS model of larger expenditure multipliers than exist in the LINK model. The METEOR effects are also larger than the LINK effects, and except for Canada, the MM effects are slightly larger than the LINK effects. For the two-year results, the METEOR effects are again larger than the LINK effects. For Canada, the RDX2-MPS effects are considerably smaller than both the LINK and METEOR effects. For the three-year or more results, the METEOR effects are quite large relative to the others. The MM, LINK, and OECD effects for Canada are fairly close, as are the LINK and OECD effects for Japan. The RDX2-MPS effects for Canada are small for the six-year period, but fairly large and negative for the eight-year period. In general, the results for the RDX2-MPS

TABLE 1

Percentage Income Change of Country Induced by a Sustained One Percent Autonomous Increase in U.S. Income

Model	Time Period (years)	U.S.	Canada	Austria	Belgium	France	Germany
				Country			
MM	1	1.56	.27				
LINK	1	1.18	.31	.05	.05	.02	.04
DS	1	2.00	.72	.13	.39	.13	.19
METEOR	1	2.42	.65		.23	.12	.19
LINK	2	1.87	.56	.11	.09	.04	.08
METEOR	2	2.86	1.29		.46	.30	.43
RDX2-MPS[a]	2	2.11	.10				
RDX2-MPS[b]	2	2.11	.10				
RDX2-MPS[c]	2	2.12	.19				
RDX2-MPS[d]	2	2.11	.21				
MM	Long Run	4.11	.93				
LINK	3	2.58	.86	.24	.15	.06	.14
METEOR	5	8.33	4.19		1.66	1.49	1.81
OECD	Long Run	2.00	.70				.50
RDX2-MPS[a]	6	1.65	.14				
RDX2-MPS[b]	6	2.02	-.21				
RDX2-MPS[c]	6	1.66	.01				
RDX2-MPS[d]	6	1.99	.09				
RDX2-MPS[a]	8	-.83	-.29				
RDX2-MPS[b]	8	-.95	-1.06				
RDX2-MPS[c]	8	-.85	-.83				
RDX2-MPS[d]	8	-.92	-.80				

a. Migration and all capital flows suppressed; fixed exchange rate.
b. Migration and all capital flows suppressed; flexible exchange rate.
c. Full transmission; fixed exchange rate.
d. Full transmission; flexible exchange rate.

TABLE 1 (continued).

Model	*Time Period (years)*			*Country*			
		Italy	*Sweden*	*U.K.*	*Japan*	*Australia*	*Netherlands*
MM	1			.14	.19		
LINK	1	.08	.10	.08	.13	.03	
DS	1	.15	.19	.19	.17	.12	.33
METEOR	1	.15		.19	.22		.17
LINK	2	.17	.19	.21	.27	.09	
METEOR	2	.34		.45	.45		.36
RDX2-MPS[a]	2						
RDX2-MPS[b]	2						
RDX2-MPS[c]	2						
RDX2-MPS[d]	2						
MM	Long Run			.14	.87		
LINK	3	.31	.33	.35	.40	.24	
METEOR	5	1.38		1.83	1.79		1.38
OECD	Long Run				.35		
RDX2-MPS[a]	6						
RDX2-MPS[b]	6						
RDX2-MPS[c]	6						
RDX2-MPS[d]	6						
RDX2-MPS[a]	8						
RDX2-MPS[b]	8						
RDX2-MPS[c]	8						
RDX2-MPS[d]	8						

TABLE 2

Percentage Price Change of Country Induced by a Sustained One Percent Autonomous Increase in U.S. Income

Model	Time Period (years)	U.S.	Canada	Austria	Belgium	France	Germany
LINK	1	.310	.000	.010	.000	.010	.050
METEOR	1	-.084	-.029		.005	.006	-.000
KWACK	1	-.013	-.003	.000	.000		.000
LINK	2	.290	.170	.010	-.010	.000	.110
METEOR	2	.152	.040		.053	.035	.031
RDX2-MPS[a]	2	.210	-.060				
RDX2-MPS[b]	2	.210	-.050				
RDX2-MPS[c]	2	.210	-.020				
RDX2-MPS[d]	2	.210	-.010				
LINK	3	.690	.640	.040	-.010	.020	.230
METEOR	5	1.050	.916		.627	.453	.480
KWACK	6	.373	.267	.003	.027		.027
RDX2-MPS[a]	6	2.770	.450				
RDX2-MPS[b]	6	2.910	.220				
RDX2-MPS[c]	6	2.780	.440				
RDX2-MPS[d]	6	2.880	.540				
RDX2-MPS[a]	8	5.090	1.000				
RDX2-MPS[b]	8	5.640	.060				
RDX2-MPS[c]	8	5.110	1.140				
RDX2-MPS[d]	8	5.550	1.140				

a. Migration and all capital flows suppressed; fixed exchange rate.
b. Migration and all capital flows suppressed; flexible exchange rate.
c. Full transmission; fixed exchange rate.
d. Full transmission; flexible exchange rate.

TABLE 2 (continued)

Model	*Time Period (years)*	*Italy*	*Sweden*	*U.K.*	*Country* *Japan*	*Australia*	*Netherlands*
LINK	1	-.010		.000	-.020	-.010	
METEOR	1	.009		.008	.003		.018
KWACK	1	.000	.000	.000	-.003	.000	.000
LINK	2	-.020		-.060	-.030	-.030	
METEOR	2	.053		.058	.041		.076
RDX2-MPS[a]	2						
RDX2-MPS[b]	2						
RDX2-MPS[c]	2						
RDX2-MPS[d]	2						
LINK	3	.010		.000	[illegible]	-.100	
METEOR	5	.565		.629	[illegible]		.647
KWACK	6	.030	.023	.020	.107	.107	.047
RDX2-MPS[a]	6						
RDX2-MPS[b]	6						
RDX2-MPS[c]	6						
RDX2-MPS[d]	6						
RDX2-MPS[a]	8						
RDX2-MPS[b]	8						
RDX2-MPS[c]	8						
RDX2-MPS[d]	8						

TABLE 3

Sources for the Results in Tables 1 and 2

Each number in Table 1 is $\Delta Y_j/Y_j \div \Delta A_i/Y_i$, and each number in Table 2 is $\Delta P_j/P_j \div \Delta A_i/Y_i$, where

ΔA_i = autonomous change in U.S. real income,
ΔY_j = induced change in the real income of country j ,
Y_j = level of real income of country j ,
ΔP_j = induced change in the price level of country j ,
P_j = price level of country j .
(country j may be the U.S.)

Model	*Source*	*Discussion*
MM	[16]	One-year values of $\Delta Y_j/\Delta A_i$ taken from Table 5 (p. 325); long-run values of $\Delta Y_j/\Delta A_i$ taken from Table 6 (p. 325); and values of Y_j taken from Table 8 (p. 328). Model is linear, so starting point does not matter. Values of Y_j are for 1964.
LINK	[13]	Numbers in Table 1 taken directly from Tables 2-4 (pp. 211-213), and numbers in Table 2 taken directly from Tables 6-8 (pp. 218-220). Starting point was 1973.
DS	[5]	The numbers in Table 1 are the numbers in Table 9 (p. 10a) multiplied by 2.0, the U.S. expenditure multiplier. The results are based on the following assumptions: use of Houthakker-Magee [14] estimated income elasticities of the demand for imports in each country; use of a U.S. expenditure multiplier of 2.0; and use of an expenditure multiplier for each of the other countries of 1.5. The year was 1974.
METEOR	[4]	Numbers in Table 1 taken directly from Table 28 (p. 96), and numbers in Table 2 taken directly from Table 29 (p. 97).
OECD	[20]	Numbers in Table 1 taken directly from Table 8B (p. 34). The effects are assumed here to be long-run, although no time period is given in [20].
RDX2-MPS	[11]	Full transmission numbers in Tables 1 and 2 taken directly from Table 1-A (p. 259) with signs reversed, and numbers in Tables 1 and 2 when migration and all capital flows are suppressed taken directly from Table 5-A (p. 273) with signs reversed. Starting point was 1963.

TABLE 3 (continued)

Model	*Source*	*Discussion*
KWACK	[15]	The numbers in Table 2 are the numbers in Table 9 (p. 27) divided by -3.0. The effects in Table 9 are for a one percentage point increase in the unemployment rate, and from the Okun's Law equation for the U.S. in Table 6 (p. 20), a one percentage point increase in the unemployment rate corresponds roughly to a three percent decrease in real output. Starting point was 1968.

model show evidence of a considerable amount of cycling.

For the one-year results in Table 2, the price effects are all fairly small except perhaps for the LINK effect for the U.S. This is also true for the two-year results. For the three-year or more results, on the other hand, the METEOR and RDX2-MPS effects are fairly large, as are the LINK effects for the U.S. and Canada. The KWACK effects are still small, and the three-year LINK effects for Belgium, Japan, and Australia are still negative.

To conclude, it is partly a matter of judgment whether one feels that the differences in Tables 1 and 2 are large or small. Clearly, however, the five-year METEOR results are quite different from the others, as are the RDX2-MPS results for Canada in Table 1. On the other hand, the MM and LINK differences in Table 1 seem fairly small, and many of the differences for the one-year results in Table 1 are also small.

With respect to the possible sensitivity of a model's properties to the treatment of capital flows and exchange rates, it should be noted that for the RDX2-MPS model the two- and six-year results in Tables 1 and 2 are not very sensitive to the treatment of capital flows and the exchange rate, but the eight-year results are. For the case in which migration and capital flows are suppressed, the eight-year income effect for Canada is -0.29 in the fixed-exchange-rate case and -1.06 in the flexible-exchange-rate case. The corresponding price effects are 1.000 and 0.060. For the case in which migration and capital flows are not suppressed, the eight-year effects are not sensitive to the treatment of the exchange rate. The income effect is 1.140 in both the fixed- and flexible-exchange-rate cases, and the price effect is -0.83 in the fixed-exchange-rate case and -0.80 in the flexible-exchange-rate case. The overall results for the RDX2-MPS model are thus somewhat mixed with respect to the sensitivity question.

III. A Quasi-Empirical Two-Country Model

As mentioned in the Introduction, the model discussed in this section (Model A) is an attempt to bridge in part the current gap between theoretical and econometric work in international economics. The fact that the properties of Model A turn out to be quite sensitive to the treatment of capital flows and the exchange rate casts some doubt on the reliability of the results presented in Section II. The properties of Model A also cast some doubt on the reliability of the results from the standard two-country theoretical models in the literature. In particular, the price links among countries that these standard models ignore appear to be important, at least as reflected in Model A.

Since Model A was constructed by linking the 84-equation U.S. econometric model in Fair [7] to itself, the U.S. is half of the world in the model. Space limitations prevent a detailed description of Model A here. The model is discussed and analyzed in Fair [9] and [10], and an appendix to [10] is available that contains a complete list of the 180 equations of the model. The following is a brief discussion of some of the key features of Model A.

There are four sectors per each of the two countries in the model: household, firm, bank, and government. All flows of funds among the eight sectors are accounted for. This means that any financial saving or dissaving of a sector in a period results in the change in at least one of its assets or liabilities, that any financial asset of one sector is a corresponding liability of some other sector, and that the government budget constraints of the two countries are accounted for. The model is one in which stock and flow effects are completely integrated. The exchange rate, for example, has an effect on both stock and flow variables, and in the flexible-exchange-rate case it is simultaneously determined along with the other endogenous variables. As discussed in [9], this integration of stock and flow effects is not true of other approaches to the balance of payments and is one of the main distinctions between Model A and other models.

Each country specializes in the production of one good, and the goods are traded. In addition to the obvious links between the two countries with respect to capital and goods flows, there are important price links between the two countries: in each country the price of the imported good has an effect on the price of the home-produced good. In other words, a price change in one country has a direct effect on the price change in the other country and vice versa. Wages are also endogenous in the model, and prices affect wages as well as vice versa.

One important feature of the model with respect to prices is that prices have, other things being equal, a negative effect on demand. One would expect for the usual microeconomic reasons the demand for a good to be a negative function of its price, and this is in fact the case in the U.S. econometric model upon which Model A is based. Although this may seem to be an obvious characteristic for a model to have, in most macroeconometric models consumption is not a direct function of prices, but only of income terms and the like. The consumption equations in Model A differ from the usual consumption equations in macroeconometric models in having explanatory variables that are consistent with microeconomic theory.[5]

[5] See Section 1.1 in [7] for a discussion of the differences between the consumption equations in the 84-equation U.S. econometric model (and thus in Model A) and the consumption equations in other macroeconometric models. One of the three main features of the theoretical model in [6], upon which the econometric model in [7] is based, is the derivation of the decisions of the individual agents in the economy from the assumption of maximizing behavior. (The other two main features of the model are an explicit treatment of possible disequilibrium effects and the accounting for all flows of funds in the system.)

In [10] the properties of Model A were analyzed in four different regimes: the regimes of (1) zero capital mobility and a fixed exchange rate, (2) zero capital mobility and a flexible exchange rate, (3) perfect capital mobility and a fixed exchange rate, and (4) perfect capital mobility and a flexible exchange rate. A summary of the results from this exercise is presented in Table 4. Two basic experiments were performed for these results: a monetary-policy experiment and a fiscal-policy experiment. For the monetary-policy experiment the amount of government securities outstanding of country 1 was decreased, a standard expansionary open market operation on the part of the monetary authorities of country 1.[6] For the fiscal-policy experiment the value of goods purchased by the government of country 1 was increased. For this latter experiment no change in the amount of government securities outstanding was made, which means that any government deficit that results from the increase in purchases is financed by an increase in non-borrowed reserves (high powered money). In other words, this latter experiment is a money-financed fiscal-policy change. The effects of these two experiments after three quarters are presented in Table 4 for two variables for each country, real output and the domestic price level.

Results are also presented in Table 4 for two versions of the import-demand equations. The first version is the actual estimated equation for the demand for imports in the U.S. econometric model in [7]. In this version there are price lags: prices affect the demand for imports with a lag of

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[6]The experiment in [10] was actually one in which the amount of government securities outstanding was increased (a contractionary action) rather than decreased. All the results in [10] are in fact for contractionary monetary and fiscal actions. Given the results in Section II of this paper, it seemed best in the present section to talk about expansionary rather than contractionary actions, and so for purposes of the discussion in this section all the signs in [10] have been reversed.

TABLE 4

Results for Model A: Effects After Three Quarters

I. Monetary-Policy Experiment (Decrease in government securities outstanding of country 1 of 1.25.)

(Each cell is split diagonally: the upper-right value is for Country 1, the lower-left value is for Country 2.)

Real Output Change

Capital Mobility	Price Lags in Import Equations, Exchange Rate: Fixed (Country 1 / Country 2)	Price Lags in Import Equations, Exchange Rate: Flexible (Country 1 / Country 2)	No Price Lags in Import Equations, Fixed (Country 1 / Country 2)	No Price Lags in Import Equations, Flexible (Country 1 / Country 2)
Zero	0.94 / 0.20	1.39 / -0.18	0.86 / 0.25	0.63 / 0.41
Perfect	0.62 / 0.59	-0.41 / 1.68	0.63 / 0.59	0.51 / 0.74

Price Level Change

Capital Mobility	Price Lags, Fixed	Price Lags, Flexible	No Price Lags, Fixed	No Price Lags, Flexible
Zero	-0.001 / -0.016	-0.159 / 0.173	0.016 / -0.026	0.117 / -0.165
Perfect	-0.015 / -0.014	0.867 / -0.904	-0.015 / -0.014	0.158 / -0.185

II. Fiscal-Policy Experiment (Money-financed increase in government purchases of goods of country 1 of 1.25.) These results are comparable to those in Tables 1 and 2.

Real Output Change

Capital Mobility	Price Lags, Fixed	Price Lags, Flexible	No Price Lags, Fixed	No Price Lags, Flexible
Zero	1.90 / 0.33	2.80 / -0.58	1.89 / 0.34	1.50 / 0.63
Perfect	1.80 / 0.46	0.92 / 1.35	1.82 / 0.44	1.81 / 0.47

Price Level Change

Capital Mobility	Price Lags, Fixed	Price Lags, Flexible	No Price Lags, Fixed	No Price Lags, Flexible
Zero	0.053 / -0.001	-0.523 / 0.599	0.066 / -0.006	0.494 / -0.475
Perfect	0.041 / 0.009	0.953 / -0.910	0.043 / 0.011	0.247 / -0.190

TABLE 4 (continued)

Notes: 1. The results are taken from Fair [10] except for the Fixed/Zero and Fixed/Perfect results in the no-price-lag case. These latter results are taken from an earlier version of the paper: "A Model of the World Economy," Cowles Foundation Discussion Paper No. 430, April 27, 1976.

2. The results for the monetary-policy experiment have not been adjusted except for the change in sign discussed in footnote 6 and for multiplication of the price level changes by 100. The numbers are merely the total induced changes in real income after three quarters and the total induced changes in the price level after three quarters (ΔY_j and ΔP_j in the notation in Table 3).

3. The results for the fiscal-policy experiment have been adjusted and are as in Tables 1 and 2. The real output changes are $\Delta Y_j/Y_j \div \Delta A_i/Y_i$ and the price level changes are $\Delta P_j/P_j \div \Delta A_i/Y_i$, where the value of ΔA_i is -1.25. For these calculations the values for Y_i and Y_j were taken to be 169.4, the actual U.S. value in 1971IV, and the values for P_i and P_j were taken to be 1.26, also the actual U.S. value in 1971IV.

4. The starting quarter for the experiments was 1971I, a quarter that is at or near the bottom of a contraction.

one or two quarters. In the second version these lags have been eliminated: imports respond in the current quarter to a change in prices.

The following is a brief discussion of some of the main features of the results in Table 4. Space limitations again prevent an extensive discussion here, and the reader is referred to [10] for more detail. It should be stressed that the following discussion is somewhat loose. Reference is sometimes made to a change in one endogenous variable "causing," "leading to," or "resulting in" a change in another endogenous variable or variables. This discussion, while useful pedagogically, is loose because the model is simultaneous. Strictly speaking, each endogenous variable in the model affects and is affected by all the other endogenous variables. It should also be stressed that the effects in Table 4 are effects after only three quarters; they are of the nature of short-run effects. It is particularly important to keep this in mind when comparing the price-lag and no-price-lag cases. While the results for these two cases are sometimes quite different in Table 4, these differences are likely to lessen as the length of the period after the change increases.

Consider first in Table 4 the monetary-policy experiment. In the Fixed/Perfect regime the monetary-policy change has almost identical effects on the two countries. In this regime it makes no difference with respect to the aggregate effects which country the open market operation takes place in. Therefore, since the two countries in the model are virtually the same, the effects on the two countries are virtually the same. In the Fixed/Zero regime, on the other hand, the effects on output in country 1 are greater than they are in country 2. This is because in this regime the monetary-policy change lowers the interest rate more in country 1 than in country 2.

In the two fixed-exchange-rate regimes the results are not sensitive to whether or not there are price lags in the import equations. In these two regimes the changes in prices are not very large, and so the results are not very sensitive to what one assumes about the price responsiveness of imports.

In the Flexible/Perfect regime the expansionary monetary policy in country 1 actually has a negative effect on country 1's real output in the case in which there are price lags in the import equations. The reason for this is as follows. The expansionary monetary policy results in this case in a depreciation of country 1's currency (which is needed to keep the interest rates in the two countries the same[7]). This then results in a higher domestic price level in country 1 (since the price of country 1's imports is higher) and a lower domestic price level in country 2 (since the price of country 2's imports is lower). As mentioned above, a higher price level in a country has, other things being equal, a negative effect on demand. It turned out in this experiment that the negative effect on output from the increase in the price level in country 1 was large enough to offset the positive effects induced by the policy change, so that there was a net contraction in output in country 1. In country 2, on the other hand, the positive effect on output from the decrease in its price level and the other positive effects induced by the policy change resulted in an expansion in output.

Remember that the results just cited are for the case in which there

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[7]Exchange-rate expectations are assumed to be static for the experiments, and so the interest rates in the two countries are the same in the perfect-capital-mobility regimes. See footnote 1 in the Appendix.

are price lags in the import equations. Because of these lags, the depreciation of country 1's currency has no direct immediate effect on decreasing its demand for imports or on increasing country 2's demand for country 1's exports. In the no-price-lag case, on the other hand, this channel is open, and in this case, as can be seen in Table 4, there is no longer a contraction in country 1's output in the Flexible/Perfect regime. The output increase is, however, smaller in country 1 than in country 2, and this is again due to the negative effect (through the price level) of the devaluation of country 1's currency on country 1's output.

The Flexible/Zero regime is unusual and probably not very realistic. In the price-lag case in this regime the expansionary monetary policy in country 1 actually leads to a contraction in country 2's output. The reason for this result is as follows. In the Flexible/Zero regime the financial saving of country 1 (its balance of payments on current account) cannot change since there is no capital mobility and no change in international-reserve holdings. If imports do not respond to current price changes, as is true in the price-lag case, then the adjustment to the expansionary monetary policy must take place through a terms-of-trade effect. Country 1's currency must appreciate to turn the terms of trade in favor of country 1 to offset the decrease in its balance of payments on current account that would otherwise have taken place as a result of country 1's expansionary monetary policy. The depreciation of country 2's currency leads to an increase in its price level, which is, other things being equal, contractionary. This contractionary effect was strong enough in the present experiment so as to lead to a net contraction in country 2's output.

In the no-price-lag case in the Flexible/Zero regime, real output in country 2 expands rather than contracts. In this case country 1's currency depreciates rather than appreciates, which results, other things being equal, in a decrease in its imports and an increase in its exports. The offset to the decrease in country 1's balance of payments on current account that would otherwise have taken place as a result of its expansionary monetary policy occurs in the no-price-lag case through a change in imports and exports rather than through a change in the terms of trade. There is thus no depreciation of country 2's currency and so no contractionary effect on its output from this source.

One further point about the results for the monetary-policy experiment in Table 4 should be noted, which is that in the fixed-exchange-rate regimes the expansionary monetary policy leads to a slight decrease in the price levels in both countries. This is explained by the fact that interest rates have, other things being equal, a positive effect on prices in the model.[8] An expansionary monetary policy leads to a decrease in interest rates and thus from this source to a decrease in prices. An expansionary monetary policy also has positive effects on prices through other sources, but the net effect on prices after three quarters is still negative for the results in Table 4.

The fiscal-policy experiment reported in Table 4 is a combination of a direct increase in the sales of country 1's good and of an expansionary monetary policy. Since the monetary-policy effects have already been discussed, the further effects from the increase in sales will not be discussed here.[9] (Again, see [10] for a detailed discussion of all the results.)

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[8] See footnote 2 in the Appendix for an explanation of this.

[9] The fiscal-policy results in Table 4 are directly comparable in terms of units to the results in Tables 1 and 2. It should be kept in mind in comparing these results, however, that fiscal-policy effects in Model A are

This completes the discussion of the results in Table 4. It is clear that the properties of Model A are sensitive to the choice of regime, which, as mentioned in the Introduction, indicates the need to make capital flows and exchange rates endogenous in multi-country econometric models before much confidence can be placed on their properties.

The discussion of the results in Table 4 also shows the importance of price effects in the model in the flexible-exchange-rate regimes, something which is generally ignored or treated very lightly in small-scale theoretical models. In Model A import prices influence domestic prices, and prices in general influence demand. These price effects can be quantitatively quite important. To give one example where they are important, consider Model A versus Mundell's two-country model [17, Appendix to Chapter 18] in the Perfect/Flexible regime. In Mundell's model in this regime an expansionary monetary policy has a positive effect on the output of the home country and a negative effect on the output of the other country. For Model A this result is either completely reversed (in the price-lag case) or else substantially modified (in the no-price-lag case). As discussed above, the depreciation of country 1's currency that the expansionary monetary policy causes in this case leads in Model A to a higher domestic price level in country 1 and then either an actual contraction in country 1's real output or else an expansion that is smaller than the expansion in country 2. It thus appears from the results in this section that Mundell's model and models of this type have omitted some potentially important price links between countries.

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sensitive to what one assumes about monetary policy (see Fair [8]). Quite different fiscal-policy effects would have been obtained for the results in Table 4 had something different been assumed about monetary policy. This sensitivity is, of course, not necessarily true of the models considered in Section II, since, as mentioned in footnote 4, many of these models have either no or a weak monetary sector.

IV. A Suggested Way of Modeling the Economic Linkages Among Countries: A Small Approach

One possible way of constructing a multi-country macroeconometric model with endogenous capital flows and exchange rates would be to estimate for each country a model as in Fair [7], in which all flows of funds among the sectors are accounted for, and then link these models together. The resulting overall model would be like Model A, only it would be completely empirical and for more than two countries. Since, as discussed in the Introduction, this is an enormous task, it may be better to start with a somewhat smaller approach. The purpose of this section is to propose such an approach. The model described in this section requires that only 5 or 6 equations be estimated per country, but yet it accounts for all the main economic links among the countries and allows for the endogenous determination of the exchange rates.

Although the model that is outlined in this section is for three countries, the generalization to more than three countries is straightforward. The model in this section is a simplified version of the three-country model in the Appendix, and the reader is assumed to have mastered the model in the Appendix before reading this section. The model in the Appendix is a three-country version of the two-country model in Fair [9]. This latter model is in turn a somewhat simplified version of Model A. Indirectly, therefore, the model in this section is a simplified version of Model A for three countries. The reason for separating the model in this section from the model in the Appendix is to make clear the simplifications that are being proposed in this section.

The 5 or 6 equations to be estimated per country for the model in this section are equations explaining (1) the demand for imports, (2) the demand for foreign securities, (3) the demand for domestic money, (4) the price of domestic goods, (5) the demand for domestic goods, and possibly (6) the domestic interest rate. The overall model consists of 20 equations per country. The notation used in this section is presented in Table 5. All domestic goods in each country are aggregated into one good, X, and all domestic financial securities (except money) in each country are aggregated into one security, B. (Liabilities correspond to negative values of B.) There is therefore only one domestic price and one domestic interest rate per country. Any possible effects on behavior of capital gains and losses on securities are ignored. The securities of the different countries are assumed not to be perfect substitutes.[10]

For those who would like to skip or skim the equations, a brief outline of them is as follows. Equations (1.1)-(1.5) and (1.7)-(1.11) are definitions: (1.1) and (1.2) define the financial savings of the private and government sectors; (1.3) and (1.4) define the budget constraints of the two sectors; (1.5) and (1.7)-(1.9) are adding-up constraints; and (1.10) and (1.11) define price and interest rate indices. Equation (1.6) explains bank reserves, and equations (1.12)-(1.17) are the 5 or 6 equations to be estimated per country. Finally, equations (1.18)-(1.21) determine the allocation of goods and securities among countries.

With respect to the equations, consider first for country 1 the aggregation of the household, firm, and bank sectors in the model in the Appendix into one private sector (denoted by a subscript p).[11] Adding the private

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[10] If they are perfect substitutes, then equation (1.13) below drops out, leaving one less equation to be estimated per country. See footnote 1 in the Appendix.

TABLE 5

Variables for the Model in Section III

(i, j = 1, 2, 3)

Number of Variables Endogenous	*Number of Variables* Exogenous	Variable	Definition
3[a] (i=j)	6	B^j_{ig}	= amount of country i's securities held by the government of country j in units of country i's money (negative values are liabilities).
9		B^j_{ip}	= amount of country i's securities held by the private sector of country j in units of country i's money (negative values are liabilities).
3		B^i_{mp}	= index of the total foreign security holdings of the private sector of country i .
	3	BO_i	= bank borrowing from the government in country i .
3		BR_i	= bank reserves in country i .
1[b]		e_2	= price of country *2*'s money in terms of country *1*'s money.
1[b]		e_3	= price of country *3*'s money in terms of country *1*'s money.
	9	M^j_{ig}	= amount of country i's money held by the government of country j in units of country i's money.
3 (i=j)	6	M^j_{ip}	= amount of country i's money held by the private sector of country j in units of country i's money.
3		M_{ib}	= total money supply in country i (total deposits in the bank sector of country i).
3		P_i	= price of the good of country i in units of country i's money.
3		P^i_m	= price index of the total imports of country i in units of country i's money.
1	2[c]	Q_i	= amount of the international reserve held by country i (price = 1.0).
3		R_i	= interest rate for country i's securities.
3		R^i_m	= interest-rate index for the total foreign security holdings of country i .

TABLE 5 (continued)

Number of Variables Endog-enous	Exog-enous	
	3	RD_i = discount rate in country i .
	3	RR_i = reserve requirement ratio in country i .
3		S_{ig} = financial saving of the government of country i .
3		S_{ip} = financial saving of the private sector of country i .
	3	V_{ip} = taxes paid by the private sector of country i .
	9	X_{ig}^j = real value of the good of country i purchased by the government of country j .
9		X_{ip}^j = real value of the good of country i purchased by the private sector of country j .
3		X_{mp}^i = index of the total real value of imports of the private sector of country i .
3		X_{if} = total real value of sales of the good of country i .
60	44	

a. Exogenous if no reaction functions of the monetary authorities are specified (equations (1.17), (1.17)', (1.17)").
b. Exogenous in fixed-exchange-rate regime.
c. Endogenous in fixed-exchange-rate regime.

saving equations, (13), (27), and (31), yields:

$$(1.1)\qquad S_{1p} = P_1(X_{1f} - X^1_{1p}) - e_2P_2X^1_{2p} - e_3P_3X^1_{3p} + R_1B^1_{1p} + e_2R_2B^1_{2p}$$

$$+ e_3R_3B^1_{3p} - V_{1p} - RD_1BO_1 \text{ . [saving of the private sector]}$$

The government saving equation, (33), remains unchanged except to note that $V_{1h} + V_{1f} = V_{1p}$:

$$(1.2)\qquad S_{1g} = V_{1p} + RD_1BO_1 - P_1X^1_{1g} - e_2P_2X^1_{2g} - e_3P_3X^1_{3g} + R_1B^1_{1g}$$

$$+ e_2R_2B^1_{2g} + e_3R_3B^1_{3g} \text{ . [saving of the government sector]}$$

Adding the private budget-constraint equations, (14), (28), and (32), yields:

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[11]With the exception of X_{1f} and M_{1b}, all the h , f , and b subscripts in the Appendix have been changed in this section to p , even when a variable in the Appendix pertains to only one or two of the three individual sectors. As examples of the change of notation, $B^1_{1p} = B^1_{1h} + B^1_{1f} + B^1_{1b}$, $X^1_{1p} = X^1_{1h} + X^1_{1f}$, and $M^1_{1p} = M^1_{1h}$. Also, government purchase of labor (L_{ig}) has been dropped as an explicit variable in the model and has instead been taken to be part of the good of country i . In other words, W_iL_{ig} has been taken to be part of $P_iX^i_{ig}$.

(1.3) $$0 = S_{1p} + \Delta(M_{1b} - M^1_{1p}) - e_2\Delta M^1_{2p} - e_3\Delta M^1_{3p} - \Delta B^1_{1p} - e_2\Delta B^1_{2p} - e_3\Delta B^1_{3p} - \Delta(BR_1 - BO_1) \, .$$ [private sector budget constraint]

The government budget-constraint equation, (34), remains unchanged:

(1.4) $$0 = S_{1g} + \Delta(BR_1 - BO_1) - \Delta M^1_{1g} - e_2\Delta M^1_{2g} - e_3\Delta M^1_{3g} - \Delta B^1_{1g} - e_2\Delta B^1_{2g} - e_3\Delta B^1_{3g} - \Delta Q_1 \, .$$ [government sector budget constraint]

Equation (35) also remains unchanged except for the replacement of h by p :

(1.5) $$M_{1b} = M^1_{1p} + M^1_{1g} + M^2_{1p} + M^2_{1g} + M^3_{1p} + M^3_{1g} \, .$$ [total deposits in the bank sector]

Equation (30) remains the same:

(1.6) $$BR_1 = RR_1 \, M_{1b} \, .$$ [bank reserves]

Equation (36) in the new notation is

(1.7) $$0 = B^1_{1p} + B^1_{1g} + B^2_{1p} + B^2_{1g} + B^3_{1p} + B^3_{1g} \, .$$ [supply of the bond of country 1 equals the demand for it]

Equation (109) remains unchanged:

(1.8) $$0 = \Delta Q_1 + \Delta Q_2 + \Delta Q_3 \, .$$ [no change in total world reserves]

Equation (24) in the new notation is

(1.9) $$X_{1f} = X^1_{1p} + X^1_{1g} + X^2_{1p} + X^2_{1g} + X^3_{1p} + X^3_{1g} \, .$$ [total sales of the good of country 1]

Let X_{mp}^{1} denote an index of the total imports of country 1's private sector from countries 2 and 3, i.e., some weighted average of X_{2p}^{1} and X_{3p}^{1} ; and let B_{mp}^{1} denote an index of the total foreign security holdings of country 1's private sector, i.e., some weighted average of B_{2p}^{1} and B_{3p}^{1} . Also, let P_{m}^{1} be a price index of the total imports of country 1 in the units of country 1's money:

(1.10) $$P_{m}^{1} = \alpha_{1}^{1} e_{2} P_{2} + \alpha_{2}^{1} e_{3} P_{3} ,$$ [price index of the total imports of country 1]

where α_{1}^{1} and α_{2}^{1} are some appropriately chosen weights. Similarly, let R_{m}^{1} be an interest-rate index for the total foreign security holdings of country 1:

(1.11) $$R_{m}^{1} = \beta_{1}^{1} R_{2} + \beta_{2}^{1} R_{3} ,$$ [interest-rate index for the total foreign security holdings of country 1]

where β_{1}^{1} and β_{2}^{1} are also some appropriately chosen weights.

The stage is now set for explaining the 5 or 6 equations to be estimated for country 1. In what follows, Z_{1} denotes a vector of all the exogenous and lagged endogenous variables that help explain the LHS variable in the equation. The variables in Z_{1} may, of course, differ for different equations. The following 6 equations are meant to be approximations to the equations that would be estimated were the complete model in the Appendix being estimated:

(1.12) $X^1_{mp} = f_{12}(P_1, P^1_m, X_{1f}, Z_1)$, [demand for imports by the private sector of country 1]

(1.13) $B^1_{mp} = f_{13}(R_1, R^1_m, Z_1)$, [demand for foreign securities by the private sector of country 1]

(1.14) $M^1_{1p} = f_{14}(R_1, P_1, X_{1f}, Z_1)$, [demand for domestic money by the private sector of country 1]

(1.15) $P_1 = f_{15}(P^1_m, R_1, X_{1f}, Z_1)$, [price of the good of country 1]

(1.16) $X^1_{1p} = f_{16}(P_1, P^1_m, X_{1f}, R_1, Z_1)$, [demand for the good of country 1 by the private sector of country 1]

(1.17) $R_1 = f_{17}(R^1_m, P_1, X_{1f}, Z_1)$. [reaction function of the monetary authorities of country 1]

The total level of sales of the good of country 1, X_{1f} , is used as the aggregate real income or activity variable of country 1 in equations (1.12) and (1.14)-(1.17). Equation (1.12) explains the demand for imports as a function of the two prices, real income, and other (non-endogenous) variables. This equation is an approximation to equations (3), (4), (18), and (19) in the Appendix. Equation (1.13) explains the demand for foreign securities as a function of the two interest rates and other variables. It is an approximation to equations (9) and (10) in the Appendix. In the actual estimation of equation (1.13), one may want to take as the LHS variable $B^1_{mp}/(B^1_{mp} + B^1_{1p})$, the share of foreign securities in the total portfolio of the private sector.

Equation (1.14) explains the demand for domestic money as a function of the interest rate, price level, real income, and other variables. It is an approximation to equation (5) in the Appendix. Equation (1.15) explains the price of domestic goods as a function of the import price index, the interest rate, aggregate real activity, and other variables. It is an approximation to equation (15) in the Appendix. The price of domestic goods is assumed to be set by the firm sector.[12] Equation (1.16) is a combination of the consumption and investment demands for domestic goods for country 1; it is an approximation to equations (2) and (17) in the Appendix. In this equation, the demand for the good of country 1 by country 1's private sector is a function of the two prices, real income, the interest rate, and other variables. Finally, equation (1.17) explains the interest rate of country 1; it is a reaction function of the monetary authorities of country 1. As discussed at the end of the Appendix, this is an optional equation. If it is specified, then B^1_{1g} is endogenous; if it is not, then B^1_{1g} is exogenous.

Equations (1.12)-(1.17) are the key behavioral equations of the model for country 1, and these are the equations where it is suggested that most of the estimation work is focused. Regarding equations (1.12) and (1.13), however, it is still necessary once X^1_{mp} and B^1_{mp} have been explained, to explain the division of these variables into X^1_{2p}, X^1_{3p}, B^1_{2p}, and B^1_{3p}. This can be done by the following "share" equation:

(1.18) $$\frac{X^1_{2p}}{X^1_{mp}} = f_{18}\left(\frac{e_2P_2}{e_3P_3}, Z_1\right),$$ [share of the imports of country 1's private sector from country 2]

(1.19) $$\frac{X^1_{3p}}{X^1_{mp}} = f_{19}\left(\frac{e_2P_2}{e_3P_3}, Z_1\right),$$ [share of the imports of country 1's private sector from country 3]

(1.20) $$\frac{B^1_{2p}}{B^1_{mp}} = f_{20}\left(\frac{R_2}{R_3}, Z_1\right),$$ [share of country 2's securities in the foreign security holdings of country 1's private sector]

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[12] See footnote 2 in the Appendix for a discussion of the inclusion of the interest rate in the price equation.

(1.21) $$\frac{B^1_{3p}}{B^1_{mp}} = f_{21}\left(\frac{R_2}{R_3}, Z_1\right) .$$ [share of country 3's securities in the foreign security holdings of country 1's private sector]

The lagged value of the share in each equation is an obvious variable to include in Z_1. These share equations should probably be estimated directly, although with a large number of countries, this is tedious, and one may instead want to assign parameter values to many of these equations without direct estimation.[13]

This completes the basic outline of the model. Equations (1.1)-(1.7) and (1.9)-(1.21) also hold for countries 2 and 3, with appropriate change of notation. Let a single prime denote the equations for country 2, and let a double prime denote the equations for country 3. This gives 61 equations, one of which is redundant. As in the Appendix, it will be convenient to **drop** equation (1.8). The remaining equations for which there are no obvious LHS variables are (1.3), (1.4), (1.7), and the corresponding equations for countries 2 and 3. To equations (1.3), (1.3)', and (1.3)" can be matched B^1_{1p}, B^2_{2p}, and B^3_{3p}. To the government budget-constraint equations, (1.4), (1.4)', and (1.4)", can be matched either R_1, R_2, and R_3 if no reaction functions of the monetary authorities are specified or B^1_{1g}, B^2_{2g}, and B^3_{3g} if such functions are specified. Finally, to equation (1.7) can be matched Q_1, and to equations (1.7)' and (1.7)" can be matched either e_2 and e_3 or Q_2 and Q_3, depending on whether there are flexible or **fixed exchange rates.**

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[13] There are clearly a number of ways in which one can model the allocations of goods and securities among countries. The present model is not restricted to one particular way. See Hickman [12] for a discussion of the allocation of goods among the various countries in the LINK model.

To summarize, if a model like the one outlined in this section were estimated, it would account for the main economic linkages among countries. In addition to the obvious capital-flow and exchange-rate linkages, there are linkages through the price equations, (1.10), (1.10)', (1.10)", (1.15), (1.15)', (1.15)", through the interest-rate equations (when reaction functions of the monetary authorities are specified), (1.11), (1.11)', (1.11)", (1.17), (1.17)', (1.17)", and through the total-sales equations, (1.9), (1.9)', (1.9)".

A few further points about this model should be noted. First, if for a given country a reaction function of the monetary authorities is not specified, then the interest rate for that country is implicitly determined. The solution value of the interest rate is, speaking loosely, the rate that makes equation (1.4), the government budget constraint, hold. If the interest rate is instead explained by a reaction function, then B^1_{1g} ,the (negative of) the amount of government securities outstanding, is taken to be endogenous. In this case the solution value of B^1_{1g} is, again speaking loosely, the value that makes equation (1.4) hold.

Second, in the regime of flexible exchange rates, the exchange rates are also implicitly determined. In the above discussion, e_2 and e_3 were matched to equations (1.7)' and (1.7)", the equations that equate the supply of securities of countries 2 and 3 to the demand for them. In the regime of fixed exchange rates, the international-reserve holdings of the countries are implicitly determined. In this case, Q_2 and Q_3 are the variables matched to equations (1.7)' and (1.7)". It is also possible, if desired, to add equations explaining e_2 and/or e_3 to the model and interpret these equations as reaction functions of some particular government authority or authorities. If this is done, then Q_2 and/or Q_3 must be taken to be endogenous. This procedure is analogous to the procedure of estimating equations explaining R_1, R_2, and R_3; interpreting these equations as reaction functions of the monetary authorities; and taking B^1_{1g}, B^2_{2g} and B^3_{3g} to be endogenous.

Third, even though the present model is relatively small, it is not an easy task to collect the necessary data for it. The data first of all must satisfy equations (1.1)-(1.9), which requires for each country linking its national-income and flow-of-funds accounts. For the U.S. this is fairly straightforward to do, as described in [7], but for countries that have poorer data than does the U.S., some data may have to be made up. Also, for most pairs of countries data on B^j_{ig} and B^j_{ip} do not exist, although it is generally possible to get data on a country's total foreign security holdings. The same holds true for M^j_{ig} and M^j_{ip}. Much of the data on the allocation of a country's total holdings of foreign securities and foreign money among the individual foreign countries will thus have to be made up.

Finally, it should be noted that one important feature of the single-country model in [7] that is lost in the model in this section is an explicit treatment of disequilibrium effects. Disequilibrium effects are present in the model in this section in that the price of the good of each country is assumed to be set by its firm sector (equations (1.15), (1.15)', and (1.15)"), rather than being such as to clear the goods markets each period. Also, the use of the aggregate activity variables, X_{1f}, X_{2f}, and X_{3f}, in equations (1.16), (1.16)', and (1.16)", respectively, can be assumed in part to be accounting for disequilibrium effects. Nevertheless, it should be clear from comparing the model in [7] to the model in this section that disequilibrium effects are only crudely accounted for here, and this is probably one of the first restrictions that should be relaxed if the model in this section is expanded. Within the present model one can include in the Z_1 vector variables that may pick up disequilibrium effects, but any variables so included must be taken to be exogenous.

It is, of course, a matter of judgment whether or not one wants to restrict the model in the Appendix in the ways proposed in this section, and if desired, it is fairly straightforward to lessen some or all of these restrictions. It is clearly an open question whether an estimated version of the model in this section would be a more accurate representation of the economic linkages among countries than, say, some future version of the LINK model. Given, however, the enormous task of accounting for all the flows of funds in the LINK model or in a similar model, it does seem worthwhile to try the small approach suggested in this paper. After the model proposed in this section has been estimated and analyzed, one can then be concerned, within the context of this basic model, with further disaggregation.[14]

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[14]At this stage, further disaggregation and expansion could include 1) disaggregation of goods by type, 2) disaggregation of securities by type and maturity, 3) accounting more explicitly for disequilibrium effects, 4) accounting for the effects of capital gains and losses on behavior, 5) and generally making more variables endogenous. Except for 1) and 2), Model A is expanded in this way.

APPENDIX

A Three-Country Model of the Balance of Payments

The model presented in this Appendix is a three-country version of the two-country model of the balance of payments in [9]. Different versions of the two-country model were considered in [9], and the one used for present purposes is the one in which there is a bank sector and in which the labor and goods markets are not always in equilibrium. The notation used here differs from the notation in [9] because of the need to keep track of three countries rather than two. The countries are numbered 1, 2, and 3. A subscript number for a variable denotes that the variable pertains to the particular country, and a superscript number for a variable denotes that the variable is held or purchased by the particular country. There are four sectors per country: household, firm, bank, and government. Subscripts h , f , b , and g will be used to denote these sectors, respectively. Each country specializes in the production of one good (X) . Labor (L) is homogeneous within a country, and there is no labor mobility among countries. Each country has its own money (M) , which takes the form of demand deposits in the bank sector, and its own bond (B) . The bonds are one-period securities. If a sector is a debtor with respect to a bond (i.e., a supplier of the bond), then the value of B for this sector is negative. The bank sector of each country holds bank reserves with its government (BR) , some of which are borrowed (BO) . The reserve requirement ratio is RR , and the discount rate is RD . Prices, wage rates, and interest rates are denoted P , W , and R , respectively. e_2 is the price of country 2's money in terms of country 1's money, and e_3 is the price of country 3's money in terms of country 1's money. The government of each country holds a positive amount of the international reserve (Q) , whose price is 1.0, and it taxes its citizens using a vector (T) of tax parameters. The three goods are assumed not to be perfect substitutes, and likewise for the three monies and the three bonds.[1]

Consider country 1. The household sector is assumed to determine jointly its labor supply and its demand for the three goods, the three monies, and the three bonds. It takes as given the wage rate, the three prices, the three interest rates, the tax parameters, the two exchange rates, and all lagged values. The vector of all relevant lagged values will be denoted at Z_{1h} . These decisions are assumed to be derived from a multiperiod maximization problem. Expectations of various future values, which are needed for such problems, are assumed to be a function of current and lagged values. The equations representing the decisions for the current period will be written as:

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[1] If the three bonds are perfect substitutes, then equations (9) and (10) below drop out (and likewise (9)', (9)", (10)', and (10 "), and the interest rates in the three countries are the same after adjusting for exchange rate expectations. See [9] for a discussion of the perfect substitution case in the two-country model. The "perfect mobility" regimes for Model A in Section III of this paper are regimes in which the bonds in the two countries are perfect substitutes and in which exchange rate expectations are static. It would be straightforward, as discussed in [9], to add equations explaining exchange rate expectations to the model, but for present purposes it is sufficient just to note that expectations in the model are assumed to be a function of current and lagged values.

(1) $L_{1h} = f_1(W_1, P_1, P_2, P_3, R_1, R_2, R_3, T_1, e_2, e_3, Z_{1h})$ [supply of labor]

(2) $X^1_{1h} = f_2($ " $)$ [demand for the good of country 1]

(3) $X^1_{2h} = f_3($ " $)$ [demand for the good of country 2]

(4) $X^1_{3h} = f_4($ " $)$ [demand for the good of country 3]

(5) $M^1_{1h} = f_5($ " $)$ [demand for the money of country 1]

(6) $M^1_{2h} = f_6($ " $)$ [demand for the money of country 2]

(7) $M^1_{3h} = f_7($ " $)$ [demand for the money of country 3]

(8) $B^1_{1h} = f_8($ " $)$ [supply of (-) or demand for the bond of country 1]

(9) $B^1_{2h} = f_9($ " $)$ [demand for the bond of country 2]

(10) $B^1_{3h} = f_{10}($ " $)$. [demand for the bond of country 3]

These ten equations are not independent, since they must satisfy a budget constraint. The taxable income of the household sector (Y_{1h}) is assumed to be

$$Y_{1h} = W_1 L_{1h} + R_1 B^1_{1h} + e_2 R_2 B^1_{2h} + e_3 R_3 B^1_{3h} , \tag{11}$$ [taxable income]

where the first term on the RHS is wage income, the second term is interest income or interest payments on the domestic bond, and the third and fourth terms are interest income on the foreign bonds. Net taxes paid by the household sector (V_{1h}) is assumed to be a function of Y_{1h} and T_1 :

$$V_{1h} = f_{12}(Y_{1h}, T_1) . \tag{12}$$ [net taxes paid]

The financial saving of the household sector (S_{1h}) is then

$$S_{1h} = Y_{1h} - V_{1h} - P_1 X^1_{1h} - e_2 P_2 X^1_{2h} - e_3 P_3 X^1_{3h} , \tag{13}$$ [saving of the household sector]

where the last three terms are expenditures on goods. Finally, the budget constraint is

$$0 = S_{1h} - \Delta M^1_{1h} - e_2 \Delta M^1_{2h} - e_3 \Delta M^1_{3h} - \Delta B^1_{1h} - e_2 \Delta B^1_{2h} - e_3 \Delta B^1_{3h} , \tag{14}$$ [household sector budget constraint]

which says that any nonzero level of saving of the household sector must result in the change in at least one of its assets or liabilities.

For simplicity, the firm sector is assumed to hold no foreign bonds and no money. It is assumed to determine jointly its price (P_1), its production (X^*_{1f}), its demand for the three goods for investment purposes $(X^1_{1f}, X^1_{2f}, X^1_{3f})$, its wage rate (W_1), the maximum amount of labor that it will employ in the period (L_{1f}), and its supply of (-) or demand for the bond of country 1 (B^1_{1f}). It takes as given R_1, T_1, P_2, P_3, e_2, e_3, and all lagged values (Z_{1f}). These decisions are also assumed to be derived from a multiperiod maximization problem, with the equations representing the decisions for the current period written as:[2]

(15) $P_1 = f_{15}(R_1, T_1, P_2, P_3, e_2, e_3, Z_{1f})$ [price of the good of country 1]

(16) $X^*_{1f} = f_{16}($ " $)$ [production of the good of country 1]

(17) $X^1_{1f} = f_{17}($ " $)$ [demand for the good of country 1]

(18) $X^1_{2f} = f_{18}($ " $)$ [demand for the good of country 2]

(19) $X^1_{3f} = f_{19}($ " $)$ [demand for the good of country 3]

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[2] See Chapter 3 in [6] for a detailed discussion and analysis of this type of model of firm behavior. For present purposes, the production-function constraint on the firm sector should be assumed to be incorporated into the decision equations (15)-(22). For the model in [6], a production function was postulated explicitly, and the possibility that it may at times be optimal for a firm to hold excess labor and excess capital was considered.

It should also be noted that in the theoretical model in [6] the interest rate has a positive effect on the price that a firm sets, and so R_1 is included as an explanatory variable in equation (15). In the empirical work in [7] the bond rate did have a significant and positive effect on the price variable of the firm sector (equation 9 in Table 2-3 in [7]).

(20) $W_1 = f_{20}($ ") [wage rate of country 1]

(21) $L_{1f} = f_{21}($ ") [maximum amount of labor that the firm sector will employ in the period]

(22) $B^1_{1f} = f_{22}($ ") [supply of (-) or demand for the bond of country 1]

Disequilibrium in the labor market is handled as follows. First, note that $L_{1f} + L_{1g}$ is the maximum amount that the household sector can work in the period, where L_{1g} is the amount of labor employed by the government. (The bank sector is assumed to employ no labor.) It is assumed that the firm and government sectors make their decisions regarding L_{1f} and L_{1g} before the household sector makes its decisions and that the household sector takes this possible labor constraint into account in making its decisions. Equations (1)-(10) are thus assumed to represent the household sector's decisions that incorporate this possible labor constraint, so that L_{1h} in (1) is always less than or equal to $L_{1f} + L_{1g}$.

Consider now the firm sector's adjustment to disequilibrium in the labor market. If L_{1h} is strictly less than $L_{1f} + L_{1g}$, then the firm sector is assumed to get only the amount $L_{1h} - L_{1g}$ of labor in the period. Call this amount L'_{1f} :

(23) $L'_{1f} = L_{1h} - L_{1g}$. [actual amount of labor employed by the firm sector in the period. $L'_{1f} \leq L_{1f}$.]

In the case in which $L'_{1f} < L_{1f}$, the firm sector is assumed to change its production decision during the period, and so equation (16) should be interpreted as reflecting this fact.

With respect to the goods market, the total amount of sales of the firm sector (X_{1f}) is

(24) $$X_{1f} = X^1_{1h} + X^1_{1f} + X^1_{1g} + X^2_{1h} + X^2_{1f} + X^2_{1g} + X^3_{1h} + X^3_{1f} + X^3_{1g} \ .$$ [total sales of the good of country 1]

The firm sector is assumed to hold inventories of the good (I_{1f}), so that any difference between production and sales in the period results in a change in inventories:

(25) $$\Delta I_{1f} = X^*_{1f} - X_{1f} \ .$$ [change in inventories of the good of country 1]

The lagged value of inventories $(I_{1f_{-1}})$ is one of the variables in Z_{1f} that affects the firm sector's current decisions.

The equations for the firm sector also must satisfy a budget constraint. The value of taxes paid by the firm sector (V_{1f}) is assumed to be a function of T_1 and of variables that determine profits:

(26) $$V_{1f} = f_{25}(T_1, P_1, X^*_{1f}, X^1_{1f}, X^1_{2f}, X^1_{3f}, W_1, B^1_{1f}, L'_{1f}, R_1, P_2, P_3, e_2, e_3, Z_{1h}) \ .$$ [taxes paid]

The financial saving of the firm sector (S_{1f}) is

(27) $$S_{1f} = P_1 X_{1f} - P_1 X^1_{1f} - e_2 P_2 X^1_{2f} - e_3 P_3 X^1_{3f} - W_1 L'_{1f} + R_1 B^1_{1f} - V_{1f} \ ,$$ [saving of the firm sector]

and its budget constraint is

(28) $$0 = S_{1f} - \Delta B^1_{1f} \ .$$ [firm sector budget constraint]

The main characteristic of the bank sector is that it takes in deposits

(M_{1b}) and makes loans (B^1_{1b}) . The bank sector is assumed for simplicity to employ no labor, buy no goods, pay no taxes, and hold no foreign bonds and monies. Its borrowing from the government is assumed to be a function of R_1 and the discount rate (RD_1) :

(29) $$BO_1 = f_{29}(R_1, RD_1) \; . \quad \text{[bank borrowing from the government]}$$

The bank sector is assumed to hold no excess reserves, so that bank reserves are determined as

(30) $$BR_1 = RR_1 \, M_{1b} \; , \quad \text{[bank reserves]}$$

where RR_1 is the reserve requirement ratio. The financial saving of the bank sector (S_{1b}) is

(31) $$S_{1b} = R_1 \, B^1_{1b} - RD_1 \, BO_1 \; , \quad \text{[saving of the bank sector]}$$

and its budget constraint is

(32) $$0 = S_{1b} - \Delta B^1_{1b} + \Delta M_{1b} - \Delta(BR_1 - BO_1) \; . \quad \text{[bank sector budget constraint]}$$

Equation (31) states that the saving of the bank sector equals the difference between the interest revenue on its loans and the interest payments to the government on its borrowing. Equation (32) states that the change in bank loans plus unborrowed reserves $(\Delta B^1_{1b} + \Delta(BR_1 - BO_1))$ equals saving plus the change in deposits $(S_{1b} + \Delta M_{1b})$.

The government is assumed to purchase labor from its own citizens (L_{1g}) and all three goods $(X^1_{1g}, X^1_{2g}, X^1_{3g})$. It also holds the three monies $(M^1_{1g}, M^1_{2g}, M^1_{3g})$ and the three bonds $(B^1_{1g}, B^1_{2g}, B^1_{3g})$, in addition to the international reserve (Q_1). Its financial saving (S_{1g}) is

(33) $$S_{1g} = V_{1h} + V_{1f} - W_1L_{1g} - P_1X^1_{1g} - e_2P_2X^1_{2g} - e_3P_3X^1_{3g} + R_1B^1_{1g} + e_2R_2B^1_{2g} + e_3R_3B^1_{3g} ,$$ [saving of the government sector]

and its budget constraint is

(34) $$0 = S_{1g} + \Delta(BR_1 - BO_1) - \Delta M^1_{1g} - e_2\Delta M^1_{2g} - e_3\Delta M^1_{3g} - \Delta B^1_{1g} - e_2\Delta B^1_{2g} - e_3\Delta B^1_{3g} - \Delta Q_1 .$$ [government sector budget constraint]

The first two terms on the RHS of (33) are tax revenue, the next four terms are purchases of labor and goods, and the last three terms are interest income or payments. Equation (34) states that any nonzero value of government saving must result in the change in at least one of the government's assets or liabilities.

Two further equations complete the model for country 1. The total amount of deposits in the bank sector (M_{1b}) is:

(35) $$M_{1b} = M^1_{1h} + M^1_{1g} + M^2_{1h} + M^2_{1g} + M^3_{1h} + M^3_{1g} ,$$ [total deposits in the bank sector]

and the supply of the bond of country 1 equals the demand for it:

(36) $$0 = B^1_{1h} + B^1_{1f} + B^1_{1b} + B^1_{1g} + B^2_{1h} + B^2_{1g} + B^3_{1h} + B^3_{1g} .$$ [supply of the bond of country 1 equals the demand for it]

Equations (1)-(36) also hold for countries 2 and 3, with appropriate changes of numerical subscripts and superscripts and with appropriate modifications of e_2 and e_3. Call these equations (1)'-(36)' and (1)"-(36)". The overall model is then closed by the following equation:

(109) $0 = \Delta Q_1 + \Delta Q_2 + \Delta Q_3$. [no change in total world reserves]

Of the 109 equations, 7 are redundant. The redundant equations are: one from the household equations (1)-(14), one from the firm equations (15)-(28), the same for countries 2 and 3, and one because the savings of all sectors sum to zero: $S_{1h} + S_{1f} + S_{1b} + S_{1g} + e_2(S_{2h} + S_{2f} + S_{2b} + S_{2g}) + e_3(S_{3h} + S_{3f} + S_{3b} + S_{3g}) = 0$. It will be convenient to drop (8), (22), the same for countries 2 and 3, and (109). This leaves 102 equations. If all the government variables (i.e., all the variables with subscript g) except S_{1g} , S_{2g} , and S_{3g} are taken to be exogenous and if all lagged values are taken to be predetermined, then there are 104 variables left. Therefore, two further variables must be taken to be exogenous in order for the model to be determined. These variables are e_2 and e_3 in the fixed-exchange-rate regime and Q_2 and Q_3 in the flexible-exchange-rate regime.

It may be helpful to consider the matching of variables and equations to see that all variables are accounted for. The equations for which there are no obvious LHS variables are (14), (28), (32), (34), (36), and the corresponding equations for countries 2 and 3. To the three budget-constraint equations, (14), (28), and (32), can be matched B^1_{1h} , B^1_{1f} , and B^1_{1b} , and similarly for countries 2 and 3. To the three government-budget-constraint equations, (34), (34)', and (34)", can be matched R_1 , R_2 , and R_3 . To (36) can be matched Q_1 , which then leaves (36)' and (36)" to be matched to e_2 and e_3 or Q_2 and Q_3 .

In the model as just outlined the interest rates are matched to the government budget constraints and therefore implicitly determined. Another possibility is to 1) assume that the monetary authority of each country behaves by controlling the domestic interest rate, 2) estimate a "reaction function" for each monetary authority with the domestic interest rate as the LHS variable, and 3) close the model by taking each government's holdings of domestic securities (B^1_{1g} , B^2_{2g} , and B^3_{3g}) to be endogenous. This was done in [8] for the single-country model in [7], and the properties of this version of the model were compared to the properties of the version without the reaction function.

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