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AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

Martin Shubik

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1. INTRODUCTION

A key problem in economic theory is how to integrate macro and microeconomics. It has been recognized for some time that an understanding of the roles of money, information and other financial instruments and institutions is critical to the needed integration.

Macroeconomic theory clearly involves dynamics, uncertainty, money and financial institutions. General equilibrium theory, which in many ways is one of the major achievements in microeconomic theorizing is essentially static, scarcely deals with information conditions and has no explicit essential role for money or financial institutions. A critique of general equilibrium theory has been given elsewhere [1], [2].

In the past ten years there have been several approaches adopted to reconcile micro and macroeconomics and to introduce finance into closed microeconomic models. Clower [3] and Lejonhufvud [4] have offered a reassessment of Keynes which leads to a consideration of models of adjust-

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ment with quantity rationing. Radner [5] has raised serious questions concerning the existence of a competitive equilibrium in markets with different levels of information [5]. Arrow and Hahn [6], Grandmont [7], Grandmont and Laroque [8] and several others have been concerned with what is now known as Temporary General Equilibrium Theory, where economies without perfect futures markets are studied with an emphasis placed upon the possible existence of a sequence of short term equilibria.

Grandmont has presented a comprehensive survey of the work in Temporary General Equilibrium Theory in an excellent article in Econometrica [9].

The approach adopted here is different from, but complementary with those noted above. And, as the prime purpose here is to sketch the approach and the results obtained no detailed critique of other approaches is attempted.

2. BASIC MODELLING CONSIDERATIONS

2.1. On Questions

There are questions and there are answers. In the development of any science it is important to examine both questions and answers for their relevance and accuracy. Without doing so, different approaches to essentially different questions can be cast in opposition to each other where, in fact, no opposition exists. For example, it should be clear to even a casual observer that general equilibrium theory and macroeconomics cannot be reconciled easily. For most questions that they are meant to answer there is no need to reconcile them.

There are many questions that the microeconomic theorist concerned with the role of money and financial institutions may find to be of interest.

For example, why did individuals ever start to use a specific item as a means of exchange? What is the relationship between search in markets and the use of prices and money? How are money and the technology of exchange related? In multiperiod trade where individuals have different levels of information how does the knowledge of prices influence the spread of information?

Given the questions, the model builder is faced with selecting the simplifying assumptions he intends to make in order to catch the essential features of reality yet product a tractable model.

The first specific question I asked that led me to the approach sketched below, had on its surface, apparently nothing to do with money. It was how can one embed oligopolistic competition into a static general equilibrium model?

The next more general set of questions lead into an explanation of what I mean by "Mathematical Institutional Economics." Can we find natural minimal conditions under which we would expect money, other financial instruments and financial institutions to appear in a society? What is the essence of the commercial banking function? What is the essence of insurance? Can fiat money or other credit instruments exist without bankruptcy and other laws covering default?

My thesis is that the natural way to embed oligopolistic competition into a general equilibrium model is by introducing a distinguished commodity to be used as a means of payment. Having done that, if we require that the outcome from trade be Pareto optimal it is necessary to have both a large number of competitors and that all traders hold enough of the means of payment. This requires defining what is meant by enough of the means of payment. Given that this is done we may observe that if there is not enough, then Pareto optimality* is lost. In order to restore it, credit must be introduced. But if credit is permitted there may be the possibility that an individual will be unable to pay back that which he owes. In which case bankruptcy laws must be specified.

Starting with the general equilibrium model of Walras, Arrow, Debreu it is possible to break out of the static nonprocess straightjacket of competitive equilibrium theory by viewing the economy as a game in strategic form and solving for noncooperative equilibria. The competitive equilibria are a special case of the noncooperative equilibria. Starting with a well defined game in strategic form we can specify the conditions under which noncooperative equilibria are identical with, approach, or differ from competitive equilibria. In exploring the relationship between competitive and noncooperative equilibria we are led to discovering the conditions for the existence of new financial instruments and institutions.

General equilibrium theory represents a masterful attempt to describe certain powerful limiting static properties of an economic system without having to specify either economic process or institutions. As soon as any attempt is made to fully define even the most abstract model of trade as a game in strategic form, the logical necessity of describing the state of the system for all positions of equilibrium or disequilibrium calls for a process description. The rules required to guide the process can be interpreted as providing a description of rudimentary institutions.

Mathematical institutional economics deals with the invention of

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*More precisely we mean Pareto optimality in the general equilibrium model. In the game model the lack of enough means of payment restricts the set of feasible trades, hence the Pareto optimal set may be changed.

institutions as formal mathematical rules of a game which are necessary and/or sufficient to guide economic processes to meet certain criteria.

2.2. The Approach Adopted

In the approach adopted here many alternative full process models are formulated and examined. In doing so certain precepts of modelling are adhered to. In particular:

(1) Every model must be a playable game.

(2) It is treated mathematically as a game in strategic form.

(3) It is solved for its noncooperative equilibria.

(4) Models are considered in general for few and many traders; i.e. oligopolistic features of markets are considered explicitly.

(5) The noncooperative equilibria are compared with and related to competitive equilibria.

(6) Symmetric and nonsymmetric information conditions are considered.

(7) Where relevant, exogenous uncertainty is introduced.

The "playable game" Criterion

For a game to be playable (at least without eventual fighting or negotiating) all rules must be known. The game theorist concerns himself with making sure that in his models this is the case. Yet a well defined game is not necessarily one that can be played by a human being of even superior abilities in a reasonable amount of time. However financial and other economic institutions are designed so that they can be operated by relatively ordinary humans.

If one defines a game (or selects a solution concept for a game) which is mathematically well defined yet is operationally unreasonable (i.e. it cannot be played reasonably as an experimental game) the odds are that it is not a good model of any existing institutions and, unless the analysis provides techniques which can convert it into a playable game it will not provide a model for new institutions. For example if there are m stocks being traded at a stock market it is possible to consider a trading strategy of an individual as being a function of $2m$ variables (quantity and price of each stock traded), yet even casual observation tells us that individuals in general do not hand to their brokers enormous books of instructions with contingent statements spelling out in detail millions of complex "if-then" chains.

The Game in Strategic Form

A game in strategic form is one in which the players each hand in a strategy to the referee. This strategy covers all contingencies. If there are few contingencies to be covered it is relatively easy to play such a game. Otherwise although it is mathematically possible to formulate the game and to prove certain theorems it is usually not practicable to play it.

Setting aside the playability consideration by using the strategic formulation of a game we are able to establish all of the properties that have been established for competitive market models, and other results pertaining to oligopolistic markets and to markets with nonsymmetric information can also be obtained.

Our results are both positive and negative. In particular although we can prove theorems about equilibria in various models and relate them to results in general equilibrium or temporary general equilibrium theory they are unsatisfactory when viewed from a playability criterion. This seems to suggest that economists simply do not have a good behavioral model of multistage decisionmaking. One may invent rules of thumb or *ad hoc* methods for updating subjective probabilities yet there is little hard evidence that this is the way people behave, and when one remembers that the economy poses a multi-person optimization problem there appears to be little of normative content in the updating of subjective probabilities.

When markets have many small participants, the idea that they behave using some sort of myopic short term maximization rule-of-thumb procedure is appealing; if for no other reason than it is simpler than most other alternatives. However as is clearly indicated by the empirically oriented literature in macroeconomics what are in fact the rules-of-thumb which are employed is a difficult empirical question.

Noncooperative Equilibria

The concept of a noncooperative equilibrium first appeared in economic application in the great work of Cournot [10]. Its most general form was given by Nash [11]. Cournot formulated his study of duopoly and oligopoly essentially as a game in strategic form. In his discussion of what happens to price as numbers increase he essentially sketched out the idea of replication and showed in the context of an open (or partial equilibrium) analysis how the noncooperative equilibrium approach the competitive equilibrium. The strategic game model provides a natural way for formulating the study of competition with few or many competitors. The competitive equilibrium results can be obtained as a limiting or special case of the noncooperative equilibrium when each individual alone has little power to influence the market.

Although an open or onesided model of economic competition is easy to formulate as a game in strategic form, a closed model presents a difficulty in the sense that an explicit description of what happens in the market at prices other than equilibrium prices calls for the specification of a rationing device. Several ways are suggested in Section 3 below.

Not only is the noncooperative equilibrium solution suited to the study of markets with few competitors it is also well defined for markets where competitors have different levels of information. In such an instance the competitive equilibrium may not even exist.

3. MODELS AND RESULTS

In following the approach sketched above it is convenient to divide difficulties by building a great number of models of trade and production in order to be able to isolate related but different aspects of an economic system guided or controlled by financial institutions. We consider:

Market mechanisms,

Money and credit,

Multiperiod models,

and Information and exogenous and endogenous uncertainty.

3.1. Market Mechanisms

For simplicity remarks are first confined here to markets with trade only. The problem of modelling trade in a closed economy as a noncooperative game is tantamount to producing a model of monopolistic or oligopolistic competition in a closed economy. There are several different models which can be constructed which use all commodities as a means of payment or only one commodity as a means of payment.

Consider an economy with n individuals trading in $m+1$ commodities. Each individual has a preference ordering over all bundles of commodities he may possess. His preferences can be represented by a utility function $\varphi^i(x_1^i, \ldots, x_{m+1}^i)$ where x_j^i is the amount of commodity j held by trader i. Each individual i starts with an endowment of $(a_1^i, \ldots, a_{m+1}^i)$ where $a_j = \sum_{i=1}^{n} a_j^i > 0$ for $j = 1, \ldots, m+1$.

<u>Barter, Bargaining and General Equilibrium</u>

Assuming all of the conditions supplied by Debreu [12], it can be shown that there exists at least one set of competitive equilibrium prices $p_1, \ldots, p_{m+1}$. The existence of these prices tell us nothing about their formation. We could imagine that trade takes place via recontracting, by a sequence of barter trades or haggling or by some central agency announcing prices to all, with each individual being required to submit a list of requests to buy and sell at the announced prices.

If we look at a market as a playable game, even for 100 traders exchanging say 30 commodities straight barter would present enormous confusion and require much time for trading. As Jevons noted instead of using barter for the $m+1$ commodities exchange could be much simplified by setting up $m(m+1)/2$ [13] markets where there is one post for the exchange of each pair of commodities. An alternative game can be constructed by declaring one commodity as a means of payment in the sense that we set up only m markets where the special commodity is exchanged against all of the other m commodities.

Mathematically any of these models can be well defined. It is suggested here that for an analysis of most modern mass economies the most realistic assumption and best approximation for many purposes is that there is one distinguished commodity which is exchanged for all others.

If the distinguished commodity has a utilitarian worth in consumption apart from its use as a means of exchange then we may regard it as a commodity money. If it has no use beyond serving as a medium of exchange it has the property of a money but not the property of commodity of intrinsic worth. We return to problems of fiat money and credit in 3.2.

Model 1. Sell-All

A simple and practically playable game is as follows. Each of the n traders is required to offer all of his goods for sale. He has as his strategy the selection of his bid in each of the m markets. This for trader i is a vector $(b_1^i, \ldots, b_m^i)$ where $\sum_{j=1}^{m} b_j^i \leq a_{m+1}^i$ where the $m+1^{st}$ commodity is the means of payment. Each trader gives in his strategy to the referee who then calculates the price of each commodity in terms of the $m+1^{st}$ commodity:

$$p_j = b_j/a_j \ , \quad j = 1, \ldots, m \ .$$

Each trader i receives

$$x_j^i = b_j^i a_j/b_j \ , \quad \text{for} \quad j = 1, \ldots, m \ .$$

Each trader i, after trade receives as income payment for the goods he has sold, or $\sum_{j=1}^{m} p_j a_j^i$.

We use the symbol $\overline{b}^i$ to stand for the vector $(b_1^i, \ldots, b_m^i)$ then the payoff to trader i in the game is:

$$\Pi^i(\bar{b}^1, \bar{b}^2, \ldots, \bar{b}^n) = \varphi^i(x_1^i, \ldots, x_m^i; x_{m+1}^i)$$

where $x_{m+1}^i = a_{m+1}^i - b^i + \sum_{j=1}^{m} p_j a_j^i$.

This game has been formulated and studied in several variants in a series of papers [14], [15], [16], [17].

When a game with $m(m+1)/2$ markets is defined, as the number of traders becomes large the competitive equilibria can be obtained as limiting noncooperative equilibria. All goods are used as a means of exchange and there is never a shortage of "means of exchange."

When exchange is modelled using a commodity money with only m markets for trade the amount of the means of exchange and its distribution influences the outcome of trade. If there is a money shortage, even with many traders the limit noncooperative equilibrium is <u>not</u> the general equilibrium but one that is constrained by a <u>cash constraint</u>.

If the commodity that is used as the means of exchange is not in adequate supply then if the economy with only m markets is to be able to achieve competitive exchange the supply of the means of exchange must be supplemented by some form of fiat money or credit.

<u>Model 2. Bid-Offer</u>

Although Model 1 calls for simple strategies, an immediate weakness in it can be seen if we contemplate multiperiod trade. Why should individuals be forced to offer everything for sale? The "sell-all" model is an accountant's and tax collector's dream because all trade is monetized. It is usually not practical in a world with assets and inventories which are held for some time. If transactions were costless and timeless we might wish to force an open revaluation of all assets every year.

A slightly more complicated model than model 1 accounts for inventories. A strategy is a vector of $2m$ dimensions of the form $(b_1^i, q_1^i; b_2^i, q_2^i, \ldots, b_m^i, q_m^i)$ where $b_j^i \geq 0$, $q_j^i \geq 0$, $\sum_{j=1}^{m} b_j^i \leq a_{m+1}^i$, $q_j^i \leq a_j^i$, for $j = 1, \ldots, m$.

The price for commodity j in terms of the $m+1^{st}$ commodity is given by

$$p_j = b_j/q_j \quad (0 \text{ if } q^j = 0).$$

Each trader i obtains

$$x_j^i = b_j^i q_j/b_j \quad \text{for } j = 1, \ldots, m .$$

He receives as income $\sum_{j=1}^{m} p_j q_j^i$. The payoff to trader i is given by:

$$\Pi^i(\bar{b}^1, \bar{q}^1, \ldots, \bar{b}^n, \bar{q}^n) = \varphi^i(\hat{x}_1^i, \ldots, \hat{x}_m^i, \hat{x}_{m+1}^i)$$

where $\hat{x}_j^i = a_j^i - q_j^i + x_j^i$ for $j = 1, \ldots, m$ and $\hat{x}_{m+1}^i = a_{m+1}^i - b^i + \sum_{j=1}^{m} p_j q_j^i$.

We have shown that for many traders the noncooperative equilibria of this game approach the competitive equilibria [16]. When traders are few however a new phenomenon appears. Noncooperative equilibria may involve <u>wash sales</u>, i.e. if traders overtrade then in aggregate the markets are thicker and an equilibrium may appear with the property that each sells to and then buys back from the market at the same price.

An important feature contrasting Models 1 and 2 is the difference in the quantity of money needed for finance. In Model 1 the total amount of money needed is $\sum_{j=1}^{m} \sum_{i=1}^{n} p_j a_j^i$, in Model 2 it is $\sum_{j=1}^{m} \sum_{i=1}^{n} p_j q_j^i$. The latter may be considerably less than the former. In fact if trade starts at equilibrium in Model 2 there will be no trade at all. Another important interpretation for Model 2 is that $\sum_{j=1}^{m} \sum_{i=1}^{n} p_j q_j^i = F$ is the amount of the float required to finance trade. In this exchange model it is also a measure of GNP, i.e. GNP = F .(as there is no depreciation here GNP = NNP). The amount $\sum_{j=1}^{m} \sum_{i=1}^{n} p_j a_j^i = W$ measures total wealth of the economy.

By the definition of strategy in these models with one period of simultaneous trade the velocity of money must be less than or equal to one. There is only one period of trade and each individual spends all he has or less.

As simple as Model 2 may be, when compared to actual trade it has the virtue that although it is a microeconomic model some of the macroeconomic monetary measures of an economy can be precisely identified.

Model 3. Price Quantity, Bid Offer

Students or others when confronted with having to play either of the games noted above frequently complain that it is not reasonable or realistic to bid in a market for a set of goods when they do not know prices. But the chief virtue of an enterprise system is that it is meant to provide a competitive means for prices to be formed. This is what these models provide. The true source of the complaint is probably that most markets are neither new nor do they last for only a single period. The price of an item yesterday is frequently used in planning what you will bid today. Furthermore in many markets individuals make or name prices rather than take the only price named at the start.

In order to introduce the naming of price as part of strategic behavior, Model 3 has a strategy for each individual i which consists of a vector of $4m$ numbers of the form $(p_1^i, x_1^i, r_1^i, q_1^i, \ldots, p_m^i, x_m^i, r_m^i, q_m^i)$ where $0 \leq p_j^i$ is the price suggested by i for selling part of his supply of j.

$0 \leq x_j^i \leq a_j^i$ is the quantity of j offered for sale by i.

$0 \leq r_j^i$ is the price suggested by i at which he will buy j.

$0 \leq q_j^i$ is the amount of j, i is willing to buy at r_j^i.

All bids and offers are given to a referee who can draw a supply and demand histogram for each market. An example is illustrated in Figure 1. DED' shows the demand conditions, SES' the supply. We now define

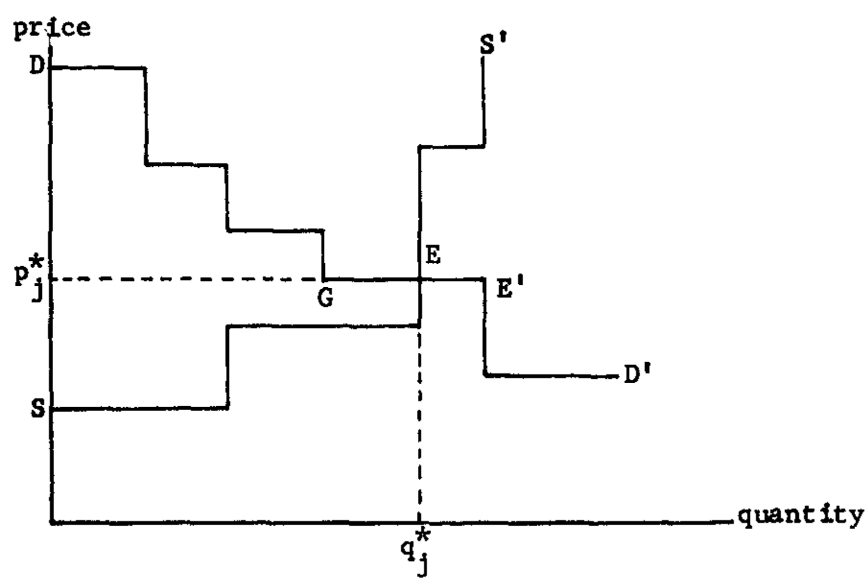

FIGURE 1

an extremely simple market mechanism which determines trade. In each market j trade takes place at a price p_j^* determined by the intersection of supply offered and demand bid.

Any effort to convert this model into a playable game brings up two new problems. They concern rationing or the serving of traders when bids do not quite match offers; and they also concern credit conditions and limitations on acceptable bids.

In Figure 1 at price p^* the marginal demand is GG' , but the marginal supply is only GE . How do we ration the GE ? Several conventions have been suggested such as pro rate the supply among the marginal buyers, or randomize the order of service [17].

The second difficulty in using this market mechanism concerns limitations on bidding. Suppose individual i has a^i_{m+1} units of a commodity money. Should his bids be restricted to $\sum_{j=1}^{m} p^i_j x^i_j \leq a^j_{m+1}$? If we do so then as it is quite likely that some bids to buy must fail, an individual bidder will be limited to a highly conservative bidding policy. If we do not limit his bids to sum to less than his "cash on hand" even though it may not be likely, it will be possible that he may buy more than he can immediately pay for. If this were to happen rules would be needed to specify the treatment of offers to pay which cannot be met. Another alternative would be to have an institution sell insurance or guarantee payment for a fee. Both of these methods have been utilized.

An analysis of Model 3 shows that at equilibrium excess supply or demand cannot be larger than the excess supply or demand of a single individual [17]. Furthermore the competitive equilibria are noncooperative equilibrium points of this model after a single replication, i.e. even with few atomic traders as long as no one is a monopolist or monopsonist.

On Markets in General

In essence bids and offers are characterized by combinations of statements involving quantities of goods, money and magnitude of prices. A market is a mechanism which takes as inputs bids and offers and the initial distribution of resources and acts on these to produce final prices and a final distribution of resources.

Questions of economic realism and relevance may have much to do with the specifics of information patterns and details of market mechanisms. If, as a first approximation we wish to consider markets with all individuals moving simultaneously with a mechanism that forms only a single price for each commodity where strategies consist of bids or offers involving quantities of goods or money and/or prices; then there are only a handful of market mechanisms that we need to consider. Three of them have been described above and some variants are presented elsewhere [18].

Although the three market models noted are all playable as games the true lack of symmetry among those who enter the markets as consumers, traders, brokers or manufacturers is not brought out.

Even casual empiricism should be sufficient to convince us that much of the worth of macroeconomic models rests upon whether the mechanisms which guide the markets are sufficiently well represented in aggregate that they enable us to predict overall behavior adequately.

The key point in this section is that the necessary first step in connecting micro and macroeconomics is to construct explicit process models of trade and production. It is possible to build many game models which have static solutions which connect noncooperative equilibria with competitive equilibria in related general equilibrium models. Yet because they are full process models they provide models which are naturally consistent with macroeconomic theorizing. Logical consistency, relevance and empirical worth are not the same thing, but at least these types of models provide a basis for empirical work.

The second point of importance in this section is that the explicit consideration of markets without universal trust and instant clearinghouses calls for the creation of monetary and credit mechanisms which facilitate the functioning of the markets. Financial instruments and institutions together with the laws governing contracts and the behavior of economic entities are frequently logical necessities when we attempt to formulate market mechanisms as playable games.

An Aside on Production and Corporations

Before we turn to fiat money and credits we note some of the difficulties and new problems which appear in the introduction of production and corporations with shares and managers to carry out the production.

Given individuals who own resources and corporations; and corporations which buy the resources they need as inputs, produce and sell outputs it is difficult to avoid constructing less than a three stage model of trade and production involving trade (including purchase and sale of inputs), production and then trade. When we construct such a model a strategy becomes a relatively complex plan if individuals are informed about any aspects of the earlier moves.

We can show [19] that a multistage noncooperative game can be constructed which has noncooperative equilibria which coincide with competitive equilibria under certain reasonable circumstances, but three new phenomena appear in the model. (1) Even with nonatomic traders noncooperative equilibria other than competitive equilibria are present. They can best be interpreted in terms of threats and historical strategies such as those which could be provided by the communication network and enforcement mechanism and common will of a cartel, trade association or social custom. (2) Even though we may consider the multistage model as encompassing only one time period; i.e. any consumer good has the same worth to a consumer whether purchased in stage 1 or stage 3; production conditions could require a different price for the same item serving as an input in stage 1 and an output in stage 3. (3) The need for firms to buy raw materials, the possibility that ownership in the firms is represented by shares and the fact that there are two market stages considerably complicates the need for cash. In particular the firms need working capital and the size of the float will depend in detail upon the pattern of payments and production.

Given the presence of firms it is usually postulated that they have profit maximizing managers. Furthermore at least, in general equilibrium theory the voting and control aspects of corporate stock play no role. Problems concerning the worth of voting and control are intimately related to the everyday facts of corporate life. The literautre from Berle and Means [20] onwards gives adequate indication that for most purposes the model of the corporate manager as a faceless, selfless mechanism is inadequate. In separate studies Lesourne [21] and we have considered some of the implications of utility maximizing managers [22].

3.2. Money and Credit

When a commodity is used as a means of payment unless it is in sufficient supply it may limit trade. A natural way to avoid this limitation is to introduce some symbolic form of money or credit. In order to do this societal rules of the game must be introduced which specify who are to be permitted to create the new instruments and what constraints are to be imposed upon their use. There are at least two relatively natural ways to do this.

We may formalize the rules of the game to characterize two types of mechanisms which we might describe as an outside bank and an inside bank. The first mechanism issues a fixed amount of special paper which without too much poetic license we might consider as fiat money. The second mechanism issues credit.

In both instances a natural question to ask is what supports the value of the credit instrument? The simple answer is the rules, laws and customs of the society. In particular these cover two critical aspects of any economy. They are the covenants and contracts governing trade involving trust or any credit means of payment and the rules concerning the treatment of those who fail to honor their contracts.

In a game of strategy where payments are to be made using a credit instrument an individual may expect to obtain an income sufficient to cover the claims against him, but he may not be certain until the markets have cleared. Furthermore it is frequently possible that through his mistakes,,or the mistakes of others he may fail to earn enough to cover his debts.

If there are no penalties against ending up in debt and if credit has some worth then it will pay the individual to obtain as much credit as possible knowing that default is costless. From this basic consideration, if we wish to design a system which uses fiat money and/or credit we must introduce a penalty against default. In the papers noted below we have called the penalty the bankruptcy penalty, but it must be noted that words such as bankruptcy and insolvency tend to have extremely special legal meanings which vary across national boundaries; the reader is warned that not too much detailed institutional meaning should be attached to the bankruptcy penalty as used here.

We could graft our credit mechanisms onto any of the models presented in Section 3.1, however as Model 1 is the simplest we use it as our basis for two new models here.

Model 4. Sell-All with Credit

In Model 1 a strategy was a vector $(b^i_1, \ldots, b^i_m)$ where the bids were required to meet the additional constraint that $\sum_{j=1}^{m} b^i_j \leq a^i_{m+1}$. In a market with credit we may remove this constraint and imagine instead that there is a "bank" or some device which costlessly grants any amount of credit. Initially each individual holds a vector of resources $(a^i_1, \ldots, a^i_m, 0)$ where the $m+1^{st}$ commodity is a credit means of payment which also serves as numeraire hence has $p_{m+1} = 1$.

An individual will be dissuaded from demanding arbitrarily large amounts of credit by a penalty for indebtedness. This can be modelled as follows. We modify the utility function of the individual to have the form:

$$\varphi^i(x^i_1, \ldots, x^i_m, \min(0, (-b^i + \sum_{j=1}^{m} p_j a^i_j))) .$$

We observe that in this one period model of trade there will be no worth to be attached to having positive amounts of credit left over after trade. However a good bankruptcy law should impose a negative worth to being in debt. An optimal bankruptcy rule will be a set of penalties which make strategic bankruptcy unprofitable.

A simple example of a sell-all model where an individual has preferences for one real commodity and credit is shown in Figure 2. The ver-

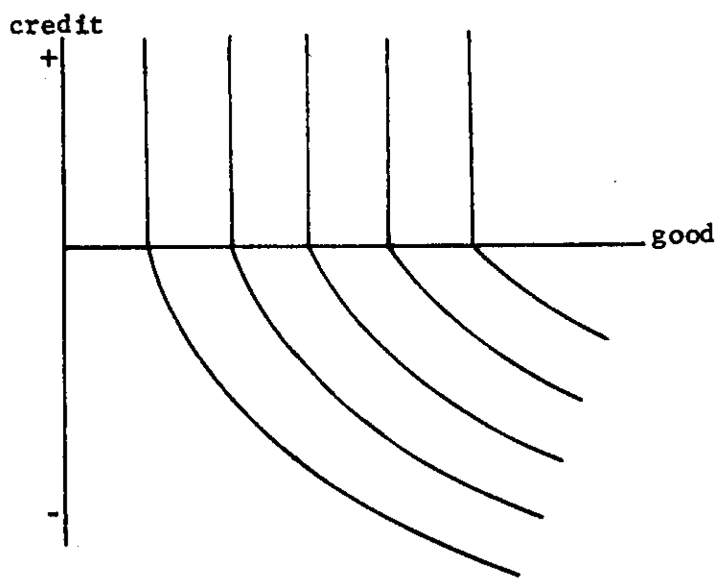

FIGURE 2

tical indifference curves indicate that for a fixed amount of the real good extra credit adds no value. The curved indifference curves show that as debt increases there is a distinct tradeoff between debt and the amount of the real good held. In this market residual credit has no intrinsic worth but residual debt has negative worth. The commodity space for an economy with commodity money was defined on R^{+}_{m+1} ; for an economy with fiat money or credit it is defined on $R^{+}_{m} \times R_1$.

By merely extending the definition of the indifference curves into the negative orthant in Figure 2 we have avoided being specific about the nature of the penalty imposed on the debtor. This could be noneconomic such as time in debtors' prison (England), a period in slavery (Babylon) or could involve confiscation of goods, garnishing of future salary and so forth.

This model has been considered by Shapley and Shubik [23], Postlewaite and Schmeidler [24] and Dubey and Shapley [25]. It can be shown that for a continuum of traders the noncooperative equilibria coincide with the competitive equilibria for any set of bankruptcy penalties. The setting of the bankruptcy penalties essentially fixes the price level. The more lenient they are the higher will be the price level.

Model 5. Sell All with a Fixed Issue of Fiat

Suppose that an outside agency supplies a fixed amount M of credit or fiat. Each individual i obtains a portion of the money supply by bidding an I.O.U. note for it. If individual i bids u^i he obtains an amount $M(u^i/u)$ which is used to bid for goods. A strategy, at its simplest is a vector of $m+1$ numbers $(u^i, b^i_1, \ldots, b^i_m)$ where $\sum_{j=1}^{m} b^i_j \leq M(u^i/u)$.

In this model the level of prices is bounded by $\sum_{j=1}^{m} p_j a_j \leq M$. After trade each individual i will have as his supply of fiat

$$\frac{u^i}{u}M - b^i + \sum_{j=1}^{m} p_j a^i_j$$

and he owes u^i , thus

$$x^i_{m+1} = \frac{u^i}{u}M - b^i + \sum_{j=1}^{m} p_j a^i_j - u^i \; .$$

Because there is clearly a relationship between the harshness or leniency of the bankruptcy penalty and the willingness to end in debt, the penalty influences the price level. However so does the money supply. It has been shown elsewhere [26], [27] that there will exist at least one set of optimal bankruptcy penalties which enable us to define a game where all individuals trade using a fixed supply of fiat money which has noncooperative equilibria coinciding (in relative prices and the distribution of resources) with the competitive equilibria of the economy described by the game.

The optimal bankruptcy penalties are intimately related to the Lagrangian multipliers of the competitive equilibrium solution. When other penalties are chosen, if they are too lenient some players may find that it is strategically profitable to go bankrupt. When the penalties are too harsh hoarding may take place.

If the penalties are optimal we obtain the result that no one goes bankrupt and $M = u$. This implies that the rate of interest in the economy is zero. u is the amount pledged by the traders to be paid in return for obtaining M hence $1+r = u/M$; if $u = M$ then $r = 0$.

A zero interest rate is consistent with our expectations of the role of a money or credit in one period trade. It is as though the government, referee or outside bank has supplied an institutionalized form of acceptable paper to <u>finance the float</u> (i.e. the gap between payments and receipt of income). As the cost of doing so is approximately zero (to a first approximation given that a society is already organized) there is no economic need for the price of this service to be above zero.

When the penalties are low it is possible that $u > M$ hence $r > 0$ and the resultant noncooperative equilibrium does <u>not</u> coincide with the competitive equilibrium. The positive rate of interest may best be regarded as a tax or loss reserve which guarantees that the bank gets back M even though there are defaults.

Comments

In both Models 4 and 5 there is only one bank or source of money or credit. It is possible but relatively messy to construct models with more than one bank in competition with each other. In doing so however either the price (the rate of interest) or the quantity of credit extendable must be given. If price is given then competition is of a Cournot variety, if credit is limited then a Bertrand-Edgeworth model emerges. A simple model of these types of competition among banks has been investigated [28].

When banks are permitted an active role in the granting of credit, information conditions become of great importance. For example an accurate knowledge of the purpose of all loans enables a banking system to estimate future returns far better than if none of this information were available.

Models 4 and 5 are clearly gross simplifications yet they both are full process models described for all positions of equilibrium or disequilibrium. They both are sufficiently simple that they can be played as experimental games.

3.3. Multiperiod Models

The essence of money and financial instruments appears to be closely associated with process. Although the models described in 3.1 and 3.2 are clearly process models they have been presented as though the economy trades only in a single period.

The Arrow Debreu model of an economy with time dated commodities and full futures markets is essentially a single period trading model. The work on temporary general equilibrium represents a departure from single period trading as does our work on models of trade as multistage noncooperative games.

Suppose that we were to take a model such as Model 5 and consider a finite sequence of say k markets with spot trades in each market. Suppose furthermore that penalties for failure to repay are levied at final settlement. Do we obtain results which differ from Model 5? Dubey and Shubik [29] have considered this. It is fairly clear to see that if a fixed amount M of money were issued by an outside bank at the start of trade, then in general trade at efficient prices could only take place with hoarding to adjust the supply of money. The amount M serves as an upper bound for prices but that is all. For this model there is a logically correct solution giving a noncooperative equilibrium with a zero rate of interest and hoarding needed to adjust intertemporal prices. This solution is neither intuitively nor empirically satisfactory. What we see and should be able to derive from an adequate model of the economy is an interest rate usually greater than zero and little hoarding except under special circumstances.

Two natural ways to vary the money supply from period to period are by having just a credit system for each period of the type indicated in Model 4. Another way is to introduce an outside bank which issues M units of fiat at the start of the economy. Once this money is in the hands of the traders an inside bank is authorized and it sells shares in exchange for fiat. Then the inside bank may grant credit. If it earns a profit this profit is paid to stockholders.

It can be shown that in a finite stage model [29] unless at the end of all trade there is some capital left over which is judged to be of positive worth in the settling of debts then this model can have no efficient noncooperative equilibrium solution with a positive rate of interest. If there is a positive rate of interest someone will go bankrupt.

If we build a new model, where Deus ex Machina a salvage value is attached to assets remaining at the end of the k^{th} period we find efficient solutions with a positive rate of interest. The higher the value assigned to capital left over after trade the higher a rate of interest can be sustained.

The details of the strategies of the traders and producers in this model are relatively complex. The interested reader is referred to the article already noted [29].

Comments

When we reduce a multiperiod model to a single period model the need for two types of banks disappears. In a one period model money is needed only to finance the float and if fixed in quantity it fixed the price level. In the multiperiod model intertemporal trade must also be financed.

The artifact of introducing a salvage value for left over capital at the end of k periods was needed to support a positive rate of interest

Yet this scheme is not so arbitrary as it might at first appear to be. In particular it is a simple way of modelling the fact that dynamic economies do not have a known finite termination date. The salvage value tells us in a parametric manner what is the worth of the ongoing economy to the "next generation."

A finite multistage model of an economy with a fixed number of economic actors who never die is not a satisfactory model for the study of many of the basic aspects of the financial infrastructure of an economy. In particular the care of the young; private and public schemes for the care of the old; private and public motivations for bequests and the passing on of the means of production to the next generation are all aspects of an economy for which financial institutions and instruments are of critical importance.

The work of Samuelson [30], [31], Yaari [32], Arthur and McNichols [33] and several others on economies with many generations shows a growing interest in microeconomic models with birth and death processes taken into account. But much remains to be done in considering both individual motivation and public devices for the intergenerational transfer of assets. Once several generations are to be considered even items such as the definition of private property become considerably more subtle than when there is only one generation.

The multigenerational models not only raise questions concerning inheritance and altruism but lead naturally to the type of modelling considerations of concern in the newly developing area of sociobiology. An excellent discussion of its relevance to economics is given by Hirshleifer [34].

3.4. Information and Exogenous and Endogenous Uncertainty

The very essence of any financial system is information and trust. A key concern of microeconomists recently has been how well does the price system reflect differences in information. It has been argued elsewhere [13] that the general equilibrium model is based on symmetric information conditions. When information conditions are not symmetric Radner has shown that competitive equilibria may not exist [5]. We have shown however that for the models described in Section 3.1 it is possible to modify the markets to encompass any level of futures trading and to solve for noncooperative equilibria [35], [36]. These equilibria reflect the differences in information in the market prices which emerge.

The basic idea is simple and depends upon the difference between a move and a strategy. Given the market structure,the way trade actually takes place is defined by the moves. However individuals with more information than others can vary their moves as a function of this information. If one individual is unable to distinguish between two states his move will be the same regardless of state; if he can make a distinction, his plan or strategy may call for action that is state dependent.

It can be shown that when full futures markets exist then for all traders having total lack of knowledge of any exogenous random variable the noncooperative equilibria will coincide with the competitive equilibria in relative prices and distribution of goods [35]. For all other conditions the noncooperative equilibria will differ from the competitive equilibria.

Although markets may reflect differences in information in the prices of the goods, it is also possible that there may be direct markets for information. Experts are often called upon for their knowledge and information, advisory services are available and in most economies there is an information industry apart from the markets for goods. In any attempt to model the market for information directly problems are encountered in the definitions of the amounts of information and proprietory rights and resale conditions [37].

The Arrow Debreu treatment of exogenous uncertainty involved the use of futures markets. An institutionally different way than this is to introduce trade in money and then instead of futures contract trading have the individuals buy insurance policies which involve the trading of money for various sums of money to be paid upon the occurrence of certain contingencies.

In a world with uncertainty and money insurance institutions offer an efficient alternative to futures markets.

Knowledge about the value of an exogenous random variable and knowledge about the moves made by others are qualitatively somewhat different. The usual discussions of the information conditions behind the general equilibrium model are not particularly explicit on who knows what. Limiting ourselves to markets without exogenous uncertainty we can show the rather surprising and general result that although as information conditions change so will the set of noncooperative equilibria; the competitive equilibria are always attainable as noncooperative equilibria in a nonatomic market game for any information conditions [38].

The remarks made and results obtained with markets with trade in money and with various forms of uncertainty are minute in comparison with the problems that remain. In particular, for example, in the context of a closed economic model what is the role of insurance company reserves? Common sense and experience tells us that insurance companies need reserves.

But there may be several different basic reasons why this is so; and it does not appear to be easy to sort them out and weigh their relative importance in the context of a closed microeconomic model.

At the most abstract, economic activity is nothing more than a process of mapping a set of resources into a different set of resources. The observer from outer space would see only the activities not the utility functions being maximized. The analogy between purposeful economic activity and local entropy reduction is appealing as are the attempts to devise measures for that which constitutes economic information. The reader is referred to the stimulating works of Perroux [39] and Georgescu Roegen [40] for a discussion in depth of these possibilities. The approach adopted here has been somewhat more restricted than those mentioned inasmuch as the prime concern has been with the appearance of specific financial instruments and information in a decision or game theoretic sense.

4. MATHEMATICAL INSTITUTIONAL ECONOMICS

The work and results discussed here represent the beginnings of a new approach toward constructing a viable theory of money and financial institutions. The major departure suggested here is in the modelling. The twin criteria that the model must be a well defined game and that it should be playable guarantee that a full process model has to be defined and that it should meet certain relevance and feasibility tests as well.

The noncooperative equilibrium solution has been selected not because of any great faith in noncooperative equilibrium theory *per se*, but because it seems to be the right relatively modest step in the direction of dynamics. Furthermore it is clearly more general than general equilibrium theory and appears to be capable of giving all the results obtained with general equilibrium theory as well as many others.

These types of models are naturally suited to including oligopolistic competition. It appears that the basic nature of many financial institutions will tend to call for fewer institutions to exist than would be required for completely competitive behavior. Thus it is important that any theory of sufficient power to characterize the behavior of financial institutions should be able to consider competition among the few.

Finally it must be stressed that the style of modelling suggested here is much more open than that presented in general equilibrium theory, but hopefully, less ad hoc then is usually found in macroeconomics. In particular the thesis is that general equilibrium theory in its development as a logically consistent and parsimonious abstraction lost touch with the important structural limitations on economic process caused by institutions which guide the process but are themselves formed not merely by economic forces but by history, society, politics and law.

In contrast with the general equilibrium approach the game theory approach is at least as mathematically rigorous but the modelling requirements force the specification and justification of devices to carry process and these devices may be interpreted as rudimentary institutions.

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