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EQUILIBRIUM AND DISEQUILIBRIUM INTERPRETATIONS OF THE IS-LM MODEL

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by

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There is a considerable tension in neoKeynesian macroeconomics as to whether the familiar IS-LM analysis should be interpreted as a description of equilibrium or disequilibrium. The present paper shows how the IS-LM model can be used for either purpose. The first section provides the preliminaries for the model of disequilibrium behavior that I will use. The second section lays out the sectoral notional and effective demands. The third section gives the equilibrium solution while the fourth and fifth sections give excess supply and excess demand solutions respectively.

I. Disequilibrium Preliminaries

Walras described a tatonnement process in which a central auctioneer repeatedly announces a complete set of tentative prices and records the demands and supplies at those prices. If all markets clear, then transactions take place. If not, then all offers are off and auctioneer announces a new set of prices. This continues until the equilibrium prices are found. This artificial construct faithfully apes an iterative mathematical solution of a set of demand and supply equations but is a seriously

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flawed depiction of the process by which prices are actually determined.

In reality, prices are set by a very large number of persons with limited and imperfect knowledge, and it would be quite surprising if they were very often set at the precise razor's edge of demand and supply. Each of these prices generally represents a genuine offer to buy or sell at the stated price and a considerable number of transactions may take place before it is realized that it is a "false" or disequilibrium price. Thus as prices find their way to market clearing levels, much of the demand and supply may already have been satisfied at other prices. The final prices at which the remaining demands and supplies are equated may consequently differ considerably from those that would have been established by a tatonnement process.

Evidence of this imperfection is that prices are often quite sluggish and revised infrequently. Wages are for example typically set for at least a year, and some interest rates are legally set or bounded for much longer periods. This may represent the completely rational preferences of market participants; the point here is that it is difficult to view such prices as equilibrium prices unless new information also arrives in sluggish and infrequent batches.

A contrary view is that when an agent sets a price, the short run demand or supply curve is horizontal in that the agent is willing to buy or sell any amount at that price. There is consequently never any disequilibrium. And if prices are legally fixed then markets are cleared by other means. If for example there is a ceiling on interest rates paid on deposits then the financial institutions will give away merchandise, offer cheaper loan rates or free safe deposit boxes to depositors, or invent other inducements to equilibrate the market. It is clearly true

that prices are not the only means of clearing markets and that ex post markets are always cleared in that the quantity bought cannot differ from the quantity sold. Nonetheless, it seems fruitful to distinguish between ex ante demands reflecting agents' pure preferences on the assumption that they could buy or sell as much as they wish, and the actual transactions that occur. While ex post demand and supply will surely be reconciled, the divergences between ex ante demands and supplies (or between transactions and ex ante preferences) are presumably what underlie the eventual adjustment of prices.

The simplest way of beginning a formal disequilibrium analysis is to discuss a situation in which there are only two agents (labeled 1 and 2) and two traded items. For concreteness one item will be a commodity (C) and the other money (M). The notional flow demands (labeled with an asterisk) represent each agent's preferred transactions if they could buy or sell as much as they wish at the terms of trade P (the price of the commodity in dollars) subject only to the budget constraint that their money holdings will be reduced by the amount that they spend on the commodity.

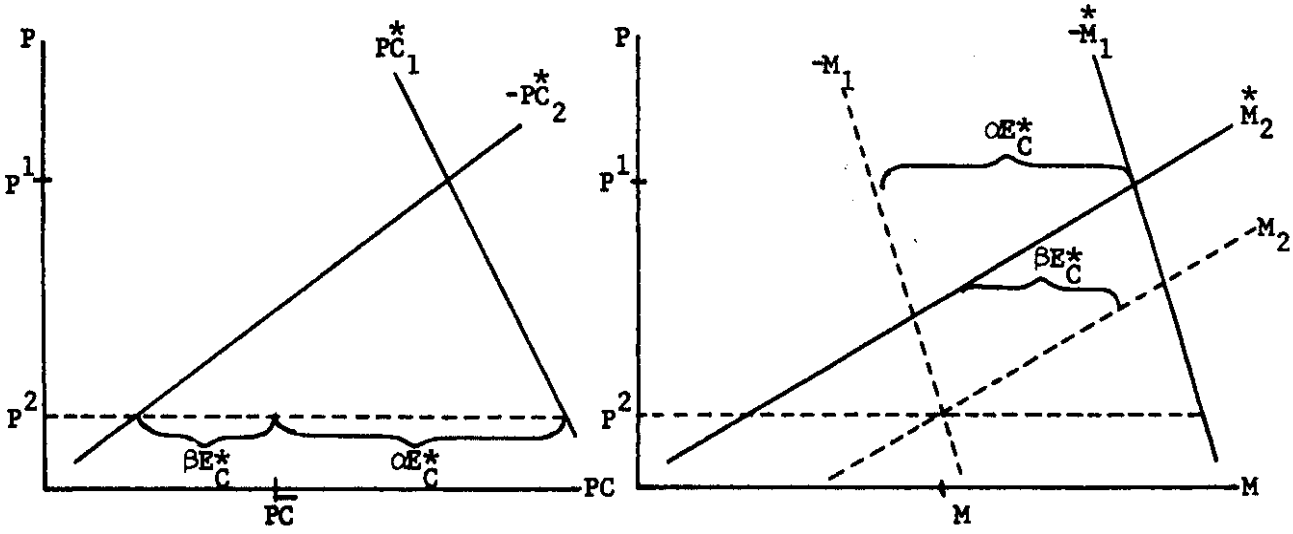
$$PC_1^*[P] + M_1^*[P] \equiv 0$$

$$PC_2^*[P] + M_2^*[P] \equiv 0 .$$

From society's budget constraint

$$0 \equiv P(C_1^* + C_2^*) + (M_1^* + M_2^*) \equiv E_C^* + E_M^*$$

the excess notional demands E^*_C and E^*_M (in dollars) sum to zero. Thus an equilibrium price P^1 which clears commodity demands will simultaneously clear money demands.



If on the other hand the price is set at P^2 then there will be an excess demand for commodities $(PC^*_1 > -PC^*_2)$ and an equal excess supply of money $(M^*_2 < -M^*_1)$. The actual amount of commodities traded $(\bar{C})$ will presumably be within the bounds set by the demand and supply curves at P^2, since otherwise both agents would prefer to trade more (or less) of the commodity. Some authors insist that the minimum of supply and demand (here C^*_2) will prevail since agents cannot be compelled to buy or sell more than they wish. I will here leave open the possibility that both sides of the market can be off their schedules, with actual dollar transactions αE^*_C less than the first sector's demand and βE^*_C greater than the second sector's supply. Since the discrepancy between demand and supply must be fully absorbed, there is a cross sector adding up restriction that $\alpha+\beta = 1$. The dominance of supply (since it is less than demand) corresponds to the special case $\beta = 0$, $\alpha = 1$.

Values of β other than zero (and even close to one!) might be used to depict the willingness of banks to satisfy loan demand, or suppliers to meet commodity demand out of inventories, or of firms to retain personnel who would otherwise be unemployed. The appropriate values of α and β will almost certainly depend upon whether there is excess demand or supply.

In a multiagent environment one has to decide not only whether the disequilibrium is absorbed by demanders or suppliers but also which particular demanders and suppliers absorb the disequilibrium. If for example banks do not meet all loan demands, then each loan may be prorated or the rejections may fall disproportionately on such categories as car loans, education loans, vacation loans, poor families, middle-income families, wealthy families, small businesses, or large corporations. The importance of this distinction is due to the presence of spillover demands from a disequilibrium market into other markets. If an agent cannot purchase as much of some commodity as it wishes, then it will have excess funds which must be spent elsewhere or held as cash. Similarly if an agent cannot sell as much as it wishes then it will have to reduce its spending elsewhere. In our example, how spending is reduced may depend critically upon who is unable to sell promissory notes to banks.

In this simple two item model, there is no choice in one's spillover demands. Sector 1 will increase its cash holdings (relative to its notional demands) by αE_C^* and sector 2 will increase its planned cash holdings by βE_C^*

$$\bar{PC} = \overset{*}{PC}_1 - \alpha E_C^* , \quad \bar{PC} = \overset{*}{PC}_2 - \beta E_C^*$$

$$M_1 = \overset{*}{M}_1 + \alpha E_C^* , \quad M_2 = \overset{*}{M}_2 + \beta E_C^* .$$

Those demands M_1 and M_2 which take into account the disequilibrium rationing in other markets are known as effective demands. Walras' Law will continue to hold for the nonrationed markets in that the excess effective demands in these markets will sum to zero:

$$M_1 + M_2 \equiv \overset{*}{M}_1 + \overset{*}{M}_2 + (\alpha+\beta)E_C^* = -P\overset{*}{C}_1 - P\overset{*}{C}_2 + E_C^* = 0 .$$

Since there is only one other market in this simple model, Walras' Law states that the effective demands and supplies of money automatically coincide.

One implication of spillover demands is that in the analysis of a particular market one must be alert for influences from disequilibrium markets. The demand for housing may be critically dependent upon rationing in the mortgage market. The corporate bond market may be influenced by disequilibria in labor, commodity, and loan markets.

Now summarizing and generalizing this discussion, a disequilibrium model with n traded items requires the following specifications:

(1) Notional demands for the agents;

(2) which k markets are rationed and which $n-k$ markets are cleared by freely moving variables;

(3) which $n-k-1$ "prices" adjust to clear the equilibrium markets and how the sluggish non-market clearing prices are determined;

(4) which agents absorb the market disequilibrium in the k rationed markets; and

(5) how these absorptions are financed (i.e., how the effective demands are influenced by quantity restrictions in the rationed markets).

II. The Model

I will now reinterpret the IS-LM model as a description of an equilibrium or disequilibrium situation. In the disequilibrium version wages and prices will be assumed to be set (in an unspecified way) at other than equilibrium levels. The labor and commodity markets will be cleared by nonprice means and the interest rate will float to clear financial markets. A discrete model will be used here since this framework allows financial markets to be immediately influenced by spillovers from labor and commodity markets.

The basic framework is displayed in Table I which lists the notional demands (again labeled with asterisks). For simplicity, taxes have been omitted from the model.

TABLE I. Real Notional Demands

	Households	Corporations	
Labor	$-\frac{\bar{w}}{P}\overset{*}{N}^{S}\left[\frac{\bar{w}}{P}\right]^{+}$	$\frac{\bar{w}}{P}\overset{*}{N}^{D}\left[\frac{\bar{w}}{P}\right]^{-}$	
Commodities	$\overset{*}{C}[\bar{r}, \bar{P}, \frac{\bar{w}}{P}^{+}, Div^{+}]$	$\overset{*}{I}[\bar{q}] - \overset{*}{Q}\left[\frac{\bar{w}}{P}\right]^{-}$	$\overset{*}{G}$
Money	$\overset{*}{L}[\bar{r}, \bar{P}, \frac{\bar{w}}{P}^{+}, Div^{+}]$		$-\overset{*}{M}^{S}/P$
Bonds + Equities	$\overset{*}{V}[\bar{r}^{+}, \bar{P}, \frac{\bar{w}}{P}^{+}, Div^{+}]$	$-P_E\overset{*}{E}^{S}[\bar{q}]/P$	$-\overset{*}{B}^{S}/P$
	$\frac{M+B+r(-1)B+P_EE}{P} + Div$	$-\frac{P_EE}{P} - Div$	$-\frac{M+B+r(-1)B}{P}$

Household notional demands will be depicted as simultaneously determined functions of the interest rate, the price level, the real wage rate, and the level of dividends. The predetermined variables M, B, K, and $r(-1)$ have been suppressed as arguments since they will be held constant. The adding up restrictions on the arguments of the notional demands are that the sum of the induced changes in $-\frac{w}{P}N^{*S}$, C^{*}, L^{*}, and V^{*}, equal the sum of the induced changes in $(M+B+r(-1)B+P_E E)/P$ plus Div.

Allowing for disequilibrium in the labor and commodity markets, household effective demands can be denoted by

$$(1)\qquad \begin{cases} -\frac{w}{P}N^{S} = -\frac{w}{P}N^{S}\left[-\frac{w}{P}N^{*S},\ C+I+G-Q\right] \\ C = C\left[C^{*},\ \frac{w}{P}(N^{D}-N^{S})\right] \\ L = L\left[L^{*},\ C+I+G-Q,\ \frac{w}{P}(N^{D}-N^{S})\right] \\ V = V\left[V^{*},\ C+I+G-Q,\ \frac{w}{P}(N^{D}-N^{S})\right] . \end{cases}$$

With a fixed capital stock and production function competitive profit maximizing firms will want to employ labor and produce output up until the point where the marginal product of labor is equal to the real wage, $F_N = w/P$. All profits are assumed to be paid out as dividends. The notional demands for output, labor, and dividends can consequently be written as

$$\overset{*}{Q} = \overset{*}{Q}\left[\frac{w}{P}\right] \qquad \frac{d\overset{*}{Q}}{d\frac{w}{P}} = \frac{F_N}{F_{NN}}$$

$$\frac{w}{P}\overset{*D}{N} = \frac{w}{P}\overset{*D}{N}\left[\frac{w}{P}\right] \qquad \frac{d\frac{w}{P}\overset{*D}{N}}{d\frac{w}{P}} = \overset{*D}{N} + \frac{F_N}{F_{NN}} < 0 \quad \text{if} \quad -\frac{\partial F_N}{\partial N}\frac{\overset{*D}{N}}{F_N} < 1$$

$$\overset{*}{Div} = \overset{*}{Div}\left[\frac{w}{P}\right] \qquad \frac{d\overset{*}{Div}}{d\frac{w}{P}} = \frac{d\left(\overset{*}{Q} - \frac{w}{P}\overset{*D}{N}\right)}{d\frac{w}{P}} = -N^*$$

The sale of new equity to finance investment depends upon q, the market value of the capital stock relative to its replacement cost.

$$\overset{*}{I}[q] = P_E(\overset{*S}{E}[q] - E)/P \ .$$

Effective demands are given by

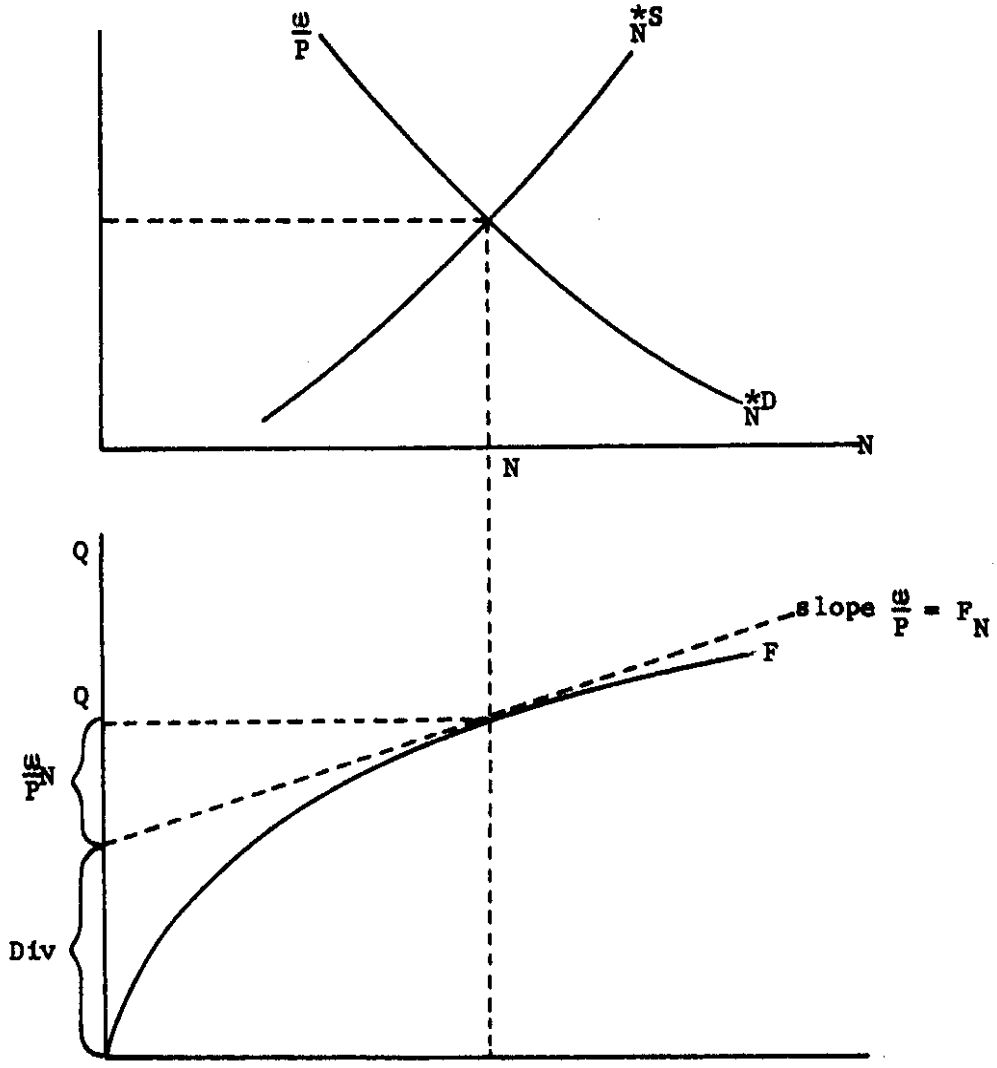
$$(2) \qquad \begin{cases} Q = Q\left[\overset{*}{Q},\ \frac{w}{P}(N^D - N^S)\right] \\ \frac{w}{P}N^D = \frac{w}{P}N^D\left[\frac{w}{P}\overset{*D}{N},\ C + I + G - Q\right] \\ Div = Div\left[\overset{*}{Div},\ \frac{w}{P}(N^D - N^S),\ C + I + G - Q\right] \\ I \equiv P_E(E^S - E)/P = I\left[\overset{*}{I},\ \frac{w}{P}(N^D - N^S),\ C + I + G - Q\right] \ . \end{cases}$$

It will be assumed that the government always realized its desired purchases G. This and the money supply M^S it chooses determines the bond supply

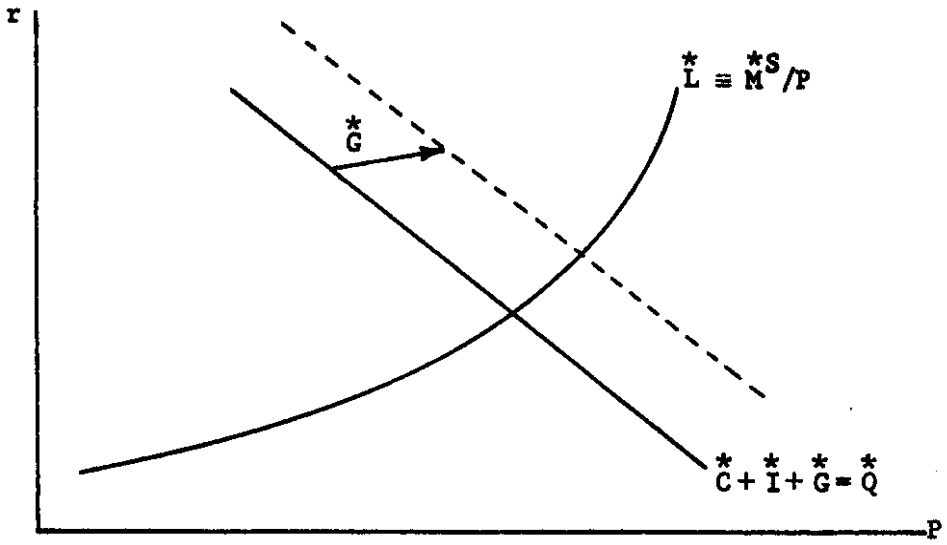
$$B^S \equiv \frac{B + r(-1)B + M - M^S}{P} + G \ .$$

III. Equilibrium

The equilibrium solution of the system can be displayed in a variety of ways. Since labor market equilibrium depends only upon the real wage, this equilibrium is usually considered separately since it alone determines the real wage, employment, output, and dividends.

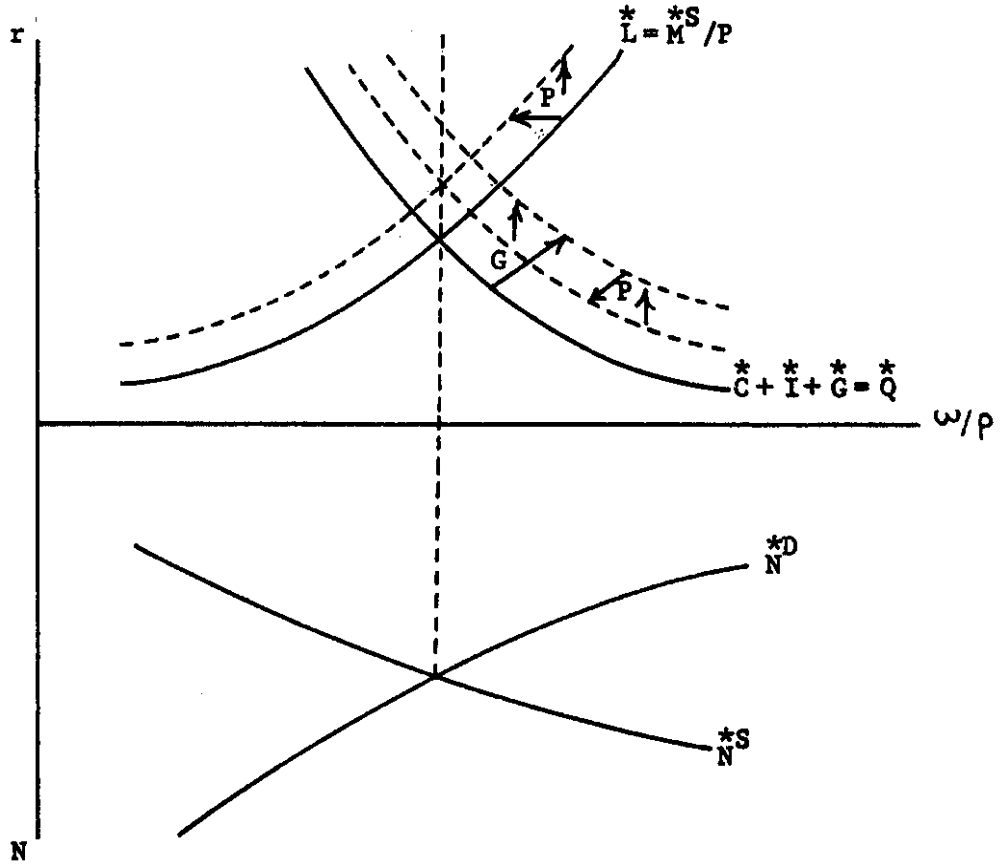


Given the real wage which clears the labor market, commodity and money equilibria will determine r and P . A bond financed increase in government spending will for example increase the interest rate and the



price level without altering the real wage, employment, or output.

An obvious alternative depiction is



where the $\overset{*}{L} = \overset{*S}{M}/P$ and $\overset{*}{C}+\overset{*}{I}+\overset{*}{G} = \overset{*}{Q}$ locii both depend upon P. The effects of a bond financed increase in government spending are again displayed.

With a little effort, this can be redrawn in another more familiar form. First we can redefine the demands as sequential, being preceded by the labor supply decision:

$$\overset{*}{C} = \overset{*}{C}\left[r,\ P,\ \frac{w}{P},\ Div\right] = \overset{**}{C}\left[r,\ P,\ \frac{w}{P}\overset{*S}{N},\ Div\right]$$

$$\overset{*}{L} = \overset{*}{L}\left[r,\ P,\ \frac{w}{P},\ Div\right] = \overset{**}{L}\left[r,\ P,\ \frac{w}{P}\overset{*S}{N},\ Div\right]$$

where the partial derivative are unchanged with the exceptions of

$$\overset{**}{C}_3 = \overset{*}{C}_3 \Big/ \left(\frac{\frac{\partial w}{P}\overset{S}{N}}{\frac{\partial w}{P}}\right) \quad \text{and} \quad \overset{**}{L}_3 = \overset{*}{L}_3 \Big/ \left(\frac{\frac{\partial w}{P}\overset{*S}{N}}{\frac{\partial w}{P}}\right) .$$

Now assuming $\overset{**}{C}_3 = \overset{**}{C}_4$ and $\overset{**}{L}_3 = \overset{**}{L}_4$, and defining $Y = \frac{w}{P}\overset{*S}{N} + Div$, we can graph (for any price level) the combinations of r and Y such that $\overset{*}{L} = \overset{*S}{M}/P$ and $\overset{*}{C}+\overset{*}{I}+\overset{*}{G} = Y$ (or $I = S$).*

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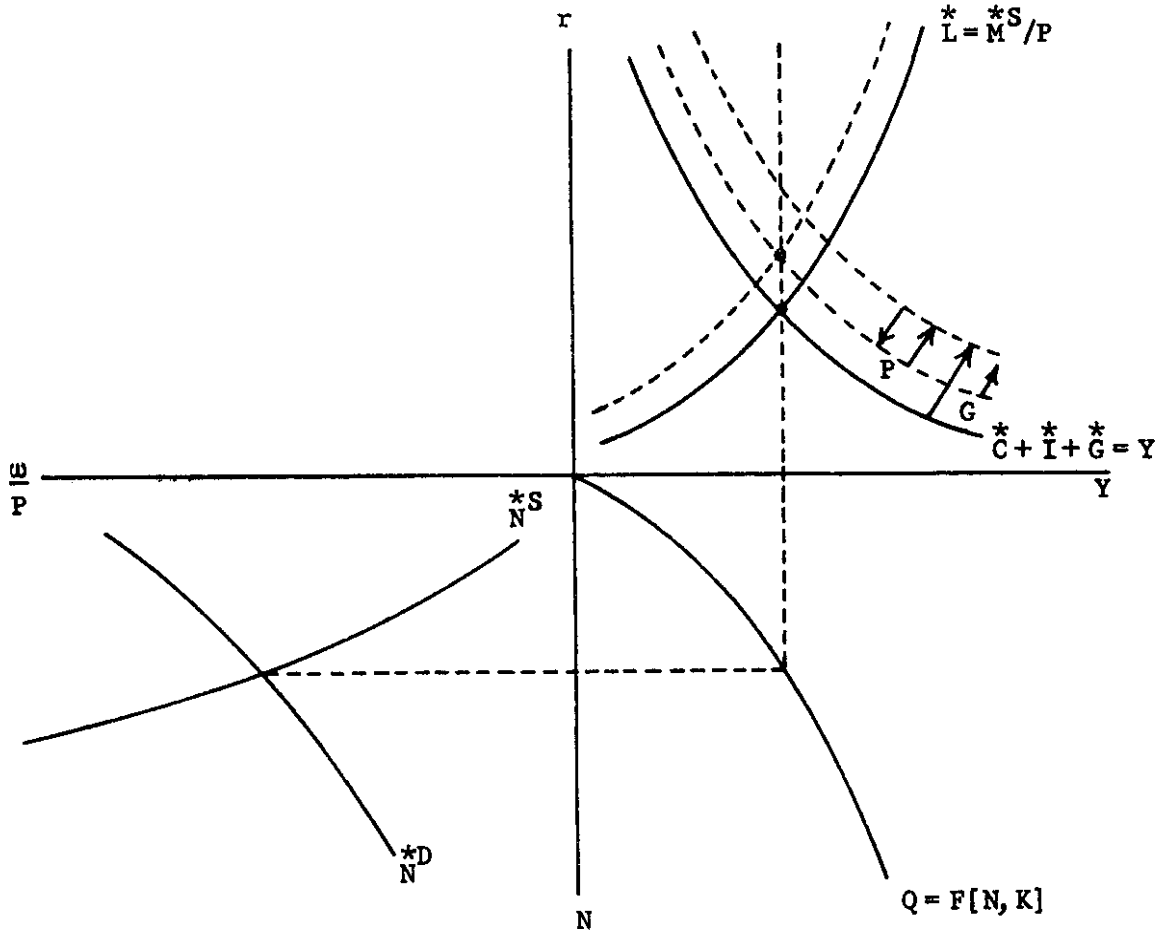
* $\overset{*}{C}+\overset{*}{I}+\overset{*}{G} = Y$ is not a market equilibrium, but it can be rewritten as

$$(\overset{*}{C}+\overset{*}{I}+\overset{*}{G}-\overset{*}{Q}) + \frac{w}{P}(\overset{*D}{N}-\overset{*S}{N}) = 0 ,$$

a condition that the excess demands for commodities and labor sum to zero. This condition and labor market equilibrium imply commodity market equilibrium.



This provides the interpretation of the IS-LM apparatus as part of an equilibrium model. I could at this point work through the general disequilibrium analysis, and then impose parametric assumptions for special cases. Unfortunately the math and notation become so formidable and tedious that they obscure the analysis. I will consequently instead work directly with the special cases.

IV. Excess Supply

We will first consider a situation in which the effective demands for both labor and commodities are less than the effective supplies. We will impose the usual assumption that actual transactions are determined by effective demands.

$$\bar{N} = N^D$$

$$\bar{Q} = C + I + G \ .$$

Since households are able to realize their effective demands for commodities, their other demands are not affected by spillovers from this market. Their effective demands (1) can consequently be rewritten as

$$(3) \quad \begin{cases} -\frac{\omega}{P}N^S = -\frac{\omega}{P}^*N^S\left[\frac{\omega}{P}\right] \\ C = C\left[\overset{*}{C},\ \frac{\omega}{P}N^D - \frac{\omega}{P}N^S\right] = c\left[r,\ P,\ \frac{\omega}{P},\ Div,\ \frac{\omega}{P}N^D\right] \\ L = L\left[\overset{*}{L},\ \frac{\omega}{P}N^D - \frac{\omega}{P}N^S\right] = \ell\left[r,\ P,\ \frac{\omega}{P},\ Div,\ \frac{\omega}{P}N^D\right] \\ V = V\left[\overset{*}{V},\ \frac{\omega}{P}N^D - \frac{\omega}{P}N^S\right] = v\left[r,\ P,\ \frac{\omega}{P},\ Div,\ \frac{\omega}{P}N^D\right] \ . \end{cases}$$

One relatively minor issue is whether an increase in real wages that is offset by a fall in employment such that wage income is unaltered will affect the allocation of wage income. If not, then $c_3 = \ell_3 = v_3 = 0$. Since the scenario generally entails a redistribution of income, it is easy to accept the possibility that these partial derivatives are not equal to zero. Another distributional issue is whether dividend and

labor income are allocated identically.

Corporations on the other hand are able to realize their effective demand for labor but unable to find buyers for their effective supply of commodities. Since labor demands are realized we can write the effective demands (2) as

$$(4)\qquad \begin{cases} Q = \overset{*}{Q} \\ \frac{\omega}{P}N^D = \frac{\omega}{P}N^D\left[\frac{\omega^*}{P}N^D,\ C+I+G-\overset{*}{Q}\right] \\ Div = Div[\overset{*}{Div},\ C+I+G-\overset{*}{Q}] \\ I = P_E(E^S-E)/P = I[\overset{*}{I},\ C+I+G-\overset{*}{Q}]\ . \end{cases}$$

Now with actual sales $C+I+G$ less than notional sales $\overset{*}{Q}$, there will be insufficient revenue to pay notional wages and dividends. One extreme alternative is to maintain employment and production, and finance the production and accumulation of unsold goods by reducing dividends (or by reducing investment or selling more equity if the equality of dividends and earnings were broken--which would surely be necessary if sales fell below wage payments).

Another extreme alternative (that I will use here) is to reduce employment in order to produce only as many commodities as are demanded. In this case, actual output and the effective demands for labor and dividends can be written as

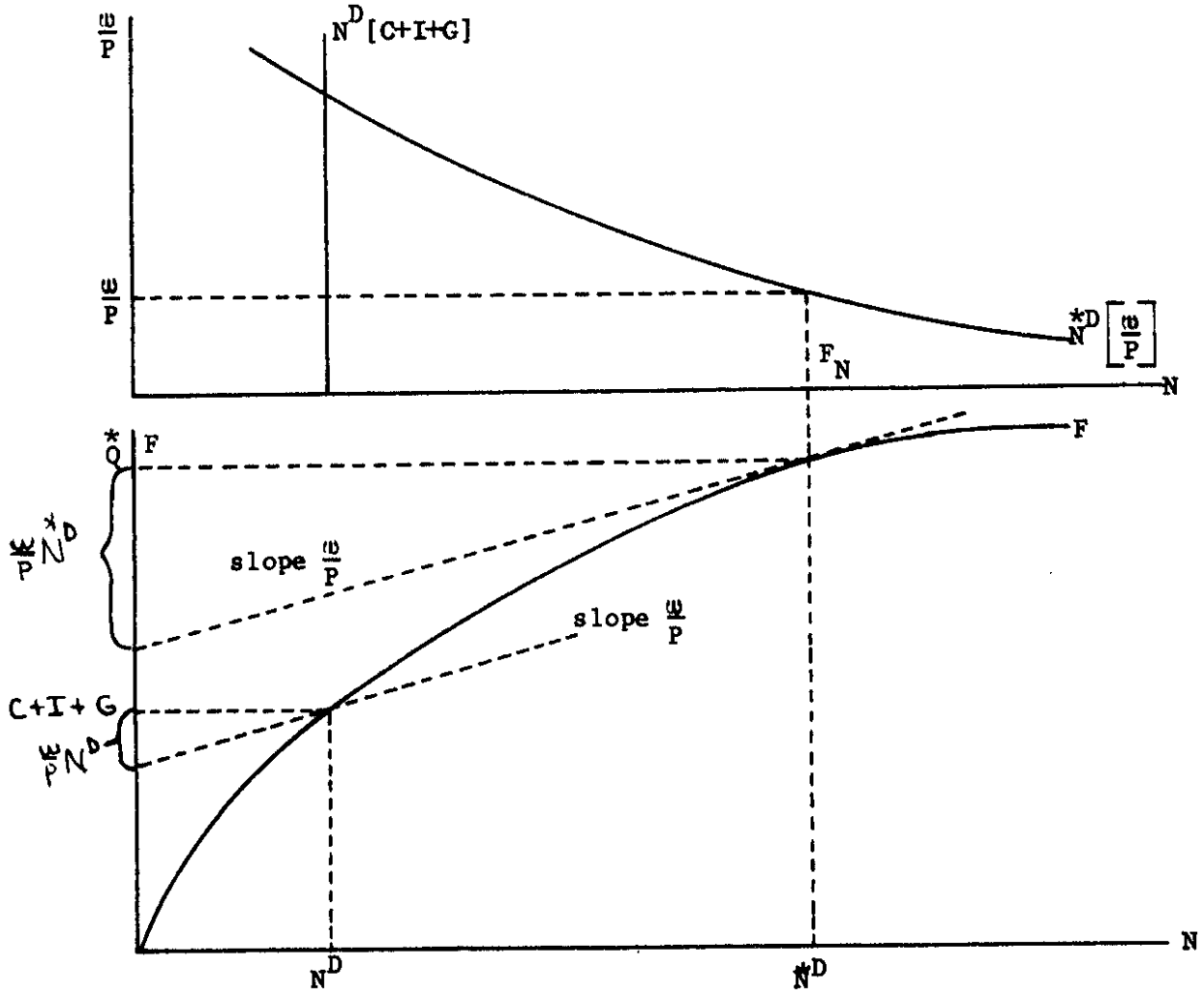
$$(4') \qquad \begin{cases} \bar{Q} = C + I + G \\ \frac{\bar{w}}{P} N^D = \frac{\bar{w}}{P} f^{-1}[C+I+G] \\ Div = C + I + G - \frac{\bar{w}}{P} f^{-1}[C+I+G] \end{cases}$$

where

$$f[N] = F[N,K] \ , \quad \frac{\partial f}{\partial N} = F_N$$

so that

$$\frac{\partial \frac{\bar{w}}{P} N^D}{\partial \frac{\bar{w}}{P}} = N^D \ , \quad \frac{\partial \frac{\bar{w}}{P} N^D}{\partial (C+I+G)} = \frac{\frac{\bar{w}}{P}}{F_N} < 1 \ .$$



Since the marginal product of labor is greater than the real wage rate, a change in output will change revenue by more than labor costs; therefore both wages and dividends will rise and fall with demand. One nagging question in this formulation is why firms don't try to exploit the differential between marginal revenue and marginal cost by expanding employment and output. At the macro level, this exercise may be futile, but myopic competitive firms are presumably interested in micro rather than macro consequences. If employers will not respond to such incentives then they must not be competitive profit maximizing price takers. A model is consequently needed which explains why wages and prices are set for significant periods of time with firms hiring and producing only as much output as they can sell. In many sectors of the American economy, wages and prices (often as a simple mark-up over normal costs) are established for extended periods and variations in aggregate demand do cause variations in employment and output rather than wages and prices. While the model described here is consequently not without interest, it slights the micro question of how this situation can arise and be sustained.

As for investment, one extreme alternative is that the profitability of capital is a longrun calculation which is largely unaffected by contemporary changes in economic activity. If this is true, then investment can only be influenced by changes in the required rate of return which capitalizes these profits. At the opposite extreme, we could imagine completely elastic expectations in which profit expectations coincide with contemporary profits:

$$q = \frac{Div}{rK} = \frac{F[N,K] - \frac{w}{P}N}{rK} .$$

At the level of notional supply $\overset{*}{Q}$, $w/P = F_N$ and

$$q = \frac{F_K[\overset{*}{N}, K]}{r} .$$

In the situation now being considered where $N < N^*$ and $F_N > w/P$

$$q = \frac{F - F_N N + \left(F_N - \frac{w}{P}\right)N}{rK} = \frac{F_K K + \left(F_N - \frac{w}{P}\right)N}{rK} > \frac{F_K}{r}$$

and

$$\frac{\partial q}{\partial N} = \frac{F_N - \frac{w}{P}}{rK} > 0$$

so that

$$\frac{F_K[N, K]}{r} < q < \frac{F_K[\overset{*}{N}, K]}{r} .$$

Thus q is lower than it would be if firms were producing their notional supply, but (since workers are being paid less than their marginal product) somewhat higher than q would be if capital were only being paid its current marginal product. The low level of employment and output reduces q, and an increase in employment and output would raise q.

Here we will allow some elasticity of profit expectations

$$I = i\left[\bar{r}, \frac{\bar{w}}{P}, C + \overset{+}{I} + G\right] .$$

The model can now be written as

$$(5)\qquad \begin{cases} \frac{\bar{w}}{P}N^D = \frac{\bar{w}}{P}N^S \\ c\left[r,\ P,\ \frac{\bar{w}}{P},\ Div,\ \frac{\bar{w}}{P}N^S\right] + i\left[r,\ \frac{\bar{w}}{P},\ c+i+G\right] + G = \bar{Q} \\ \ell\left[r,\ P,\ \frac{\bar{w}}{P},\ Div,\ \frac{\bar{w}}{P}N^S\right] = \frac{M^S}{P} \\ v\left[r,\ P,\ \frac{\bar{w}}{P},\ Div,\ \frac{\bar{w}}{P}N^S\right] = \frac{B^S}{P} + \frac{P_E E^S}{P} \end{cases}$$

where $N^D = f^{-1}[\bar{Q}]$ and the sectoral budget constraints are

$$(6)\qquad \begin{cases} Div + \frac{\bar{w}}{P}N^D \equiv \bar{Q} \\ i \equiv P_E(E^S - E)/P \\ G + r(-1)B/P \equiv (M^S - M + B^S - B)/P \\ c + \ell + v \equiv \frac{\bar{w}}{P}N^S + Div + (M+B+r(-1)B+P_E E)/P\ . \end{cases}$$

Summing these budget constraints gives

$$\frac{\bar{w}}{P}(N^D - \bar{N}^S) + (c+i+G-\bar{Q}) + \left(\ell - \frac{M^S}{P}\right) + \left(v - \frac{B^S + P_E E^S}{P}\right) \equiv 0$$

so that the four equations in (5) are linearly dependent. Substituting in the requirement that the labor and commodity market clear <u>ex post</u> $(N^D - \bar{N}^S = c + i + G - \bar{Q} = 0)$, we obtain Walras' law for the effective demands in the nonrationed markets,

$$\left(\ell - \frac{M^S}{P}\right) + \left(v - \frac{B^S + P_E E^S}{P}\right) \equiv 0\ .$$

$$(5)\qquad \begin{cases} \frac{\bar{w}}{P} N^D = \frac{\bar{w}}{P} N^S \\ c\left[r,\ P,\ \frac{\bar{w}}{P},\ Div,\ \frac{\bar{w}}{P} N^S\right] + i\left[r,\ \frac{\bar{w}}{P},\ c+i+G\right] + G = \bar{Q} \\ \ell\left[r,\ P,\ \frac{\bar{w}}{P},\ Div,\ \frac{\bar{w}}{P} N^S\right] = \frac{M^S}{P} \\ v\left[r,\ P,\ \frac{\bar{w}}{P},\ Div,\ \frac{\bar{w}}{P} N^S\right] = \frac{B^S}{P} + \frac{P_E E^S}{P} \end{cases}$$

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$$\frac{\bar{w}}{P}(N^D - \bar{N}^S) + (c + i + G - \bar{Q}) + \left(\ell - \frac{M^S}{P}\right) + \left(v - \frac{B^S + P_E E^S}{P}\right) \equiv 0$$

so that the four equations in (5) are linearly dependent. Substituting in the requirement that the labor and commodity market clear <u>ex post</u> $(N^D - \bar{N}^S = c + i + G - \bar{Q} = 0)$, we obtain Walras' law for the effective demands in the nonrationed markets,

$$\left(\ell - \frac{M^S}{P}\right) + \left(v - \frac{B^S + P_E E^S}{P}\right) \equiv 0 \ .$$

$$(5)\qquad \begin{cases} \frac{\bar{w}}{P}N^D = \frac{\bar{w}}{P}N^S \\ c\left[r,\ P,\ \frac{\bar{w}}{P},\ Div,\ \frac{\bar{w}}{P}N^S\right] + i\left[r,\ \frac{\bar{w}}{P},\ c+i+G\right] + G = \bar{Q} \\ \ell\left[r,\ P,\ \frac{\bar{w}}{P},\ Div,\ \frac{\bar{w}}{P}N^S\right] = \frac{M^S}{P} \\ v\left[r,\ P,\ \frac{\bar{w}}{P},\ Div,\ \frac{\bar{w}}{P}N^S\right] = \frac{B^S}{P} + \frac{P_E E^S}{P} \end{cases}$$

where $N^D = f^{-1}[\bar{Q}]$ and the sectoral budget constraints are

$$(6)\qquad \begin{cases} Div + \frac{\bar{w}}{P}N^D \equiv \bar{Q} \\ i \equiv P_E(E^S - E)/P \\ G + r(-1)B/P \equiv (M^S - M + B^S - B)/P \\ c + \ell + v \equiv \frac{\bar{w}}{P}N^S + Div + (M+B+r(-1)B+P_E E)/P\ . \end{cases}$$

Summing these budget constraints gives

$$\frac{\bar{w}}{P}(N^D - \bar{N}^S) + (c+i+G-\bar{Q}) + \left(\ell - \frac{M^S}{P}\right) + \left(v - \frac{B^S + P_E E^S}{P}\right) \equiv 0$$

so that the four equations in (5) are linearly dependent. Substituting in the requirement that the labor and commodity market clear ex post $(N^D - \bar{N}^S = c + i + G - \bar{Q} = 0)$, we obtain Walras' law for the effective demands in the nonrationed markets,

$$\left(\ell - \frac{M^S}{P}\right) + \left(v - \frac{B^S + P_E E^S}{P}\right) \equiv 0\ .$$

That is, taking into account the constrained transactions in the nonequilibrium markets, the budget constraints insure that the excess demands in the equilibrium markets sum to zero.*

The complete system of four equations could be solved for $\bar{N}^S$, $\bar{Q}$, and the equilibrating variable r. Alternatively, $\bar{N}^S$ can be substituted out, leaving three equations

$$(7)\qquad \begin{cases} c\left[r,\ P,\ \frac{w}{P},\ \bar{Q} - \frac{w}{P}f^{-1}[\bar{Q}],\ \frac{w}{P}f^{-1}[\bar{Q}]\right] + i\left[r,\ \frac{w}{P}[\bar{Q}]\right] + G = \bar{Q} \\ \ell\left[r,\ P,\ \frac{w}{P},\ \bar{Q} - \frac{w}{P}f^{-1}[\bar{Q}],\ \frac{w}{P}f^{-1}[\bar{Q}]\right] = \frac{M^S}{P} \\ v\left[r,\ P,\ \frac{w}{P},\ \bar{Q} - \frac{w}{P}f^{-1}[\bar{Q}],\ \frac{w}{P}f^{-1}[\bar{Q}]\right] = \frac{B^S}{P} + \frac{P_E E^S}{P} \end{cases}$$

which can be solved for $\bar{Q}$ and r. In this form the model is virtually identical to the classroom IS-LM model in which wages and prices are held constant.

The IS-LM equations

$$C + I + G = Y$$

$$L = M/P$$

can consequently be interpreted two ways. In the first interpretation, these equations are part of an equilibrium model and the condition $C + I + G = Y$ is a requirement that the sum of the excess demands for commodities and labor be zero,

---

*Since we have distinguished between effective demand and actual transactions in the nonequilibrium markets the sum of the excess effective demands across all four markets is not zero.

$$(C + I + G - Q) + \frac{w}{P}(N^D - N^S) = 0 \ .$$

When supplemented by a labor market equilibrium condition, this model will determine the equilibrium values of r, P, and w/P. In the second interpretation, the two IS-LM equations are a complete description of an excess supply disequilibrium situation. The equation $C + I + G = Y$ states that output is demand determined, and can be combined with $L = M/P$ to determine the value of r that clears financial markets.

The selected interpretation does affect the nature of the functional arguments, since the demands are notional in the first case and effective in the second. Consider for example the notional and effective consumption functions

$$\overset{*}{C} = \overset{**}{C}\left[r,\ P,\ \frac{w}{P}\overset{*S}{N}\left[\frac{w}{P}\right],\ Div\right]$$

$$C = c\left[r,\ P,\ \frac{w}{P},\ Div,\ \frac{w}{P}N^D\right] ,$$

where for simplicity $c_3 = 0$, as discussed earlier. Now an increase in labor income in the first case represents a desired increase as real wages rise or preferences change, while in the second case workers are passively accepting an increase in real wages or employment. In the first case, they have chosen to earn more while in the second they have no control over labor income. The allocation of such income clearly could differ in the two situations. In empirically estimating such equations it is also important to distinguish between arguments which are predetermined and those which are simultaneously determined.

A final set of questions involves the effects of changes in P

and ω/P . A fall in P increases the real supply of money and the purchasing power of monetary balances, stimulating demand and hence output. At given levels of Q and r , a fall in real wages increases dividends at the expense of labor income. This stimulates investment but has ambiguous effects on consumption and money demands.

V. Excess Demand

I will now consider levels of P and ω/P such that the effective demands for labor and commodities are greater than the effective supplies. As in the preceding analysis I will make the simplifying assumption that the short side of the market prevails so that here actual transactions are determined by effective supplies

$$\bar{N} = N^S$$

$$\bar{Q} = Q .$$

The composition of output will depend upon which sectors are unable to make purchases.

Households realize their effective labor supply but there are spillovers from their unfulfilled commodity demands, so that (1) becomes

$$
(8)\qquad
\begin{cases}
\dfrac{\bar{\mathfrak{M}}}{P}^{S} = \dfrac{\bar{\mathfrak{M}}}{P}^{S}\left[\dfrac{\bar{\mathfrak{M}}}{P},\ C+I+G-Q\right] \\[2ex]
C = \overset{*}{C}\left[\bar{r},\ \bar{P},\ \dfrac{\overset{+}{\mathfrak{M}}}{P},\ \overset{+}{Div}\right] \\[2ex]
L = L[\overset{*}{L},\ C+I+G-Q] = \ell\left[\bar{r},\ \bar{P},\ \dfrac{\overset{+}{\mathfrak{M}}}{P},\ \overset{+}{Div},\ C+\overset{?}{I}+G-Q\right] \\[2ex]
V = V[\overset{+}{V},\ C+\overset{+}{I}+G-Q] = v\left[\overset{+}{r},\ \bar{P},\ \dfrac{\overset{+}{\mathfrak{M}}}{P},\ \overset{+}{Div},\ C+\overset{+}{I}+G-Q\right]
\end{cases}
$$

An inability to purchase all of the effective demand C will lead to a reduction in labor supply and/or the diversion of the unspent funds to the acquisition of financial assets.

It is uncertain whether the demand for money will rise (because the excess funds are held as money) or fall (because of diminished transactions needs). Actual consumption is given by

$$\bar{C} = C - \lambda(C+I+G-Q) .$$

Corporations are able to sell their effective supply of commodities but cannot purchase their effective demand for labor, so that (2) becomes

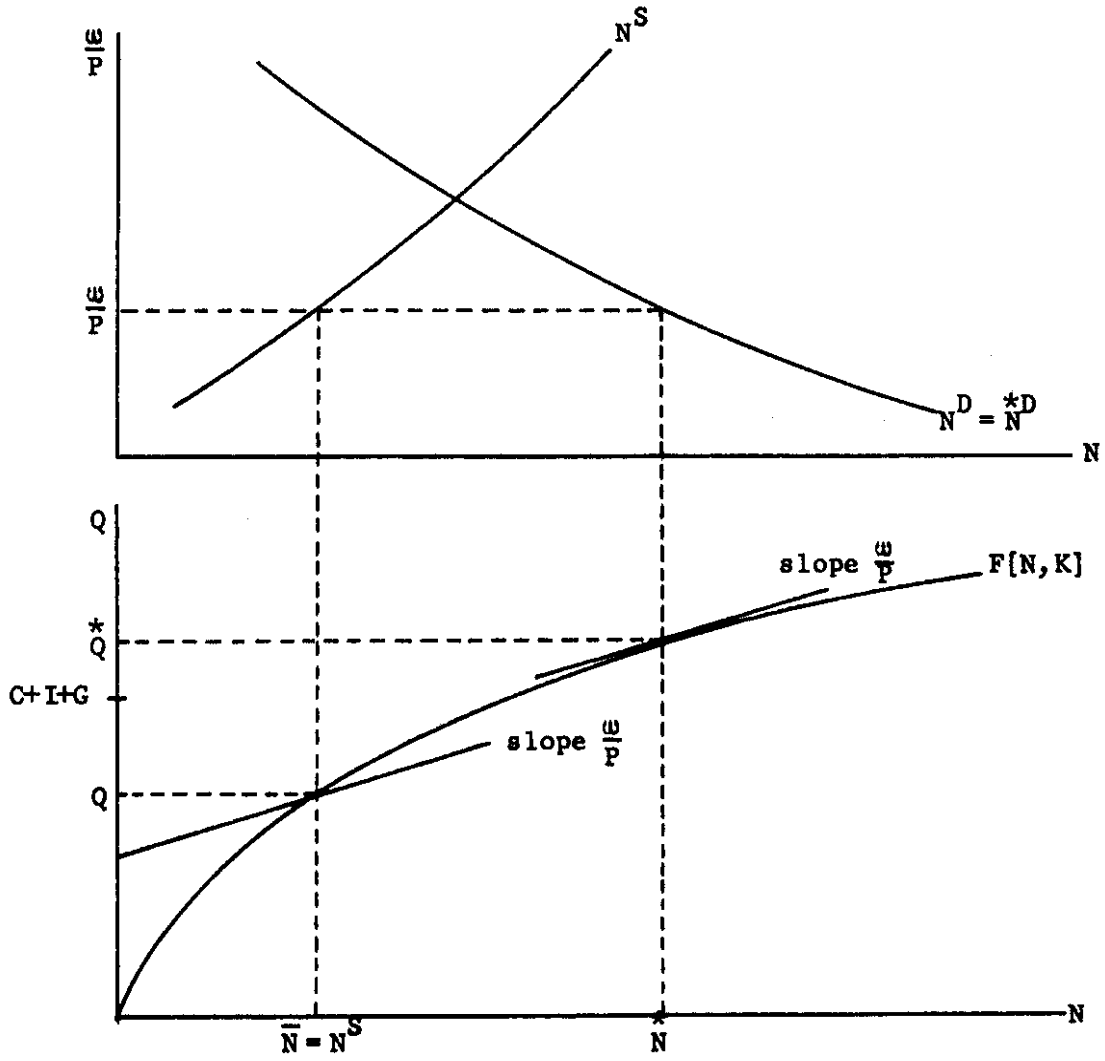
$$
(9)\qquad
\begin{cases}
Q = Q\left[\overset{*}{Q},\ \dfrac{\bar{\mathfrak{M}}}{P}^{D} - \dfrac{\bar{\mathfrak{M}}}{P}^{S}\right] \\[2ex]
\dfrac{\bar{\mathfrak{M}}}{P}^{D} = \dfrac{\bar{\mathfrak{M}}}{P}^{*D} \\[2ex]
Div \equiv Q - \dfrac{\bar{\mathfrak{M}}}{P}^{D} = Div\left[\overset{*}{Div},\ \dfrac{\bar{\mathfrak{M}}}{P}^{D} - \dfrac{\bar{\mathfrak{M}}}{P}^{S}\right] \\[2ex]
I \equiv P_E(E^S - E)/P = I\left[\overset{*}{I},\ \dfrac{\bar{\mathfrak{M}}}{P}^{D} - \dfrac{\bar{\mathfrak{M}}}{P}^{S}\right] \\[2ex]
\bar{I} = I - (1-\lambda)(C+I+G-Q) .
\end{cases}
$$

Unfulfilled investment demand is assumed here to affect only the supply of equity. A natural assumption regarding the spillover from the unrealized labor demand to commodity supply is that supply is determined by the production function using realized employment

$$Q = F[N^S, K]$$

$$Div = Q - \frac{\omega}{P}N^S \ .$$

This would not necessarily be true if we introduced inventories as buffer stocks or for speculative hoarding.



Notice that again actual output is less than notional output $\overset{*}{Q}$, and the marginal product of labor is greater than the real wage. The problem now is not a hesitancy to increase output when aggregate demand is slack but rather an inability to find sufficient employees.

Turning to investment, with fully elastic expectations

$$q = \frac{F[N^S, K] - \frac{w}{P}N^S}{rK}$$

so that an increase in the supply of labor will increase dividends and q

$$\frac{\partial q}{\partial N^S} = \frac{F_N - \frac{w}{P}}{rK} > 0$$

while an increase in real wages has an ambiguous effect

$$\frac{\partial q}{\partial \frac{w}{P}} = \frac{\left(F_N - \frac{w}{P}\right)\frac{\partial N^S}{\partial \frac{w}{P}} - N^S}{rK} \gtrless 0 .$$

An increase in real wages directly raises labor costs at the expense of dividends; however, the labor supply also expands raising dividends. The labor supply is probably usually sufficiently inelastic so that $\partial q/\partial(w/P) < 0$. With this assumption

$$I = I\left[\bar{r}, \frac{\bar{w}}{\bar{P}}, \bar{N}\right] .$$

The complete model is now

$$(10)\qquad \begin{cases} \frac{\bar{w}}{P}\bar{N} = \frac{\bar{w}}{P}N^S\left[\frac{\bar{w}}{P},\ C+I+G-Q\right] \\ \bar{C}+\bar{I}+G = Q \\ \ell\left[\bar{r},\ \bar{P},\ \frac{\bar{w}}{P},\ Div,\ C+I+G-Q\right] = \frac{M^S}{P} \end{cases}$$

where

$$Q = F[\bar{N},\ K]$$

$$C+I+G = \overset{*}{C}\left[\bar{r},\ \bar{P},\ \frac{\bar{w}}{P},\ Div\right] + I\left[\bar{r},\ \frac{\bar{w}}{P},\ \bar{N}\right] + G$$

$$\bar{C} = \overset{*}{C} - \lambda(C+I+G-Q)$$

$$\bar{I} = I - (1-\lambda)(C+I+G-Q)\ .$$

Substituting out Q, these two equations (11) determine $\bar{N}$ and r,

$$(11)\qquad \begin{cases} \bar{N} = N^S\left[\frac{\bar{w}}{P},\ \overset{*}{C}\left[r,\ P,\ \frac{\bar{w}}{P},\ Div\right] + I\left[r,\ \frac{\bar{w}}{P},\ \bar{N}\right] + G - F[\bar{N},K]\right] \\ \ell\left[r,\ P,\ \frac{\bar{w}}{P},\ Div,\ \overset{*}{C}\left[r,\ P,\ \frac{\bar{w}}{P},\ Div\right] + I\left[r,\ \frac{\bar{w}}{P},\ \bar{N}\right] + G - F[\bar{N},K]\right] = \frac{M^S}{P}\ . \end{cases}$$

The first equation implies a (normally) positive relationship between $\bar{N}$ and r:

$$\frac{\Delta\bar{N}}{\Delta r} = \frac{\frac{\partial N^S}{\partial(C+I+G-Q)}\left(\frac{\partial \overset{*}{C}}{\partial r} + \frac{\partial I}{\partial r}\right)}{1 - \frac{\partial N^S}{\partial(C+I+G-Q)}\left(\frac{\partial I}{\partial \bar{N}} - F_N\right)}\ .$$

An increase in the interest rate reduces excess commodity demand, increasing employment since the supply of labor is inversely related to the amount

of excess commodity demand. If there are no spillovers from commodity market disequilibrium to the labor supply, then the level of employment depends only upon the real wage

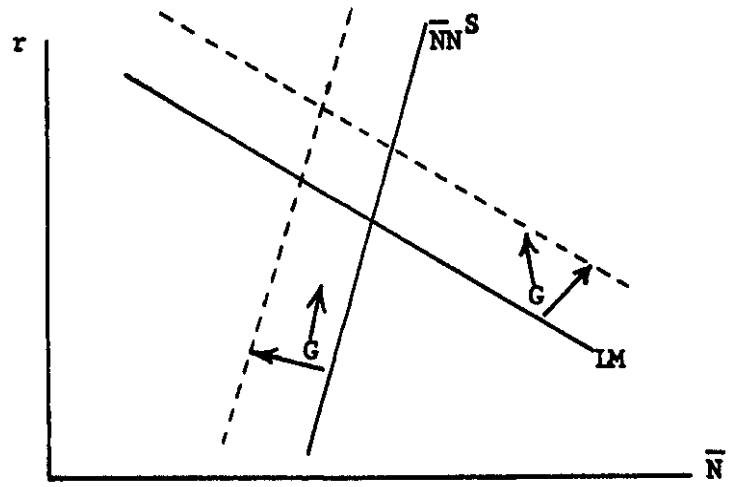
$$\bar{N} = \overset{*S}{N}\left[\frac{\omega}{P}\right] , \quad \frac{\Delta\bar{N}}{\Delta r} = 0 .$$

The equilibrium of the demand and supply of money implies a second relationship between $\bar{N}$ and r :

$$\frac{\Delta\bar{N}}{\Delta r} = \frac{\overset{-}{\frac{\partial \ell}{\partial r}} + \overset{?}{\frac{\partial \ell}{\partial(C+I+G-Q)}}\left(\overset{*}{\frac{\partial C}{\partial r}} + \overset{-}{\frac{\partial I}{\partial r}}\right)}{\left(F_N - \frac{\partial I}{\partial N}\right)\overset{?}{\frac{\partial \ell}{\partial(C+I+G-Q)}}} .$$

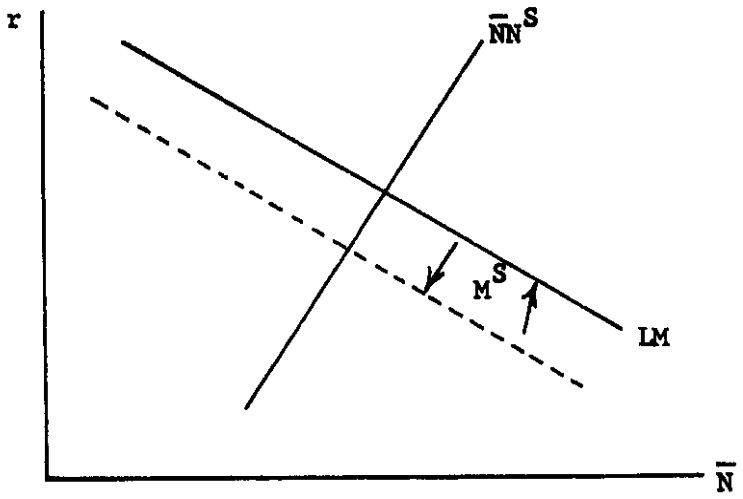
If $\partial \ell/\partial(C+I+G-Q)$ is positive, then the relationship is positive. An increase in employment reduces the excess demand for commodities and therefore the spillover into money demand. Money demand is increased directly by a fall in r and also indirectly since a decline in r increases the excess demand for commodities and some of this will spill over into money demand.

The intersection of these two relations is displayed below.



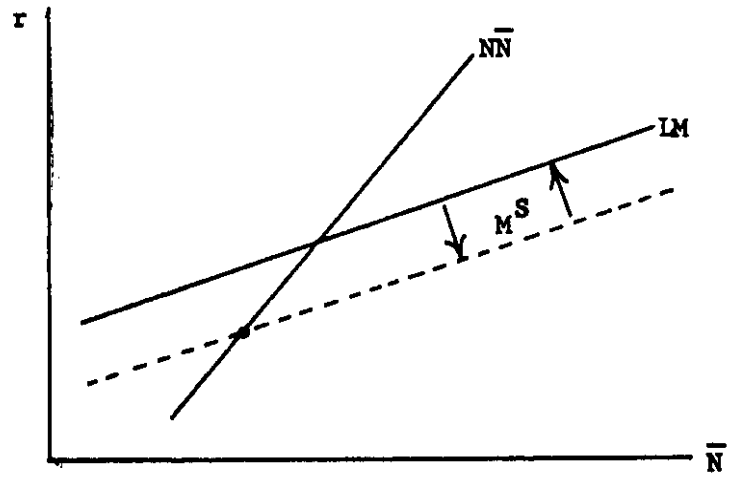
The indicated bond financed increase in government spending increases r and has an ambiguous effect on $\bar{N}$. Intuitively, the excess demand for commodities is exacerbated which spills over into a reduced labor supply (contractionary) and an increased money demand (expansionary).

An increased money supply reduces r and $\bar{N}$, since the increased

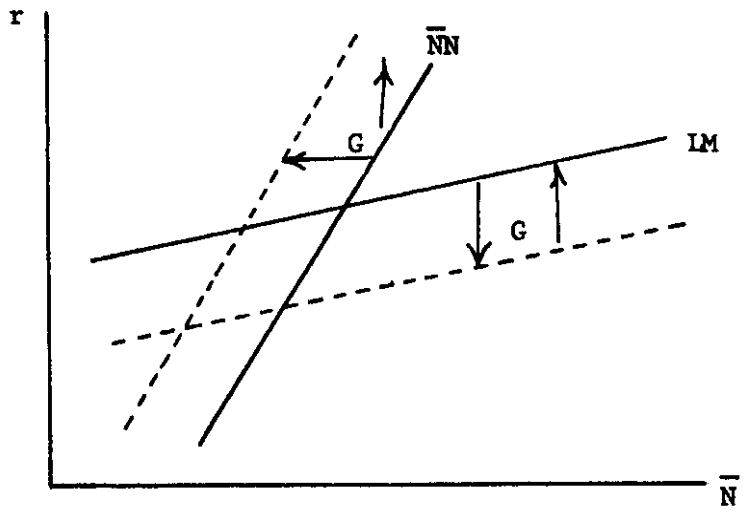


excess demand crowds out some consumption and reduces the labor supply.

If instead $\partial \ell / \partial (C + I + G - Q)$ is negative, then the LM curve may become positively sloped. For illustrative purposes, we will draw it flatter than the $\bar{N}\bar{N}$ curve. An increase in the money supply will again



reduce r , increasing the excess demand and reducing labor supply. An increase in government spending also reduces employment by increasing



excess demand directly and indirectly through the lower r that is required to offset the reduced demand for money.

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