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MODELING TECHNOLOGICAL CHANGE:

USE OF MATHEMATICAL PROGRAMMING MODELS IN THE ENERGY SECTOR

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by

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I. *Introduction*

As the United States prepares for a radical new departure in its energy policy, it is useful to take a careful look at the underlying technological assumptions. With the exception of nuclear power, most of the basic *processes* which are used in energy are several decades old: crude petroleum, natural gas, and coal form 90 percent of energy use, yet they are fuels which date to the 19^{th} century. It is now widely accepted that over the next 25 to 50 years, oil and gas will decline, and coal will either decline or be used in ways which require further processing (such as gasification). The energy sector, therefore, faces over the next decade the likelihood of a major shift in the fuels used at the very center of its processes.

From an economic point of view, the major fact about the new processes which will be used is that they are at the present time unproven at a societal scale. There simply are no clean, generally acceptable, technologies which both meet projected demands and have been demonstrated on a large scale. The time of reckoning, it should be emphasized, is neither immediate nor immutable, as there remain considerable reserves of oil and gas in the ground; moreover, we can temporize by using limited or not-completely-acceptable resources. But we cannot avoid the conclusion that a technological transition must be made over the next few decades.

In the present paper we focus on a specific aspect of the transition: the fact that development of new technologies (R&D for short) is risky, expensive, and contains potential inefficient levels. The fact that R&D is *expensive* means that a significant economic choice must be made: for example, should we postpone development of the breeder *option* because of the cost-benefit ratio? The *risky* nature of R&D is a reflection of the fact that the outcome of R&D is uncertain: thus the future cost of solar or fusion energy is extraordinarily uncertain.

Technological change may be treated as an economic activity in that it creates an output--new basic and applied knowledge--of value to society, and in doing so, employs inputs--skilled manpower, research materials, and equipment--which are scarce and have alternative social uses. Like all economic activities, technological change should be expanded until the additional social benefits yielded by its expansion no longer exceed the additional social costs, including among costs the opportunity cost of the inputs, i.e. their value to society in alternative use. The ability of a market economy to conform to this criterion depends upon: sufficient divisibility of inputs and outputs in the production of technical change; reasonable foresight about market costs; and a correspondence between the technology's social costs and benefits and those accruing to the producing firm. If these conditions should hold, an industry of competitive research firms producing and selling technology to an industry of competitive manufacturers would be able to provide knowledge and inventions in the amounts desired by society as a whole.

In the provision of technological change, it is unlikely that the assumptions required for efficient competitive organization of markets will be fulfilled. The locus of the difficulty lies in the technological nature of knowledge. Such knowledge is generally expensive to produce but cheap to reproduce; hence it is difficult for its producer to profit from the production. Examined more closely technological knowledge resembles other public goods and natural monopolies in exhibiting several economic characteristics which produce market failure. The sources of the market failure are: (1) inappropriability of the product by the producer; (2) the presence of non-insurable uncertainty; and (3) indivisibilities in inputs and in products. The first two aspects of energy technology pose no serious analytical problems, and have been treated elsewhere. In the present paper we discuss the third problem as the most likely cause of market failure in the energy sector.

Development of new technologies involves significant economies of scale because of the indivisibility or "setup cost" of once-and-for-all costs of performing the research, testing equipment, building small scale and prototype projects, rejecting uneconomic designs and so forth. Once the R&D has been performed, however, the benefits of the new technology may be obtained by others at much lower costs by the subsequent firms or nations than those incurred by the originator. For this reason, it is said that new technologies are *expensive to produce but cheap to reproduce*.

Indivisibilities in the production of knowledge follow from the fact that knowledge is expensive to produce and seldom is produced in single bits. Typically, introducing a new product or process to the market requires substantial expenditures. A number of small firms might in theory competitively supply different components of a new process or piece of equipment to one another; in practice, however, the work of scientists, technicians, and engineers requires coordination within a single enterprise of communication and command.

A further implication of the economies of scale in the production of technological change is that there will generally be a "funding problem." The funding problem arises because the marginal cost of using technological change is well below its average cost, so that either R&D must be priced inefficiently high, or R&D must be subsidized by other goods. In the latter case, a free-rider problem and possibilities for "cream-skimming" arise if the cross-subsidization is not set carefully. This is an extremely important issue for technological change.

A more general form of economy of scale in technology is the concept known as "learning by doing." The concept of learning by doing has widespread application in social and economic behavior. One of the most important is the area of research and development of new technologies. In its narrowest sense, learning by doing refers to the observed phenomenon that average costs fall as a function of cumulative output of a product; more broadly, the process of research and development can also be thought of as a learning process.

In the early studies of airframe manufacture, the following relation was found to hold:

$$\log(Cost) = a - b \log(Q) , \quad b = .3$$

where cost = average cost of production and Q is cumulative output of airframes. This relation has been tried in other areas and is thought to be representative of the way that learning affects productivity.

The phenomenon of learning by doing has important economic implications because it is a form of dynamic economy of scale.* As is well

*See Arrow (1962).

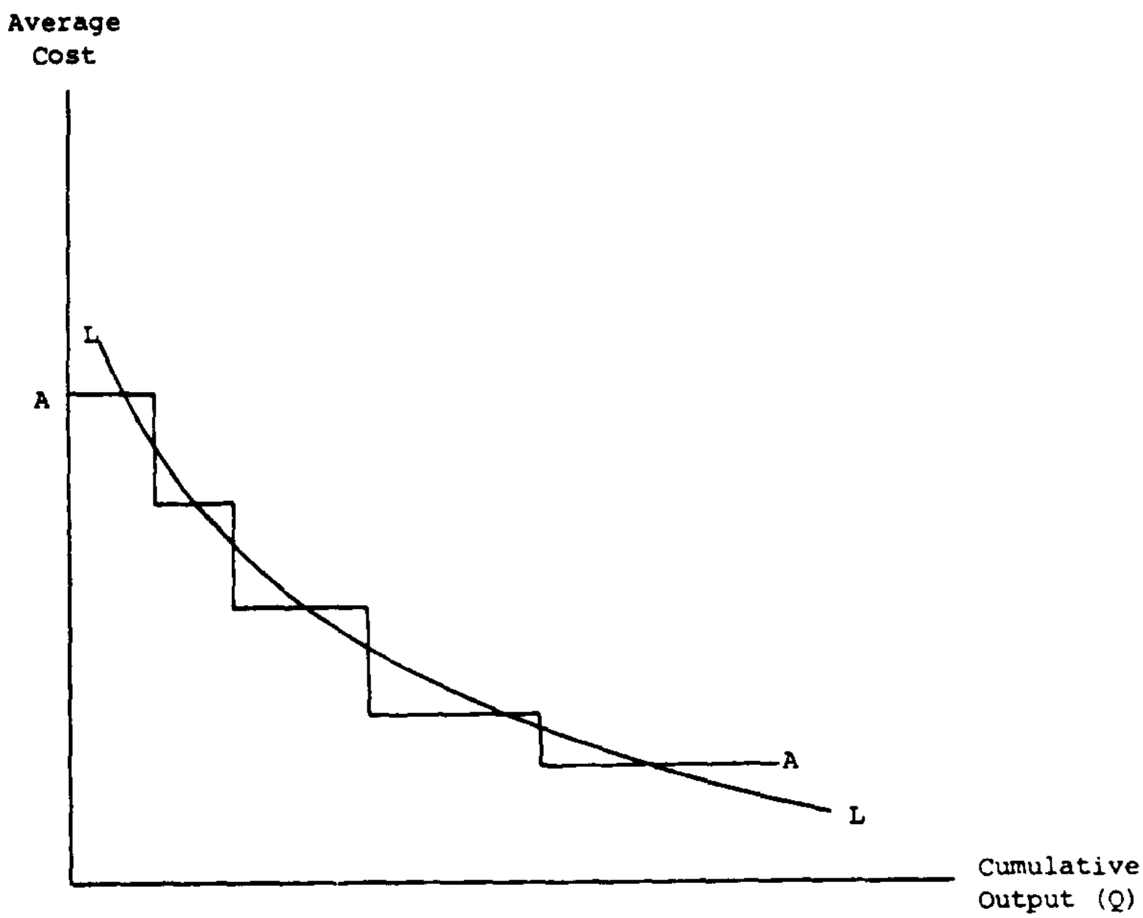

FIGURE 1.1. A "true learning curve," LL , and its stepwise linear approximation, AA .

known, under conditions of economies of scale, competitive economies will generally function inefficiently. It is thus important to analyze how competitive or other allocations will differ from efficient allocations. It is most likely that divergences between market behavior and efficient behavior will occur in the earliest stages of a product life (especially before full-scale production occurs).

Consider the following simplified example. Figure 1.1 shows a "true" learning curve as line LL which represents the average cost of production as a function of cumulative output. In order to use mathematical programming, we have shown as an approximation to the "true" learning curve the line AA , which is a step function. Introducing the linear approximation would be a straightforward problem in linear programming, using the linear approximation to the non-linear function, except that the cost function is not a concave function. This of course is the mathematical reflection of the difficulty that the market mechanism has in establishing an efficient solution.

How can we solve a problem involving learning using existing computational techniques? It is helpful to formulate the problem in static terms, and to simplify the problem by approximating the learning function by only two steps. The problem can be visualized in Figure 1.2. We have shown two processes, one an old, established process with constant cost, c_1 . The new process is described as having a setup cost, c_0 , after which the marginal cost is c_2 . Thus average cost for process 2 is $c_2 + (c_0/x_2)$.

We can then write an example as follows:

$$\min_{\{x_1, x_2, \delta\}} 2x_1 + x_2 + 5\delta$$

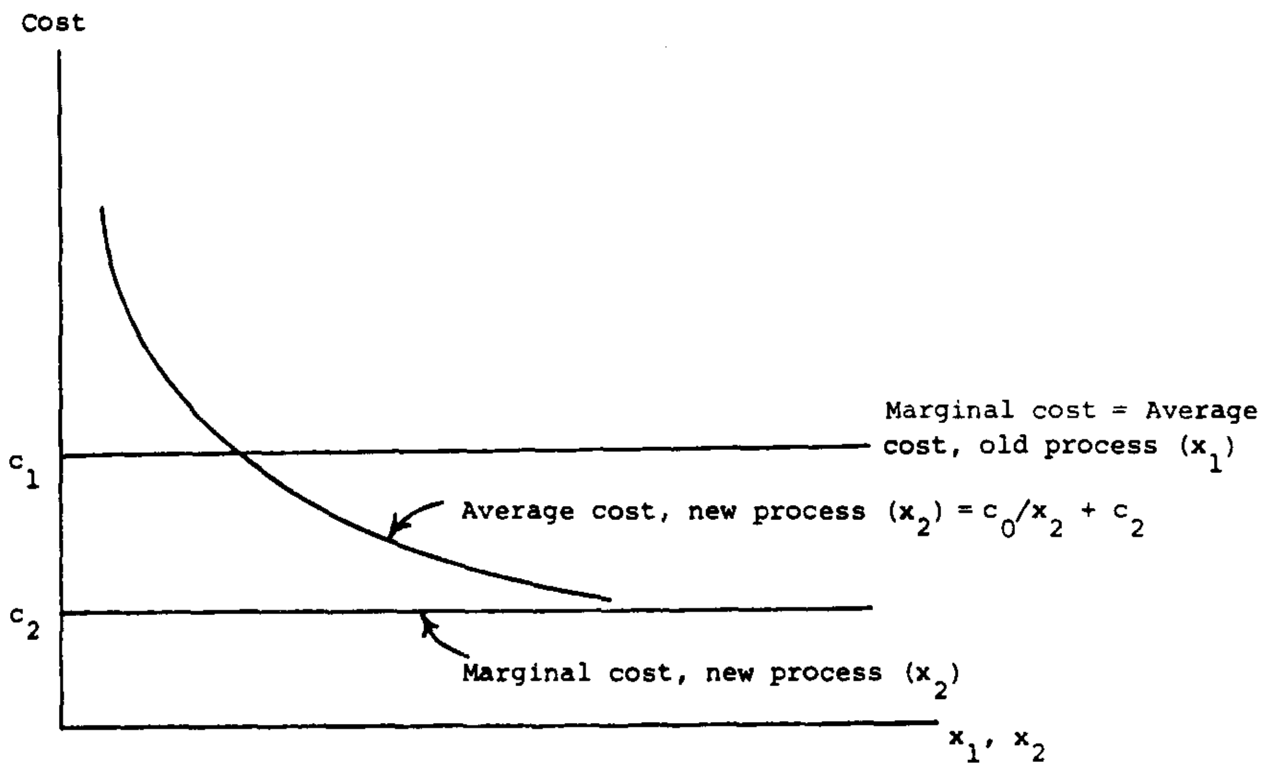

FIGURE 1.2. A reformulation of the learning by doing problem
(a) in static (or one-period) model, and
(b) with only two steps

where

$$x_1 + x_2 \geq b$$

$$\delta = \begin{cases} 0 & \text{if} \quad x_2 = 0 \\ 1 & \text{if} \quad x_2 > 0 \; . \end{cases}$$

Clearly the solution will be to use only x_1 if $b < 5$ and only x_2 if $b > 5$. The problem can easily be formulated to be an integer programming problem with δ as the integral variable. It is obvious that for $b < 5$ the associated linear program (ignoring the variable δ) will be incorrect.

In the present study, we have developed a technique for analyzing the development of new technologies, with special reference to the energy sector. Section II discusses in more detail the problem of modeling R&D in mathematical programming models. Section III discusses the use of integer variables. Sections IV and V describe the computer runs and computational experience. Section VI presents the results.

II. Modeling R&D with Integer Variables

Our starting point for examining the R&D issues is the energy model developed by Nordhaus,* which we briefly sketch here. The model is a *mathematical program*** performing an optimization designed to simulate the functioning of a competitive market for energy goods. It can be represented as:

$$\text{(P1)} \quad \begin{aligned} & \max \quad U(x') - C(x') \\ & \text{subject to (s.t.)} \quad F(x') \leqq R' \ . \end{aligned}$$

In this problem x' is the vector of inputs and outputs of the energy sector (e.g. extraction and processing of fossil and nuclear fuels, energy consumption, ...), $U(x')$ is the preference function, $F(x')$ is the cost function, R' is the vector of parameters representing resource endowment and other constraints, and $C(x')$ is the cost function. As is the case with most non-linear programs of reasonable size, a linear approximation is sought which will be computationally tractable:

$$\text{(P2)} \quad \begin{aligned} & \max \quad r \cdot x - c \cdot x \ , \\ & \text{s.t.} \quad Ax \leqq R \ . \end{aligned}$$

In this process of approximating non-linear functions, supplementary activities and constraints might have to be introduced, which we note by distinguishing the decision variables in the programs (P1) and (P2).

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*For a more detailed description of an early version of the model, we refer the reader to Nordhaus (1973). The version used in our analysis is described in Nordhaus (1977b).

**For more details on mathematical programs, see Section III.

The preference function is derived from market demand data, while the technology, or constraint set, and the cost function are derived from engineering and geological data on the resource availability, the costs of extraction, transportation, and conversion. This activity analysis model is then solved by a linear programming algorithm, generating an output in terms of the activity levels (e.g. the production of coal in a given period), as well as the value of the dual variables (to be interpreted as shadow prices, opportunity costs, or, in a competitive framework, as the simulation of competitive prices). Finally, we add that a *horizon* has to be determined for the optimization. The concept that is relevant to this question is the *'backstop technology,'** which is defined as a set of processes capable of meeting demand requirements and which has virtually infinite resource base (e.g. solar electricity). Ultimately, the economy will be based on these abundant resources, prices will remain stable and the periods beyond the backstop's full introduction can be ignored in the computation. The horizon T is then determined so as to have all energy produced with the backstop technology at the end of period T.

This mathematical programming representation of the economy must be somehow extended if it is to apply to R&D assessment. Up to now, the standard technique for analyzing the benefits of government-sponsored R&D has been the very laborious one of inserting new techniques into the old constraint set one by one and then examining the resulting solution. This technique is extremely expensive from a computational point of view, since the combinatorial possibilities with even 10 different R&D projects

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*This concept was originally introduced in Nordhaus (1973).

are very large. As a result, very few options are examined. In addition, the standard approach ignores the fact that at least some R&D projects will be performed by industrial sources if they are profitable from a private vantage point.

To be more specific, we consider the introduction of R&D activities. These are best interpreted as special variables, d, which make available new activities, y, in the technology. Thus, as in the case described above, the research and development on the LMFBR,* if successful, would add a new method for generating electricity and producing plutonium as a by-product. Thus we modify the problem analyzed above as follows. The new problem is:

$$\max_{\{x,y,z\}} \; v \cdot x - c \cdot x + w \cdot y - d \cdot z$$

$$\text{(P3)} \quad \begin{array}{lll} (1) & s.t. \; Ax + By & \leqq R \,, \\ (2) & \quad Dy - Ez & \leqq S \,, \\ (3) & \quad z = (0 \text{ or } 1) \,. & \end{array}$$

In this formulation, the decision problem adds two new activities, y and z which have unit cost w and d, respectively. In addition, the constraints have been augmented to reflect the new activities.

With respect to the integral nature of the decision, the research and development activities can be seen as buying a new tecynology and for this reason they are inherently lumpy in nature. It does no good to produce only the fuselage of a new airplane, or the core-cooling com-

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*LMFBR = Liquid Metal Fast Breeder Reactor.

ponent of a new reactor. The entire research and development must be performed before the new technology can become available. Together with (3), (2) indicates whether the new activity y is available or not. If $z_i = 0$, then the R&D is not performed and constraints in (2) will imply that the new technology or activity is not used. On the other hand, if $z_i = 1$, the R&D is performed and from (2) we know that the technology is available up to certain limits. Note that the cost of R&D, d, is a once-and-for-all or setup cost.

To illustrate what sort of inequalities define the constraint set, we consider in more detail the modeling of the electricity generation by breeder reactors, which is one of the nuclear subactivities. This example will also explain how integer variables are introduced to specify the R&D opportunities. First consider how we might model the introduction of new tecynologies when R&D is not explicitly considered. Let $y_{k,t}$ be the electricity generated by breeder reactors in period t, $a(k)$ be the earliest period the activity is securable, and let A_k be its availability in that period. Then the time path for this activity is described as follows:

(4) $$y_{k,a(k)} \leqq A_k ,$$

(5) $$-g_k \cdot y_{k,t-1} + y_{k,t} \leqq \beta A_k ,$$

for $$t = a(k)+1,\ a(k)+2,\ \ldots,\ T ,$$

where g_k is the maximal rate of growth of production of the technology, β is a small but positive number permitting a postponement in the introduction of the breeder tecynology. The inequalities thus allow for an exponential growth of the resource at rate g_k, with a "lump" of technology becoming available at period $a(k)$.

Next consider how we explicitly introduce the "production" of new technologies. Our aggregative approach to R&D modeling is to introduce integer variables $z_{k,t}$ equal to 1 if the R&D on technology k is completed by period t and equal to 0 otherwise. Technically R&D on technology   comprises prototype and early commercial plants up to an installed capacity at level R_k . The inequalities (4) and (5), characterizing the evolution of activity k over time, are therefore replaced by:

$$y_{k,a(k)} - R_k \cdot z_{k,a(k)} \leq 0 ,$$

$$-\gamma_k \cdot y_{k,t-1} + y_{k,t} - R_k \cdot z_{k,t} \leq 0 \quad \text{for} \quad t = a(k)+1,\ a(k)+2,\ \ldots,\ T ,$$

$$z_{k,t} = 0 \text{ or } 1 .$$

An additional constraint has to be introduced reflecting the fact that R&D is carried out only once:

$$\sum_{t=a(k)}^{T} z_{k,t} \leq 1 .$$

In the technique used here, there is assumed to be no uncertainty about the success of the R&D attempt or in the costs involved. If we recognize that such uncertainty exists, our analysis defines a *state of the world* as one resolution of these uncertainties. The purpose of the R&D analysis is then to investigate the best possible decision *given* a state of the world. This information, together with judgmental probability estimates of the uncertainties, serves as input to obtain the distribution of benefits of each of the decisions on R&D. This technique is more fully described in the MRG report (1977) and in Nordhaus (1977a).

The introduction of endogenous R&D activities makes the constraint set *nonconvex*, which poses serious computational problems, in that the standard linear programming techniques will not succeed in finding optimal solutions. Therefore, the solution of the R&D problem requires different computational techniques, as will be explained in Section III.

III. An Introduction to Mathematical Programming with Integer Variables

For the purposes of this paper, *mathematical* programs can be defined as the maximization of a real-valued function over a subset of R^n. It can be formulated as:

$$\text{(P4)} \qquad \begin{array}{ll} \max & f(v)\,, \\ s.t. & v \in V\,. \end{array}$$

The function f is the *objective function* of the program and V the *constraint set*. Any element $v \in V$ is called a *(feasible) solution*. We are looking for an *optimal* solution, that is for any feasible solution v^* satisfying:

$$f(v^*) \geq f(v) \quad \text{for all} \quad v \in V\,.$$

An important concept in mathematical programming is that of *relaxation*, whereby instead of considering (P4) we consider the relaxed program:

$$\text{(P5)} \qquad \begin{array}{ll} \max & f(v)\,, \\ s.t. & v \in W\,, \end{array}$$

with $V \subset W \subset R^n$. It should be clear that by solving program (P5), one obtains an optimal solution v^*_W satisfying:

(6) $$f(v_W^*) \geq f(v_V^*)$$

where v_V^* is the optimal solution to (P4). Note also that v_W^* is an optimal solution for (P4) if $v_W^* \varepsilon V$, that is, if v_W^* is feasible in the original program.

The most widely used representative of this class of programming problems is the *linear program*, where V is a polyhedral subset of R^n and f is a linear function:

$$f(v) = c \cdot v \text{ ,}^*$$

$$V = \{v | Av \leq b\} \text{ ,}$$

where c is a vector in R^n, A is a matrix and b is a vector whose dimension equals the number of rows of the matrix A. Efficient algorithms exist for this problem, and these linear programs can be solved numerically for "large" programs where "large" refers to the column or row size of A. Unfortunately, most *nonlinear programs* are much harder to solve numerically, and often one introduces an *iterative* procedure where one solves the nonlinear program by *successive approximations* using linear programs. This will be the case for the programs obtained in Section II, called *mixed integer linear programs* (MILP). This terminology is due to the fact that the decision variables are a combination of integer and continuous variables, with the integrality requirements constituting the only nonlinearity of the program. Therefore any MILP can be written:

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* $c \cdot v$ = inner product of c and $v = \sum_{i=1}^{n} c_i v_i$.

$$\max_{\{y,z\}} a \cdot y + b \cdot z \,,$$

(P6) $$\text{s.t.} \quad A \cdot y + B \cdot z \leqq c \,, \quad y \geqq 0 \,, \quad z \geqq 0 \,, \quad z \text{ integer.}$$

Letting $S = \{z \mid \text{there exists a vector } y \text{ s.t. } (y,z) \text{ is feasible for (P6)}\}$, we denote the cardinality of the set S by $|S|$.* It is reasonable to assume that each integer variable z_j is bounded above by some integer u_j, so that z_j can take at most (u_j+1) different values. Hence $|S|$, the number of possible values for the vector z, is finite and smaller than or equal to $\prod_j (u_j+1)$. We could thus imagine solving the MILP by enumerating *explicitly* the $|S|$ possible values of the integer vector z and considering the linear programs resulting from (P6) by assigning successively the $|S|$ different values to z. This approach is impractical due to the large values $|S|$ usually takes. An *implicit* enumeration scheme is therefore proposed. The result of the enumerative process is more easily described if related to a *tree*, consisting of *nodes* and *branches*. Each node represents a subset S_j of the set S, with the first node of the tree, S_0, representing the set S itself. From any node S_j (we identify nodes with the subsets of S they represent), we determine its successors $S_{j_1}, S_{j_2}, \ldots, S_{j_m}$ so that they represent a *separation* of S:

$$S_j = \bigcup_{k=1}^{m} S_{j_k} \,,$$

*The cardinality of a set = the number of elements in the set.

and we draw a branch from node S_j to every node S_{j_k} . This procedure is illustrated in Figure 3.1, where $S = S_0 \cup S_1 \cup S_5$, $S_1 = S_2 \cup S_3 \cup S_4$, and $S_5 = S_6 \cup S_7$.

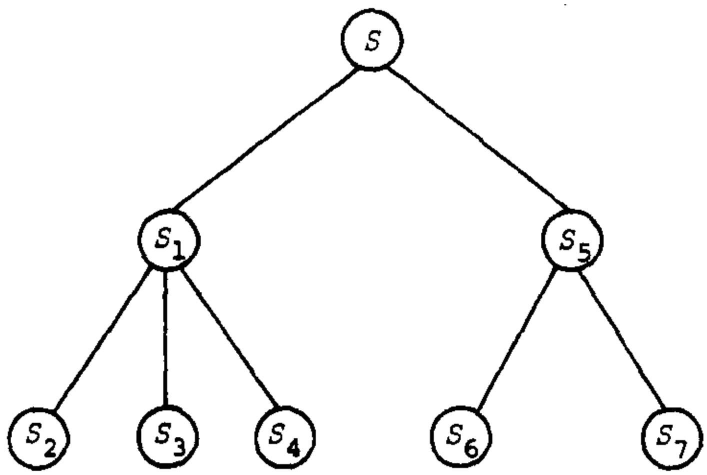

FIGURE 3.1

Illustration of a tree used for the separation of a set S

Enumerating the values of the integer variables implicitly, we solve a sequence of linear programs to reach the optimal solution of the MILP by successive approximations. We start with solving the linear program obtained from (P6) by relaxing the integrality constraints and considering the set:

$$T = \{(z, y) : A \cdot y + B \cdot z \leqq c, \; z \geqq 0, \; y \geqq 0\} .$$

Assume that $f^0 = a\, y^0 + b\, z^0$, where

$$a \cdot y^0 + b \cdot z^0 = max\{a \cdot y + b \cdot z \mid (y, z) \in T\} .$$

If z^0 is integer, we solved our MILP. If one of the components of z^0 , say z_j^0 , has a fractional value, we enumerate implicitly the values taken by z_j by considering the sets:

$$T_1 = \{(y,z) \mid (y,z) \in T,\ z_j \leqq [z_j^0]\}\ ,^*$$

$$T_2 = \{(y,z) \mid (y,z) \in T,\ z_j \geqq [z_j^0] + 1\}\ .$$

Hence two branches sprout from node S, each branch corresponding to a constraint as is shown in Figure 3.2. Assume now that both approximating linear programs at nodes S_1 and S_2 are solved, so that we have

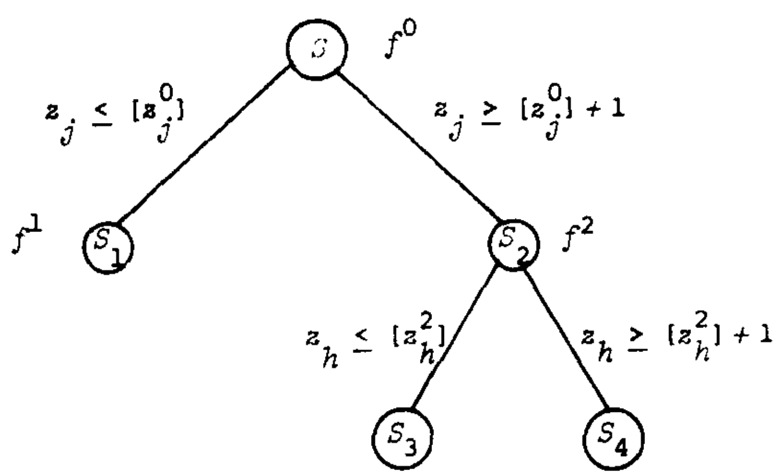

FIGURE 3.2

Illustration of the branching procedure

obtained the values f^i, where

$$f^i = a \cdot y^i + b \cdot z^i = max\{a \cdot y + b \cdot z \mid (y,z) \in T^i\} \quad \text{for} \quad i = 1,\ 2\ ,$$

and that z^1 is integer. Then node S_1 is said to be *fathomed by integrality*, by which we mean that no further exploration of that node is necessary since the solution of the relaxed problem is feasible and thus optimal for the problem considered at that node. If z^2 is not all-

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* $[x]$ = greatest integer less than or equal to x . Thus $[3/2] = 1$.

integer, then further *branching* of that node on a variable z_h with fractional value z_h^2 is necessary. Two branches are added to the tree, corresponding to the constraints $z_h \leq [z_h^2]$ and $z_h \geq [z_h^2] + 1$, and leading us to nodes S_3 and S_4, where one solves the approximate problems whose constraint sets are given by:

$$T_3 = \{(y,z) \mid (y,z) \in T,\ z_j \geq [z_j^0] + 1,\ z_h \geq [z_h^2]\},$$

$$T_4 = \{(y,z) \mid (y,z) \in T,\ z_j \geq [z_j^0] + 1,\ z_h \geq [z_h^2] + 1\}.$$

Note that there is a unique path P_j from node S to every node S_j generated in this way, and that T_j can be thought of as the intersection of the set T with the set of points satisfying the constraints given by the branches in P_j. The finiteness of this implicit enumeration procedure is proved by considering the function g, which gives an upper bound to the number of values integer variables can take at a node. Hence $g(S_0) = \prod_j (1 + u_j)$. If node S_j has two successors, S_h and S_k, then by construction of the tree

$$g(S_h) < g(S_j),$$

$$g(S_k) < g(S_j),$$

so that every path from S can contain at most $g(S_0)$ branches. Note also that when $g(S_j) = 1$, then node S_j is *fathomed by integrality*: at that point all integer variables have been assigned integer values by the algorithm.

However, during the enumeration useful information is obtained which may indicate that further exploration of certain nodes cannot produce improved solutions. Returning to the example of Figure 3.2, we assumed that z^1 was all-integer, so that (y^1, z^1) is a feasible solution to the MILP. Since T_2 is a relaxation for all the nodes emanating from S_2, we may apply (6) to conclude that f^2 is an upper bound for the value of all feasible solutions of S_2. If $f^1 \geq f^2$, no feasible solution in S_2 will provide an improved objective function value as compared to (y^1, z^1) and no further exploration of node S_2 is desired. We say that S_2 is *fathomed by bounds.*

A last remark about MILP's containing only binary integer variables, e.g. when they only take values 0 or 1. Our branching scheme is then simplified. If at a node S_k, we decide to branch on a bivalent variable $z_j = z_j^k$, the two branches emanating from S_k will definitively fix the values of z_j as is illustrated in Figure 3.3. Note also that the tree will contain paths with at most n branches, if n is the number of binary integer variables.

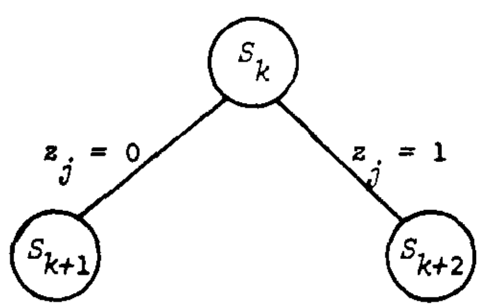

FIGURE 3.3

Illustration of implicit enumeration with binary variables

Sophisticated methods exist for the selection of a branching variable at a node and for deciding which node to branch upon next. We refer the reader interested in these matters to Garfinkel and Nemhauser (1972) or Salkin (1975), where these issues are amply discussed and illustrated. Our aim was only to illustrate a technique not often seen in the economic literature. We conclude this section with a typical run of the algorithm, shown in Figure 3.4.

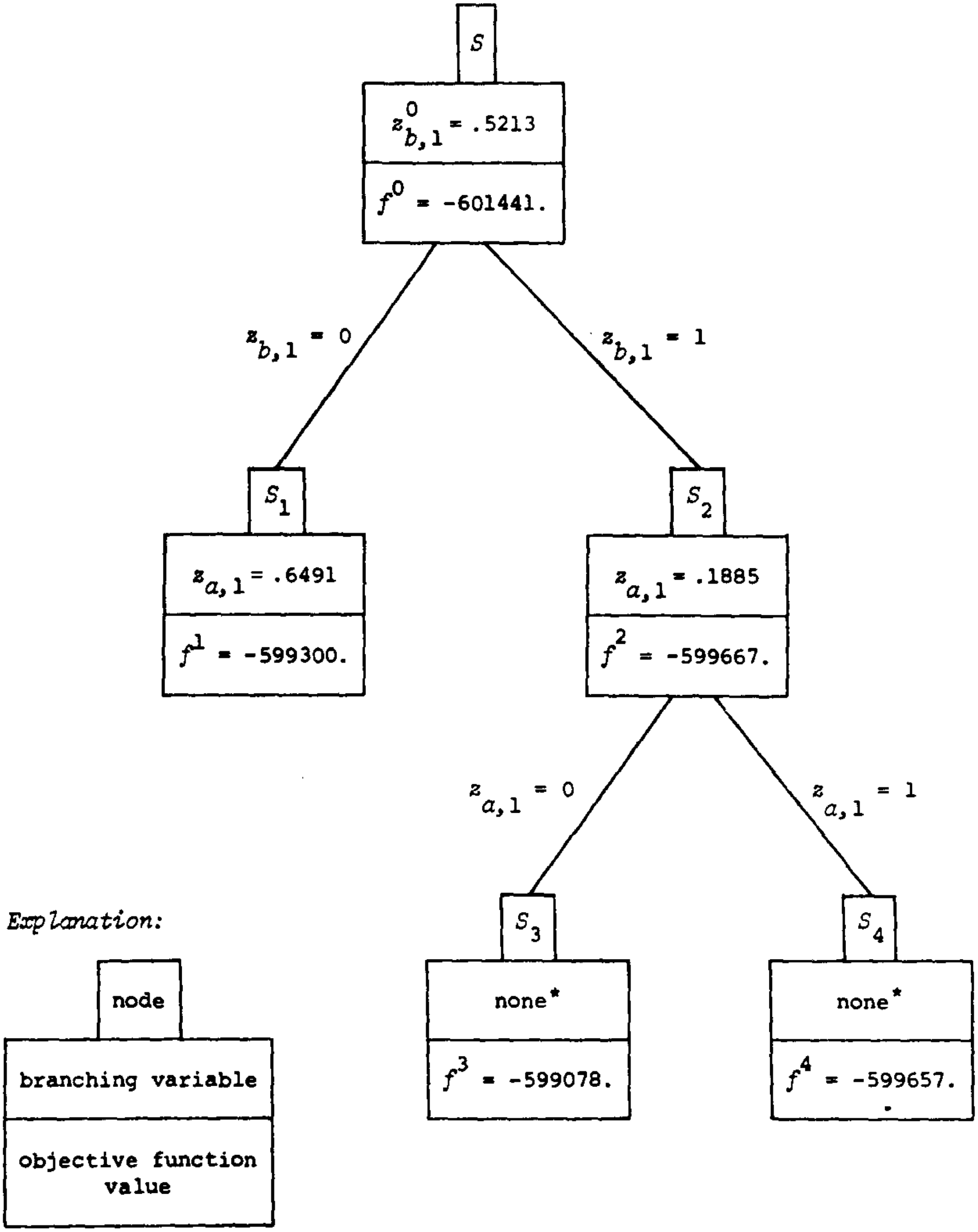

FIGURE 3.4

A typical run of the branch-and-bound procedure

*All integer variables assume integer values, so no further branching is necessary.

Example (see Figure 3.4):

Step 1. Solve the LP at node S . The solution is not feasible, hence select a branching variable, say $z_{b,1}$.

Step 2. Solve the LP's at nodes S_1 and S_2 ; since no feasible solutions (for the original MILP) are obtained, select a branching variable at one of the nodes, say $z_{a,1}$ at node S_2 .

Step 3. Solve the LP's at nodes S_3 and S_4 ; both have all-integer solutions. Hence nodes S_3 and S_4 are fathomed by integrality.

Step 4. Return to node S_1 , which is not yet fathomed. Fathom node S_1 by bounds, since $f^3 > f^1$. The optimal solution for the original MILP is given by the optimal solution at node S_3 .

IV. Description of Computer Simulations

In this section we describe the computer simulations produced during our R&D modeling work. The first series of simulations were presented to the Modeling Resource Group (MRG) and are concerned with the R&D options on two nuclear technologies only, the Liquid Metal Fast Breeder Reactor (LMFBR) and the Advanced Converter (AC). In a second series of computer runs, we broaden the issue by introducing R&D options on synthetic fuels (shale oil, coal oil, and coal gas) and on two backstop technologies (one electric and one nonelectric Alternative Energy System) as well. Finally, a last series of simulations are motivated by the consideration of *learning effects.*

If we do not take learning effects into account to begin with, the setup cost for the introduction of a new tecynology is L-shaped as shown in Figure 4.1, which tells us that the new technology is available after t_{av} time periods at a cost c_{oo} . Introducing learning effects, Figure 4.2 approximates the cost function in a smoother way. The technology now becomes available in period $t_{av} - 1$ at a cost kc_{oo} $(k > 1)$, which can be taken as the cost of a "crash" program for the technology. Expenditures decrease exponentially at learning rate ρ thereafter until they reach the level c_{oo} (costs are in undiscounted form):

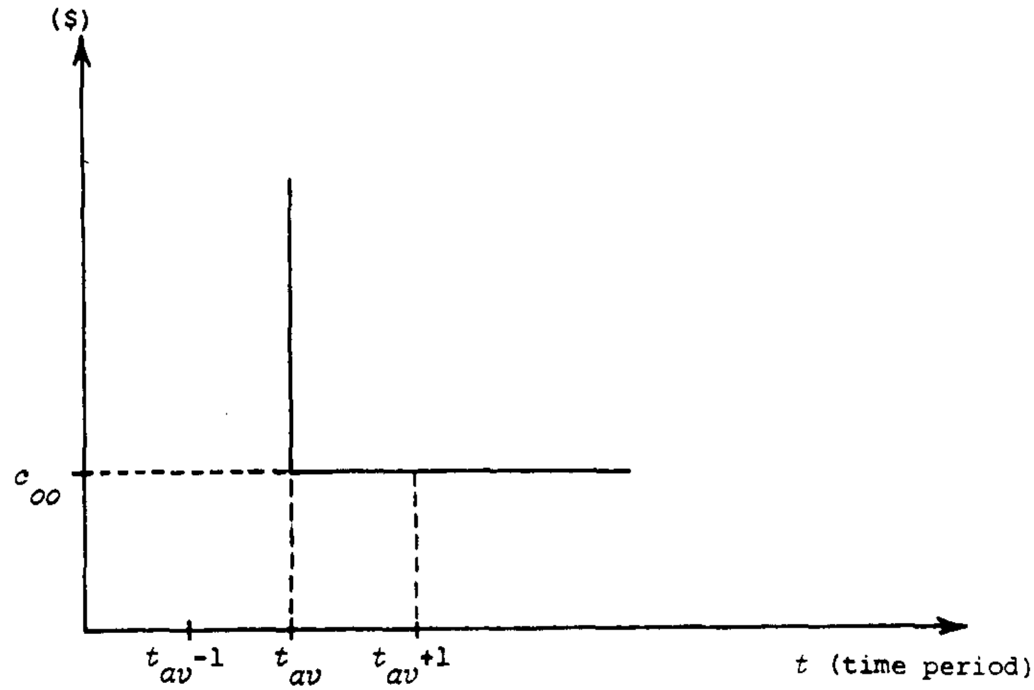

FIGURE 4.1

Setup cost for the introduction of a new technology (*without* learning)

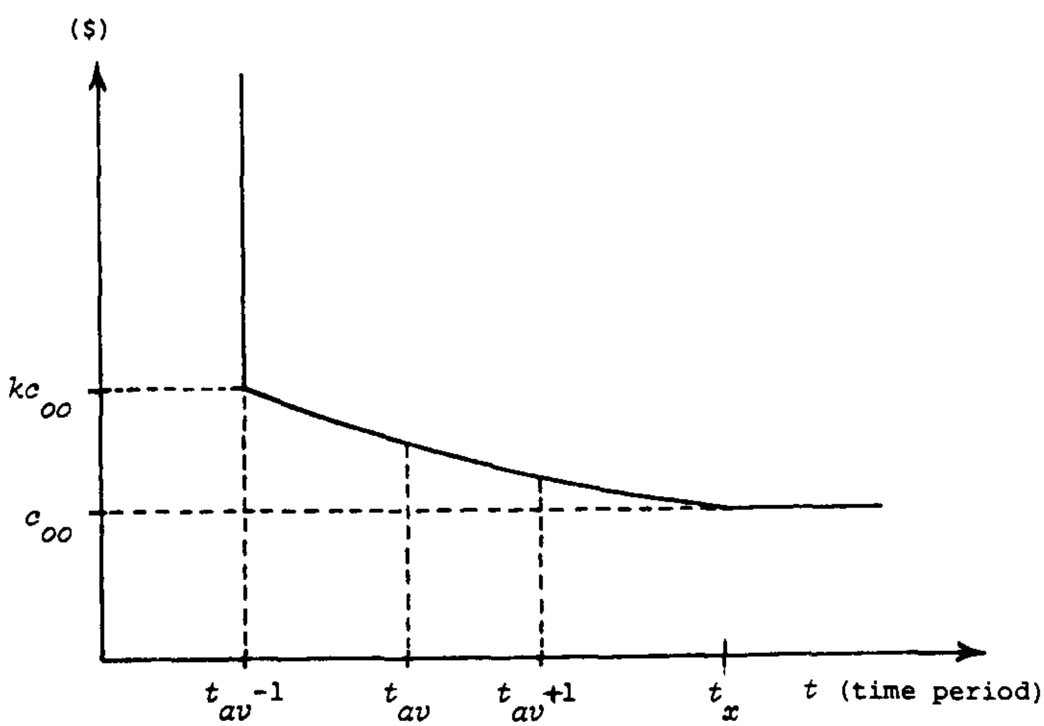

FIGURE 4.2

Setup cost for the introduction of a new technology (*with* learning)

$$c(t) = +\infty \quad \text{for} \quad t \le t_{av} - 1 \ ,$$

$$= kc_{oo}(1+\rho)^{-(t-t_{av})-1} \quad \text{for} \quad t_{av} - 1 < t \le t_x \ ,$$

$$= c_{oo} \quad \text{for} \quad t_x < t \ ,$$

time t_x being such that $k(1+\rho)^{-(t_x-t_{av})-1} = 1$.

Following the assumptions of the MRG study, the R&D cost c_{oo} comprises a development cost c_1 and a commercialization cost c_2 , occurring 15 and 5 years before the first utilization of the technology. If we denote the consumption discount rate by r (= .06 in our study), the total R&D expenditure is defined by:

$$c_{oo} = c_1(1+r)^{15} + c_2(1+r)^5 \ .$$

The numbers associated with the cost coefficients (c_1, c_2, c_{oo}) , the cost multipliers (k) , the learning factors (ρ) , and the earlier dates of availability $(1980 + 10(t_{av} - 1))$ are exhibited in Table 4.1.

We repeat that our attention is on the *optimal timing* for the introduction of new technologies. This evaluation is heavily dependent on assumptions about the realization of uncertainties like the acceptability of nuclear power, the supply of Uranium, or the rate of growth of energy demand. One approach, described by Nordhaus (1977a), consists of giving the future events a judgmental probability distribution so that the realization of these uncertainties gives rise to a decision tree whose endpoints correspond to a deterministic environment where all uncertainties have been solved. The optimal decision can then be computed using standard mathematical programming techniques.

Table 4.1. Cost Coefficients (c_1, c_2, c_{oo}), Cost Multipliers (k), Learning Factors (ρ), and Earliest Dates of Availability ($1980+10(t_{av}-1)$) of the New Technologies

Type of Technology	c_1 (billions of dollars, 1975 prices)	c_2	c_{oo}	k	ρ (per annum)	$1980+10(t_{av}-1)$
1. Backstop Technologies:						
1a. Elec AES	15.	20.	62.71	4.	.072	2010
1b. Nonelec AES	15.	20.	62.71	5.	.084	2010
2. Nuclear Technologies:						
2a. LMFBR	10.	10.	37.35	2.	.035	2010
2b. FBR	5.	10.	25.36	2.	.035	2010
3. Synthetic Fuels:						
3a. Shale Oil	5.	10.	25.36	3.	.056	2000
3b. Coal Oil	5.	10.	25.36	3.	.056	2000
3c. Coal Gas	3.	8.	17.90	2.	.035	1990

A different path is followed here. Given the experience provided by the MRG study, we distinguish five *scenarios* which sum up the extreme possibilities in the resolution of the uncertainties. Each scenario is described in Table 4.2. The base case is analogous to the base case of the MRG report (1977). Of the 8 uncertainties mentioned in the report as important to the decision-making, 4 uncertainties are considered here:

1. GNP: the rate of energy demand.
2. Coal/Shale Limits: the availability of coal and shale for large-scale future deployment.
3. Uranium: the supply curve for Uranium.
4. Nuclear Moratorium: the acceptability of nuclear reactor designs from an economic and environmental viewpoint.

Furthermore, the uncertainties take the form of binary random variables, whose judgmental probability distributions are deduced from the answers to the questionnaires circulated to a group of specialists in the CONAES study, and mentioned in the MRG report (1977).

From the probabilities given in Table 4.3 it becomes easy to compute the probability of each scenario. For example, the probability q_1 that all 4 random variables assume their base values is equal to $p_1 p_2 p_3 p_4$. Similarly, we obtain the probabilities Q_2, Q_3, Q_4, and Q_5 so that, given our assumption on the realization of one of the five scenarios, the probability P_i that scenario i is realized is equal to $Q_i / \sum_{i=1}^{5} Q_i$. The numerical computations are summarized in Table 4.4.

Table 4.2. List of Scenarios

RUN(1,J)	Base Case (see MRG Report (1977)).
RUN(2,J)	Coal/Shale Limits, Base Case otherwise.
RUN(3,J)	Coal/Shale Limits, Nuclear Moratorium, Base Case otherwise.
RUN(4,J)	Coal/Shale Limits, High GNP, Low Uranium, Base Case otherwise.
RUN(5,J)	High GNP, Low Uranium, Base Case otherwise.

$J = 1$: R&D on nuclear technologies only,
No learning.

$J = 2$: R&D on the seven technologies mentioned in Table 4.1,
No learning.

$J = 3$: R&D on the seven technologies mentioned in Table 4.1,
Learning.

Table 4.3. Probability Distributions of the Uncertainties

1. GNP

    1a. Base value: $p_1 = 0.84$

    1b. High value: $1 - p_1 = 0.16$

2. Coal/Shale Limits

    2a. Base value (No): $p_2 = 0.38$

    2b. Yes: $1 - p_2 = 0.62$

3. Uranium

    3a. Base value: $p_3 = 0.15$

    3b. Low: $1 - p_3 = 0.85$

4. Nuclear Moratorium

    4a. Base value (No): $p_4 = 0.60$

    4b. Yes: $1 - p_4 = 0.40$

Table 4.4. Probabilities of Realization of Each Scenario

0 = Base value, 1 = Otherwise

Scenario	*Values of Binary Random Variables*	Q_i	P_i
1	(0,0,0,0)	.0287	.152
2	(0,1,0,0)	.0469	.249
3	(0,1,0,1)	.0312	.166
4	(1,1,1,0)	.0506	.269
5	(1,0,1,0)	.0310	.164
Total		.1884	1.000

$$Q_i = p_1^{1-i_1} p_2^{1-i_2} p_3^{1-i_3} p_4^{1-i_4} q_1^{i_1} q_2^{i_2} q_3^{i_3} q_4^{i_4} \quad \text{if} \quad i = (i_1, i_2, i_3, i_4)$$

$$P_i = Q_i / \sum_{i=1}^{5} Q_i$$

V. *Computational Experience and Results*

The computer simulations were made at the Yale Computer Centre. This installation operates an IBM 370 computer and supports the Mathematical Programming System-Extended (MPSX) which is an IBM Program Product with its own language and its own compiler. It includes a set of procedures for the solution of linear programs. The Mixed Integer Programming (MIP) feature of MPSX provides the ability to solve mixed integer linear programs as well. Both are described in length in two IBM Program Description Manuals--SH20-0968 for MPSX and SH20-0908 for MIP.

Given the size of the programs under consideration (Table 5.1), a considerable amount of work is needed to translate the data consisting of cost coefficients and utility function for the objective function, and technological information for the constraint matrix into the rigid format required by MPSX. This is handled by a Fortran program, named BULLDOG, whose output constitutes the input of the MPSX procedures.

The computational experience is summarized in Table 5.2. The simulations were computed sequentially, although not necessarily in the order of their appearance in Table 5.2. Where possible, the optimal basis of a simulation served as a starting basis for the next simulation. We note that our branching strategy consisted of branching on the integer variables associated with the early time periods first. This explains why the runs of the lower part of Table 5.2 required a greater amount of computation. The algorithm explores first the possibility of an expensive "crash" program but realizes, as seen in Table 5.18, that in most cases this is too expensive. We also emphasize that the heuristic rule of rounding the optimal LP solution to the nearest integer is inappropriate for this class of problems. We have encountered R&D decision variables leaving

Table 5.1. Size of Mixed Integer Programs

Simulation	*Number of Continuous Variables, Including Slack Variables*	*Number of Integer Variables*	*Number of Rows*
RUN(I,1)	1578	12	264
RUN(I,2)	1656	46	276
RUN(I,3)	1689	53	283

Comment: The index I takes the values 1 to 5, corresponding to the cases mentioned in Table 4.2.

Table 5.2. Computational Experience

Simulation	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
RUN(1,1)	2.33	0.23	1073	7	3	2	2	3	1
RUN(2,1)	1.48	0.32	311	29	4	2	2	4	1
RUN(3,1)*	+	0.00	+	0	1	1	1	1	0
RUN(4,1)	2.76	0.08	910	0	1	1	1	1	0
RUN(5,1)	2.80	0.36	1165	34	6	6	1	6	2
RUN(1,2)	2.82	0.38	884	26	6	4	2	6	2
RUN(2,2)	4.47	1.45	927	135	21	12	2	21	3
RUN(3,2)	1.51	0.31	387	17	7	6	2	7	1
RUN(4,2)	3.02	0.27	912	12	3	2	2	3	1
RUN(5,2)	2.06	0.41	477	44	6	6	1	6	2
RUN(1,3)	3.29	0.51	1451	40	10	8	2	10	2
RUN(2,3)	3.67	1.26	858	122	25	18	2	20	3
RUN(3,3)	3.08	1.12	749	121	25	18	4	25	3
RUN(4,3)	3.96	1.52	786	154	26	12	3	18	4
RUN(5,3)	4.50	0.81	1256	86	14	14	1	14	3

+Data lost.

*All integer variables are fixed at 0; there is no MIP component to the computation.

Symbols: X_1 = CPU time for total search (including job management by the computer, setting up the problem by MPSX procedures, saving the basis, ...).

X_2 = CPU time for MIP search after the optimal LP solution is found.

X_3 = Number of iterations to reach optimal LP solution.

X_4 = Number of iterations during MIP search.

X_5 = Number of nodes explored.

X_6 = Node number at which first integer solution is found.

X_7 = Number of integer nodes found.

X_8 = Node number of optimal MIP solution.

X_9 = Number of integer variablps having fractional values in the optimal LP solution.

the LP optimization at levels as low as .08 to assume the value 1 in the optimal MIP solution for a certain state of the world and the value 0 for a different state of the world. Finally, the relatively short search for integer solutions is due to the special structure of the problem in its integer variables.

A last program was written to summarize the considerable amount of information provided by the output of the MPSX program (values of decision variables, dual variables or prices, ...). This is accomplished by the SUMMARY program whose output is reported in Tables 5.3 to 5.17. Table 5.19 then reviews the main components in the data handling and computation of each simulation.

VI. "Solfus," Nuclear Power, and Synthetic Fuels*

As compared with the runs made for the MRG Report, the simulations called RUN(I,2) and RUN(I,3) emphasize that the electric AES and the two synthetic fuels, shale oil, and liquefied coal, come out as suboptimal energy sources in all five scenarios. More concretely, Table 6.1 exhibits the probabilities that the technologies are part of an optimal decision, while the time paths are given in Figures 6.1 to 6.7. It appears from these numbers and figures that the coal gas technology and a nonelectric AES are likely to be asked for in the next fifty years. Furthermore, among the nuclear technologies, the decision to introduce the AC seems to be a wiser choice than the decision to support the development of the FBR. From Tables 5.3 to 5.17, we note that electricity is generally produced with the coal reserves (note the second round of coal in scenario 1), and when allowed for, by nuclear energy.

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*Term introduced by A. S. Manne to denote the backstop technologies, which could be solar energy or nuclear fusion.

TABLE 5.3. RUN(1,1)

	1980	*1990*	*2000*	*1990*	*2000*	*2010*	*2020*
1. Objective Function = 47743.679 billions of 1975 dollars							
2. Oil and Gas Imports	15.5	16.1	14.9	21.3	25.8	12.9	1.6
3. Total Energy Consumption	70.5	87.5	92.2	107.4	129.7	157.5	190.1
4. Domestic Energy Production by Source, Net of Exports	55.0	71.4	77.2	86.1	104.0	144.6	188.5
4.1. Solar Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.2. Solar Non-Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.3. Oil and NGL	12.7	6.1	5.5	5.7	14.1	21.1	14.2
4.4. Shale	0.0	0.0	0.0	0.0	0.0	1.9	1.9
4.5. Coal	15.2	36.1	31.2	29.0	39.6	70.4	128.3
4.6. Natural Gas	24.8	19.5	21.1	17.4	8.5	3.3	1.3
4.7. Nuclear	2.4	9.6	19.5	34.0	41.8	47.9	43.0
LWR	2.4	9.6	19.5	34.0	41.8	47.9	43.0
LMFBR	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Advanced Converters	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5. Electricity Generation by Source, Quads	18.4	23.1	26.3	34.0	41.8	57.8	79.9
5.1. % Oil and Gas	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.2. % Coal	87.1	58.6	25.8	0.0	0.0	17.1	46.9
5.3. % Solar Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.4. % Nuclear	12.9	41.4	74.2	100.0	100.0	82.9	53.8
6. Domestic Prices							
6.1. Coal ($/10E6 BTU)	2.06	1.86	1.31	0.75	0.75	0.75	0.75
6.2. Gas ($/MCF)	3.24	3.00	2.94	3.07	3.37	3.68	3.79
6.3. Oil ($/BBL)	15.40	14.23	14.72	15.81	17.48	20.01	20.40
6.4. Electricity ($/1000 KWH)	33.56	31.63	25.76	19.80	19.87	20.32	20.85
7. Energy Consumption	46.0	47.7	57.0	75.8	92.2	105.1	127.4
7.1. Residential and Commercial							
7.1.1. Specific Electric	2.7	2.9	3.8	5.6	7.0	8.0	9.8
7.1.2. Non-Electric	11.8	13.7	16.2	21.6	27.0	30.6	37.4
7.2. Industrial							
7.2.1. Specific Electric	2.9	3.1	4.1	6.0	7.6	8.7	10.6
7.2.2. Non-Electric	16.9	14.6	18.5	25.4	31.0	36.6	43.5
7.3. Transportation							
7.3.1. Specific Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7.3.2. Non-Electric	11.7	13.5	14.5	17.1	19.6	21.2	26.0

Assumptions: Base Case, Nuclear R&D only.

TABLE 5.4. RUN(1,2) and RUN(1,3)

	1980	*1990*	*2000*	*2010*	*2020*	*2030*	*2040*
1. Objective Function = 47735.208 billions of 1975 dollars							
2. Oil and Gas Imports	15.5	16.7	15.2	20.5	24.3	13.4	2.8
3. Total Energy Consumption	70.5	88.0	92.6	108.2	130.8	158.0	190.2
4. Domestic Energy Production by Source, Net of Exports	55.0	71.4	77.4	87.6	106.5	144.6	187.4
4.1. Solar Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.2. Solar Non-Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.3. Oil and NGL	12.7	6.1	5.5	5.7	14.1	21.1	14.2
4.4. Shale	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.5. Coal	15.2	36.1	31.6	30.6	41.9	72.3	129.0
4.6. Natural Gas	24.8	19.5	20.8	17.3	8.7	3.4	1.3
4.7. Nuclear	2.4	9.6	19.5	34.0	41.8	47.9	43.0
LWR	2.4	9.6	19.5	34.0	41.8	47.9	43.0
LMFBR	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Advanced Converters	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5. Electricity Generation by Source, Quads	18.4	23.8	26.6	34.0	41.8	47.9	66.6
5.1. % Oil and Gas	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.2. % Coal	87.1	59.8	26.7	0.0	0.0	0.0	35.5
5.3. % Solar Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.4. % Nuclear	12.9	40.2	73.3	100.0	100.0	100.0	64.5
6. Domestic Prices							
6.1. Coal ($/10E6 BTU)	2.06	1.86	1.30	0.75	0.75	0.75	0.75
6.2. Gas ($/MCF)	3.24	2.99	2.94	3.06	3.38	3.73	3.82
6.3. Oil ($/BBL)	15.43	14.29	14.81	15.97	17.77	20.74	20.95
6.4. Electricity ($/1000 KWH)	33.52	31.59	25.73	19.80	19.87	20.32	20.85
7. Energy Consumption	56.3	65.3	67.1	75.7	89.4	102.0	118.6
7.1. Residential and Commercial							
7.1.1. Specific Electric	3.0	3.9	4.3	5.5	6.8	7.8	9.2
7.1.2. Non-Electric	13.2	17.8	18.6	21.6	26.1	29.5	34.3
7.2. Industrial							
7.2.1. Specific Electric	3.2	4.2	4.7	6.0	7.3	8.4	10.0
7.2.2. Non-Electric	19.6	20.5	21.8	25.1	29.8	35.3	40.3
7.3. Transportation							
7.3.1. Specific Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7.3.2. Non-Electric	17.3	18.9	17.8	17.6	19.4	21.0	24.8

<u>Assumptions</u>: RUN(1,2) = Full R&D, Base Case, Full R&D.
RUN(1,3) = Base Case, Full R&D, Learning.

TABLE 5.5. RUN(2,1)

	1980	1990	2000	2010	2020	2030	2040
1. Objective Function = 47619.729 billions of 1975 dollars							
2. Oil and Gas Imports	15.5	24.0	18.3	17.4	15.5	12.9	8.5
3. Total Energy Consumption	70.5	84.4	90.9	108.3	131.7	150.7	181.4
4. Domestic Energy Production by Source, Net of Exports	55.0	60.4	72.6	91.0	116.2	137.8	173.0
4.1. Solar Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.2. Solar Non-Electric AES	0.0	0.0	0.0	0.0	0.0	2.9	25.8
4.3. Oil and NGL	12.7	6.9	6.1	6.2	14.8	20.7	13.1
4.4. Shale	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.5. Coal	15.2	24.4	31.8	51.0	62.8	62.8	62.8
4.6. Natural Gas	24.8	19.5	20.6	17.2	8.9	3.4	1.3
4.7. Nuclear	2.4	9.6	14.2	16.6	29.7	47.9	70.0
LWR	2.4	9.6	14.2	16.6	28.3	41.2	42.8
LMFBR	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Advanced Converters	0.0	0.0	0.0	0.0	1.3	6.7	27.2
5. Electricity Generation by Source, Quads	18.4	23.1	27.2	34.9	41.8	61.1	84.6
5.1. % Oil and Gas	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.2. % Coal	87.1	58.5	47.9	52.5	29.0	21.6	17.3
5.3. % Solar Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.4. % Nuclear	12.9	41.5	52.1	47.5	71.0	78.4	82.7
6. Domestic Prices							
6.1. Coal ($/10E6 BTU)	2.05	2.08	o.33	0.60	0.73	1.07	1.48
6.2. Gas ($/MCF)	3.22	3.26	3.13	3.24	3.67	4.21	4.88
6.3. Oil ($/BBL)	15.22	15.61	15.64	61.46	18.23	22.16	23.22
6.4. Electricity ($/1000 KWH)	33.41	33.69	26.63	19.65	19.88	20.34	20.88
7. Energy Consumption	46.0	46.7	56.6	76.0	90.5	98.6	115.1
7.1. Residential and Commercial							
7.1.1. Specific Electric	2.7	2.9	3.9	5.7	7.0	8.0	9.8
7.1.2. Non-Electric	11.8	13.2	15.5	20.6	25.5	28.7	33.0
7.2. Industrial							
7.2.1. Specific Electric	2.9	3.1	4.2	6.2	7.6	8.7	10.6
7.2.2. Non-Electric	16.9	14.1	18.8	26.6	30.9	33.0	36.6
7.3. Transportation							
7.3.1. Specific Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7.3.2. Non-Electric	11.7	13.5	14.2	16.8	19.6	20.3	25.1

Assumptions: Coal/Shale Limits, Base Case otherwise.

TABLE 5.6. RUN(2,2)

	1980	*1990*	*2000*	*2010*	*2020*	*2030*	*2040*
1. Objective Function = 47603.918 billions of 1975 dollars							
2. Oil and Gas Imports	15.5	24.0	17.6	14.8	16.5	15.5	8.1
3. Total Energy Consumption	71.3	84.4	90.7	107.9	129.6	146.8	171.5
4. Domestic Energy Production by Source, Net of Exports	55.8	60.4	73.0	93.1	113.2	131.4	163.4
4.1. Solar Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.2. Solar Non-Electric AES	0.0	0.0	0.0	0.0	0.0	1.3	6.7
4.3. Oil and NGL	13.5	6.9	6.4	9.4	17.7	18.5	10.0
4.4. Shale	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.5. Coal	15.2	24.4	31.8	51.0	62.8	62.8	62.8
4.6. Natural Gas	24.8	19.5	20.2	17.1	9.2	3.6	1.4
4.7. Nuclear	2.4	9.6	14.6	15.7	23.4	45.2	82.5
LWR	2.4	9.6	14.6	15.7	22.1	38.4	55.3
LMFBR	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Advanced Converters	0.0	0.0	0.0	0.0	1.3	6.7	27.2
5. Electricity Generation by Source, Quads	18.4	23.1	27.2	34.9	41.8	50.0	82.5
5.1. % Oil and Gas	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.2. % Coal	87.1	58.5	46.1	55.2	43.9	9.6	0.0
5.3. % Solar Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.4. % Nuclear	12.9	41.5	53.9	44.8	56.1	90.4	100.0
5. Domestic Prices							
6.1. Coal ($/10E6 BTU)	2.07	2.09	1.34	0.60	0.62	1.09	1.71
6.2. Gas ($/MCF)	3.25	3.27	3.18	3.34	3.86	4.55	5.25
6.3. Oil ($/BBL)	15.30	15.51	15.66	16.66	18.60	22.81	24.32
6.4. Electricity ($/1000 KWH)	33.64	33.78	26.67	19.62	19.83	20.21	21.04
7. Energy Consumption	56.3	64.0	66.2	76.0	89.0	93.8	102.9
7.1. Residential and Commercial							
7.1.1. Specific Electric	3.0	3.7	4.4	5.7	6.8	8.1	9.5
7.1.2. Non-Electric	13.2	17.3	17.9	20.4	23.6	25.8	28.9
7.2. Industrial							
7.2.1. Specific Electric	3.2	4.1	4.8	6.1	7.3	8.8	10.3
7.2.2. Non-Electric	19.6	20.0	22.0	26.8	31.9	31.8	31.0
7.3. Transportation							
7.3.1. Specific Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7.3.2. Non-Electric	17.3	18.9	17.2	17.0	19.4	19.3	23.1

Assumptions: Coal/Shale Limits, Base Case otherwise, Full R&D.

TABLE 5.7. RUN(2,3)

	1980	*1990*	*2000*	*2010*	*2020*	*2030*	*2040*
1. Objective Function = 47603.918 billions of 1975 dollars							
2. Oil and Gas Imports	15.5	24.0	17.6	14.8	16.5	15.5	8.1
3. Total Energy Consumption	71.3	84.4	90.7	107.9	129.6	146.8	171.5
4. Domestic Energy Production by Source, Net of Exports	55.8	60.4	73.0	93.1	113.2	131.4	163.4
4.1. Solar Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.2. Solar Non-Electric AES	0.0	0.0	0.0	0.0	0.0	.3	6.7
4.3. Oil and NGL	13.5	6.9	6.4	9.4	17.7	18.5	10.0
4.4. Shale	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.5. Coal	15.2	24.4	31.8	51.0	62.8	62.8	62.8
4.6. Natural Gas	24.8	19.5	20.2	17.1	9.2	3.6	1.4
4.7. Nuclear	2.4	9.6	14.6	15.7	23.4	45.2	82.5
LWR	2.4	9.6	14.6	15.7	22.1	38.4	55.3
LMFBR	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Advanced Converters	0.0	0.0	0.0	0.0	1.3	6.7	27.2
5. Electricity Generation by Source, Quads	18.4	23.1	27.2	34.9	41.8	50.0	82.5
5.1. % Oil and Gas	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.2. % Coal	87.1	58.5	46.1	55.2	43.9	9.6	0.0
5.3. % Solar Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.4. % Nuclear	12.9	41.5	53.9	44.8	56.1	90.4	100.0
6. Domestic Prices							
6.1. Coal ($/10E6 BTU)	2.07	2.09	1.34	0.60	0.62	1.09	1.71
6.2. Gas ($/MCF)	3.25	3.27	3.18	3.34	3.86	4.55	5.25
6.3. Oil ($/BBL)	15.30	15.51	15.66	16.66	18.60	22.81	24.32
6.4. Electricity ($/1000 KWH)	33.64	33.78	26.67	19.62	19.83	20.21	21.04
7. Energy Consumption	56.3	64.0	66.2	76.0	89.0	93.8	102.9
7.1. Residential and Commercial							
7.1.1. Specific Electric	3.0	3.7	4.4	5.7	6.8	8.1	9.5
7.1.2. Non-Electric	13.2	17.3	17.9	20.4	23.6	25.8	28.9
7.2. Industrial							
7.2.1. Specific Electric	3.2	4.1	4.8	6.1	7.3	8.8	10.3
7.2.2. Non-Electric	19.6	20.0	22.0	26.8	31.9	31.8	31.0
7.3. Transportation							
7.3.1. Specific Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7.3.2. Non-Electric	17.3	18.9	17.2	17.0	19.4	19.3	23.1

<u>Assumptions</u>: Coal/Shale Limits, Base Case otherwise. Full R&D, Learning.

TABLE 5.8. RUN(3,1)

	1980	1990	2000	2010	2020	2030	2040
1. Objective Function = 47334.263 billions of 1975 dollars							
2. Oil and Gas Imports	15.5	24.0	16.4	14.6	19.9	15.7	5.9
3. Total Energy Consumption	72.1	80.8	80.5	94.0	114.9	143.8	203.7
4. Domestic Energy Production by Source, Net of Exports	56.6	56.8	64.1	79.4	94.9	128.1	197.8
4.1. Solar Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.2. Solar Non-Electric AES	0.0	0.0	0.0	0.0	7.3	47.8	124.4
4.3. Oil and NGL	16.6	12.9	10.6	10.8	16.9	14.4	5.7
4.4. Shale	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.5. Coal	15.2	24.4	31.8	51.0	62.8	62.8	62.8
4.6. Natural Gas	24.8	19.5	21.7	17.7	7.9	3.1	1.2
4.7. Nuclear	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LWR	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LMFBR	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Advanced Converters	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5. Electricity Generation by Source, Quads	18.4	23.1	23.7	30.1	36.2	37.1	40.0
5.1. % Oil and Gas	11.8	0.0	0.0	0.0	0.0	0.0	0.0
5.2. % Coal	88.2	100.0	100.0	100.0	100.0	100.0	90.7
5.3. % Solar Electric	0.0	0.0	0.0	0.0	0.0	0.0	9.3
5.4. % Nuclear	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6. Domestic Prices							
6.1. Coal ($/10E6 BTU)	2.26	2.14	2.06	1.54	1.47	2.31	3.00
6.2. Gas ($/MCF)	3.27	3.34	3.24	3.44	4.04	4.82	5.15
6.3. Oil ($/BBL)	15.38	15.90	16.05	17.19	19.54	24.51	28.10
6.4. Electricity ($/1000 KWH)	35.44	34.35	33.57	28.56	27.92	35.93	42.48
7. Energy Consumption	46.0	45.6	50.9	68.1	81.7	83.3	96.3
7.1. Residential and Commercial							
7.1.1. Specific Electric	2.7	2.9	3.4	4.9	6.1	6.2	7.1
7.1.2. Non-Electric	11.8	12.7	15.2	19.8	23.9	26.0	32.2
7.2. Industrial							
7.2.1. Specific Electric	2.9	3.1	3.6	5.3	6.6	6.8	7.7
7.2.2. Non-Electric	16.9	14.1	15.1	21.5	26.4	25.8	28.3
7.3. Transportation							
7.3.1. Specific Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7.3.2. Non-Electric	11.7	12.7	13.6	16.5	18.7	18.5	20.9

Assumptions: Coal/Shale Limits, Nuclear Moratorium; Base Case otherwise. Nuclear R&D.

TABLE 5.9. RUN(3,2) and RUN(3,3)

	1980	1990	2000	2010	2020	2030	2040
1. Objective Function = 47312.576 billions of 1975 dollars							
2. Oil and Gas Imports	15.5	24.0	16.4	12.2	16.3	18.0	9.6
3. Total Energy Consumption	72.2	81.2	80.2	92.8	111.1	125.5	189.1
4. Domestic Energy Production by Source, Net of Exports	56.7	57.2	63.8	80.6	94.8	107.5	179.5
4.1. Solar Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.2. Solar Non-Electric AES	0.0	0.0	0.0	1.3	6.7	27.2	110.0
4.3. Oil and NGL	16.7	13.3	10.9	10.8	16.8	14.2	5.5
4.4. Shale	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.5. Coal	15.2	24.4	31.8	51.0	62.8	62.8	62.8
4.6. Natural Gas	24.8	19.5	21.1	17.4	8.4	3.3	1.3
4.7. Nuclear	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LWR	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LMFBR	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Advanced Converters	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5. Electricity Generation by Source, Quads	18.4	23.1	23.7	30.1	36.2	37.1	40.0
5.1. % Oil and Gas	11.8	0.0	0.0	0.0	0.0	0.0	0.0
5.2. % Coal	88.2	100.0	100.0	100.0	100.0	100.0	100.0
5.3. % Solar Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.4. % Nuclear	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6. Domestic Prices							
6.1. Coal ($/10E6 BTU)	2.27	2.19	2.16	1.61	1.47	2.31	3.16
6.2. Gas ($/MCF)	3.28	3.40	3.36	3.69	4.52	5.78	5.81
6.3. Oil ($/BBL)	15.45	16.02	16.21	17.48	20.14	25.53	28.83
6.4. Electricity ($/1000 KWH)	35.56	34.80	34.49	29.24	27.92	35.93	44.03
7. Energy Consumption	56.3	62.8	60.4	66.1	75.8	76.2	83.7
7.1. Residential and Commercial							
7.1.1. Specific Electric	3.0	3.7	3.9	4.9	5.9	6.0	6.5
7.1.2. Non-Electric	13.2	16.9	17.1	18.6	20.7	21.6	26.7
7.2. Industrial							
7.2.1. Specific Electric	3.2	4.1	4.2	5.3	6.4	6.5	7.0
7.2.2. Non-Electric	19.6	20.0	18.4	21.2	25.2	24.5	24.5
7.3. Transportation							
7.3.1. Specific Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7.3.2. Non-Electric	17.3	18.2	16.9	16.2	17.7	17.6	19.0

Assumptions: Coal/Shale Limits, Nuclear Moratorium, Base Case otherwise.
RUN(3,2) = Full R&D.
RUN(3,3) = Full R&D, Learning.

TABLE 5.10. RUN(4,1)

	1980	*1990*	*2000*	*2010*	*2020*	*2030*	*2040*
1. Objective Function = 84496.209 billions of 1975 dollars							
2. Oil and Gas Imports	15.7	27.8	18.7	13.8	22.5	14.5	1.0
3. Total Energy Consumption	72.6	93.2	102.6	129.2	176.4	268.2	433.3
4. Domestic Energy Production by Source, Net of Exports	56.9	65.4	83.8	115.4	153.9	253.7	432.3
4.1. Solar Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.2. Solar Non-Electric AES	0.0	0.0	0.0	1.3	17.8	94.3	230.0
4.3. Oil and NGL	14.6	11.9	11.3	17.3	19.3	10.2	3.9
4.4. Shale	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.5. Coal	15.2	24.4	31.8	51.0	62.8	62.8	62.8
4.6. Natural Gas	24.8	19.5	22.8	18.1	7.0	2.7	1.0
4.7. Nuclear	2.4	9.6	18.0	27.7	47.0	83.7	134.6
LWR	2.4	9.6	18.0	25.0	33.5	29.3	10.9
LMFBR	0.0	0.0	0.0	1.3	6.7	27.2	98.2
Advanced Converters	0.0	0.0	0.0	1.3	6.7	27.2	25.5
5. Electricity Generation by Source, Quads	18.6	26.4	31.2	42.4	56.9	105.9	167.2
5.1. % Oil and Gas	0.1	0.0	0.0	0.0	0.0	0.0	0.0
5.2. % Coal	87.2	63.7	42.5	34.8	17.4	21.0	19.5
5.3. % Solar Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.4. % Nuclear	12.7	36.3	57.5	65.2	82.6	79.0	80.5
6. Domestic Prices							
6.1. Coal ($/10E6 BTU)	2.37	2.33	1.69	1.17	1.75	2.54	2.74
6.2. Gas ($/MCF)	3.39	3.57	3.73	4.30	5.38	6.59	6.06
6.3. Oil ($/BBL)	15.96	16.87	17.49	19.83	24.72	33.51	31.60
6.4. Electricity ($/1000 KWH)	36.49	36.12	30.03	25.08	27.99	30.26	30.75
7. Energy Consumption	51.2	54.7	66.8	93.5	119.6	144.3	219.9
7.1. Residential and Commercial							
7.1.1. Specific Electric	3.0	3.5	4.7	7.3	10.0	13.2	19.2
7.1.2. Non-Electric	13.6	16.1	19.4	27.5	37.0	48.1	82.0
7.2. Industrial							
7.2.1. Specific Electric	3.3	3.8	5.1	7.9	10.8	14.4	20.8
7.2.2. Non-Electric	18.6	16.5	21.6	31.2	38.1	43.5	58.0
7.3. Transportation							
7.3.1. Specific Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7.3.2. Non-Electric	12.7	14.8	15.9	19.7	23.7	25.1	40.0

<u>Assumptions</u>: Coal/Shale Limits, High GNP, Low Uranium, Base Case otherwise, Nuclear R&D.

TABLE 5.11. RUN(4,2)

	1980	*1990*	*2000*	*2010*	*2020*	*2030*	*2040*
1. Objective Function = 84380.247 billions of 1975 dollars							
2. Oil and Gas Imports	15.7	27.8	18.6	13.1	21.3	15.1	2.3
3. Total Energy Consumption	72.5	91.7	99.8	123.7	157.5	205.5	328.3
4. Domestic Energy Production by Source, Net of Exports	56.8	63.9	81.2	110.5	136.3	190.4	326.0
4.1. Solar Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.2. Solar Non-Electric AES	0.0	0.0	0.0	1.3	6.7	27.2	110.0
4.3. Oil and NGL	14.4	10.4	9.8	15.4	19.9	12.5	4.8
4.4. Shale	0.0	0.0	0.0	0.0	0.4	0.9	0.9
4.5. Coal	15.2	24.4	31.8	51.0	62.8	62.8	62.8
4.6. Natural Gas	24.8	19.5	21.9	17.7	7.8	3.0	1.2
4.7 Nuclear	2.4	9.6	17.7	25.1	38.6	84.0	146.4
LWR	2.4	9.6	17.7	22.4	25.2	29.6	14.1
LMFBR	0.0	0.0	0.0	1.3	6.7	27.2	96.6
Advanced Converters	0.0	0.0	0.0	1.3	6.7	27.2	35.7
5. Electricity Generation by Source, Quads	18.6	25.5	30.8	42.4	54.5	87.5	146.4
5.1. % Oil and Gas	0.1	0.0	0.0	0.0	0.0	0.0	0.0
5.2. % Coal	87.2	62.5	42.4	40.9	29.1	4.1	0.0
5.3. % Solar Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.4. % Nuclear	12.7	37.5	57.6	59.1	70.9	95.9	100.0
6. Domestic Prices							
6.1. Coal ($/10E6 BTU)	2.46	2.48	1.83	1.35	1.85	3.13	3.62
6.2. Gas ($/MCF)	3.48	3.74	3.86	4.66	6.27	8.14	8.32
6.3. Oil ($/BBL)	16.42	17.71	18.80	22.05	28.90	41.45	40.50
6.4. Electricity ($/1000 KWH)	37.34	37.49	31.34	26.79	31.55	38.31	37.66
7. Energy Consumption	56.5	69.4	72.4	84.8	102.5	114.8	159.9
7.1. Residential and Commercial							
7.1.1. Specific Electric	3.0	4.1	5.0	6.9	8.8	10.9	15.2
7.1.2. Non-Electric	13.3	19.2	20.2	24.0	29.5	35.2	52.4
7.2. Industrial							
7.2.1. Specific Electric	3.3	4.5	5.4	7.5	9.6	11.8	16.4
7.2.2. Non-Electric	19.7	21.8	23.5	28.4	33.9	36.4	44.2
7.3. Transportation							
7.3.1. Specific Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7.3.2. Non-Electric	17.3	19.8	18.3	18.1	20.7	20.6	31.8

<u>Assumptions</u>: Coal/Shale Limits, High GNP, Low Uranium, Base Case otherwise. Full R&D.

TABLE 5.12. RUN(4,3)

	1980	*1990*	*2000*	*2010*	*2020*	*2030*	*2040*
1. Objective Function = 84456.738 billions of 1975 dollars							
2. Oil and Gas Imports	15.7	27.8	17.1	13.2	25.1	15.1	0.0
3. Total Energy Consumption	72.6	93.5	104.8	135.2	179.8	261.6	399.0
4. Domestic Energy Production by Source, Net of Exports	56.9	65.8	87.7	121.9	154.7	246.6	399.0
4.1. Solar Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.2. Solar Non-Electric AES	0.0	0.0	0.0	1.3	6.7	27.2	110.0
4.3. Oil and NGL	14.6	12.3	13.0	19.8	18.6	8.0	3.1
4.4. Shale	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.5. Coal	15.2	24.4	31.8	51.0	62.8	62.8	62.8
4.6. Natural Gas	24.8	19.5	22.6	18.0	7.1	2.8	1.1
4.7. Nuclear	2.4	9.6	20.3	31.8	59.4	145.8	222.1
LWR	2.4	9.6	18.9	23.8	25.5	29.8	14.0
LMFBR	0.0	0.0	1.3	6.7	27.2	88.9	172.0
Advanced Converters	0.0	0.0	0.0	1.3	6.7	27.2	36.1
5. Electricity Generation by Source, Quads	18.6	26.4	32.3	45.5	69.4	145.8	222.1
5.1. % Oil and Gas	0.1	0.0	0.0	0.0	0.0	0.0	0.0
5.2. % Coal	87.2	63.7	37.3	28.7	13.5	0.0	0.0
5.3. % Solar Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.4. % Nuclear	12.7	36.3	62.7	70.0	85.6	100.0	100.0
6. Domestic Prices							
6.1. Coal ($/10E6 BTU)	2.36	2.31	1.59	0.91	o.19	1.82	2.42
6.2. Gas ($/MCF)	3.37	3.55	3.59	4.20	5.46	6.12	5.87
6.3. Oil ($/BBL)	15.88	16.75	17.53	19.71	24.81	31.11	28.55
6.4. Electricity ($/1000 KWH)	36.35	35.96	29.07	22.61	24.16	24.94	23.56
7. Energy Consumption	56.6	71.0	76.0	93.1	114.9	141.1	194.5
7.1. Residential and Commercial							
7.1.1. Specific Electric	3.0	4.3	5.2	7.4	10.0	13.7	19.1
7.1.2. Non-Electric	13.4	19.8	21.2	25.6	31.7	43.2	64.4
7.2. Industrial							
7.2.1. Specific Electric	3.3	4.6	5.7	8.0	10.8	14.8	20.7
7.2.2. Non-Electric	19.7	21.8	24.6	31.4	38.7	43.9	50.4
7.3. Transportation							
7.3.1. Specific Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7.3.2. Non-Electric	17.3	20.5	19.3	20.7	23.7	25.5	39.9

Assumptions: Coal/Shale Limits, High GNP, Low Uranium, Base Case otherwise. Full R&D, Learning.

TABLE 5.13. RUN(5,1)

	1980	*1990*	*2000*	*2010*	*2020*	*2030*	*2040*
1. Objective Function = 84920.378 billions of 1975 dollars							
2. Oil and Gas Imports	15.7	25.5	21.2	23.9	20.9	5.5	0.0
3. Total Energy Consumption	71.2	97.9	109.9	145.8	205.7	298.6	415.9
4. Domestic Energy Production by Source, Net of Exports	55.6	72.4	88.7	121.9	184.8	293.1	415.9
4.1. Solar Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.2. Solar Non-Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.3. Oil and NGL	13.2	7.2	7.0	13.3	21.6	16.2	6.3
4.4. Shale	0.0	0.0	0.0	0.0	3.0	14.4	11.4
4.5. Coal	15.2	36.1	41.3	63.3	131.9	226.2	308.7
4.6. Natural Gas	24.8	19.5	22.8	18.1	7.0	2.7	1.0
4.7. Nuclear	2.4	9.6	17.6	27.1	21.4	33.7	88.5
LWR	2.4	9.6	17.6	25.8	14.6	6.5	16.8
LMFBR	0.0	0.0	0.0	1.3	6.7	27.2	71.8
Advanced Converters	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5. Electricity Generation by Source, Quads	18.6	26.4	32.3	45.5	64.2	88.0	145.4
5.1. % Oil and Gas	0.1	0.0	0.0	0.0	0.0	0.0	0.0
5.2. % Coal	87.2	63.7	45.7	41.4	66.7	61.7	39.1
5.3. % Solar Energy	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.4. % Nuclear	12.7	36.3	54.3	59.6	33.3	38.3	60.9
6. Domestic Prices							
6.1. Coal ($/10E6 BTU)	2.29	1.93	1.34	0.77	0.78	0.80	0.84
6.2. Gas ($/MCF)	3.27	3.08	3.16	3.38	3.73	3.87	3.84
6.3. Oil ($/BBL)	15.53	14.64	15.30	16.85	19.35	21.38	20.84
6.4. Electricity ($/1000 KWH)	35.70	32.28	26.73	21.22	21.34	21.54	21.91
7. Energy Consumption	56.9	73.4	79.8	101.4	133.9	180.5	248.2
7.1. Residential and Commercial							
7.1.1. Specific Electric	3.0	4.3	5.2	7.4	10.4	14.3	19.7
7.1.2. Non-Electric	13.4	20.3	22.7	30.3	40.8	59.6	87.1
7.2. Industrial							
7.2.1. Specific Electric	3.3	4.6	5.7	8.0	11.3	15.5	21.4
7.2.2. Non-Electric	19.8	22.9	25.8	33.0	43.7	56.7	73.8
7.3. Transportation							
7.3.1. Specific Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7.3.2. Non-Electric	17.5	21.3	20.4	22.8	27.7	34.4	46.2

<u>Assumptions</u>: High GNP, Low Uranium, Base Case otherwise. Nuclear R&D.

TABLE 5.14. RUN(5,2) and RUN(5,3)

	1980	1990	2000	2010	2020	2030	2040
1. Objective Function = 84896.764 billions of 1975 dollars							
2. Oil and Gas Imports	15.7	24.6	18.5	16.5	23.7	13.3	0.0
3. Total Energy Consumption	71.2	98.1	109.9	143.0	200.9	301.6	420.9
4. Domestic Energy Production by Source, Net of Exports	55.6	73.5	91.4	126.5	177.2	288.2	420.9
4.1. Solar Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.2. Solar Non-Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.3. Oil and NGL	13.2	8.2	8.8	17.2	21.5	12.5	4.8
4.4. Shale	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.5. Coal	15.2	36.1	42.2	64.1	127.3	239.4	326.6
4.6. Natural Gas	24.8	19.5	22.8	18.1	7.0	2.7	1.0
4.7. Nuclear	2.4	9.6	17.6	27.1	21.4	33.7	88.5
LWR	2.4	9.6	17.6	25.8	14.6	6.5	16.8
LMFBR	0.0	0.0	0.0	1.3	6.7	27.2	71.8
Advanced Converters	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5. Electricity Generation by Source, Quads	18.6	26.4	32.3	45.5	64.2	88.0	145.8
5.1. % Oil and Gas	0.1	0.0	0.0	0.0	0.0	0.0	0.0
5.2. % Coal	87.2	63.7	45.7	40.4	66.7	61.7	39.2
5.3. % Solar Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.4. % Nuclear	12.7	36.3	54.3	59.6	33.3	38.3	60.8
6. Domestic Prices							
6.1. Coal ($/10E6 BTU)	2.32	1.99	1.38	0.77	0.78	0.80	0.84
6.2. Gas ($/MCF)	3.30	3.15	3.22	3.52	4.06	4.08	3.84
6.3. Oil ($/BBL)	15.70	15.04	15.85	17.84	21.13	22.53	20.85
6.4. Electricity ($/1000 KWH)	36.02	32.86	27.02	21.22	21.34	21.55	21.92
7. Energy Consumption	56.9	73.4	79.4	98.7	131.1	179.8	248.0
7.1. Residential and Commercial							
7.1.1. Specific Electric	3.0	4.3	5.2	7.4	10.4	14.3	19.7
7.1.2. Non-Electric	13.4	20.3	22.3	28.7	39.2	58.9	86.8
7.2. Industrial							
7.2.1. Specific Electric	3.3	4.6	5.7	8.0	11.3	15.5	21.4
7.2.2. Non-Electric	19.8	22.9	25.8	33.0	43.7	56.7	73.8
7.3. Transportation							
7.3.1. Specific Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7.3.2. Non-Electric	17.5	21.3	20.4	21.6	26.5	34.4	46.2

<u>Assumptions:</u> High GNP, Low Uranium, Base Case otherwise.
RUN(5,2) = Full R&D.
RUN(5,3) = Full R&D, Learning.

TABLE 5.16. RUN(5,2)

	1980	*1990*	*2000*	*2010*	*2020*	*2030*	*2040*
1. Objective Function = 84896.764 billions of 1975 dollars							
2. Oil and Gas Imports	15.7	24.6	18.5	16.5	23.7	13.3	0.0
3. Total Energy Consumption	71.2	98.1	109.9	143.0	200.9	301.6	420.9
4. Domestic Energy Production by Source, Net of Exports	55.6	73.5	91.4	126.5	177.2	288.2	420.9
4.1. Solar Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.2. Solar Non-Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.3. Oil and NGL	13.2	8.2	8.8	17.2	21.5	12.5	4.8
4.4. Shale	.0	0.0	0.0	0.0	0.0	0.0	0.0
4.5. Coal	15.2	36.1	42.2	64.1	127.3	239.4	326.6
4.5. Natural Gas	24.8	19.5	22.8	18.1	7.0	2.7	1.0
4.7. Nuclear	2.4	9.6	17.6	27.1	21.4	33.7	88.5
LWR	2.4	9.6	17.6	25.8	14.6	6.5	16.8
LMFBR	0.0	0.0	0.0	1.3	6.7	27.2	71.8
Advanced Converters	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5. Electricity Generation by Source, Quads	18.6	26.4	32.3	45.5	64.2	88.0	145.8
5.1. % Oil and Gas	0.1	0.0	0.0	0.0	0.0	0.0	0.0
5.2. % Coal	87.2	63.7	45.7	40.4	66.7	61.7	39.2
5.3. % Solar Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.4. % Nuclear	12.7	36.3	54.3	59.6	33.3	38.3	60.8
6. Domestic Prices							
6.1. Coal ($/10E6 BTU)	2.32	1.99	1.38	0.77	0.78	0.80	0.84
6.2. Gas ($/MCF)	3.30	3.15	3.22	3.52	4.06	4.08	3.84
6.3. Oil ($/BBL)	15.70	15.04	15.85	17.84	21.13	22.53	20.85
6.4. Electricity ($/1000 KWH)	36.02	32.86	27.02	21.22	21.34	21.55	21.92
7. Energy Consumption	56.9	73.4	79.4	98.7	131.1	179.8	248.0
7.1. Residential and Commercial							
7.1.1. Specific Electric	3.0	4.3	5.2	7.4	10.4	14.3	19.7
7.1.2. Non-Electric	13.4	20.3	22.3	28.7	39.2	58.9	86.8
7.2. Industrial							
7.2.1. Specific Electric	3.3	4.6	5.7	8.0	11.3	15.5	21.4
7.2.2. Non-Electric	19.8	22.9	25.8	33.0	43.7	56.7	73.8
7.3. Transportation							
7.3.1. Specific Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7.3.2. Non-Electric	17.5	21.3	20.4	21.6	26.5	34.4	46.2

<u>Assumptions:</u> High GNP, Low Uranium, Base Case otherwise. Full R&D.

TABLE 5.17 UN(5,3)

	1980	1990	2000	2010	2020	2030	2040
1. Objective Function =							
2. Oil and Gas Imports	15.7	24.6	18.5	16.5	23.7	13.3	0.0
3. Total Energy Consumption	71.2	98.1	109.9	143.0	200.9	291.8	411.2
4. Domestic Energy Production by Source, Net of Exports	55.6	73.5	91.4	126.5	177.2	278.4	411.2
4.1. Solar Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.2. Solar Non-Electric AES	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.3. Oil and NGL	13.2	8.2	8.8	17.2	21.5	12.5	4.8
4.4. Shale	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.5. Coal	15.2	36.1	42.2	64.1	127.3	229.6	316.8
4.6. Natural Gas	24.8	19.5	22.8	18.1	7.0	2.7	1.0
4.7. Nuclear	2.4	9.6	17.6	27.1	21.4	33.7	88.5
LWR	2.4	9.6	17.6	25.8	14.6	6.5	16.8
LMFBR	0.0	0.0	0.0	1.3	6.7	27.2	71.8
Advanced Converters	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5. Electricity Generation by Source, Quads	18.6	26.4	32.3	45.5	64.2	105.6	163.3
5.1. % Oil and Gas	0.1	0.0	0.0	0.0	0.0	0.0	0.0
5.2. % Coal	87.2	63.7	45.7	40.4	66.7	68.1	45.8
5.3. % Solar Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.4. % Nuclear	12.7	36.3	54.3	59.6	33.3	31.9	54.2
6. Domestic Prices							
6.1. Coal ($/10E6 BTU)	2.31	1.99	1.38	0.77	0.78	0.80	0.84
6.2. Gas ($/MCF)	3.30	3.15	3.22	3.52	4.06	4.08	3.84
6.3. Oil ($/BBL)	15.63	14.91	15.67	17.51	20.54	21.06	19.76
6.4. Electricity ($/1000 KWH)	35.88	32.86	27.02	21.22	21.33	21.54	21.91
7. Energy Consumption	56.9	73.4	79.4	98.7	131.1	181.9	250.1
7.1. Residential and Commercial							
7.1.1. Specific Electric	3.0	4.3	5.2	7.4	10.4	14.3	19.7
7.1.2. Non-Electric	13.4	20.3	22.3	28.7	39.2	59.2	87.1
7.2. Industrial							
7.2.1. Specific Electric	3.3	4.6	5.7	8.0	11.3	15.5	21.4
7.2.2. Non-Electric	19.8	22.9	25.8	33.0	43.7	56.7	73.8
7.3. Transportation							
7.3.1. Specific Electric	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7.3.2. Non-Electric	17.5	21.3	20.4	21.6	26.5	36.2	48.0

<u>Assumptions</u>: High GNP, Low Uranium, Base Case otherwise. Full R&D, Learning.

TABLE 5.18. *Optimal R&D Decisions**

Simulation	*Assumptions*	*FBR*	*AC*	*Electric AES*	*Nonelectric AES*	*Shale Oil*	*Coal Oil*	*Coal Gas*
RUN(1,1)	Base Case, Nuclear R&D	∞	∞	∞	∞	2030	∞	2030
RUN(1,2)	Base Case, Full R&D	∞	∞	∞	∞	∞	∞	2000
RUN(1,3)	Base Case, Full R&D, Learning	∞	∞	∞	∞	∞	∞	2000
RUN(2,1)	Coal/Shale Limits, Base Case Otherwise Nuclear R&D	∞	2020	∞	2030	∞	∞	2030
RUN(2,2)	Coal/Shale Limits, Base Case Otherwise Full R&D	∞	2020	∞	2030	∞	∞	1990
RUN(2,3)	Coal/Shale Limits, Base Case Otherwise Full R&D, Learning	∞	2020	∞	2030	∞	∞	2000
RUN(3,1)	Coal/Shale Limits, Nuclear Moratorium, Base Case Otherwise Nuclear R&D	--	--	2040	2020	∞	∞	∞
RUN(3,2)	Coal/Shale Limits, Nuclear Moratorium, Base Case Otherwise Full R&D	--	--	∞	2010	∞	∞	∞
RUN(3,3)	Coal/Shale Limits, Nuclear Moratorium, Base Case Otherwise Full R&D, Learning	--	--	∞	2010	∞	∞	∞
RUN(4,1)	Coal/Shale Limits, High GNP, Low Uranium, Base Case Otherwise	2010	2010	∞	2010	∞	∞	2020
RUN(4,2)	Coal/Shale Limits, High GNP, Low Uranium, Base Case Otherwise, Full R&D	2010	2010	∞	2010	2020	∞	2000
RUN(4,3)	Coal/Shale Limits, High GNP, Low Uranium, Base Case Otherwise, Full R&D, Learning	2000	2010	∞	2010	∞	∞	2000
RUN(5,1)	High GNP, Low Uranium, Base Case Otherwise Nuclear R&D	2010	∞	∞	2030	∞	∞	2010
RUN(5,2)	High GNP, Low Uranium, Base Case Otherwise, Full R&D	2010	∞	∞	∞	∞	∞	1990
RUN(5,3)	High GNP, Low Uranium, Base Case Otherwise, Full R&D, Learning	2010	∞	∞	∞	∞	∞	1990

*The entry, ∞ , indicates that R&D is not performed for this technology within the time span, 1970-2060.

TABLE 5.19. Computer Manipulations

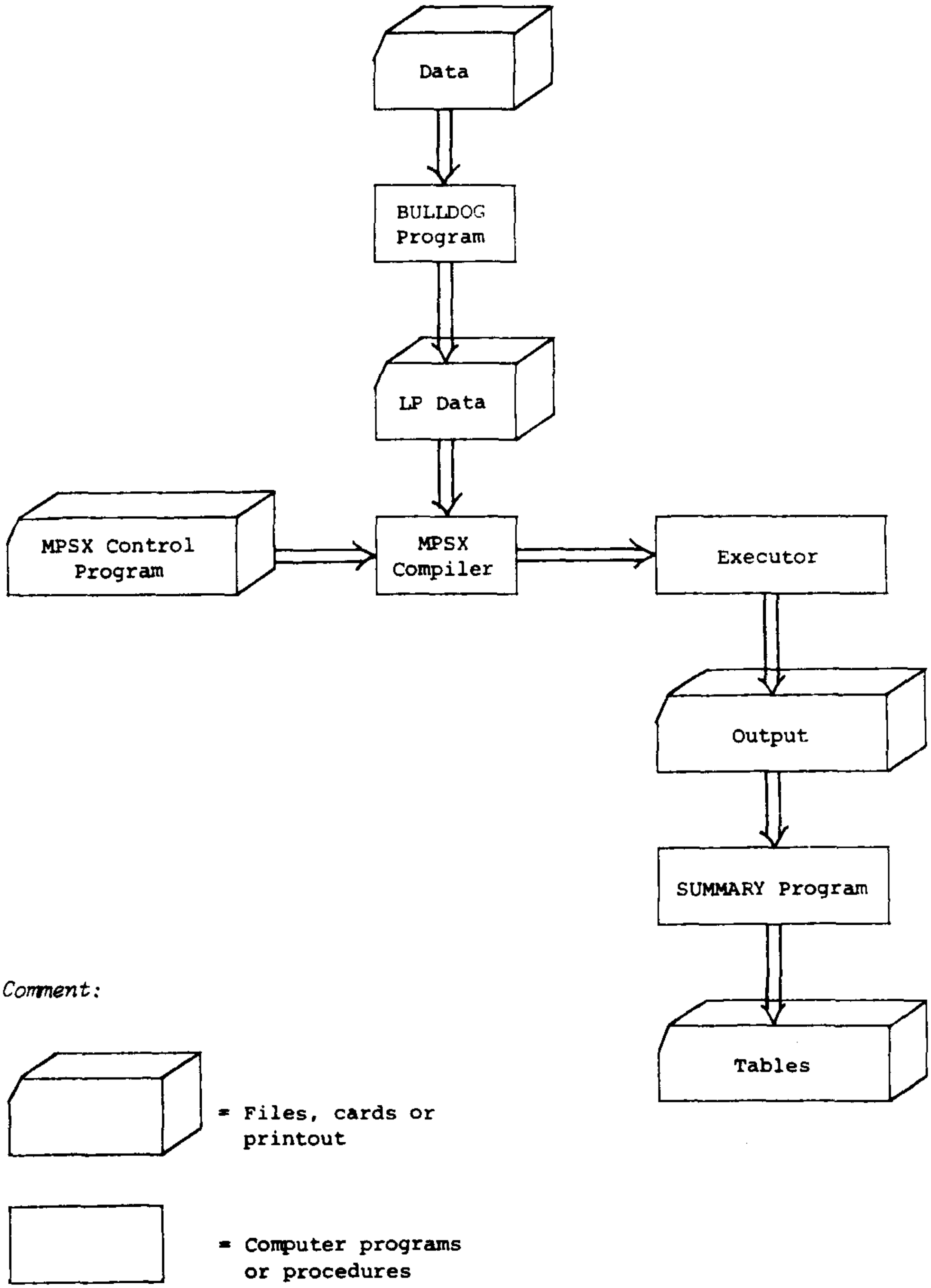

Table 6.1. Probabilities That a Technology Is Part of an Optimal Decision (optimality taking into account the learning effects).

Technology	*Probability*
1. Backstop Technologies:	
1a. Elec AES	.000
1b. Nonelectric AES	.684
2. Nuclear Technologies:	
2a. LMFBR	.433
2b. AC	.518
3. Synthetic Fuels:	
3a. Shale Oil	.000
3b. Coal Oil	.000
3c. Coal Gas	.834

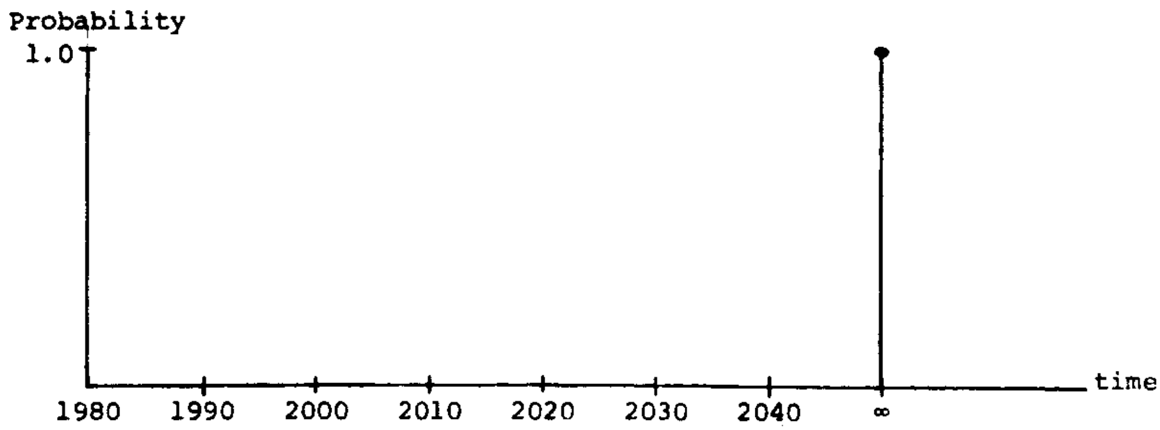

FIGURE 6.1. Probability that completion of R&D of electric AES in year t is part of an optimal decision.

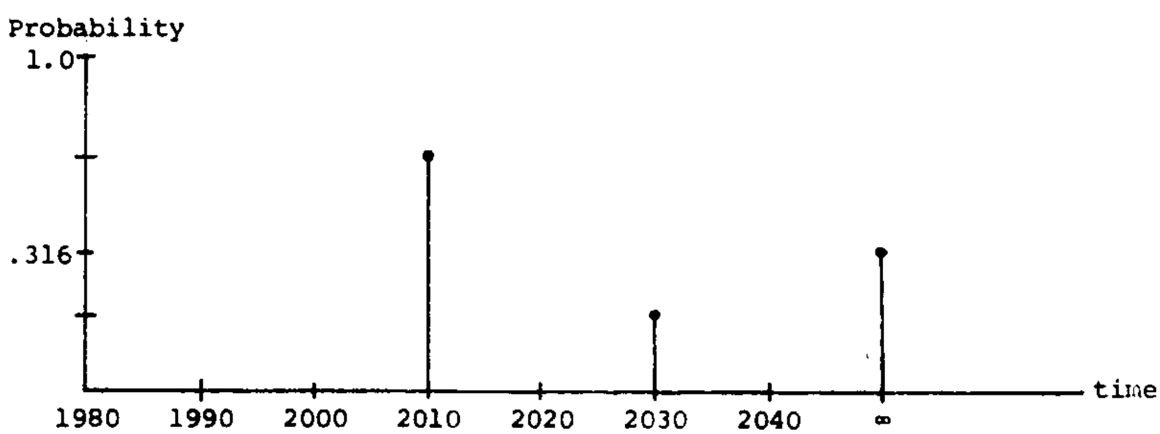

FIGURE 6.2. Probability that completion of R&D of nonelectric AES in year t is part of an optimal decision.

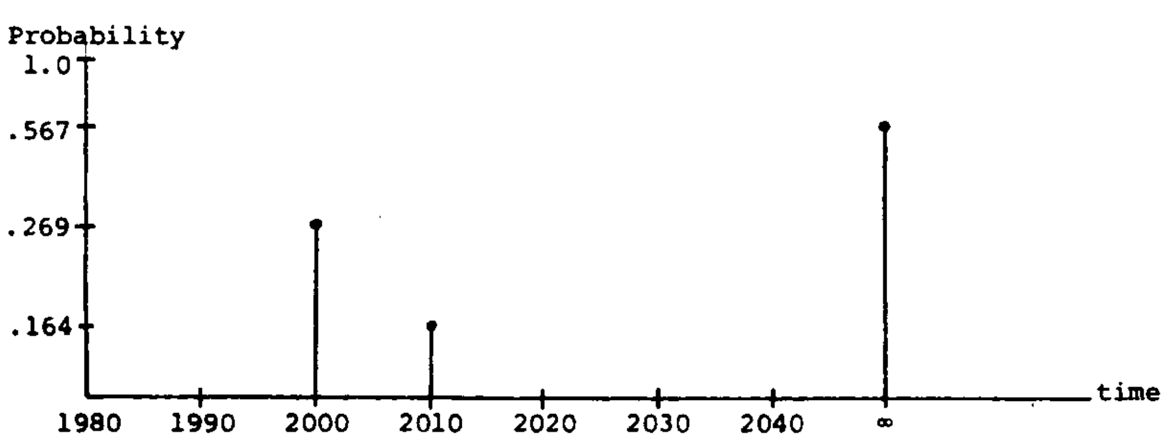

FIGURE 6.3. Probability that completion of R&D of FBR in year t is part of an optimal decision.

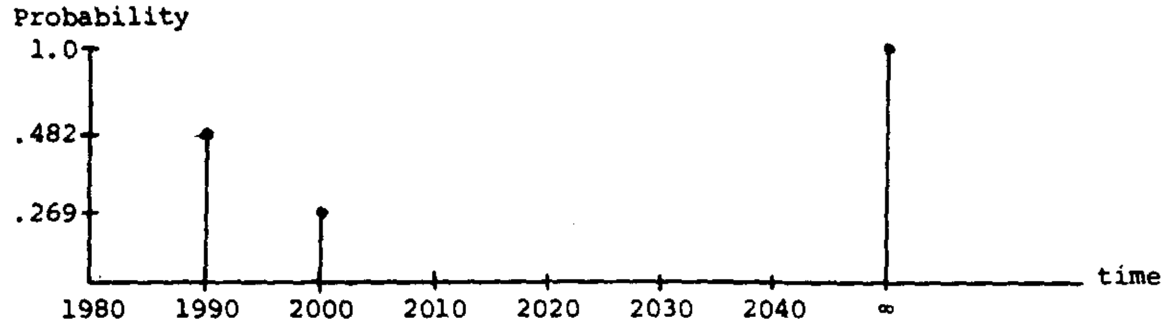

FIGURE 6.4. Probability that completion of R&D of AC in year t is part of an optimal decision.

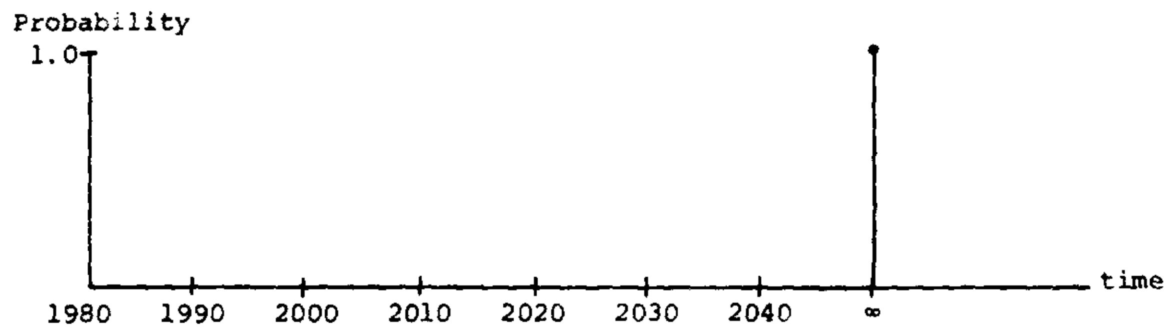

FIGURE 6.5. Probability that completion of R&D of Shale Oil in year t is part of an optimal decision.

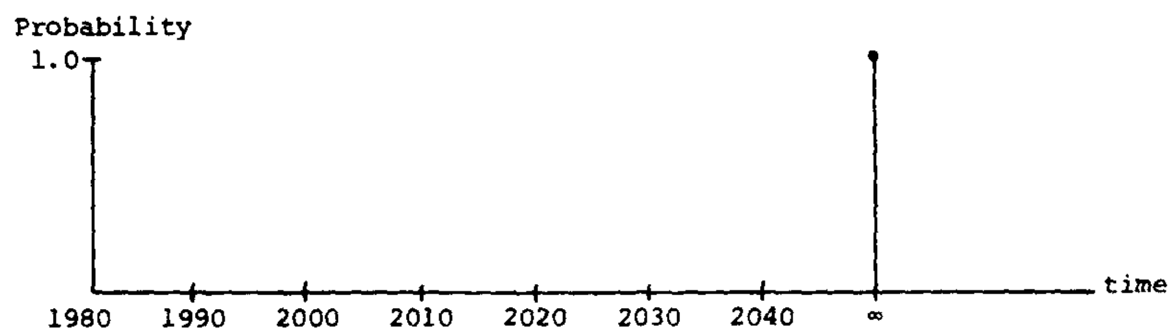

FIGURE 6.6. Probability that completion of R&D of Coal Oil in year t is part of an optimal decision.

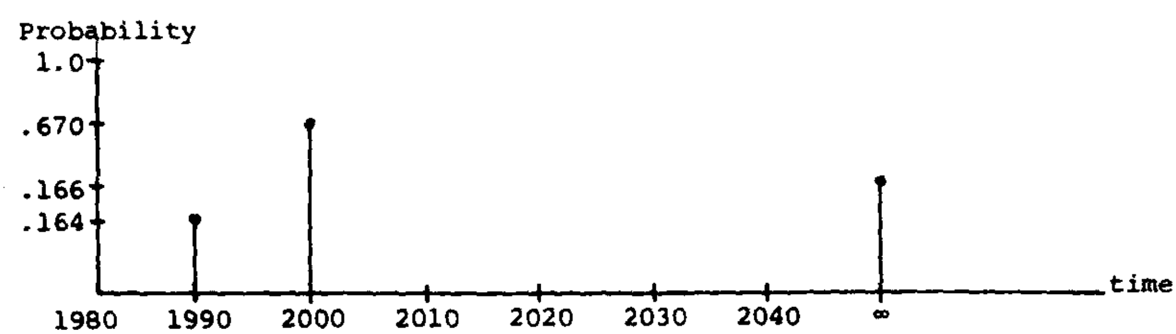

FIGURE 6.7. Probability that completion of R&D of Coal Gas in year t is part of an optimal decision.

The introduction of learning effects does not change the optimal decisions significantly. In scenario 2, coal gas appears one period later because of higher costs. The other difference occurs in scenario 4 with the FBR being implemented one period earlier and the shale oil technology leaving the optimal decision.

A caveat is in order. Saying that a decision is suboptimal does not mean that the decision is unsound or unacceptable. A given decision may be optimal for a certain environment, but concurrently may reveal a poor judgment given other states of the world. Conversely, a policy may be sufficiently close to optimality in any future state of the world that, considering the uncertainties, that policy should be regarded less risky than other policies which are optimal in certain scenarios, but further away from optimality in others. What would be helpful is an enumeration of the best solutions for the set of scenarios chosen to represent all future states of the world. We then would be able to select a decision with the knowledge of how well it performs given any realization of the uncertainties. Unhappily the branch-and-bound technique used in this paper does not rank all integer nodes; it terminates as soon as it finds an optimal solution without attempting to search for a second best solution. The integer nodes found during the search can be ranked using the objective function values, but nothing has been said about the unexplored nodes.

We hope that our ideas will contribute to the solution of the diffcult problem of huge investments on future technologies when so much remains uncertain. The techniques exposed in this paper present the computation of efficient decisions given the resolution of the uncertainties, and deduce some probabilistic estimates of efficient decisionmaking before the uncertainties are resolved. Striving for an optimal decision given a state of knowledge, expertise, and social values, we have to keep in mind that there might not exist a single, definite answer [Arrow (1974)]:

> There are moments in history when we must simply act, fully knowing our ignorance of possible consequences, but to retain our full rationality we must sustain the burden of action without certitude, and we must always keep open the possibility of recognizing past errors and changing course.

REFERENCES

Arrow, K. J. (1962), "The Economic Implications of Learning by Doing," *The Review of Economic Studies*, vol. 29(3), no. 80.

Arrow, K. J. (1974), *The Limits of Organization*, Norton and Company, New York.

Garfinkel, R. S. and G. L. Nemhauser (1972), *Integer Programming*, Wiley, New York.

Nordhaus, W. D. (1973), "The Allocation of Energy Resources," *Brookings Papers on Economic Activity*, 3:1973.

Nordhaus, W. D. (1977a), "What Is the Value of Advanced Nuclear Power?" Cowles Foundation Discussion Paper No. 449, Yale University.

Nordhaus, W. D. (1977b), *The Efficient Use of Energy Resources*, forthcoming.

Report of the Modeling Resource Group of the Synthesis Panel (1977), National Academy of Sciences, forthcoming.

Salkin, H. M. (1975), *Integer Programming*, Addison-Wesley, Reading, Mass.