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A SHORTRUN MACROECONOMIC MODEL OF AN OPEN ECONOMY

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by

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Macroeconomic theory is primarily concerned with the domestic consequences of shocks to a closed economy. The international consequences of internal economic policies are ignored as are the domestic implications of economic developments in other countries. Efforts to extend this analysis to open economies have generally involved fairly minor modifications of the basic model. Typically domestic demands for foreign products and foreign demands for domestic products are introduced but price adjustments are neglected. International financial transactions are usually ignored or specified in ad hoc and questionable ways. Two notable exceptions to this characterization are Henderson [1] and Henderson and Sargent [2] who provide fully specified and consistent models of economic agents and international markets. However, Henderson and Sargent assume that physical capital is perfectly mobile and always valued at cost, and Henderson continues to maintain the IS-LM assumption of a single homogeneous commodity whereas the essence of an open economy would seem to be the trading of commodities.

International trade theory has on the other hand, focused on the production, relative prices, and trade of different commodities, but has generally paid insufficient attention to many shortrun macro issues such

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as sticky wages and prices, immobile factors of production, and even the existence of financial markets.

The present paper attempts to fill a small part of this gap between macroeconomics and international trade. It essentially extends the standard Keynesian shortrun macroeconomic model of a closed economy by allowing for two commodities and two such economies which interact through fully specified commodity and financial markets. In many ways this construct is a description of a world economy which is complicated by the existence of two currencies and restrictions on the mobility of labor. The broad conclusion of the paper is that opening the economy in this fashion does not fundamentally alter the effects of monetary and fiscal policies. Instead it provides a broader (if somewhat more complicated) perspective from which one can observe the international consequences of domestic policies.

As always, there are a number of basic assumptions which limit the generality of this analysis but provide more concrete results. The two commodities that I have chosen to separately identify are consumption and investment goods. In part this division is motivated by the belief that consumption and investment demands respond in markedly different ways to changes in wages, prices, and interest rates. In addition, the fraction of national output that is consumed rather than hoarded as reproductive capital has important implications for the future course of an economy. I also assume that the existing stock of physical capital is immobile. While there is no market for the used capital which is already in place, new capital is freely traded. Indeed, it is the divergence between the market value of existing capital and the price of new capital which motivates investment. Labor can move between firms but not between

countries. All interest bearing financial assets are perfect substitutes. Commodity prices are flexible and exhibit purchasing power parity. The flexible wage (Section II) and fixed nominal wage (Section III) cases are both analyzed. Flexible and fixed exchange rate regimes are considered in each of these sections. The basic model is more fully described in Section I.

I. The Basic Model

The two nations in this model are distinguished by labelling one with a star or asterisk (*). The balance sheets for the model and much of the notation are displayed in Table 1. All entries in this table are current period flows measured in the currency of the unstarred country. This is a discrete time model in which the beginning of period stocks of financial assets and physical capital are labelled with a (-1). Each country produces consumer and investment goods* and also has household and government sectors. Labor is mobile between firms but not between countries. Each business sector has a constant returns to scale production function, pays labor its marginal revenue product, and distributes all profits to its equity holders. Investment is financed by the sale of new equity. Household saving can be used to accumulate bonds, equity, and domestic currency. Government spending is financed through the printing of domestic currency and the sale of single period bonds; taxes have been omitted for notational simplicity. There are a sufficient number

*The case of international specialization could be easily handled here by setting a sector's capital stock equal to zero.

TABLE 1: Balance Sheets

	Country				Country*			
	Households	Consumer Goods Firms	Investment Goods Firms	Government	Households	Consumer Goods Firms	Investment Goods Firms	Government
Labor	WS	$-wN_C$	$-wN_I$					
Labor*					ew^*S^*	$-ew^*N_C^*$	$-ew^*N_I^*$	
Dividends and Interest	D+R	$-P_C \frac{\partial Q_C}{\partial K_C} K_C (-1)$	$-P_I \frac{\partial Q_I}{\partial K_I} K_I (-1)$	$-r(-1)B(-1)$	eD^*+eR^*	$-eP_C^* \frac{\partial Q_C^*}{\partial K_C^*} K_C^* (-1)$	$-eP_I^* \frac{\partial Q_I^*}{\partial K_I^*} K_I^* (-1)$	$-er(-1)B^*(-1)$
Consumer Goods	$-P_C C$	$P_C Q_C$		$-P_C G_C$	$-eP_C^* C^*$	$eP_C^* Q_C^*$		$-eP_C^* G_C^*$
Investment Goods		$-P_I I_C$	$P_I Q_I - P_I I_I$	$-P_I G_I$		$-eP_C^* I_C^*$	$eP_I^* Q_I^* - eP_I^* I_I^*$	$-eP_I^* G_I^*$
Currency	$M(-1) - P_C L$			$M - M(-1)$				
Currency*					$eM(-1) - eP_C^* L^*$			$eM^* - eM^*(-1)$
Bonds and Equity	$P_C V - P_C H$	$P_{E_C} [E_C - E_C(-1)]$	$P_{E_I} [E_I - E_I(-1)]$	$B - B(-1)$	$eP_C^* V^* - eP_C^* H^*$	$eP_{E_C}^* [E_C^* - E_C^*(-1)]$	$eP_{E_I}^* [E_I^* - E_I^*(-1)]$	$eB^* - eB^*(-1)$
SUM	0	0	0	0	00	0	0	0

e = exchange rate ($= P_C/P_C^* = P_I/P_I^*$) ; V, V^* = current market value of beginning of period bonds plus equity.

of households who consider domestic bonds and domestic equity perfect substitutes, and a sufficient number who consider domestic and foreign bonds perfect substitutes, so that all bonds and equity are perfect substitutes.

Letting the citizens of the unstarred country own fractions α and β respectively of the equity of the consumer and investment goods industries of their country (with α^* and β^* the corresponding fractions for the citizens of the starred country) real factor incomes are

$$\begin{aligned}
 Y = & \frac{w}{P_C} S + \alpha \frac{\partial Q_C}{\partial K_C} K_C (-1) + \beta \frac{P_I}{P_C} \frac{\partial Q_I}{\partial K_I} K_I (-1) + (1-\alpha^*) \frac{\partial Q_C^*}{\partial K_C^*} K_C^* (-1) \\
 & + (1-\beta^*) \frac{P_I}{P_C} \frac{\partial Q_I^*}{\partial K_I^*} K_I^* (-1)
 \end{aligned}
 \tag{1}$$

$$\begin{aligned}
 Y^* = & \frac{w^*}{P_C^*} S^* + (1-\alpha) \frac{\partial Q_C}{\partial K_C} K_C (-1) + (1-\beta) \frac{\partial Q_I}{\partial K_I} K_I (-1) + \alpha^* \frac{\partial Q_C^*}{\partial K_C^*} K_C^* (-1) \\
 & + \beta^* \frac{P_I}{P_C} \frac{\partial Q_I^*}{\partial K_I^*} K_I^* (-1) .
 \end{aligned}$$

Letting the citizens similarly own fractions γ and γ^* of their nation's bonds, the current period real value of beginning of period financial wealth (including interest payments) for the households of these countries are

$$\begin{aligned}
W &= \frac{M(-1) + [\gamma B(-1) + e(1-\gamma^*)B^*(-1)][1+r(-1)]}{P_C} + \alpha \frac{\partial Q_C}{\partial K_C} \frac{K_C(-1)}{r} \\
&\quad + \beta \frac{P_I}{P_C} \frac{\partial Q_I}{\partial K_I} \frac{K_I(-1)}{r} + (1-\alpha^*) \frac{\partial Q_C^*}{\partial K_C^*} K_C^*(-1) + (1-\beta^*) \frac{P_I}{P_C} \frac{\partial Q_I^*}{\partial K_I^*} K_I^*(-1) \\
(2) \quad W^* &= \frac{eM^*(-1) + [e\gamma^*B^*(-1) + (1-\gamma)B(-1)][1+r(-1)]}{P_C} + (1-\alpha) \frac{\partial Q_C}{\partial K_C} \frac{K_C(-1)}{r} \\
&\quad + (1-\beta) \frac{P_I}{P_C} \frac{\partial Q_I}{\partial K_I} \frac{K_I(-1)}{r} + \alpha^* \frac{\partial Q_C^*}{\partial K_C^*} K_C^*(-1) + \beta^* \frac{P_I}{P_C} \frac{\partial Q_I^*}{\partial K_I^*} K_I^*(-1)
\end{aligned}$$

The supply of labor is assumed to depend upon the real wage measured in terms of consumer goods

$$(3) \quad S = S[w/P_C] \quad , \quad S^* = S^*[w^*/P_C^*]$$

where both functions have positive slopes. Household real demands for consumption, money, and bonds plus equity are:

$$\begin{aligned}
&\quad - \quad + \quad + \\
C &= C[r, Y, W] \\
&\quad - \quad + \quad + \\
L &= L[r, Y, W] \\
&\quad + \quad + \quad + \\
H &= H[r, Y, W] \\
(4) \quad &\quad - \quad + \quad + \\
C^* &= C^*[r, Y^*, W^*] \\
&\quad - \quad + \quad + \\
L^* &= L^*[r, Y^*, W^*] \\
&\quad + \quad + \quad + \\
H^* &= H^*[r, Y^*, W^*] .
\end{aligned}$$

Since these household demands are constrained by income and the market value of beginning of period wealth, they are subject to the adding

up restrictions that the partials with respect to the interest rate sum to zero, and that the partials with respect to income or wealth sum to one.

For each corporate sector in each country, the fixed capital stock means that the demand for labor and the supply of output depend (inversely) only upon the real wage measured in terms of the commodity produced by that sector

$$\begin{aligned}
 N_C &= N_C[\bar{w}/\bar{P}_C] & Q_C &= Q_C[\bar{w}/\bar{P}_C] \\
 N_I &= N_I[\bar{w}/\bar{P}_I] & Q_I &= Q_I[\bar{w}/\bar{P}_I] \\
 N_C^* &= N_C^*[\bar{w}^*/\bar{P}_C^*] & Q_C^* &= Q_C^*[\bar{w}^*/\bar{P}_C^*] \\
 N_I^* &= N_I^*[\bar{w}^*/\bar{P}_I^*] & Q_I^* &= Q_I^*[\bar{w}^*/\bar{P}_I^*] .
 \end{aligned}
 \tag{5}$$

A firm's decision to sell equity and purchase investment goods is assumed to depend (positively) upon a comparison of the cost of investment goods with the market value of the profits that they will yield

$$\begin{aligned}
 I_C &= I_C \left[\frac{P_C}{rP_I} \frac{\partial Q_C}{\partial K_C} \right]^+ \\
 I_I &= I_I \left[\frac{1}{r} \frac{\partial Q_I}{\partial K_I} \right]^+ \\
 I_C^* &= I_C^* \left[\frac{P_C}{rP_I} \frac{\partial Q_C^*}{\partial K_C^*} \right]^+ \\
 I_I^* &= I_I^* \left[\frac{1}{r} \frac{\partial Q_I^*}{\partial K_I^*} \right]^+ .
 \end{aligned}
 \tag{6}$$

Each government determines its domestic money supply and the quantities of consumer and investment goods to purchase, with bond sales residually set so as to satisfy the government's budget constraint. Thus, the government policies that we are explicitly analyzing are the printing of money to purchase bonds and bond financed purchases of commodities. The consequences of other policies are of course implicit in this analysis.

Finally, the equilibrium conditions are:

$$\begin{aligned}
 N_C + N_I &= S \\
 N_C^* + N_I^* &= S^* \\
 C + C^* + G_C + G_C^* &= Q_C + Q_C^* \\
 (7) \quad I_C + I_I + I_C^* + I_I^* + G_I + G_I^* &= Q_I + Q_I^* \\
 L &= M/P_C \\
 L^* &= M^*/P_C^* \\
 H + H^* &= \frac{B + eB^* + P_{E_C} E_C + P_{E_I} E_I + eP_{E_C^*} E_C^* + eP_{E_I^*} E_I^*}{P_C}
 \end{aligned}$$

From the budget constraints, one of these equilibrium conditions is redundant since it is necessarily met when the remaining conditions are satisfied. We will follow the IS-IM tradition of deleting the market for bonds plus equity.

II. Flexible Wages

Using the purchasing power parity implication that $P_I/P_C = P_I^*/P_C^*$, the demands for labor from [5] can be consolidated as:

$$(8) \quad N_C \left[\frac{w}{P_C} \right] + N_I \left[\frac{w}{P_I} \right] \equiv N \left[\frac{w}{P_C}, \frac{P_I}{P_C} \right]$$

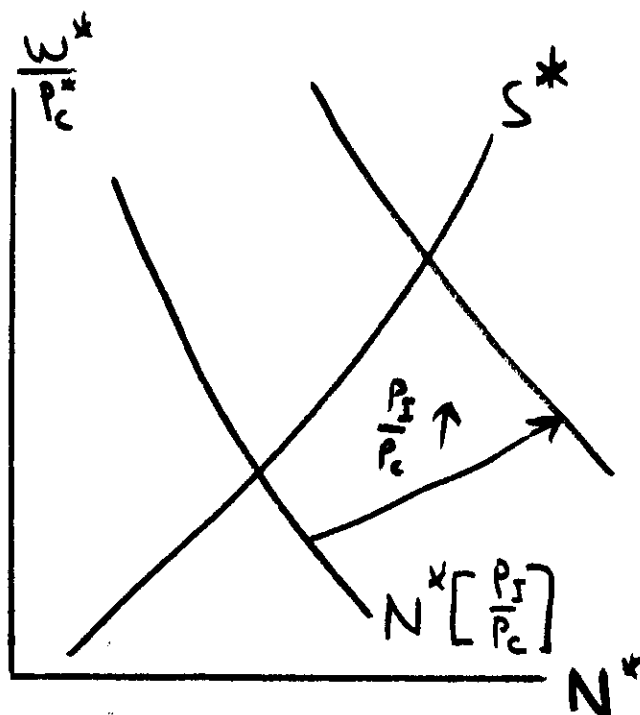
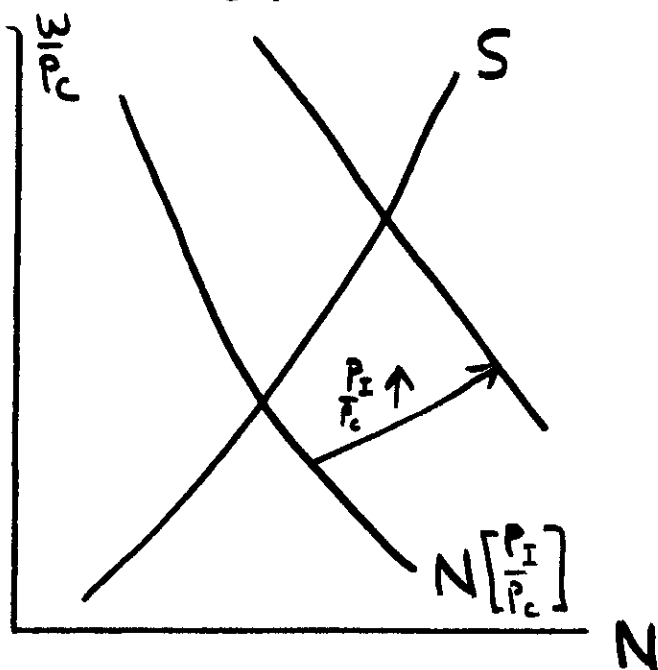
$$N_C^* \left[\frac{w^*}{P_C^*} \right] + N_I^* \left[\frac{w^*}{P_I^*} \right] \equiv N^* \left[\frac{w^*}{P_C^*}, \frac{P_I^*}{P_C^*} \right].$$

The labor market equilibrium conditions from (3), (7), and (8)

$$N \left[\frac{w}{P_C}, \frac{P_I}{P_C} \right] = S \left[\frac{w}{P_C} \right]$$

$$N^* \left[\frac{w^*}{P_C^*}, \frac{P_I^*}{P_C^*} \right] = S^* \left[\frac{w^*}{P_C^*} \right]$$

are graphed below:



Thus given the relative price of investment and consumer goods, each country's labor market will be cleared by the domestic real wage in terms of consumer goods. An increase in the price of investment relative to consumer goods will increase each country's real wages, employment, and the output of investment goods and decrease each country's output of consumer goods. The relative magnitudes of these changes will depend upon each country's production functions, endowments of capital, and labor supply. Since the key endogenous variable for the supply side is P_I/P_C , I will keep explicit track of it when solving the demand side of the model.

Using equations (1), (5) and these results from the supply side, it is now clear that current period real factor income depends only upon the relative price ratio P_I/P_C . An increase in P_I/P_C will increase total labor income and profits in the investment goods industry; however, to the extent that the marginal product of capital in the consumer goods sector falls, profits in this sector will decline. If all equity is domestically held then it can be shown that the net effect of an increase in P_I/P_C will be to raise both Y and Y^* (see Smith-Starnes [3]). I will assume here that the distribution of stock ownership is not so exotic as to reverse this result.

The substitution of the supply sides results into equations (2) yields

$$W = W(r, P_C, e, P_I/P_C)$$

$$W^* = W^*(r, P_C, e, P_I/P_C) .$$

The net effect of an increase in P_I/P_C on financial wealth is ambiguous, but will be positive if the changes in the marginal product of capital

are minor. I will here assume that if an increase in P_I/P_C does reduce wealth while increasing income, that the former will not be large enough to reverse the implications of the latter.

The supply side results also imply that investment demand (6) is a function only of r and P_I/P_C .

$$I_C + I_I = \bar{I}[r, P_I/P_C]$$

$$I_C^* + I_I^* = \bar{I}^*[r, P_I/P_C] .$$

An increase in P_I/P_C will directly lower the profitability of purchasing investment goods in order to produce consumption goods. There are also indirect effects in that the induced changes in employment will lower the marginal physical product of capital in the consumer goods sector but raise it in the investment goods sector. I assume that on balance an increase in P_I/P_C will increase the production of investment goods relative to their demand.

A. Flexible Exchange Rate

The labor market equilibrium conditions have now been substituted out of the four commodity and money equilibria, and these can be solved for four endogenous variables. Because of the two purchasing power parity conditions ($P_C = eP_C^*$ and $P_I = -eP_I^*$), there are actually six remaining endogenous variables: r , P_C , P_I , P_C^* , P_I^* , e . I have chosen to substitute out the purchasing power parity relations so as to explicitly represent the model in terms of r , P_C , e , and P_I/P_C . The other endogenous variables can of course be determined from these variables and some of them are presented in the comparative statics results. The four

equilibrium conditions from (7) are now:

$$\begin{aligned}
 & C^+ [P_I/P_C, P_C, r, e] + C^{*-} [P_I/P_C, P_C, r, e] + G_C + G_C^* \\
 & = Q_C^- [P_I/P_C] + Q_C^{*-} [P_I/P_C] \\
 (9) \quad & I^? [P_I/P_C, r] + I^{*-} [P_I/P_C, r] + G_I + G_I^* = Q_I^+ [P_I/P_C] + Q_I^{*-} [P_I/P_C] \\
 & L^+ [P_I/P_C, r, P_C, e] = M/P_C \\
 & L^{*+} [P_I/P_C, r, P_C, e] = eM^*/P_C .
 \end{aligned}$$

This total differentiation of this system is displayed below in matrix form with each a_{ij} positive:

$$(10) \quad \begin{bmatrix} a_{11} & -a_{12} & -a_{13} & a_{14} \\ -a_{21} & 0 & -a_{23} & 0 \\ a_{31} & a_{32} & -a_{33} & a_{34} \\ a_{41} & a_{42} & -a_{43} & -a_{44} \end{bmatrix} \begin{bmatrix} \Delta P_I/P_C \\ \Delta P_C \\ \Delta r \\ \Delta e \end{bmatrix} = \begin{bmatrix} -\Delta G_C - \Delta G_C^* \\ -\Delta G_I - \Delta G_I^* \\ \Delta M/P_C \\ e \Delta M^*/P_C \end{bmatrix} .$$

The only new assumption embodied in this sign pattern is the conventional IS-LM assumption that an increase in the price level will reduce the supply of money relative to the demand. Specifically I am assuming that

$$\begin{aligned}
 & \left. \frac{\partial (L - M/P_C)}{\partial P_C} \right|_{P_I/P_C, r, e} > 0 \\
 & \left. \frac{\partial (L^* - M^*/P_C^*)}{\partial P_C^*} \right|_{P_I/P_C, r, e} > 0 .
 \end{aligned}$$

The realism of these assumptions rests upon the price induced fall in wealth not having a large effect on the demand for money. More precisely, I am assuming that

$$\frac{\partial L}{\partial W} < \frac{M}{M(-1) + [\gamma B(-1) + e(1-\gamma^*)B^*(-1)][1+r(-1)]}$$

$$\frac{\partial L^*}{\partial W^*} < \frac{M^*}{M^*(-1) + [\gamma^* B^*(-1) + (1-\gamma)B(-1)/e][1+r(-1)]} .$$

Now after some manipulation it can be shown that the Jacobian determinant is negative and that the comparative statics multipliers are those displayed in Table 2. The final row in this table is the balance of trade from the standpoint of the unstarred country measured in terms of consumer goods.

$$BOT = (Q_C - C - G_C) + \frac{P_I}{P_C}(Q_I - I - G_I) .$$

The table entries in parentheses are the comparative statics multipliers for the single commodity model in which consumer and investment goods are perfect substitutes. (Algebraically, P_I/P_C is held constant and the consumer and investment goods markets are combined.) When the one and two commodity models give qualitatively the same result, no parenthetical entry has been made.

The multipliers displayed in Table 2 are generally plausible and straightforward. I will discuss them briefly in a progression from the well known neoclassical single commodity closed economy to the single commodity open economy to the two commodity open economy. In the familiar single commodity closed economy, employment and output are determined on

TABLE 2. Comparative Statics Multipliers with Flexible Wages
and a Flexible Exchange Rate

	ΔG_C	ΔG_C^*	ΔG_I	ΔG_I^*	ΔM	ΔM^*
$\Delta(P_I/P_C)$	- (0)	- (0)	+ (0)	+ (0)	+ (0)	+ (0)
Δr	+	+	+	+	-	-
ΔP_C	+	+	? (+)	? (+)	+	?
ΔP_C^*	+	+	? (+)	? (+)	?	+
Δe	?	?	?	?	+	-
$\Delta C + \Delta G_C$	+	-	? (-)	? (-)	?	?
$\Delta C^* + \Delta G_C^*$	-	+	? (-)	? (-)	?	?
$\Delta Q_C + \Delta Q_C^*$	+	+	-	-	-	-
$\Delta I + \Delta G_I$? (-)	? (-)	+	-	? (+)	? (+)
$\Delta I^* + \Delta G_I^*$? (-)	? (-)	-	+	? (+)	? (+)
$\Delta Q_I + \Delta Q_I^*$	-	-	+	+	+	+
ΔY	- (0)	- (0)	+ (0)	+ (0)	+ (0)	+ (0)
ΔY^*	- (0)	- (0)	+ (0)	+ (0)	+ (0)	+ (0)
ΔBOT	? (-)	? (+)	? (-)	? (+)	?	?

the supply side and spending and portfolio decisions determine only the price level and the interest rate. An expansionary fiscal policy raises the price level and the interest rate, fully crowding out an equal amount of private spending. An expansionary monetary policy raises the price level; if there are no wealth effects on the demands for money and commodities, then the real money supply, the interest rate, and the composition of output are all unchanged. With wealth effects the price increase is less than proportional to the expansion of the money supply; the interest rate falls and commodities are diverted from consumption to investment. (Even without such wealth effects, an increase in the money supply through increased government spending is not neutral; the increased government demand for commodities will crowd out an equal amount of private demand through an increased interest rate, and the resulting reduced demand for money implies an increase in prices that is more than proportionate to the increase in the money supply.)

The open economy analogue displayed parenthetically in Table 2 gives quite similar results. Each country's output is supply determined. ← A bond financed increase in government spending by either nation raises the interest rate and each nation's price level, reducing private consumption and investment in both nations. The composition of output will turn in favor of the type of commodity purchased by the government and the balance of trade will turn against the nation whose government increases purchases. Without wealth effects on money and commodity demands either country's expansionary monetary policy drives up its own price level proportionately without affecting the interest rate or the other country's price level. With wealth effects, an expansionary

monetary policy will lower the interest rate, encourage investment at the expense of consumption and raise domestic relative to foreign prices. While the domestic price level must rise, the foreign price level may actually fall.

The explanation for this is as follows. Without Pigou effects on consumption, the real demand for commodities depends only upon income and the interest rate. With income and the supply of commodities fixed by supply conditions, the world demand is reconciled to this supply by the interest rate. If there are no wealth effects on the demand for money, then the domestic price level will equate each country's real supply of money to the real demand that is predetermined by income and the interest rate. Since a bond financed increase in the money supply does not directly affect the demand or supply of commodities, it does not change the real demand for money; the domestic price level will consequently rise to maintain a constant real supply of money. With Pigou effects on commodity demands, an open market purchase is nonneutral. The domestic increase in prices now reduces the world demand for commodities and hence reduces the interest rate which equates the demand and supply of commodities. Increased investment offsets reduced consumption. In addition, the foreign demand for money is now increased by the lower interest rate, and a compensating increase in the real supply is achieved through a fall in the foreign price level. That is, an inflationary domestic monetary policy reduces foreign prices. Finally, if we also allow wealth effects on the demand for money, then an inflationary monetary policy may raise the foreign price level. Given the foreign price level, an increase in domestic prices will reduce the purchasing power of domestic bonds. If this leads foreigners who hold domestic bonds to reduce their demand for

foreign money by more than the lower interest rate is increasing demand, then the new equilibrium will be with a lower real supply of foreign money and hence a higher foreign price level.

Most of this rather complicated story carries over to the two commodity open economy. One difference is of course that the flexibility of relative commodity prices means that monetary and fiscal policies can change the level as well as the composition of output. The endogenous output level does not reverse the direction of the changes in consumption and investment, but does make the balance of trade effects ambiguous. An increase in government purchases crowds out foreign purchases while expanding foreign output of the commodity purchased; this worsens the balance of trade. It is uncertain though how domestic purchases of the other commodity will change relative to the reduced supply.* Also the changed relative price ratio will directly alter the balance of trade; if for example the country was a net importer of investment goods, then an increase in P_I/P_C will worsen the balance of trade measured in consumer good units.

Wealth effects continue to be an important issue in the two commodity model. Without wealth effects on the demands for money and commodities, monetary policy can only alter domestic prices and the exchange rate. With wealth effects on commodity demands but not on money demands, an expansionary monetary policy will raise domestic prices while unambiguously lowering the foreign price of consumer goods.

For example, an increase in G_C or G_C^ will lower P_I/P_C , increasing consumer goods output. If the marginal physical productivity of capital does not change much, then the lower P_I/P_C will increase the demand for investment goods. The interest rate will rise by enough to reduce the total demand for investment goods by the same amount as supply as fallen; however, either country could on balance increase its demand both relative to supply and absolutely.

B. Fixed Exchange Rate

In a deterministic model, either government could use any of its policy instruments to peg the exchange rate. I will concentrate on the case where the unstarred government allows its domestic money supply to become endogenous by committing itself to engage in whatever open market operations are necessary to fix the exchange rate. For illustrative purposes I will also briefly mention a regime in which the unstarred government uses fiscal policy to peg the exchange rate. In both cases, the starred government uses open market operations to fix its domestic money supply. There are of course many alternative combinations of exchange rate stabilization policies and responses by the other nation which can all be analyzed as linear combinations of the policies explicitly discussed here.

When M is endogenously set so as to peg the exchange rate, the total differentiation of the four demand side equilibrium conditions (10) can be rewritten in matrix form as

$$\begin{bmatrix} a_{11} & -a_{12} & -a_{13} & 0 \\ -a_{21} & 0 & -a_{23} & 0 \\ a_{31} & a_{32} & -a_{33} & -1 \\ a_{41} & a_{42} & -a_{43} & 0 \end{bmatrix} \begin{bmatrix} P_I \\ \Delta P_C \\ \Delta r \\ \Delta M/P_C \end{bmatrix} = \begin{bmatrix} -\Delta G_C - \Delta G_C^* - a_{14} \Delta e \\ -\Delta G_I - \Delta G_I^* \\ -a_{34} \Delta e \\ e \Delta M^*/P_C + a_{44} \Delta e \end{bmatrix}$$

The comparative statics multipliers are contained in Table 3. These multipliers are qualitatively identical to those in Table 2 for the flexible exchange rate case, with one exception: $\Delta P_C / \Delta M^*$ is now positive whereas before it was ambiguous.

The analytics of the situation are as follows. Let Z be some

TABLE 3. Comparative Statics Multipliers with Flexible Wages
and Exchange Rate Fixed by M

	ΔG_C	ΔG_C^*	ΔG_I	ΔG_I^*	ΔM^*	Δe
$\Delta(P_I/P_C)$	- (0)	- (0)	+ (0)	+ (0)	+ (0)	+ (0)
Δr	+	+	+	+	-	-
ΔP_C	+	+	? (+)	? (+)	+	+
ΔP_C^*	+	+	? (+)	? (+)	+	?
ΔM	?	?	?	?	+	+
$\Delta C + \Delta G_C$	+	-	? (-)	? (-)	?	?
$\Delta C^* + \Delta G_C^*$	-	+	? (-)	? (-)	?	?
$\Delta Q_C + \Delta Q_C^*$	+	+	-	-	-	-
$\Delta I + \Delta G_I$? (-)	? (-)	+	-	? (+)	? (+)
$\Delta I^* + \Delta G_I^*$? (-)	? (-)	-	+	? (+)	? (+)
$\Delta Q_I + \Delta Q_I^*$	-	-	+	+	+	+
ΔY	- (0)	- (0)	+ (0)	+ (0)	+ (0)	+ (0)
ΔY^*	- (0)	- (0)	+ (0)	+ (0)	+ (0)	+ (0)
ΔBOT	? (-)	? (+)	? (-)	? (+)	?	?

endogenous variable, X some exogenous variable, and let

$$\frac{de}{dM}, \quad \frac{de}{dX}, \quad \frac{dZ}{dX}, \quad \frac{dZ}{dM}$$

be the multipliers in the flexible exchange rate case. Since the change in e when both M and X change will be

$$\Delta e = \frac{de}{dX} \Delta X + \frac{de}{dM} \Delta M,$$

every unit change of X must be accompanied by the following change in M if e is to remain constant

$$\frac{\Delta M}{\Delta X} = - \frac{de/dX}{de/dM}.$$

Thus the fixed exchange rate multiplier will be

$$\frac{\Delta Z}{\Delta X} = \frac{dZ}{dX} - \frac{dZ}{dM} \frac{de}{dX} \left/ \left(\frac{de}{dM} \right) \right.$$

Thus, an increase in M^* pushes the exchange rate downward. To offset this, the unstarred nation increases M , pushing e back up to its original level. This induced increase in M reinforces the consequences of increasing M^* , with one exception: an increase in M^* would have an uncertain effect on P_C if the exchange rate were allowed to fall; however when M increases by enough to prevent this decline in e , it is also sufficient to insure that on balance P_C will rise.

The effects in Table 3 of an increase in e are of course identical to the flexible exchange rate effects of an increase in M , since it is through an increase in M that e is increased. Increases in government spending have ambiguous effects on e and hence ambiguous

implications for M . While it is unclear whether fiscal policy will be countered or reinforced, it can be shown that the effects will not be reversed. Since this is somewhat unexpected, it is worth discussing why government spending has an ambiguous effect on the exchange rate. The simpler single commodity story will be told here; with two commodities, the story is more complicated but similar. A bond financed increase in the demand for commodities will increase the interest rate and the price level in both nations. The higher interest rate will reduce the demands for both currencies while the higher price level will increase the nominal demand for each currency. Which price level rises the most (i.e., which direction the exchange rate moves) depends upon the relative interest and price elasticities for the two currencies.

The regime in which fiscal policy is used to peg e is completely ambiguous because of the indeterminate effect of government spending on e . I will arbitrarily assume that G_C is used to peg e with $de/dG_C > 0$; if de/dG_C were negative all of the reported comparative statics signs would be reversed. These comparative statics multipliers are displayed in Table 4. Expansionary monetary policy by the unstarred country provokes a contractionary fiscal policy. On balance, the monetary policy succeeds in shifting output toward investment while the fiscal policy succeeds in lowering the price level. An expansionary monetary policy by the starred country induces an expansionary fiscal policy by the unstarred country. The fiscal policy succeeds in discouraging investment while the monetary policy pushes prices upward.

An increase in consumer goods purchases by the starred country is fully offset by a decrease in the starred country's purchases. In a single commodity world, the same holds true of investment goods purchases. In

TABLE 4. Comparative Statics Multipliers with Flexible Wages
and Exchange Rates Fixed by G_C (with $\Delta e/\Delta G_C > 0$)

	Δe	ΔG_C^*	ΔG_I	ΔG_I^*	ΔM	ΔM^*
$\Delta(P_I/P_C)$	- (0)	0 (0)	? (0)	? (0)	+ (0)	- (0)
Δr	+	0	? (0)	? (0)	-	+
ΔP_C	+	0	? (0)	? (0)	-	+
ΔP_C^*	+	0	? (0)	? (0)	-	+
ΔG_C	+	-1	? (-1)	? (-1)	-	+
$\Delta C + \Delta G_C$	+	-1	? (-1)	? (-1)	-	+
$\Delta C^* + \Delta G_C^*$	-	+1	? (0)	? (0)	+	-
$\Delta Q_C + \Delta Q_C^*$	+	0	? (-1)	? (-1)	-	+
$\Delta I + \Delta G_I$? (-)	0	? (+1)	? (0)	? (+)	? (-)
$\Delta I^* + \Delta G_I^*$? (-)	0	? (0)	? (+1)	? (+)	? (-)
$\Delta Q_I + \Delta Q_I^*$	-	0	? (+1)	? (+1)	+	-
ΔY	- (0)	0	? (0)	? (0)	+ (0)	- (0)
ΔY^*	- (0)	0	? (0)	? (0)	+ (0)	- (0)
ΔBOT	? (-)	+	? (0)	? (+1)	? (+)	? (-)

a two commodity world, investment goods continue to have an uncertain effect on the exchange rate and hence on G_C .

III. Fixed Nominal Wages

I will now examine the situation in which nominal wages are fixed at levels such that there are excess supplies of labor in both countries. Employment and output are determined by the demand for labor and increase as rising product prices lower the real wage rate paid by employers. I will consequently now concentrate on absolute rather than relative prices. The substitution of these supply considerations into the definitions of income (1) and wealth (2), yields

$$Y = Y[r, P_I, P_C]$$

$$Y^* = Y^*[r, P_I^*, P_C^*]$$

$$W = W[r, P_I, P_C, e]$$

$$W^* = W^*[r, P_I^*, P_C^*, e]$$

Now substituting these into the demand side equilibria in (7) yields

$$(11) \quad \begin{aligned} C[r, P_C, P_I, e] + C^*[r, P_C^*, P_I^*, e] + G_C + G_C^* &= Q_C[P_C] + Q_C^*[P_C^*] \\ I[P_C, P_I, r] + I^*[P_C^*, P_I^*, r] + G_I + G_I^* &= Q_I[P_I] + Q_I^*[P_I^*] \\ L[r, P_C, P_I, e] &= M/P_C \\ L^*[r, P_C^*, P_I^*, e] &= M^*/P_C^* \end{aligned}$$

A. Flexible Exchange Rate

Using purchasing power parity, and totally differentiating (11) to solve for the equilibrating variables r , P_C , P_I , and e :

$$(12) \quad \begin{bmatrix} -b_{11} & -b_{12} & b_{13} & b_{14} \\ -b_{21} & b_{22} & -b_{23} & b_{24} \\ -b_{31} & b_{32} & b_{33} & b_{34} \\ -b_{41} & b_{42} & b_{43} & -b_{44} \end{bmatrix} \begin{bmatrix} \Delta r \\ \Delta P_C \\ \Delta P_I \\ \Delta e \end{bmatrix} = \begin{bmatrix} -\Delta G_C - \Delta G_C^* \\ -\Delta G_I - \Delta G_I^* \\ \Delta M/P_C \\ e \Delta M^*/P_C \end{bmatrix}$$

The signs of b_{32} and b_{42} repeat the earlier previous assumption that an increase in the price of consumer goods reduces the real supply of money relative to demand; an extra complication here is that an increase in the price of consumer goods not only reduces real wealth, but also alters real income by increasing the output of consumer goods and reducing the purchasing power of income from the investment goods industry. The sign of b_{24} states that when the prices of consumer and investment goods rise by equal percentage amounts, the stimulus to investment demand by both sectors will be less than the induced increase in the output of investment goods. The signs of b_{44} and b_{23} follow from the assumed signs of b_{42} and b_{24} .

The sign of b_{14} is somewhat more difficult:

$$b_{14} = \left(\frac{\partial C}{\partial W} \right) \left(\frac{\partial W}{\partial e} \right) + \left(\frac{\partial C^*}{\partial W^*} \right) \left(\frac{\partial W^*}{\partial e} \right) + \left(\frac{P_C}{e} \frac{\partial Q_C^*}{\partial P_C^*} \right) \left(1 - \frac{\partial C^*}{\partial Y^*} \right) - \left(\frac{P_I^2}{e} \frac{\partial C^*}{\partial Y^*} \right) \left(\frac{\partial Q_I^*}{\partial P_I^*} \right)$$

Where all terms within parentheses are positive. If the sign of b_{14} is left indeterminate, then the sign of the Jacobian is ambiguous. Even with a sign assumption for the Jacobian, the only unambiguous comparative

statics multipliers are $\Delta r/\Delta G_C$, $\Delta r/\Delta G_I$, and $\Delta P_C/\Delta G_C$.

I have consequently made the seemingly reasonable assumption that b_{14} is positive. If the output price elasticities are the same (ϵ) for each sector then

$$b_{14} = \left(\frac{\partial C}{\partial W}\right)\left(\frac{\partial W}{\partial e}\right) + \left(\frac{\partial C^*}{\partial W^*}\right)\left(\frac{\partial W^*}{\partial e}\right) + \frac{E}{P_C} \left[\left(1 - \frac{\partial C^*}{\partial Y^*}\right) P_C^* Q_C^* - \frac{\partial C^*}{\partial Y^*} P_I^* Q_I^* \right].$$

Now, b_{14} will be positive if the fraction of nominal income arising from the consumption goods sector is at least as large as the marginal propensity to consume; if there were no trade, this would state that the household MPC is now greater than the APC for society. This is of course a sufficient condition; the necessary condition is somewhat weaker, depending upon how important the Pigou effects are.

With these assumptions and a considerable amount of tedious manipulation, one can show that the Jacobian determinant is positive and that the signs of the comparative statics multipliers are as displayed in Table 5. These multipliers are surprisingly similar to those for the flexible wage case. One significant difference is of course that increases in aggregate demand may stimulate both sectors whereas before a demand induced increase in the output of one sector necessarily reduced the output of the other sector. A second difference is that expansionary monetary policy need not stimulate the other country's output of either commodity. Since in this rigid wage model an increase in output is synonymous with an increase in prices, this result can be restated as an observation that an increase in prices created by an expansionary monetary policy need not be exported. Unfortunately these two ambiguities create a great deal of uncertainty about the course of aggregate demand for a particular

TABLE 5: Comparative Statics Multipliers with Fixed Wages and a Flexible Exchange Rate

	ΔG_C	ΔG_C^*	ΔG_I	ΔG_I^*	ΔM	ΔM^*
Δr	+	+	+	+	-	-
ΔP_C	+	+	? (+)	? (+)	+	?
ΔP_C^*	+	+	? (+)	? (+)	?	+
ΔP_I	? (+)	? (+)	+	+	+	?
ΔP_I^*	? (+)	? (+)	+	+	?	+
Δe	?	?	?	?	+	-
$\Delta C + \Delta G_C$?	?	?	?	?	?
$\Delta C^* + \Delta G_C^*$?	?	?	?	?	?
$\Delta I + \Delta G_I$?	?	?	?	? (+)	?
$\Delta I^* + \Delta G_I^*$?	?	?	?	?	? (+)
ΔY	? (+)	? (+)	? (+)	? (+)	? (+)	?
ΔY^*	? (+)	? (+)	? (+)	? (+)	?	? (+)
ΔBOT	?	?	?	?	?	?

commodity in a particular country.

B. Fixed Exchange Rate

We will now consider the case where the unstarred country uses its monetary policy to peg the exchange rate. The system (12) can be rewritten as follows:

$$\begin{bmatrix} -b_{11} & -b_{12} & b_{13} & 0 \\ -b_{21} & b_{22} & -b_{23} & 0 \\ -b_{31} & b_{32} & b_{33} & -1 \\ -b_{41} & b_{42} & b_{43} & 0 \end{bmatrix} \begin{bmatrix} \Delta r \\ \Delta P_C \\ \Delta P_I \\ \Delta M/P_C \end{bmatrix} = \begin{bmatrix} -\Delta G_C - \Delta G_C^* - b_{14} \Delta e \\ -\Delta G_I - \Delta G_I^* - b_{24} \Delta e \\ -b_{34} \Delta e \\ e \Delta M^*/P_C + b_{44} \Delta e \end{bmatrix}$$

The Jacobian is negative and the comparative statics multipliers are displayed in Table 6.

Since an increase in the exchange rate is achieved through expansionary domestic monetary policy, its comparative statics effects are identical to those of an expansionary domestic monetary policy. An expansionary foreign monetary policy puts downward pressure on the exchange rate and thereby commits the domestic country to an expansionary monetary policy to stabilize the exchange rate; as a consequence the decline in the interest rate is more pronounced and the prices and outputs of both commodities in both countries increase. Since fiscal policies have ambiguous effects on the exchange rate, the induced monetary policy is uncertain. However, it can be shown that this monetary policy will not reverse the basic fiscal thrusts of increasing the interest rate and both nations' output of the commodity that is increasingly purchased by the government.

TABLE 6: Comparative Statics Multipliers with Fixed Wages and the Exchange Rate Fixed by M

	ΔG_C	ΔG_C^*	ΔG_I	ΔG_I^*	ΔM^*	Δe
Δr	+	+	+	+	-	-
ΔP_C	+	+	? (+)	? (+)	+	+
ΔP_C^*	+	+	? (+)	? (+)	+	?
ΔP_I	? (+)	? (+)	+	+	+	+
ΔP_I^*	? (+)	? (+)	+	+	+	?
ΔM	?	?	?	?	+	+
$\Delta C + \Delta G_C$?	?	?	?	?	?
$\Delta C^* + \Delta G_C^*$?	?	?	?	?	?
$\Delta I + \Delta G_I$?	?	?	?	? (+)	? (+)
$\Delta I^* + \Delta G_I^*$?	?	?	?	? (+)	?
ΔY	? (+)	? (+)	? (+)	? (+)	? (+)	? (+)
ΔY^*	? (+)	? (+)	? (+)	? (+)	? (+)	?
ΔBOT	?	?	?	?	?	?

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