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What Is the Value of Advanced Nuclear Power?

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WHAT IS THE VALUE OF ADVANCED NUCLEAR POWER?*

by

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The United States is currently at a critical decision point for its energy policy: should it continue to push ahead rapidly with the development of nuclear power or not? If the answer is yes, which of the many competing designs for advanced nuclear reactors should be developed?

The present paper is intended to provide a partial answer only to the second question posed above. In doing so it makes no presumption about the answer to the first, nor is there any examination of the environmental, military, safeguards, or moral aspects of the nuclear dilemma. The question posed here is simply: What are the net economic costs or benefits of developing advanced nuclear power reactors?**

Up to now the evaluation of different energy technologies has proceeded on the assumption of a deterministic environment, and with little

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**There is a very long line of similar studies going back to the classical Marschak and Schurr [1952]. One of the later studies is [1977], and this contains many references.

attention to the competitive aspects of different technologies or to the optimal timing of the introduction of different technologies. In the present work, we wish to analyze in some detail the economic benefits to the introduction of advanced nuclear reactors, with attention only on the United States. By contrast with most other work in this area the following R&D analysis greatly simplifies the decision structure, while the uncertainties will be treated in considerable detail.

As far as the decision structure is concerned, the questions we ask are: first, what are the economic benefits, in terms of the discounted sum of costs and benefits, of alternative decisions net of R&D costs but before accounting for environmental costs? In what follows, we will call this accounting concept the "net benefits." The decision concerns the economic benefits of alternative research, development, and commercialization strategies (abbreviated as R&D) for advanced nuclear systems. The decisions analyzed are whether either or both of the fast breeder breeder reactor (LMFBR) and an intermediate technology, the advanced converter (AC), should be pursued. The two possible reactors will be called "advanced nuclear" in the present work. For both reactors, the options analyzed are R&D efforts pursued in such a way that they will be completed in one of five periods, 2000, 2010, 2020, 2030, and "never." Thus the problem is to calculate the discounted sum of net benefits for each of 25 possible decisions.

In the analysis presented here, the attention is primarily on the "downstream" decisions. That is, what are the benefits of having R&D completed by one of the dates mentioned above. The R&D comprises all research, prototype or demonstration plants, and early commercial plants up to having 6 GW^e installed capacity. Thus the analysis is relatively aggregated, and does not investigate the important but detailed engineering and logistical planning problems involved in reaching the 6 GW^e level. This approach can be rationalized as follows: before a detailed examination of exactly how a given option should be implemented (where should plants be built, who will be the vendors, what form of pricing or taxation should be used to pay for the R&D, etc.?) an aggregative assessment of each option, like that in MRG [1977], should be made. If the aggregative assessment is definite one way or the other, the results will give a good indication as to what detailed implementation plans are likely to be fruitful.

In the R&D analysis, the uncertainties which face decisionmakers are highlighted and analyzed in great detail. In earlier work, it was seen that there are at least eight important variables which influence the net benefits of having an advanced nuclear technology:[1]

1. What will be the rate of growth of energy demand?
2. Will coal and shale be available for large-scale future deployment?
3. What will be the cost of alternative "backstop" technologies for generating electricity?
4. What will be the cost of alternative "backstop" technologies for replacing oil and gas?
5. What will be the Uranium supply curve?
6. What will be the capital cost of the LMFBR?
7. What will be the capital costs of the advanced converters?
8. What nuclear reactors designs, if any, will be acceptable from an economic and environmental viewpoint?

[1] See MRG [1977], especially Sections II and III.

The exact meaning of these questions is discussed below, pp. 11.

In what follows, we will refer to each joint realization or resolution of the uncertainties as a state of the world. It should be stressed not only that by explicitly treating these 8 uncertainties, we have opened up the analysis to an enormous number of possible outcomes or states of the world; but also, that there are other uncertainties which are not treated and will affect the outcome (such as the possible breakup of OPEC, breakthroughs in abatement techniques for generating electricity from coal, or a large-scale war). In some cases we have explicitly tested for importance of variables; in others, omission is due either to prior expectation or simple lack of time and imagination.

To summarize then, the purpose of the R&D analysis is to ask: what is the net benefit of each of the possible decisions about advanced nuclear technologies in each state of the world? and, given a set of judgmental probabilities, what is the expected value of the benefit of each of the decisions?

A. General Description of the Technique for Estimating Benefits

We will first outline the detailed steps involved in making the calculations, with further details given in Section B. The process of estimating the value of decisions about advanced nuclear reactors involved four different steps:

First, among the 25 x 10,368 different outcomes (25 decisions and 10,368 states of the world) a small number of runs were chosen, giving eventually approximately 100 "observations." The runs were chosen so as to minimize the expected error in our calculation, which meant sampling more heavily those runs where the expected value of the advanced nuclear was high and those runs which were relatively likely. In some cases the choice was judgmental, while others were clearly important from past runs or more likely according to the preliminary probability questionnaires. The chosen runs were then made using a slightly modified version of the earlier BULLDOG model.[1] The main modification was the introduction of an integer-programming code, so that in each state of the world the calculation would always find the decision which yielded the highest net benefit. From these runs were obtained the estimated value of R&D decisions conditional on the state of the world.

Second, it was necessary to interpolate between different runs for the missing observations. This step proceeded in two stages: first, from a priori information about the structure of the underlying economic problem, a small model was constructed which gave the "predicted value" of the decision conditional on the state of the world. Because the small model had much smaller dimensionality than the large model, the "predicted values" had errors. In a second stage, a regression analysis was performed in which the "actual values" (i.e., the values coming from the BULLDOG model runs) were regressed on the "predicted values" as well as decision and uncertain variables. This stage tightened up the fit considerably, so that in a weighted logarithmic regression, the standard error of estimate was approximately 10 percent of the actual value. In making the runs, then, the predicted values from the regression model was used in estimating the value of a decision in each state of the world. It is important to note that the regression technique assures that the weighted

[1] See MRG [1977] for a brief description. A fuller discussion will be forthcoming in Nordhaus [1977].

sum of the prediction errors in the regression is exactly zero, so no systematic errors are introduced by this procedure.

The third step of the analysis used the information from the mathematical programming and regression analysis discussed in steps one and two above with judgmental probability estimates to calculate the expected value of the decisions. The procedure described in step two allows us to estimate the value of all decisions for all the different states of the world. We use the results of the judgmental probability questionnaires[1] to provide estimates of the probability of each state of the world. Combining these, then we obtain the distribution of the net benefits of each of the decisions net of R&D costs but before considering environmental costs.

A final detail in the procedure is the question of the sequential revelation of the outcomes of uncertain events. Each of the eight uncertainties shown above will be resolved at some point, and once that uncertainty has been resolved the expected value of decisions will generally change and the rank ordering of decisions may also change. At present we have treated them in a very simple manner: it is assumed that <u>none</u> of the uncertainties will be resolved before the decision about advanced nuclear systems is made. For example, if we wish to decide about capital cost of the LMFBR, it is assumed that we must proceed through the R&D on the LMFBR before we can "buy" that knowledge. Similarly, it is assumed that the acceptability of the advanced converters will not be known before the full R&D decision has been made.

It is clear that the assumption about resolution of uncertainty is extreme; in particular, it is probable that a great deal will be learned

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[1] Reported from the Second Round, see MRG [1977], Section 4.

about uranium supplies, demand growth, and coal technologies in the next 20 years. At a later stage in the analysis, in sections D and E, we present further analysis to test the difference which earlier resolution of uncertainties might make. We do, however, have some general idea of the importance of resolution of uncertainty. For example, the uncertainty about energy demand functions, as well as coal and other nonelectric technologies, will probably be resolved independently of the decision about advanced nuclear power, and these variables are thus exogenous uncertainties. In these cases, then, there is value in postponing decisions, for the "insurance premia" will decline. Other uncertainties--such as the two mentioned in the last paragraph--are probably integrally linked to the outcome of the R&D effort itself. These R&D uncertainties will not completely evaporate over time and there is less value to postponing decisions than in the case of the exogenous uncertainties.

In general then, the assumption of no revelation of uncertainties is likely to bias the estimates in favor of early decisions. A tentative judgment on the importance of this assumption is given in Section D.

It will be helpful to write the decision problem more explicitly. The economy consists of a set of exogenous or driving variables, $x(t) = [x_1(t), \ldots, x_n(t)]$, which, by assumption, evolve over time independently of the energy sector. In addition, there are a number of decision variables which are determined in part by market forces, in part by political processes. Of these decision variables, we will be considering explicitly here only the R&D decision for advanced nuclear, $d(t) = [d_1(t), d_2(t)]$. In what follows, $d_i(t)$ represents the decision whether to undertake R&D so as to have reactor type i at a 6 GW^e (6,000 megawatt electric) by year t. If $d_i(t) = 0$, this implies the R&D

is not undertaken while if $d_i(t) = 1$ the R&D is undertaken. The variable $d_i(t)$ is integer and clearly $\sum_{t=1}^{5} d_i(t) \leq 1$. In this formulation $d_i(t)$ represents the decision to have the research, development, and commercialization on a particular reactor type at an installed capacity of 6 GW^e by the year t.

In the decision analysis to follow, the two decisions which we investigate are the question of when, if ever, to perform the R&D on two particular reactor designs, where reactor 1 is an "advanced converter," and reactor 2 is a "breeder." These two decisions are represented, respectively by a High-Temperature Gas-Cooled Reactor with a conversion ratio of 0.82, and a Liquid Metal Fast Breeder Reactor, such as that envizaged in the Clinch River and later developments.[1] The time periods investigated are $t = 1$ for 2000, $t = 2$ for 2010, $t = 4$ for 2030, and $t = 5$ for the decision not to implement the R&D, or alternatively to implement it in 2040 or later.

In the analysis that follows, we will investigate 8 exogenous random variables, which are listed on page 3 above and described in Table 1 below. As a first approximation we assume that the variances are drawn from a discrete probability distribution $P(x = \tilde{x})$, where the tilde ($\sim$) indicates that $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_8)$ is a realization of the vector x. The distribution P is then characterized by the numbers $P^j = P(x = \tilde{x}^j)$, where $j = 1, 2, \ldots, S$ runs over all S states of the world considered in the analysis.

The procedure for estimating the value of having one or two advanced

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[1] See particularly ERDA-1 for a discussion of the characteristics and fuel cycle of each of these designs.

nuclear reactor types starts with an estimate of the value in each state of the world. Let $M(d, \tilde{x}^j)$ be an estimate of the present value of a "program" which develops advanced nuclear technology according to the decision d, where the exogenous random variables take values $\tilde{x}^j$. The function M is estimated from the mathematical programming energy model developed above, and the present value represents the optimized or maximized (hence M) value of the objective function subject to the demand and resource constraints. In this calculation, we normalize by setting $d = 0$ as the "base," i.e. that where no R&D is performed and where, therefore, no new nuclear technologies are developed. Then the value of a particular decision about nuclear R&D is $V(d, \tilde{x}^j)$, where V is defined as the difference in the optimized values of the two decisions:

$$V(d, \tilde{x}^j) = M(d, \tilde{x}^j) - M(0, \tilde{x}^j) \tag{1}$$

V is therefore the value of decision d <u>conditional on the state of the world</u>.

The objective of the R&D analysis is to estimate the expected value of the decision over all states of the world. Thus let $\overline{V}(d)$ be the expected value of the decisions. Then

$$\overline{V}(d) = \sum_{j=1}^{S} P^j V(d, \tilde{x}^j) . \tag{2}$$

The final purpose, of course, is to calculate the optimal decision, d^*, where

$$d^* = \{d^* \text{ such that } \overline{V}(d^*) \geqq V(d), \text{ all } d\} . \tag{3}$$

The procedure just outlined suffers from the difficulty that it is infeasible to calculate all--or even a sizable fraction--of the states of the world. In a complete analysis, there are $N = 25 \times S$ different pairs of a policy and a state of the world, to be called from here on "policy-state combinations." In the estimation that follows, we will estimate directly the value of advanced nuclear reactors in a small subset of the policy-state combinations, and will rely on statistical estimation for the rest.

More precisely, let Ω_1 be a subset of the first N integers for which direct calculations are made, and Ω_2 be the rest of the first N integers. If we designate each of the N policy-state combinations by one of the first N integers, we have the following procedure: For all policy-state combinations whose index lies in Ω_1 we perform a direct calculation of the value of an advanced nuclear reactor decision. Let $V^i = V(d^i, \tilde{x}^i)$ be the estimated value of policy-state combination $(d^i, \tilde{x}^i)$, where $i \in \Omega_1$. For these, the exact value of the estimate, V, is known.

For the other policy-state combinations, statistical estimates of the estimates are prepared. These predict from a nonlinear regression the value in a policy-state combinations by

(4) $$\hat{V} = H(d,x) ,$$

where H is a suitable, nonlinear function.

Finally, the expected value of policies are calculated from the actual (in (1)) or predicted (in (4)) values of decisions:

$$(5) \qquad \hat{\hat{V}}(d) = \sum_{j \in \Omega_1} P^j V(d^j, x^j) + \sum_{j \in \Omega_2} P^j H(d^j, x^j) .$$

To summarize, the estimates of the value of the decisions are drawn from estimates of the value of decisions in a set of states of the world. The estimates of the values in each state of the world are obtained from the mathematical programming model, while the values of the probabilities and thence the calculations are drawn from the questionnaires prepared by the Modeling Resource Group of CONAES.

B. Detailed Description of Estimates of Value of Advanced Nuclear[1]

1. The judgmental probability estimates

We now present the details of the calculations for the value of advanced nuclear summarized in a formal way above. The first question of importance deals with the uncertainties to be taken into account. In the course of preparing the MRG report discussed above, it was found necessary to estimate the probabilities of different outcomes. Under the guidance of Dr. Harry Davitian of Brookhaven, a set of questionnaires was prepared and circulated to a small group of specialists in the CONAES study. Of the 14 questions for which responses were gathered, these were compressed to the eight which are shown in Table 1, with the note indicating how the compression occurred. Table 2 then shows the mean probability estimates associated with each random variable.

Among the results, the following appear the most notable. First, according to the group, the future rate of growth of GNP will be consider-

[1]The present section is extremely detailed and may be skipped by readers interested only in results.

TABLE 1. Questions in Probability Assessment for Advanced Nuclear

1.A. What is the growth in GNP, 1975-2010, as a proxy variable for Energy Demand?

1) 1.2 percent annually
2) 3.1 percent annually
3) 4.1 percent annually

B. Non-price induced conservation; linearly from 1975 to 2010

1) 10% decrease in constant of demand function
2) No change in constant of demand function

2. What will be the asymptotic upper limits on U.S. coal and shale annual production, aside from resource availabilities?

1) infinity
2) 62 quads per year

3. What will be the cost (and price) of "clean synfuels," or an alternative inexhaustible liquid or gaseous fuel?

1) $2 per million Btu
2) $5 per million Btu
3) $8 per million Btu

4. Public acceptance, safety, and technical feasibility of nuclear:

1) No nuclear or LWR* with Pu recycle acceptable
2) Advanced converters and LWR acceptable
3) Breeders and LWR acceptable
4) All nuclear acceptable

5. Uranium resources, up to marginal cost of $150 per pound

1) 2.1 million short tons
2) 5.0 million short tons
3) 9.2 million short tons
4) 13.5 million short tons

6. Capital cost differential between LWR and LMFBR

1) 0
2) 25%
3) 50%

7. Capital cost differential between LWR and advanced converter

1) 0
2) 10%
3) 25%

8. Capital cost of inexhaustible electric energy, as fraction of LWR capital cost, the best of solar or fusion:

1) 1.25 times LWR capital cost
2) 1.6 times LWR capital cost
3) 2.7 times LWR capital cost
4) 4.0 times LWR capital cost

*LWR = light water reactor.

TABLE 1 (continued)

Notes: In general, see MRG [1977] for a complete discussion of background definitions.

Question 1: In MRG [1977], the energy demand question was a compounded one involving not only GNP growth but also shift factors, income elasticities, price elasticities, and population growth. As the elasticities were estimated separately, only the GNP relation was used in the present study. Question 1.B was then combined with 1.A by modifying the probabilities of the GNP growths so that mean energy demand was the same. This leads to the estimate shown in Table 2.

A final modification for the simplified model was to use GNP growth 1975-2030, as the later part of the period is more important for demand for advanced nuclear.

Question 3 omits a question on oil imports for period 1990-2010. In trial runs this variable appeared to be unimportant for advanced nuclear decisions.

Question 4 combines no nuclear with LWR only as these both imply that no advanced nuclear is allowed.

Question 5 combined all resources below $150 per pound for presentation (but not in the calculations), and defines in terms of marginal cost rather than "selling price." The first category was determined by fitting graphically a lognormal distribution through the three other points.

Question 8 is derived by estimating the probability that the <u>minimum</u> of solar of fusion costs--each estimated separately--will equal the given figure. The estimates for each source are derived by interpolating linearly between point estimates.

TABLE 2. Values of Critical Random Variables Used in R&D Analysis

I Index	Variable	Values and Probability SOW(I,1)	SOW(I,2)	SOW(I,3)	SOW(I,4)
1	GNP growth				
	Average growth rate 1975-2030, percent per annum	1.2	3.1	4.1	
	Probability: mean	.26	.58	.16	
	s.d.	(.28)	(.24)	(.14)	
2	Coal/shale limits				
	Production limit (quad/yr)	∞	62.		
	Probability: mean	.38	.62		
	s.d.	(.26)	(.26)		
3	Clean synfuels				
	Cost, $ per mmbtu	$2	$5	$8	
	Probability: mean	.18	.56	.27	
	s.d.	(.13)	(.16)	(.18)	
4	Public acceptance of advanced nuclear				
	Event	No advanced nuclear	AC only	LMFBR only	All
	Probability: mean	.30	.16	.14	.40
	s.d.	(.12)	(.07)	(.15)	(.27)
5	Uranium resources				
	10^6 tons up to $150 lb	2.1	5.0	9.2	13.5
	Probability: mean	.06	.17	.40	.37
	s.d.	n.a.	(.17)	(.20)	(.25)
6	Capital cost, AC				
	Cost as multiple of LWR	1.0	1.1	1.25	
	Probability: mean	.15	.52	.33	
	s.d.	(.09)	(.07)	(.14)	
7	Capital cost, LMFBR				
	Cost as multiple of LWR	1.0	1.25	1.5	
	Probability: mean	.10	.56	.34	
	s.d.	(.07)	(.11)	(.14)	
8	Electric AES (Best of solar or fusion)				
	Capital cost as multiple of LWR	1.25	1.6	2.7	4.0
	Probability: mean	.44	.25	.23	.08
	s.d.	n.a.	n.a.	n.a.	n.a.

Source: MRG probability assessment questionnaire, as in MRG [1977].

n.a. = not available.
s.d. = standard deviation.

ably lower than the historical growth rate. Second, the group felt that there was approximately a two in three chance that there would be serious constraints on the growth of shale and coal production. Perhaps the most surprising result was that the group felt that there was about a 50 percent chance that either solar or fusion would turn out to have essentially the same cost structure as nuclear or coal generated electricity. Finally, the group thought that there were six chances in ten that some kind of public acceptance constraints would exist on the deployment of nuclear power.

It should be noted that the standard deviations of the responses were fairly large. It is easily confirmed that the maximum divergence of opinion is for a standard deviation of 0.50 (when half the respondents have 0 and half have 1). Thus if the standard deviations in Table 2 are doubled, this provides a rough estimate of the interquartile range of the judgmental probability estimates. As can be seen, fairly high conformity is obtained for clean synfuels, the capital cost of the advanced converter and the LMFBR, and for public acceptance of advanced nuclear. Conformity is however relatively low on GNP growth, coal and shale limits, and Uranium resources. In summing up the results of the probability assessments, the MRG report states as follows (MRG [1977]):

> The range in the individual responses...is large. These observations indicate that a close consensus was not achieved among the respondents after two rounds. Accordingly, the mean judgmental probability estimates as incorporated in the R&D decision tree should not be interpreted as providing a precise definition of the uncertainties.

This reservation applies equally to the R&D decisions analysis presented in this paper.

2. Choice of runs

Armed with these probability estimates, the next question concerns the subset of policy-state-pairs for which the values are computed directly, with the remainder to be computed by the "small model" alluded to in Section A. How should the sample be chosen? As far as sampling is concerned, the method is taken to be subjectively unbiased, and the objective of the sampling is to choose a set of runs which bestows minimum variance on the estimate of the value of decision used here.

Let $\textcircled{\Sigma}_d$ be the judgmental variance of the estimator $\hat{\bar{V}}(d)$ of $\bar{V}(d)$, i.e.

$$\begin{aligned}
\textcircled{\Sigma}_d &= E\{[\hat{\bar{V}}(d) - \bar{V}(d)]^2\} \\
&= E\{[(\sum_{j\in\Omega_2} P^j H(d^j, \tilde{x}^j)) - (\sum_{j\in\Omega_2} P^j V(d^j, \tilde{x}^j))]^2\} \\
&= \sum_{j\in\Omega_2} (P^j)^2 E\{H(d^j, \tilde{x}^j) - V(d^j, \tilde{x}^j)\}^2 .
\end{aligned}$$

If the judgmental variance of different points is independent, our problem is to sample so as to minimize

$$\textcircled{\Sigma}_d = \sum_j (P^j)^2 \hat{\sigma}^2(\hat{V}^j) ,$$

where $\hat{\sigma}^2(\hat{V}^j)$ = (judgmental) variance of the estimate of V in policy-state-combination j .

This analysis suggests a procedure whereby the sampling is directed to those SOW, where the product of judgmental probability and forecast error would be high if these SOW were left out of the sample.

In the actual choosing of runs, a list of possible states of the world, with probabilities, values, and subjective variances was written down. It was envisaged that approximately 100 runs would be made, and these were concentrated on the high probability and the high variance states of the world. It should be noted that, since the V functions are inherently non-negative, and because a great deal of the structure was apparent, the value of advanced nuclear in many of the states of the world was obviously zero, so no estimates were necessary.

It was anticipated that, after a first round of observations was obtained, further runs would be made. This plan was dropped when it became obvious that the difficulty was with the statistical analysis (Section 3 below). Therefore, after the initial set of direct estimates, no more were made.

Table 3 shows the values of the observations found in the runs. No data other than those shown were collected.

3. Statistical analysis

The most difficult part of the estimation turned out to be the estimation of the nonlinear function $H(d,x)$ in equation (4). The basic problem was that the actual function is highly nonlinear in the exogenous and policy variables, while regression analysis thrives only in a linear culture.

The journey to the final form was a pilgrimage, but the criterion was simply to find the best fit which also has the proper *a priori* signs. Further, in order to preclude the (remotely) possible chance of selecting a regression which conformed to *a priori* ideas about what the outcomes should be, the following procedure was decided on and followed: (a) In the first stage, the statistical analysis would proceed blindly in the

Table 3. Results of Direct Caclulations of Value of Advanced Nuclear in Different Policy States of World

X_1	X_2	X_3	X_4	X_5	X_6	X_7	D_1	D_2	V	wt
0	3.1	2.	5.	5.	1.0	1.25	1	1	16.5	1.2
0	3.1	2.	5.	5.	1.0	1.25	5	5	0.01	5.2
0	3.1	2.	5.	9.2	1.25	1.0	2	2	2.0	1.5
0	3.1	2.	5.	2.1	1.1	1.0	2	2	2.4	1.5
0	3.1	1.25	5.	9.2	1.1	1.25	1	1	2.1	1.5
0	3.1	1.25	5.	9.2	1.1	1.25	5	5	0.01	5.
1	4.1	1.25	5.	1.9	1.1	1.25	1	1	5.	1.
1	4.1	1.25	8.	1.9	1.1	1.25	1	1	10.	1.
1	4.1	1.25	8.	1.9	1.1	1.25	5	5	0.01	2.
1	4.1	2.	5.	9.2	1.1	1.0	1	1	170.	1.
1	4.1	2.	5.	9.2	1.0	1.25	1	1	121.	1.
1	4.1	2.	5.	9.2	1.1	1.5	1	1	39.5	1.
1	4.1	2.	5.	9.2	1.25	1.25	1	1	79.5	1.
1	4.1	2.	5.	9.2	1.0	1.25	5	1	117.	1.
1	4.1	2.	5.	9.2	1.25	1.25	5	1	42.	1.
1	4.1	2.	5.	9.2	1.1	1.0	1	5	185.	1.
1	4.1	2.	5.	9.2	1.1	1.5	1	5	15.	1.
1	3.1	2.	5.	2.1	1.1	1.25	5	5	.01	1.
1	3.1	2.	5.	2.1	1.1	1.25	5	1	33.7	1
1	3.1	2.	5.	2.1	1.1	1.25	2	5	38.3	1.
1	3.1	2.	5.	2.1	1.1	1.25	1	5	45.6	1.
0	3.1	2.	5.	9.2	1.1	1.25	5	1	1.5	1.
0	3.1	2.	5.	9.2	1.1	1.25	1	5	0.01	1.
0	3.1	2.	5.	9.2	1.1	1.25	5	5	0.01	1.
0	3.1	2.	5.	9.2	1.1	1.25	5	5	0.01	1.
1	3.1	2.	5.	9.2	1.1	1.25	2	5	4.51	1.
1	3.1	2.	5.	9.2	1.1	1.25	3	5	4.61	1.
1	3.1	2.	5.	9.2	1.1	1.25	5	5	0.01	1.
1	3.1	2.	5.	9.2	1.1	1.25	5	1	7.72	1.
1	3.1	2.	5.	9.2	1.1	1.25	5	2	7.09	1.
1	4.1	4.	8.	2.1	1.1	1.25	1	1	556.	.01
1	4.1	4.	8.	2.1	1.1	1.25	1	5	526.	.01
1	4.1	4.	8.	2.1	1.1	1.25	5	1	167.	.01
1	4.1	4.	8.	2.1	1.1	1.05	5	1	601.	.01
1	4.1	4.	8.	2.1	1.1	1.25	5	1	451.	.01
1	4.1	4.	8.	2.1	1.0	1.25	1	5	184.	.01
1	4.1	4.	8.	2.1	1.25	1.25	1	5	149.	.01
1	4.1	4.	8.	2.1	1.1	1.25	2	5	335.	.01
1	4.1	4.	8.	2.1	1.1	1.25	5	5	.01	.1
1	4.1	2.	5.	2.1	1.1	1.25	1	1	290.	.1
1	4.1	2.	5.	2.1	1.1	1.25	5	1	68.1	.1
1	4.1	2.	5.	2.1	1.1	1.25	1	5	269.	.1
1	4.1	2.	5.	2.1	1.1	1.25	2	5	205.5	.1
1	1.2	3.	5.	2.1	1.1	1.25	2	5	2.1	.1
1	1.2	3.	5.	2.1	1.1	1.25	3	5	2.1	.1
1	1.2	3.	5.	2.1	1.1	1.25	4	5	1.8	.1
1	1.2	3.	5.	2.1	1.1	1.25	5	1	3.7	.1
1	1.2	3.	5.	2.1	1.1	1.25	5	2	3.5	.1
1	1.2	3.	5.	2.1	1.1	1.25	5	3	2.7	.1
1	1.2	3.	5.	2.1	1.1	1.25	5	3	2.7	.1

Symbols:

X_1 = 1 if coal and shale limits, = 0 if no coal and shale limits.

X_2 = GNP growth rate, % per annum, 1975-2030.

X_3 = best of solar or fusion capital costs (x .74 xLWR capital costs)

X_4 = clean synfules cost, $ per million Btu

(continued on next page)

Table 3. (continued)

X_1	X_2	X_3	X_4	X_5	X_6	X_7	D_1	D_2	V	wt
1	3.1	2.	8.	9.2	1.1	1.25	5	1	16.30	2.
1	3.1	2.	8.	9.2	1.1	1.25	1	5	12.	2.
1	3.1	2.	8.	9.2	1.1	1.25	2	5	12.1	2.
1	3.1	2.	8.	9.2	1.1	1.25	3	5	10.6	2.
1	3.1	2.	8.	9.2	1.1	1.25	5	5	0.01	2.
1	3.1	2.	8.	9.2	1.1	1.25	5	5	0.01	2.
0	1.2	2.	8.	5.	1.1	1.25	1	1	.01	6.7
0	1.2	2.	5.	5.	1.1	1.25	1	1	.01	6.7
1	1.2	3.	8.	5.	1.1	1.25	1	1	.01	6.7
1	3.1	2.	8.	9.2	1.	1.25	5	1	[illegible]	2.
1	3.1	2.	8.	9.2	1.25	1.25	5	1	.01	2.
1	3.1	2.	8.	9.2	1.1	1.	1	5	[illegible]	2.
1	3.1	2.	8.	9.2	1.1	1.5	1	5	.01	2.
1	3.1	2.	8.	9.2	1.1	1.	2	5	[illegible]	2.
1	3.1	2.	8.	9.2	1.1	1.5	2	5	.01	2.
1	3.1	2.	8.	9.2	1.1	1.	3	5	22.	2.
1	3.1	2.	8.	9.2	1.1	1.5	3	5	.01	2.
0	3.1	2.	5.	9.2	1.1	1.0	1	1	10.9	2.4
0	3.1	2.	5.	9.2	1.1	1.0	1	1	10.9	.01
1	4.1	1.5	5.	2.1	1.1	1.25	1	1	20.	.001
1	4.1	1.0	5.	2.1	1.1	1.25	1	1	2.	.001
0	3.1	2.	5.	2.1	1.1	1.25	1	1	16.	1.
1	3.1	2.	5.	2.1	1.1	1.25	1	1	30.	1.
0	4.1	2.	5.	9.2	1.1	1.25	1	1	6.	8.
0	4.1	2.	5.	9.2	1.1	1.25	5	1	5.	8.
1	4.1	3.	8.	9.2	1.1	1.25	1	1	231.8	.7
1	4.1	3.	8.	9.2	1.1	1.25	5	1	187.9	.7
1	4.1	3.	8.	9.2	1.1	1.25	1	5	311.5	.7
1	4.1	3.	8.	9.2	1.1	1.25	2	5	263.4	.7
1	4.1	3.	8.	9.2	1.1	1.25	5	2	116.2	.7
1	4.1	3.	8.	9.2	1.1	1.25	2	2	258.4	.7
1	3.1	2.	8.	9.2	1.1	1.25	5	5	.01	8.
1	3.1	2.	5.	9.2	1.1	1.25	5	5	.01	10.
0	3.1	2.	5.	9.2	1.1	1.25	5	5	.01	1.2
0	1.2	2.	8.	5.	1.1	1.25	5	5	.01	6.7
0	1.2	2.	5.	5.	1.1	1.25	5	5	.01	6.7
1	1.2	3.	8.	5.	1.1	1.25	5	5	.01	6.7
1	3.1	2.	8.	9.2	1.	1.25	5	5	.01	2.
1	3.1	2.	8.	9.2	1.25	1.25	5	5	.01	2.
1	3.1	2.	8.	9.2	1.1	1.	5	5	.01	6.
1	3.1	2.	8.	9.2	1.1	1.5	5	5	.01	6.
0	3.1	2.	5.	9.2	1.1	1.25	5	5	.01	6.
0	3.1	2.	5.	9.2	1.1	1.	5	5	.01	2.4
0	3.1	2.	5.	1.0	1.1	1.25	5	5	.01	6.
1	3.1	2.	8.	1.0	1.1	[illegible]	5	5	.01	6.
0	4.1	2.	5.	9.2	1.1	1.25	5	5	.01	16.
1	4.1	2.	8.	9.2	1.1	1.25	5	5	.01	4.2
0	3.1	2.	5.	9.2	1.1	1.25	5	5	.01	1.2
0	1.2	2.	5.	9.2	1.1	1.25	5	5	.01	1.2
1	4.1	2.	5.	9.2	1.1	1.25	1	5	99.6	1.
1	4.1	2.	5.	9.2	1.1	1.25	2	1	96.8	1.
1	4.1	2.	5.	9.2	1.1	1.25	1	1	104.	1.
1	4.1	2.	5.	9.2	1.1	1.25	5	1	87.2	1.
1	4.1	2.	5.	9.2	1.1	1.25	2	5	57.6	1.
0	3.1	2.	5.	9.2	1.25	1.25	1	1	0.01	1.2
0	3.1	2.	5.	9.2	1.0	1.25	1	1	12.5	2.4
0	3.1	2.	5.	9.2	1.25	1.25	5	1	0.01	1.2
0	3.1	2.	5.	9.2	1.0	1.25	5	1	16.5	1.2

Symbols (cont.):

X_5 = Uranium resources, 10^6 tons up to $150/lb.

X_6 = capital cost advanced converter (x LWR capital cost)

X_7 = capital cost LMFBR (x LWR capital cost)

D_1 = decision on advanced converter } 1 = 2000, 2 = 2010, 3 = 2020,

D_2 = decision on L M F B R } 4 = 2030, 5 = 2040 or later

V = gross value of new technology (= 0.01 for no advanced nuclear)

wt = probability weight x 100.

sense of attempting to get the best fit for $H(d,x)$ in equation (4) without using any runs to calculate the value of R&D decisions. (b) Once the statistical analysis was satisfactory, and the approximation function was chosen, that function H, would be set in concrete and not again be changed. Only after the approximation function was chosen would the value of decisions be estimated as in equation (5), and no changes in the approximation function would be allowed after the first run was made.

The statistical analysis proceeded in two steps. In the first step a small *a priori* model was used to estimate the value of advanced nuclear reactor designs. This small model is designated by $L(d,x)$. This small model essentially estimated the demand for advanced nuclear designs (as a function of GNP growth and whether there were coal and shale limits) and the supply function of alternative electric systems (whether LWR, coal, or solar/fusion). The value of advanced nuclear designs in this small model was then estimated as the product of the demand for advanced nuclear designs times the cost advantage, if any, of advanced nuclear designs over the alternative electric system. The results of the "little model" can be summarized as follows: the little model produced estimates of the predicted value of advanced nuclear designs in each policy-state combination, where the little-model estimate is denoted by $L(d,x)$. A regression of the actual estimate from the programming model $V(d,x)$, on the predicted values from the little model $L(d,x)$, yields the following results:

$$\log[V(d,x)] = .498 + \underset{(.0037)}{.944} \log[L(d,x)]$$

$\bar{R}^2 = .928$, S.E.E. = 0.53

This indicates that a good deal of the variance was picked up by the small model, but that the error was still uncomfortably large.

In the second stage, then, the small model was improved by a multiple regression analysis. In addition to the predicted value from the small model, $L(d,x)$, non-linear transformations of both the exogenous and the decision variables were entered in a regression analysis. After some experimentation, it became clear that little was added by going beyond the first-order terms in exogenous variables, and second order terms in the decision variables: when further terms were entered, not only was there no significant improvement in the degree of approximation, but also it became impossible to make sure that the a priori signs on the variables were those obtained. Therefore, the final regression used first-order terms in the exogenous variables, first and second order terms in the decision variables, and one interaction term between the two decision variables. Table 4 shows the regression results that were found and used in the subsequent analysis. Figure 1 shows the predicted and actual values for the observations, as well as plots of the residuals.

The regression is not of great interest in itself. The major points to note are the following: first, the final equation leads to a precise and accurate determination of the values. A total of 97% of the weighted variance is explained, with a weighted standard deviation of approximately ten percent. This is in the order of $1 billion for the runs presented below. Second, it should be noted that by the nature of regression analysis, the sum of the errors is precisely zero; this means that the regression analysis has not introduced any systematic error into the estimation procedure.

TABLE 4

[illegible] REGR FOR CONAES [illegible] MODELING 11/28/76

FILE NONAME (CREATION DATE = 11/28/76)

* M U L T I P L E R E G R E S S I O N * * * * * *

DEPENDENT VARIABLE.. LV........................log of gross value of advanced nuclear (i. e. before subtracting R&D costs.)

VARIABLE(S) ENTERED ON STEP NUMBER 1..

- LX1 ...0 if no coal/shale constraint, 1 if coal/shale constraint
- LX2 ...log of GNP growth
- LX3 ...log of best of capital costs of solar or fusion
- LX4 ...log of clean synfuels cost
- LX5 ...-log of Uranium resources
- LX6 ...-log of capital cost of advanced converter
- LX7 ...-log of capital cost of L M F B R
- D1 ... 5.01 - decision on L M F B R
- D2 ... 5.01 - decision on advanced converter
- D3 ... log ((D1 + D2)/2.)
- D4 ...D1 x D1
- D5 ...D2 x D2
- VHL ...value of advanced nuclear predicted from small model

MULTIPLE R	0.98294
R SQUARE	0.96617
ADJUSTED R SQUARE	0.96608
STANDARD ERROR	0.36470

---------------- VARIABLES IN THE EQUATION ----------------

VARIABLE	B	BETA	STD ERROR B	F
LX1	-0.17399	-0.03240	0.02177	63.890
LX2	0.34191	0.12528	0.01125	924.322
LX3	0.82062	0.07920	0.04540	326.710
LX4	0.45268	0.05319	0.03655	153.580
LX5	0.25407	0.07143	0.01163	477.298
LX6	4.41793	0.09291	0.13154	1128.020
LX7	1.66491	0.07048	0.06717	614.405
D1	-0.02650	-0.02473	0.02170	1.491
D2	-0.48214	-0.46900	0.03216	224.694
D3	0.23402	0.22922	0.00590	1575.138
D4	-0.00313	-0.01154	0.00520	0.363
D5	0.10949	0.42620	0.00817	179.788
VHL	0.73722	0.75209	0.00624	13959.606
(CONSTANT)	0.01144			

ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
REGRESSION	13.	19092.23921	1468.63379	11042.153?
RESIDUAL	5026.	668.47048	0.13300	

FIGURE 1

WEGHT REGRE FOR CONAES RD MODELING   12/01/76

FILE   NCNAME   (CREATION DATE = 12/01/76)

* M U L T I P L E   R E G R E S S I O N * * * * * * * * * * *

DEPENDENT VARIABLE: LV   FROM   VARIABLE LIST 1
REGRESSION LIST 1

SECNUM	OBSERVED LV	PREDICTED LV	RESIDUAL	PLOT OF STANDARDIZED RESIDUAL (-2.0 -1.0 0.0 1.0)
1	2.824943	2.490155	.3347874	I *
2	2.484906	2.175271	.3096344	I *
3	2.493205	2.156824	.3363805	I *
4	2.360854	2.104935	.2559192	I*
5	.0	-.3168247E-01	.3168340E-01	*
6	.0	-.3168247E-01	.3168340E-01	*
7	.0	.7260012	-.7259995	* I
8	.0	.5132387	-.5132384	* I
9	.0	.9847454	-.8847447	* I
10	3.572346	3.300878	.2714681	I*
11	.0	.4766895	-.4766878	* I
12	3.453156	3.492288	-.3913139E-01	*
13	.0	.7494853	-.7494835	* I
14	3.446808	3.473840	-.2703283E-01	*
15	.0	.7310380	-.7310363	* I
16	3.091043	3.421950	-.3309085	* I
17	.0	.6791481	-.6791466	* I
18	2.388762	2.752798	-.3640364	* I
19	2.388762	2.752798	-.3640364	* I
20	2.995732	4.240707	-1.244976	* I
21	.6931472	1.797133	-1.103987	* I
22	2.772589	1.383272	1.389318	I *
23	3.401197	3.693093	-.2918961	*I
24	1.791759	1.349854	.4419055	I *
25	1.609438	1.344586	.2648510	I*
26	5.804532	5.642637	.1618935	I*
27	5.235909	5.637370	-.4014618	* I
28	5.741400	5.643013	.9838516E-01	*
29	5.573673	5.624565	-.5089417E-01	*
30	4.755313	5.283949	-.5286362	* I
31	5.698435	5.337509	.3609248	I *
32	.0	-.3168247E-01	.3168340E-01	*
33	.0	-.2444431	.2444444	I*
34	.0	-.7045442E-01	.7045543E-01	*
35	.0	-.3524003	.3524004	I *
36	.0	-.5651609	.5651615	I *
37	.0	-.1936542	.1936552	I*
38	.0	.3833893	-.3893383	* I
39	.0	-.5964448	.5964450	I *
40	.0	.3398318	-.3398303	* I
41	.0	-.3352314	.3352320	I *
42	.0	-.7045442E-01	.7045543E-01	*
43	.0	.3010598	-.3010582	* I
44	.0	.3303002	-.3302990	* I
45	.0	.1563116	-.1563100	*I

FIGURE 1, continued

WEGHT REGRE FOR CONAES RD MODELING 12/01/76

46	.0	.2714548	-.2714540
47	.0	.6429615	-.6429603
48	.0	-.7045442E-01	.7045543E-01
49	.0	-.7200826	.7200834
50	4.601162	4.597429	.3732083E-02
51	4.572647	4.620093	-.4744663E-01
52	4.644391	4.602771	.4161929E-01
53	4.468204	4.597505	-.1293001
54	4.580877	4.578982	.1894606E-02
55	.0	.4431847	-.4431828
56	2.525728	2.459990	.6573796E-01
57	.0	.4379175	-.4379157
58	2.803360	2.454722	.3486369
59	2.803360	2.423005	.3803543
60	.0	.5055391	-.5055382
61	.6931472	1.882909	-1.189761
62	.8754687	2.822996	-1.947528
63	.7419371	.6222497	.1196891
64	.0	-.4561517	.4561520
65	1.609438	3.646443	-2.037005
66	2.302585	3.859204	-1.556620
67	.0	.3252842	-.3252841
68	5.135798	5.089783	.4601552E-01
69	4.795791	5.078609	-.2828188
70	3.676300	4.299223	-.6229230
71	4.375757	4.026352	.3494046
72	4.762174	5.073342	-.3111687
73	3.737669	3.942116	-.2044469
74	5.220356	5.096098	.1242574
75	2.708050	4.140126	-1.432076
76	.0	.1308823	-.1308820
77	3.655839	3.522047	.1337910
78	3.645450	3.680962	-.3551241E-01
79	3.819907	3.699409	.1204979
80	.4054651	1.002678	-.5972123
81	.0	1.014260	-1.014259
82	.0	-.7045442E-01	.7045543E-01
83	.0	-.7045442E-01	.7045543E-01
84	1.506297	1.944063	-.4377664
85	1.528228	1.892174	-.3639460
86	.0	-.2444431	.2444444
87	2.043814	2.277393	-.2335804
88	1.958685	1.923971	.3471300E-01
89	6.320768	6.488791	-.1680257
90	6.265302	6.495109	-.2298080
91	5.117993	6.427835	-1.309842
92	6.398595	6.718119	-.3195247
93	6.111467	6.069694	.4177158E-01
94	5.214935	6.916181	-1.701245
95	5.003946	5.930346	-.9264019
96	5.814131	6.476662	-.6625318
97	.0	1.254367	-1.254366
98	5.669881	4.966440	.7034402
99	4.220977	4.797830	-.5768536

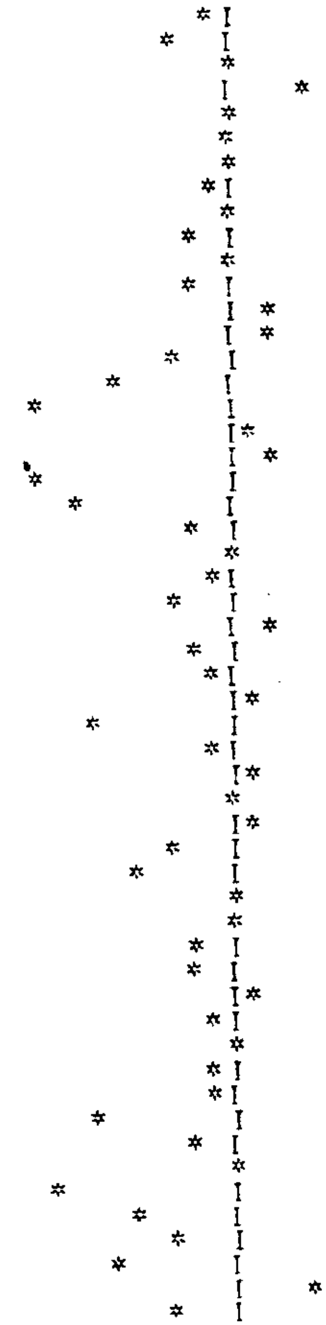

FIGURE 1, continued

WEGHT REGRE FOR CONAES RD MODELING 12/01/76

100	5.594711	4.972755	.6219550	I *
101	5.325446	4.954309	.3711370	I *
102	.7419371	.8802582	-.1383194	*I
103	.7419371	.8283683	-.8642977E-01	*
104	.5877863	.7044226	-.1166355	*I
105	1.308332	.8871212	.4212114	I *
106	1.251763	.5326993	.7190640	I *
107	.9932517	.3720826	.6211705	I *
108	.9932517	.3720826	.6211705	I *

C. Results for the Basic Run

We next present the results for the basic run. Recall that these estimates assume that none of the uncertainties are resolved before decisions are made. Table 5 shows the undiscounted and discounted costs assumed for R&D on the different reactors. The costs are assumed to be independent. These figures are subtracted from the gross benefits of each technology to obtain the net benefits. Note that the net benefits with alternative costs of R&D may be easily calculated below by simply adding or subtracting the difference between any assumed value and that in Table 5 below.

Finally, Table 6 summarizes the net benefit estimates for each of the 25 combinations of decisions. On the top of the table are the decisions about the date of introduction of the LMFBR. In the center is given results for the discounted net benefit of different decisions. All costs and benefits are discounted at a rate of 6% per annum. For each entry is given the expected (or mean) for each of the decisions. Although there is no exact way of calculating the "standard error" of the estimates from the regression analysis, as noted above the judgmental estimate is that the standard error of estimate is approximately $1 billion.

The overall picture that emerges from Table 6 is that, given the assumptions of the present analysis, there appears to be good reason to postpone implementation of R&D on either of the two advanced reactors. The best case for immediate development--where "immediate" means having advanced nuclear on line by 2000--is to develop the advanced converter but not the LMFBR. According to Table 6, the net benefits of the advanced

TABLE 5. R&D Costs of Decisions to Implement Different Reactor Types, billions of dollars, 1975 prices

	Advanced Converter	LMFBR
Undiscounted Costs:		
R&D	5	10
Commercialization	10	10
Discounted Costs:		
Discounted to 1975 (1975 prices) for 6 GW^e in place by:		
2000	5.9	8.7
2010	3.3	4.9
2020	1.8	2.7
2030	1.0	1.5
2040-or-later	0.0	0.0

Source: K. Hoffman, D. Cope, and R. Richels for the undiscounted value. The discounted values make the following assumptions:

(a) Commercialization occurs during the period from 10 to 0 years before the availability date. The cash flows are assumed all to be concentrated five years before the availability.

(b) Research and development is assumed to occur from 20 to 10 years before the availability date. The cash flows are assumed to be concentrated 15 years before the availability date.

(c) Any "overhead" expenditures are reoptimized in such a way that they are delayed by the time period equal to that for research, development, and commercialization.

TABLE 6

ECONOMIC BENEFITS OF DECISIONS ABOUT ADVANCED NUCLEAR REACTORS
EXCLUSIVE OF ENVIRONMENTAL COSTS BUT SUBTRACTING R AND D COSTS
DISCOUNTED TO 1975, 1975 PRICES, MILLIONS OF DOLLARS

EXPECTED VALUES OF DECISIONS

LMFBR DECISION: AVAILABLE AT 6 GWE	ADVANCED CONVERTER DECISION: AVAILABLE AT 6 GWE BY:				
CAPACITY BY	2000	2010	2020	2030	NEVER*
2000	-7.442	-6.435	-5.402	-4.301	-2.587
2010	-3.597	-2.628	-1.613	-0.538	1.111
2020	-1.443	-0.501	0.489	1.516	3.033
2030	-0.427	0.495	1.450	2.594	3.630
NEVER*	0.293	1.186	2.089	2.850	0.0

*NEVER SHOULD BE INTERPRETED AS 2040 OR LATER

NOTE THAT THE NEVER-NEVER DECISION IS THE REFERENCE.
THEREFORE IT HAS ZERO VALUE IN ALL CASES.

TABLE 7

ECONOMIC BENEFITS OF DECISIONS ABOUT ADVANCED NUCLEAR REACTORS
EXCLUSIVE OF ENVIRONMENTAL COSTS BUT SUBTRACTING R AND D COSTS
DISCOUNTED TO 1975, 1975 PRICES, BILLIONS OF DOLLARS

STANDARD DEVIATION OF OUTCOME

LMFBR DECISION: AVAILABLE AT 6 GWE	ADVANCED CONVERTER DECISION: AVAILABLE AT 6 GWE BY:				
CAPACITY BY	2000	2010	2020	2030	NEVER*
2000	22.033	17.323	16.067	16.924	19.901
2010	22.174	17.325	15.996	16.785	19.548
2020	22.045	17.045	15.619	16.281	18.588
2030	21.533	16.332	14.754	15.183	16.496
NEVER*	19.974	14.206	12.204	12.109	0.0

*NEVER SHOULD BE INTERPRETED AS 2040 OR LATER

SINCE THE REFERENCE POINT IS ALWAYS THE PATH WITHOUT ANY ADVANCED NUCLEAR, THE (NEVER,NEVER) DECISION HAS A ZERO STANDARD DEVIATION. ALL OTHER PATHS THUS MEASURE THE VARIABILITY OF THE OUTCOME RELATIVE TO THE (NEVER,NEVER) DECISION converter in 2000 are $0.3 billion, while the net benefits of the LMFBR are -$2.6 billion.[1] A crash program to develop both advanced reactors is the least economic of the decisions, having a net benefit of -$7.4 billion.

Using only the criterion of net economic benefits, the highest expected value comes from a decision which implements the LMFBR in 2030, while the advanced converter is not developed. This strategy has an expected net benefit of $3.6 billion. Other strategies in the neighborhood of the optimal one, however, are hardly less economic: the decision to speed up the LMFBR ten years from the optimal point loses only $0.6 billion, while the decision to develop the advanced converter rather than the breeder in 2030 loses $0.7 billion.

It is interesting to note that the current U.S. strategy--to develop the LMFBR by 2000 but not to develop the advanced converter--ranks 19th out of the 25 decisions which were considered.

A second feature of the analysis which is worth attention is the degree of variability of the outcome. Table 7 shows the standard deviation of the outcome for each of the decisions relative to the no-advanced-nuclear path. The standard deviations are quite large--in the order of $20 billion--indicating that the effect of the policy on the discounted value of real income has a large dispersion across states of the world.[2]

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[1] The normalization used here, it must be emphasized, is that a decision to forego any advanced nuclear is arbitrarily set at zero. Therefore all net benefit calculations are the difference between the benefits with the decision under consideration and the benefits of no advanced nuclear.

[2] In retrospect, choosing the never-never decision as a normalization for calculating the standard deviation was probably a poor choice. It would have been much more interesting to determine the extent to which a particular decision has a lower mean and a high dispersion relative to the optimal decision rather than a suboptimal one. It is likely that there is a very high covariance between the outcome of each decision across states of the world. A rough guess is that the standard deviation of

Figure 2 - Distribution of net benefits in optimal decision

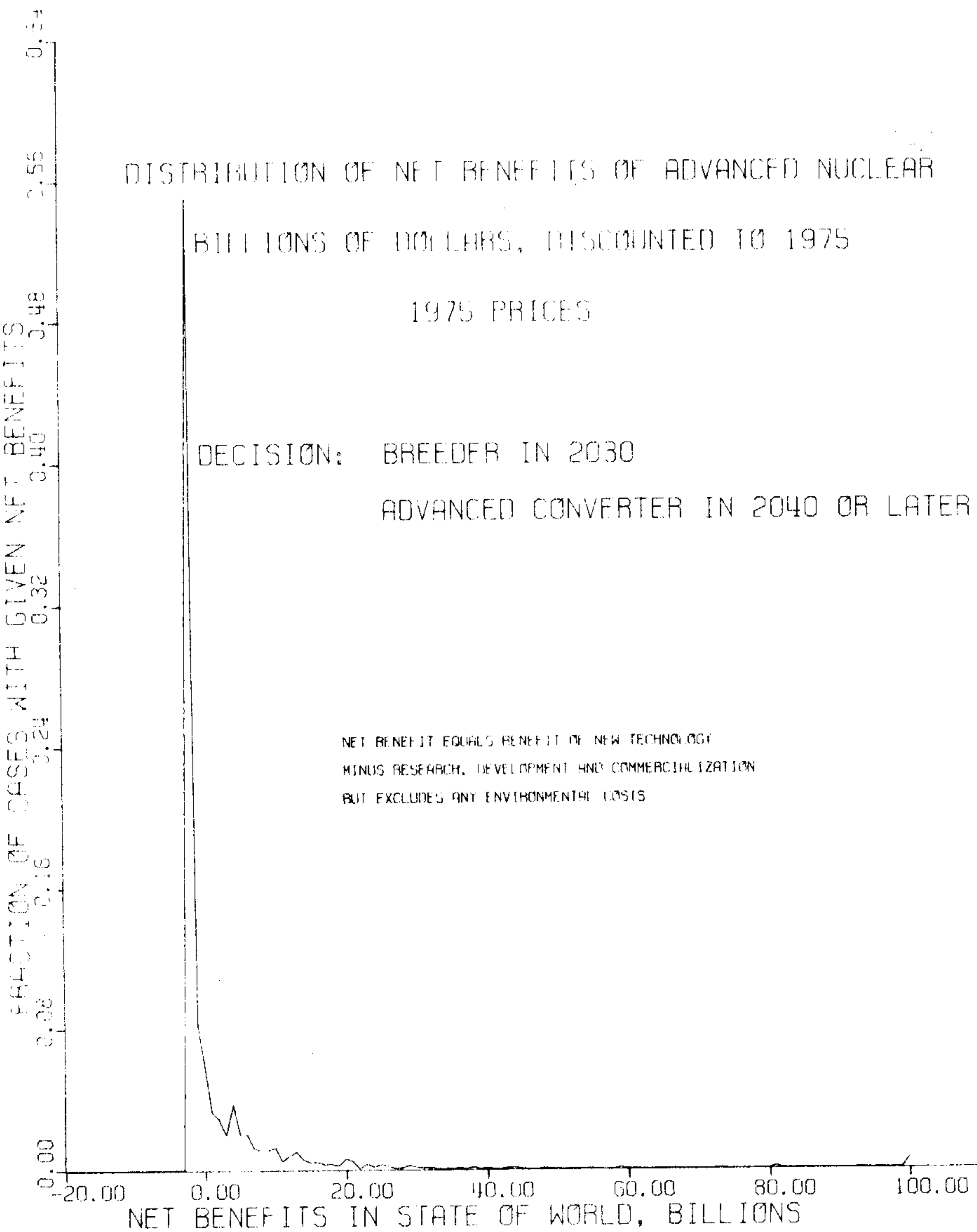
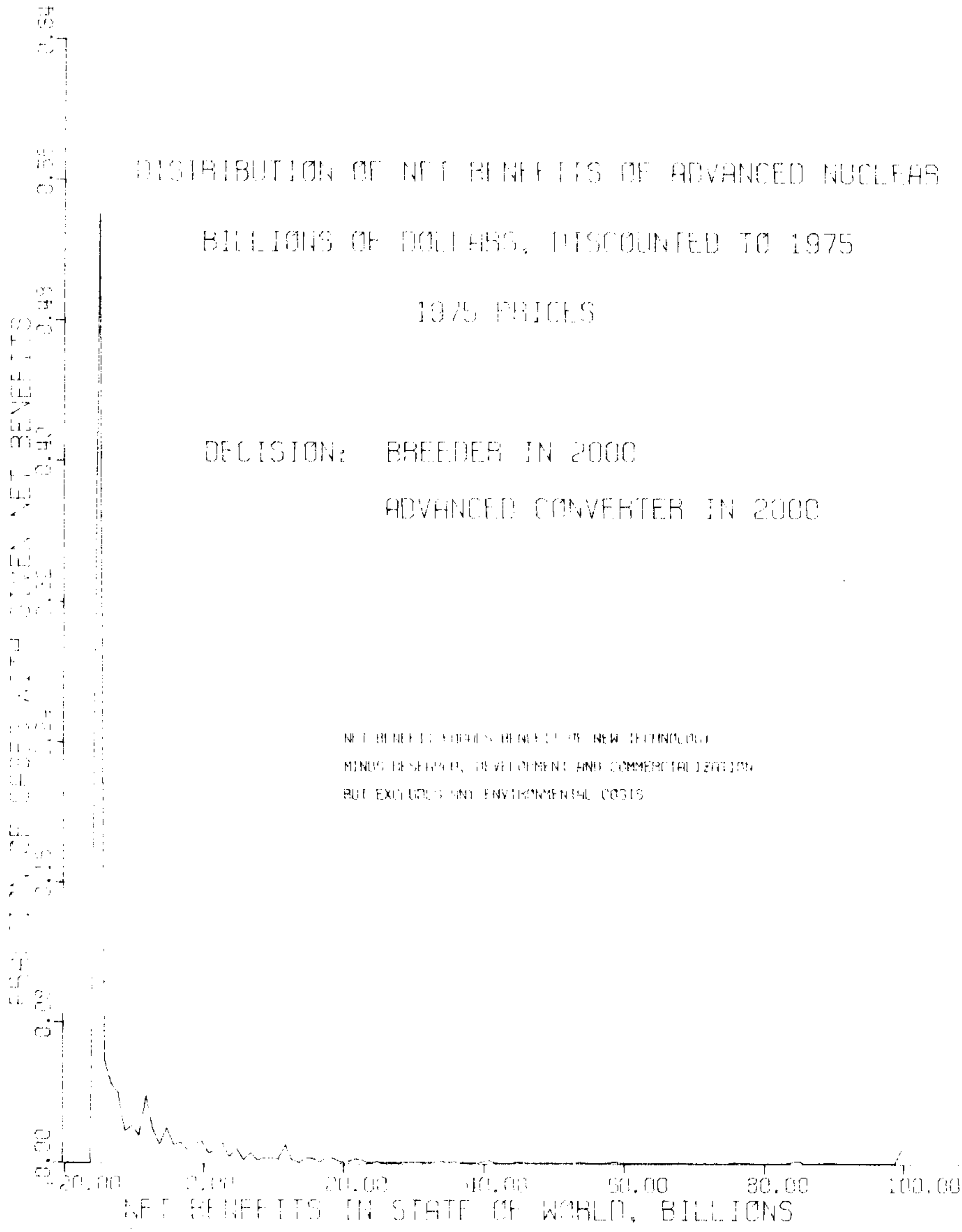


Figure 3 - Distribution of net benefits for decision to implement both types immediately



The dispersion of the outcomes are shown graphically in Figures 2 and 3. Each of the figures shows the frequency distribution of net benefits according to the probability assessments of the group reported above. Figure 2 shows the frequency distribution for the optimal decision (LMFBR in 2030, advanced converter in 2040-or-later). As in all the decisions, there is a very sizable fraction of the distribution at the far left; this indicates in Figure 2, for example, that there is a 65 percent judgmental probability that the value of the new technology is zero before subtracting the R&D costs. Put differently, even in the most economic case, the probability of having a net loss is estimated to be 65 percent, while the probability of having a discounted net benefit over $100 billion is about 1 percent. Under particular configurations of outcomes, the values of advanced nuclear reactors is estimated to rise as high as $1000 billion, although, to be sure, these are highly unlikely cases.

Figure 3 shows the frequency distribution for the decision to implement both advanced nuclear reactor types by the year 2000. The major difference between this frequency distribution and that for the optimal decision in Figure 2 is that the distribution is displaced to the left (i.e. toward the negative values) by approximately $10 billion. This displacement is less than the R&D savings, which amount to about $13 billion.

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the difference between the optimal and any of close decisions is an order of magnitude lower than those in Table 7. Therefore the dispersion estimates presented in Table 7 are misleading.

Second, several readers have enquired why no estimates of the "risk premia" on decisions are calculated given the very large risk in the optimal decision. At the present stage of the analysis, it is impossible to calculate risk premia because there is no obvious base from which to calculate risk. Different states of the world reflect differences in endowments, in tastes, and in productivity growth; therefore, it is not clear whether the never-never decision or the optimal decision is one which is "riskier."

D. The Value of Early Information

It was noted in the earlier discussion that the analysis used up to now assumes that the knowledge about the uncertain variables is revealed only after the decisions have been made. In certain cases, this is probably an extreme assumption. In the case of demand growth, for example, a great deal will be revealed before the decision to implement the optimal strategy is far along.

In what follows we will examine the expected value of early information (E.V.E.I.). Early information implies that knowledge is obtained about the realization of one or all random variables before decisions are made.

In order to test for the importance of the assumption about the sequential resolution of uncertainty, a further set of runs was made in which it was assumed that uncertainties were resolved before the decision was taken. This procedure can be understood as follows: there continue to be uncertainties about the exact state of the world that will hold in the future. Assume, however, that these uncertainties were resolved in the near future, and before any substantial investment in R&D had been made. Then, upon learning which state of the world we are in, decision-makers could at that point choose the optimal decision in a deterministic framework.

The question we ask, then, is: what is the value of having early information about the value of uncertain variables. The technique for calculating the value of information can be explained briefly, using Uranium as an example. Let us assume that by doing a certain amount of drilling, we can resolve the uncertainty about Uranium resources. It

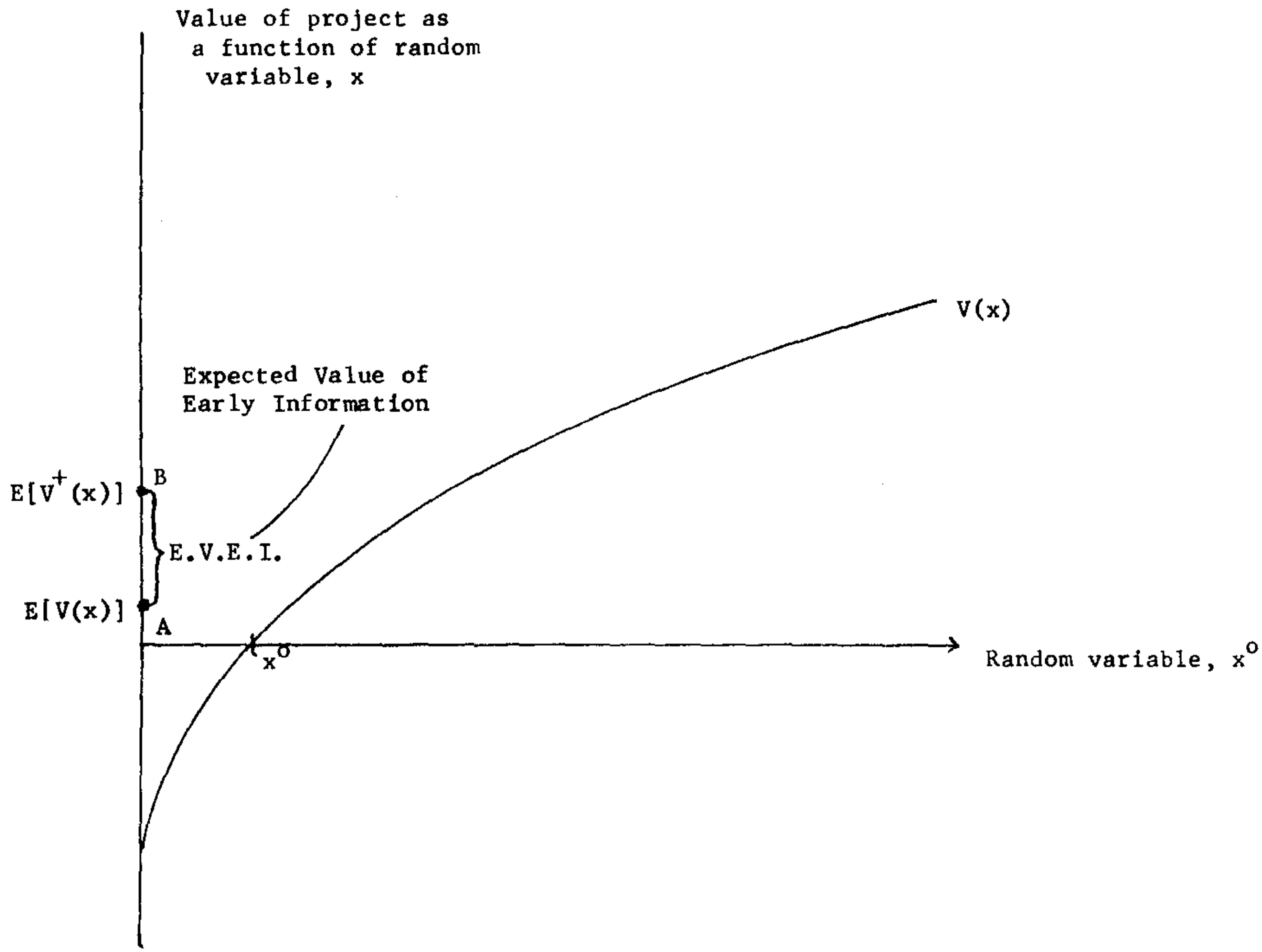


Figure 4 . Illustration of the value of early information in decision. E[V(x)] is the value of the project when x is now known in advance, while $E[V^+(x)]$ is value of project when value x is known in advance. Difference = E.V.E.I. = $E[V^+(x)] - E[V(x)]$ = expected value of early information.

is assumed, of course, that the probability of each outcome is given by the judgmental probabilities estimated above. Thus, say that there are six equally Uranium figures, each of which is (judgmentally) equally likely. Then it is as if nature were to roll a die and the die would show the outcome; the value of early information is that we could make our decisions after knowing the outcome of the roll of the die rather than being forced to make our decision before knowing the outcome.

The value of information is illustrated in Figure 4. For simplicity, we assume there is only one decision, whether to go ahead with a R&D project or not, and abstract away from the problem of timing. There is an uncertain variable, x, and Figure 4 shows the distribution of the value of the decision as a function of the random variable. In the case where the decision must be made before the value of x is known, the value of the decision is the mean of $V(x)$, $E[V(x)]$, shown on the figure. In the case of early information, the value of x is known before the decision is made; consequently, the decision to undertake the project will be positive only when x is greater than x^0, where x^0 is that value of x where the value of the R&D project is first non-negative.

The expected value of the decision when early information is not available is shown as point A, $E[V(x)]$. This is simply the expectation of $V(x)$, given the probability distribution of x. The value of the project with early information is obtained by taking the expected value of a function $V^+(x) = \max[0, V(x)]$, and is shown as point B; this implies that the project is undertaken only when the value is positive. The value of early information (E.V.E.I.) is then the difference between $E[V^+(x)]$ and $E[V(x)]$, shown as E.V.E.I. in Figure 4. If all uncertainties concerning the state of the world were resolved by knowledge of x , the difference $E[V^{+}(x)] - E[V(x)]$ will be called the value of perfect information.

The procedure for estimating the value of reduction of uncertainty is as follows: say that uncertain variable x_1 takes values $\{x_1 = 2.$ with probability $p_{1,1} = .3\}$, $\{x_1 = 3.$ with probability $p_{1,2} = .4\}$, and $\{x_3 = 4.$ with probability $p_{1,3} = .3\}$. Then three runs are made; in the first run the value of advanced nuclear is calculated with $x_1 = 2.$, giving a vector of values of decisions, $V_{1,1}(d)$, with similar techniques for the second and third values of x_1 . Finally, the value with early information is determined by calculating the expected value of the decision: $V_1^{max} = p_{1,1} \max_d \{V_{1,1}(d)\} + p_{1,2} \max_d \{V_{1,2}(d)\} + p_{1,3} \max_d \{V_{1,3}(d)\}$, where the "max" operator takes the maximum of the values of the elements in the $V_{i,j}(d)$ vector for different values of d . By comparing the optimized value, V_1^{max} , with that calculated when the decisions are the same in all three states of the world for variables 1, we can estimate the value of reducing the uncertainty about variable 1.

In a first set of runs, we calculate the value of early information when all uncertainties are resolved before the decisions are made. This implies that separate decisions are possible for each of the 10,328 states of the world. Table 8 shows the results of this calculation at the top of the table. According to the assumption used in the present section, the expected value of the decision, when the decision is optimized for each state of the world, is \$5.7 billion. Figure 5 shows the distribution of net benefits for this case.

TABLE 8. Value of Early Information about Uncertain Variables, billions of dollars, 1975 prices

A. Value of early information about all variables:

Expected Value of Decision, with Early Information	$ 5,749 million
Expected Value of Optimal Decision (see Table 6)	3,630 million
Expected Value of Early Information (E.V.E.I.)	$ 2,119 million

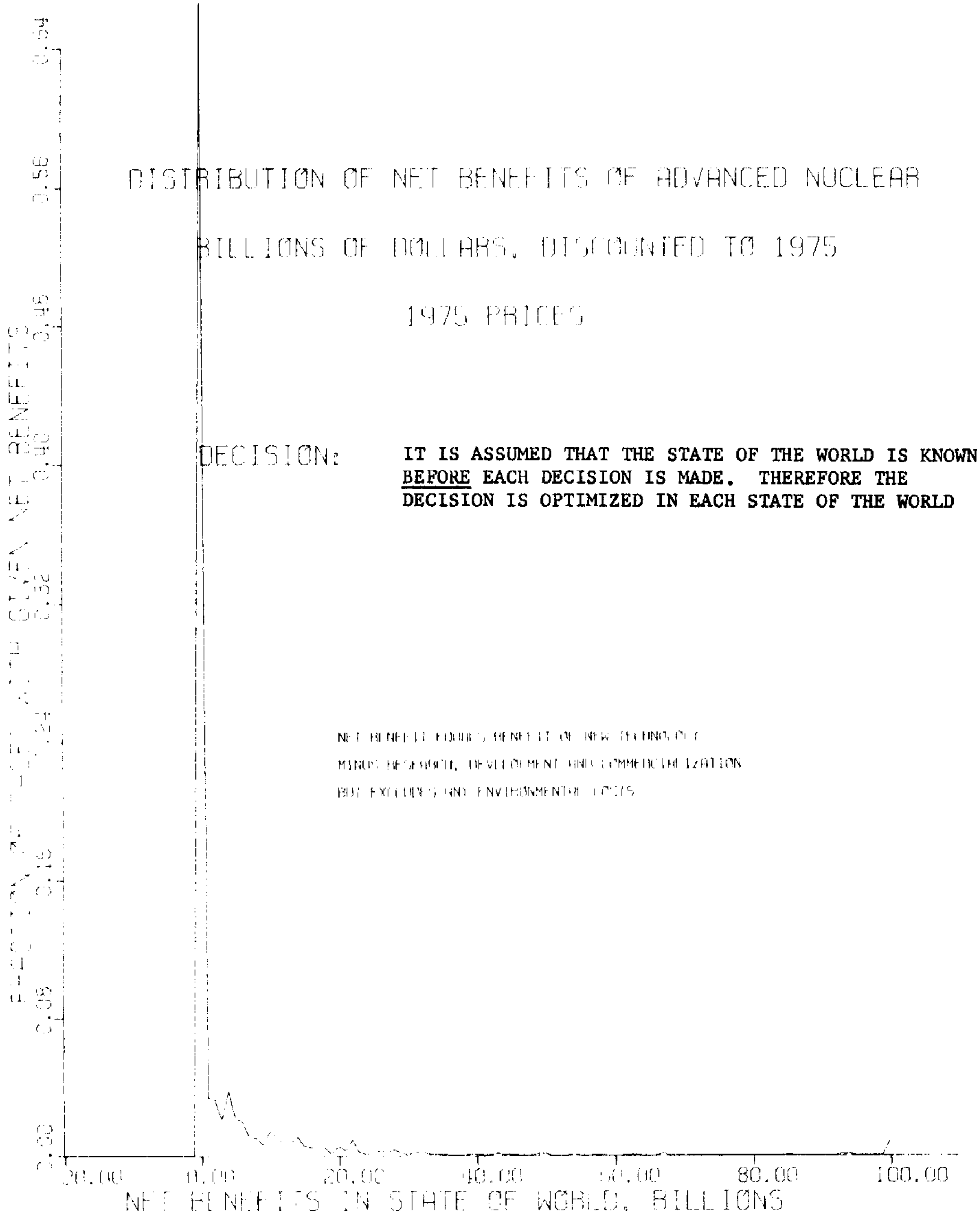
B. Value of early information about variables, when taken one at a time and all other variables are uncertain:

1. GNP growth	$ 466	million
2. Coal/shale limits	52	
3. Clean synfuels	0	
4. Uranium resources	0	
5. Capital cost, AC	1	
6. Capital cost, LMFBR	17	
7. Solar or Fusion Capital Costs	27	
8. Nuclear acceptability	934	
Total, taken one at a time and summed	$1,497	million

How are we to interpret this result? This says that the value of perfect information is $2.1 billion. That is, if instead of investing so as to make the LMFBR available in 2030 and the advanced converter in 2040-or-later, we had perfect information--this would provide an increase of the expected value of the benefits from $3.6 billion to $5.7 billion, a difference of $2.1 billion. As can be seen by comparing Figures 2 and 5, the main value of perfect information is to prevent investing in R&D for those cases when the net benefits are negative.

Several readers have expressed surprise that the value of perfect information is so small. In understanding where the value of perfect information comes from, it is useful to divide the gain into two parts. First, there are some states of the world where advanced nuclear has a gross value of zero. This is the illustration shown in Figure 4. According to Figure 5, this amounts to approximately two thirds of all cases. The first benefit of perfect information, then, is to allow the decisionmaker to stop the advanced nuclear program when it has negative value. The second source of gain from perfect information is that, when advanced nuclear has a positive value, the decisionmaker may gain from either a speedup or slowdown of the program depending on the state of the world. Thus in those states of the world where advanced nuclear is extremely valuable (say high GNP growth, low Uranium supplies, and high cost alternative energy systems), the decisionmaker could develop the advanced nuclear option earlier than the stochastic "optimal" decision when no information is available.

Figure 5 - Distribution of net benefits when perfect information is available



It is not possible to separate out precisely the $2.2 billion into the two sources, but a rough estimate is available. We see from Figure 2 that the probability that advanced nuclear has negative net benefits is two-thirds. Since in the optimal decision shown in Figure 2 R&D costs are $1.5, perfect knowledge allows the decisionmaker to save 0.66 x $1.5 billion = $1.0 billion in these cases. The remainder which is about $1.2 billion, comes from reshuffling the timing of the R&D program between different time periods.

<u>The value of early information for individual variables</u>

The estimates given above provide the value of information for all exogenous variables. The calculation suggests that the value of information is low relative to the enormous uncertainties involved. It might be the case, however, that a good deal of the costs of uncertainty are attached to individual variables for which the uncertainty is relatively easy to reduce. To ascertain whether this is the case, we have estimated the value of early information for each of the eight exogenous random variables.

The results of the estimates of the value of early information are shown in Tables 8 and 9. They indicated that the uncertainty for which the resolution would have major value is the nuclear debate about reactor acceptability. The value of knowing in advance which reactors are acceptable is almost $1.0 billion. This very large value results from the fact that, if a reactor is not acceptable, the R&D on that reactor is simply wasted. The main implication for policy is that, if an R&D program is undertaken, great care should be taken to design it so that the public acceptability questions are identified and resolved quite early.

The only other uncertainty for which early resolution would have a significant payoff is that regarding GNP growth, in which early information would be worth $466 million. All other uncertainties would pay between $0 and $50 million for early resolution.

The sum of the individual components for the value of early information, taken one at a time, is $1,497 million. This compares with an estimate of $2,119 for all eight calculated simultaneous. This indicates that the interaction between uncertainties imposes further costs to decisions.

On the right hand side of Table 9 is shown the effect of the early information on the decisions. The most important change is for early information about nuclear acceptability, where the shift to the advanced converter in 2030 is made when the Breeder is not acceptable but the ACR is. In addition, there are several cases when the decision is either postponed or accelerated by 10 years, but for no case is the decision accelerated by more than 10 years. It should be noted, of course, that in some cases when more than one variable is known in advance there is much more acceleration; for example, with high GNP, coal and shale limits, and low Uranium, the economical decision is to develop the Breeder as soon as possible.

TABLE 9. Value and Effect on Optimal Decision of Early Information on Uncertain Variables

	Variable	Value of Early Information	Effect on Decision*
1.	GNP growth	$466 million	No advanced nuclear with low GNP growth; decision is base with medium GNP growth; FBR accelerated 10 years with high GNP growth.
2.	Coal/shale limits	$52 million	No advanced nuclear if there are no coal and shale limits; decision unchanged if there are coal and shale limits.
3.	Clean synfuels	$0 million	No change in decisions.
4.	Uranium resources	$0 million	No change in decisions.
5.	Capital cost, advanced converter	$1 million	No change in decision for high or medium capital cost; for low capital cost, FBR accelerated by 10 years.
6.	Capital cost, Breeder	$17 million	No change in decision for high or medium capital cost; for low capital cost, FBR accelerated by 10 years.
7.	Solar or fusion capital costs	$27 million	No change in decision with low or medium solar-fusion capital costs; for high solar-fusion capital costs, the FBR decision is accelerated 10 years.
8.	Acceptability of advanced nuclear	$934 million	No advanced nuclear R&D when both unacceptable; decision is ACR only when ACR only acceptable; FBR only when FBR only or all acceptable.

*Recall that "base" decision is FBR in 2030 and ACR in 2040-or-later. "Unchanged" means that optimal decision is "base."

E. Calculation of Insurance Premia for Advanced Reactors

In the analysis performed up to this point, we have analyzed the expected value of decisions, where this was calculated as the average value of each decision in each state of the world, weighted by the judgmental probabilities in each state of the world. In the present section, we analyze the question: how much is the economic insurance premium for each decision?

The estimate of the value of the insurance premium on a new technology rests on the asymmetrical nature of the payoff. This is illustrated in Figure 6. Let us call $V(x)$ the value of a new technology, where x is a random variable. For simplicity, assume that the variable x takes the values $x = x_1$ with probability 1/2 and $x = x_2$ with probability 1/2 ; therefore the expected value is $\bar{x}$. The <u>insurance premium</u> is defined as the difference between the expected value of the technology, taken by averaging the value of the technology across different states of the world (to be called the "<u>stochastic value</u>" henceforth), and the value of the technology when all variables take their expected value (to be called the "<u>deterministic value</u>" henceforth). Hence the "insurance premium" on development of the new technology is $E[V(x)] - V[E(x)]$.

The calculation of the insurance premium is relatively straightforward in the simple example described above and shown in Figure 6. The deterministic value of the new technology is $V(\bar{x})$, shown as point B in the Figure; this value is found by simply reading off the value of the expected value of x . The stochastic value of the technology is found by taking the expected value of the benefit, $E[V(x)] = \frac{1}{2}V(x_1) + \frac{1}{2}V(x_2)$, this is shown as point A in Figure 6. Finally, the difference between the stochastic and the deterministic value of the new technology, AB, is the

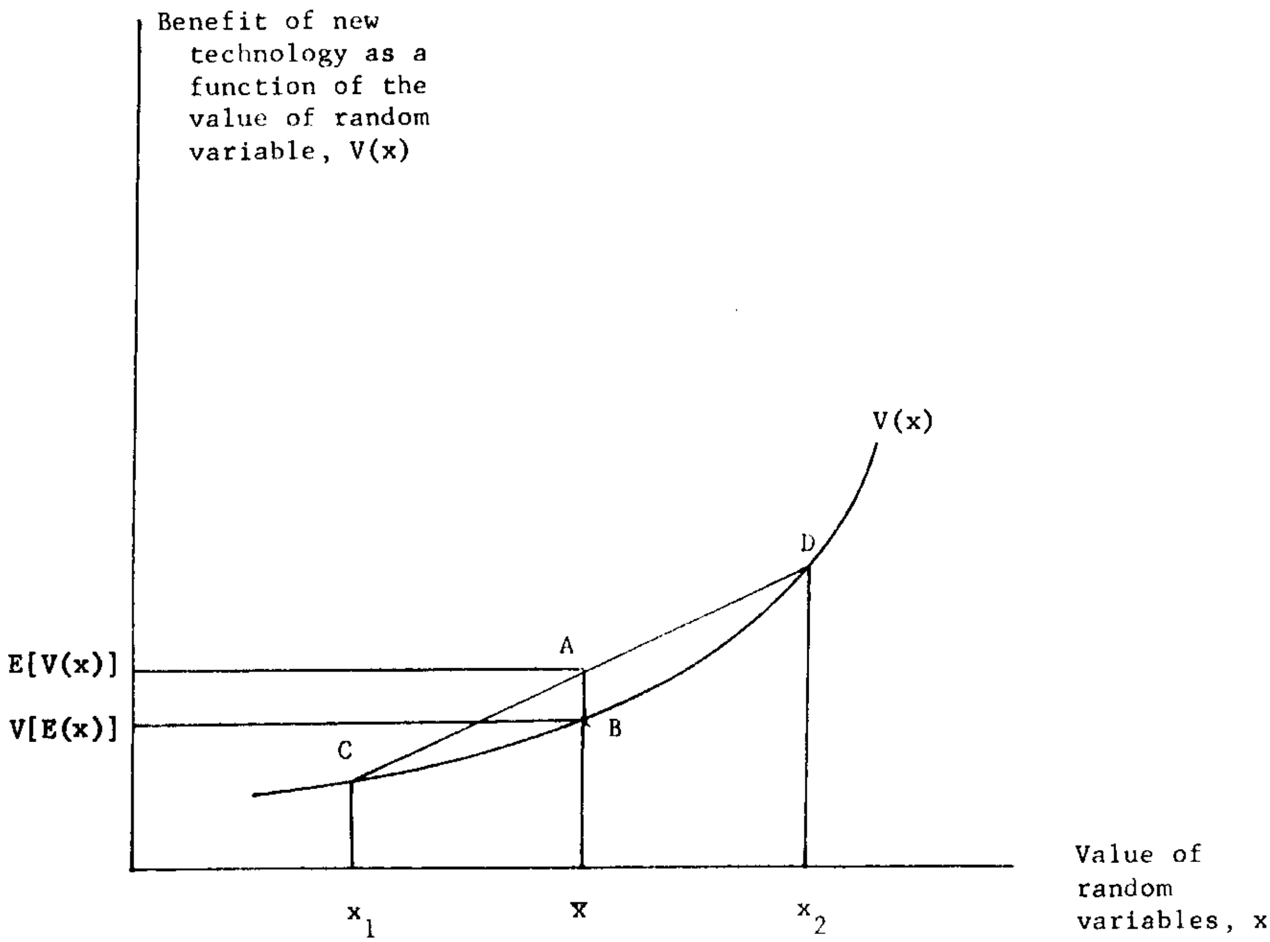


Figure 6. . Figure showing the method for calculation of the insurance premium on new technology. Point A is the deterministic value of technology, while point B is the stochastic value; difference = E[V(x)] - V[E(x)] = AB = insurance premium on the new technology.

insurance premium on the new technology.

A word should be stated about the interpretation of the insurance premium and its difference from ordinary calculations. The general interpretation of the insurance premium on new technologies is that they are, to a certain extent, a hedge against future uncertainties. Thus it might be that along the expected value (deterministic) path, there would be absolutely zero value of a breeder or solar energy. On the other hand, in the case of unfortunate realizations of uncertainties--such as low uranium supplies, worse than anticipated health effects of coal combustion, and high demand--the value of one of the new technologies might be very high. It would be prudent policy, then, to assign some value to the new technology in order to account for the uncertainty.

At the same time, it should be noted that there is no need for the insurance premium to be positive. In Figure 6, the premium was necessarily positive because the $V(x)$ function was convex. Although we would ordinarily expect the $V(x)$ functions to be convex in many cases, convexity is not necessarily in the structure of the problem, or in the theory of convex programming.

Yet another interpretation of the insurance premia is that they are the value of changing the distribution of the variables from one with a given mean and variance to one with the <u>same</u> mean but a zero variance; this differs, of course, from the process of gaining information about the exact state of the world, investigated in Section D above. What does it mean to reduce the variance to zero? This is rather hard to conceptualize in general, but a simple example will give the flavor: suppose that the only uncertainty is the outcome of research and development on solar energy. Say that the deterministic value of a new technology is $2 billion and the stochastic value is $4 billion, so that the insurance premium is $2 billion. Now say, we could, for certain, make an improvement or new technology which either would reduce the uncertainty of solar to zero guaranteeing the mean value or that we find a new technology which had the same expected value as solar energy with a zero variance. Then the value of this improvement in solar or of the new technology would be $2 billion.

We have calculated insurance premia in two steps: first we present the premia for all quantifiable uncertainties (that is, all but the outcome of the nuclear acceptability); then we present insurance premia for one uncertainty at a time.

Table 10 shows the <u>deterministic value</u> of decisions about advanced nuclear for the case where all exogenous uncertainties[1] are replaced by their expected values. This should be compared with the stochastic values shown in Table 6 above. Also shown in Table 11 is the difference between the two, that is the insurance premium for each decision.

It is clear from these results that the cost of uncertainty is very substantial: for all decisions, the insurance premium is between $4 and $7 billion, for example, the decision to implement both the breeder and advanced converter by 2000 has an insurance premium of $7.2 billion (= -$7.4 - (-$14.6) billion). The insurance premia fall off slightly in absolute terms as decisions are delayed, mainly due to the discounting of the value. The large size of the insurance premia reflects the very substantial uncertainty about the exogenous variables, as well as the high degree of nonlinearity in the benefit function.

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[1]We note here, but will not repeat, that "all uncertainties" refers to those seven of the eight exogenous uncertainties which are quantifiable; it excluded nuclear acceptability.

In addition to estimating the insurance premia which would be economical on reducing all seven uncertainties, we have estimated the insurance premia which would be earned by reducing each the uncertainties one by one. These are summarized in Table 12, where the insurance premia on each of the seven variables for two decisions are shown--those for the early program (both reactors in 2000) and those for the stochastic optimum (FBR in 2030, AC never). From these results, it is clear that the significant uncertainties are energy demand and the question of coal and shale limits. These two uncertainties are each valued at approximately \$6 billion for early decisions and \$4 to \$5 for the stochastic optimum, indicating that it would be worth approximately this amount to reduce the uncertainty to the expected value. The insurance premia on the other variables lie between \$0.5 billion and \$2 billion. The major surprise, perhaps, is that the uncertainty about Uranium resources is the least important, in the sense that the insurance premium on that uncertainty is less than on any of the others.

At the bottom is shown the sum of the insurance premia for all uncertainties taken both one at a time and together. It is interesting to note that there is a strong interaction between the uncertainties in the sense that the sum value of the individual premia taken one at a time is considerably larger than the premium for all taken together. This is, in fact, one of the fundamental results of portfolio theory, that there are strong diminishing returns to diversification or to reducing uncertainty.[1]

[1] See Tobin [1958]. If the covariances between the random variables were zero, then the sum of the variances of the individual components would equal the variance of the total. If the insurance premia are proportional to the standard deviations, then the sums of the squares of the individual insurance premia should be equal to the square of the insurance premium on the total. In fact the sum of the square of the insurance premia is 83.3, while the squared value of the overall is 7.2 x 7.2 = 51.8.

TABLE 10. CALCULATION OF VALUE OF DECISIONS IN DETERMINISTIC CASE: ALL VARIABLES (EXCEPT FOR NUCLEAR ACCEPTABILITY) AT EXPECTED VALUES

ECONOMIC BENEFITS OF DECISIONS ABOUT ADVANCED NUCLEAR REACTORS
[illegible] ENVIRONMENTAL COSTS BUT SUBTRACTING R AND D COSTS
DISCOUNTED TO 1975, 1975 PRICES, BILLIONS OF DOLLARS

EXPECTED VALUES OF DECISIONS

BREEDER DECISION: AVAILABLE AT 6 GWE

ADVANCED CONVERTER DECISION: AVAILABLE AT 6 GWE BY:

CAPACITY BY	2000	2010	2020	2030	NEVER*
2000	-14.564	-11.972	-10.474	-9.673	-8.669
2010	-10.764	-8.172	-6.674	-5.273	-4.870
2020	-8.564	-5.972	-4.475	-3.674	-2.671
2030	-7.365	-4.773	-3.276	-2.475	-1.474
NEVER*	-5.869	-3.277	-1.780	-0.981	0.0

*NEVER SHOULD BE INTERPRETED AS 2040 OR LATER

NOTE THAT THE NEVER-NEVER DECISION IS THE REFERENCE.
THEREFORE IT HAS ZERO VALUE IN ALL CASES.

VMAX = 0.012

TABLE 11. CALCULATED INSURANCE PREMIA ON ADVANCED NUCLEAR, ALL EXOGENOUS UNCERTAINTIES (EXCEPT NUCLEAR ACCEPTABILITY) ELIMINATED

ECONOMIC BENEFITS OF DECISIONS ABOUT ADVANCED NUCLEAR REACTORS
EXCLUSIVE OF ENVIRONMENTAL COSTS BUT SUBTRACTING R AND D COSTS
DISCOUNTED TO 1975, 1975 PRICES, BILLIONS OF DOLLARS

LMFBR DECISION: AVAILABLE AT 6 GWE	ADVANCED CONVERTER DECISION: AVAILABLE AT 6 GWE BY:				
CAPACITY BY	2000	2010	2020	2030	NEVER*
2000	7.2	5.6	5.0	5.3	6.1
2010	7.2	5.6	5.1	5.4	6.0
2020	7.2	5.5	5.0	5.3	5.7
2030	6.0	5.3	4.8	4.9	5.1
NEVER*	6.2	4.5	3.9	3.9	0.0

*NEVER SHOULD BE INTERPRETED AS 2040 OR LATER

NOTE THAT THE NEVER-NEVER DECISION IS THE REFERENCE.
THEREFORE IT HAS ZERO VALUE IN ALL CASES.

*Insurance premia equal difference between deterministic and stochastic values of decision. All economic benefits of decisions about advanced nuclear reactors are exclusive of environmental costs but subtract R and D costs.

TABLE 12. Insurance Premia* for Reducing Uncertainty on Exogenous Variables Taken One by One, Discounted to 1975, 1975 prices, billions of dollars

Variable	Early Decision (AC and LMFBR in 2000)	Stochastic Optimum (LMFBR in 2030, AC never)
1. GNP growth	6.6	4.7
2. Coal/Shale limits	5.5	3.9
3. Clean synfuels	1.5	1.0
4. Uranium resources	0.6	0.4
5. Capital cost, AC	1.8	1.2
6. Capital cost, LMFBR	1.2	0.8
7. Solar or fusion costs	1.5	1.0
Total, one by one (sum of 1 through 7)	18.7	12.0
Total, simultaneously calculated (Table 9)	7.2	5.1

*Insurance premia equal difference between stochastic and deterministic values of decision. All economic benefits of decisions about advanced nuclear reactors are exclusive of environmental costs but subtract R and D costs.

It is important to ask whether reducing the uncertainty exogenous variables changes any decisions. In the case of reducing all uncertainties, shown in Table 8 above, it turns out that the optimal decision is not to develop any advanced nuclear. The stochastic optimum (AC never and LMFBR in 2030) moves in the rank order of projects from number 1 to number 3. For the case of reducing other uncertainties one at a time, it was found that the stochastic optimum remains optimal when uncertainty is reduced, one at a time, for the capital cost of advanced converters, for Uranium, for synfuels, and for solar/fusion. In the cases of reducing the dispersion in demand and coal/shale limits, however, the optimal decision changes to no advanced nuclear (just as in the full deterministic case), with the stochastic optimum moving to third place in each case. For the three cases where the stochastic optimum is displaced, the second place decision is to develop the advanced converter in 2030 and the Breeder never.

Conclusions

The conclusions which can be drawn from the present analysis are as follows: First, the expected benefit of developing the advanced nuclear option is in the order of \$3 billion when development takes the form of either an advanced converter or breeder after 2020. Among the decisions, there is a slight edge toward developing the breeder in 2030 and no advanced converter.

Second, the worst decision among the 25 analyzed here was a program to develop both the advanced converter and the breeder very quickly; any decision to develop both reactor types before 2020 has an estimated net loss of between \$3 and \$7 billion. A strategy to develop the breeder by 2000 also is estimated to have a net loss.

Third, from the estimates made about the value of perfect information, it is probable that the estimates made here would be slightly more favorable to developing advanced nuclear if a substantial improvement in knowledge about the uncertain driving variables were attained. In the optimal decision, adding complete certainty about driving variables adds approximately $2 billion to the net benefits. It is somewhat surprising that the value of perfect information is so small. Given this small value, however, it seems likely that the assessment of the value of advanced nuclear reactors would not be substantially changed by further refinements of the nature of the revelation of the uncertainties.

Fourth, it was found that the value of advanced nuclear has a substantial element of insurance against adversity. The value of reducing the exogenous uncertainties was estimated, and it was found that the value ranged from a low of about 0.5 billion for Uranium, to very substantial magnitudes, in the order of $5 billion, for demand and coal and shale limits. The total insurance premium in the optimal decision was estimated to be $5 billion.

Finally, it should be emphasized that all the estimates in this analysis are made without weighing the environmental costs of advanced nuclear reactor strategies into the calculations.

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