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PRICE INFORMATION AND THE ECONOMICS OF CONSUMERISM:

A MODEL OF STOCHASTIC EQUILIBRIUM

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PRICE INFORMATION AND THE ECONOMICS OF CONSUMERISM:

A MODEL OF STOCHASTIC EQUILIBRIUM[1]

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A model of a quasi-competitive industry is constructed, in which the firm's sales are described by a random variable whose expected rate of change depends on price. It is shown that a stationary (non-degenerate) distribution of prices results, so that price differences persist over time.

It is further shown that, as consumers become more aware of, and responsive to, price differences between firms, the average price set by the (profit maximising) firms tends to decrease, implying a reduction in the degree of monopoly in the industry.

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The present paper is concerned with the development of a model of industry equilibrium in which consumers are imperfectly informed as to firms' current prices, and in which firms choose their optimal price strategies in the knowledge that this is the case. A difficulty associated with certain earlier models of this kind has been, to quote Rothschild [4],

> Variety and volatility of prices, the commonplace of our experience. . . disappear from the scene once equilibrium has been established. This is unfortunate, for they do not seem to be disappearing from the world.

It is generally agreed that the appropriate equilibrium concept in such models is a stochastic one. Such an equilibrium may be obtained by assuming, for example as in Green and Majumdar [3] , that the economy is subject to random shocks. A more natural line of approach is to look for an endogeneous explanation of persistent price differences. One way of doing this is to suppose that firms may charge different prices because, due to imperfect information, at least some of them set a "wrong" (suboptimal) price [5].

The present paper describes a stochastic model of industry equilibrium in which consumers move randomly between firms, in such a manner that low price firms have a higher probability of increasing their sales. It is shown that the optimal strategy of firms is to lower their price as sales fall, and vice versa. Steady state distributions of sales, and prices, result, which correspond to the stationary state of the underlying stochastic process. Thus price differences persist for purely endogeneous reasons (even though firms do not make mistakes).

Little work of an analytical kind has as yet been directed towards the effect on prices of an improvement in the quality of price information available to consumers. The scope of such action however, by both private and public agencies, has, in many economies, been widening in recent years. (In the U.K., for example, many local authorities now publicize centrally the prices charged by major local retailers for a wide range of branded goods.) It might plausibly be conjectured that such an improvement in information might reduce the *dispersion* of prices: what is of interest however, is rather, whether it can affect *average* price, and it is by no means clear *a priori* whether this is so. The latter question is examined in the final section. We consider two technically identical industries, with the same level of aggregate demand, but in which the degree of responsiveness by consumers to price differences is allowed to differ. The analysis suggests that an improvement in price information tends not only to affect the dispersion of prices, but also to reduce the average price set by profit maximizing firms. Thus increasing information to consumers increases consumer welfare not only insofar as their utility maximizing purchases at prevailing prices may better be determined, but also, via its effects on the optimal price strategy of firms, exerts a redistributive effect in favor of consumers which is tantamount to a lowering of the degree of monopoly.

1. THE DYNAMICS OF CONSUMER FLOW

We will be concerned throughout with an industry consisting of many identical firms selling a homogeneous product to a large number of consumers.

We consider a firm whose output x is a random variable defined over a set of (equally spaced) values lying in the range $o \leq x \leq M$, where we identify M with "full capacity".

If output takes the value j in the current period, it takes values $(j + 1)$, j and $(j - 1)$ with probabilities θ_j, $1 - \theta_j - \phi_j$ and ϕ_j respectively in the next period; the upper and lower bounds to output are then characterised simply by $\theta_M = 0$ and $\phi_o = 0$ respectively.

The process thus described is a random walk between reflecting barriers at o and M. Given any initial value, the probability distribution of output after a time lapse of η periods is readily calculated.

Irrespecitve of its initial value, the probability distribution of output converges over time to a stationary distribution; it is this stationary distribution, as will be seen in Section II, which is relevant to the present study. It is easily shown that this distribution is given by

$$\text{Prob}(1) = C, \quad \text{Prob}(j+1) = \text{Prob}(j) \; . \; (p_j/q_{j+1}) \quad j=1,M-1 \tag{1a}$$

where C is a normalising constant.

Rather than work with a discrete formulation, however, we allow the spacing Δx between successive output states, and the time interval between transitions Δt, to vanish. In order to guarantee a sensible result, it is necessary that

$$\Delta x \to o, \quad \Delta t \to o, \text{ while } (\Delta x)^2 = A\Delta t$$

where A is a constant. ([1] Chap. 5.5)

We may now define the instantaneous mean, or drift, of the process, as

$$\mu(x) = \lim_{\Delta t \to o} \frac{E\{x\ (t + \Delta t) - x(t)\ | x(t) = x\}}{\Delta t}$$

The instantaneous variance of the process $\sigma^2(x)$, is defined in an exactly similar manner. In what follows, we will take this variance to be a constant, σ^2, independent of the value of x. We will be concerned primarily with the behaviour of the drift, which we shall take to be dependent both on current output x, and on the price set by the firm.

The process thus described as a Wiener process between reflecting barriers. From the above definitions of μ and σ^2, the probability distribution function of x over time may be shown to be described by the so called forward differential equation of the process; it will be shown in Section II below, by reference to this equation that the stationary distribution of the process is given by

$$\text{Prob }(x) = C \exp. \{\frac{2}{\sigma^2} \int_o^x \mu(w)dw\} \tag{1b}$$

being the continuous analogue of (1a) above.

We now turn to the determination of the value of drift. We assume that any given consumer may, or may not, search for a lower price during any time interval. Such consumers as do seek a new supplier may choose any firm, including their previous supplier. Thus we may conveniently decompose the drift μ at any level of sales x into two contributions. The first, μ_o, is associated with the departure of existing consumers. We assume that some fraction γ of sales is lost, per unit period, so that

$$\mu_o\ (x) = - \gamma\ x \tag{1}$$

If all firms charged the same price, consumers might choose their new supplier at random. If, however, this is not the case, consumers are taken to be partially informed as to prices, so that a firm which charges a lower price enjoys a greater expected rate of increase of sales to new customers. We will assume, in general, that returns to price reductions are diminishing, in the sense that the associated contribution to drift, which we will label μ_1, rises as price falls, but at a diminishing rate. As price rises, conversely, we assume that the higher the price, p, the more likely a given increase in price is to be noticed by consumers, so that μ falls at an increasing rate as p rises. In fact, as might be anticipated intuitively, the analysis which follows indicates that since, as firm's output level approaches full capacity, the benefit - in terms of possible increases in sales - associated with charging a lower price than one's competitors ("buying goodwill") declines towards zero. It follows that the optimal strategy involves increasing price indefinitely as full capacity is reached, so long as the rate of response of drift to price remains finite. We thus require the assumption that our response curve becomes vertical at some "ceiling price," p_o, as shown in Fig. 1.

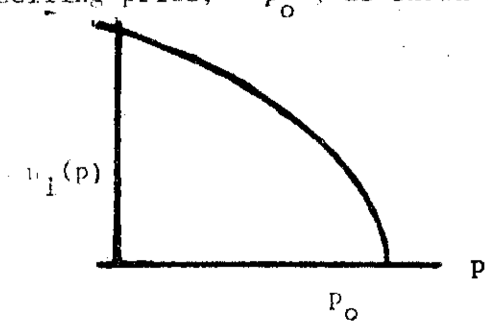

Fig. 1: Drift associated with the arrival of new customers, as a function of price.

The simplest relationship which embodies the characteristics just discussed is the quadratic, which may, moreover, be regarded as an approximation to any curve of this type. We take,

$$\mu_1(p) = \sqrt{\frac{p_o - p}{\alpha}} \quad , \; p \leq po \tag{2}$$

We assume that μ goes to minus infinity for all p exceeding po .

Combining (1), (2) we have

$$\mu(x,p) = \mu_o \;(x) + \sqrt{\frac{p_o - p}{\alpha}} \tag{3a}$$

$$= -\gamma x + \sqrt{\frac{p_o - p}{\alpha}} \tag{3b}$$

or equivalently

$$p = po - \alpha(\mu - \mu_o(x))^2 \tag{3c}$$

The loss of generality implied by our choice of a particular (quadratic) form for the response function should, of course, be borne in mind in interpreting the results which follow.

It should perhaps be stressed that the above formulation has been developed in terms of the absolute price, rather than a price relative such as p/p_o or $p/\overline{p}$. This underlines the fact that our present model is directed towards the analysis of industry equilibrium, rather than aggregate analysis. The aggregate price level depends on the prices prevailing in all industries, and it is this which determines the value of a given absolute change in the price of the product of our industry: the amount of search warranted in seeking a 5% reduction in the price of a tube of toothpaste is very much less than that warranted for a similar fractional reduction in the price of a house.

The response curve, written above as $\mu(x,p)$, depends, of course, on the underlying parameters α, γ, σ^2 and p_o . All but the last of these will be suppressed throughout; where it is important to stress the identity of a particular response curve, parameterized by p_o , however, it will be written as

$$\mu(p_o;x,p) \ .$$

We will see below that increases in ceiling price, p_o , are associated with increases in average price, $\bar{p}$, and average output, $\bar{x}$. Thus the family of curves may alternatively be parameterized by $\bar{p}$, or $\bar{x}$. The ceiling price p_o is more appropriately regarded as a derived parameter, indeed, for given $\bar{p}$ or $\bar{x}$, so that the latter parameterization will be used below (Section III). In studying the optimal price policy of the firm, however, in which we take the response curve as a given, it will be more convenient to work in terms of p_o .

II. THE OPTIMAL PRICING POLICY

The expectations of the firm regarding the probability distribution of sales after any time lapse τ in the above model, depend only on current sales, being independent of present time, i.e., the process is Markovian. Hence, the optimal price depends only on x , and is time independent. We may therefore describe a price strategy as a function $p(x)$ defined on $o \leq x \leq M$; our problem is to find the function which maximizes expected per period profits. We are thus assuming the firm to be a risk neutral, long run, profit maximizer, **with zero discount rate.**

We will confine ourselves to the case where marginal cost is constant up to full capacity; without further loss of generality we may then take marginal cost to be zero over the range $o \leq x \leq M$, and maximize expected revenue. The optimal price strategy for any nonzero level of marginal cost is then obtained by simply adding that value to the price described below, at each x .

We now consider the probability distribution of sales associated with any price strategy. Let $y\,(x_o,x;t)$ be the conditional probability that sales lie in the interval $(x,x+\Delta x)$ at time t , given that $x = x_o$ at $t = o$. Then $y\,(x_o,x;t)$ is described by the forward differential equation of the Wiener process

$$\frac{\sigma^2}{2}\frac{\partial^2 y}{\partial x^2} - \frac{\partial}{\partial x}\{\mu y\} = \frac{\partial y}{\partial t} \tag{4a}$$

with boundary conditions for reflecting barriers at $x = o, M$,

$$\frac{\sigma^2}{2}\frac{\partial y}{\partial x} - \mu y = o;\ x = o,M \tag{4b}$$

where the latter equation corresponds to the requirement that x takes values in the interval $o \leq x \leq M$ only ([1], p.223).

To obtain the stationary distribution, which is independent of the initial condition, we set the time derivative to zero; it is then readily verified that, as noted earlier (equation 1b),

$$y(x) = C \exp\{\frac{2}{\sigma^2}\int_o^x \mu(w)dw\} \tag{5}$$

is a solution of (5), where C is a normalizing constant. For constant μ, the expression (5) reduces to the well known result that the stationary distribution is a uniform distribution if $x = o$, and a truncated exponential distribution otherwise.

The optimal price strategy may now be obtained as follows. We choose as our independent variable

$$y(x) = C\, e^{\frac{2}{\sigma^2}\int_o^x \mu(w)dw} \tag{6}$$

so that, on differentiating,

$$\frac{\dot{y}}{y} = \frac{2}{\sigma^2}.\mu(x) \tag{7}$$

Our problem is to choose $y(x)$, such that expected per period profits are maximized, viz

$$\max \int_o^M xp(x)y(x)dx$$

$$\text{subject to } \int_o^M y(x)dx = 1$$

where $p(x)$ depends on our choice of $y(x)$ as specified by (3c) and (7). Substituting for $p(x)$ we obtain the maximand in explicit form as:

$$\int_o^M x\ \{p_o - \alpha\ [\frac{\sigma^2}{2}.\frac{\dot{y}}{y} - \mu_o(x)]^2\}ydx\ .$$

This is an isoperimetric problem in the calculus of variations, with

Lagrangian

$$G = x\{p_o - \alpha[\frac{\sigma^2}{2} \cdot \frac{\dot{y}}{y} - \mu_o(x)]^2\}y - \lambda y \tag{8}$$

where the Lagrange multiplier λ is a constant. The Euler-Lagrange condition for an optimum is

$$\frac{d}{dx} G_{\dot{y}} = G_y \tag{9}$$

By differentiating (8), and using (7), and its first order derivative,

$$\dot{\mu}(x) = \frac{\sigma^2}{2}\left(\frac{\ddot{y}}{y} - \left(\frac{\dot{y}}{y}\right)^2\right) \tag{10}$$

to transform the resulting expressions, we obtain

$$\frac{d}{dx} G_{\dot{y}} = \frac{\sigma^2}{2} \{-2\alpha(\mu-\mu_o) - 2\alpha x(\dot{\mu}-\dot{\mu}_o)\}$$

and

$$G_y = x\{p_o - \alpha(\mu-\mu_o)^2\} + 2\alpha x\mu(\mu-\mu_o) - \lambda \ .$$

The Euler-Lagrange equation may then be written in the form

$$x(\dot{\mu}-\dot{\mu}_o) + \frac{1}{\sigma^2} x(\mu^2-\mu_o^2) + (\mu-\mu_o) = \frac{2}{\sigma^2} \cdot \frac{\lambda - xp_o}{2\alpha} \tag{11}$$

The natural boundary condition,

$$G_{\dot{y}} = o \quad \text{at} \quad x = o,M$$

takes the form

$$-\frac{\sigma^2}{2} \ . \ 2\alpha x(\mu-\mu_o) = o \ ; \ x = o,M \tag{12}$$

This condition is satisfied identically at $x = o$ for any solution nonsingular at the origin; for any such solution, moreover, we obtain on substituting $x = o$ in (12), the initial conditions

$$\mu(o) = \mu_o + \frac{2}{\sigma^2}\frac{\lambda}{2\alpha} \tag{13}$$

and, by taking the limit of (11) as x approaches zero, and applying ℓ'Hopitals Rule,

$$\mu'(o) = -\frac{1}{(2\sigma^2)^3}\left(\frac{\lambda}{2\alpha}\right)^2 .$$

For $x = M$, we have immediately from (12),

$$\mu(M) = \mu_o(M) \tag{14}$$

The solution $\mu(x)$ is then completely specified as the solution to the first order differential equation (11), with initial condition (13), where λ is determined by the requirement that equation (14) be satisfied.

The differential equation (11) is of the Generalized Riccati type, and may, by means of the substitution

$$u = \sqrt{y}$$

be transformed into a second order linear equation, which can then be solved in series form.[1] It is more convenient in the present instance however to proceed directly from (11). We note at this point, that the Lagrange multiplier λ, which equals the expected per-period profit[2] corresponding to the optimal choice of $\mu(x)$, is strictly positive for any $p_o > o$. This follows from the fact that the strategy

$$p(x) = p_o , \qquad o \leq x \leq M$$

is feasible, and is readily shown to yield a positive profit. The nature of the solutions to the differential equation (11) may conveniently be examined by rearranging the terms of the equation as follows:

$$\dot{\mu}-\dot{\mu}_o = -\frac{1}{\sigma^2}(\mu^2-\mu_o^2) + \frac{1}{x}\cdot\left\{\frac{2}{\sigma^2}\frac{\lambda}{2\alpha} - (\mu-\mu_o)\right\} - \frac{2}{\sigma^2}\frac{p_o}{2\alpha} \qquad (11)'$$

The initial conditions (13) ensure that μ is positive, and declining, at $x = o$. Of the three terms in (11)', the first is negative for $\mu^2 > \mu_o^2$. The second is initially positive, as is readily established by application of ℓ'Hopital's Rule, and appeal to the initial conditions (13). As μ declines, an upper bound to the value of this term in the region $\mu(o) \leq \mu \leq \mu_o(x)$ is given by $\frac{1}{x}\cdot\frac{2}{\sigma^2}\cdot\frac{\lambda}{2\alpha}$. The third term is constant, and negative.

The form of the solutions to the equation are illustrated in Figs 2,3. As noted above μ is declining initially; the first and third terms in (11)' are negative, while the second is positive. We may easily establish, moreover, that μ is decreasing <u>throughout</u> the region $\mu(o) \leq \mu \leq -\mu_o(x)$. For, if $\dot{\mu}$ increases to zero at any point x_o, it is readily established that the second derivative of μ is negative, since, at such a point, the first and second terms are stationary, while the second - positive - term is strictly decreasing, due to the appearance of the factor $1/x$.

In the region $\mu < \mu_o$ all three terms are negative and μ is strictly decreasing.

We now turn to the behavior of μ in the region

$- \mu_o \geq \mu \geq \mu_o$

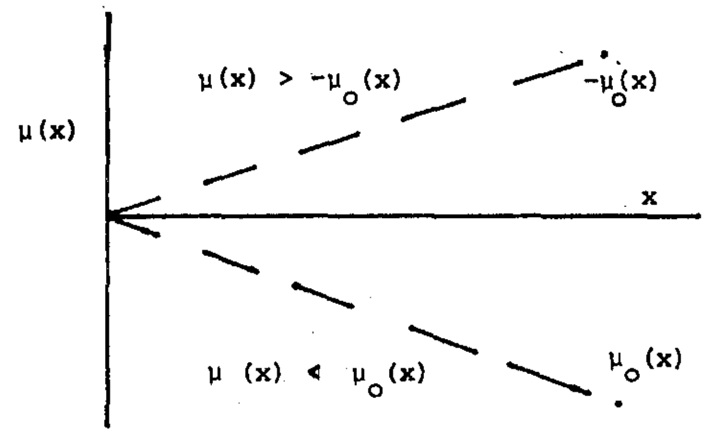

Figure 2

Now μ is declining initially in this region, and it will continue to decline throughout unless the terms

$$- \frac{1}{\sigma^2} (\mu^2 - \mu_o^2) + \frac{1}{x} \frac{2}{\sigma^2} \cdot \frac{\lambda}{2\alpha}$$

which are positive, exceed the remaining terms

$$- \frac{1}{x} (\mu - \mu_o) - \frac{2}{\sigma^2} \frac{p_o}{2\alpha}$$

in absolute value.

We consider the family of solutions parameterized by λ as shown in Figure 3. For $\lambda = o$, we have from our initial condition that $\mu(o) = o$ so that $\mu(x)$ is strictly decreasing, since it remains in the region $\mu \leq \mu_o(x)$ throughout. As λ increases, the latter of our two positive terms increases so that for sufficiently large λ, the curve begins to increase at some point; it is easily shown that such a value of λ exists. We now consider the nature of the limiting case for which $\mu(x)$ becomes tangent to $\mu_o(x)$ at some point M^c, as shown in the diagram (Fig. 3).

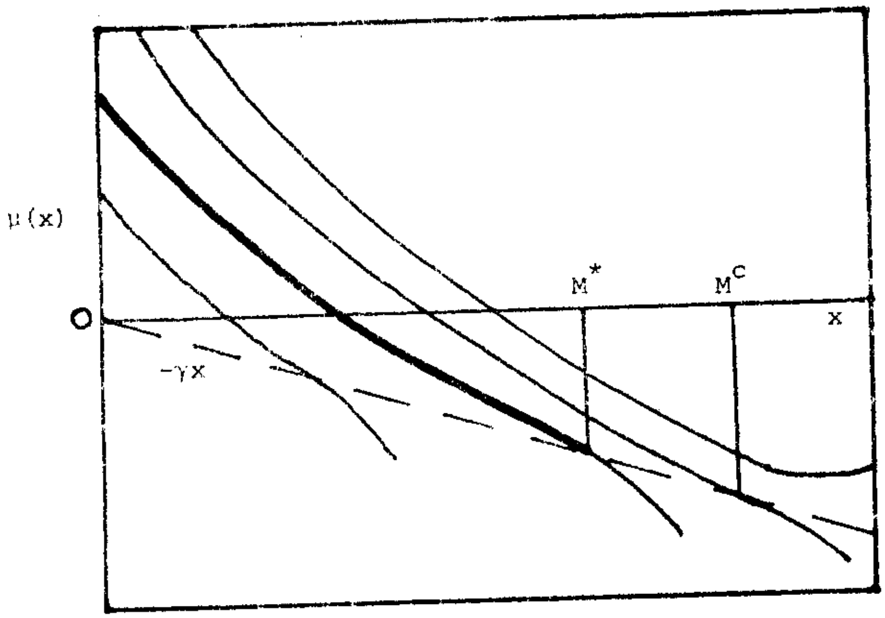

Fig 3

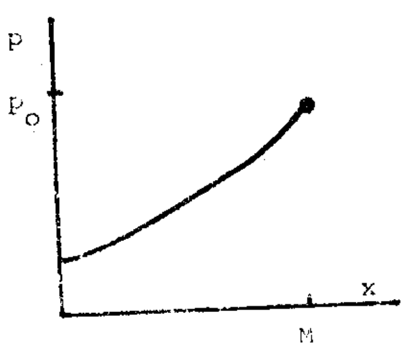

Fig 4a

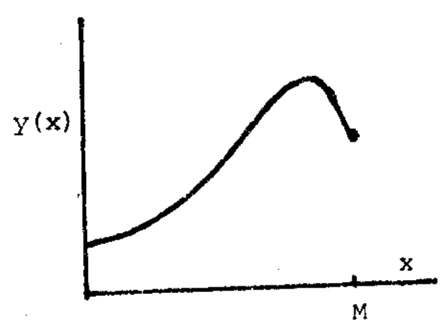

Fig 4b

This implies that

$$\mu = \mu_o ,$$

$$\dot{\mu} = \dot{\mu}_o ,$$

and so, from equation (11)',

$$\frac{1}{x} \cdot \frac{2}{\sigma^2} \cdot \frac{\lambda}{2\alpha} = \frac{2}{\sigma^2} \frac{p_o}{2\alpha} , \quad \text{at} \quad x = M^c$$

or, equivalently,

$$\lambda = p_o M^c . \tag{15}$$

Equation (15) corresponds in fact to the upper bound of λ, associated with the limiting case where the maximum price p_o is always charged, and output coincides always with the full capacity level. Before interpreting (15) fully, we first consider the form of the solution for a value of M lying in the range $o < M < M^c$. Such a case is illustrated by the capacity level M^* shown in Fig. 4a. The boundary condition

$$\mu(M) = \mu_o(M) = - \gamma M$$

is used to identify the solution (heavy line) and the associated value of expected per period profits λ^*. It is seen that drift falls, so that price rises, with output, as shown in Fig. 4a, which illustrates the associated price strategy $P(x)$, derived by substitution of our solution $\mu(x)$ into the equation of the response curve (equation (3)). The probability distribution of sales, $y(x)$, is obtained from $\mu(x)$, using equation (6). (Figure 4b)

The solutions for $\mu(x)$, $p(x)$ and $y(x)$ are readily obtained for any particular values of M, α, σ and γ by numerical methods. It has been found in practice that an efficient computational procedure is to choose an arbitrary value of λ initially (say $\frac{1}{2} p_o M$) , and estimate $\mu(\lambda;x)$ by using the Runge-Kutta method to solve equation (11). This yields a value of $\mu(M)$ which in general differs from $-\gamma M$. The value of λ is then increased or decreased according as $\mu(M)$ is greater than, or less than, $-\gamma M$. It has been found that the value of λ converges rapidly to λ^* .[3]

We now return to the limiting case of the equation represented by condition (15), corresponding to a value of output which always coincides with full capacity, while price always remains at p_o . It is readily shown that for any $\alpha > o$, that λ is always less than $p_o M$, since the requirement that output be equal to the full capacity level on average implies that $\mu \to + \infty$. This can only occur in the limit $\alpha \to o$, $\sigma \to o$, where consumers respond infinitely rapidly to price reductions. Thus equation (15) is satisfied only in the limiting case corresponding to perfect competition, in which consumers respond infinitely rapidly, and with certainty, to price reductions. Under these circumstances a (positive output) equilibrium is possible only at full capacity operation, corresponding to the elementary analysis of the L-shaped cost curve analyzed here (Figure 5a). We see from our initial condition (13) that, in this case, since $\alpha \to o$, and $\sigma \to o$, we have $\mu(o) \to \infty$. Under these circumstances, and not otherwise, our solution may be characterized by tangiency as illustrated at $x = M^c$ in Figure 3 . The behavior of the probability distribution of output as $\alpha \to o$, $\sigma \to o$ is illustrated in Figure 5b; $y(x)$ converges pointwise to the Dirac delta function centered on $x = M$, while the price function degenerates to the single point $(x = M, p = p_o)$.

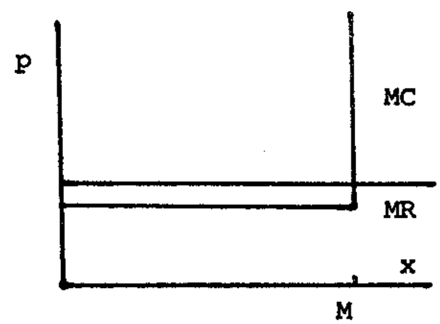

Fig 5a

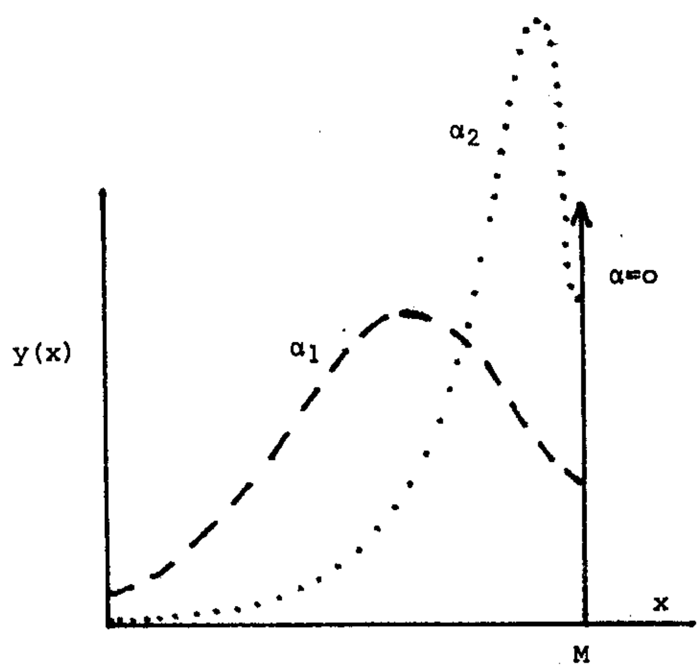

Fig 5b

III. THE INDUSTRY SUPPLY CURVE

We now consider the effect of changing the value of p_o on the quantities

$$\text{Average output, } \bar{x} = \int_o^M xy(x)dx$$

and

$$\text{Average price, } \bar{p} = \int_o^M xp(x)y(x)dx \;/ \int_o^M xy(x)dx \; ,$$

$$= \lambda \;/\; \bar{x} \; .$$

The parameters γ (describing the rate of turnover of consumers), α (their rate of response to price differences) and σ^2 (the variance associated with that response), are fixed data in respect of any given group of consumers. For such a group, we now consider the relationship between maximum price p_o , average price $\bar{p}$, and average output $\bar{x}$.

In order to establish our theorem, we first need two preliminary results. The first of these establishes the relationship between the drift function $\mu(x)$ and the average values of x and p .

<u>Lemma I</u>

For two strategies, such that $\mu_1(x) > \mu_2(x)$ everywhere in [o,M], except possibly the endpoints, where $\mu_1(x) \geq \mu_2(x)$, then $\bar{x}_1 > \bar{x}_2$.

Our second result provides us with a condition sufficient to guarantee that the requirement of the above Lemma is met in the examples which follow. Since an explicit solution of our differential equation in closed form may not be obtained the requirement $\mu_1(x) > \mu_2(x)$ can not be verified directly. The values of $\mu(x)$ at the endpoints are however known, so that the following condition is easily applied.

We represent the value of the expression for the derivative $\dot{\mu}$ given by equation (11), at any point in the μ,x plane, by $\dot{\mu}(\mu,x)$. We then have:

Lemma II

Let two strategies be such that

$$\mu_1(o) > \mu_2(o) \ ; \ \mu_1(M) \geq \mu_2(M)$$

and either $\quad \dot{\mu}_1(x,\mu) > \dot{\mu}_2(x,\mu) \qquad$ (15a)

or $\quad \dot{\mu}_1(x,\mu) < \dot{\mu}_2(x,\mu) \qquad$ all $\mu, x,$ $\qquad$ (15b)

then $\quad \mu_1(x) > \mu_2(x) \qquad$ all $x, \ o < x < M$.

The usefulness of Lemma II lies in the fact that the condition required may be verified directly from the differential equation (11); the point (x,μ) in (15) may or may not lie on the curve $\mu(x)$.

We now apply the above results to establish the following theorem.

Theorem I

For any two industries described by the same set of parameters α, σ, γ, μ_o , but different insofar as $p_{o_1} = p_{o_2} + \Delta p_o$, $\Delta p_o > o$; then

(i) $\quad \bar{x}_1 > \bar{x}_2$

(ii) $\quad \bar{p}_1 > \bar{p}_2$

(where bars denote expected values).

Proof

Let $p_2(x)$ be the price strategy optimal in II, and construct the strategy $p_2(x) + \Delta p_o$; using this strategy in I earns a profit of

$$\lambda_2 + \bar{x}_2 \, \Delta p_o$$

so that we have, $\qquad$ for the strategy optimal in I,

$$\lambda_1 \geq \lambda_2 + \bar{x}_2 \, \Delta p_o > \lambda_2 \tag{16}$$

The relationship between the two optimal strategies is illustrated in Figure 6.

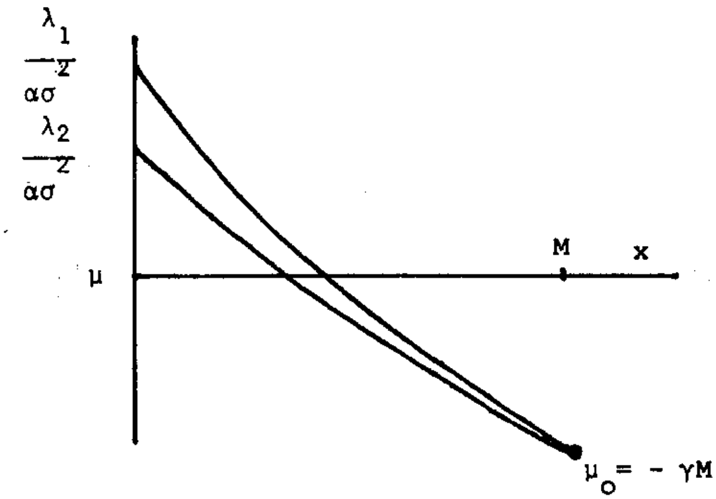

Figure 6

Moreover, rearranging our differential equation (11), $x(\dot{\mu}-\dot{\mu}_o) = -\frac{1}{\sigma^2} \cdot$ $x(\mu^2-\mu_o^2) - (\mu-\mu_o) + \frac{2}{\sigma^2}\frac{\lambda - xp_o}{2\alpha}$.

We now examine the final term of the equation; as shown in Figure 7, there is at most one value x^* where the two final terms are equal. Call this x^* .

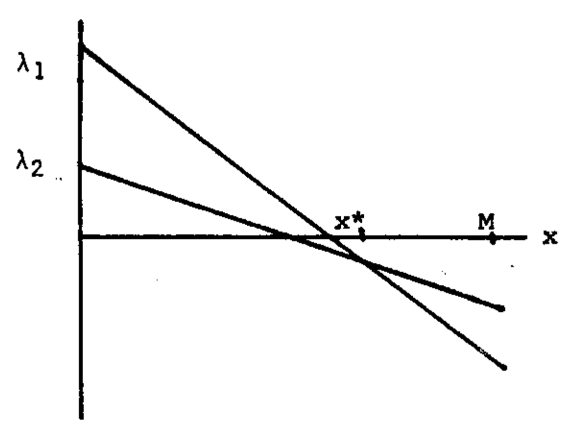

Figure 7

Now

$$\dot{\mu}_1(o) > \dot{\mu}_2(o)$$

and, to the left of x^* ,

$$\dot{\mu}_1(x,\mu) > \dot{\mu}_2(x,\mu)$$

from which we have immediately

$$\mu_1 > \mu_2 , \qquad o \leq x \leq x^*$$

where the strict inequality for the point x^* follows from continuity. We now consider the region $x^* \leq x \leq M$. We have from the above argument

$$\mu_1(x^*) > \mu_2(x^*)$$

Moreover

$$\mu_1(M) = \mu_2(M) = -\gamma M .$$

Thus, since,

$$\dot{\mu}_1(x,\mu) < \dot{\mu}_2(x,\mu)$$

it follows that, by Lemma 2,

$$\mu_1 > \mu_2 \qquad\qquad x^* \leq x < M .$$

Thus we have established that $\mu_1 > \mu_2$ for all $x < M$, so that we have immediately from Lemma 1

$$\bar{x}_1 > \bar{x}_2$$

which establishes (i). In order to establish (ii), we suppose the contrary, i.e.,

$$\bar{p}_1 \leq \bar{p}_2 .$$

We begin by establishing that the first relation in (16) above in fact holds with the strict inequality, i.e.,

$$\lambda_1 > \lambda_2 + \bar{x}_1 \Delta p_o \tag{17}$$

This follows from the first part of the theorem; for consider a third economy similar to the others except that

$$p_{o_3} = p_{o_2} + \frac{1}{2} \Delta p_o .$$

We then have, applying (16), and using the fact that $\bar{x}_3 > \bar{x}_2$ by the first part of the theorem,

$$\lambda_1 \geq \lambda_3 + \frac{1}{2} \bar{x}_3 \Delta p_o$$

$$\geq \lambda_2 + \frac{1}{2} \bar{x}_2 \Delta p_o + \frac{1}{2} \bar{x}_3 \Delta p_o$$

$$> \lambda_2 + \bar{x}_2 \Delta p_o$$

which establishes (17).

We thus have,

$$\bar{p}_1 \bar{x}_1 > \bar{p}_2 \bar{x}_2 + \Delta p_o \bar{x}_2$$

$$\bar{x}_1 > \frac{\bar{p}_2}{\bar{p}_1} \bar{x}_2 + \frac{\Delta p_o}{\bar{p}_1} \bar{x}_2 .$$

We are assuming that $\bar{p}_1 \leq \bar{p}_2$. Write $\bar{p}_2 - \bar{p}_1$ as $\Delta \bar{p}$ so that $\bar{p}_1 = \bar{p}_2 - \Delta \bar{p}$ where $\Delta \bar{p}$ is positive. Thus

$$\bar{x}_1 > \{1 + \frac{\Delta \bar{p}}{\bar{p}_1}\} \bar{x}_2 + \frac{\Delta p_o}{\bar{p}_1} \bar{x}_2$$

$$\frac{\Delta \bar{x}}{\bar{x}_2} = \frac{\bar{x}_1 - \bar{x}_2}{\bar{x}_2} > \frac{\Delta \bar{p}}{\bar{p}_1} + \frac{\Delta p_o}{\bar{p}_1} .$$

We now proceed to construct an alternative price strategy for industry II, by setting $p(x)$ equal to the value optimal in I, less Δp, at each x. This implies that $\mu(x)$, and ~~so~~ $\bar{x}$, coincide with the values obtained in I using the optimal strategy. We then have, for our alternative strategy, which we label "a",

$$\lambda^a = \bar{p}^a \bar{x}^a = (\bar{p}_1 - \Delta p_o)\cdot\bar{x}_1$$

$$= (\bar{p}_2 - \Delta\bar{p} - \Delta p_o)\ (\bar{x}_2 + \Delta\bar{x})$$

$$> \bar{p}_2\bar{x}_2 = \lambda_2$$

since

$$\frac{\Delta\bar{x}}{\bar{x}_2} > \frac{\Delta\bar{p} + \Delta p_o}{\bar{p}_2} \quad .$$

Thus our alternative strategy is superior to the optimal strategy, contradicting our hypothesis. Hence $\bar{p}_1 > \bar{p}_2$, which establishes the second part of our theorem.

We may now apply Theorem I to investigate the supply curve of the industry, and the nature of industry equilibrium.

We assume throughout that the parameters γ, describing the level of search activity of consumers, α, their rate of response to price differences and σ^2, the associated variance of the process, are fixed in respect of the group of consumers in question. The family of response curves $\mu(x,p)$ which firms may face are then conveniently parameterized by p_o, as in the earlier development. By virtue of Theorem I however, we have established that p_o, $\bar{p}$ and $\bar{x}$ all increase together, so that we may parameterize our curves equivalently in terms of average price, or average output.

We assume throughout that all learning has been done already by firms and consumers in respect of α, γ and σ^2, which will be taken as known with certainty by all agents.

We begin by defining the industry supply curve as the locus of all points $(\bar{p}, N\bar{x})$ traced out as p_o goes from zero to infinity. Theorem I ensures that the supply curve is upward sloping, and that there is a 1:1 correspondence between values of p_o, and points on the supply curve.

A series of industry equilibria then exist which are characterised as follows: each firm anticipates correctly, via past learning, the response curve $\mu(\bar{p};x,p)$ which it faces. It sets an optimal price policy $p(\mu;x)$ and correctly anticipates average sales $\bar{x}\ (\mu)$, so that the price charged is $\bar{p}\ (\mu)$. Moreover, given this price policy on the part of firms, average aggregate demand by consumers coincides with average aggregate supply $N\bar{x}\ (\mu)$.

An increase in demand leads to a new equilibrium corresponding to an upward movement along the supply curve, and vice versa. The learning process by which firms discover the response curve which they face, and adjust to a change in demand, is outside our present scope, and will be the subject of a separate paper.

It is an important feature of the present model that prices do <u>not</u> converge over time to adjust to a unique price; firms' sales fluctuate over time, and price is adjusted accordingly. The weak condition proposed by Fisher [2], which guarantees covergence to a unique price, does not hold. It is not necessarily the case that if Firm A charges a lower price than Firm B, then it will have sales no less than B's. It is, of course, true that its expected sales after a time lapse Δt are greater; so long as the variance of the process is finite, however, Fisher's condition will not hold.

IV. THE EFFECT OF INCREASING INFORMATION

In the present section we examine the effect of a higher degree of price responsiveness by consumers (a lower value of α), due to improved information concerning prices.

We wish to compare two economies with all parameters except α equal, and in which the (expected) levels of aggregate output - and so of average output for each of our identical firms - coincide.

We begin by comparing two industries I, II with equal values of γ, σ ; we wish to assume that industry II has a lower value of α . If both

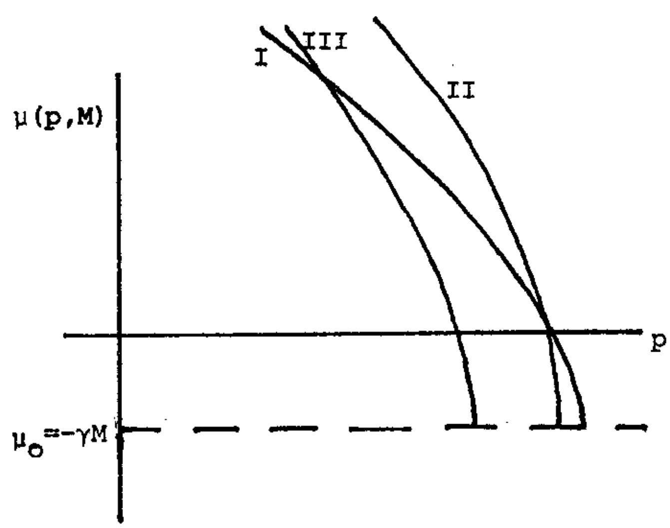

Figure 8

industries had the same value of p_o , it is easily shown that the industry with the lower value of α would have a larger value of λ . We construct an industry, II, with a value of p_o such that I and II have <u>equal</u> values of λ ; using Theorem I, and noting that $\lambda = \bar{x}\,\bar{p}$ by virtue of our definition of $\bar{p}$, this implies that industry II must have a lower value of p_o . (Figure 8)

Thus:

$$\begin{cases} \alpha_2 < \alpha_1 \\ \lambda_2 = \lambda_1 \\ p_{o_2} < p_{o_1} \end{cases} , \quad \text{or} \quad \frac{\lambda_2}{\alpha_2} > \frac{\lambda_1}{\alpha_1}$$

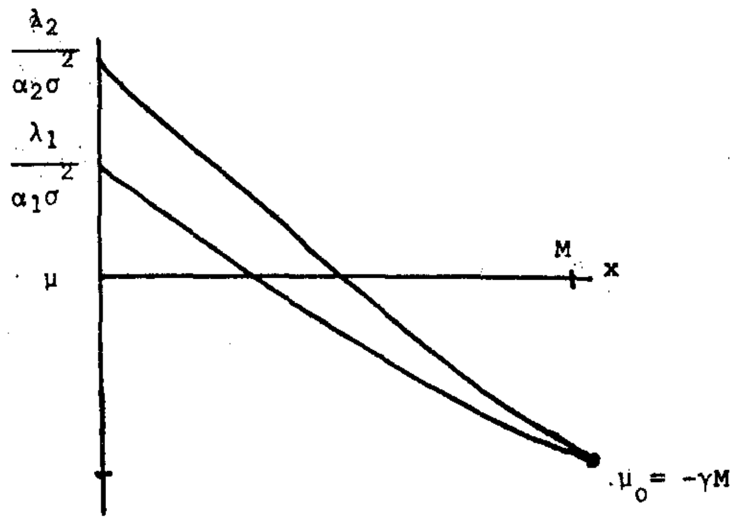

Figure 9

Hence $\mu_2(o) > \mu_1(o)$ as shown in Figure 9. Referring to our differential equation

$$x(\dot{\mu}-\dot{\mu}_o) = -\frac{1}{\sigma^2}\, x(\mu^2-\mu_o^2) - (\mu-\mu_o) + \frac{2}{\sigma^2}\,\frac{\lambda-xp_o}{2\alpha} \tag{11}$$

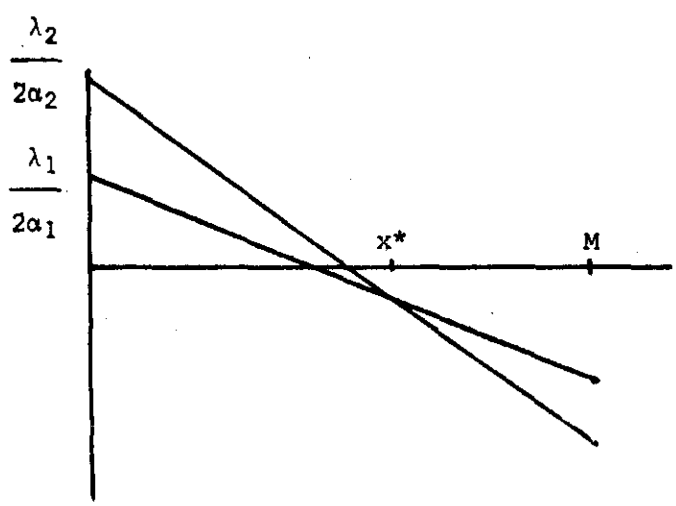

Figure 10

We note that the third terms are equal at, at most, one point x^* ; to the left of x^*

$$\dot{\mu}_2(\mu,x) > \dot{\mu}_1(\mu,x)$$

so that

$$\mu_2 > \mu_1 \quad , \quad o \leq x \leq x^*$$

where the strict inequality at x^* follows from continuity. (Figure 10) To the right of x^* , we have

$$\dot{\mu}_2\ (\mu,x) < \dot{\mu}_1\ (\mu,x)\ , \qquad x^* < x < M$$

Moreover $\mu_2\ (x^*) \ > \mu_1\ (x^*)$

$$\mu_2\ (M) \ = \mu_2\ (M)$$

Hence $\mu_2\ (x) \ > \mu_1\ (x) \quad , \quad x^* \leq x < M\ .$

Thus we have proved that $\mu_2 > \mu_1$ for all $x < M$ and so by Lemma I,

$$\overline{x}_2 > \overline{x}_1$$

But $\lambda_2 = \overline{p}_2\ \overline{x}_2 = \overline{p}_1\ \overline{x}_1 = \lambda_1$

Thus $\overline{p}_2 < \overline{p}_1$.

Hence, for two industries with σ , γ equal, and earning equal profits, the more competitive industry has a higher average level of output and lower average price.

Our theorem now follows easily; for if our hypothetical economy II, with a lower value of α , produces more output at a lower price, λ being fixed, then if we consider an economy with the same level of output as our original economy, but with $\alpha = \alpha_2$, this clearly corresponds (by Theorem I) to a lower value of p_o , and so (again by Theorem I) to a value of $\bar{p}$ which is lower still than that of I. Formally, we define economy II as follows:

$$\alpha_3 = \alpha_2 ,$$

We require $\bar{x}_3 = \bar{x}_2$, so by Theorem I ,

$$p_{o_3} < p_{o_2}$$

Therefore $\bar{p}_3 < \bar{p}_2$

But $\bar{p}_2 < \bar{p}_1$

$\therefore$ $\bar{p}_3 < \bar{p}_1$

which establishes our result, viz Theorem II: for any two industries I, II, with equal values of γ , σ , but with $\alpha_2 < \alpha_1$, then, the average price associated with a given value of average output is lower in the more competitive[4] industry II.

CONCLUSIONS

The above result suggests that in an economy in which perfect information on prices is lacking, the degree of monopoly is closely related to the quality of price information. Specifically, as information improves, the aggregate supply curve shifts downwards , implying a lower average price level for any level of aggregate demand. Thus the quality of information affects not only the dispersion of prices in the system - an effect generally recognized since Stigler's classic study [6] - but affects also the average industry price level. The reason for the existence of such an effect lies in the fact that the payoff to firms associated with a given price reduction is enhanced, as price responsiveness improves, while its cost, in terms of losses on unit margins, remains fixed: thus firms choose to purchase more of the now relatively cheaper "strategy" - lower prices, and higher sales.

APPENDIX

The present appendix provides proofs of the two Lemmata stated in the text.

Lemma I

If $y(x) = C \exp \int_o^x \mu(w)dw$ is the probability density function of x, and if

$$\mu_1 (w) > \mu_2 (w) , \text{ all } w , a < w < b \qquad \text{(i)}$$

$$\mu_2 (w) \geq \mu_2 (w) , w = a,b \qquad \text{(ii)}$$

then,

$$\bar{x}_1 > \bar{x}_2 \qquad \text{(iii)}$$

Proof

Let y_1 , y_2 be the p.d.f.'s of x_1 , x_2 respectively. Then, from (i), we have immediately,

$$y_1 (x) , y_2 (x) > o , \text{ all } x \qquad \text{(iv)}$$

From (ii), it follows that

$$\int_o^x \mu_1 (w) \, dw - \int_o^x \mu_2 (w) \, dw$$

is a strictly increasing function of x. Thus,

$$y_1 (x) / y_2 (x) = (c_1/c_2) \exp \int_o^x [\mu_1(w)-\mu_2(w)] \, dw$$

is a strictly increasing function of x. Hence, there is some value of x, say x_c, such that

$$y_2 (x) - y_1 (x) \lesseqgtr o \quad \text{as} \quad x \lesseqgtr x_c . \qquad \text{(v)}$$

Moreover, since the integral of each of y_1 , y_2 is unity, over the interval [a,b] ,

$$\int_a^b [y_1(w)-y_2(w)] = o$$

$$\int_a^{x_c} [y_2(w)-y_1(w)]\, dw = \int_{x_c}^b [y_2(w)-y_1(w)]\, dw \qquad \text{(vi)}$$

Combining (iv), (vi) and (vii) we then have,

$$\begin{aligned}\int_{x_c}^b w\, [y_2(w)-y_1(w)]\, dw &> x_c \int_{x_c}^b [y_2(w)-y_1(w)]\, dw \\ &= x_c \int_o^{x_c} [y_2(w)-y_1(w)]\, dw \\ &> \int_o^{x_c} w\, [y_2(w)-y_1(w)]\, dw\end{aligned}$$

Thus, using (v), we have

$$\int_a^b w\, [y_2(w)-y_1(w)]\, dw > o \qquad \text{(vii)}$$

We then have,

$$\begin{aligned}\overline{x}_2 &= \int_a^b w\, y_2(w)\, dw \\ &= \int_a^b w\, y_1(w)\, dw + \int_a^b w[y_2(w)-y_1(w)]\, dw\end{aligned}$$

The first term here equals $\overline{x}_1$ however, while the second term is strictly positive by (vii), so that our result, (iii), follows immediately.

<u>Lemma II</u>

Let μ_1,μ_2 be two functions of x , differentiable on [o,M], with derivatives $F_1(\mu,x)$, $F_2(\mu,x)$ respectively. Then if $\mu_1(o) > \mu_2(o)$; $\mu_1(M) \geq \mu_2(M)$, and if either $\quad F_1(\mu,x) < F_2(\mu,x)$ (i)

or $\quad F_1(\mu,x) > F_2(\mu,x)$ (ii)

then

$$\mu_1(x) > \mu_2(x) \ , \ o < x < M \ .$$

We will prove case (i); the proof follows in a similar manner for (ii).

<u>Proof</u>

Suppose $\exists\, x_o$, $o < x_o < M$, st $\mu_1(x_o) < \mu_2(x_o)$. Construct the (continuous) function $\mu_1(x) - \mu_2(x) = f(x)$, say, with derivative $f'(x) < o$ by (i). Now $f(x_o) < o$, $f(M) \geq o$. Then, by the Mean Value Theorem, we can find c_1 , $x_o \leq c_1 \leq M$, such that $f'(c_1) > o$, contradicting the negativity of $f'(x)$. Hence $\mu_1(x) \geq \mu_2(x)$, $o < x < M$.

We now extend this result to establish the strict inequality.

Suppose $\mu_1(x_1) = \mu_2(x_1)$, $o < x_1 < M$. Then since μ_1 , μ_2 are continuous, and since $F_1(\mu,x) < F_2(\mu,x)$, it follows that there exists some point x_o in the neighborhood of x_1 such that $\mu_1(x_o) < \mu_2(x_o)$. The above proof then follows as before. Hence we have established that $\mu_1(x) > \mu_2(x)$, $o < x < M$.

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Footnotes

1. It is in fact a generalization of Sharpe's differential equation, and reduces to the latter for certain values of the parameters. The boundary condition (14) ensures that only the first branch of the function, which has a countable number of singularities, appears in our solution.

2. In the isoperimetric problem, if the objective functional is linear,

$$F(ky(x)) = kF(y(x))$$

then it is easily shown that the value of the Lagrange multiplier coincides with the optimal value of F. It follows immediately by inspection of (8) that this condition holds in the present case.

3. For example, about five iterations were sufficient to estimate λ to within 2-3%.

4. This statement of Theorem II assumes that the search parameter γ remains unaltered. It might seem reasonable to define our "more competitive" economy, II, by $\alpha_2 < \alpha_1$, $\gamma_2 \geq \gamma_1$. It may be shown that our result continues to hold for this formulation.