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A MODEL OF INSURANCE MARKETS WITH ASYMMETRIC INFORMATION

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by

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1. Introduction

The question with which this paper is concerned is how a competitive insurance market will allocate insurance policies when firms cannot distinguish among different risk classes of consumers.[1] To analyze this problem, I have chosen the simplest model I could find which still captures at least some of its essential features. Consumers have a random endowment which takes one of only two possible values. All consumers have the same attitude toward risk but may differ in their individual probability of receiving a low value as their endowment. Insurance firms provide insurance policies which permit consumers to supplement their endowment when it is low in exchange for a reduction of their endowment when it is higher. There are no transactions costs. The problem to be answered in this paper, then, is which policies firms will make available if firms cannot directly determine the probability of receiving a low endowment of each individual consumer.

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[1] Much of the basic analysis presented here was developed earlier by Rothschild and Stiglitz in a preliminary draft of [2], although most of my results were developed independently of their work. The main point of divergence between our work is my introduction of an equilibrium concept with non-static expectations. Pauly [1] has also used this basic model to analyze some issues in public provision of insurance, but puts additional restrictions on the policies firms can offer which are not made here.

There are several important features to be noted about this problem. First, there is no way that firms can distinguish among consumers before offering them a set of policies. Secondly, if an insurance policy is expressed as a two-dimensional vector with a premium on one axis and an indemnity (i.e. payment in case of a low endowment) on the other, it can be demonstrated that different risk types have different preference orderings over the set of policies. This in turn leads to additional complications in the problem of the firm.

Assuming firms can monitor the total amount of insurance a consumer has purchased, firms may have an incentive to restrict the amount of insurance which can be purchased at any given price (i.e. premium/indemnity ratio)--aside from any problems of moral hazard. By limiting the amount of insurance available at a low price, and offering more insurance at a higher price, firms may succeed in attracting high risk consumers to the high priced insurance and low risk consumers to the lower priced insurance. If the amount of insurance which can be purchased at the lower price is increased, however, high risk consumers may be attracted to the lower priced policy, making it unprofitable for firms to offer it.

The possibility just outlined also points to another problem which firms may face. Since the profitability of a policy depends upon which risk types purchase it, and the policies which each risk type purchases depends upon which policies are offered, the profitability of one firm's offer of policies may depend upon the offers of other firms. Besides the possibility of being undersold by other firms--for instance, lowering the premium for the same level of insurance, there is also the possibility that when the policies which high risk types prefer are withdrawn, the high risk types will be attracted to policies which are profitable only when sold to low risk types. Consequently, the profitability a given policy offer may depend critically upon the actions of other firms. This possibility, in turn, leads to fundamental problems with the existence of an equilibrium.

In analyzing the behavior of firms in this model, I avoid any possibility of explicit collusion among firms and also rule out any consideration of sophisticated dynamic threat strategies on the part of firms. Rather, a firm's expectations about how other firms will respond to its actions are based upon simple rules of thumb which, presumably, have been suggested by the firm's experience in the market. If one is trying to approximate a world with informational and computational limitations, I think that this may not be an unreasonable approach. Accordingly, I start with the presumption that although firms have learned the profitability of each set of policies it could offer, it has essentially static expectations about the response of other firms to changes in its own policy offer. That is, each firm anticipates that other firms will not change their policy offers in response to any changes which it may make in its own policy offer. Equilibrium then refers to a set of policies which, if already being offered in the market, no firm has an incentive to change. It will be demonstrated, however, that with static expectations, or something close to it, a robust class of examples can be constructed where there is no stationary equilibrium. In response to this result, I then introduce a different equilibrium concept which implicitly incorporates a non-static expectation rule on the part of firms designed to reflect the responses that firms would observe if they used the static expectation rule. Under the assumptions of this model, it is demonstrated that this modification does always lead to the existence of an equilibrium.

The firms' expectations are modified by assuming that each firm will correctly anticipate which policies in the offers of other firms will become unprofitable as a consequence of any changes in its own offer. It assumes that these policies will be withdrawn and then calculates the profitability of its new offer accordingly. It will have an incentive to offer the new set of policies only if it makes positive profits after the other firms have made the anticipated adjustments in their policy offers.

In general, my approach to this problem has been the following. Start with simple expectation rules and determine the equilibrium. If none exists, modify the expectations in a way which might be suggested by the firms' experiences and then search for a new equilibrium. Continue the process until an equilibrium exists.

Even if one concedes that it is not feasible for firms to calculate dynamic non-cooperative equilibria for the model, there at least is one serious objection to this approach. The assumption about how firms will revise their expectations at any given point is, of necessity, rather _ad hoc_ and not necessarily unique. Given the same experience, firms could have learned different lessons about the consequences of their actions. As a consequence, different expectation rules might emerge which would lead to very different equilibria. Since I do not even attempt to describe the market behavior when it has not reached a stationary state, I find this objection particularly compelling.

On the other hand, I think one can argue that the validity of using a given expectation rule may well depend on whether or not it is consistent with a stationary state. It does not necessarily follow that expectation _rules_ will be revised simply because they have proved to be incorrect in a few instances. If a firm is going to reject one rule, it must adopt another. Therefore, it seems most likely that a rule will be rejected when two conditions are realized: (i) The rule is consistently and continually violated; and (ii) A pattern emerges which suggests a correction to the rule. Since the longer a rule is violated the more likely it may be that a pattern will be recognized, these conditions are somewhat interdependent. Furthermore, they are most likely to be met when no stationary equilibrium exists. In that case the market could never reach a point where expectations were consistently confirmed, and it is in such a case that I look for revisions in the expectation rule which would lead to an equilibrium.

Having defined an equilibrium which exists, I then turn my attention to the welfare properties of the allocation of policies which it implies. Even with the implicit information constraints taken into account, I find that generally the market allocation of policies in this model will not be Pareto optimal. That is, it is often possible to devise procedures which allocate policies in such a way as to make everyone better off and which do not require that the risk classes be identified beforehand. It should be emphasized from the outset, however, that these results depend critically upon the expectation rules that firms are assumed to follow.

The formal analysis will proceed by representing the problem of the firm as its choice of the set of policies it will offer to consumers. Consumers will then choose their best policy from among the union of sets which are offered. An equilibrium will correspond to a set of policies which no firm has an incentive to change, given its expectations.

Although the ideas presented in this paper are rather straightforward, I have found it difficult to present them both precisely and simply. Therefore, the formal analysis of most of the important points has also been illustrated with graphical examples. In addition, some of the obtuseness of the presentation may be reduced if the reader remembers that a set of policies in this model corresponds roughly to a market price in more familiar models of competitive markets or monopolistic competition.

2. Consumers and Market Demand

The economy consists of one consumption good and a flow of consumers who enter the market continuously.

Assumption 1: Each consumer owns a random endowment which takes on one of two values: y or x where $y > x > 0$.

Definition 1: A consumption vector (c_x, c_y) is an element of R_+^2 which denotes the consumption of a consumer when the value of his endowment is x and y respectively.

Assumption 2: The consumers can be partitioned into a finite number of types indexed by $\{1, \ldots, I\} = I^*$, with $I \geqq 1$. P_i denotes the probability of type i consumers receiving x as the value of their endowment, and $i > j$ implies $P_i < P_j$. Furthermore $0 < P_I < P_1 < 1$.

If $i > j$, then type i consumers will sometimes be referred to as lower risk types than type j consumers.

Assumption 3: The rate at which each type of consumer enters the market will be denoted by vector $a \in R_+^I$ where a_i denotes the rate of flow of the i^{th} type consumer. The vector a is assumed to be constant over time, and $a_i > 0$ for each $i \in I^*$.

Each consumer's attitude toward risk is given by an identical utility function, $u(\cdot)$, defined over all non-negative values of consumption.

Assumption 4: $u(\cdot)$ is a strictly increasing, strictly concave, twice continuously differentiable function from R_+ to R for which the expected utility theorem holds.

Given assumptions 1 and 4, there may be an incentive for consumers to exchange their random endowments for another consumption vector. I will assume that consumers may make such an exchange only by purchasing insurance policies from firms.

Definition 2: An insurance policy is a two-dimensional vector $s = (s_1, s_2)$. If a consumer purchases s, his consumption vector becomes $(x - s_1 + s_2, y - s_1)$. The value, s_1, will be referred to as the premium and s_2 as the indemnity.

Clearly, for any consumption vector there is a unique insurance policy by which the consumer can attain that consumption vector. Since the utility function is only defined over non-negative levels of consumption, the set of insurance policies which can be considered are restricted to those which generate non-negative levels of consumption in both state x and y.

Definition 3: The space of insurance policies is:

$$\bar{S} = \{s \in R^2 : x - s_1 + s_2 \geqq 0,\ y - s_2 \geqq 0\} .$$

Since the consumption vector of each consumer can be represented by the insurance policy he purchases, the preference ordering of each type i over the set of consumption vectors implied by the utility function, u, and the probability, P_i, of receiving x as an endowment can also be represented by an indirect utility function, v^i, defined over the set of insurance policies.

<u>Definition 4</u>: $v^i(s) = P_i u(x - s_1 + s_2) + (1 - P_i)u(y - s_1)$ for all $s \in \bar{S}$.

In what follows, it will be more convenient to work directly with the indirect utility function since it is insurance policies and not consumption vectors which firms directly sell to consumers.

From the definition of v^i some useful properties of that function follow immediately:

<u>Lemma 1</u>: For each i , $v^i(\cdot)$ is a concave twice differentiable function throughout $\bar{S}$.

<u>Proof</u>: The lemma follows upon examining the second derivative of $v^i(\cdot)$ and noting that the sufficient conditions for concavity are satisfied.

Q.E.D.

The slope of an indifference curve through policy s for type i consumers can be found by taking the total derivative of $v^i(s)$ with respect to s and setting it equal to zero:

$$(1) \qquad \left.\frac{ds_1}{ds_2}\right|_{\bar{v}^i(s)} = -\frac{v_2^i}{v_1^i} = \frac{P_i u'(x - s_1 + s_2)}{P_i u'(x - s_1 + s_2) + (1 - P_i)u'(y - s_1)} ,$$

where v_j^i represents the partial derivative of v^i with respect to the j^{th} argument and $u'(\cdot)$ represents the first derivative of u . Two important implications follow from equation (1).

<u>Lemma 2</u>: $\left. ds_1/ds_2 \right|_{\bar{v}^i(s)} \gtreqless P_i$ if and only if $s_2 \lesseqgtr (y-x)$.

<u>Proof</u>: The result follows from inspection of equation (1).

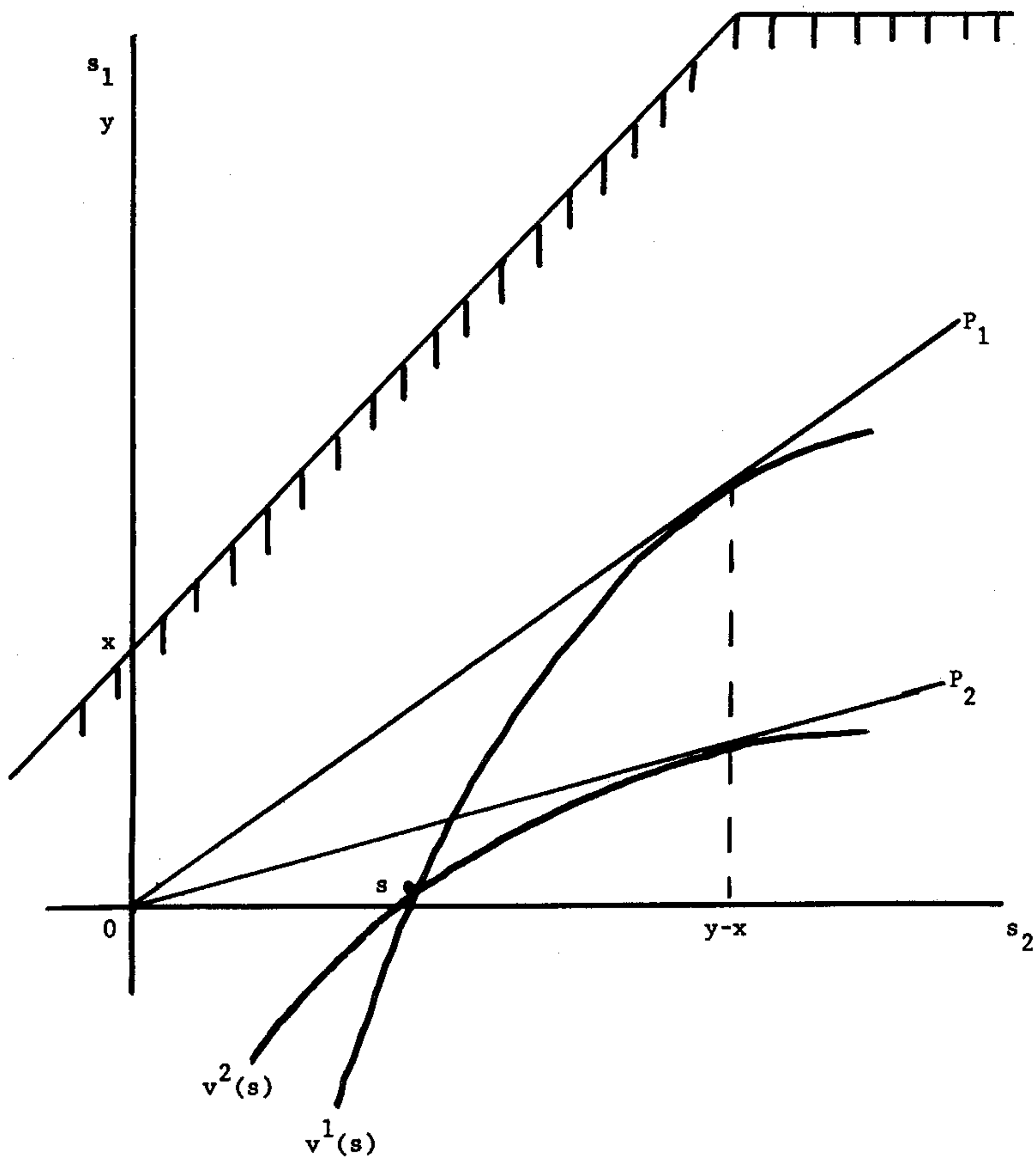
<u>Lemma 3:</u> $P_j > P_i$ implies $ds_1/ds_2\big|_{\bar{v}^j(s)} > ds_1/ds_2\big|_{\bar{v}^i(s)} > 0$.

<u>Proof</u>: The result follows from inspection of equation (1).

Lemma 2 states that individuals are willing to pay more than the expected value of an incremental increase in s_2 up to the point where s_2 implies that an individual's consumption is independent of his endowment. This result will be useful later in characterizing the market equilibrium. Lemma 3 states that when s_2 is measured on the horizontal axis, high risk indifference curves are always steeper than low risk indifference curves. Intuitively, this result should not be surprising since higher risk types with their correspondingly higher probability of having an accident are more likely to receive any added increment to s_2 , and hence, will be willing to increase their premium, s_1 , by a larger amount than lower risk types. This is really the critical result of this section. It establishes that the preferences of the different type consumers are naturally ordered in a way which corresponds to the probability of their receiving x as the endowment.

These results are summarized in Figure 1 for types 1 and 2. The line $\overline{OP}_1$, has slope P_1 . It represents the set of policies for which type 1 consumer's expected value of an indemnity, s_2 , is equal to the premium s_1 . The line $\overline{OP}_2$ represents the same set of policies of type 2 consumers. Lemma 1 implies that the indifference curves labelled, $v^1(s)$ and $v^2(s)$, are concave. Lemma 2 implies that the indifference curves of types 1 and 2 are tangent to the $\overline{OP}_1$ and $\overline{OP}_2$ lines respectively at the value $s_2 = (y-x)$. Finally, Lemma 3 implies that the slope of the type 1 indifference curve is steeper than the slope of the type 2 indifference curve at their point of intersection, policy s . The hashed line in the upper right hand part of the graph

FIGURE 1.



represents the boundary of $\bar{S}$. Note that utility for both types increases as s_2 rises and s_1 falls.

The objective of consumers is to choose the best policy from a given set of policies offered by firms.

Definition 5: $v^{i*}(S) = \max\{v^i(s) : s \in S\}$ for all compact subsets $S \subset \bar{S}$ which contain 0 .[2]

Definition 6: $S_i^*(S) = \{s \in S : v^i(s) = v^{i*}(S)\}$ for all $S \subset \bar{S}$.

Given an offer of policies S , a consumer of type i , who maximizes $v^i(\cdot)$, will purchase a policy in $S_i^*(S)$ and reach utility level, $v^{i*}(S)$.

Lemma 4: Let $i > k > j$. If $s^i \in S_i^*(S)$ and $s^j \in S_j^*(S)$, then $s^i \leqq s^j$. Furthermore, if $S_i^*(S) \cap S_j^*(S) \neq \emptyset$ then $S_i^*(S) \cap S_j^*(S)$ contains exactly one policy and $S_k^*(S) = S_i^*(S) \cap S_j^*(S)$.

Proof: By hypothesis, $v^i(s^i) \geqq v^i(s^j)$ and $v^j(s^i) \leqq v^j(s^j)$. Therefore, by definition 4,

(2) $$P_i u(x - s_1^i + s_2^i) + (1 - P_i)u(y - s_1^i) \geqq P_i u(x - s_1^j + s_2^j) + (1 - P_i)u(y - s_1^j) ,$$

and

(3) $$P_j u(x - s_1^i + s_2^i) + (1 - P_j)u(y - s_1^i) \leqq P_j u(x - s_1^j + s_2^j) + (1 - P_j)u(y - s_1^j) .$$

Subtracting (3) from (2) yields:

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[2] For the remainder of the paper all subsets of $\bar{S}$ that are considered are to be regarded as compact and to contain 0 , unless otherwise specified.

$$(4) \qquad (P_i - P_j)[u(x - s_1^i + s_2^i) - u(y - s_1^i)] \geqq (P_i - P_j)[u(x - s_1^j + s_2^j) - u(y - s_2^j)]$$

or

$$(5) \qquad u(x - s_1^i + s_2^i) - u(y - s_1^i) \leqq u(x - s_1^j + s_2^j) - u(y - s_1^j) \ .$$

But (5) and (2) can only be satisfied if $u(y - s_1^i) \geqq u(y - s_1^j)$ which implies $s_1^j \geqq s_1^i$. But then equation (3) implies that $s_2^i \leqq s_2^j$. This proves the first statement in the lemma.

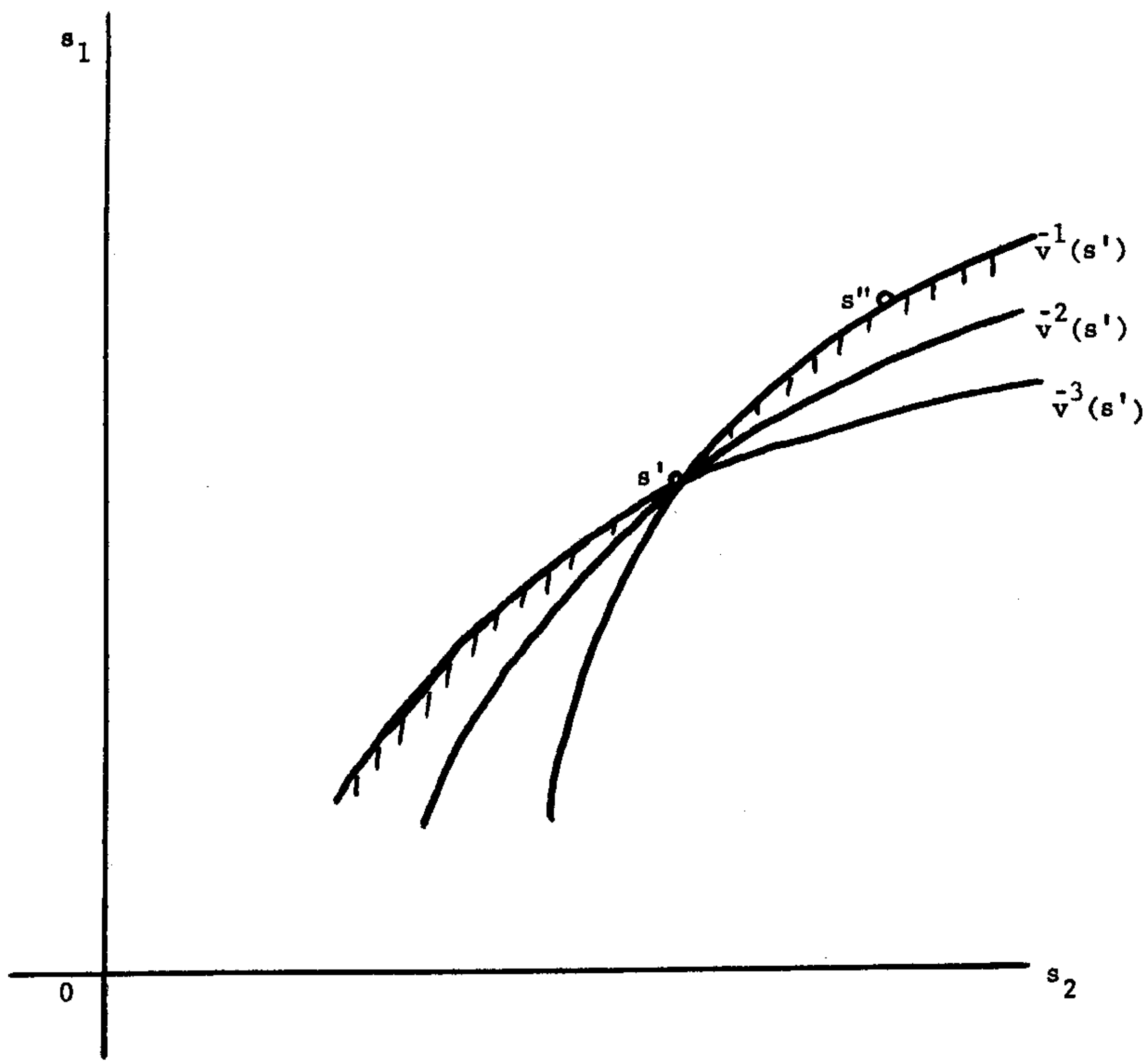
The second statement follows by noting that $s^i, s^j \in S_i^*(S)$ must imply $s^i \geqq s^j$ or $s^j \geqq s^i$. But if $s^i, s^j \in S_j^*(S)$ as well, the statement just proved implies that $s^i \leqq s^j$ and $s^i \geqq s^j$ or $s^i = s^j$. And if $i > k > j$, and $s^k \in S_k^*(S)$, then by the first part of the lemma, $s^k \geqq s^j \in S_j^*(S)$ and $s^k \leqq s^i \in S_i^*(S)$. Thus $s^k = s^i$.

Q.E.D.

Lemma 4 gives the most important implication of the assumption that all consumers have the same utility function over consumption. It states that when faced with the same menu of policies, higher risk types will always prefer policies which have at least as large indemnities and premium levels as lower risk types. Furthermore, only one policy can be in the most preferred set of both types and if such a policy exists, it must be the only policy in the preferred set of types with intermediate riskiness (i.e. with P such that $P_i < P < P_j$).

Lemma 4 is illustrated in Figure 2 for types 1, 2, and 3. If $v^1(s'') = v^1(s')$, then since type 1 indifference curves are steeper, $v^2(s'') < v^2(s')$. Therefore if $s' \in S$, s'' cannot be in $S_2^*(S)$. Secondly, if s' is most preferred by both types 1 and 3, then no policy below the

FIGURE 2.



hashed line can be available. Therefore s' must also be the most preferred policy for type 2 consumers.

Definition 7: A demand function, d, is a function from $\bar{S}$ to R_+^I for which $d_i(s) = a_i$ for one $s \in \bar{S}$ and $d_i(s) = 0$ otherwise.[3] The set of demand functions will be denoted by $\bar{D}$.

A demand function describes how many consumers of each type purchase each policy in $\bar{S}$ per unit of time. By definition, it only describes a state where all consumers of the same type purchase an identical policy.

Not all demand functions are of much interest. In a market economy, each consumer is given the opportunity to choose his most preferred contract from among all those available. This leads to the notion of an admissible demand function.

Definition 8: A demand function, $d \in \bar{D}$, is said to be admissible if there is a set $S \subset \bar{S}$ for which $d_i(s) = a_i$ implies $s \in S_i^*(S)$. D^* will represent the set of admissible demand functions.

Definition 9: Let $S \subset \bar{S}$. $D^*(S)$ will refer to a subset of D^* having the property that $d \in D^*(S)$ if and only if $d_i(s) = a_i$ implies $s \in S_i^*(S)$.

Clearly, $D^*(S)$ is never empty. If all consumers are offered a set of contracts, S, the market demand would have to be described by a demand function in $D^*(S)$. In general, $D^*(S)$ is analogous to the value of a demand correspondence for a given price vector. Changes in S correspond to changes in the prices which consumers face.

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[3] $d_i(s)$ refers to the i^{th} component of d evaluated at policy s.

Lemma 5 describes how the risk class of a consumer affects the amount of insurance he demands. Essentially it is merely a restatement of Lemma 4 in terms of the demand function.

Lemma 5: Let $S \subset \bar{S}$, $d \in D^*(S)$, and $i_1 > i_3 > i_2$. If $d_{i_1}(s^1) = a_{i_1}$ and $d_{i_2}(s^2) = a_{i_2}$, then $s^1 \leqq s^2$. If $s^1 = s^2$, then $d_{i_3}(s^1) = a_{i_3}$.

Proof: The proof follows immediately upon the application of Lemma 4 to definition 8.

Q.E.D.

For the remainder of the paper, I will be concerned with describing how a market allocates contracts. It is assumed that the demand by consumers depends only on the union of the sets of contracts being offered by each firm. Since there may be more than one admissible demand function for this set, a market demand function will have to be selected from the set of admissible demand functions.

Definition 10: The market demand function for S , $d^*(\cdot;S)$, is a function from $\bar{S}$ to R_+^I . It is defined for each $S \subset \bar{S}$.

Assumption 5: $d^*(\cdot;S) \in D^*(S)$ for all $S \in \bar{S}$.

The market demand function gives the number of consumers of each type purchasing each contract in $\bar{S}$ for each offer of contracts. That information will be necessary for firms to determine their optimal policy offers.

3. The Profit Function

By profits, I will always mean expected profits. Furthermore, any transactions costs or set-up costs are assumed to be zero. Therefore, when a type i consumer purchases policy s, the profit to the firm is equal to the premium, s_1, minus the expected value of indemnity payments $P_i s_2$. The profit function extends this definition to cover an arbitrary demand vector for each policy in $\bar{S}$.

Definition 11: For each $s \in \bar{S}$ and $b \in R_+^I$, the value of the profit function $R(s,b)$, is

$$R(s,b) = \sum_{i \in I^*} b_i (s_1 - P_i s_2) .$$

The profit function gives the total flow of profits accruing to firms offering policy s, when a vector of b consumer types are purchasing that policy per unit of time. It is clearly continuous in s and linear in b.

The continuity of the profit function will be useful later by ensuring that optimal policy offers exist. But the linearity property is equally important. It essentially plays the role of a constant returns to scale production function. There are no economies to be gained from increasing the number of sales or from mixing different types of consumers.

Let e_i stand for the i^{th} unit vector in R_+^I.

Lemma 6: If $i > j$, then for any policy, $s = (s_1, s_2) \in \bar{S}$, $R(s, e_i) \gtreqless R(s, e_j)$ if and only if $s_2 \gtreqless 0$.

Proof: The proof follows immediately from the convention that $i > j$ implies $P_i < P_j$ and inspection of the relation: $R(s, e_i) = s_1 - P_i s_2$.

Q.E.D.

Lemma 6 summarizes the second critical property which, along with the results in Lemma 4, is necessary to ensure the existence of an equilibrium in Section 7. It establishes that consumers can be ranked according to their profitability to firms. Furthermore, this ranking corresponds to the ranking of preference orderings implied by Lemma 3.

One implication of the assumption that market demand functions are admissible will be that in equilibrium all policies purchased by consumers will lie in the positive orthant. That result will be based on the following lemma. Throughout the remainder of the paper $R(s, a_i)$ will be short hand notation for $R(s, (0, \ldots, a_i, \ldots, 0))$.

<u>Lemma 7</u>: Let $S \subset \bar{S}$ and $d \in D^*(S)$. If $d_i(s) = a_i$ for some $s \in \bar{S}$ for which $s_2 \leqq 0$, then $R(s, a_i) \leqq 0$.

<u>Proof</u>: By convention $0 \in S$ and therefore $v^{i*}(S) \geqq v^i(0)$. Assume there exists an s such that $s_2 \leqq 0$ and $d_i(s) = a_i$ for some $d \in D^*(S)$. Then $v^i(s) \geqq v^i(0)$, and by the concavity of $u(\cdot)$, and the fact that $y > x$,

$$
\begin{aligned}
v^i(s) &= P_i u(x - s_1 + s_2) + (1 - P_i)u(y - s_1) \\
&\leqq P_i u(x - s_1 + P_i s_2) + (1 - P_i)u(y - s_1 + P_i s_2) \\
&= v^i(s_1 - P_i s_2, 0) .
\end{aligned}
$$

Therefore, if $v^i(s) \geqq v^i(0)$, $s_1 - P_i s_2 \leqq 0$. But by the definition of R , this means that $R(s, a_i) \leqq 0$.

Q.E.D.

Given the profit function and the market demand function, it is possible to compute the total profits resulting from each offer of contracts. This can be summarized in the market profit function.

Definition 12: The market profit function for S, $R^*(\cdot;S)$, is defined by: $R^*(s;S) = R(s; d^*(s;S))$ for all $s \in \bar{S}$.

If S is the market offer, then $R^*(s;S)$ gives the total flow profits to firms from offering policy s.

4. Pareto Optimality

In this section, the concept of Pareto optimality is formally defined in terms of the set of demand functions. In later sections, this concept will be used, not only to evaluate the equilibrium allocations, but also to provide necessary and sufficient conditions for the existence of an equilibrium.

Definition 13: Let $d \in \bar{D}$. Define $w^i(d) = v^i(s)$ for the unique s for which $d_i(s) = a_i$.

The function $w^i(d)$ gives the utility type i consumers attain from any given demand function.

Definition 14: A demand function d^1 is Pareto superior to another demand function d^2 if:

(i) $w^i(d^1) \geqq w^i(d^2)$ for each $i \in I^*$; and

(ii) $\sum_s R(s, d^1(s)) \geqq \sum_s R(s, d^2(s))$;[4]

with strict inequality holding in at least one relation in (i) or in relation (ii).

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[4] $\sum_s$ will be taken to mean $\sum_{s \in \bar{S}}$.

Definition 15: A demand function, d' , is Pareto optimal with respect to a set of demand functions, D , if:

(i) $d' \in D$; and

(ii) there is no $d \in D$ which is Pareto superior to d' .

There are two points to be emphasized about these definitions. First, not only is the welfare of each class of consumers considered, but the profits of firms are included as well. One demand function is Pareto superior to another only if each consumer is at least as well off and the total profits to all firms are at least as large under the first demand function as they are under the second.[5] The profits of individual firms are not considered separately.

I think this may be a useful definition for policy implications since the aggregate profits to firms do represent a gain to the economy, which a policy maker may want to take into account in evaluating two different demand functions.

Secondly, the concept of Pareto optimality is defined only with respect to a given set of demand functions. The concept will have no meaning unless the set of demand functions being considered is first specified. However, it is possible in principle to consider not only admissible demand functions, but also others as well. Non-admissible demand functions may be useful for comparison if it is assumed that some agency is able to distinguish among the different risk types of consumers. I will return to this issue again in Section 9.

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[5]This is an attempt to capture the general equilibrium welfare implications of the model without an explicit general equilibrium model. It is analogous to using producer surplus to assign gains to other sectors of the economy. I am indebted to L. McKenzie for emphasizing this point.

5. El Equilibrium

In this and the following sections, I will consider two different definitions of market equilibrium for this model. Both definitions are intended to describe a stationary market allocation of policies for an economy in which firms can costlessly enter the market and costlessly change their policy offers in response to the actions of other firms. Firms are assumed to have learned the amount of profit which accrues to each policy for every possible set policies which can be offered. The definitions will differ, however, in their implicit treatment of the expectations which firms have about the responses of other firms to their policy offers.

Elsewhere [5], I have analyzed the equilibrium, where firms have completely static expectations--that is, each firm assumes that the aggregate set of policies offered by all other firms does not change as a response to its own offer. In this section, however, I am going to modify the firm's expectations slightly. With static expectations, firms may have an incentive to offer a set of policies which results in some individual policies earning negative profits while some other policy earns sufficiently positive profits to make the aggregate profits of the firm positive. This incentive will be eliminated, however, if the firm believes that other firms will respond by trying to undercut those policies earning positive profits, leaving the first firm to offer the unprofitable policies by itself. In the first definition of equilibrium to be presented, therefore, it will be assumed that a firm has an incentive to offer a new set of policies if and only if the aggregate return to those policies is positive and if each individual policy earns non-negative profits. I have shown in [5] that this restriction does not change the nature of the equilibrium, but does insure the existence of an equilibrium in a larger number of cases.

<u>Definition 16</u>: S^* is an E1 equilibrium if:

(i) $R(s; S^*) \geqq 0$ for all $s \in \bar{S}$; and

(ii) There is no S such that $R^*(s; S^* \cup S) \geqq 0$ for all $s \in S$ with $(>)$ for some $s \in S - S^*$.

Condition (i) requires that in equilibrium, each policy earns non-negative profits. Condition (ii) states that if a different set of policies can be offered which, given the existing market offer of policies, earns non-negative profits for all policies in that set with positive profits for some policy which is in the new set but not in the existing market offer, then the existing market offer cannot be an E1 equilibrium. This reflects the presumption that if there are positive profits to be earned, some firm will have an incentive to try to capture those profits if each policy it offers will earn non-negative profits.[6]

Lemma 8 states the first important result which follows from the definition of an E1 equilibrium. Not only does it establish that in equilibrium, each policy must earn zero profits, but that the profits generated by each type of consumer must be zero.

<u>Lemma 8</u>: If S^* is an E1 equilibrium, then $R(s, d_i^*(s; S^*)) = 0$ for all $s \in \bar{S}$ and $i \in I^*$.

<u>Proof</u>: Assume that it has been demonstrated that $R(s, d_i^*(s; S^*)) \leqq 0$ for all $i \in I^*$ and $s \in \bar{S}$. Then since the definition of equilibrium requires

[6] If each firm is assumed to be permitted to offer only one policy then the non-negative profit condition is automatically satisfied and it can be shown that the equilibrium corresponds to a stationary state with static expectations on the part of each firm. This is the definition used by Rothschild and Stiglitz [2].

that $\sum_{i \in I^*} R(s, d_i^*(s; S^*)) = R(s, d^*(s; S^*)) \geqq 0$ for all $s \in S^*$, it follows immediately that $R(s, d_i^*(s, S^*)) = 0$ for all $s \in \bar{S}$ and $i \in I^*$.

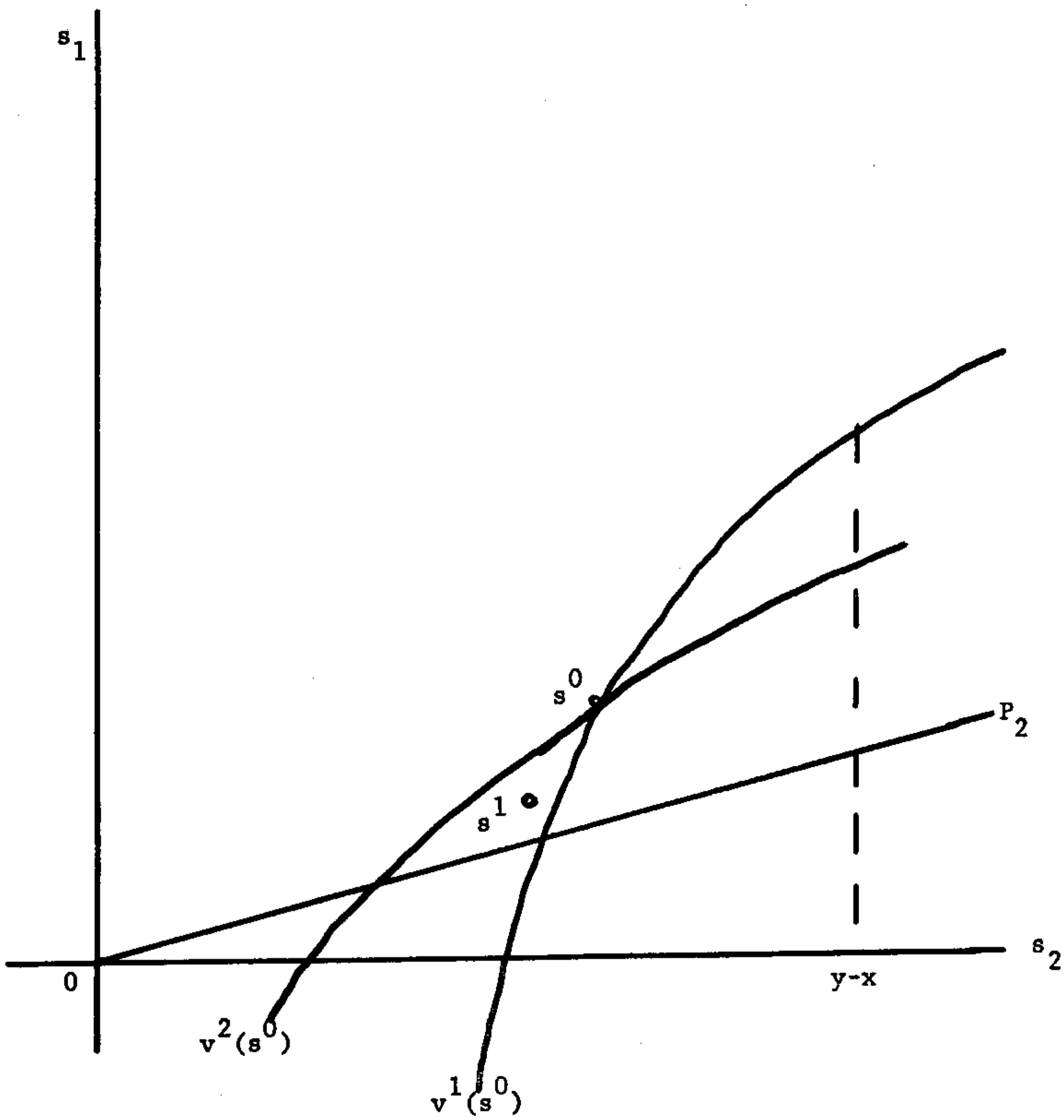
Therefore, we need only show that $R(s, d_i^*(s; S^*)) \leqq 0$ for all $i \in I^*$ and $s \in \bar{S}$. Suppose $R(s', d_{i'}^*(s'; S^*)) > 0$ for some $i' \in I^*$ and $s' \in \bar{S}$. By Lemma 7, $s' > 0$. Therefore by Lemma 3 and the continuity of $R(\cdot)$, there is an s'' such that: (i) $v^{i'}(s'') > v^{i'}(s')$; (ii) $v^i(s'') < v^i(s') \leqq v^{i*}(S^*)$ for $i < i'$; and (iii) $R(s'', a_{i'}) > 0$. Let $S = \{s''\}$. Then $d^*(\cdot; S^* \cup S)$ has the following properties: (i) $d_i^*(s''; S^* \cup S) = 0$ if $i < i'$; and (ii) $d_{i'}^*(s''; S^* \cup S) = a_{i'}$. Therefore, by Lemma 6, $R^*(s''; S^* \cup S) > 0$. Since $s'' \notin S^*$, condition (ii) of the definition of an E1 equilibrium is violated.

Q.E.D.

The proof of Lemma 8 is illustrated in Figure 3. Suppose there are only two types of consumers, and that type 2 consumers are purchasing a policy, s^0, which would earn positive profits if sold only to type 2 consumers. The $\overline{OP_2}$ line represents the zero profit policies for type 2 consumers. The lines $v^1(s^0)$ and $v^2(s^0)$ represent indifference curves through policy s^0 for types 1 and 2 respectively. Note that since s^0 is offered type 1 consumers must be purchasing a policy on or below the $v^1(s^0)$ line. Therefore, since the type 1 indifference curve is steeper than the type 2 indifference curve, there is a policy such as s^1 which lies above the $\overline{OP_2}$ line, attracts only type 2 consumers, and thus earns a positive profit. Therefore, $\{s^0\}$ cannot be an E1 equilibrium.

Using Lemma 8, it is possible to describe a procedure for constructing the unique market demand function which can support an E1 equilibrium. Lemma 9 essentially outlines this procedure and describes some properties that the

FIGURE 3.



equilibrium demand function must have. Theorem 1 will establish that there is indeed only one possible E1 equilibrium demand function.

The equilibrium demand function is constructed as follows. Type 1 consumers are assigned their most preferred policy among those which earn non-negative profits for type 1 consumers. Type 2 consumers then are assigned their most preferred policy from among those which earn non-negative profits for type 2 consumers and which are not preferred by type 1 consumers to their best policy. The process continues until the lowest risk type is reached.

Lemma 9: Define $d^{1*} \in \overline{D}$, by the following rule:

(i) Let $s^1 = (P_1(y-x), y-x)$ and let $d_1^{1*}(s^1) = a_1$;

(ii) For $i > 1$, let $S^i = \{s : R(s, e_i) \geqq 0$ and $v^{i-1}(s) \leqq v^{i-1}(s^{i-1})\}$, and choose s^i so that $d_i^{1*}(s^i) = a_i$ implies $s^i \in S^i$ and $v^i(s^i) \geqq v^i(s)$ for all $s \in S^i$.

The following properties hold for all $i > 1$: (a) $s^1 > s^{i-1} > s^i > 0$; (b) $R(s^i, e_i) = 0$; (c) $v^{i-1}(s^{i-1}) = v^{i-1}(s^i)$; and (d) s^i is unique. Furthermore if $S = \{s^1, \ldots, s^I\}$, then $d^{1*} \in D^*(S)$.

Proof: The proof of statements (a) through (d) will proceed by induction. Suppose for some $i > 1$: (i) $s^1 > s^{i-1} > s^i > 0$; (ii) $R(s^i, e_i) = 0$; (iii) $v^{i-1}(s^i) = v^{i-1}(s^{i-1})$; and (iv) s^i is unique. I will prove that the same statements hold for $i+1$.

First show that $s^{i+1} \leqq s^i < s^1$. Since it is easy to verify that $s^i \in S^{i+1}$, it follows that $v^{i+1}(s^{i+1}) \geqq v^{i+1}(s^i)$. But if $s^{i+1} > s^i$, then Lemma 3 implies $v^i(s^{i+1}) > v^i(s^i)$ which contradicts $s^{i+1} \in S^{i+1}$. Inspection of the definition of $v^i(\cdot)$ reveals, therefore, that $s^{i+1} \leqq s^i < s^1$.

Suppose $R(s^{i+1}, e_{i+1}) > 0$, then by the continuity of $R(\cdot)$ and Lemma 3, there is an s' such that: $v^i(s') < v^i(s^{i+1}) < v^i(s^i)$; $v^{i+1}(s') > v^{i+1}(s^{i+1})$; $R(s', e_{i+1}) > 0$. Thus $s' \in S^{i+1}$ and since $v^{i+1}(s') > v^{i+1}(s^{i+1})$, the definition of s^{i+1} is violated. This proves (b). Also since $R(s^i, e_i) = 0$ and $R(s^{i+1}, e_{i+1}) = 0$, it follows from (i) that $s^i > s^{i+1}$.

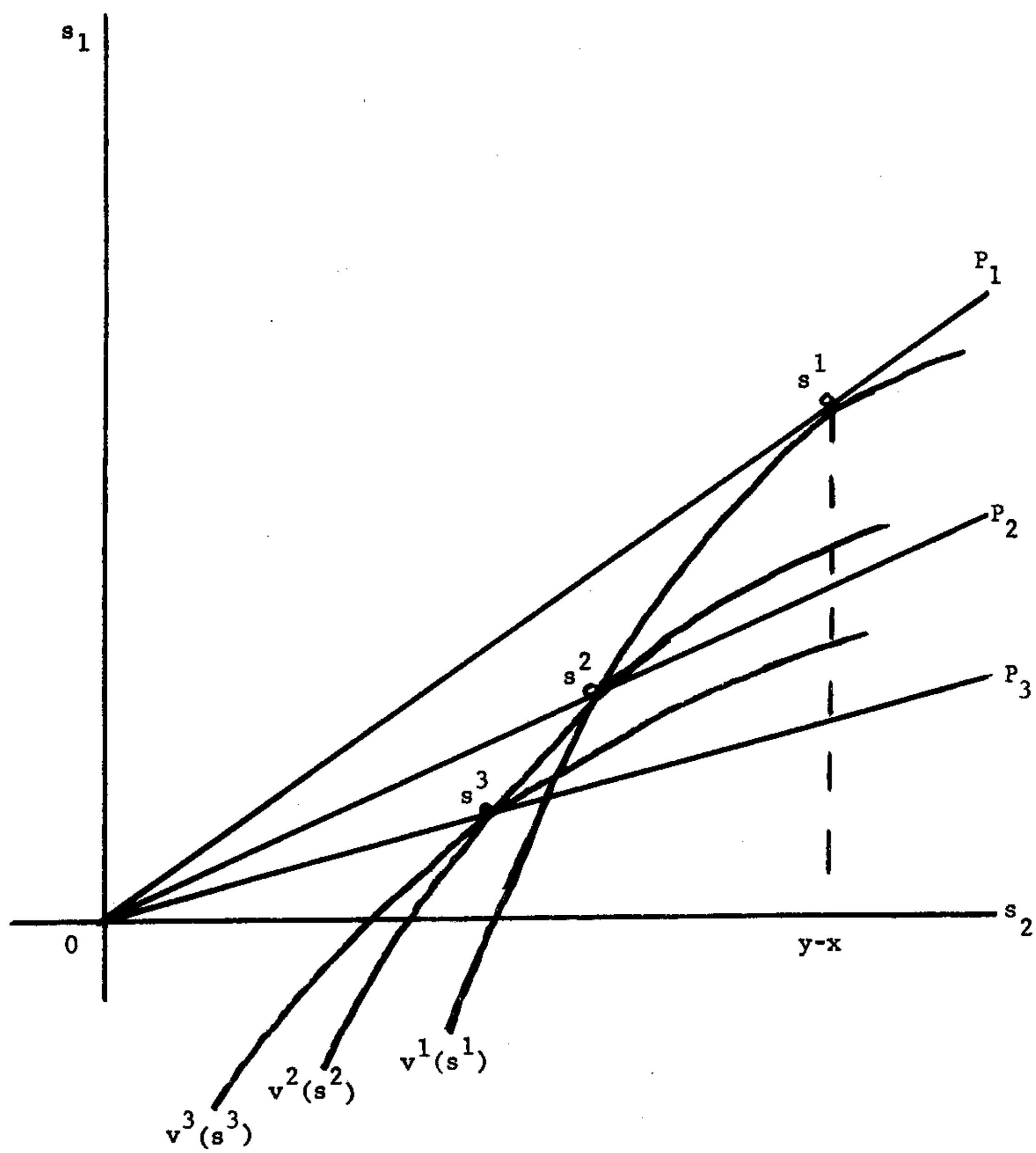
To prove (c), note that $R(s^{i+1}, e_{i+1}) = 0$ implies $s^{i+1} = (P_{i+1}\epsilon, \epsilon)$ for some real ϵ . By Lemma 2, $[\partial v^{i+1}(P_{i+1}\epsilon, \epsilon)]/\partial\epsilon > 0$ for all $\epsilon < y-x = s_2^1$. Therefore, $s^{i+1} = (P_{i+1}\epsilon^*, \epsilon^*)$ where ϵ^* is the largest ϵ such that $v^i((P_{i+1}\epsilon, \epsilon)) \leqq v^i(s^i)$. Clearly, $v^i(0) < v^i(s^i)$ by induction hypothesis (i) and Lemma 2. Therefore (a) is proved, (c) is proved and since ϵ^* is unique, (d) is proved. It is easy to verify that the same method proof applies for $i = 2$. Using Lemma 4 and that fact that $v^i(s^i) \geqq v^i(s^{i-1})$ and $v^{i-1}(s^{i-1}) \geqq v^{i-1}(s^i)$, it is easy to verify that $d^{1*} \in D^*(S)$.

Q.E.D.

An illustration of the d^{1*} demand function is given in Figure 4 for 3 types of consumers. For each i , s^i refers to the policy purchased by that type. Note that the policy purchased by each type lies on the $\overline{OP}_i$ line and thus earns zero profits. Higher indexed types purchase smaller policies --but all policies are strictly positive. Each type consumer is indifferent to his policy and the policy purchased by the next highest type. Finally, only type 1 consumers are fully insured with $s_2^1 = y-x$.

<u>Theorem 1</u>: If S^* is an E1 equilibrium, then $d^*(\cdot; S^*) = d^{1*}$.

FIGURE 4.



<u>Proof</u>: For each $i \in I^*$, let s^i be defined by $d_i^{1*}(s^i) = a_i$. Let s'^i be defined by $d_i^*(s'^i; S^*) = a_i$. Let j be the smallest $i \in I^*$ such that $s'^i \neq s^i$. Then by Lemma 8 and the construction of d^{1*}, $v^{j-1}(s'^j) \leqq v^{j-1}(s^{j-1})$, and $R(s'^j, d_j^*(s'^j; S^*)) = 0$. But from part (ii,d) of Lemma 10, $s'^j \neq s^j$ implies $v^j(s'^j) < v^j(s^j)$. Define $s''^j = (s_1^j + \epsilon, s_2^j)$. Then for $\epsilon > 0$ sufficiently small the following properties hold: (i) $v^i(s''^j) < v^{i*}(S^*)$ for $i < j$; (ii) $v^j(s''^j) > v^j(s'^j)$; (iii) $R(s''^j, a_j) > 0$. Let $S = \{s''^j\}$. Then $d_i^*(s''^j; S^* \cup S) = 0$ for $i < j$; $d_j^*(s''^j; S^* \cup S) = a_j$; and, therefore, by Lemma 6, $R^*(s''^j; S^* \cup S) > 0$. Since $s''^j \in S - S^*$, condition (ii) of the definition of equilibrium E2 is violated.

Q.E.D.

The final set of results in this section relate some welfare properties of the d^{1*} demand function to the existence of an E1 equilibrium. Corollary 1 establishes that the E1 equilibrium demand function is Pareto optimal with respect to all admissible demand functions earning non-negative profits for each type of consumer. However, Theorem 2 establishes that the existence of an E1 equilibrium requires that the d^{1*} demand function be Pareto optimal with respect to a larger set of demand functions. For an E1 equilibrium to exist, there must be no admissible demand function which is Pareto superior to d^{1*} and which earns non-negative profits for each policy with strictly positive profits for some policy. Hence, the possibility that a set of policies can be offered in which two or more different risk types purchase the same policy that may lead to the elimination of an E1 equilibrium. In the next section, this possibility will be examined in more detail.

<u>Corollary 1</u>: The E1 equilibrium demand function, d^{1*}, is Pareto optimal with respect to $\{d \in D^* : R(s, d_i(s)) \geqq 0$ for all $s \in \bar{S}$ and $i \in I^*\}$.

Proof: The corollary follows immediately from the definition of d^{1*}. Q.E.D.

Theorem 2: An E1 equilibrium exists if and only if there is an $S^* \subset \overline{S}$ such that $d^{1*} = d^*(\cdot; S^*)$ [7] and there is no $d \in D^*$ such that: (i) d is Pareto superior to d^{1*}; and (ii) $R(s, d(s)) \geqq 0$ for all $s \in \overline{S}$ with ($>$) for at least one $s \in \overline{S}$.

Proof: I will prove the "only if" part first. Theorem 1 requires that $d^{1*} = d^*(\cdot; S^*)$ for some $S^* \subset \overline{S}$. Suppose $d \in D^*$ has the property that: $w_i(d) \geqq w_i(d^{1*})$ for all $i \in I^*$; $R(s, d(s)) \geqq 0$ for all $s \in \overline{S}$; and $R(s', d(s')) > 0$ for some $s' \in \overline{S}$. Let $J^* = \{i : d_i(s) > 0$ implies $d_{i+1}(s) = 0\}$. There are two cases: (a) $w_i(d) > w_i(d^{1*})$ for some $i \in J^*$; (b) $w_i(d) = w_i(d^{1*})$ for all $i \in J^*$.

Take case (a) first. Let j be the smallest $i \in J^*$ such that $w_i(d) > w_i(d^{1*})$, and let s^j be the s for which $d_j(s^j) = a_j$. If there is an $i < j$ such that $d_i(s^j) = 0$, let j' be the largest such i. Now define $s'' = (s_1^j + \epsilon, s_2^j)$. Then for $\epsilon > 0$ sufficiently small, the selection of j implies $v^j(s'') > v^{j*}(S^*)$, and $v^{j'}(s'') < v^{j'*}(S^*)$. Therefore, by Lemma 4, $v^i(s'') < v^{i*}(S^*)$ for all $i \leqq j'$. Also by Lemma 7, $s^j > 0$; therefore by construction, $s'' > 0$. Let $S = \{s''\}$. Then, by assumption 2, $d_j^*(s''; S^* \cup S) = a_j$ and $d_i^*(s''; S^* \cup S) = 0$ for $i < j'$. Furthermore, by Lemma 5, if $d_i^*(s''; S^* \cup S) = a_i$ for $i < j$, then $d_k^*(s''; S^* \cup S) = a_k$ for $j \geqq k \geqq i$. Therefore, by Lemma 6, ϵ can be chosen sufficiently small so that $R^*(s''; S^* \cup S) > 0$. But since $s'' \in S - S^*$ condition (ii) of the definition of an E1 equilibrium is violated.

[7] This is a technical requirement, which is not necessarily satisfied by the assumption that the market demand function is admissible. It is discussed in more detail in [5].

For case (b), consider any s' for which $R(s', d(s')) > 0$. Let j be the smallest i for which $d_i(s') > 0$. By Lemma 3, a policy s'' can be chosen so that $v^i(s'') > v^i(s')$ for $i \geqq j$; and by Lemma 4, $v^i(s'') < v^i(s') \leqq v^{i*}(S^*)$ for $i < j$. Let $S = \{s''\}$. Then by Assumption 2, $d_i^*(s''; S^* \cup S) = a_i$ for all i such that $d_i(s') = a_i$; and $d_i^*(s''; S \cup S_1') = 0$ for $i < j$. Since Lemma 7 implies $s' > 0$, we may choose $s'' > 0$, and therefore by Lemma 6, we may choose s'' so that $R^*(s''; S^* \cup S) > 0$. Since $s'' \in S - S^*$, condition (ii) of the definition of equilibrium is again violated.

To prove the "if" part, assume there is an S such that $R^*(s; S^* \cup S) \geqq 0$ for all $s \in S$ with $(>)$ for some $s \in S$. Let $d = d^*(\cdot; S^* \cup S)$. Construct d' as follows. If $w_i(d) > w_i(d^{1*})$, or if $w_i(d) = w_i(d^{1*})$ and $R(s, d_i(s)) \geqq 0$ for all $s \in \overline{S}$, then let $d_i = d_i'$. If $w_i(d) = w_i(d^{1*})$ and $R(s, d_i(s)) < 0$ for some $s \in \overline{S}$, then let $d_i = d_i^{1*}$. Clearly $d' \in D^*(S \cup S_1')$ and $R(s, d'(s)) \geqq R(s, d(s))$ for all $s \in \overline{S}$. Therefore, $w_i(d') \geqq w_i(d^{1*})$ for all $i \in I^*$, and $R(s, d'(s)) \leqq 0$ for all $s \in S$ with $R(s, d'(s)) > 0$ for some $s \in \overline{S}$.

Q.E.D.

6. An Example with Two Types of Consumers

For the case where there are only two types of consumers, it is easy to construct an example where no E1 equilibrium exists. In fact, one may rewrite that necessary and sufficient conditions for the existence of an E1 equilibrium directly in terms of the utility functions, v^i , the probabilities, P_i , and the proportion of consumer types given by a .

<u>Theorem 3</u>: Let $I = 2$. Suppose there is an S^* for which $d^{1*} = d^*(\cdot; S^*)$. Then an equilibrium exists if and only if there is no $s \in S$ $(s = (s_1, s_2))$ such that:

(i) $w_i(d^{1*}) \leqq v^i(s)$ for $i = 1, 2$; and

(ii) $a_1(s_1 - P_1 s_2) + a_2(s_1 - P_2 s_2) > 0$.

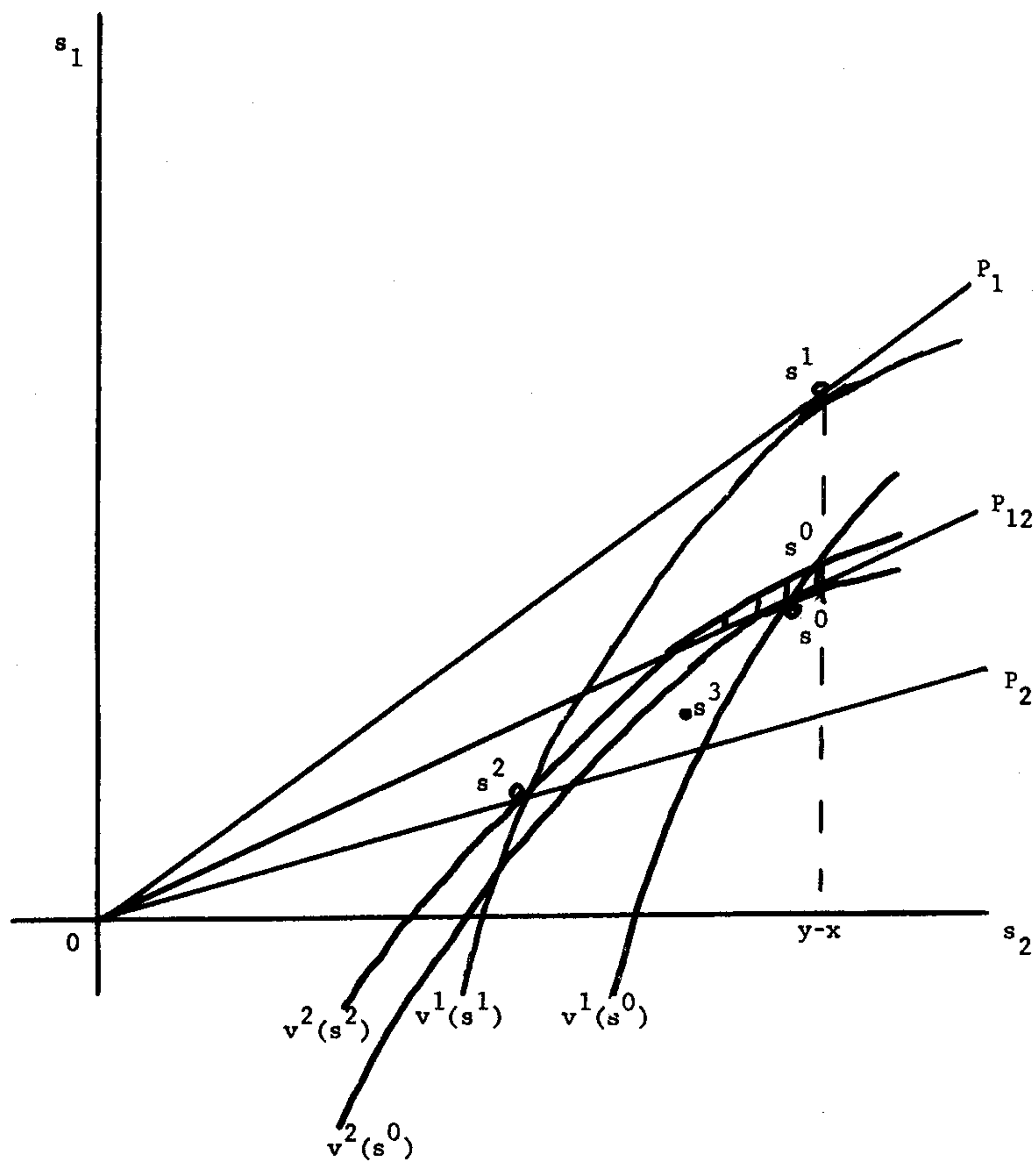
<u>Proof</u>: Suppose an E1 equilibrium does not exist. Then, by Theorem 2, there exists a $d \in D^*$ such that: $w_i(d) \geqq w_i(d^{1*})$ for $i = 1, 2$; $R(s, d(s)) \geqq 0$ for all $s \in \bar{S}$; and $R(s, d(s)) > 0$ for some $s \in \bar{S}$. But by Corollary 1, any such d must have the property that $d_1(s') = a_1$ if and only if $d_2(s') = a_2$. Therefore $v^i(s') \geqq w_i(d^{1*})$ for $i = 1, 2$ and $R(s', (a_1, a_2)) > 0$. But by the definition of $R(\cdot)$, that is just condition (ii) of this theorem. This proves the "only if" part. The converse is established by the reserve chain of argument.

Q.E.D.

Theorem 3 reduces the problem of the existence of an equilibrium to conditions which guarantee that no single policy can be found which makes both types at least as well off as they are under the d^{1*} demand function and which still earns positive profits when both types purchase it.

Consider Figure 5. The lines, $\overline{OP}_1$, $\overline{OP}_2$, and $\overline{OP}_{12}$ represent those policies which earn zero profits if purchased by types 1, 2, or 1 and 2 respectively. If an E1 equilibrium is to exist, Theorem 1 requires that type 1 consumers purchase policy s^1. Type 2 consumers then purchase the policy which lies on the $\overline{OP}_2$ line and on the type 1 indifference curve through policy s^1. Any other demand function leaves an incentive for a new set of policies to be offered. Theorem 3 states that an E1 equilibrium will exist if and only if the type 2 indifference curve through policy s^2 lies everywhere on or

FIGURE 5.



below the $\overline{OP}_{12}$ line. Consequently, the example shown in Figure 5 is a case where no E1 equilibrium exists. If, for instance, policies s^1 and s^2 are being offered, some firm could offer a policy in the shaded region which would attract all of the consumers and still earn positive profits.

Furthermore, if all firms offer a policy such as s^0 which is purchased by both types, that would not be an E1 equilibrium either. In that case, another firm could offer a policy such as s^3 which attracts only the low risk types and, because it lies above the $\overline{OP}_2$ line, earns positive profits. Since that would leave only high risk types at policy s^0, that policy would cease to earn positive profits. But if it is withdrawn and all high risk types purchase policy s^3, then s^3 will earn negative profits. Hence no E1 equilibrium exists. In general, there is no E1 equilibrium if it is profitable to make both typss better off by selling them the same policy--thus permitting high risk types to be subsidized by low risk types. This is reflected in the necessary and sufficient conditions for an equilibrium stated in Theorem 2.

7. E2 Equilibrium

In this section, a second definition of equilibrium is introduced which essentially incorporates into the expectations of the firms the responses by other firms which tended to eliminate the E1 equilibrium. Under the assumptions of this model, it is demonstrated that an E2 equilibrium will always exist.

In the preceeding section, it was demonstrated that an E1 equilibrium may not exist, in part at least, because: (i) firms do not correctly anticipate the responses of other firms; and (ii) those responses can so alter the market demand function, that the profits to a firm can change from non-negative to strictly negative. Since firms can do better by withdrawing all policies rather than offering unprofitable policies, firms which do not take into account the impact of their offers on the profits of other firms will not correctly anticipate that some policies will be withdrawn as a consequence of their actions. In some instances, therefore, they will consistently anticipate non-negative profits only to find that their profits become negative after other firms respond. The E2 equilibrium concept will "correct" the firms' expectations by adding the rule that firms only offer policies which will earn positive profits after other firms have responded by withdrawing their unprofitable policies.

To see the effect of this modification of the expectation rule, return for a moment to Figure 5 of Section 6. An E1 equilibrium does not exist when there is a policy which attracts both types away from s^1 or s^2 and still earns positive profits--that is, when the type 2 indifference curve through s^2 lies above the $\overline{OP}_{12}$ line for some range of s_2. In the previous section, it was argued that s^0 could not be an E1 equilibrium because some firm could offer a policy like s^3 and, since it will attract only type 2 consumers, earn positive profits.

Now consider a firm's action when it first accounts for the withdrawal of unprofitable policies. Again $\{s^1, s^2\}$ could not be an equilibrium because some policy in the shaded region could be offered which attracts both types. Suppose that policy were different from s^0 --the most preferred policy on or above the $\overline{OP}_{12}$ line by type 2 consumers. Then another policy near s^0 but above $\overline{OP}_{12}$ line could be offered which will attract the type 2 consumers. It will therefore make positive profits even if it attracts the type 1 consumers as well. However, it must ultimately attract the type 1 consumers since if they purchase a policy different from s^0 it must lie below the $\overline{OP}_1$ line and earn negative profits. Therefore, such a policy would be withdrawn. This establishes that the only possible E2 equilibrium is one where both types purchase policy s^0.

Suppose all firms were offering s^0, would any firm have an incentive to offer a new policy? Since any new policy must earn positive profits, it must attract the type 2 consumers but not the type 1 consumers. Therefore, it must lie in the region above the $\overline{OP}_2$ line, below the type 2 indifference curve through s^0, but above the type 1 indifference curve through s^0. But if such a policy is offered--say s^3, only type 1 consumers will be left at policy s^0. It will therefore earn negative profits and be withdrawn. As a consequence, type 1 consumers will move to the new policy. But since it lies below the $\overline{OP}_{12}$ line, it will earn negative profits. Since firms are assumed to anticipate this response, s^3 would not be offered, and consequently, s^0 is an E2 equilibrium.

In the remainder of this section, the E2 equilibrium is formally defined and a general existence result is established. I also present some welfare results and some general characterizations of the E2 equilibrium.

The first step is to define precisely the set of policies which firms anticipate will remain after a new set of policies is offered. This is not trivial since there may be more that one subset of policies which can be removed to keep the remaining policies profitable. It is also important to guarantee that some policies are not withdrawn needlessly. For instance, a reasonable assumption might guarantee that if all policies remain profitable no policies are removed. However additional restrictions are also needed to ensure that enough policies remain. One possible set of assumptions is given in Assumption 6. Interpret $Q^*(S, S')$ as the set of policies which remain from an initial offer, S, when some firm offers set S'.

Assumption 6: Given S , S' , $Q^*(S, S')$ has the following properties:

(a) $R^*(s; Q^*(S, S') \cup S') \geqq 0$ for all $s \in Q^*(S, S')$.

(b) There is no S'' such that:

(i) $R^*(s; S'' \cup S)$ for all $s \in S''$;

(ii) If $v^{i*}(Q^*(S, S') \cup S') \geqq v^{i*}(S)$, then $v^{i*}(S'' \cup S') \geqq v^{i*}(S)$ for all $i \in I^*$;

(iii) $v^{i*}(Q^*(S, S') \cup S') < v^{i*}(S'' \cup S') = v^{i*}(S)$ for some $i \in I^*$.

(c) $Q^*(S, S') = S$ if conditions (a) and (b) are not violated.

Since there are only a finite number of consumer types, a standard induction argument will establish the existence of a set $Q^*(S, S')$ satisfying the conditions of assumption 6 for any pair of subsets, S and S' . Essentially, assumption 6 guarantees that a minimal number of types of consumers are made worse off when a new set is offered.

Definition 17: S^* is an E2 equilibrium if:

(i) $R(s; S^*) \geqq 0$ for all $s \in S^*$.

(ii) There is no $S \subset \bar{\bar{S}}$ such that $R^*(s; Q^*(S^*, S) \cup S) \geqq 0$ for all $s \in S$ with $(>)$ for some $s \in S - S^*$.

The definition of an E2 equilibrium differs from the definition of an E1 equilibrium only with respect to the set of policies which a firm takes as given when it tests the profitability of a new policy offer. As before, condition (i) states that each policy in the equilibrium set must earn non-negative profits. Condition (ii) states that there must not be a new set of policies which will earn non-negative profits for each policy and strictly positive profits for some new policy after the policies in the original set which have become unprofitable are withdrawn.

Before proceeding to a discussion of the existence of an E2 equilibrium and some of its properties, one additional restriction is needed on the market demand function. In general, whenever a consumer type is indifferent among two or more policies being offered by firms, there are several admissible demand functions for that set. The following assumption insures that the "correct" market demand function is chosen.

<u>Definition 18</u>: For each $S \subset \overline{S}$, let $D^{**}(S) = \{d \in D^*(S) :$ $\{s : R(s, d(s)) < 0\} \leqq \{s: R(s, d'(s)) < 0\}$ for all $d' \in D^*(S)\}$.

<u>Assumption 7</u>: $d^*(\cdot;S) \in D^{**}(S)$ for all $S \subset \overline{S}$.

The set of demand functions, $D^{**}(S)$, contains those admissible demand functions for S which minimize the number of policies earning negative profits. Since at most a finite number of policies are purchased, this set is well defined. Assumption 7 requires that the market demand function have the smallest possible number of unprofitable policies. The most obvious necessity for this assumption is to ensure that an equilibrium is not eliminated because some policy unnecessarily earns negative profits. But it is also used in the proof of Lemma 11 where I need to show that the market demand function for some market offers has only one unprofitable policy. This lemma and the one which follows are the key results used to prove the existence of an E2 equilibrium.

Although the proof of Lemma 11 is rather tedious, the result itself is straightforward. It guarantees that if a new set is offered which attracts a given risk type, the welfare of less risky consumers will not be affected by the consequent withdrawal of policies from the original set.

<u>Lemma 9</u>: Let S have the property that $R^*(s;S) \geqq 0$ for all $s \in \bar{S}$. If $R^*(s; Q^*(S, S') \cup S') \geqq 0$ for all $s \in \bar{S}$, and if i' is the largest i such that $u_i^*(S') > u_i^*(S)$, then $u_j^*(Q^*(S, S') \cup S') = u_j^*(S)$ for all $j > i'$.

<u>Proof</u>: Suppose the lemma is false and that $v^{j*}(Q^*(S, S') \cup S') < v^{j*}(S)$ for some $j > i'$. For notational convenience, let $d^1 = d^*(\cdot; Q^*(S, S') \cup S')$. Let $S_1 = \{s \in Q^*(S, S') : d_i^1(s) = a_i \text{ for some } i < i'\}$. Let $S_2 = \bigcup_{i>i'} S_i^*(S)$. Let $S_3 = S_1 \cup S_2$. Note that since $S_{i'}^*(S' \cup S_3) \cap S = \emptyset$, Lemma 4 implies that $S_i^*(S' \cup S_3) \cap S_2 = \emptyset$ for all $i \leqq i'$, and hence $S_2 \cap S_1 = \emptyset$.

Consider any $d^2 \in D^*(S_3 \cup S')$. By assumptions 6 and 7, $R(s, d^2(s)) < 0$ for some $s \in \bar{S}$, otherwise $Q^*(S, S')$ has reduced the utility of some type unnecessarily. Let s' be the policy for which $d_{i'}^2(s') = a_{i'}$. Let i_1 and i_2 be the largest and smallest i respectively such that $d_i^2(s') = a_i$. Now define d^3 as follows:

$$d_i^3 = d_i^*(\cdot;S) \quad \text{for } i > i_1 ;$$

$$d_i^3 = d_i^2 \quad \text{for } i_1 \geqq i > i_2 ;$$

$$d_{i_2}^3 = d_i^2 \quad \text{if } S_{i_2}^*(Q^*(S,S') \cup S') \cap S_3 = \emptyset ;$$

$$d_{i_2}^3 = d_i^1 \quad \text{otherwise;}$$

$$d_i^3 = d_i^1 \quad \text{for } i < i_2 .$$

It can be verified that $d^3 \in D^*(S_3 \cup S')$. Furthermore, Lemma 4 implies that if $d_i^1(s) = a_i$ for $s \in S_1$ then $i \leqq i_2$. Since $R(s, d^1(s)) \geqq 0$ and $R(s, d^*(s;S)) \geqq 0$ for all $s \in \bar{S}$, Lemmas 6 and 7 imply that s' is the only s for which $R(s, d^3(s)) < 0$. Furthermore, by Lemmas 6 and 7, it follows that $R(s', d^2(s)) \leqq R(s', d^3(s)) < 0$. Therefore, it has been shown that if $d \in D^*(S_3 \cup S')$, then $d_{i'}(s) = a_{i'}$ implies $R(s, d(s)) < 0$, and if $d \in D^{**}(S_3 \cup S')$, then $R(s, d(s)) \geqq 0$ for all $s \in S_3$. Therefore, by assumption 7, $R^*(s; S_3 \cup S^*) \geqq 0$ for all $s \in S_3$. But since $v_i^*(S_3 \cup S') \geqq v_i^*(Q^*(S, S') \cup S')$ for all $i \in I^*$ and $v_j^*(S_3 \cup S') = v_j^*(S)$ $> v_j^*(Q^*(S, S') \cup S')$ for some $j > i'$, assumption 6 is violated since S_3 could serve as $Q^*(S, S')$ instead.

Q.E.D.

Lemma 12 provides a method of constructing what will be proved to be an E2 equilibrium set of policies:

<u>Lemma 12</u>: Let $\mathcal{L}_+ = \{S : R^*(s;S) \geqq 0$ for all $s \in S\}$. Then there is an $S^* \in \mathcal{L}_+$ such that if $S \in \mathcal{L}_+$ and $u_i^*(S) > u_i^*(S^*)$ for some $i \in I^*$, then $u_j^*(S) < u_j^*(S^*)$ for some $j > i$.

<u>Proof</u>: The proof is by induction. Note that $\emptyset \in \mathcal{L}_+$, and let $\bar{v}_I = \sup\{v^{I^*}(S) : S \in \mathcal{L}_+\}$. It is easy to verify that all sets in $\mathcal{L}_+$ are contained in a compact subset. Therefore, there is a sequence $\{S^t\}_{t=1,2,\ldots}$ such that: (i) $S^t \in \mathcal{L}_+$ for all t ; (ii) $v^{I^*}(S) \to \bar{v}_I$ as $t \to \infty$; and (iii) $s^{i,t} \to s^{i,0}$ as $t \to \infty$ for all $i \in I^*$, where $s^{i,t}$ is defined by $d^*(s^{i,t}; S^t) = a_i$. Let $S_I = \{s : s = s^{i,0}$ for some $i \in I^*\}$. Define d^0 by $d_i^0(s^{i,0}) = a_i$ for all $i \in I^*$. I will show that $d^0 \in D^*(S_I)$ and that $R(s, d^0(s)) \geqq 0$ for all $s \in \bar{S}$. By assumption 7, this shows that $R^*(s; S_I) \geqq 0$ for all $s \in \bar{S}$.

For each $t > 0$ and $i \in I^*$, it must be true that $v^i(s^{i,t}) \geqq v^i(s^{j,t})$ for all $j \in I^*$. By the continuity of v^i , it then follows that $v^i(s^{i,0}) \geqq v^i(s^{j,0})$ for all $j \in I^*$ or $v^i(s^{i,0}) \geqq v^{i*}(S_I)$.

To show that $R(s, d^0(s)) \geqq 0$ for all $s \in \bar{S}$, let $J(s) = \{i : d_i^0(s) = a_i\}$ for each $s \in \bar{S}$. It then follows from the continuity of $R(\cdot)$ in s and the linearity of $R(\cdot)$ in $d(s)$, that $\lim_{t \to \infty} \sum_{i \in J(s)} R(s^{i,t}, d_i^*(s^{i,t}, S^t))$ $= R(s, \sum_{i \in J(s)} \lim_{t \to \infty} d^*(s^{i,t}; S^t)) = R(s, d^0(s)) \geqq 0$ for all $s \in \bar{S}$. This establishes that $S_I \in \mathcal{S}_+$.

Now consider some type $i < I$. Let $\mathcal{S}_+^i$ represent the class of sets, S, satisfying: (a) $S \in \mathcal{S}_+$, and (b) there is no S' for which: (i) $u_j^*(S') > u_j^*(S)$ for some $j \geqq i$; (ii) $u_k^*(S') \geqq u_k^*(S)$ for all $k > j$. The argument above proved the existence of a set $S_I \in \mathcal{S}_+^I$. The induction assumption is to suppose that there exists a set $S_i \in \mathcal{S}_+^i$. By using the existence of a set $S_i \in \mathcal{S}_+^i$, a similar argument can be constructed to show the existence of a set $S_{i-1} \in \mathcal{S}_+^{i-1}$. Therefore, by induction, there is a set $S_1 \in \mathcal{S}_+^1$. But $S^* = S_1$ satisfies the conditions of the lemma.

Q.E.D.

<u>Theorem 4</u>: Under assumptions 1-7, an E2 equilibrium exists.

<u>Proof</u>: Let S^* satisfy the conditions of Lemma 11, and let $S^{**} = \{s : v^i(s) \leqq v^{i*}(S^*)$ for all $i \in I^*\}$. Then S^{**} also satisfies the conditions of Lemma 11, and I claim that S^{**} is an E2 equilibrium.

Suppose not. Then there is an S' such that $R^*(s; Q^*(S^{**}, S') \cup S') \geqq 0$ for all $s \in \bar{S}$ with $(>)$ for some $s \in S' - S^{**}$. Therefore, $v^i(S') > v^{i*}(S^{**})$ for some $i \in I^*$. Otherwise, $S' \subset S^{**}$. Let i' be the largest such i. Then by Lemma 11, $v^{i*}(Q^*(S^{**}, S') \cup S') = v^{i*}(S^{**})$ for all $i > i'$. But by the construction of S^{**}, this implies that $R^*(s'; Q^*(S^{**}, S') \cup S') < 0$ for some $s \in \bar{S}$. Since $R^*(s; Q^*(S^{**}, S') \cup S') \geqq 0$ for all $s \in Q^*(S^{**} \cup S')$, it follows that $s' \in S'$ which contradicts the initial hypothesis.

Q.E.D.

The proof of Theorem 4 also provides a significant general welfare result for the E2 equilibrium.

Corollary 2: There exists an E2 equilibrium for which the market demand function is Pareto optimal with respect to $\{d \in D^* : R(s, d(s)) \geqq 0 \text{ for all } s \in \bar{S}\}$.

Proof: The result is immediate from the construction of S^{**} in Theorem 4.

Q.E.D.

Corollary 2 states that there is always an E2 equilibrium for which there is no admissible demand function which earns non-negative profits for each policy and which is also Pareto superior to the equilibrium market demand function. However, this condition is a little too strong to hold for all E2 equilibria. In general, all that can be said is that there is no admissible demand function which is Pareto superior to the equilibrium market demand function and which earns positive profits for some policy as well as non-negative profits for all policies. This result is summarized in Theorem 5.

Theorem 5: If S^* is an E2 equilibrium, then there is no $d \in D^*$ such that: (i) $w_i(d) \geqq w_i(d^*(\cdot; S^*))$ for all $i \in I^*$; (ii) $R(s, d(s)) \geqq 0$ for all $s \in \bar{S}$ with $(>)$ for some $s \in \bar{S}$.

Proof: Suppose such a d exists. Then $d \in D^*(S')$ for some $S' \subset \bar{S}$ with $R(s', d(s')) > 0$ for some $s' \in \bar{S}$. Let j be the smallest $i \in I^*$ such that $d_j(s') = a_j$. Then by Lemma 3, there is an s'' such that $v^i(s'') > v^i(s')$ if $i \geqq j$, and $v^i(s'') < v^i(s')$ if $i < j$.

Let $S'' = S' \cup \{s''\}$. Then by assumption 5, $d_i^*(s''; S^* \cup S_1') = 0$ for $i < j$; $d_i^*(s''; S^* \cup S_1') = a_i$ if $d_i(s') = a_i$; and $d_i^*(s''; S^* \cup S_1') = a_i$ implies $d_k^*(s''; S^* \cup S_1') = a_k$ for $i \geqq k \geqq j$. By Lemma 7, $s' > 0$. Therefore s'' can be chosen so that $s'' > 0$. Therefore, by Lemma 6 and the continuity of $R(\cdot)$, s'' can be chosen so that $R(s'', d^*(s''; S^* \cup S'')) > 0$. Let $d_i^1 = d_i$ if $d_i^*(s''; S^* \cup S_1') = 0$, and $d_i^1 = d_i^*(\cdot; S^* \cup S'')$ otherwise. By Lemma 7, $d(s) > 0$ implies $s \geqq 0$, and therefore Lemma 6 implies $R(s, d^1(s)) \geqq 0$ for all $s \in \overline{S}$. Note that $d^1 \in D^*(S^* \cup S'')$. Therefore, by Assumption 7, $R^*(s; S^* \cup S_1') \geqq 0$ for all $s \in \overline{S}$, and hence $Q^*(S^*, S'') = S^*$. Thus, $R^*(s; S^* \cup S_1') = R(s; Q^*(S^*, S'') \cup S'') \geqq 0$ for all $s \in \overline{S}$. Since $s'' \notin S^*$, and $R^*(s''; Q^*(S^*, S'') \cup S'') > 0$, this shows that S^* is not an E2 equilibrium.

Q.E.D.

The zero profit condition then follows as an immediate consequence of Theorem 5.

<u>Corollary 3</u>: If S^* is an E2 equilibrium, then $R^*(s; S^*) = 0$ for all $s \in S^*$.

<u>Proof</u>: The corollary follows from Theorem 5.

Q.E.D.

The final result of this section establishes the relationship between an E1 equilibrium and an E2 equilibrium.

<u>Theorem 6</u>: Suppose $d^*(\cdot; S^*) = d^{1*}$ (defined in Lemma 9). Then S^* is an E2 equilibrium if and only if it is an E1 equilibrium.

<u>Proof</u>: To prove the "if" part, suppose S^* is an E1 equilibrium. Then, from Lemma 10, $R(s, d_i^*(s; S^*) = 0$ for all $s \in \overline{S}$ and $i \in I^*$. Therefore, given any $S' \subset \overline{S}$, $Q^*(S^*, S') = S^*$ by assumptions 6 and 7. But then Theorems 2 and 5 imply that $R^*(s; S^* \cup S') = 0$ for all $s \in \overline{S}$ if $R^*(s; S^* \cup S') \geqq 0$ for all $s \in \overline{S}$. Thus S^* is an E2 equilibrium.

To prove the "only if" part, suppose S^* is not an E1 equilibrium. Then from Theorem 2, there is an S' such that: (i) $w_i(d^*(\cdot; S')) \geqq w_i(d^*(\cdot; S^*))$ for all i ; (ii) $R^*(s; S') \geqq 0$ for all $s \in \bar{S}$ with ($>$) for some $s' \in \bar{S}$. But by Theorem 5, this implies that S^* is not an E2 equilibrium either.

Q.E.D.

This result is consistent with my arguments made at the beginning of Section 6 for introducing a new equilibrium reflecting different expectations rules on the part of firms. If an E1 equilibrium exists, the same equilibrium is an E2 equilibrium. Unfortunately, as I shall demonstrate in the next section, the E2 equilibrium is not necessarily unique. Thus, even when the E1 equilibrium exists, there may be other E2 equilibria as well.

8. Uniqueness of the E2 Equilibrium

Although, in the simple example presented at the beginning of Section 7, it can be demonstrated that the E2 equilibrium is unique, that need not generally be the case. Even with only two types of consumers, there is a kind of knife-edge case where two equilibria may exist. One is Pareto superior to the other, but it is not possible to find a Pareto superior demand function to either equilibria which also earns positive profits for some policy and non-negative profits for each policy. In Figure 5, the reader may verify that this circumstance would arise if the type 2 indifference curve passing through policy s^0 also passes through policy s^2 . In this case both $\{s^0\}$ and $\{s^1, s^2\}$ would be E2 equilibria even though type 1 consumers are made strictly better off under the $\{s^0\}$ equilibrium. This example also demonstrates that the E2 equilibrium demand function need not be Pareto optimal with respect to all admissible demand functions earning non-negative profits for each policy.

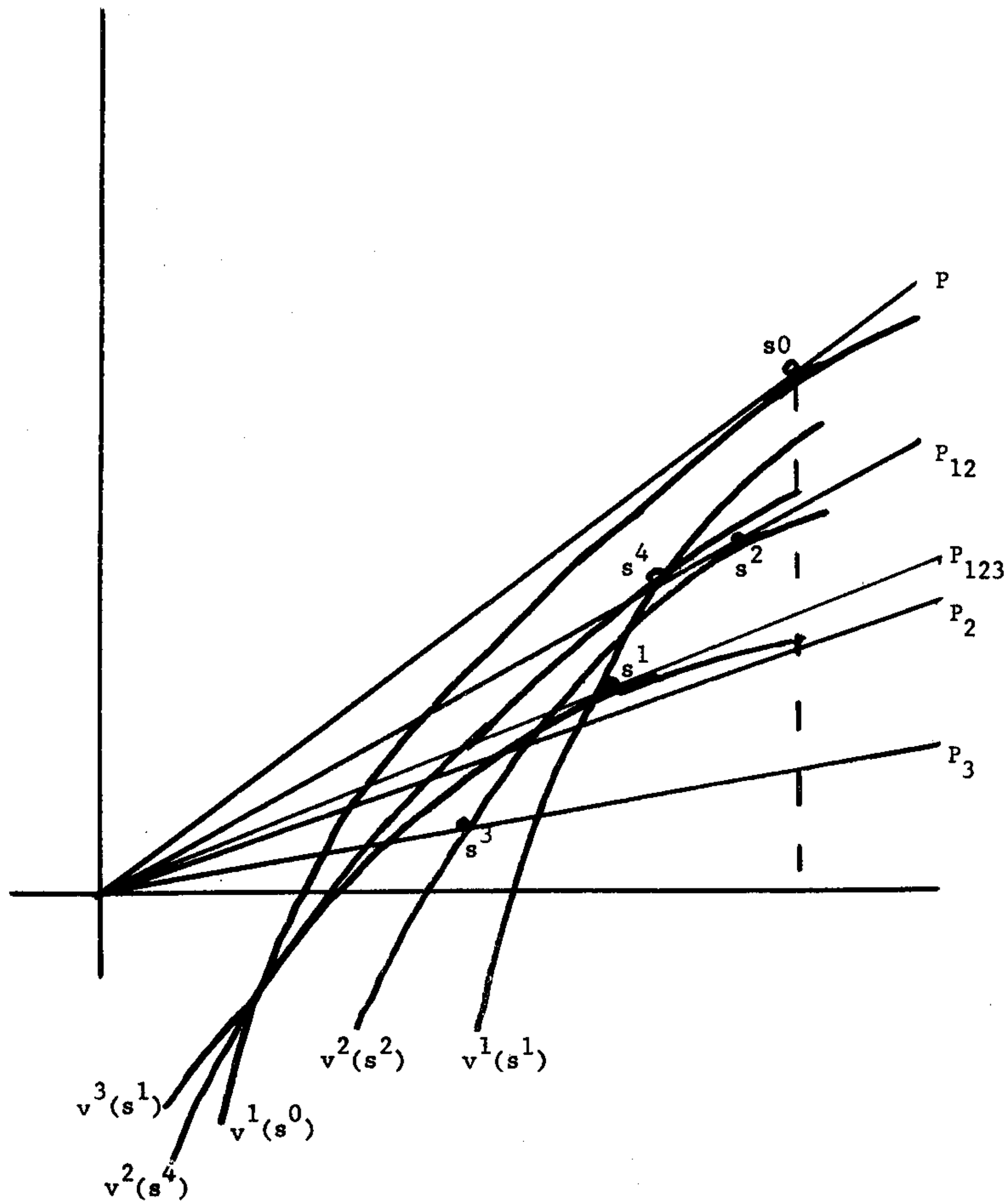
Although such an example establishes that the E2 equilibrium may not be unique, one still might be led to believe that an E2 equilibrium, market demand function must make the lowest risk type consumer as well off as any other admissible demand function which earns non-negative profits for each policy. That this is not the case can be seen from the following example. Assume there are three types of consumers with indifference curves given in Figure 6. The subscript on each $\overline{OP}_x$ line refers to the class of consumer types for which policies on that line earn zero profits. For instance, the OP_{12} line are policies which earn zero profits when purchased by types 1 and 2. I claim that there are two E2 equilibria in this example: $\{s^1\}$ and $\{s^2, s^3\}$.

Consider first the $\{s^1\}$ equilibrium. Suppose only s^1 were offered and all types purchased it. Could some firm offer a new set of policies and earn non-negative profits on each policy with strictly positive profits on some policy after all unprofitable policies were withdrawn? The answer is no.

The argument I use to establish this uses three basic results: (1) in order to create negative profits for an otherwise profitable offer, the new offer must include a policy which is at least indifferent to some consumer to any policy in the original offer; (2) from Lemma 5, if types 1 and 3 purchase the same policy, type 2 consumers must purchase that policy also; and (3) after the unprofitable policies in the original offer are withdrawn, all policies remaining must earn non-negative profits.

Consider now the set of policies which could emerge after a new set is offered and any unprofitable policies in the original set are withdrawn. The first and third results given above guarantees that type 2 consumers must purchase the same policy as type 1 consumers, otherwise all policies lie outside the $v^1(s^0)$ indifference curve. Furthermore, type 3 consumers must be purchasing

FIGURE 6.



a different policy; otherwise, in order to earn positive profits, the policy they purchase would lie above the $\overline{OP}_{123}$ line and hence be less preferred to s^1. But, given the example in Figure 6, result 2 insures that as long as type 3 consumers purchase s^1 it must earn non-negative profits and hence remain in the final offer.

Therefore, the policy which will be purchased by type 2 consumers must lie on or above the $\overline{OP}_{12}$ line and, consequently, be less preferred by them to s^1. This means that s^1 has been withdrawn which can only happen if the new policy purchased by type 3 consumers is at least as preferred as s^1, and the new policy purchased by type 1 consumers is no better than s^1. Since that policy must lie above the $\overline{OP}_{12}$ line and be most preferred by type 2 consumers to any policy the final set, there can be no policy in the new set which lies below the $v^2(s^4)$ indifference curve. But that also eliminates any policy above the $\overline{OP}_3$ line which is preferred by type 3 consumers to s^1, which contradicts a previous argument. Therefore, $\{s^1\}$ is an E2 equilibrium.

A similar argument could be given to show that $\{s^1, s^3\}$ is an E2 equilibrium, but that is unnecessary because it is easy to verify that $\{s^2, s^3\}$ corresponds to the set S^* defined in Lemma 11 which is used in Theorem 4 to establish the existence of an E2 equilibrium.

I have not tried to find general conditions under which the E2 equilibrium is unique, but the previous counter-example is clearly robust to small changes in the utility functions and the P_i's .

9. Further Welfare Results

In the previous sections, the E1 and E2 equilibria were at least partially characterized by their welfare properties. In this section I want to explore further some of the welfare properties of the E2 equilibrium and investigate some procedures for improving the market allocation.

When discussing the welfare implications of the equilibrium policy sets, it is important that the limitations implied by the information structure on any procedure for allocating policies be recognized. For instance, if we assume that it is possible to distinguish among different risk types, there may well be an allocation of policies which is superior to the equilibrium allocation. What must be of prcatical importance, however, is whether or not a procedure can be defined which does not require that it be possible to distinguish among the different risk types and which still generates a superior allocation. Rather than deal with this question directly, however, I will first evaluate the equilibrium allocation as if there were no information constraints and then introduce restrictions until all of the limitations generated by the information problem have been captured.

In the absence of information constraints, each type of consumer can be treated separately. Therefore, it is quite feasible to assign one type of consumer a policy which would be preferred by some other type. In this case it is straightforward to characterize the Pareto optimal demand functions.

Theorem 7: A demand function, d , is Pareto optimal with respect to $\tilde{D}$, the set of all demand functions, if and only if $d_i(s^i) = a_i$ implies $s_2^i = y-x$.

Proof: The proof is an immediate consequence of Lemma 2 and the definition of $R(\cdot)$.

Q.E.D.

Theorem 7 states that, in the absence of a self-selection problem, it is always possible to make some type better off without hurting any other type or lowering aggregate profits if and only if each type is fully insured. Note that the resource constraints are implicitly considered by including the aggregate profits in the definitions of Pareto optimality.

The result stated in Theorem 7 should not be surprising. The type i iso-profit line through any policy s can be thought of as a transformation schedule for a type i consumer when s corresponds to an initial allocation of resources. The marginal rate of transformation of s_1 into s_2 is then the slope of that line-- P_i . Since we know that Pareto optimality requires that the marginal rate of transformation equal the marginal rate of substitution, it follows immediately that if an allocation is to be Pareto optimal, the policy purchased by a type i consumer must be at the point where his indifference curve is tangent to a type i iso-profit line. Since such lines must have slope P_i , Lemma 2 implies that he must be completely insured. Looked at another way, since all individuals are risk averse and there is no aggregate uncertainty, Pareto optimality must require that all risk be eliminated. For each level of aggregate profit, different allocations along the Pareto optimal frontier then correspond to the assignment of different premium levels among the types of consumers.

The introduction of the self-selection problem imposes a constraint on which demand functions can be attained. Since different type consumers cannot be distinguished directly, the only way of guaranteeing that each consumer type is assigned the policy intended is to offer each consumer the identical set of policies. The resulting demand function will then be in the admissible set D^* . Therefore, when considering the self-selection problem, it is reasonable to restrict comparisons with the equilibrium demand function to the class of admissible demand functions.

From Theorem 7, it is clear that there is only one admissible demand function which is Pareto optimal with respect to all demand functions: $s^* = (\bar{P}(y-x), y-x)$, where $\bar{P}$ is the average probability for all types. Furthermore, this demand function will never be Pareto superior to the E2 equilibrium demand function. This can be seen very clearly when there are only two types of consumers. Refer back to Figure 5. Regardless of the positions of the indifference curves, type 2 consumers will always be at least as well off as they are at their most preferred policy on the $\overline{OP_{12}}$ line. Since that policy will always be smaller than $(P_{12}(y-x), y-x)$, it follows that they are better off under the E2 equilibrium demand function than under one which assigns them policy $(P_{12}(y-x), y-x)$. Therefore, with an information constraint, any feasible Pareto superior improvement on the equilibrium demand function will still not be Pareto optimal with respect to all demand functions.

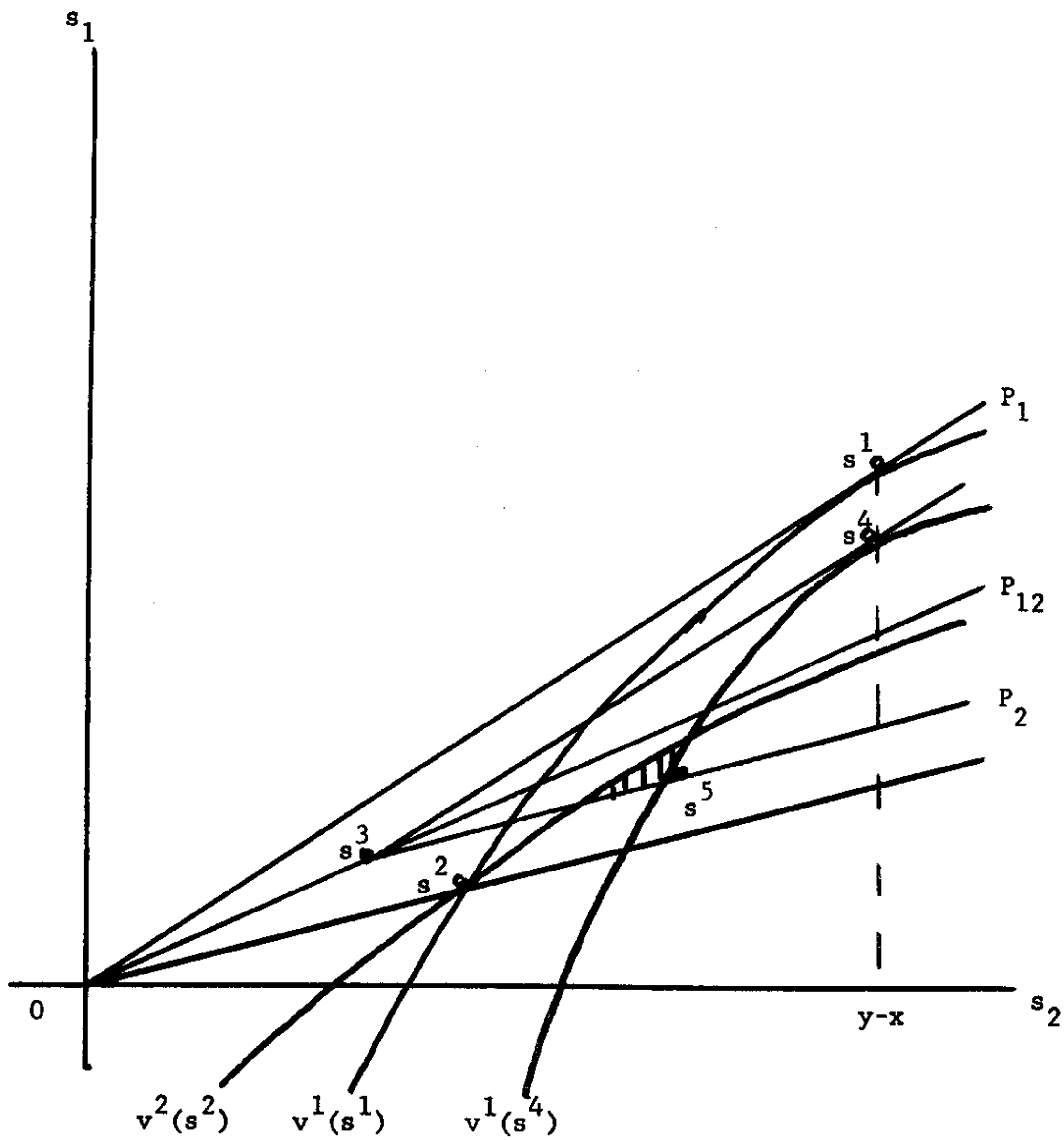
Corollary 2 of Section 7 established that there is always an E2 equilibrium which is Pareto optimal with respect to the set of all admissible demand functions earning non-negative profits for each policy, and Theorem 5 established that for any E2 equilibrium; there is no demand function in this class which earns strictly positive profits and which is Pareto superior to the equilibrium demand function. However, the requirement that each policy earn non-negative profits is clearly an additional constraint not implied by the self-selection problem. Rather, it came about because of the expectations which firms were assumed to have about the effect of an unprofitable policy on their aggregate profits.

I mentioned at the end of Section 6 that an E1 equilibrium fails to exist when it is possible to move to a Pareto superior demand function which permits type 2 consumers to subsidize type 1 consumers by attracting both types to the same policy. However, firms do not have an incentive to offer policies which force low risk types to subsidize high risk types if the two types purchase different policies. Consequently, it is the possibility that both types might be made better off by permitting low risk types to subsidize high risk types by purchasing different policies. This leads to the possibility that there may be an admissible demand function which is Pareto superior to either E1 or E2 equilibria.

Consider first the case where an E1 equilibrium exists. From Theorem 6, we know that there is also an E2 equilibrium with the same demand function. In Figure 7, the E1 equilibrium is represented by policies s^1 and s^2. The type 2 indifference curve lies below the $\overline{OP}_{12}$ line so $\{s^1, s^2\}$ is an E1 equilibrium with both policies earning zero profits. Suppose now that policies s^4 and s^5 are offered. Since the type 1 iso-profit line through s^4 intersects the $\overline{OP}_{12}$ line at policy s^3 and the type 2 iso-profit line through s^4 intersects the $\overline{OP}_{12}$ line at s^3, it is straightforward to verify that if type 1 consumers purchase s^4 and type 2 consumers purchase s^5, the aggregate profits will remain at zero. Clearly both types are made better off. Therefore, an example has been constructed in which the E1 equilibrium demand function is not Pareto optimal with respect to the set of all admissible demand functions. In general, one can check for superior demand functions by letting policy s^3 in Figure 7 move along the $\overline{OP}_{12}$ line. This will generate a new policy s^4 and a new policy s^5. Then there is a Pareto superior demand function only if for some policy s^3, the shaded area emerges--that is, there is a set of policies above the $v^1(s^4)$ indifference curve and above the type 2 iso-profit line through s^3 but below the $v^2(s^2)$ indifference curve.

In the case where no E1 equilibrium exists, an even stronger result emerges. In this case, the E2 equilibrium is policy s^0 in Figure 8. Since s^0 must

FIGURE 7.



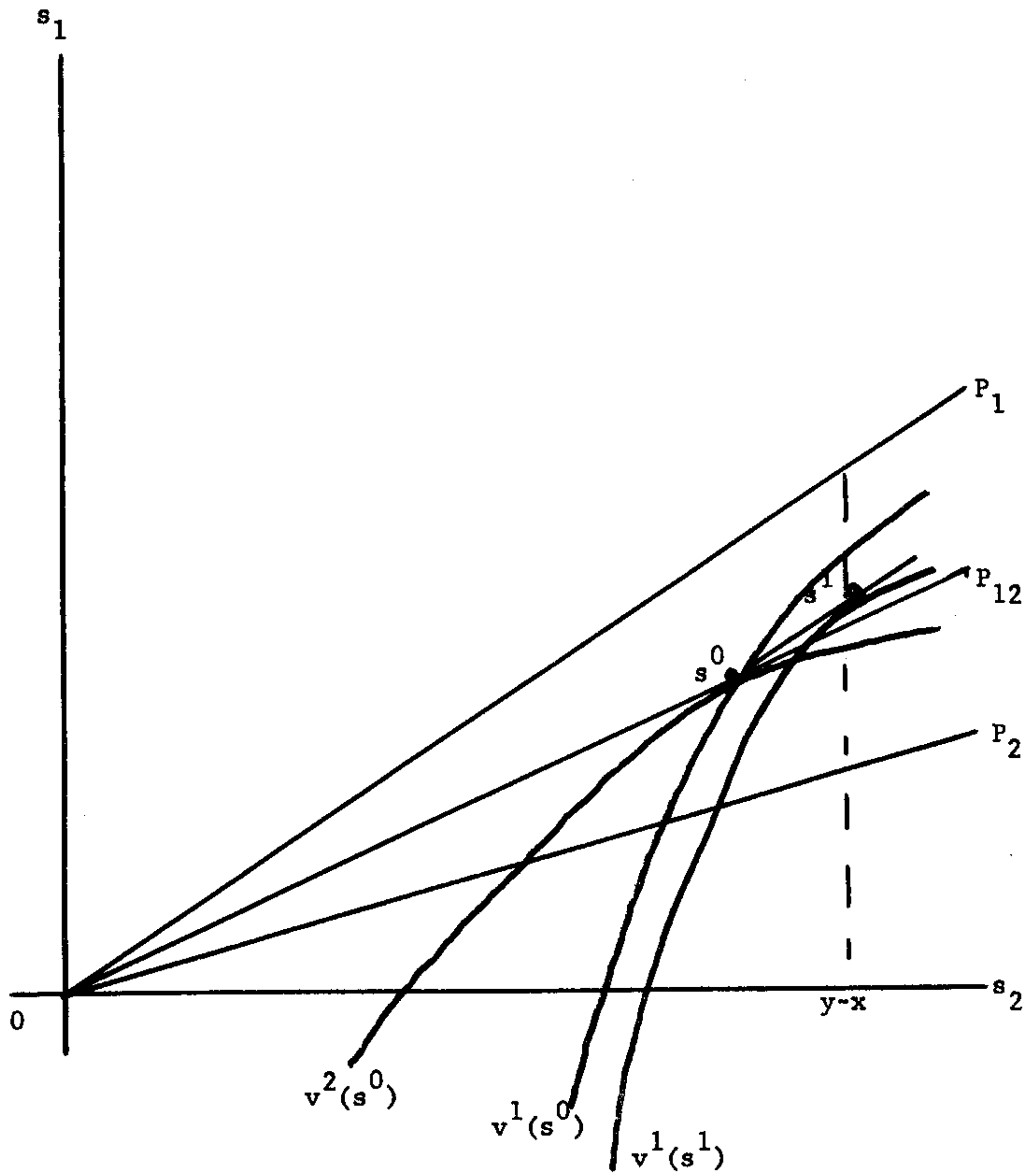
be the most preferred policy for type 2 consumers of all the policies on the $\overline{OP}_{12}$ line, Lemma 2 and the fact that $P_{12} > P_2$ implies that $s_2^0 < y-x$. Suppose, now that a supplementary policy, $(P_1(y-x-s_2^0),\ y-x-s_2^0)$ is offered, which, when combined with s^0 yields policy $(s_1^0 + P_1(y-x-s_2^0),\ y-x)$.

In Figure 8, this represents a movement from s^0 along a type 1 iso-profit line with slope P_1 to policy s^1. By construction the new policy earns the same profit from high risk types as policy s^0. Furthermore, s^1 is strictly preferred to s^0 by high risk types but not by low risk types. Therefore, if s^0 and s^1 are offered, profits remain zero and type 1 consumers are made better off. The reader may also verify that type 2 consumers could also be made better off if s^1 is offered by offering a slightly larger policy on the type 2 iso-profit line through policy s^0 which is still less preferred to s^1 by type 1 consumers. Theorem 8 summarizes this result for any number of types of consumers.

<u>Theorem 8</u>: Let S^* be an E2 equilibrium. If $d_1^*(s;\ S^*) = a_1$ implies $d_2^*(s;\ S^*) = a_2$, then $d^*(\cdot;\ S^*)$ is not Pareto optimal with respect to D^*.

<u>Proof</u>: Let s^1 be the policy such that $d_1^*(s^1;\ S^*) = a_1$. I will first show that if $s_2^1 < y-x$, $d^*(\cdot;\ S^*)$ is Pareto inferior to some $d \in D^*$. Consider policy $s^0 = (s_1^1 + P_1(y-x-s_2^1),\ y-x)$. By construction of s^0 and Lemma 2, $v^1(s^0) > v^1(s^1)$. Also by Lemma 7, $d^*(s;\ S^*) \neq 0$ implies $s \geqq 0$. Since Corollary 3 implies that $R(s;\ S^*) = 0$ for all $s \in \overline{S}$, it follows from the definition of $R(\cdot)$, Lemmas 5 and 6 and the construction of s^0 that $R(s^0,\ a_i) \geqq R(s,\ d_i^*(s;\ S^*))$ for all $i \in I$. Therefore, there exists a $d \in D^*(S^* \cup \{s^0\})$ such that $w_i(d) \geqq w_i(d^*(\cdot;\ S^*)$ for all $i \in I$; $w_1(d) > w_1(d^*(\cdot;\ S^*))$; and $\sum_s R(s,\ d(s)) \geqq \sum_s R^*(s;\ S^*)$. So $d \in D^*$ is Pareto superior to $d^*(\cdot,\ S^*)$.

FIGURE 8.



Therefore, we need only show that if $d_2^*(s^1; S^*) = a_2$, then $s_2^1 < y-x$. Suppose not. Let i' be the largest i such that $d_i^*(s^1; S^*) = a_i$ and let $P' = \sum_{i=1}^{i'} a_i P_i$. Then for $\varepsilon_1, \varepsilon_2 > 0$ chosen sufficiently small Lemma 2 implies that $s^0 = (s_1^1 - P'\varepsilon_1 + \varepsilon_2, s_2^1 - \varepsilon_1)$ has the property that $v^{i'}(s^0) > v^{i'}(s^1) = v^{i'*}(S^*)$. Furthermore, $R(s^0; d^*(s^1; S^*)) > 0$. Let $\{s^0\} = S'$. It is then easy to check that for any value of $Q^*(S^*; S')$, $R^*(s^0; Q^*(S^*; S') \cup S') > 0$. Therefore S^* is not an E2 equilibrium.

Q.E.D.

Theorem 8 can be explained by the fundamental principle, appealed to in our earlier discussion, that in the absence of any additional constraints, Pareto optimality requires that the marginal rate of substitution be equal to the marginal rate of transformation. If type 1 consumers share a policy with any other type, then this condition is not satisfied for any risk class. But the discussion preceding Theorem 8 demonstrates that it is always possible to move along the iso-subsidy line of the high risk types to the policy for which the marginal efficiency conditions are satisfied without affecting the welfare of the low risk types. Even when we are restricted to the class of admissible demand functions Pareto optimality still requires that the marginal efficiency conditions be satisfied by the high risk types.

The method used here for establishing when the equilibrium allocation can be improved also suggests a procedure for reaching a superior allocation. By itself, the market may be inefficient because it does not permit low risk types to be subsidized. However, the necessary subsidies could be generated if all consumers were required to purchase the appropriate policy on the $\overline{OP}_{12}$ line, say s^3 in Figure 6. If we then permit the private market to provide any additional policies demanded by consumers, we will reach a modified equilibrium which, combined with the required policy, s^3, generates policies s^4 and s^5. Furthermore, if s^3 is chosen correctly, the resulting allocation will be Pareto optimal with respect to all admissible allocations. Note also that if all types purchase the same policy in equilibrium, the demand function can always be improved upon by requiring everyone to purchase the equilibrium policy, and then allowing the market to provide additional insurance coverage.

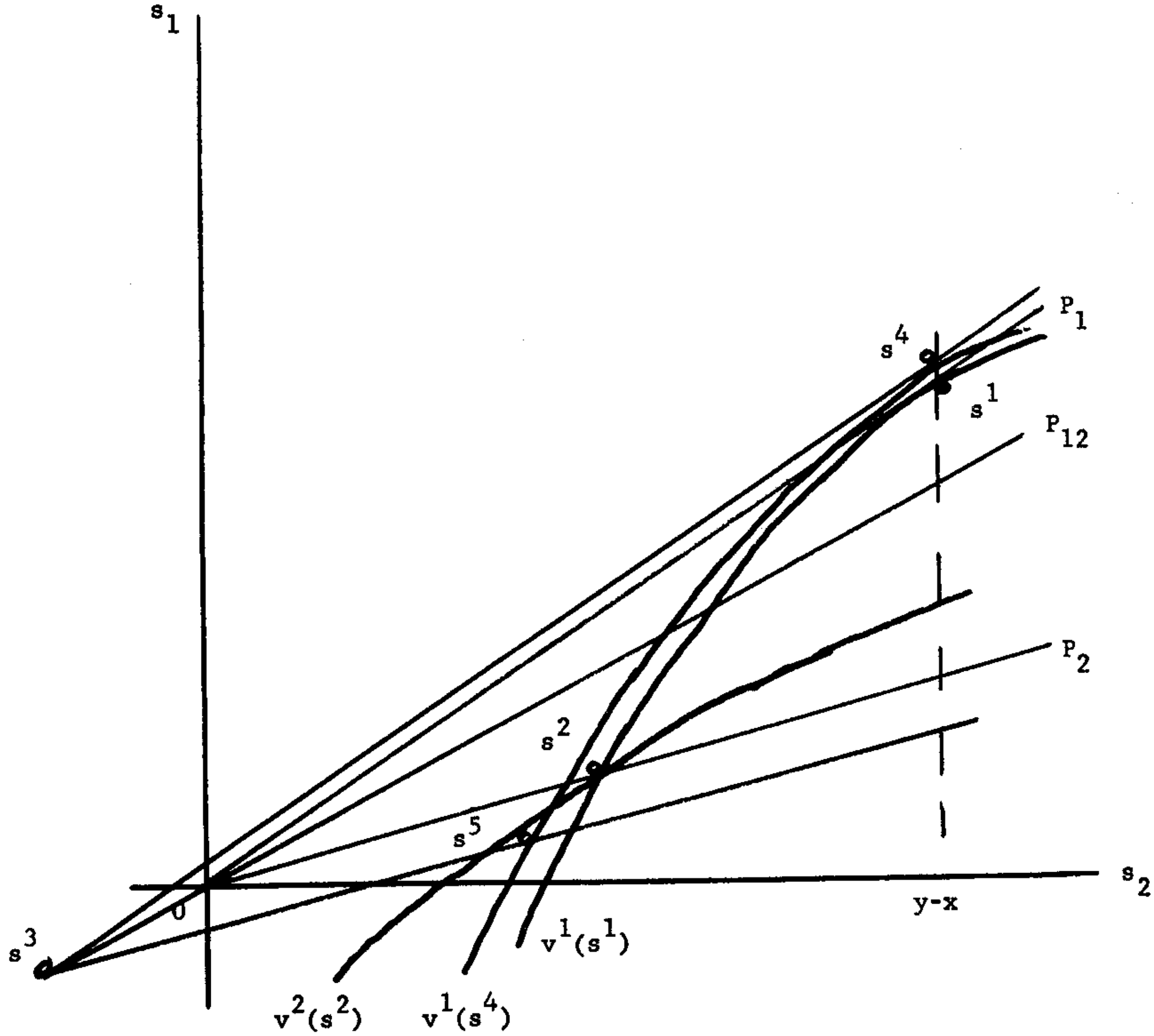
A final comment about the policy implications of those results should be made. Throughout this section only the rather weak welfare concept of Pareto optimality has been used to to evaluate the equilibrium allocation. And, as is true in general, there may be several Pareto optimal demand functions even when our attention is restricted to admissible demand functions. Which of these allocations should be chosen obviously will depend upon further value judgments not yet incorporated into the model.

It is clear that the presence of low risk types may make high risk types better off than they would have been otherwise--and never any worse off. The presence of high risk types, on the other hand, must make low risk types worse off. Nevertheless, it is still always true that the expected utility of low risk types must exceed that of high risk types in equilibrium. This can be verified immediately by noting that in equilibrium: (1) high risk types are indifferent between their policy and the policy of the low risk types; and (2) the policy purchased by low risk types always has an indemnity less than the loss incurred in the low endowment state. Therefore, decreasing the probability of getting a smaller endowment must raise the expected utility of a consumer.

Thus even if the market allocation is Pareto optimal with respect to admissible allocations, it may still not be a socially desirable allocation depending on the interpretation we give to the two risk classes. If we think of low risk types as being safe drivers and the high risk types as being irresponsible reckless drivers, we may feel that the market allocation is unfair, or at least unfortunate, because the high risk types are creating an externality for the low risk types. If, on the other hand, the high risk types are the nadicapped or otherwise disadvantaged members of society who are more likely to incur the lower endowment, x, than the low risk types, the market allocation might be considered unfair because it permits the low risk types to escape some of their responsibility for subsidizing the high risk types. The unique unconstrained Pareto optimal admissible policy, $s^* = (P_{12}(y-x), y-x)$ might be considered best in this case since then all types will have the same expected utility.

Even if we are concerned primarily with the welfare of the low risk types --for example, they are the safe drivers--the market allocation still may not be socially optimal. Depending upon the proportion of each type of consumer and the degree of risk aversion, it may be possible to force the high risk types to subsidize the low risk types and make them better off. An example is shown in Figure 9. This time we require all risk types to purchase a negative policy on the $\overline{OP}_{12}$ line, s^3. When an accident occurs, each individual must pay out an extra amount, $-s_2^3$, whenever he has an accident. If the market is then permitted to provide supplemental insurance coverage, an allocation is reached at (s^4, s^5) at which the low risk types are made better off. Cases can even be constructed, by increasing the proportion of high risk types, where the low risk types can be made better off than if no high risk types had existed at all.

FIGURE 9.



10. Some Final Remarks

Although I think this model successfully captures the essential problems which are generated by adverse selection in an insurance market, I do not think the solution I have proposed for determining the market equilibrium is completely satisfactory. It seems reasonable to me that imperfect knowledge about the objectives and beliefs held by other firms as well as imperfect information about the demand function the firm faces may make it infeasible for a firm to compute a non-cooperative solution at the outset of the game or necessarily believe that other firms have computed the solution as well. In these cases, the anticipated responses of other firms must be based upon the firm's experience. What is missing in this model, however, is an adequate description of how the firm's expectations are formed and what it costs the firm to change its policy offers. None of this is made explicit in this model. Furthermore, it seems essential that if expectations are to be based on the firm's experience, the model should be capable of describing the market before it has reached a stationary state. This in turn seems to require a more complete description of the existing firms and their histories as well as the possibility of new entrants and a specification of their beliefs.

Finally, I should note that the particular equilibrium concept which generates a solution in this model may not work in general. By varying the attitude toward risk of the various consumer types, Stiglitz [3] has constructed a counter-example for which no E2 equilibrium exists--at least if firms are restricted to offering only one policy to consumers. I suspect that with suitable modification the counter-example can be extended to the case where firms can offer several policies as well. I should point out, however, that I do not necessarily regard this result as a lethal blow to the E2 equilibrium concept. Following the methodology adopted in this paper, it simply means that in the in the cases where an E2 equilibrium does not exist, firms must further revise their expectations. The result does help emphasize the point I made earlier, however, that in the absence of a more complete description of the process by which firms form their expectations, the approach I have employed here is very _ad hoc_. Other expectation rules might have been tried which lead to completely different results.

In the light of the preceeding remarks, I am forced to conclude that the problem which was addressed in the beginning of this paper was not satisfactorily resolved. However, the problems which I believe prevent an adequate solution, are problems which apply to all models of oligopoly and even monopolistic competition. Perhaps what is most disappointing is that in the self-selection problem these problems do not necessarily disappear by making "competitive" assumptions about firm behavior.

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