COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

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COWLES FOUNDATION DISCUSSION PAPER NO. 413

A GENERAL EQUILIBRIUM MODEL OF WORLD TRADE

PART II

THE EMPIRICAL SPECIFICATION

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December 8, 1975

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THE EMPIRICAL SPECIFICATION

by

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1. Introduction

The aim of this paper is to give a full description of GEM WT, a general equilibrium model of the world trade between developing and developed countries. This model is the basis of the computational and economic results given in [3], [6], and [7].

The paper will be organized as follows. We start in Section 2 with the theoretical formulation of competitive and tariff ridden international equilibria. Section 3 discusses the theoretical significance of a number of devices which are helpful in specifying equilibrium models. Sections 4 to 9 describe GEM. We discuss in turn:

4. linearization procedures;

5. the submodels for Latin America, Asia, and Africa;

6. the Middle East oil producers submodel;

7. the submodel for developed countries;

8. the international export import balances;

9. the relation between international prices and marginal

utilities, and the specification of balances of payments.

Several appendices, available on request, are devoted to (a) the numerical data used in constructing the model, (b) a detailed description of the computer programs which have been written and (c) particular technical aspects.

2. Formulation and Solution of International Trade General Equilibrium Problems

2.1. Formulation of an International Competitive Equilibrium

In the formulation of the model, it is assumed that each country (or group of countries) acts as an economic agent, who, at the same time, produces, consumes, exports and imports; this departs from the usual formulation in which one considers consumers and firms within each country. We assume in this section that no country invests in another country, and that the profits of a country are not distributed to the others.[1]

Let y_i, x_i, e_i, m_i and $\bar{w}_i$ be r-components vectors of production, consumption, exports, imports and initial endowments in country i (i = 1, 2, ..., m) . Let Y_i, X_i, E_i and M_i respectively denote the production, consumption, exports and imports sets of i . Assume the r^1 first commodities are traded and the remaining $r^2 = r - r^1$ are not; then E_i and M_i are the sets of vectors of which the first r^1 components are non-negative and the remaining r^2 components are zero. Partition accordingly the vectors y_i, x_i and $\bar{w}_i$. Let $p_i = (p \ p_i^2)$ be the vector of prices; the r^1 first components p are the international prices while p_i^2 represents the domestic prices of non-traded goods in country i . Assume finally that the preferences of country

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i can be represented by a continuous real valued utility function $U_i(x_i)$.

Then we define an __international competitive equilibrium__ as:

__Definition 2.1__: The allocation $\{\bar{y}_i\}$, $\{\bar{x}_i\}$, $\{\bar{e}_i\}$, $\{\bar{m}_i\}$ supported by the non-negative price vectors $\{\bar{p}_i\}$ is an international equilibrium if the following conditions hold (for i = 1, 2, ..., m)

(a) Equality of exports and imports for non-free traded goods on the international market

$$\sum_i \bar{m}_i - \sum_i \bar{e}_i \leq 0 ; \quad \bar{p}_i \cdot (\sum_i \bar{m}_i - \sum_i \bar{e}_i) = 0 .$$

(b) Utility maximization of each country: $\bar{x}_i$ maximizes $U_i(x_i)$ subject to the following constraints:

(b1) feasibility: $x_i \in X_i$; $y_i \in Y_i$; $e_i \in E_i$; $m_i \in M_i$

(b2) balance of payments constraint: $\bar{p}_i \cdot (m_i - e_i) \leq 0$

(b3) supply-demand equilibrium constraints on national markets:

$$x_i + e_i - y_i - m_i - \bar{w}_i \leq 0 .$$

__Remark__: By constructing the Lagrangean of the maximization problem defined by condition (b), it can easily be shown that the following conditions will be satisfied (for i = 1, 2, ..., m):

(c1) zero price for free goods on national markets

$$\bar{p}_i \cdot (\bar{w}_i + \bar{m}_i + \bar{y}_i - \bar{e}_i - \bar{x}_i) = 0$$

(c2) $\bar{y}_i$ maximizes profits $\bar{p}_i \cdot y_i$ subject to $y_i \in Y_i$

(c3) $\bar{x}_i$ maximizes $U_i(x_i)$ subject to the budget constraint

$$\bar{p}_i \cdot (x_i - y_i - \bar{w}_i) \leq 0 , \quad \text{and} \quad x_i \in X_i .$$

2.2. Formulation of a Tariff Ridden Equilibrium

The concept of a tariff ridden equilibrium is more complex. Tariffs distort prices as perceived by economic agents, so that the decisions are not based on the true prices paid by the country for its exports and imports. It is therefore necessary, in defining equilibria, to work with true and tariff ridden prices, and two budget constraints: the true balance of payments, and the distorted balance of payments which reflects the tariff ridden prices perceived by agents.[2]

For the sake of concreteness, it is assumed that tariffs are ad valorem.[3] Let tariff rates on exports and imports of country i be defined by t_i^e, t_i^m, with components t_{ik}^e and t_{ik}^m, possibly zero and less or equal to 1 in absolute value. Let p_i^1 represent the domestic prices in country i of the r_1 first traded goods so that we can define $p_i = (p \; p_i^2)$ as before and $p_i^* = (p_i^1 \; p_i^2)$. It will prove convenient to define also vectors p_i^e and p_i^m with components $(1 - t_{ik}^e)p_{ik}$ and $(1 + t_{ik}^m)p_{ik}$; p_i^e and p_i^m are respectively international prices reduced by the export tariff and increased by the import tariff. Finally we define tariff receipts as $t_i^* = (p_i^m - p_i) \cdot m_i - (p_i^e - p_i) \cdot e_i$. Then:

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[2] Tariffs may also affect decisions through their impact in increasing the income of the government and reducing that of consumers. This second impact is not relevant in the context of GEM, since consumers and governments are aggregated into a single decision unit with a single budget constraint and preference function. To treat this other effect of tariffs rigorously in a general equilibrium context, it would be necessary to consider the governments and consumers of countries as separate agents with distinct budget constraints and preference functions. This is quite feasible, in principle.

[3] The formulation is easily modified to take account of specific tariffs, or of price equalizing tariffs such as those used in the EEC Common Agricultural Policy.

<u>Definition 2.2</u>: The allocation $\{\bar{y}_i\}$, $\{\bar{x}_i\}$, $\{\bar{e}_i\}$, $\{\bar{m}_i\}$, supported by the non-negative price vectors $\{\bar{p}_i\}$ is a tariff ridden equilibrium if the following conditions hold (for $i = 1, 2, \ldots, m$):

(a) Equality of exports and imports for non-free traded goods on the international market

$$\sum_i \bar{m}_i - \sum_i \bar{e}_i \leq 0 \; ; \quad \bar{p}_i \cdot (\sum_i \bar{m}_i - \sum_i \bar{e}_i) = 0 \; .$$

(b) Utility maximization of each country: $\bar{x}_i$ maximizes $U_i(x_i)$ subject to the following constraints:

(b1) feasibility: $x_i \in X_i$; $y_i \in Y_i$; $e_i \in E_i$; $m_i \in M_i$

(b2) perceived balance of payments constraint: $\bar{p}_i^m \cdot m_i - \bar{p}_i^e \cdot e_i \leq \bar{t}_i^*$

where $\bar{p}_i^m$, $\bar{p}_i^e$ and $\bar{t}_i^*$ are the values of p_i^m, p_i^e and t_i^* corresponding to the equilibrium solution.

(b3) supply-demand equilibrium constraints on national markets:

$$x_i + e_i - y_i - m_i - \bar{w}_i \leq 0 \; .$$

<u>Remark 1</u>: The (b2) constraint implies that agents in country i regard $\bar{t}_i^*$ as independent of their importing and exporting decisions in maximizing their utility. The true budget constraint with which the country is faced is of course $\bar{p}_i \cdot (m_i - e_i) \leq 0$. It is easy to check that, at equilibrium this constraint is implied by (b2).

<u>Remark 2</u>: The Lagrangean of the maximization problem defined by condition (b2) is

$$L_i(\cdot) = U_i(x_i) + p_i^* \cdot (\bar{w}_i + m_i + y_i - e_i - x_i) + \lambda_i (\bar{t}_i^* - \bar{p}_i^m \cdot m_i + \bar{p}_i^e \cdot e_i)$$

At the maximum point, we have:

(c1) zero price for free goods on national markets

$$\vec{p}_i^* \cdot (\vec{w}_i + \vec{m}_i + \vec{y}_i - \vec{e}_i - \vec{x}_i) = 0 \ .$$

From the saddle point conditions, it is easy to show that

(c2) $\vec{y}_i$ maximizes profits $\vec{p}_i^* \cdot y_i$ subject to $y_i \in Y_i$.

(c3) $\vec{x}_i$ maximizes $U_i(x_i)$ subject to the budget constraint

$\vec{p}_i^* \cdot (x_i - y_i - \vec{w}_i) \leq 0$ and $x_i \in X_i$.

(c4) $\vec{e}_i$ maximizes profits $(\lambda_i \vec{p}_i^e - \vec{p}_i^*) \cdot e_i$ subject to $e_i \in E_i$.

(c5) $\vec{m}_i$ maximizes profits $(\vec{p}_i^* - \lambda_i \vec{p}_i^m) \cdot m_i$ subject to $m_i \in M_i$.

In (c4) and (c5), λ_i can be interpreted as the exchange rate of country i .

Remark 3: The last two conditions (c4) and (c5) imply the well-known decision rule that exporters (importers) trade only if the difference between the export (import) and the domestic price of a commodity k covers the profit (cost) of the corresponding tariff. This is usually expressed as:

(c6) $\lambda_i (1 - t_{ik}^e) \vec{p}_{ik} \leq \vec{p}_{ik}^*$; equality holds if $\vec{e}_{ik} > 0$.

(c7) $\lambda_i (1 + t_{ik}^m) \vec{p}_{ik} \geq \vec{p}_{ik}^*$; equality holds if $\vec{m}_{ik} > 0$.[4]

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[4] In the international equilibrium it is unnecessary to specify profit maximization for exporters and importers, as the other conditions imposed ensure that their profits are zero. If the Lagrangean of the international equilibrium is written as:

$$L(\cdot) = U_i(x_i) + p_i^* \cdot (\vec{w}_i + m_i + y_i - e_i - x_i) + \lambda_i \vec{p}_i \cdot (e_i - m_i)$$

it would follow that

$\vec{e}_i$ maximizes $(\lambda_i \vec{p}_i - \vec{p}_i^*) \cdot e_i$ s.t. $e_i \in E_i$, or $\lambda_i \vec{p}_i \leq \vec{p}_i^*$

$\vec{m}_i$ maximizes $(\vec{p}_i^* - \lambda_i \vec{p}_i) \cdot m_i$ s.t. $m_i \in M_i$, or $\lambda_i \vec{p}_i \geq \vec{p}_i^*$

implying $\lambda_i \vec{p}_i = \vec{p}_i^*$ and zero profits for both importers and exporters.

2.3. Computational Procedures for Tariff Ridden Equilibria

In [6], we defined a "master program" which can be associated with a general equilibrium, and showed that if $\bar{q}$ are the prices which enter the budget constraints of this master program and if the Lagrange multipliers associated to the resource balance constraints at the optimum are proportional to $\bar{q}$, then, the solution of the master program is an equilibrium. We then discussed three procedures designed to find the desired price vector $\bar{q}$.

A similar approach can be used to solve a tariff ridden equilibrium. Consider the following:

<u>Master Program</u>:   max $\sum_i [\alpha_i U_i(x_i) + \bar{\lambda}_i(\bar{p}_i^e - \bar{p}_i)\cdot e_i + \bar{\lambda}_i(\bar{p}_i - \bar{p}_i^m)\cdot m_i]$ subject

to $x_i + e_i - y_i - m_i - \bar{w}_i \leq 0$; $x_i \in X_i$; $y_i \in Y_i$; $e_i \in E_i$; $m_i \in M_i$;

$\bar{p}_i\cdot(m_i - e_i) \leq 0$ for all i   and $\sum_i m_i - \sum_i e_i \leq 0$.

Let $\bar{v}_i = (\bar{v} \; \bar{v}_i^2)$, where $\bar{v}$ and $\bar{v}_i^2$ are respectively the Lagrange multipliers associated to the r^1 first constraints $\sum_i m_i - \sum_i e_i \leq 0$ and to the r^2 last constraints $x_i + e_i - y_i - m_i - \bar{w}_i \leq 0$ at the optimal point; let also $\bar{\pi}_i$ be the Lagrange multipliers associated to $\bar{p}_i\cdot(m_i - e_i) \leq 0$ at the optimal point. Then, a straightforward application of the reasoning used in Theorem 2.1 of [6] will prove

<u>Theorem</u>:   If $\bar{v}_i = k\bar{p}_i$ where k is a positive scalar (or, more simply, if $\bar{v} = k\bar{p}$) and if $\bar{\lambda}_i = k + \bar{\pi}_i$, the solution $\{\bar{x}_i\}$, $\{\bar{y}_i\}$, $\{\bar{e}_i\}$, $\{\bar{m}_i\}$, $\{\bar{v}_i\}$, $\{\bar{\pi}_i\}$ of the master program is a tariff ridden equilibrium.

The three computational procedures described in [6] are thus applicable to tariff ridden equilibria. There are however several remarks

which have to be made:

(1) At each iteration, a value has to be set for k. A reasonable choice would be $k = \sum_\ell v_\ell / \sum_\ell p_\ell$. In practice, since v and p are close together, $k = 1$ gives good results.

(2) The terms $\bar{\lambda}_i(\bar{p}_i^e - \bar{p}_i) = -\bar{\lambda}_i t_i^e \cdot \bar{p}_i$ and $\bar{\lambda}_i(\bar{p}_i - \bar{p}_i^m) = -\bar{\lambda}_i t_i^m \cdot \bar{p}_i$ appearing in the objective function of the master program have to be re-evaluated at every iteration of the computational procedure.

(3) In the third procedure defined in [6] (indirect computation in utility space), the balance of payments constraints do not appear in the program to be solved so that $\pi_i = 0$ and $\lambda_i = k$ for all i.

(4) Since, at equilibrium, the balance of payments constraints will be satisfied as equalities, nothing prevents us from writing them down as equalities. This, together with $\bar{p} \cdot (\sum_i \bar{m}_i - \sum_i \bar{e}_i) = 0$ shows that one of the balances can be dropped.

3. Formulation of General Equilibrium Models. Methodological Remarks

3.1. Reduced Production Sets and Trade Welfare Functions

Due to lack of data or to cost considerations, it may be difficult to quantify the international equilibrium problem in full detail. We discuss here ways of reducing the size of the problem, compatible with the goal of obtaining price and allocations which meet fully the conditions of equilibrium.

3.1.1. Reduced production sets

It is a general observation that some commodities are used by one group of production processes only, and cannot be consumed, exported, or imported. For example, agriculture is the sole user of land; the steel industry is the sole user of blast furnaces. This means that the production vector y_i and set Y_i can be written as:

$$y_i = y_i^1 + y_i^2 \ ; \quad y_i^1 \ \epsilon \ Y_i^1 \ ; \quad y_i^2 \ \epsilon \ Y_i^2$$

where the last r^{**} coordinates of $y_i^1 \ \epsilon \ Y_i^1$ are identically zero. Partitioning y_i^2 into y_i^{2*}, y_i^{2**}, where y_i^{2**} contains the last r^{**} coordinates of y_i^2, the national balance constraints imply that any solution of the equilibrium problem must satisfy

$$y_i = y_i^1 + y_i^2 \ ; \quad y_i^1 \ \epsilon \ Y_i^1 \ ; \quad y_i^2 \ \epsilon \ Y_i^2 \ ; \quad -y_i^{2**} \leq w_i^{**}$$

where w_i^{**} contains the r^{**} last coordinates of w_i . This leads to defining the reduced production set Y_i^* composed of $(r - r^{**})$ dimensional vectors as:

$$Y_i^*(w_i^{**}) = \left[y_i^* \Big| y_i^* = y_i^{1*} + y_i^{2*} ; \begin{pmatrix} y_i^{1*} \\ 0 \end{pmatrix} \epsilon \ Y_i^1 ; \begin{pmatrix} y_i^{2*} \\ y_i^{2**} \end{pmatrix} \epsilon \ Y_i^2 ; \ -y^{2**} \leq w^{**} \right] .$$

Replacing Y_i by Y_i^* and y_i by y_i^* in the definition of a general equilibrium in no way changes the nature of the problem posed,[5] but reduces

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[5] The treatment of the energy sector in [8] is an excellent example of this type of production set reduction procedure. They do not however give the general formulation of the approach which we provide here.

the number of commodities which must be covered.

3.1.2. Trade welfare function

International trade theorists have for a long time been using, as a matter of routine, an even more drastic reduction of the production sets considered in the general equilibrium problem. The theoretical derivation of this concept is analogous to that of the reduced production sets. The function $U_i^*(w_i; e_i^*, m_i^*)$ defined by

$$\max \ U_i(x_i)$$

subject to:

$$x_i + e_i - y_i - m_i - \bar{w}_i \leq 0$$

$$m_i \leq m_i^* \ ; \quad -e_i \leq -e_i^*$$

$$x_i \in X_i \ ; \quad y_i \in Y_i \ ; \quad e_i \text{ and } e_i^* \in E_i \ ; \quad m_i \text{ and } m_i^* \in M_i \ ,$$

is the (obviously concave) trade welfare function of country i .[6]

3.2. Reduction to a One Period Problem

Any modeling exercise requires that the future be truncated at an appropriate point, beyond which the model builder feels that he does not have enough information to predict the course of the economy. There are many ways in which this may be done in the framework of an equilibrium model, according to the way in which time enters into the analysis. We confine our attention to one period models covering several years.

In this theoretical section it will be assumed that the model builder

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[6] See [11] for further details.

is able to make a forecast of total investment in the terminal year, and assume that production capacities of all activities can be increased by investing a single capital good. Once this "capital putty" is embodied in specific production capacities, it becomes clay and cannot be used for any other purposes.

Knowledge of initial and terminal levels of investment is of course not sufficient to determine the amount of capital which accumulates over the whole span of the model. The indeterminacy disappears, however, if the time slope of the investment path is known. Many assumptions are possible, see e.g. [9], [10], [12]; the simplest is to follow Sandee [12] and assume that the investment path of each activity is linear.

Then we can first calculate what initial capacities will be in the terminal year T if investment fell to zero in that year. Assuming, and this leads to useful arithmetic simplification, that there is no lag between investment and the resulting increase in capacity, the accumulated investment and the resulting increase in capacity of activity k will be $\frac{T}{2} I_k^0$ and $\frac{T}{2} \frac{I_k^0}{\kappa_k}$ respectively, where κ_k is the capital coefficient. If production in the initial year is z_k^0 and if capacity was fully employed in that year, then the initial capacity available in the terminal year is

(1) $$\bar{\imath}_k^0 = z_k^0 + \frac{T}{2} \frac{I_k^0}{\kappa_k} .$$

The linear investment increase assumption therefore implies that, unless investment in the base year $I_k^0 = 0$, the available capacity in the terminal year will be higher than in the base year. This is reasonable,

though whether the linear investment path assumption is correct is of course debatable.

Part of the capital accumulated during the period is absorbed by the linearly declining sectoral investment levels described above. The amount of investment which is not so committed will clearly grow from zero in the initial year to I^T in the terminal year, where $I^T = \sum_k I_k^T$ is aggregate investment. The linear investment path implies that a stock of "capital putty"

$$(2) \qquad K = \frac{TI^T}{2}$$

will accumulate which can be allocated to activities which need to expand their capacity above the initial capacity $\bar{i}_k^0$ defined above. The expressions (1) and (2) can be used to define the initial resources of production capacities and of capital putty which can be introduced in the one stage equilibrium model defined to describe equilibria in year T .

3.3. Linear Programming Representation of Production Sets and Utility Functions

3.3.1. Linearization of the underlying non-linear functions

In [5] it was shown that any convex production set or concave utility function can be represented to any degree of approximation by linear programming constraints. Linear programming provides therefore an entirely general framework for the study of general equilibrium. The only restriction is that utility functions need to be concave rather than merely quasi-concave; this is harmless from an empirical point of view.

In [5] we discussed only the cases of general production sets and

utility functions, as well as the homothetic and separable cases. In
Section 4 we describe the specific linearization procedures used in GEM.

3.3.2. The approximation region concept

Clearly, it is useless to approximate functions over the whole
range of values which variables can take, if a guess can be made of an
approximate solution. It is enough to linearize the equations in a neigh-
borhood of this solution. This way of proceeding has the advantage of
reducing the number of variables and constraints.

The results of the model of course become doubtful on the borders
of the approximation zone, which will, in practice, be truncated by ad hoc
constraints; these are unrelated to the non-linear specification which is
approximated and will distort both prices and quantities, and render the
solutions meaningless.[7]

When this happens (usually because of a poor guess of the solution),
the approximation region should be extended further. This can be done
either in an ad hoc way as we have done, or by incorporating into the
"revision phase" of the solution algorithms, instructions which modify
the linearizing constraints or add new ones whenever the optimal solution
lies at the border of the approximated region.

3.4. Value constraints

As is apparent from Section 2.3, the master program associated with
the equilibrium contains constraints and objective function terms which
include both primal variables and (dual) prices. Such value constraints
will also appear (see Section 4.2.3) in the treatment of investment; and
they are, of course, present in many second best models.

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[7] For an excellent analysis of these distortions, see L. Taylor [13].

It could be thought that the problem is due to the use of linear programming, which cannot handle bilinear constraints, and that more general non-linear algorithms would solve it. Clearly, this is not the case. The difficulty is that Kuhn-Tucker's mathematical programming theory does not cover the case where multipliers enter the definition of the feasible set.

The way we solved the difficulty was to alternate between solving mathematical programs giving market clearing prices and revising the bilinear constraints by plugging in the market clearing prices. If this procedure converges, it gives an answer to the problem.

4. The Model. An Overview

4.1. A Non-Technical Description

The model is designed to study the relations between the developing countries of Latin America, Asia, and Africa and the rest of the world economy, divided into the Middle East oil producers, and developed countries. As the emphasis is on the first three zones, only they are represented by fairly detailed, though still highly aggregated models. The economy of the oil producers is represented by a simplified set of equations, since at the time the model was constructed their trade and other relations with developing countries were still relatively small. Finally, to avoid constructing a detailed representation of the economy of developed countries their behavior is described by a trade welfare function.

As explained above, each region acts as an economic agent who, at the same time, produces, consumes, exports, and imports; capital flows net of debt service into Latin America, Asia, and Africa are supposed to be exogenous. In the case of oil producers, on the other hand, it is assumed that, because of the pull exerted by financial markets in developed

countries, and their large foreign assets, their surpluses or deficits
are covered by building up or drawing down their accumulated financial
assets. Only one balance of payments needs to be specified to cover both
the Middle East oil producers and developed countries.

Traded goods are aggregated into 13 categories: six primary com-
modities (tropical competing foodstuffs; tropical non-competing foodstuffs;
temperate foodstuffs; agricultural raw materials; ores and non-ferrous
metals; oil) and seven manufactures (food products; textiles; chemicals;
non-metallic products; basic metals; machinery; other industries). How-
ever, since commodities produced by different regions are not considered
identical, the number of traded goods is close to fifty.[8] In Latin America,
Asia, and Africa, non-traded goods include construction, services, capital,
13 import substituting commodities, 15 production capacities, 13 export
capacities, and urban labor. The agricultural production sets are reduced
sets from which land and labor constraints are eliminated. These goods
are therefore not treated explicitly as commodities. The reduction of the
developed countries model to a trade welfare function means that there
are no non-traded goods in that region. Finally, there is only one non-
traded good in the Middle East.[9]

The production sets of Latin America, Asia, Africa, and the Middle
East are partly linear, and partly non-linear. The latter are approximated
by appropriate linear programming objective function terms and constraints.

The non-linear utility functions are based on a specification,

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[8]Regions do not necessarily export all goods, so that the number of traded
goods is less than 5 x 13 = 65.

[9]A full list of commodities produced by each region is given in the Ap-
pendix 1.

which makes it possible to represent certain non-separable functions by a small number of variables and constraints. Utility in Latin America, Asia, Africa and the developed world will be represented in this way, while the function is linear for the Middle East. An unorthodox feature for Latin America, Asia, and Africa is that the functions include terms which impute psychological costs to certain types of labor migration.

Our feeling is that it is not appropriate to use general equilibrium models to describe short-run economic behavior which is dominated by expectational phenomena and other types of disequilibria, difficult to insert into equilibrium analysis.[10] The 10-year span of GEM is, however, long enough for equilibrium forces to assert themselves. The model describes economic behavior in a one period framework. This requires that some way is found to cut loose a meaningful static equilibrium from the dynamic fabric of possible future developments. This problem is dealt with using the ideas presented in Section 3.2.

Agents and goods are highly aggregated. We have sandwiched into a mere five zones the multitude of countries whence come the throngs which congregate at the UN General Assembly and similar meetings. The spectrum of goods exported by each zone is aggregated into 13 commodities. There is only 1 type of labor in each major region, etc. An aggregation problem is likewise involved in the representation of the behavior of all agents in a region by a single utility function. It is well-known that aggregation of utilities is permissible only under extremely strict assumptions which are unlikely to be realized. Though it is impossible, from a practical point of view, to represent the behavior of each agent by a utility

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[10]The temporary equilibrium concept, however, offers a promising approach to the description of this behavior.

function, it would be extremely desirable--and quite feasible, in prin-
ciple, with our approach--to specify separate utility functions and bud-
get constraints for different social groups in each region.

Given these drastic simplifications, compounded by the defects
and lacunae of the data, the emphasis on the general equilibrium under-
pinning GEM, should be taken as a parable rather than as an assertion
that world economic equilibrium is truly described. We found the parable
useful in that it forced us to adopt theoretically sound formulations of
relations. The benefit from the general equilibrium framework is not
principally that the results are more accurate than those obtained by
less rigorous approaches. There is some gain in accuracy, but of much
greater importance is the intellectual discipline which forces the model
builder to incorporate only theoretically significant constraints, avoid-
ing the use of the crude heuristics which play such a large role in de-
velopment planning models. The real gain is that the results have a clear
meaning and provide a sound and rigorous basis for further analysis of
problems.

All econometric models are in any case parables which stress the
relevant features of the situation and omit irrelevant ones. Ours empha-
sizes the substitution mechanisms which determine adjustments in foreign
trade, and omits details which do not affect them. The aspects of reality
stressed are:

(a) competition between less developed countries on developed
    countries' markets is reflected in the specification of the
    trade welfare function of developed countries, describing
    a demand behavior in which overall demand for each commodity
    is somewhat inelastic, whereas there is strong substitution

between different suppliers of each commodity. As transport

costs are neglected, the competition between developing areas

is not influenced by freight rates.

(b) inelasticity of supply of traded commodities in developing

countries: for non-agricultural commodities supply is assumed

to be elastic, apart from the fact that, beyond a certain

level, increases in exports require additional investment to

extend harbor facilities, etc. Constraints on available land

recognize the inelasticity of supply of agricultural goods,

and also the high cross elasticities of supply which imply

that the increase in price of any agricultural good shifts

upwards the supply curve of other goods.

(c) import substitution: Latin America, Asia, and Africa are

assumed to be capable of replacing imported by domestically

produced commodities. The cost of this import substituting

production increases however quite rapidly as output expands.

(d) capital-labor substitution is assumed to operate very weakly.

Labor enters costs of production only in industry, at a level

reflecting a rough estimate of urbanization which raises the

cost of city labor above the (supposedly zero) marginal pro-

ductivity of labor in rural areas. Except in agricultural

production, processes involve fixed coefficients, so that sub-

stitution between labor and capital is possible only through

shifts in the commodity composition of output.

(e) substitution in consumption is reflected by the utility maxi-

mizing behavior of consumers described in the models for Latin

America, Asia, and Africa (and implicitly by the developed

countries' trade welfare function).

(f) <u>psychological costs of migration</u>: It seemed worthwhile to

recognize the psychological sacrifices involved in shifting

labor between activities. These are thought to be especially

significant for movements between primary sectors, which are

often geographically far removed from each other; these costs

are taken into account by appropriate utility function terms.

The events the model is designed to study are mainly changes in

import tariffs by developing and developed countries, and changes in the

level and distribution of aid. The model can also be used to examine

the impact of changes in the growth rate of developing and developed coun-

tries, in growth of particular agricultural commodities and in changes of

oil prices (which are quasi exogenous).

4.2. <u>Technical Devices Used in Constructing the Equations</u>[11]

4.2.1. <u>Production processes</u>

GEM uses extensively activity analysis. Let z_j represent the

activity level of the jth production process; then the inputs and out-

puts[12] of commodities $k = 1, 2, \ldots, r$ by process j can be simply

---

[11]Notations are defined when used first. A complete list of notations
is given in Appendix 2. Let us simply state here a few general rules;
subscripts i , j and k will be used respectively for the country,
the production process and the commodity; subscript i will usually be
omitted when it is clear we deal with one country; subscript h will
be used as a local summation index; superscripts refer to the linear
piece of an approximation. Except for technical coefficients (a_{kj})
and labor coefficients (ℓ) we use greek letters for coefficients.
Latin letters represent variables; when upperbarred, they are right hand
side constants. Functions are usually represented by capital letters.

[12]We use the familiar convention that negative y_{kj} describe inputs,
positive y_{kj} , outputs.

described by functions $y_{kj}(z_j)$. A process j will be linear--or con-
cave--if $y_{kj}(z_j)$ is linear--or concave--for every k ; j will be strictly
concave if at least one $y_{kj}(z_j)$ is strictly concave. From an economic
point of view, linear, concave, and strictly concave processes correspond
to the cases of constant, non increasing and decreasing returns to scale.

No detail needs to be given about linear activity analysis processes,
which can be used as such in linear programs by simply setting $y_{kj}(z_j) = \delta_{kj} z_j$.
Non-linear processes must be linearized. Assuming that they are concave,
it is possible to break up the range of variation of the activity variables
into a series of steps z_j^s , over which $y_{kj}(z_j)$ is approximated by a
piecewise linear function. The formulation is

$$z_j^s \leq \overline{z}_j^s , \quad s = 1, 2, \ldots, s^*$$

$$-y_{kj} = \overline{y}_{kj} + \sum_s \delta_{kj}^s z_j^s$$

where the δ_{kj}^s are average values of the derivatives dy_{kj}/dz_j over
the intervals s . These average values and $\overline{y}_{kj}$ are chosen to ensure
that $y_{kj}(0) = \overline{y}_{kj}$; $y_{kj}(\sum_s \overline{z}_j^s) = \overline{y}_{kj} + \sum_s \delta_{kj}^s \overline{z}_j^s$. As a result of con-
cavity, step s will enter the solution before step $s+1$.

More complex processes were used only in agriculture, to take ac-
count, in an ad hoc way, of the competition for land between different
crops. This formulation is described in Section 5.4.

4.2.2. Utility functions

The other extensively used device is a utility function which has
a number of nice properties. Though non-separable, this function involves
few parameters and can be approximated by a small number of linear con-

straints and variables. It is easy to derive parameter values which cor-
respond in a rough way to the results of econometric studies.[13] As these
functions do not seem to have been used by other authors, they will be
referred to as "GEM functions."[14]

GEM utility functions have the form

$$U(x) = V(\sum_k x_k) - \sum_k W_k [x_k - \gamma_k - \beta_k \sum_h (x_h - \gamma_h)]$$

where V is concave and the W_k are non-negative convex functions which
go through the origin. Also $\sum_h \beta_h = 1$.

The distinctive property of these functions is that all indifference
curves can be obtained by sliding a single indifference curve along a ray
going through γ . This is depicted in the diagram below.
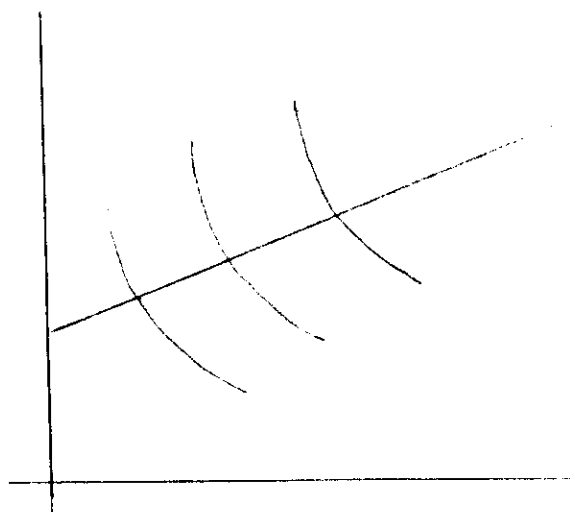
[13] The procedure which we have used to quantify these functions is described
in the "data" appendix of the paper.

[14] It is quite similar in purpose, but both simpler and less rigorous than
the procedure suggested by Fourgeaud and Nataf in an interesting, but little
quoted article [2].

It follows that:

    (a)   the Engel line corresponding to a situation of equal prices has the equation

(1)
$$x_k = \gamma_k + \beta_k \sum_h (x_h - \gamma_h)$$

    (b)   along this line the W_k vanish so that $U(x) = V(\sum_h x_k)$

    (c)   if $V(\cdot)$ is linear the other Engel lines are parallel to (1)

    (d)   defining V', W_k' as derivatives $dV/d(\sum x_k)$ and $dW_k/d(x_k - \gamma_k - \beta_k(\sum_h x_h - \gamma_h))$, the marginal utilities are

(2)
$$\frac{\partial U}{\partial x_k} = V' - (W_k' - \sum_h \beta_h W_h') \ .$$

It will be noted that the function is not additively separable. The property (c) appears to be thought unfortunate by persons with whom we have discussed the model and who seem to have picked up the idea that homotheticity of preferences is an attractive property not because it is mathematically convenient but because it is an economically realistic assumption. Since the function is used to represent preferences in the neighborhood of equilibrium only and not in the large, homotheticity is not an important issue in the present context. The function happens to be computationally convenient because its linearization involves upper bounds which are cheaper to handle by linear programming codes, than are constraints.

GEM functions can be approximated by the following linear programming objective function terms and constraints, where subscript k ($k = 1, 2, \ldots, r$) refers to commodity k and superscript s to the sth linear piece:

$$(3) \qquad U(x^*) = \max_s [\sum_s \omega^s x^s - \sum_k \sum_s \omega_k^s (x_k^{s+} + x_k^{s-})]$$

subject to

$$(4) \qquad \sum_s x^s - \sum_k x_k = 0 \qquad\qquad (w)$$

$$(5) \qquad x_k - \beta_k (\sum_h x_h - \sum_h \gamma_h) - \gamma_k = \sum_s (x_k^{s+} - x_k^{s-}) \qquad (w_k)$$

$$(6) \qquad x_k^{s+} \leq \overline{x}_k^{s+} \qquad\qquad (w_k^{s+})$$

$$(7) \qquad x_k^{s-} \leq \overline{x}_k^{s-} \qquad\qquad (w_k^{s-})$$

$$(8) \qquad x^s \leq \overline{x}^s \qquad\qquad (w^s)$$

$$(9) \qquad x_k = x_k^* \qquad\qquad (v_k)$$

where we have written, at the right of each constraint, the dual variable associated with the constraint. The first term of the objective function corresponds to $V(\sum_k x_k)$: see constraint (4). The ω^s are approximations of V' over the steps s . Because of the concavity of V , $\omega^{s+1} \leq \omega^s$; this ensures that steps enter the solution in successive order.

The second term of the objective function corresponds to $\sum_k W_k [x_k - \gamma_k - \beta_k \sum_h (x_h - \gamma_h)]$: see (5). The ω_k^s are approximations of the derivatives W_k' . Once again the convexity of the W_k ensures that $\omega_k^{s+1} \geq \omega_k^s$ and that the steps enter the solution in successive order.

The marginal utilities of the goods are obviously equal to the vector v of dual variables associated with the constraint (9). This leads to studying the dual constraints

$$w + w^s \qquad\qquad \geq \omega^s \qquad (x^s)$$

$$-w + w_k - \sum_h \beta_h w_h + v_k \geq 0 \qquad (x_k)$$

$$w_k^{s+} - w_k \qquad\qquad \geq -\omega_k^s \qquad (x_k^{s+})$$

$$w_k^{s-} + w_k \qquad\qquad \geq -\omega_k^s \qquad (x_k^{s-})$$

where the primal variable which corresponds to each constraint is noted in brackets at the right of the constraint.

By complementary slackness, non-negativity of a primal variable implies that the corresponding dual constraint is exactly satisfied, and non-negativity of a dual variable implies the same for the corresponding primal constraint. For the sake of simplicity, we will assume that at the optimal point, $0 < x_k^{s+} < \bar{x}_k^{s+}$ (or[15] $0 < x_k^{s-} < \bar{x}_k^{s-}$) for a step s^+ (or s^-) and $0 < x^s < \bar{x}^s$ for a step s , so that $w^s = w^{s+}$ (or w^{s-}) $= 0$ by complementary slackness.[16]

Then, the dual variables must obey

$$(10) \qquad w = \omega^s$$

$$(11) \qquad v_k = w - (w_k - \sum_h \beta_h w_h)$$

$$(12) \qquad w_k = \omega_k^s \text{ or } w_k = -\omega_k^s \text{ according to whether } x_k^{s+} > 0 \text{ or } x_k^{s-} > 0 .$$

Since ω^s and ω_k^s are approximations of the derivatives V' and W_k' of the GEM function, it is clear that expression (11) is identical

---

[15] It is easily shown that if $x_k^{s+} > 0$ then $x_k^{s-} = 0$.

[16] The reasoning is easily extended if these primal constraints are satisfied as equalities, but the expressions become more complicated.

to (2) which defined marginal utility of consumer goods for that function.

4.2.3. Formulation of the problem as a one stage economic equilibrium

In the methodological Section 3.2 we explained how it is possible, given a forecast of investment levels in a future year and an assumption on the time path of investment, to specify a one stage equilibrium problem which provides what amounts to a snapshot of a dynamically growing economy. This section explains how the idea was implemented.

We had to face two problems which were disregarded in the methodological section. Investment is composed of two goods, machinery and construction; and its level of future years is not exogenous, but depends on the overall growth of the economy. This raises three difficulties:

(a) Even if total investment is known, its distribution between different capital goods depends on which sectors invest; this affects sales of capital goods in the terminal year. Housing for example consists of construction only and ships solely of "machines."

(b) The stock K of capital goods available for investment then consists not of one, but of several goods. This could be taken to imply that several capital balances should be created.

(c) The feasible investment in the terminal year is probably best defined by a current price constraint which matches saving plus capital imports to investment at current prices; saving itself is a function of national income at current prices. Such a constraint, which involves both dual and primal variables, cannot be formulated in a linear programming framework.

There are several ways in which these problems can be solved; none is perfect but all are good enough for practical purposes. In GEM the following solution was adopted. The model solved in the first "near equilibrium" iteration of the three solution procedures contains a saving constraint which relates saving to national income at constant prices, and equates this saving to investment at constant prices.

For each process two capital coefficients $K_{12,j}$ and $K_{14,j}$ are defined, one for construction and the other for machinery. Knowing initial investment, it is then easy to calculate what the initial capacities $\bar{i}_j^0$ of each process would be if investment declined linearly to zero from its initial year level.

The solution of the near equilibrium model will give figures of terminal year sales of machinery and construction, shadow prices u_{12}, u_{14} of these goods and the part K_{12}, K_{14} of capital accumulation in these goods which is not absorbed by creation of the initial capacities $\bar{i}_j^0$.

The construction and machinery coefficients are then aggregated using these shadow prices: $K_j = u_{12}K_{12,j} + u_{14}K_{14,j}$. The same is done for the capital stocks: $K = u_{12}K_{12} + u_{14}K_{14}$. This reduces the two capital balances to one. At the same time the terminal year investments in machinery and construction are made exogenous and shifted to the right hand side of the constraints; the investment activity is suppressed. In [4], the whole procedure is outlined in more detail.

In the equilibrium iterations of the computational procedures this capital balance and these exogenous investment demands are left unchanged. The equilibrium is thus calculated as though the capital stock and these

investments were independent of the equilibrium solution. To the extent

that the final equilibrium is close to the initial near equilibrium solu-

tion, this procedure does not introduce a significant inconsistency in

the solution.[17]

5. The Submodels for Latin America, Asia, and Africa

5.1. Production Sets Outside Agriculture and the Oil Industry

These sets are represented by activity analysis processes. For

each commodity there are defined processes which produce:

(1) a "home commodity";

(2) a commodity which replaces imports;

(3) additional capacities needed for production of the home

commodity;

(4) additional capacities needed for production of the im-

port substituting commodity.

Each of these activities produces one commodity (the corresponding

output coefficient is normalized to be unity); and consumes one or more

of the following commodities:

(1) 15 "input-output" goods, referred to hereafter as "goods";

(2) one type of urban labor;

(3) 15 production capacities for home goods;

(4) 13 production capacities for import substituting commodities;

(5) 13 imported commodities;

(6) one type of undifferentiated capital good.

---

[17]We feel that, given the crudeness of the linear investment time path
assumption, or of any other time path assumption, there was no point in
refining this procedure.

The product groups identified in the model are broad aggregates of many different commodities. Of these there will be several for which the country has a competitive advantage; they are the "home goods." Others can find a profitable market only if they benefit from protection or if the domestic exchange rate drops sharply; they are the "import substituting goods."[18]

The parable which we used to represent this situation is that, for each type of commodity, the home goods can satisfy only a part of the market. The rest of the market can be filled either by imports, or by import substituting goods; the latter however are produced under diminishing returns to capital, and have higher capital requirements at all output levels, than the corresponding home goods.

This leads to representing the production sets for home and import substituting goods by linear activity analysis processes using production capacities (the input coefficients corresponding to those capacities are normalized to 1) and other commodity inputs. The activities which "produce" production capacities use the undifferentiated capital good. The output coefficient is unity; the input coefficient is the marginal capital output ratio. Capacities for production of home and import substituting goods are created by linear and strictly concave processes, respectively.

Let z_{j_1} and z_{j_2} be the processes which produce home goods and import substituting goods and let i_{j_1} and i_{j_2} be the processes which

---

[18] Ideally import substitution could be represented by a very detailed model based on information on the structure of costs of the goods which can be produced. This data is lacking, and anyway such a detailed treatment of production would have led to a model which could not be built and handled with available means.

produce the respective capacities. Then if k is the commodity involved, the production set is described by:

Y_k = set of vectors y_k such that their non zero elements are (k = 5 and 7 to 15)

production of goods $\leq a_{kj_1} z_{j_1} + a_{kj_2} z_{j_2}$

input of urban labor $\leq -\ell_{j_1} z_{j_1} - \ell_{j_2} z_{j_2}$

input of j_1 capacity $\leq -z_{j_1} + i_{j_1}$

input of j_2 capacity $\leq -z_{j_2} + \sum_s i^s_{j_2}$

net input of imported goods $\leq -u_{j_1} z_{j_1} + (1 - u_{j_2}) z_{j_2}$

input of undifferentiated capital $\leq -\kappa_{j_1} i_{j_1} - \sum_s \kappa^s_{j_2} i^s_{j_2}$

subject to the linearization bounds $i^s_{j_2} \leq \bar{i}^s_{j_2}$. Inputs are represented by negative outputs, and the output coefficient is normalized to be unity.

5.2. Production Sets for Oil

Here again it is necessary to specify commodities and production processes in a way which takes account of the excessive aggregation of the model. In each of the three developing zones, some countries are capable of producing oil at costs that are comparable to those involved in the Middle East: Venezuela in Latin America, Indonesia in Asia and Algeria in Africa. The remaining countries which have oil reserves can only exploit them at relatively high costs (e.g. Brazil or Tunisia).

To capture this aspect of reality, it was decided to treat the exported oil of the low cost producers and the import substituting oil of the high cost countries as different products. It is then assumed that the low cost oil is demanded only abroad, while the domestic market is

supplied either by the import substituting oil or by imports. As will be seen later, exported oil is affected by high export tariffs, which increase its cost to foreign importers.

If z_{j_1} and z_{j_2} represent processes which export and substitute for import and i_{j_1} and i_{j_2} represent processes corresponding to the respective capacities, the production set is defined as

Y_6 = set of vectors y_6 such that their non-zero elements are:

$$
\begin{array}{ll}
\text{production of goods} & \le a_{kj_1} z_{j_1} + a_{kj_2} z_{j_2} \\[2ex]
\text{input of urban labor} & \le -\ell_{j_1} z_{j_1} - \ell_{j_2} z_{j_2} \\[2ex]
\begin{array}{l}\text{input of } j_1 \\ \text{capacity}\end{array} & \le -z_{j_1} + i_{j_1} \\[2ex]
\begin{array}{l}\text{input of } j_2 \\ \text{capacity}\end{array} & \le -z_{j_2} + \sum_s i^s_{j_2} \\[2ex]
\begin{array}{l}\text{net input of} \\ \text{imported oil}\end{array} & \le -\mu_{j_1} z_{j_1} + (1 - \mu_{j_2}) z_{j_2} \\[2ex]
\begin{array}{l}\text{input of undiffer-} \\ \text{entiated capital}\end{array} & \le -\kappa_{j_1} i_{j_1} - \sum_s \kappa^s_{j_2} i^s_{j_2}
\end{array}
$$

subject to linearization bounds $i^s_{j_2} \le \overline{i}^s_{j_2}$.

5.3. Production Sets for Agricultural Goods

The representation of agricultural production sets is far more difficult than for manufacturing and service activities. Agriculture uses a large number of types of land which differ in climate, location, and natural fertility. Most types of land can serve alternative uses, and a principal goal of any model will be to find out the optimal allocation of land between different crops.

Mathematical programming offers an excellent framework for analysis

of agriculture, as shown by a number of studies.[19] But successful imple-
mentation of this approach requires a large and costly effort even for
a single country. Construction of a world agricultural model was not
feasible with the means at our disposal.

The representation of agriculture in GEM is the result of a brutal
simplification of the complex underlying reality. To understand the equa-
tions, we can start from a picture of agricultural production as the sum
of productions on numerous types of land, to each of which corresponds a
different production function, say

$$f^a(z_j^a,\ i_j^a,\ L_j^a) \leq \bar{a}^a \tag{1}$$

where the subscript j runs over the four agricultural commodities, the
superscript a refers to the type of land; $\bar{a}^a$ is the amount of land
a which is available; L stands for labor.

The goal is to describe very crudely by a few constraints the re-
duced production set obtained by eliminating all the land constraints
(1) from the agricultural production set. The linear constraints which
are designed to approximate this set reflect the following ideas about
its structure:

(a) There are different qualities of land. We distinguish only
two.

(b) The more fertile lands will be fully used. As a result, each
crop grown on fertile land can be increased only at the ex-
pense of other crops.

---

[19] E.g. see J. Duloy and R. Norton [1].

(c) There seems to be a substantial supply of marginal land, even in Asia. There is less likelihood therefore that the expansion of one crop on these lands will limit the possible expansion of other crops.

(d) The yield (and to a certain extent the area) of fertile land can be increased by investment in irrigation, drainage, and fertilizer production.

(e) On marginal land there is no similar choice between capital and land intensive production techniques.

(f) Labor is overabundant and has zero marginal productivity. This is a hotly debated question and the assumption may be wrong; but the opposite view would also draw criticisms.

(g) Because marginal lands are less fertile they require higher capital and intermediate goods inputs per unit of production than fertile lands.

These ideas are embodied in the following constraints. Let j_f and j_m be indexes designating the processes which produce an agricultural commodity on fertile and marginal land respectively and let j_p be the index of the process producing the corresponding import substituting commodity.[20] Let similarly i_{j_f}, i_{j_m} and $i_{j_p}^s$ $(s = 1, \ldots, s_*)$ be production capacities of the processes where, as the notation indicates, capacities for producing import substituting commodities are broken down into steps $s = 1, \ldots, s_*$. Let finally $i_{j_f}^*$ be activities which increase the amount or the productiveness of fertile land. Since there

---

[20]For the sake of simplicity it is convenient to assume that import substituting goods are produced on one type of land only.

are four agricultural commodities in the model, we have $j = 1, 2, 3,$ and 4. With these notations, the agricultural production set can be described by:

Y_k = set of vectors y_k such that their non-zero elements are: (k = 1 to 4)

Output of goods $\leq \sum_{j=1}^{4} (a_{kj_f} z_{j_f} + a_{kj_m} z_{j_m} + a_{kj_p} z_{j_p})$

Input of j_f capacities $\leq -z_{j_f} + i_{j_f}$

Input of j_m capacities $\leq -z_{j_m} + i_{j_m}$

Input of j_p capacities $\leq -z_{j_p} + \sum_{s} i_{j_p}^s$

Input of imported goods $\leq -\sum_{j=1}^{4} (\mu_{j_f} z_{j_f} + \mu_{j_m} z_{j_m} - (1 - \mu_{j_p})z_{j_p})$

Input of undifferentiated capital $\leq -\sum_{j=1}^{4} (\kappa_{j_f} i_{j_f} + \kappa_{j_m} i_{j_m} + \sum_{s} \kappa_{j_p}^s i_{j_p}^s + \kappa_{j_f}^* i_{j_f}^*)$

Y_k is subject to the following constraints:

(a) Linearization constraints, reflecting the non-linear relation between capacity and capital intput in import substituting activities $i_{j_p}^s \leq \bar{i}_{j_p}^s$.

(b) Constraints reflecting the fact that the amount of fertile land suitable for a particular crop is limited $z_{j_f} - i_{j_f}^* \leq \bar{z}_{j_f}$; also, irrigation possibilities are limited: $i_{j_f}^* \leq \bar{i}_{j_f}^*$.

(c) Constraints recognizing the same as in (b) for marginal lands: $z_{j_m} \leq \bar{z}_{j_m}$.

(d) The three last constraints recognize that there is competition for fertile land between different crops. Since land

may be used for several crops, the maximum output of both tropical competing $(j = 1)$ and non-competing $(j = 2)$ is less than the maximum output of either crop

$$z_{1_f} + z_{2_f} - i_{1_f} - i_{2_f} \leq \theta_1(\bar{z}_{1_f} + \bar{z}_{2_f}) \; ; \quad \theta_1 < 1 \; .$$

Likewise, for temperate foodstuffs $(j = 3)$ and agricultural raw materials $(j = 4)$

$$z_{3_f} + z_{4_f} - i_{3_f} - i_{4_f} \leq \theta_2(\bar{z}_{3_f} + \bar{z}_{4_f}) \; ; \quad \theta_2 < 1$$

and for $j = 1$, 2 and 4

$$z_{1_f} + z_{2_f} + z_{4_f} - i_{1_f} - i_{2_f} - i_{4_f} \leq \theta_3(\bar{z}_{1_f} + \bar{z}_{2_f} + \bar{z}_{4_f}) \; ; \quad \theta_3 < 1 \; .$$

5.4. Production Set for Urban Labor

Increasing the supply of urban labor involves an urbanization cost which reflects infrastructure and "skilling" costs. For the sake of simplicity, these costs are assumed to consist only of undifferentiated capital. The relation between the production of urban labor and the required capital input is described by a strictly concave activity analysis process. This is defined by

$$Y_{16} = \text{set of vectors } y_{16} \text{ such that:} \quad \left| \begin{array}{l} \text{production of urban labor} \quad \leq \sum_s z_{16}^s \\ \\ \text{input of undifferentiated capital} \leq -\sum_s \kappa_{16}^s z_{16}^s \end{array} \right.$$

subject to $z_{16}^s \leq \bar{z}_{16}^s$

5.5. Export and Import Sets

In GEM goods imported into a country and goods available on international markets are considered as different commodities; the same holds for exported goods and goods available on these markets. There exist thus import and export processes which "transport" goods between domestic and international markets.

For each traded good, an import process is defined in each region, which purchases the commodity in fixed proportions on international markets and delivers it to the domestic market as an "imported good." There is therefore no direct price substitution between suppliers of a commodity to developing countries; as will be seen in Section 7.2, the same is not true on developed countries' markets. This, we feel, is not a serious weakness of the model, since the share of developed countries on developing countries' markets is so large that substantial shifts in the distribution of these imports are unlikely over the period covered by the model.

For exports, likewise, we adopt the convention that there exist export processes which buy each traded good on the domestic market and transform it into an international good. To reflect the infrastructure bottlenecks which so often hamper the growth of developing countries we assume that these processes require export capacities, which can be increased by drawing on the available stock of capital putty.

The import and export sets are then determined as

M_k = set of $\begin{pmatrix} m_k^d \\ \\ m_k \end{pmatrix}$ the non-zero

    elements of which are

    for the m_k^d component:

imported goods made available to domestic markets	$\leq z_k^m$

    for the m_k component:

purchases by the importing process of goods produced by region i	$\geq \mu_{ik} z_k^m \ (\underset{i}{\Sigma}\ \mu_{ik} = 1)$

E_k = set of $\begin{pmatrix} e_k^d \\ \\ e_k \end{pmatrix}$ the non-zero

    elements of which are

    for the e_k^d component:

domestic goods purchased by the exporting activity	$\geq z_k^e$
input of export capacities	$\leq -z_k^e + i_k^e$
input of undifferentiated capital	$\leq -\kappa_k^e i_k^e$

    for the e_k component:

domestic goods supplied to international markets by the export process	$\leq -z_k^e$

where, as before, inputs are negative outputs.

5.6. Consumption and Migration Sets

5.6.1. Sets of feasible consumptions

For traded goods, the consumption processes involve joint inputs of the domestically produced and of the corresponding imported good. For non-traded goods only the domestic commodity is purchased.

The consumption sets are thus defined by:[21]

X_k = set of consumption vectors x_k such that their non-zero elements are:

input of domestic good	$\geq \lambda_k z_k^c$
input of imported good	$\geq (1 - \lambda_k) z_k^c$

where $\lambda_k = 1$ for non-traded goods.

---

[21] The objective function linearization constraints might appear to restrict the consumption set. We feel that this restriction should be disregarded, since it depends on the arbitrarily chosen extent of the approximation zone. As was pointed out, this zone should be extended wherever the solution is on its boundary.

5.6.2. Sets of feasible migrations between primary producing activities

These migrations can take positive, zero, or negative values.[22]

5.7. Commodity Balances

In addition to the linearization constraints defined in the pre-ceding section, the constraints set of the regional submodels contains the balance equations specified by equilibrium condition (b3). The $\bar{w}_i$ vector of this condition has zero elements except for positive initial resources of production capacities and the capital putty constraints; section 4.2.3 shows how these figures are generated. As also explained there, it is necessary to include in the balances the (exogenous) terminal year investment levels; let these be represented by the vector x_i^* whose only non-zero elements correspond to the balances for home and imported machines, and construction. The regional balances for region i can then be written simply as

$$x_i + x_i^* + e_i^d - y_i - m_i^d \leq \bar{w}_i .$$

5.8. Utility Functions

The utility functions have the form

$$U(x,L) = U^*(x) - U^{**}(L)$$

$$= \sum_k x_k - \sum_k W_k(x_k - \gamma_k - \beta_k \sum_h (x_h - \gamma_h)) - \sum_j V_j^L(L_j)$$

where W_k and V_j^L are convex non-negative functions going through the origin.

---

[22] The same remark as in footnote 21 applies for migrations.

The first component $U^*(x)$ is a GEM function the first term of which is linear. This implies no loss in generality, since it is always possible to find an increasing transformation of the utility function which is linear on the Engel curve $\gamma_k + \beta_k(\sum\limits_h x_h - \gamma_h)$.

The introduction of terms reflecting migration in primary producing activities is not usual. It accounts for an aspect of economic mechanisms which is very apparent to observers of reality, though it is usually disregarded in theoretical analysis. Primary producing activities are usually widely separated geographically, so that the labor movements in and out of these areas involve substantial psychological costs which merit recognition, as much as the satisfaction associated with the consumption of goods. This is a way of reflecting the fact that these activities face upward sloping supply curves for labor.

In the model, divergences between primary sectors' rates of growth and the overall growth of the economy are used as an indicator of these forced migrations.

The GEM function is represented in the model by a linear programming approximation. The consumption part of the function $U^*(x)$ is given by the following system

$$U^*(x) = \sum\limits_k x_k - \sum\limits_s \sum\limits_k w_k^s (x_k^{s+} + x_k^{s-})$$

subject to

$$x_k^{s+} \leq \bar{x}_k^{s+}$$

$$x_k^{s-} \leq \bar{x}_k^{s-}$$

$$x_k - \beta_k \sum\limits_h (x_h - \gamma_h) - \gamma_k = \sum\limits_s (x_k^{s+} - x_k^{s-})$$

where, since $\sum\limits_{k} \beta_k = 1$, $\sum\limits_{k} \sum\limits_{s}(x_k^{s+} - x_k^{s-}) = 0$.

The migration part of the function is:

$$(1) \qquad U^{**}(L) = \sum_{j=1}^{5} \sum_{s} \omega_j^{s\ell}(L_j^{s+} + L_j^{s-})$$

subject to

$$(2) \qquad L_j^{s+} \leq \overline{L}_j^{s+}$$

$$(3) \qquad L_j^{s-} \leq \overline{L}_j^{s-}$$

$$(4) \qquad z_{j_f} + z_{j_m} + z_{j_p} - \varepsilon_j P = \sum_s (L_j^{s+} - L_j^{s-}) \; ; \quad j = 1, \ldots, 4$$

$$(5) \qquad z_{j_1} + z_{j_2} - \varepsilon_j P = \sum_s (L_j^{s+} - L_j^{s-}) \; ; \quad j = 5$$

$$(6) \qquad P = \sum_{j=1}^{4}(z_{j_f} + z_{j_m} + z_{j_p}) + \sum_{j=5}^{15}(z_{j_1} + z_{j_2}) \; .$$

For the sake of simplicity, the model does not contain a full accounting of labor migrations. Equations (4), (5) and (6) represent these flows by proxies equal to the divergences between the growth of primary producing activities and the average growth of total production P .

5.9. Import and Export Tariffs

As explained in Section 2, the representation of a tariff ridden equilibrium requires that the tariff revenue net of subsidies of each country be deducted from the corresponding utility function. These revenues should be evaluated at say $\overline{p}$, the equilibrium prices, corrected for exchange rates, as explained in Section 2.3.

Import tariffs τ_k^m are taken as <u>ad valorem</u> tariffs; the resulting income is therefore $\sum\limits_k \tau_k^m \bar{p}_k m_k$. There are export tariffs only for oil, and they are thought of as specific. The tariff terms in the objective functions are therefore

$$t* = \sum_k \tau_k^m \bar{p}_k m_k + \tau_6^e e_6 \ .$$

The export tariff on oil accounts for the considerable taxes and royalties which are levied by governments of oil rich countries. It may also, to a certain extent, reflect the above-average profit margins which oil companies are able to earn as a result of monopolistic collusion.

6. The Middle East Oil Producers Submodel

GEM was constructed before the oil crisis, at a time when oil producers were less at the center of attention than today. The Middle Eastern oil producing countries are accordingly represented by a very simple set of equations since, from the point of view of the world economy, they appeared principally a reliable source of oil. Their role is much more important and complex today; but the path of their development is still far from clear, and it is not yet easy to see what model would adequately represent their economies.

The economy of this zone is supposed to consist of closely inter-linked industries: oil, and industries whose growth depends on the income generated by oil exports. It is thus oil which generates the forward momentum of the whole region.

All goods are aggregated into two commodities; oil, and an aggregate good used mainly locally. Oil output is not constrained by initial

capacity resources; costs of production are extremely low, and (at least at the time when the model was built) the capital required to produce more crude oil could be easily supplied by the international companies. Unlike other processes in developing countries oil production was therefore not subjected to a capacity constraint and depends only on demand.

The expansion of the non-oil modern sector depends on the initial productive capacity, increased by the accumulation of capital made possible by oil royalties and by saving in that sector. A small part of the accumulated domestic capital is, however, absorbed by investments in housing and infrastructure which are required for oil production but not financed by the oil companies.

The economy is thus described by the following constraints where the subscripts 6 and 0 refer to oil and other products.

The oil production balance is

$$-y_6 + e_6 \leq \bar{y}_6 \; .$$

Capacity in the non-oil sector is given by

$$y_0 \leq (\sigma_6 - \alpha)y_6 + \sigma_0 y_0 + \bar{y}_0 \; .$$

This expression reflects the fact that production is bounded upwards by the initial capacity $\bar{y}_0$, increased by the capacity which can be created using the savings generated by the oil and non-oil sectors (coefficients σ_6, σ_0), net of the domestic capital used by the oil industry (coefficient α). The logic of this equation is discussed in Section 3.2.

Imports are related to activity levels in oil and non-oil production.

$$m_k = u_k^6 y_6 + u_k^0 y_0 \; .$$

The supply demand balance for the non-oil commodity is

$$x_0 = y_0 - \sum_{k \neq 6} e_k$$

and welfare in that region is simply:

$$U = x_0 .$$

The model finally takes account of heavy export duties on oil, representing royalties, taxes, and to a certain extent the high profit margins made possible by the operation of the cartel. These appear as a negative term $\tau_6 e_6$ in the objective function and are treated as specific.

The tariff ridden welfare function is accordingly

$$U - \tau_6 e_6 = x_0 - \tau_6 e_6 .$$

7. Developed Countries Submodel

7.1. Import and Export Sets

Import processes are defined for every commodity originating from every developing region. These activities directly purchase on the international markets. The import set is the non-negative orthant.

Export processes are defined in the same way. They sell directly to international markets. The export set is also the non-negative orthant.

7.2. The Trade Welfare Function

As has been explained, developed countries are represented by a trade welfare function and not by a full-blown model. This implies that their production and consumption sets need not be specified

explicitly; the trade welfare function, however, depends parametrically on initial resources of non-traded goods.

The coefficients are chosen in a way which ensures that generated demands functions have income and price elasticities consistent with a priori values suggested by econometric studies and other data.

The underlying non-linear trade welfare function consists of two parts:

      (a)   a sum of GEM welfare functions for imports of each commodity (except for temperate foodstuffs for which net imports are considered). This sum enters the function with a plus sign.

      (b)   a sum of exports of developed countries, with a minus sign.

Separability and additivity seems a reasonable assumption for the broad commodity groups considered in the model. The strong aggregation implies that products which do interact--e.g. copper and aluminum--are included in the same aggregate commodities. It makes sense, therefore, to build up the trade welfare function by adding components for the different commodities imported and exported.

For exports (with the exception of temperate foods) it seemed safe to assume that, to domestic consumers, the marginal utility of exported goods did not depend on the volume exported. For almost all goods, sales from developed to developing countries are a very small fraction of the available supply; they consist mainly of manufactured goods which have highly elastic long-term cost functions. The corresponding components of the welfare functions are thus simply minus the sum of exports of zone

o : $-\sum_k e_{ok}$.

For traditional commodity imports of developed countries--and for their exports of temperate foodstuffs--the constant marginal utility

assumption was not appropriate. This judgment is based on the following reasons:

(a) These commodities represent, in most instances, a substantial share of the supply available to domestic users.

(b) Competing producers in developed countries operate in conditions of diminishing returns.

(c) These commodities have no close substitutes, so that their direct or derived demand is quite inelastic.

The products of different exporters of the same commodity are not perfectly substitutable. Partly, this is the result of the strong aggregation of the model; the "commodities" considered are not homogeneous, and the model must reflect a complex competitive mechanism in which a reduction of the average export prices of, say, Asian exports of non-ferrous metals affects tin prices more than copper prices. Even with perfect substitution for each metal, so that prices of each metal moved together in all regions, index weighting would lead to a drop of the average export prices of non-ferrous metals exported by Asia as compared with the metals exported by other regions. To reflect this, the model should treat aggregate commodities as imperfectly substitutable, even if this is not true of each of their components. The fact that, even narrowly defined, commodities cannot substitute for each other fully in all uses only strengthens the argument of course.

This leads to including for imports of each commodity k a GEM trade welfare component. We first discuss the relation between utility and imports; the g_o and tariff terms are discussed later. If

$m_{ok} = \sum_i m_{iok}$ (we restore here the zone subscript i), each term is written

$$V_{ok}(m_{ok}, g_o) - \sum_i W_{iok}(m_{iok} - \mu_{iok}m_{ok}) - \tau_{ok}^m m_{ok} .$$

As implied by the general specification of GEM functions

(a) the functions V_{ok} and W_{iok} are concave and convex non-negative functions respectively;

(b) the V_{ok} are increasing functions; the W_{iok} go through the origin.

The trade welfare components for individual commodities are not separable functions of imports from different regions--separability would indeed violate common sense here. The case of perfect substitutability between goods of different origins can be covered by making the W_{iok} functions identically equal to zero.

The preferred import baskets, when prices are unity, lie along the ray $m_{iok} = \mu_{iok}m_{ok}$. This ray has the same theoretical significance as the Engel curves of the utility functions described earlier. It is noticed, however, that unlike the Engel curve, the import distribution ray for each type of commodity is a straight line which goes through the origin; V_{ok} is strictly concave, marginal utility decreases along it.

To understand the g_o term, it is useful to refer to the theoretical discussion of trade welfare functions in Section 3.1.2. It was made clear there, that this function is a derived and not a fundamental data of the problem: it is obtained as the result of a partial optimization with respect to the supply-demand balances for non-traded goods. The trade welfare function therefore depends implicitly on initial resources of non-traded goods (capital stocks and the like), and on the corresponding parts of the production and consumption sets of developed countries.

This dependence is, in principle, very complex. In practice, all that is possible, is to reflect rather crudely the impact of changes of domestic resources and production functions on the trade welfare function. The term g_o in this function is used as a general proxy for the impact of technical progress on production sets and of capital accumulation on initial resources of non-traded goods.[23]

The tariff terms finally reflect the theoretical discussion of Section 2. For the sake of simplicity, tariffs are assumed to be specific rather than _ad valorem_. As--except for temperate agricultural goods-- these tariffs are not very high, it did not seem that the gain in accuracy would have been worthwhile. For temperate foodstuffs an adequate description of the very complex system of import regulations, which protects developed countries' markets, was in any case not possible within the simplistic framework of the model.

7.3. Linearization of the Developed Countries Submodel

The underlying tariff ridden non-linear welfare function is therefore (we drop the zone o index to lighten notations; i is the partner country index)

$$U - \sum_k \tau_k^m m_k = -\sum_k e_k + \sum V_k(m_k, g_0) - \sum_k \tau_k^m m_k$$

$$+ \sum_k \sum_i W_{ik}(m_{ik} - \mu_{ik} m_k) .$$

---

[23] Likewise, it would be possible to reflect the impact of such technical discoveries as a breakthrough in nuclear electricity generation, or a good harvest, by changing specific import components of the trade welfare functions.

This function is represented in the model by the linear approximation

$$
(1) \qquad U - \sum_k \sum_s \tau_k^m m_k^s = -\sum_k e_k + \sum_k \sum_s \omega_k^s m_k^s - \sum_k \sum_s \tau_k^m m_k^s
$$

$$
- \sum_k \sum_i \omega_{ik}(m_{ik}^+ + m_{ik}^-) + \sum_k \sum_i \omega_{ik}^o \mu_{ik}^o g_o
$$

$$
(2) \qquad m_{ik} - \mu_{ik} \sum_{i'} m_{i'k} = m_{ik}^+ - m_{ik}^- + \mu_{ik}^o g_o
$$

$$
(3) \qquad \sum_i m_{ik} - \sum_s m_k^s = 0
$$

$$
(4) \qquad m_{ik}^+ \leq \overline{m}_{ik}^+
$$

$$
(5) \qquad m_{ik}^- \leq \overline{m}_{ik}^-
$$

$$
(6) \qquad m_k^s \leq \overline{m}_k^s
$$

where it is seen that if $\sum_{i'} \mu_{i'k} = 1$ and $\sum_{i'} \mu_{i'k}^o = 0$, then $\sum_{i'} (m_{i'k}^+ - m_{i'k}^-) = 0$; notice that g_o is an exogenous variable.

The constraints (3) and (6) together with the second objective function term correspond to the term $V_k(m_k, g_o)$ of the trade welfare function; while (2), (4), (5) and the fourth objective function terms represent $\sum_k \sum_i W_{ik}(m_{ik} - \mu_{ik} m_k)$. The export term is linear in the original GEM function and is unchanged in its linearized approximation.

8. The International Balances

Since commodities of a given type exported by different regions are considered as different commodities, it is necessary to ensure balance between supply and demand of every commodity exported by each country. These balances are simply (i is the exporting country; i' the importing country):

$$e_{ik} \geq \sum_{i'=0}^{4} m_{ii'k} \; .$$

It is useful to recall that the $m_{ii'k}$ have been defined as follows (see Sections 5.5 and 7.1)

$$m_{ii'k} = \mu_{ii'k} z_{i'k}^{m} \; , \quad i' = 1, \; \ldots, \; 4$$

$$m_{iok} = \mu_{iok} \sum_{i'} m_{i'ok} + m_{iok}^{+} - m_{iok}^{-} + \mu_{iok}^{o} g_{o}$$

for all traded goods k and exporting countries i .

9. The Pricing Mechanism and the Balances of Payments

Understanding price determination in GEM is necessary for two reasons. First, prices are of course a fundamental component of general equilibrium. Second, import and export prices of regions determine their balances of payments, around which our three computational procedures are constructed. Only the second problem is of concern to us here.

We examine below, the dual at the equilibrium point of the Negishi welfare optimum, a concept which is discussed in detail in [5]. It is enough to recall here that at such an (equilibrium) optimum, balance of

payments constraints are satisfied even though they are not included in the formulation of the problem; and that the utility functions of agents are multiplied in the objective function by welfare weights α_i determined by our solution algorithms. In solving GEM and in view to normalize, the welfare weight of the developed world α_0 was set to 1 .

9.1. Marginal Utility and International Prices. Developed Countries

The marginal utility of imports and exports implied by the trade welfare function of developed countries can be defined along the lines of sections 4.2.2, 7.1 and 7.2.

We first note that the marginal utilities of exported goods (other than temperate foodstuffs) is constant and equal to 1 : nothing more need be said about them. It is trivial to show that the corresponding export prices are equal to 1 also.

For imports we can investigate any one of the groups of objective function terms and constraints defined for each commodity imported (and for net temperate foodstuffs). The mathematical program which defines the marginal utilities of imports of developed countries is (the dual variables associated to the constraints are written at the right of each constraint):

(1)
$$U(m^*) - \sum_k \sum_s \tau_k^m m_k^s = \max \; -\sum_k e_k + \sum_k \sum_s (\omega_k^s - \tau_k^m) m_k^s$$

$$- \sum_k \sum_i \omega_{ik}(m_{ik}^+ + m_{ik}^-) + \sum_k \sum_i \omega_{ik}^o \omega_{ik}^o g_o$$

subject to

(2) $\qquad m_{ik} - \mu_{ik} \sum_{i'} m_{i'k} = m_{ik}^+ - m_{ik}^- + \mu_{ik}^o g_o$ $\qquad (w_{ik})$

(3) $\qquad -\sum_i m_{ik} + \sum_s m_k^s = 0$ $\qquad (w_k)$

(4) $\qquad m_{ik}^+ \leq \bar{m}_{ik}^+$ $\qquad (w_{ik}^+)$

(5) $\qquad m_{ik}^- \leq \bar{m}_{ik}^-$ $\qquad (w_{ik}^-)$

(6) $\qquad m_k^s \leq \bar{m}_k^s$ $\qquad (w_k^s)$

(7) $\qquad m_{ik} \leq m_{ik}^*$ $\qquad (v_{ik})$.

For each k , the program decomposes into independent subproblems of the above form. In each of these, the dual variable v_{ik} of the constraint (7) is the marginal utility of good (i,k) --i.e. of commodity k exported by country i --net of the tariff τ_k .

The dual constraints are

(8) $\qquad w_k + w_k^s \qquad\qquad \geq \omega_k^s - \tau_k^m$ $\qquad (m_k^s)$

(9) $\qquad w_{ik}^+ - w_{ik} \qquad\qquad \geq -\omega_{ik}$ $\qquad (m_{ik}^+)$

(10) $\qquad w_{ik}^- + w_{ik} \qquad\qquad \geq -\omega_{ik}$ $\qquad (m_{ik}^-)$

(11) $\qquad w_{ik} - \sum_{i'} \mu_{i'k} w_{i'k} - w_k + v_{ik} \geq 0$ $\qquad (m_{ik})$.

In the dual of the full GEM model, the constraints which correspond to the activities m_k^s , m_{ik}^+ , m_{ik}^- , m_{ik} are identical to the constraints defined above, except that v_{ik} is now the dual variable

of the international balance constraints for commodity (i,k) . The same

algebra as in Section 4.2.2 shows that if good (i,k) is imported, its

marginal utility in developed countries equals its world price plus the

import tariff.

9.2. Marginal Utility and International Prices. Latin America, Asia, and Africa

The world export price of commodity (i,k) may likewise be re-

lated to marginal utility and to prices of imported goods in Latin America,

Asia and Africa by considering the dual constraints corresponding to the

export, import and consumption activities. The reasoning is merely sketched

out:

(a) As in the preceding section, it can be shown by considering

the dual constraints of the consumption activities x_k ,

x_k^{s+} and x_k^{s-} , that the product of the shadow welfare weight

by the marginal utility of a good k , consumed in positive

quantities equals a weighted average of the prices of the

home good and of the corresponding imported good.

(b) The dual constraint of the import process shows that the

price of the imported good (if imported) equals a weighted

average of the corresponding international prices, plus the

tariff levied.

(c) The dual constraint of the export process shows that the ex-

port price of a good, if exported, equals the domestic price

of the home good, increased by the shadow cost of the export

capacity used, and by the export tariff if there is one.

9.3. The Balances of Payments

We are now ready to handle the problem of the balances of payments.

Denote by v_{ik} the international price of commodity k , country i ;

then, the balance of payments constraint is simply

$$\sum_k v_{i'k}(\sum_{i'} m_{ii'k} - e_{ik}) \leq A_i \qquad (i = 1, \ 2 \text{ and } 3)$$

where A_i is the amount of aid granted to country i (given exogenously).

APPENDIX 1:

List of Commodities Represented in the Model[24]

Latin America

    101. Non-competing tropical foods

    102. Competing tropical foods

    103. Temperate foodstuffs

    104. Agricultural raw materials

    105. Mining and ores

    106. Oil

    107. Processed food

    108. Textiles and clothing

    109. Chemicals and fertilizers

    110. Non-metallic products

    111. Basic metals

    112. Machinery

    113. Other industrial goods

    114. Construction

    115. Services

    116. Urban labor

    117 to 131. Production capacities for "home goods" 101 to 115

    132 to 144. Production capacities for import substituting goods

              101 to 113

    145 to 157. Production capacities for exported goods 101 to 113

    158 to 170. Imported goods 101 to 113

    171. Undifferentiated capital good

---

[24] Some commodities are not produced, exported, imported or consumed by some regions. Complete lists are included in the Appendix on data.

Asia - Africa

The numbering is the same as for Latin America, except that it starts with 201 for Asia and 301 for Africa.

Middle Eastern Oil Producers

400. Non-oil

406. Oil

417. Production capacity for non-oil

International Markets (and Developed Countries)

1101 to 1113. Exports of Latin America

1201 to 1213. Exports of Asia

1301 to 1313. Exports of Africa

1401 to 1413. Exports of the Middle East

1501 to 1513. Exports of developed countries

APPENDIX 2:

Notation Used in GEM WT

Variables and constants usually appear with subscripts and superscripts.

Subscripts: — i (or i') refers to the country and takes values 0
to 4 . To lighten notation, it is usually omitted when
it is clear from the context that we deal with a specific
country.

— j_1 and j_2 are used for processes producing home and
import substitution goods.

— j_f, j_m and j_p are used for agricultural processes;
the first two are used for processes producing on fertile
and marginal lands; the third refers to import substitut-
ing commodities.

— k (or h) is used for commodities ($k = 1, 2, ..., 15$) .

Superscripts: — s refers to the sth linear piece of an approximation
of a nonlinear function.

Variables: are usually represented by small letters, with some exceptions,
when confusion could have been possible.

A_i : aid granted to country i

e_i : export vector, country i

e_{ik} : export of country i , commodity k

g_o : parametric variable related to growth in developed countries

i_{j_1} : capacity producing process, home goods, except agriculture

$i_{j_2}^s$: capacity producing process, step s , import substituted
goods, except agriculture

i_{j_f} : capacity producing process, agriculture on fertile lands

i_{j_m} : capacity producing process, agriculture on marginal lands

$i^s_{j_p}$: capacity producing process, step s , agriculture import substituted goods

$i^*_{j_f}$: capacity producing process, increasing area of fertile lands

i^e_k : capacity producing process, exports

K_{12} , K_{14} : capital accumulation in construction and machinery, solution of the near equilibrium, and exogenous in the equilibrium model

L^{s+}_j , L^{s-}_j : positive or negative deviation of production from "normal" level, step s , primary commodities 1 to 5

m^s_{ok} : imports of developed countries, step s , commodity k

m^+_{iok} , m^-_{iok} : positive or negative distortion from the "normal" import pattern of the developed countries, imports from country i , commodity k

m_i : import vector, country i

$m_{ii'k}$: import of country i' from country i , commodity k

$m_{ik} = \sum_{i'} m_{i'ik}$: import of country i , all origins, commodity k

P : "total production"

$\bar{p}_k$ (or $\bar{q}_k$) : international price, commodity k

t^*_i : tariff revenue, country i

u_{ik} : domestic price in country i , commodity k

v_{ik} : international price of commodity k exported by i

w : marginal utilities (dual variables associated with constraints relative to the utility functions)

x_i : consumption vector, country i

x_k : consumption, commodity k

x_k^{s+} , x_k^{s-} : positive or negative distortions from the "normal" consumption level, step s , commodity k

x_k^s : consumption, step s , commodity k

x_k^* : level of sales to investment, solution of the near equilibrium, and exogenous in the equilibrium model ($k = 12$, 14)

y_i : production vector, country i

y_k : production, commodity k

z_{j_1} : production process, home goods, except agriculture

$z_{j_2}^s$: production process, step s , import substituted goods, except agriculture

z_{j_f} : production process, home goods, agriculture on fertile lands

z_{j_m} : production process, home goods, agriculture on marginal lands

$z_{j_p}^s$: production process, step s , import substituted good in agriculture

z_k^c : consumption process, commodity k

z_k^e : export process, commodity k

z_k^m : import process, commodity k

<u>Parameters</u>: are usually represented by greek letters, with some excep-
tions. We only give in this list the most important parameters:

α_i : welfare weight, country i

a_{kj} : input (negative) or output (positive) coefficient,
commodity k , process j

κ_j : capital coefficient, process j

ℓ_j : labor coefficient, process j

$\mu_{ii'k}$: import coefficient of country i , imports of i' ,
commodity k

τ^e_{ik} , τ^m_{ik} : export or import tariffs, country i , commodity k

ω : coefficients appearing in the utility functions

<u>Right Hand Side Constants</u>: are represented by upper barred small letters.

APPENDIX 3:

Equations of the Model

1. Submodels for Latin America, Asia, and Africa

(1) Input-output goods balances (1 to 15)

$$\lambda_k z_k^c + x_k^* + z_k^e - \sum_{j=1}^{4} (a_{kj_f} z_{j_f} + a_{kj_m} z_{j_m} + a_{kj_p} z_{j_p}) - \sum_{j=5}^{15} (a_{kj_1} z_{j_1} + a_{kj_2} z_{j_2}) - z_k^m \leq 0$$

$$k = 1, \ldots, 15$$

$$x_k^* = 0 \quad \text{for all} \quad k \quad \text{except 12 and 14}$$

$$z_6^e = z_{6_1}$$

(2) Urban labor balance (16)

$$\sum_{j=5}^{15} (\ell_{j_1} z_{j_1} + \ell_{j_2} z_{j_2}) - \sum_s z_{16}^s \leq 0$$

(3) Balances for production capacities of home goods (17 to 31)

$$z_{j_m} - i_{j_m} - \overline{i}_{j_m}^o \leq 0 \qquad j = 1, \ldots, 4$$

$$z_{j_f} - i_{j_f} - \overline{i}_{j_f}^o \leq 0 \qquad j = 1, \ldots, 4$$

$$z_{j_1} - i_{j_1} - \overline{i}_{j_1}^o \leq 0 \qquad j = 5, \ldots, 15$$

(4) Balances for production capacities of import substituted goods (32 to 44)

$$z_{j_p} - \sum_s i_{j_p}^s \leq 0 \qquad j = 1, \ldots, 4$$

$$z_{j_2} - \sum_s i_{j_2}^s \leq 0 \qquad j = 5, \ldots, 13$$

(5) Balances for export capacities (45 to 57)

$$z_k^e - i_k^e - \overline{i}_k^e \leq 0 \qquad k = 1, \ldots, 13$$

(6) Import balances (58 to 70)

$$(1 - \lambda_k)z_k^c + \sum_{j=1}^{4} [u_{j_f} z_{j_f} + u_{j_m} z_{j_m} - (1 - u_{j_p})z_{j_p}] + \sum_{j=5}^{13} [u_{j_1} z_{j_1} - (1 - u_{j_2})z_{j_2}] - z_k^m \leq 0$$

$$k = 1, \ldots, 13$$

(7) Undifferentiated capital balance (71)

$$\sum_{j=1}^{4} (\kappa_{j_f} i_{j_f} + \kappa_{j_m} i_{j_m} + \sum_s \kappa_{j_p}^s i_{j_p}^s + \kappa_{j_f}^* i_{j_f}^*) + \sum_{j=5}^{15} (\kappa_{j_1} i_{j_1} + \sum_s \kappa_{j_2}^s i_{j_2}^s) + \sum_s \kappa_{16}^s z_{16}^s$$

$$+ \sum_{k=1}^{13} \kappa_k^e i_k^e - u_{12}K_{12} - u_{14}K_{14} \leq 0$$

(8) Linearization bounds

$$i_{j_p}^s \leq \overline{i}_{j_p}^s \qquad j = 1, \ldots, 4$$

$$i_{j_2}^s \leq \overline{i}_{j_2}^s \qquad j = 5, \ldots, 15$$

$$z_{16}^s \leq \overline{z}_{16}^s$$

(9) Purchases of good produced by country i

$$m_{ik} = \mu_{ik} z_k^m$$

(10) Agricultural constraints

$$z_{j_f} - i^*_{j_f} - \bar{z}_{j_f} \leq 0 \qquad j = 1, \ldots, 4$$

$$i^*_{j_f} - \bar{i}^*_{j_f} \leq 0 \qquad j = 1, \ldots, 4$$

$$z_{j_m} - \bar{z}_{j_m} \leq 0 \qquad j = 1, \ldots, 4$$

$$\sum_{j=1}^{2} (z_{j_f} - i_{j_f} - \theta_1 \bar{z}_{j_f}) \leq 0$$

$$\sum_{j=3}^{4} (z_{j_f} - i_{j_f} - \theta_2 \bar{z}_{j_f}) \leq 0$$

$$\sum_{\substack{j=1 \\ j \neq 3}}^{4} (z_{j_f} - i_{j_f} - \theta_3 \bar{z}_{j_f}) \leq 0$$

(11) Constraints relative to the utility function

$$x^{s+}_k - \bar{x}^{s+}_k \leq 0 \qquad\qquad k = 1, \ldots, 15$$

$$x^{s-}_k - \bar{x}^{s-}_k \leq 0 \qquad\qquad k = 1, \ldots, 15$$

$$x_k - \beta_k \sum_h (x_h - \gamma_h) - \gamma_k - \sum_s (x^{s+}_k - x^{s-}_k) = 0 \qquad\qquad k = 1, \ldots, 15$$

$$L^{s+}_j - \bar{L}^{s+}_j \leq 0 \qquad\qquad j = 1, \ldots, 5$$

$$L^{s-}_j - \bar{L}^{s-}_j \leq 0 \qquad\qquad j = 1, \ldots, 5$$

$$z_{j_f} + z_{j_m} + z_{j_p} - \epsilon_j P - \sum_s (L^{s+}_j - L^{s-}_j) = 0 \qquad\qquad j = 1, \ldots, 4$$

$$z_{j_1} + z_{j_2} - \epsilon_j P - \sum_s (L^{s+}_j - L^{s-}_j) = 0 \qquad\qquad j = 5$$

$$P - \sum_{j=1}^{4} (z_{j_f} + z_{j_m} + z_{j_p}) + \sum_{j=5}^{15} (z_{j_1} + z_{j_2}) = 0$$

$$t^* - \sum_k \tau^m_k \bar{p}_k m_k + \tau^e_6 e_6 = 0$$

(12) Tariff ridden utility function

$$U - t^* = \sum_k x_k - \sum_k \sum_s \omega_k^s (x_k^{s+} + x_k^{s-}) - \sum_{j=1}^{5} \sum_s (L_j^{s+} + L_j^{s-}) - t^*$$

2. Submodel for the Middle Eastern Oil Producers

(1) Oil production balance

$$-y_6 + e_6 - \bar{y}_6 \leq 0$$

(2) Non-oil production capacity balance

$$y_0 - (\sigma_6 - \alpha)y_6 - \sigma_0 y_0 - \bar{y}_0 \leq 0$$

(3) Balance for non-oil commodities

$$x_0 + \sum_{k \neq 6} e_k - y_0 = 0$$

(4) Imports

$$m_k - \mu_k^6 y_6 - \mu_k^0 y_0 = 0 \qquad k = 1, \ldots, 13$$

(5) Tariff ridden utility function

$$U - \tau_6 e_6 = x_0 - \tau_6 e_6$$

3. Submodel for the Developed World

(1) Constraints relative to the utility function   (i = 1, ..., 4 ;

k = 1, ..., 13)

$$m_{ik}^+ - \bar{m}_{ik}^+ \leq 0$$

$$m_{ik}^- - \bar{m}_{ik}^- \leq 0$$

$$m_k^s - \bar{m}_k^s \leq 0$$

$$\sum_i m_{ik} - \sum_s m_k^s = 0$$

$$m_{ik} - \mu_{ik} \sum_{i'} m_{i'k} - m_{ik}^+ + m_{ik}^- + \mu_{ik}^0 g_0 = 0$$

$$t^* - \sum_k \sum_s \tau_k^m m_k = 0$$

(2) Tariff ridden utility function

$$U - t^* = -\sum_k e_k + \sum_k \sum_s \omega_k^s m_k^s - \sum_k \sum_s \tau_k^m m_k^s - \sum_k \sum_i \omega_{ik}(m_{ik}^+ + m_{ik}^-)$$

$$+ \sum_k \sum_i \omega_{ik}^0 \mu_{ik}^0 g_0$$

4. International Balances

$$\sum_{i'=0}^{4} m_{ii'k} - e_{ik} \leq 0 \qquad i = 0, \ldots, 4 \; ; \quad k = 1, \ldots, 13$$

5. Balances of Payments

$$\sum_k v_{i'k}(\sum_{i'} m_{ii'k} - e_{ik}) \leq A_i \qquad i = 1, 2, 3 \; ; \quad i' = 0, \ldots, 4$$

6. Objective Function

$$\sum_{i=1}^{3} \alpha_i U_i + \alpha_0(U_0 + U_4) - \sum_{i=0}^{4} t_i^*$$

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