TESTING DYNAMIC SPECIFICATION IN SMALL SIMULTANEOUS SYSTEMS:

AN APPLICATION TO A MODEL OF BUILDING SOCIETY BEHAVIOR IN THE UNITED KINGDOM

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I. Introduction

The literature on estimating economic models from time-series data has revealed a considerable state of ferment in recent years. Conventional econometric approaches have been sharply questioned (see e.g. Granger and Newbold (1974)) and the forecasting ability of systems estimated by such methods has been criticized (see Cooper (1972)). Conversely, the main alternatives advocated, namely fitting Autoregressive Integrated Moving Average (ARIMA) equations (see Box and Jenkins (1970)) or the closely related "control theoretic" methods (see Astrom (1970)) are far from having unqualified support from practioners (see Chatfield and Prothero (1973) and Wall and Westcott (1974)). As argued by the last authors and by Nerlove (1972) a crucial function is left to the "Mutually supportive roles of theory and measurement."

The criteria which determine the selection of an ARIMA representation are roughly: maximize the likelihood of the model transformed to be stationary and invertible, basing the initial specification on autocorrelation analysis of the data such that the chosen form is the most parsimonious which does not violate diagnostic checks on the residuals being "white noise." For a multiple input process, considerable judgment is required.

These criteria are basically sensible but minimal. Slight problems are that some economic variables are not well represented by ARIMA models (e.g. tax rates) and there are other ways to achieve stationarity than blanket differencing. More importantly, emphasizing single output models distracts attention from the joint dependence of economic variables and by allowing almost no weight to theory or prior information accrued from other studies (relevant variables, signs and magnitudes of parameters, orders of lags, etc.) it exacerbates the difficulty of using short data series. Indeed, by placing

all the initial emphasis on the (albeit important) dynamics-autocorrelation interaction it may underplay the multivariate aspect which remains of primary concern to the economist. On the other hand, initially examining only the multivariate relationship is equally unhelpful and it seems imperative to consider both <u>ab</u> <u>initio</u> (see Hendry (1974) and Zellner and Palm (1974)). Even then it is essential that the analysis be conducted in the context of an appropriate economic theory.

Unfortunately, although interdependence and dynamics have been accorded equal importance in general theoretical analysis (see e.g. Hicks (1946)) the latter has not been successfully developed for operational use. Economic theories remain for the most part of the long run equilibrium-comparative statics variety and their practical value has also been queried (see Nerlove (1972)). In any case, this is precisely the component which differencing (to achieve stationarity) will remove!

Nevertheless this paper is an attempt to integrate a "long run equilibrium" theory of the behavior of Building Societies with a short-run control-theoretic model of their dynamic disequilibrium adjustment deliberately designed such that the equilibrium solution of the latter reproduces the former. The <u>theory</u> also suggests which data transformations may be useful (e.g. differencing, ratios, etc.). While simple, it is based on established institutional evidence and I have shown elsewhere that it includes three other extant models of Building Societies as special cases (see Hendry (1975b)).

Finally a sequential procedure for statistically testing the dynamics, autocorrelation and economic theory restrictions is developed and we jointly apply these to the model of O'Herlihy and Spencer (1972) to investigate whether the economic theory predicts any mis-specifications of variables or

dynamics, whether the statistical approach reveals any in practice (their system was estimated by two-stage least squares) and if so, how closely these coincide with our anticipations from the theory.

II. Building Societies

These non-profit making co-operative institutions (denoted BS) dominate the UK mortgage market with their 70% share, and they also hold 25% of private financial assets. Most are members of the Building Societies Association and they meet monthly to agree mortgage and deposit interest rates and act essentially as a cartel by quoting (for the most part) homogeneous figures. Mortgages have a standard life of 20-25 years while deposits can be withdrawn on demand. Changes in either interest rate affects the complete stock immediately. Since the availability and price of mortgages have been important politically BS have been subjected to considerable indirect government pressure and/or help at various times. Revell (1973) provides an excellent discussion of their structure and functioning.

Clayton et al. (1974) investigated the objectives Building Societies claim to set themselves, Ghosh and Parkin (1972) and Ghosh (1974) considered their Portfolio and Debt Behavior, O'Herlihy and Spencer (1972) built a formal econometric model of their behavior and estimated the existence and extent of credit rationing, the London Business School model of the U.K. economy contains a submodel of BS behavior (see Ch. 1 in Renton (1975)) and Riley (1974) has developed a model for the Treasury based on time-series analysis methods (see Astrom (1970)). As discussed in Hendry (1975b), none of these studies are fully satisfactory although all aid our understanding in different respects.

Since BS constitute a homogeneous sector, problems of aggregation and

measurement errors are likely to be of relatively less importance than usual.

Conversely, the Credit Rationing for which these instititions are infamous

is a serious additional complication, as it seems to be chronic (rather than

a transient "switching of regimes" type) and endogenous to the system. Thus

there may be no observations on the demand for mortgages schedule which need

not preclude estimating the effects of changes in demand but does rule out

using the conventional market clearing equation to close the system for obser-

vability. Further, as BS act as a group the equally conventional "Walrasian"

equation of prices adjusting in response to excess demand is not valid either.

Clearly "chronic" rationing entails an ability to control both price and quantity

(although "transient" rationing could be a slowly evolving disequilibrium),

and as an operational criterion we used: if an increase in the mortgage interest

rate induces a non-decrease in advances ceteris paribus then rationing was

previously present.

The lack of identification of mortgage demand was solved by O'Herlihy and

Spencer using "prior" subjective information (primarily based on newspaper

reports) formulated in dummy variables to proxy "mild" D(1) and "strict"

D(2) rationing. See Figure 1 (r_m = Mortgage Interest Rate, M = Mortgage

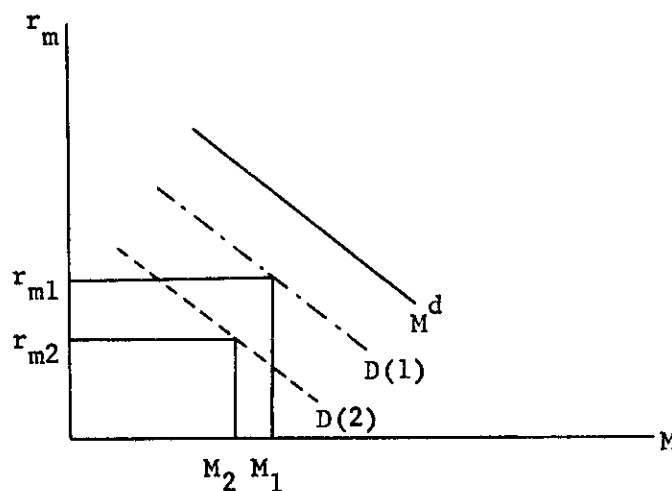Lending, M^d = Demand for Mortgages Perceived by BS). Criticisms of this

FIGURE 1

approach are its subjective nature, that it implies constant absolute magnitude effects of rationing, that the dummies are really endogenous and are not an "explanation" of rationing, hence being difficult to use for forecasting and that the method is highly specific to the BS sector. Our "guildeline" theory suggests that rationing is potentially present due to the inherent nature of how BS take their decisions. Specifically, it seems to be generated by the efforts of Building Societies to reconcile conflicting objectives rather than achieve a sole target such as (e.g.) profit maximization (compare Ghosh (1974)). To model this we adopt a "Managerial" type of theory based on the optimization of an objective function comprising several goals and subject to several (cost) constraints on the variability of the instruments to attain the targets. This is an explicitly dynamic "control theory" approach but only partly circumvents the awkward dilemma of postulating "desired" variables separately determined outside of the optimization framework. (See Theil (1964).)

We do not consider the overall portfolio allocation problem however, (see e.g. Ghosh and Parkin (1972)), but treat Other Assets as a single entity. A simple linear expository model of their long run behavior can be developed as follows.

Capital Account

(1) $A \equiv LA + OA + M \equiv D + R \equiv L$

where the symbols respectively denote Assets, Liquid Assets, Other Assets, Mortgages, Deposits, Reserves and Liabilities. We postulate that BS plan to relend a constant proportion θ_1 of Deposits as Mortgages (this being their primary function as co-operative institutions),

(2) $M^S = \theta_1 D$

and hold a constant proportion θ_2 of their assets in liquid form,

(3) $LA = \theta_2 A$

accept all deposits offered,

(4) $D = D^s$

maintain a constant ratio θ_3 of reserves to assets,

(5) $R = \theta_3 A$

and in equilibrium they realize their plans. From (1)-(5) we have

(6) $OA = (1 - \theta_2 - \theta_1(1 - \theta_3))A = \theta_4 A$

and

(7) $D = (1 - \theta_3)A$.

Finally, the private sector's equations for Mortgage Demand and Deposit Supply
are

(8) $M^d = f(y, r_m, PH)$ $\frac{\partial f}{\partial y} > 0$, $\frac{\partial f}{\partial r_m} < 0$

(9) $D^s = h(y, r_d, T, r_c, P)$ $\frac{\partial h}{\partial y} > 0$, $\frac{\partial h}{\partial r_d} > 0$, $\frac{\partial h}{\partial T} < 0$, $\frac{\partial h}{\partial r_c} < 0$, $\frac{\partial h}{\partial P} < 0$

where y is income, r_m , r_d , r_c are pre-tax mortgage, deposit and compe-
titive interest rates, PH and P are price indices for housing and all goods
and T is the tax rate. (BS pay tax on interest earnings for depositors
at a rate which differs slightly from the standard tax rate and they generally
quote after tax deposit interest rates: $r = r_d(1-T)$.)

Current Account

$$(10) \qquad r_m M + r_c OA \equiv E + r_d D + S$$

where $E = \theta_5 A$ are management expenses (including overheads does not materially affect the analysis) and S is the "surplus" planned to equilibrate the reserve ratio. Dividing by A and re-arranging:

$$(11) \qquad r_d = \theta_5 \theta_6 + \theta_1 r_m - \theta_4 \theta_6 r_c + \theta_6 S/A$$

where $\theta_6 = 1/(\theta_3 - 1) < 0$. Let $K_t = (R/A)_t$ then if $A_t \approx (1+g)A_{t-1}$ (say), $S/A = K_t - (1/1+g)K_{t-1}$ (zero in stationary equilibrium) and if BS do plan to have $K_t = \theta_3$, then for given g, r_c, K_{t-1} and θ_i (11) determines r_d as a function of r_m (the "mark-up") or *vice versa* on re-normalizing on r_m. Note that $\Delta(S/A)_t = -(1/1+g)\Delta K_{t-1}$ if $K_t = \theta_3$.

This leaves (e.g.) r_m to be determined and in the absence of a profit maximization motive, some other objective function must be postulated. The crucial point is that if BS do not equilibrate M^s and M^d then as a near monopoly which believes in keeping interest rates low (or is ordered to do so by the government), which does not seek to maximize S <u>and</u> which plans to achieve its balance sheet goals then they will ration credit to control both price and quantity. Two extreme possibilities are (i) the government dictates r_m on political grounds, (ii) BS do equilibrate M^s and M^d and so choose r_m as the implicit solution of

$$(12) \qquad f(y, r_m, PH) = \theta_1 h(y, \{\theta_5 \theta_6 + \theta_1 r_m - \theta_4 \theta_6 r_c + \theta_6 S/A\}, T, r_c) .$$

Either way, given r_m, (11) determines r_d, (9) gives D, (2) gives M and so on. Figure 2 summarizes the behavior of the model for an arbitrarily
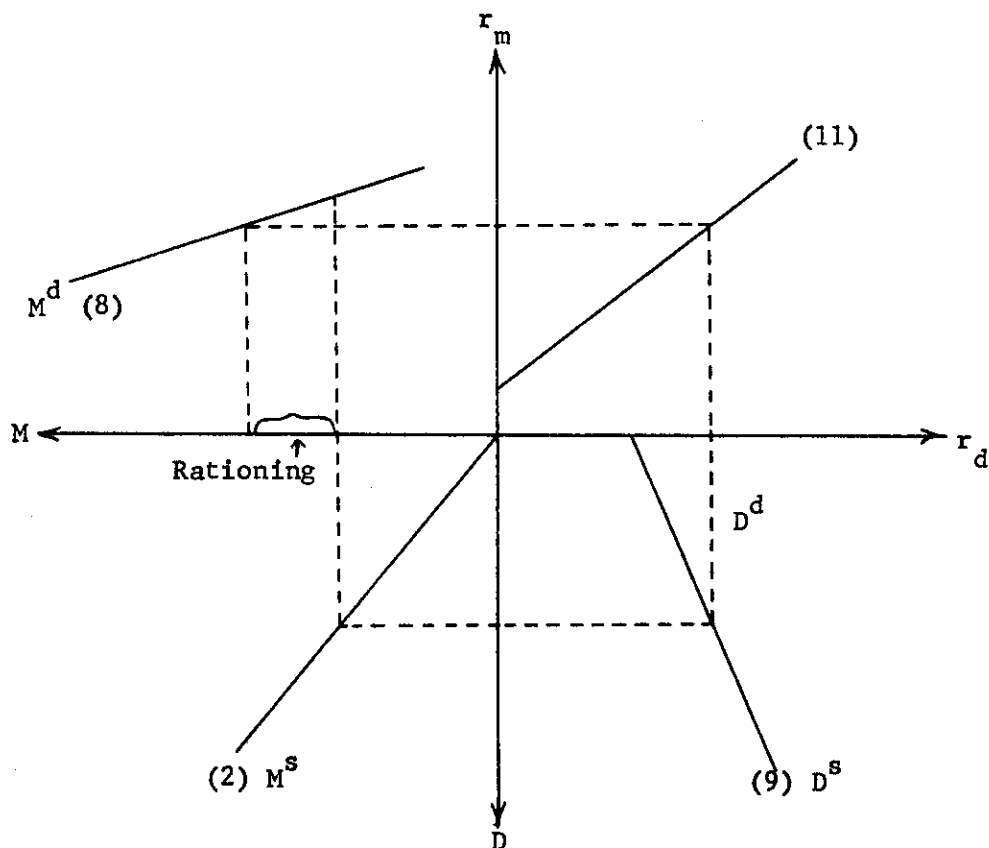
FIGURE 2

given low value r_m , and fixed y , r_c , PH , P , T . By abstracting from the dynamics we have implicitly taken

$$\Delta D = ND = 0 = NA = \Delta M \qquad (g = 0)$$

where ND and NA are Net Deposits and Net Advances and

(13)   GD = WD , GA = PR

where the symbols respectively denote Gross Deposits, Withdrawals, Gross Advances and Principal Repayments. Out of stationary equilibrium, to obtain

(14)   ND = GD - WD   and   NA = GA - PR

the four components in (13) must be modelled. Note, however, that in equilibrium

(15) $PR = \frac{1}{n}M$ if the Mortgage Life is n years.

In disequilibrium, r_m and GA will not be constant and there will be premature terminations as well as routine repayments.

Given this "long-run" framework we can formulate the objective function of BS and introduce dynamics at the same time. As argued above, their prime objective is to achieve the long-run aim of relending a relatively constant fraction of their expected total Deposits $\hat{D}_t$ as Mortgages, but now $\hat{D}_t \neq D_t$ due to the stochastic nature of cash flows, unforseen changes in y , r_c , etc. At the same time they pay some attention to satisfying "reasonable" mortgage applications and maintaining their long run reserve ratio while attaching adjustment costs to changes in their instruments (if only to avoid "Bang-Bang" control). Four other objectives/costs which do not affect the formal specification of the control rules but do alter the precise parameterization are: maximizing the growth rate of assets, minimizing lending rates, adjusting faster the more they are out of equilibrium and equilibrating interest rate differentials with the money market.

To facilitate comparison with existing models we take the objective function to be quadratic in the variables and to be one-period only. This is because BS do act as if they were myopic, but introducing $t^* > 1$ in (16) "simply" creates leads in the analysis and first-period Certainty Equivalence applies (see Hay and Holt (1975)). However, the control is also equivalent to ignoring terminal conditions and the stability properties of this are well known!

(I)   We measure the "Disutility" of "Disequilibrium" by $(M_t - \theta_1 \hat{D}_t)^2$, even though such costs are probably asymmetric away from equilibrium.

(II)   Since $M_t - M_t^d = NA_t - NA_t^d$ and $NA_t = Min(NA_t^s, NA_t^d)$ we take the dis-

utility of not supplying the required volume of housing finance to be

$\alpha_1 (NA_t - NA_t^d)^2$.

(III)   To achieve the desired reserve ratio requires the appropriate surplus,

and if θ_5 is constant, to model the BS making marginal adjustments

rather than re-evaluating the entire basis of the mark-up each period

we have; $\alpha_2 (\Delta r_{mt} - \frac{1}{\theta_1} \Delta r_{dt} - \theta_4 \theta_7 \Delta r_{ct} - \theta_7 \Delta (S/A)_t)^2$, on normalizing with

respect to r_m in (11), with $\theta_7 = 1/\theta_1 (1 - \theta_3)$. This could also be

interpreted as a cost of adjusting the differential.

(IV)   Inertia to avoid high frequency oscillations in NA_t and r_{dt} seems

desirable, but there are adjustment costs in any event arising from

processing costs and destabilizing influences on the housing market

for the former, and administrative costs (perhaps less so with computers)

and the creation of uncertainty for depositors concerning the latter.

Thus we use $\lambda_1 (\Delta NA_t)^2 + \lambda_2 (\Delta r_{dt})^2$.

The objective function which results to determine BS behavior is

(16)
$$c_t = \sum_{t=1}^{t^*} (1+r_c)^{-t} \{ (M_t - \theta_1 \hat{D}_t)^2 + \alpha_1 (NA_t - NA_t^d)^2 + \lambda_1 (\Delta NA_t)^2 + \lambda_2 (\Delta r_{dt})^2$$
$$+ \alpha_2 (\Delta r_{mt} - \frac{1}{\theta_1} \Delta r_{dt} - \theta_4 \theta_7 \Delta r_{ct} + \theta_8 \Delta K_{t-1})^2 \} \quad (\theta_8 = \theta_7/(1+g))$$

and BS seek to minimize this as a function of NA_t , r_{mt} and r_{dt} . Com-

pletely independent empirical support exists for formulating a myopic objec-

tive function (in which t^* denotes six-months) with conflicting aims and

emphasizing the goals of stable flows of funds, reserves in line with assets,

relending the maximum subject to achieving the desired liquidity ratio, attempt-

ing to satisfy the demand for housing finance and NOT maximizing profits.

This is provided by Clayton et al. (1974) who conducted a questionnaire survey of how the Building Societies Association perceived the role of BS in the housing market and they reached the conclusions noted in the previous sentence.

While c_t is not "homogeneous" in its arguments and it would be preferable to deflate M , D , and NA by A (say), we retain (16) for simplicity, with $t^* = 1$.

Minimizing c_t with respect to NA_t , r_{mt} , and r_{dt} yields the basic system (25), (27), and (28) closed by (23), (26), and (29)-(32), as follows.

$$(17) \qquad \frac{1}{2}\frac{\partial c_t}{\partial NA_t} = e_{1t} = (M_t - \theta_1 \hat{D}_t) + \alpha_1(NA_t - NA_t^d) + \lambda_1 \Delta NA_t$$

$$(18) \qquad \frac{1}{2}\frac{\partial c_t}{\partial r_{mt}} = e_{2t} = -\alpha_1\left(\frac{\partial NA^d}{\partial r_m}\right)(NA_t - NA_t^d) + \alpha_2(\Delta r_{mt} - \frac{1}{\theta_1}\Delta r_{dt} - \theta_4\theta_7\Delta r_{ct} + \theta_8\Delta K_{t-1})$$

$$(19) \qquad \frac{1}{2}\frac{\partial c_t}{\partial r_{dt}} = e_{3t} = -\theta_1\left(\frac{\partial \hat{D}}{\partial r_d}\right)(M_t - \theta_1\hat{D}_t) - \frac{\alpha_2}{\theta_1}(\Delta r_{mt} - \frac{1}{\theta_1}\Delta r_{dt} - \theta_4\theta_7\Delta r_{ct} + \theta_8\Delta K_{t-1}) + \lambda_2\Delta r_{dt}$$

Further

$$(20) \qquad \Delta M_t = NA_t$$

$$(21) \qquad \Delta D_t = ND_t$$

and

$$(22) \qquad \hat{D}_t = D_{t-1} + \widehat{ND}_t \ ,$$

where

$$(23) \qquad \widehat{ND}_t = \beta_1 + \beta_2(B)ND_t + \beta_3\Delta r_{dt} + \beta_4\Delta r_{ct} + \beta_5\Delta y_t \qquad (0 < \beta_2 \leq 1, \ 0 < \beta_3, \ \beta_4 < 0, \ \beta_5 > 0)$$

and $\beta_2(B)$ is a polynomial in the lag operator B . Note that (23) includes both rational expectations and ARMA models as special cases depending on what

values the BS use for the B_j. We obtain from these and (17),

$$(24) \quad NA_t = -\frac{1}{1+\lambda_1}(M_{t-1} - \theta_1 D_{t-1}) + \frac{\lambda_1}{1+\lambda_1}NA_{t-1} + \frac{\alpha_1}{1+\lambda_1}(NA_t^d - NA_t) + \frac{\theta_1}{1+\lambda_1}\widehat{ND}_t + \frac{\epsilon_{1t}}{1+\lambda_1}$$

or

$$(25) \quad NA_t = -\frac{1}{1+\lambda_1+\alpha_1}(M_{t-1} - \theta_1 D_{t-1}) + \frac{\lambda_1}{1+\lambda_1+\alpha_1}NA_{t-1} + \frac{\alpha_1}{1+\lambda_1+\alpha_1}NA_t^d$$

$$+ \frac{\theta_1}{1+\lambda_1+\alpha_1}\widehat{ND}_t + \epsilon_{1t} \, .$$

As required, in equilibrium (25) reproduces (2). From (16) we have
$0 < \lambda_1, \lambda_2, \alpha_1, \alpha_2 < 1$ if the first objective is the major one, and hence

$$\left(0 < \frac{1}{1+\lambda_1+\alpha_1}, \frac{\lambda_1}{1+\lambda_1+\alpha_1}, \frac{\alpha_1}{1+\lambda_1+\alpha_1}, \frac{\theta_1}{1+\lambda_1+\alpha_1} < 1 \right) \, .$$

From (18), assuming (possibly with additional lags)

$$(26) \quad NA_t^d = \delta_1 y_t + \delta_2(r_{mt}(1 - T_t) - \dot{P}_t) + \delta_3 PH_t + \delta_4 M_{t-1} + v_{1t}$$

$$(0 < \delta_1 \, , \ \delta_2 < 0 \, , \ \delta_3? , \ -1 < \delta_4 < 0)$$

we obtain (neglecting variations in T_t)

$$(27) \quad \Delta r_{mt} = \frac{1}{\theta_1}\Delta r_{dt} + \theta_4 \theta_7 \Delta r_{ct} - \theta_8 \Delta K_{t-1} + \frac{\alpha_1 \delta_2}{\alpha_2}(NA_t - NA_t^d) + e_{2t}$$

$$(1 < \frac{1}{\theta_1}, \ -\frac{\alpha_1 \delta_2}{\alpha_2} < 0 \, , \ \theta_4 \theta_7 > 0 \, , \ 0 < \theta_8)$$

Finally, from (19) and (23), and using (27) to eliminate Δr_{mt}

$$(28) \qquad \Delta r_{dt} = \frac{\theta_1 \beta_3}{\lambda_2}(M_t - \theta_1 \hat{D}_t) + \frac{\alpha_1 \delta_2}{\lambda_2 \theta_1}(NA_t - NA_t^d) + e_{3t} \qquad (0 < \frac{\theta_1 \beta_3}{\lambda_2}, \frac{\alpha_1 \delta_2}{\lambda_2 \theta_1} < 0)$$

Note that all three control equations depend on the excess demand for Mortgages. To close the system, an equation is required for D^s, which need not be (23), as the BS could be using an incorrect model. To allow for different lag responses between Deposits and Withdrawals, we used

$$(29) \qquad D_t^s = D_{t-1} + (GD_t^s - WD_t^s) + \text{interest credited } (I_t)$$

with $GD_t = GD_t^s$ and $WD_t = WD_t^s$, so that $ND_t^s = I_t + GD_t^s - WD_t^s = ND_t$.

$$(30) \qquad GD_t = \gamma_1(B)y_t + \gamma_2(B)r_{dt} + \gamma_3(B)r_{ct} + \gamma_4(B)\dot{P}_t + v_{2t}$$

$$(0 < \gamma_1 < 1, \ 0 < \gamma_2, \ \gamma_3 < 0, \ \gamma_4 < 0)$$

and similarly for WD_t. Finally, empirically

$$I_t \simeq \mu\left(\frac{r_{dt} + r_{dt-1}}{2}\right)\left(\frac{D_t + D_{t-1}}{2}\right).$$

Equivalently we have

$$(31) \qquad GA_t = NA_t + PR_t$$

where PR_t is composed of routine repayments and premature terminations. If monthly payments are held constant, an increase in r_{mt} will reduce PR_t; but for recently effected loans, this effect will hold even if the length of life of the mortgage is held constant. Increased income and/or house prices will probably increase premature repayments. Thus, allowing for (15), and with linearity, to explain changes in PR_t we propose (subject to additional lags)

(32) $\Delta PR_t = \phi_1 NA_{t-1} + \phi_2 \Delta y_t + \phi_3 \Delta r_{mt} + \phi_4 \Delta PH_t + \phi_5 (PR_{t-1} - \frac{1}{n}M_{t-1}) + v_{3t}$

$$(0 < \phi_1 < 1, \ 0 < \phi_2, \ 0 > \phi_3, \ 0 < \phi_4, \ \phi_5 < 0)$$

In equilibrium this reproduces (15). As argued in Hendry (1975b) however, dimensional analysis, homoscedasticity and stationarity all argue for logarithmic or ratio forms for all of these equations but we keep linearity for comparison with O'Herlihy and Spencer.

III. The Model of O'Herlihy and Spencer

The formulation of this is shown in Table IA (see Table II for four one-period ahead forecasts). The Gross Deposits equation is similar to (9) as is that for Withdrawals (compare (23)) and the Principal Repayments equation is (very roughly) $\Delta PR_t = \phi_1 \Delta GA_t$. Given (31) and (15) we can expect very considerable simultaneity between these and this is supported by the extremely large coefficient on GA_t .

The Gross Advances equation can be interpreted as (24) with $(GA_t - GA_t^d) = \xi(D(1), D(2))$ (see Figure 1), $\widehat{ND}_t = a(y/P)_t$ and with $(M_{t-1} - \theta_1 D_{t-1})$ omitted. Note that if this last term is reformulated in a stationary way as $[(M/D)_{t-1} - \theta_1]$ this is approximately equal to $((1 - \theta_1(1 - \theta_3)) - LQ_{t-1})/(1 - \theta_3)$ where $LQ = (A-M)/A$ which will be highly correlated with the form chosen by O'Herlihy and Spencer, namely

(33) $L_t^o = \frac{1}{2}(LQ_{t-1} - 15.5)/(19 - LQ_{t-1})(LQ_{t-1} - 12)$

as the denominator is always positive. Both the Mortgage and Deposit interest rate equations are formulated in levels; the former is made a geometric distributed lag of r so (27) predicts a number of omitted variables while the

latter depends negatively on L_t^o (see the previous sentence) and on K_t^o (a transform of K_t similar to L^o) such that the dependent variable could easily be made Δr (compare (28)). Given these theoretical predictions of mis-specification, we next develop a statistical theory for testing the formulation of the system.

IV. Statistical Testing of Dynamic Specification in Small Simultaneous Systems

The determination of an appropriate lag structure in dynamic models is still primarily treated as an empirical matter, generally resolved by experimenting with a set of lagged regressors using a relevant estimator (e.g. ordinary or two-stage least squares). This can yield very misleading results in view of the well known lagged dependent variable--autocorrelated error bias, especially in simultaneous equations systems (see Hendry (1975a)). However, an econometric approach which explicitly recognizes the interaction between the equation dynamics and the stochastic specification was developed by Sargan (1964), and can be generalized as follows.

Consider a model given by

$$(34) \qquad B\underset{\sim}{y}_t + C\underset{\sim}{z}_t + D\underset{\sim}{y}_{t-1} = \underset{\sim}{u}_t \quad \text{where} \quad \underset{\sim}{u}_t = F\underset{\sim}{u}_{t-1} + \underset{\sim}{\varepsilon}_t \, , \quad \underset{\sim}{\varepsilon}_t \sim NI(\underset{\sim}{0}, \underset{\sim}{\Sigma})$$

with restricted reduced form

$$(35) \qquad \underset{\sim}{y}_t = \underset{\sim}{\Pi}_1 \underset{\sim}{z}_t + \underset{\sim}{\Pi}_2 \underset{\sim}{y}_{t-1} + \underset{\sim}{v}_t$$

where $\underset{\sim}{v}_t = \underset{\sim}{\Phi}\underset{\sim}{v}_t + \underset{\sim}{\omega}_t$, $\underset{\sim}{\Pi}_1 = -B^{-1}\underset{\sim}{C}$ and $\underset{\sim}{\Pi}_2 = -B^{-1}\underset{\sim}{D}$, $\underset{\sim}{\Phi} = B^{-1}\underset{\sim}{FB}$ and transformed reduced form (eliminating the autocorrelation)

$$(36) \qquad \underset{\sim}{y}_t = \underset{\sim}{\Pi}_1 \underset{\sim}{z}_t + (\underset{\sim}{\Pi}_2 + \underset{\sim}{\Phi})\underset{\sim}{y}_{t-1} - \underset{\sim}{\Phi}\underset{\sim}{\Pi}_1 \underset{\sim}{z}_{t-1} - \underset{\sim}{\Phi}\underset{\sim}{\Pi}_2 \underset{\sim}{y}_{t-2} + \underset{\sim}{\omega}_t \, .$$

Equivalently, we have an <u>unrestricted</u> version of (35)

$$(37) \qquad \underset{\sim}{y}_t = \underset{\sim}{P}_1 \underset{\sim}{z}_t + \underset{\sim}{P}_2 \underset{\sim}{y}_{t-1} + \underset{\sim}{v}_t$$

where $\underset{\sim}{v}_t = \underset{\sim}{G}\underset{\sim}{v}_{t-1} + \underset{\sim}{w}_t$ and its transformation

$$(38) \qquad \underset{\sim}{y}_t = \underset{\sim}{P}_1 \underset{\sim}{z}_t + (\underset{\sim}{P}_2 + \underset{\sim}{G})\underset{\sim}{y}_{t-1} - \underset{\sim}{G}\underset{\sim}{P}_1 \underset{\sim}{z}_{t-1} - \underset{\sim}{G}\underset{\sim}{P}_2 \underset{\sim}{y}_{t-2} + \underset{\sim}{w}_t \ .$$

Finally, we could ignore the autoregressive restriction in (38):

$$(39) \qquad \underset{\sim}{y}_t = \underset{\sim}{Q}_1 \underset{\sim}{z}_t + \underset{\sim}{Q}_2 \underset{\sim}{y}_{t-1} + \underset{\sim}{Q}_3 \underset{\sim}{z}^0_{t-1} + \underset{\sim}{Q}_4 \underset{\sim}{y}_{t-2} + \underset{\sim}{e}_t$$

where $\underset{\sim}{z}^0_{t-1}$ are all the non-redundant regressors in $\underset{\sim}{z}_{t-1}$.

This leads to a sequential testing procedure for the various restrictions (see e.g. Anderson (1971), ch. 6.4). The symbol $\mathcal{L}_z$ denotes the log likelihood value when the error on the equation is z . The chosen order of testing is such that each hypothesis is investigated without conditioning on untested assumptions about a further hypothesis to be investigated i.e. from the <u>least</u> restricted hypothesis to the <u>most</u>. Despite this seeming very natural, most empirical work appears to proceed in the opposite direction and indeed investigators generally conduct tests <u>ONLY</u> within (34) and for $\underset{\sim}{F}$ either zero or diagonal (the latter being required for AIV to be valid). Appropriate technology does exist for conducting the required tests (see Hendry (1971)) and is not too expensive (see Hendry and Tremayne (1973)) Our sequence is:

H_a : the autoregressive restrictions on (38) is valid. If this is true then

$$(40) \qquad 2(\mathcal{L}_e - \mathcal{L}_w) \underset{A}{\sim} \chi^2_m \quad \text{for} \ m \ \text{regressors in} \ \underset{\sim}{z}^0_{t-1} \ .$$

If H_a is rejected then the system requires dynamic re-specification (go to

H_d). If H_a is not rejected proceed to test:

H_b : $\underset{\sim}{C} = 0$ in (38). If this is true then

(41) $2(\mathcal{L}_w - \mathcal{L}_v) \underset{A}{\sim} \chi^2_{n2}$ for n equations.

If H_b is rejected use "autoregressive" estimators, otherwise use "white noise"

estimators. Next we have:

H_c : $\underset{\sim}{\Pi}_1 = \underset{\sim}{P}_1$ (i.e. $\underset{\sim}{P}_1 = -\underset{\sim}{B}^{-1}\underset{\sim}{C}$, $\underset{\sim}{P}_2 = -\underset{\sim}{B}^{-1}\underset{\sim}{D}$). If this is true then

(42) $2(\mathcal{L}_w - \mathcal{L}_\psi) \underset{A}{\sim} \chi^2_N$ if H_b was rejected or

(43) $2(\mathcal{L}_v - \mathcal{L}_\nu) \underset{A}{\sim} \chi^2_N$ if H_b was not rejected

for N over-identifying restrictions. (Before proceeding from H_b to H_c

one could also test whether $\underset{\sim}{G}$ was diagonal: H_a and H_b assumed it unre-

stricted. If the diagonality refers to $\underset{\sim}{F}$, it could be tested after H_c .)

If H_c is rejected, the over-identifying restrictions are invalid, and the

exclusion restrictions in (34) require modification (or the underlying theory

is false). Finally if H_a is rejected one could attempt to re-specify the

dynamics by testing

H_d : $\underset{\sim}{Q}_{31} = \underset{\sim}{Q}_{41} = \underset{\sim}{0}$.

This is intended to denote testing rows and/or columns of the $\underset{\sim}{Q}_3$, $\underset{\sim}{Q}_4$ matrices

for significance and could be based on appropriate modifications of the methods

proposed in Multivariate Analysis (see e.g. Anderson (1958), ch. 8). For the

usual sample sizes and numbers of reduced form regressors in econometrics,

some "degrees of freedom" adjustments seem appropriate and those used below

are of the form:

$$(44) \qquad (T-K)^{-1}\hat{\underset{\sim}{W}}'\hat{\underset{\sim}{W}} \quad \text{for} \quad \hat{\underset{\sim}{\Omega}}_{W}$$

where $\underset{\sim}{W}' = (W_1, \ldots, W_T)$ and (for example)

$$(45) \qquad 2(1 - h/T)(\mathcal{L}_e - \mathcal{L}_v) \underset{A}{\sim} \chi^2_{(n^2+nm)}$$

for $h = K_1 + \frac{1}{2}(n + K_2 + 1)$ with $K = K_1 + K_2$ regressors in (39), $K_1 = n + k$, $K_2 = n + m$, and there are k exogenous variables.

Generalizing the analysis for longer lags in y_t in the initial formulation is straightforward; doing so for higher orders of autocorrelation is not as it raises potential non-nesting problems. However, even if n is small (3-6 say) the ARMA representations derived from (39) will have very high order lags.

Conversely, specializing the analysis to individual equations using the Autoregressive error generalization of Limited Information Maximum Likelihood, or its asymptotic equivalent Autoregressive Instrumental Variables (denoted AIV) yields the approach developed by Sargan (1964). Since this special case has already proved useful in revealing dynamic mis-specifications in empirical research (see e.g. Hendry (1974)) and is not expensive computationally, we first re-estimated O'Herlihy and Spencer by AIV to test (for each equation separately) the validity of the autoregressive restriction (H_a for a _structural_ equation), the significance of the autoregressive parameter (H_b for $n = 1$) and also the implicit hypothesis of zero residual autocorrelation of higher order (see Pierce (1971)). Taking account of this information we produced a revised system with no residual autocorrelations other than, where required, a valid first order autoregressive error, and the results are reported in Table IB.

The re-specification brings the system closer to our theoretical form
and improves its forecasting performance (other than r) but the forecast
tests indicate that significant mis-specifications remain (see Table II).
However, just re-estimating the original equations by AIV with a fourth order
autoregressive error (compare Wallis (1972)) produced an equivalent improve-
ment in the forecast accuracy as shown in Table II. We conclude that while
there is some benefit in using AIV and its related tests relative to two-stage
least squares, to yield consistent estimates all these individual equation
estimators rely on the validity of the untested hypotheses that $\underset{\sim}{F}$ is diagonal
and that both H_a and H_c are valid. The importance of this is that we
cannot really a priori expect these to hold and hence we are making the
"maintained" hypothesis have less prior plausibility than many of the "null"
hypotheses we are seeking to test--anticipating rejection!

Even when all these implicit hypotheses are valid, AIV remains inefficient
asymptotically compared to Full Information estimation (note the results in
Rotherberg (1973), ch. 5). Both arguments emphasize the potential advantages
of estimating and testing the system as a whole and although 54 observations
is hardly a large sample for a six equation model it seems worth examining the
practical value of the various tests applied to the original O'Herlihy and
Spencer specification.

The non-linearity relating GA and GA/PH precludes a direct applica-
tion of our analysis to their model, but if we revise it slightly (and quite
reasonably) to use PR/PH instead of PR so that all values are in real terms
then the model does become linear. The results for the various test statistics
are shown in Table IV.

TABLE IV

Asymptotic χ^2-Statistic Values

Equation	Unadjusted	Adjusted	df
40	76		48
41	75		36
42	204	122	56
45	151	41	84

((42) is adjusted using $K_1 = 4 + 6 + [22/6]$, $K_2 = K - K_1 = 9$.) Two points
of note are that the small sample size means that the degrees of freedom (df)
adjustment has a major impact on (45), and that the validity of the overidentify-
ing restrictions is clearly rejected (H_c) even if the results for H_a and
H_b are equivocal.

Apart from the equation for PR , the restricted reduced form of the
original model can be obtained by solution while the unrestricted reduced form
can be estimated directly. We have also done this subject to a different (but
similar) approximation from that used for Table IV, namely making PR depend
on $(GA/PH)_{t-1}$ instead of on GA_{t-1} in the reduced form. These unrestricted
and solved reduced form estimates (ignoring PR_t in the latter and GA_{t-1}
in both) are presented in Table V, and there are a number of major differences
between them. Most importantly, in the unrestricted the rationing dummies
are not significant in (GA/PH) , but are in (GD/P) , while L_t^o is signifi-
cant in (GA/PH) . Thus the dummies appear to represent unexpected downward
shifts in the supply of Deposits, and if the supply of Advances is NOT formu-
lated as a "processing" equation, the dummies naturally proxy a fall in Advances.
This conforms closely with the theory in Section II. It should be noted (when
interpreting the remaining coefficients) that r_{mt-1} and r_{t-1} are very

collinear, but many of the other coefficients in common between the two reduced forms have markedly similar values.

Conclusion

The statistical tests of the various implicit hypotheses (and such hypotheses are often made in time-series studies) revealed a number of mis-specifications in the BS model we have examined. The new estimates suggest appropriate ways of revising the formulation of the model and despite the "pre-test" problems in doing this we believe its justification is considerably enhanced by the existence of independent predictions from a theoretical analysis that such mis-specifications would indeed occur.

To efficiently estimate our own model requires a relatively complicated computer program generalizing the method in Hendry (1971) to systems with non-linear, cross-equation restrictions on the parameters. This has now been developed and we intend to report our results from it at a later date.

TABLE I

A. Original Model, as reproduced[1] (See O'Herlihy and Spencer (1972).)

(1) $GD/P^* = 39r_t - 10r_{bt} + 44T_t + .06y/P_t + .62GD/P_{t-1} - 684$ (s = 18.0)

(2) $WD/P_t^* = -23r_t + 9r_{bt} - 9T_t - .03y/P_t + .07D/P_{t-1} + 185$ (s = 11.5)

(3) $GA/PH_t^* = .72GA/PH_{t-1} + .02y/P_t - 16D(1) - 29D(2) - 6$ (s = 17.3)

(4) $PR_t^* = .20GA_t - .09GA_{t-1} + .72PR_{t-1} + 11$ (s = 3.9)

(5) $r_{mt} = .96r_t + .35r_{mt-1} + .58$ (s = .18)

(6) $r_t = -.06k_t^o - .16L_t^o + .93r_{t-1} + .31$ (s = .10)

B. Reformulated Model. (See Anderson (1974).)

(1) $GD/P_t^* = -11(r_b - r)_t + 105r_t - 69r_{t-1} + .08y/P_t - .04y/P_{t-1}$

$$+ 33T_t + .68GD/P_{t-1} - 554 \qquad (s = 18.2)$$

(2) $WD/P_t^* = 8(r_b - r)_t - .03y/P_{t-1} - 9T_t + .07D/P_{t-1} + 112$ (s = 9.6)

(4) $(PR_t - PR_{t-4}) = .17(GA_t - GA_{t-4}) - .06(GA_{t-1} - GA_{t-5})$

$$- 3(r_{mt} - r_{mt-4}) + .57(PR_{t-1} - PR_{t-5}) + 3 \qquad (s = 3.9)$$

(5) $r_{mt} = .66r_t + .55r_{mt-1} + .45 \qquad [\tilde{\rho}_1 = -.34]$ (s = .17)

(6) $(r_t - r_{t-1}) = -.03k_t^o - .18L_t^o + .04(r_{bt-1} - r_{bt-2}) + .05$

$$[\tilde{\rho}_1 = -.24] \qquad (s = .09)$$

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[1]Less four observations for forecasts. The sample size was 54 and s denotes the equation standard error. See Table III for the complete set of definitions. * denotes that seasonal dummy variables were included but are not reported. Instrumental Variable estimation was used where required. No respecification improving equation (3) was obtained, and ρ_1 denotes the first order autoregressive error parameter.

TABLE II

Forecast Errors

	GD/P			WD/P			GA/PH			PR			r_m			r		
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
1969 II	20	-14	-19	19	12	14	16	16	17	16	11	13	-.40	-.25	-.23	.39	.36	.37
1969 III	-15	-20	-38	38	26	18	3	3	3	8	2	4	-.31	-.27	-.30	-.04	-.03	.03
1969 IV	2	-6	-8	22	4	16	-17	-17	-15	7	0	1	-.03	.01	-.02	-.13	-.18	-.12
1970 I	20	14	13	16	8	-5	1	1	4	1	7	5	-.24	-.18	-.10	-.04	-.17	-.01

		GD/P	WD/P	GA/PH	PR	r_m	r
(A)	χ^2_4	3.2	19.1[+]	1.6	23.4[+]	9.4	18.6[+]
(B)	χ^2_4	2.5	9.9	1.6	12.3[+]	5.7	20.4[+]
(C)	χ^2_4	6.4	9.6	1.6	18.0[+]	4.8	16.8[+]

(A) O'Herlihy and Spencer; (B) Anderson; (C) O'Herlihy and Spencer re-estimated allowing for

$$u_t = \sum_{i=1}^{4} \rho_i u_{t-i} + \varepsilon_t$$

TABLE III

Definitions

Variable	Definition	Source
GD_t	Gross increase in shares and deposits out-standing net of interest credited to accounts during period t	Financial Statistics
WD_t	Withdrawals of shares during period t	Financial Statistics
PR_t	Repayments of mortgage principal during period t	Financial Statistics
NA_t	Net advances of principal during period t	Financial Statistics
r_{mt}	Average rate of interest charged on new mortgages during period t	Financial Statistics
r_t	Average rate of interest offered, post tax, on deposits during period t $\{= r_{dt}(1 - T_t)\}$	Financial Statistics
ND_t	GD_t + Interest credited to accounts - WD_t	Financial Statistics
LQ_t	Liquidity ratio x 100 where the liquidity ratio is cash and "non mortgage" investments as a percentage of total assets at the end of period t	Financial Statistics
r_{bt}	Average rate of interest paid on bank deposits withdrawable at seven days notice, during period t	Financial Statistics
$r_{\ell t}$	Average rate of interest paid by local authorities on short term loans during period t	Financial Statistics
T_t	Standard rate of income tax expressed as a percentage payable during period t	Inland Revenue
y_t	Personal disposable income in current money terms	Digest of Statistics
P_t	General Index of Retail Prices	Digest of Statistics
PH_t	Index of New House Prices	Digest of Statistics
L_t^0, K_t^0	Liquidity and Reserves Constructions	O'Herlihy and Spencer
GA_t	$NA_t + PR_t$	O'Herlihy and Spencer

TABLE III, continued

Variable	Definition	Source
D_t	Deposit and Shares Total at end of period t	Financial Statistics
M_t	Mortgage total outstanding at the end of period t	Financial Statistics
Q_1, Q_2, Q_3	Seasonal Dummy variables	Construction
$D(1), D(2)$	Mild and Severe Rationing Dummy Variables	O'Herlihy and Spencer

The following is taken from Financial Statistics notes April 1972 with reference to Building Societies.

1. Statistics are based on returns from a sample of about 85 societies with assets amounting to 90% of total building society assets: all societies exceeding Ŀ50 million included, 2/5 of societies with assets Ŀ10m-Ŀ50m and 1/13 of societies with assets of less than Ŀ10m. Each year figures are reconciled with statutory returns made to Registrar of Friendly Societies by all building societies.

2. Pre-1965 statistics based on annual and quarterly samples smaller than the sample size now used (33 societies), together with annual statistics of Registrar.

TABLE V

Unrestricted Reduced Form Estimates

	D(1)	D(2)	T_t	r_{bt}	$(y/P)_t$	$(D/P)_{t-1}$	L_t^o	K_t^o	$(GD/P)_{t-1}$	$(GA/PH)_{t-1}$	PR_{t-1}	r_{mt-1}	r_{t-1}
$(GD/P)^*$	-19. (2.4)+	-26. (2.6)+	45. (3.7)+	-16. (4.0)+	.06 (4.0)+	.08 (3.9)+	-21. (1.5)	-30. (3.5)+	.29 (2.2)+	-.26 (1.6)	-.71 (1.3)	-15. (1.5)	-35. (1.5)
$(WD/P)^*$	2. (0.4)	1. (0.2)	-15. (1.9)	8. (3.5)+	-.05 (5.5)+	.07 (5.3)+	2.5 (0.3)	18. (3.3)+	.06 (0.8)	-.13 (1.4)	.59 (1.8)	-7. (1.1)	8. (0.5)
$(GA/PH)^*$	7. (1.2)	-3. (0.5)	-5. (0.6)	-2. (0.5)	.02 (1.4)	-.01 (0.4)	45. (4.3)+	7.6 (1.1)	.42 (4.2)+	.39 (3.2)+	-.18 (0.4)	-.7 (0.1)	-16. (0.9)
PR^*	3. (1.3)	1. (0.4)	-9. (2.5)+	-.8 (0.7)	-.002 (0.4)	.01 (1.0)	-3.8 (0.9)	1.4 (0.6)	.18 (4.6)+	.08 (1.8)	.29 (1.9)+	2.1 (0.7)	2.6 (0.4)
$100r_m^*$	-14. (1.6)	-12. (1.0)	4. (0.3)	2.4 (0.6)	.04 (2.4)+	.04 (1.5)	-17. (1.1)	-15. (1.6)	-.2 (1.6)	.04 (0.2)	-1.2 (2.0)+	30. (2.5)+	64. (2.6)+
$100r^*$	-6. (1.1)	8. (1.1)	.2 (0.3)	2.8 (1.2)	-.003 (0.3)	.04 (3.2)+	-16. (1.7)	-5. (1.0)	-.04 (0.4)	-.13 (1.3)	-.55 (1.6)	9.4 (1.5)	40. (2.8)+

The s values (adjusted for degrees of freedom) are 15.3, 9.4, 11.9, 4.5, 0.16, .09 ('t' values in parentheses).

Solved Reduced Form Estimates

	D(1)	D(2)	T_t	r_{bt}	$(y/P)_t$	$(D/P)_{t-1}$	L_t^o	K_t^o	$(GD/P)_{t-1}$	$(GA/PH)_{t-1}$	PR_{t-1}	r_{mt-1}	r_{t-1}
$(GD/P)^*$	--	--	44	-10	.06	--	-6.	-2.	.62	--	--	--	35
$(WD/P)^*$	--	--	-9	9	-.05	.07	4.	1.	--	--	--	--	-21
$(GA/PH)^*$	-16	-29	--	--	.02	--	--	--	--	.72	--	--	--
$100r_m$	--	--	--	--	--	--	-16	-6	--	--	--	35	92
$100r$	--	--	--	--	--	--	-16	-6	--	--	--	--	93

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