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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART 23

FIAT MONEY, BANK MONEY, THE FORCE OF THE RATE OF INTEREST  
AND THE VANISHING FLOAT

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by

Martin Shubik

1. INTRODUCTION

In two previous papers it was argued that the existence of a positive money rate of interest in a dynamic general equilibrium system depends upon the presence of two types of money in the system.<sup>1,2</sup> These two types of money can be interpreted as fiat money issued by the government and bank money which is created and destroyed by the banking system.

The previous models were formulated in terms of a discrete trading market with expenditures on trade and receipts of the goods taking place at the start of each period and receipt of income from the sale of goods obtained at the start of the next period. This formulation creates a one period "float" of money which has been spent by the buyers but not received by the sellers. It was argued that in order to obtain a money rate of interest in conformity with our intuitive understanding of the relationship between a "real" or "natural" rate of interest it is necessary to issue at no interest change an amount of fiat money which precisely covers the float in the first period of trade.

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This paper is devoted to showing that this result still holds even if trade takes place continuously and hence even if the size of the float is apparently diminished. The argument is self-contained; although less discussion of the basic model is given here than in the previous papers.

## 2. THE DISCRETE TRADING MODEL

We consider an economy which functions for  $n$  periods which we will call "years." There are  $m$  traders who are identical hence we will need only talk about a representative trader.

A supply of a single perishable consumer good or "manna" appears each period. The quantity is  $m$  units each period. Each trader is the owner of one unit. Ownership in this economy has a somewhat different meaning than ownership in a barter economy. All of the good is sold for money in a market and at the start of the subsequent period each trader receives the money income derived from the sale of the unit of good he owned.

For ease in exposition and proof let us consider that each trader has a utility function of the form:

$$(1) \quad U = \sum_{t=1}^n \beta^{t-1} x_t \quad 0 \leq \beta < 1, \quad x_t \geq 0$$

where  $\beta$  is a "natural discount rate" per annum which is common to all traders.  $x_t$  is the amount of the good consumed in period  $t$ .

Although the model of trade constructed here is designed for the analysis of disequilibrium states and oligopolistic phenomena, as the purpose of this paper is to examine the money rate of interest in an equilibrium state equivalent to that of the competitive equilibrium no

further discussion of either disequilibrium or oligopolistic effects is given here. They are discussed elsewhere.<sup>3</sup>

### 2.1. The Competitive Equilibrium

It is trivially easy to observe that if we considered this  $n$  year trading economy as a single competitive trading market there is an immediate equilibrium solution where each trader merely consumes his own endowment. In this one time market, prices (which are really future prices) are

$$(2) \quad 1, \beta, \beta^2, \dots, \beta^n .$$

The initial and final endowments of each trader are  $(1, 1, \dots, 1)$  and the total utility derived by each is:

$$(3) \quad U = \frac{1 - \beta^n}{1 - \beta} .$$

### 2.2. Trade with Fiat and Bank Money

We now consider an equilibrium solution to the following "parlor game." It is played as follows:

At the start the referee gives each trader some amount of chips or counters which can be used to make purchases in the markets.

The referee announces a sequence of prices  $p_1, p_2, \dots, p_n$  which are the spot prices at which trade will take place in each period. There are no futures markets. All trade is spot for money.

The referee also announces a rate of interest  $\rho$ , and he provides the traders with an outside banking service which can create and destroy

extra chips. This service consumes no real resources.\*

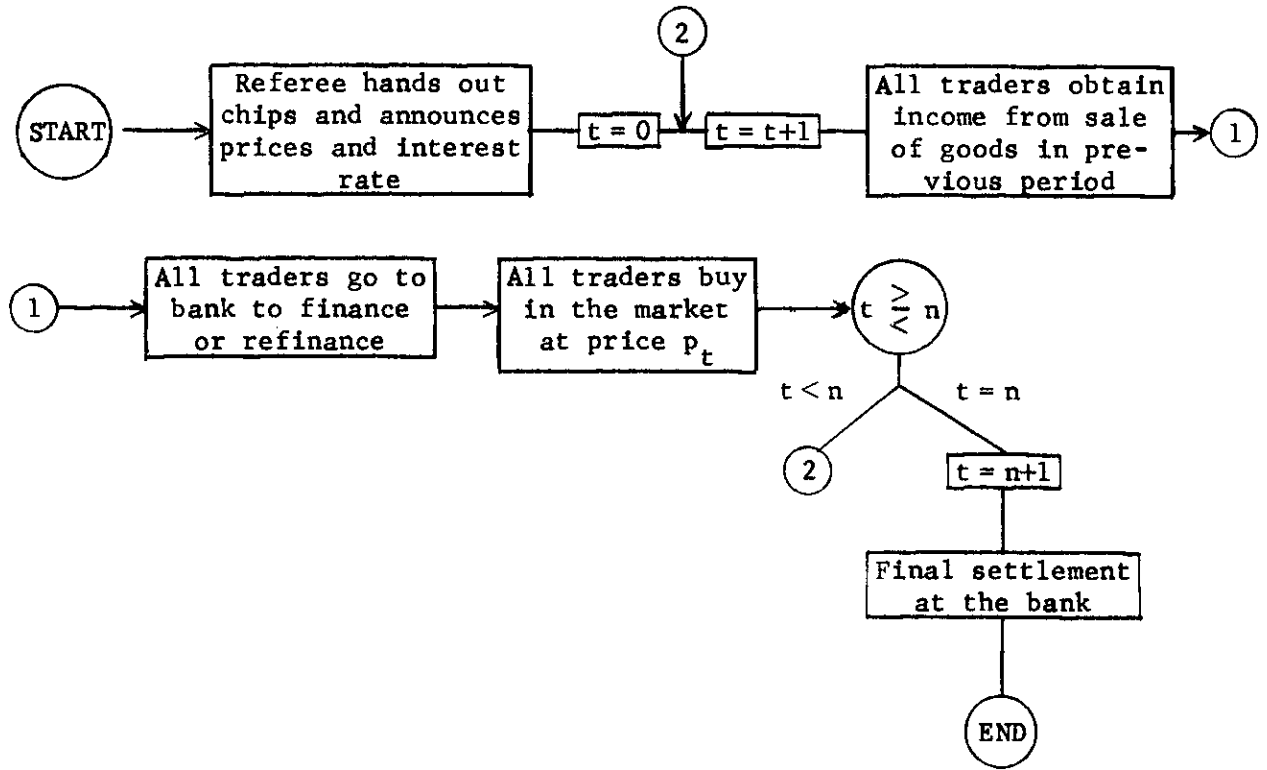


FIGURE 1

A price parameter equilibrium in this sequence of markets with money in general is:

A set of prices  $(p_1, \dots, p_n)$ , an interest rate  $\rho$ , and an initial issue of fiat  $(B^1, \dots, B^m)$  such that if all traders trade, borrow and deposit at the bank the resultant physical outcome will be the same as at a competitive equilibrium and the books will balance at final settlement.

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\*It is easy to modify the model so that the bank does consume resources, however this point is not central to the problem examined here.

### 2.3. The Float

The imposition of the requirement that individual pay for what they buy at the time they buy it, but are not paid until the start of next trading period creates a "float" or money in transit in the system equal to the total value of trade during a period.

The referee has freedom in selecting the unit of money, hence without loss of generality he can select  $p_1 = 1$  and issue enough fiat so that the amount of fiat equals the size of the float in the first period. In the simple example we study here this calls for  $m$  units of fiat issued equally to all, i.e. one unit to each trader.

### 2.4. An Example

The referee announces prices

$$(4) \quad 1, \beta(1+\rho), \beta^2(1+\rho)^2, \dots, \beta^{n-1}(1+\rho)^n ;$$

an interest rate  $\rho$  and issues 1 unit of fiat to each trader. We may proceed to solve for the  $\rho$  which will give us a price parameter equilibrium. For the books to balance at settlement date we require that the present value of the individual's expenditure must equal the present value of his income plus his initial holding of trading chips issued by the referee.

The present value of the individual's expenditures given (4) is:

$$(5) \quad \left\{ \frac{1 - \beta^n}{1 - \beta} \right\} .$$

The present value of income plus initial money stock is:

$$(6) \quad 1 + \frac{1}{1+\rho} \left\{ \frac{1 - \beta^n}{1 - \beta} \right\} .$$

The factor of  $1/(1+\rho)$  appears here because of the one period lag between income and expenditure.

Equating (5) and (6) we obtain

$$(7) \quad 1+\rho = \frac{1}{\beta} \left\{ \frac{1 - \beta^n}{1 - \beta^{n-1}} \right\} .$$

For a finite  $n$ ,  $1+\rho$  is larger than the natural discount however from (7) it follows:

$$(8) \quad n \rightarrow \infty \quad 1+\rho \rightarrow \frac{1}{\beta} .$$

### 3. THE CONTINUOUS TRADING MODEL

Consider an economy that lasts for  $n$  years. In the analysis in Section 2 it was assumed that there was one trading session or market each year. Here we consider  $k$  markets every year. In this model the manna instead of being completely sold off at the start of each year is sold in equal quantities  $k$  times during the year. The velocity of payments is now increased  $k$  fold as all individuals obtain their incomes at the start of the next market.

The total flow per annum of manna is constant, however trade becomes more evenly spread over the year as the number  $k$  of meetings of the market increase. In this model where we have assumed that the annual market is held at the start of the year, the increase in the number of market meetings lowers the overall value of trade as is shown in Figure 2. This is clearly an artifact of deciding when the single market is

be held. Figure 2 illustrates the relationship between trade in two, two year economies where in the first the market meets once a year and in the second the market meets twice a year.

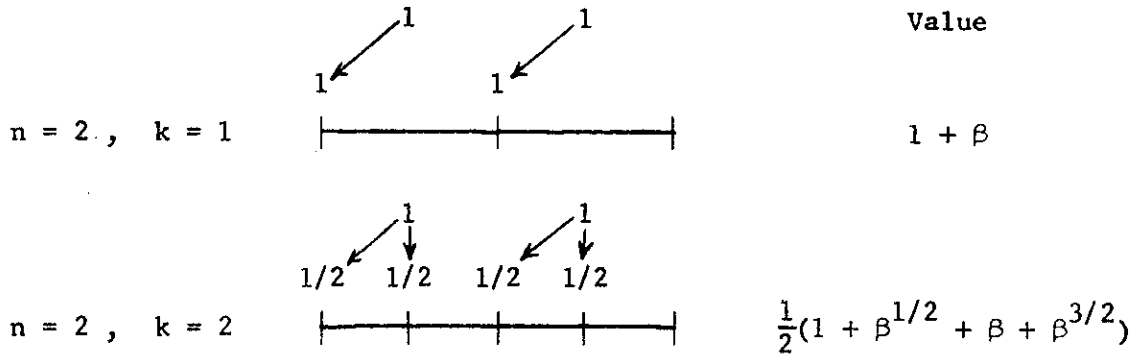


FIGURE 2

Before we calculate the effect of increasing the number of market meetings per annum, we note that for  $k = 1$  we require  $n \geq 2$ . If there were only one market and the referee had issued money to the traders then if the traders have a banking system available, no equilibrium can exist with the fiat covering the float. This is so because all traders will borrow to avoid having worthless paper at settlement period, but this would inflate prices in the first period.\*

We now consider an economy which lasts for  $n$  ( $n \geq 2$ ) years and has  $k$  trading periods per annum. The referee hands out  $1/k$  units of fiat money to each trader before the start of trade. This hand out is a "gift" it carries no interest charge and does not have to be returned to the government. In the instance of an economy with finite duration and an outside banking system it follows immediately that because the money or the chips will be worthless at period  $n+1$ , each trader should

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\*A more detailed discussion of this point is given elsewhere.<sup>3</sup>



try to be in debt to the bank by at least his final income, hence at final settlement the banking system will have sucked out the initial issue.

During each market there are  $1/k$  units of manna per capita for sale.

Suppose that along with an annual interest rate of  $\rho$  the referee announces the following prices:

$$(9) \quad 1, \beta^{1/k} \left(1 + \frac{\rho}{k}\right)^{1/k}, \beta^{2/k} \left(1 + \frac{\rho}{k}\right)^{2/k}, \dots, \beta^{kn-1/k} \left(1 + \frac{\rho}{k}\right)^{kn-1/k} .$$

The present value of the individual's expenditures given (9) is:

$$(10) \quad \frac{1}{k} \left\{ \frac{1 - \beta^n}{1 - \beta^{1/k}} \right\} .$$

The present value of income plus initial money stock is:

$$(11) \quad \frac{1}{k} + \frac{1}{(1 + \rho/k)} \frac{1}{k} \left\{ \frac{1 - \beta^n}{1 - \beta^{1/k}} \right\} .$$

Equating (10) and (11) we obtain:

$$(12) \quad 1 + \frac{\rho}{k} = \frac{\left\{ \frac{1 - \beta^n}{1 - \beta^{1/k}} \right\}}{\left\{ \frac{1 - \beta^n}{1 - \beta^{1/k}} \right\} - 1}$$

hence:

$$(13) \quad \rho = \frac{k(1 - \beta^{1/k})}{(1 - \beta^n) - (1 - \beta^{1/k})} .$$

For  $k \rightarrow \infty$  we obtain

$$(14) \quad \rho = \frac{-\log \beta}{(1 - \beta^n)} .$$

For  $n \rightarrow \infty$

$$(15) \quad \rho = \frac{k(1 - \beta^{1/k})}{\beta^{1/k}}$$

and for  $n$  and  $k \rightarrow \infty$

$$(16) \quad \rho = -\log \beta$$

but the force of interest is  $e^\rho$  which is  $1/\beta$  .

#### 4. CONCLUDING REMARKS

It is straightforward to generalize these results to trading economies with  $s$  consumable goods each year and a utility function for the  $i^{\text{th}}$  trader of the form

$$(17) \quad U^i = \sum_{t=1}^n \beta_i^{t-1} \varphi_i^i(x_{j,t}^i) \quad i = 1, \dots, m \text{ and } j = 1, \dots, s .$$

where the  $\varphi_i^i$  are homogeneous of order 1 in the  $x_{j,t}^i$  and the ownership claims of each individual  $i$  are given by a vector of the form

$$(A_{1,1}^i, \dots, A_{s,1}^i; A_{1,2}^i, \dots; \dots, A_{s,n}^i) \text{ where } A_{j,t}^i \geq 0 .$$

As the continuous model is approached, although the velocity of

the fiat is going up and the quantity down, the various financial measures all remain well defined thus items such as the total size of the float over a year can be obtained by integration even in a society in which the amount of fiat is vanishingly small.

Changes in the methods of payments, communication networks, better accounting and so forth all operate to reduce the amount of "idle" or nonworking money. Institutional arrangements, trading frictions, information and communication problems and a host of other transactions costs tend to bound the velocity of transactions. Suppose in the example of the stationary economy considered in Section 3 the velocity of money were 1, 10, 20, 50 times a year, Table 1 shows the effect of this turnover on an economy of indefinite length with  $\beta = .94$ .

k	$\rho$
1	.06383
10	.06207
20	.06197
50	.06191
$\infty$	.06187

TABLE 1

Two further simple examples show inflationary and deflationary effects. Suppose,  $\beta = .9$  in an economy with continuous trading where final settlement of all debts must be made within 20 years.

$$p_t = e^{(\log \beta + \rho)t} = e^{-(\log \beta) \left( \frac{\beta^n}{1-\beta^n} \right) t}$$

here  $p_t = e^{.0145822t}$ .

In the second example final settlement never comes but there are 20 markets a year.

$$p_t = \beta^{t-1/k} \left( 1 + \frac{k(1 - \beta^{1/k})}{\beta^{1/k}} \right)^{t-1/k}$$

here  $p_t = (.9950746)^{t-.05}$  .

Table 2 shows the prices in both cases for several years

t	Case 1	Case 2
1	1.014689	.995320
10	1.156991	.952059
20	1.338627	.906193

TABLE 2

#### REFERENCES

<sup>1</sup>Shubik, M. "A Linear Exchange Model with Financial Institutions: The Rate of Interest," CF-504-16, Cowles Foundation, April, 1975.

<sup>2</sup>Shubik, M. "Fiat Money, Bank Money, the Float and the Money Rate of Interest," CFDP No. 394, Cowles Foundation, May 1975.

<sup>3</sup>Shubik, M. "A Theory of Money and Financial Institutions," Parts 1-20, Cowles Foundation, 1972-1975.