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OKUN'S LAW REVISITED

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In 1962, Arthur Okun offered a simple rule of thumb which soon became known as Okun's Law: "In the postwar period, on the average, each extra percentage point in the unemployment rate above 4% has been associated with about a 3% decrement in real GNP."

Now a staple of macroeconomics courses, the standard classroom view of Okun's Law was well expressed by Warren Smith:

> ...since the equation is a purely empirical relation based on past behavior of the economy, it is likely to be less reliable as time passes and underlying conditions depart from those that prevailed in the period from which it is derived. But while it would be a mistake to take the equation too seriously, it does serve to illustrate certain basic relationships and ideas that are important.

Using the additional decade of data that is now available, this paper will investigate the reliability of Okun's Law and the extent to which changes in the underlying structure compel a modification or abandonment of Okun's 3 to 1 rule of thumb. In Section I, I discuss the methodological implications of one's causal interpretation of Okun's Law and some apparent structural changes of potential importance. The results of a reestimation of Okun's equations are presented in Section II. In Section III, I examine the usefulness and reliability of Okun's Law as a forecasting rule of thumb. A summary is given in Section IV.

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I

Okun's Law involves two interrelated concepts: (a) that in the short-run the unemployment rate is sensitive to fluctuations in aggregate demand, with potential output defined as the level of GNP necessary to attain 4% unemployment; and (b) that this short-run sensitivity (or this definition of potential output) follows a stable long-run trend which is governed by the secular growth of the labor force, hours, and productivity.

By assuming that the short-run sensitivity is stable, one can use the observed data on GNP and unemployment to estimate the relevant parameters and from this construct an implied path of potential output. Or one could work from the supply side and directly estimate the potential path from the secular trends of the labor force, hours, and productivity; one could then calculate the short-run gaps between observed GNP and the constructed potential series as an estimate of the sensitivity of GNP to unemployment fluctuations.

Okun followed the first route and offered the equation

$$\frac{P-A}{A} = .032(U-4)$$

(where: A = actual real GNP, P = potential GNP at 4% unemployment, and U = unemployment rate) as a subjective summary of three alternative equations which he estimated. The potential output series constructed from this relation wiggled about a 3-1/2% trend line drawn through actual output in mid-1955, and he accepted this trend line as a more reliable measure of potential output.

As an expositional device, Okun's Law can be used to illustrate a variety of different points. As a rule of thumb, it can similarly be used either to estimate the lost output for various levels of unemployment or to estimate the unemployment rate associated with various levels of GNP, and we could measure its effectiveness in either of these contexts.

Imbedded in a complete macro model in which output and employment would undoubtedly both be endogenous, Okun's Law could be used for a recursive forecast of either variable or could be used in conjunction with other equations to simultaneously forecast output and employment. Here the value of Okun's Law would depend upon its ability to mesh with the rest of the model in providing reduced form forecasts. Modified Okun's Law relations do appear in many forecasting models; however it would seem best to let these model builders speak for themselves about its usefulness in this context.

Okun apparently viewed the causal forces as running from unemployment to output, arguing that "The unemployment rate can be viewed as a proxy variable for all the ways in which output is affected by idle resources." Solow and Thurow and Taylor similarly estimated potential output by explicitly specifying production functions in which the unemployment rate hopefully captured discrepancies between measured and utilized inputs. However, an alternative Keynesian view is that Okun's Law is a statement of by how much a given insufficiency in aggregate demand will affect unemployment; that is, a fall in aggregate demand causes a less than proportionate fall in the unemployment rate because of induced labor hoarding, withdrawals from the labor force, and changes in average hours. Followers of this approach typically use a production function to derive demand equations for the production inputs. Applying the usual econometric dictum that the left hand variable should be the natural dependent variable in a causal interpretation of the equation, the derived factor demand equations are then estimated with factor inputs as regressors. This approach has been convincingly advocated by Nerlove and by Fair (1969) and widely applied in recent years (e.g., Coen and Hickman, Dhrymes in Duesenberry et al. (1969), Fair (1969 and 1971), Kuh in Duesenberry et al. (1965), and Nadri and Rosen (1969)).

Despite his causal interpretation, Okun consistently estimated his reported* equations with unemployment as the dependent variable. Because this agrees with my causal disposition and because it is convenient to statistically test relations in the form in which they are estimated, I chose to work directly with Okun's reported estimation equations, and to focus on the ability of these equations to forecast the unemployment rate.

Consistent with this causal interpretation, there have been at least three major secular changes which may have altered the sensitivity of unemployment to GNP fluctuations. The first of these is the changing composition of employment, with an increasing proportion composed of white collar, service and government employees who are largely insulated from fluctuations in aggregate demand. The second change has been in the composition of the labor force and particularly in the ranks of the unemployed. As Perry stresses, these groups contain fewer prime age males and increasing proportions composed of females and young males who traditionally work at below average wage rates for fewer than average house per week. It is consequently argued that a given unemployment rate will be associated with less idle resources in recent than in more distant years. It is not clear however whether this change has made unemployment more or less sensitive

*In some 1971 comments on George Perry's work, Okun acknowledged unreported regressions in which he used output as the dependent variable and obtained a multiplier of .025 rather than .032.

to GNP fluctuations. Since the unemployed are presumably less productive than in earlier years, it would take less of an increase in aggregate demand to absorb their potential contribution. However, they may be truly structurally unemployed so that a very large increase in GNP would be needed to create a sizable demand for their specific skills. A larger part of any increased demand for labor would consequently be met by those already employed or by new entrants into the labor force. The third plausible secular factor is an increased confidence by employers that prolonged recessions can and will be avoided; this would presumably increase their resolve to maintain a stable work force. On balance then, my priors were that unemployment has grown less sensitive to GNP fluctuations and that Okun's .032 multiplier should be increased. For a given GNP and unemployment rate, I would expect that Okun's Law has recently understated potential output in the sense of how high aggregate demand would have to rise in order to achieve 4% unemployment. This is not to deny that inflationary pressures might arise long before 4% unemployment is achieved.

II

I reestimated Okun's equations in a variety of ways using Okun's sample period (1947.I-1960.IV), the subsequent period (1961.I-1973.IV), and the total period (1947.I - 1973.IV). I first used ordinary least squares (OLS), which Okun used throughout his paper. I then used the following autoregressive transformation-instrumental variable procedure (Auto-Inst). Consider an estimation equation that is written as

$$\underset{(T\times 1)}{y} = \underset{(T\times k)}{x_1}\ \underset{(k\times 1)}{\beta_1} + \underset{(T\times q)}{x_2}\ \underset{(q\times 1)}{\beta_2} + \underset{(T\times 1)}{u}$$

where the endogenous explanatory variables are collected as x_1 and the predetermined variables are collected as x_2. Assuming that $u = \rho u_{-1} + \epsilon$, I regressed

$$y - \rho y_{-1} = [\hat{x}_1 - \rho x_1(-1)]\beta_1 + [x_2 - \rho x_2(-1)]\beta_2 + [\epsilon + (x_1 - \hat{x}_1)\beta_1]$$

by searching at .001 intervals between -1 and +1 for the estimates $\hat{\rho}$, $\hat{\beta}_1$ and $\hat{\beta}_2$ which minimized the sum of squared residuals. $\hat{x}_1$ was obtained by regressing x_1 on $y(-1)$, $x_1(-1)$, x_2, $x_2(-1)$, and a time trend. Finally I used OLS on sample periods classified according to whether real GNP increased or declined in that quarter.

Okun's estimation equations are generally of the form

$$y = \alpha + \beta x + \gamma z + \epsilon$$

where $1/\hat{\beta}$ is a reestimate of the coefficient .032 in Okun's Law. Neglecting the possibility of negative values of β or $\hat{\beta} \pm 2\hat{\sigma}_{\hat{\beta}}$, confidence intervals for $\widehat{(1/\beta)}$ can be conveniently constructed which will have equal probability of being on either side of the population parameter:

$$.95 \approx \Pr[\hat{\beta} - 2\hat{\sigma}_{\hat{\beta}} < \beta < \hat{\beta} + 2\hat{\sigma}_{\hat{\beta}}]$$

$$= \Pr\left[\frac{1}{\hat{\beta} + 2\hat{\sigma}_{\hat{\beta}}} < \frac{1}{\beta} < \frac{1}{\hat{\beta} - 2\hat{\sigma}_{\hat{\beta}}}\right].$$

The endpoints are not equidistant from $\widehat{(1/\beta)}$, nor is the interval the shortest possible; however, this type of interval seems satisfactory for our purposes.

Two further questions are whether to test the stability of β or $1/\beta$, and whether α and γ should be assumed stable, tested separately, or tested jointly with β. The most reasonable approach would appear to be to confront each set of data as if I were replicating Okun's techniques using solely that body of data. This (and convenience) would argue for tests of the estimated coefficients rather than their inverses, and for joint tests.

If this spirit is taken seriously, then it could also be argued that the statistical tests of stability are relatively unimportant. No one would expect the coefficients to be completely unchanged, and it is well known that any change no matter how slight would be detected with sufficient data. Thus the difference in our estimates might be statistically significant but small enough to be unimportant. Or the new data may not indicate a structural change but still lead us to substantially revise our estimates. The relevant question then is not whether we have enough data to detect some change, but rather if there is evidence of an important change. That question is probably best answered by a simple presentation of the point and interval estimates. Nonetheless I did calculate the statistical tests described above and report them throughout the text.

Method I

Okun's first estimated relationship is

$$\Delta U = \alpha_1 + \beta_1 \left(\frac{\Delta A}{A_{-1}} \cdot 100 \right) .$$

If we enforce the Okun's Law assumption that $\Delta U = 0$ when P/A is constant (i.e., when $\Delta P/P_{-1} = \Delta A/A_{-1}$), then

$$\frac{\Delta P}{P_{-1}} = \frac{-\alpha_1}{100\beta_1} .$$

Using this result, the first relationship can be rewritten as

$$\Delta U = (\alpha_1 - 100\beta_1)\frac{A}{P}\left(\frac{P}{A} - \frac{P_{-1}}{A_{-1}}\right)$$

as compared to the implication of Okun's Law that

$$\Delta U = \left(\frac{1}{.032}\right)\left(\frac{P}{A} - \frac{P_{-1}}{A_{-1}}\right) .$$

It follows that

$$\frac{1}{\alpha_1 - 100\beta_1}\frac{P}{A}$$

is an estimate of Okun's coefficient, .032. Notice that if the first equation is true, then the crucial Okun's Law parameter is a variable rather than a constant. Similarly, if Okun's Law is correct, then the parameters of the first relation are not both constants.

In Tables I and II, I've reported the various estimates of $1/(\alpha_1 - 100\beta_1)$. These correspond to Okun's .032 if Okun's Law is interpreted as describing the consequences of achieving full employment. In more general situations, this figure would need to be multiplied by a factor of P/A .

In comparison with Okun's reported equation, my OLS estimate is identical for Okun's sample period, almost twice as large for the subsequent quarters, and moderately larger for the period as a whole. In particular, Okun's first method applied to the period 1961 through 1973 implies that each extra percentage point in the unemployment rate has been associated with a 6.7% rather than 3.4% decrement in real GNP. Under the usual assumptions this difference is statistically significant at the 5% level.

The Auto-Inst estimates are strikingly lower than the OLS estimates.

If this is accepted as evidence that the OLS estimates are strongly biased upwards, then this indicates that fluctuations in GNP are positively correlated with the disturbance term in our regression equation. Our OLS estimates would then understate the extent to which rising GNP decreases unemployment because of the presence of omitted factors which tend to raise U and which are positively correlated with GNP changes. The OLS estimates consequently overstate the change in GNP associated with a given change in U . This is consistent for example with the hypothesis that unemployment responds to that part of the observed change in GNP which is considered to be permanent. The increased disparity between the OLS and Auto-Inst estimates in the later part of the period could be interpreted under this hypothesis as evidence that GNP fluctuations were increasingly regarded as transitory.

If Okun's Law is intended as an historical rule of thumb, then it is the correlation between observed GNP and U that is of interest rather than the structural coefficients. That is, we are seemingly interested in the gross correlation between U and GNP and not in isolating how much of this correlation is attributable to direct rather than indirect effects. This argument implies that it is the OLS estimates that are of greater interest, though they must be used with caution if the historical relationship between observed GNP and the omitted variables is altered.

For the split sample, it appears that real GNP is substantially and increasingly more sensitive to changes in unemployment during expansions than during contractions. Translated into our causal interpretation, a 1% increase in real GNP has had less effect on unemployment than a 1% decline in real GNP, and this distinction has become more pronounced in the second half of the postwar period. The differences between the up and down estimates are statistically insignificant within each sample period, while the differences between sample periods are almost significant.

Method II

In his second technique, Okun assumed that

$$P = P_0 e^{rt}$$

and regressed

$$U = \alpha_2 + \beta_2 \frac{P-A}{P}$$

using various time periods and a variety of assumed values for r, with the results judged according to goodness of fit, absence of trend in the residuals, and agreement with the principle that $P = A$ when $U = 4$. His reported estimated equation was

$$U = 3.72 + 36\left(\frac{P-A}{P}\right)$$

for the period 1953-60 with P following a 3-1/2% trend line through A in mid-1955.

Okun's estimation equation can be rewritten as

$$U = (\alpha_2 + \beta_2) + \left(\frac{\beta_2}{P_0}\right)[-Ae^{-rt}] \ .$$

From the Gauss-Markov theorem, OLS estimates of

$$\widehat{(\alpha_2 + \beta_2)} = a \quad \text{and} \quad \widehat{\left(\frac{\beta_2}{P_0}\right)} = b$$

do not depend upon whether the equation is regressed in this form or in Okun's form (or with some alternative linear rearrangement of Ae^{-rt} and a constant term), and in particular do not depend upon the choice of a benchmark P_0 . Estimates of the individual terms α_2 , β_2 , and P_0 require an *a priori* constraint and will of course depend upon the particular constraint chosen. Okun chose to fix P_0 and look for a and b such that $\hat{\alpha}_2$ would be close to 4. Instead he might plausibly have fixed α_2 at 4 and calculated P_0 residually; or he might have compromised on both, using the tradeoff

$$\frac{d\alpha_2}{dP_0} = - \frac{d\beta_2}{dP_0} = -b \ .$$

Thus, with the expected $b > 0$, Okun could have raised α_2 closer to 4 by slightly lowering P_0 ; this would have also lowered β_2 .

Consistent with the other relations, I used the three periods 1947.I-1960.IV, 1961.I-1973.IV, and 1947.I-1973.IV, and values of r at .00025 intervals from .00750 through .01125. The values of r that minimized the sum of squared residuals were respectively .00950, .00975, and .00950; however, following Okun's criteria I chose .00925 for all three periods. This corresponds to a quarterly growth rate, $\Delta P/P_{-1} = .00929$.

Tables I and II display (as Method IIA) my estimated values of $1/\beta_2$ with P_0 chosen so that (as with Okun's estimates) P would equal A in mid-1955. For OLS and $\Delta A > 0$, the estimates of α_2 range between 3.5 and 3.9; for $\Delta A < 0$ and Auto-Inst, the range is a less satisfactory 2.9 to 3.4. I've also reported as Method IIB, $1/(\beta_2 + \alpha_2 - 4)$, which would be my estimate of $1/\beta_2$ if α_2 were constrained to equal 4; as explained previously, this lowers P_0 slightly. The estimates reported as

Method IIC are the result of constraining both α_2 to be 4.0 and P to equal A in mid-1955. Another search over r revealed that .01000, .00925, and .00925 minimized the sum of squared residuals for the three respective sample periods;for convenience, I used .00925 for all three cases. (For the early sample period, the estimates of $1/\beta_2$ are not very sensitive to choices of r between .00925 and .01000; the four respective estimates are .0388, .0386, .0385, and .0390).

All of these estimates of $1/\beta_2$ correspond to Okun's .032 when P = A , since the estimation equation can be rewritten as

$$\frac{P}{A} = 1 + \left(\frac{1}{\beta_2}\frac{P}{A}\right)(U - \alpha_2) \ .$$

In more general situations these estimates should (as with method I) be multiplied by P/A .

With the exception of Auto-Inst, the estimates for IIA and IIB follow the same overall pattern as with the first method; however, the differences between sample periods are not nearly as striking. Although Okun's Law constrains U to be 4 when P = A and Okun's trend P runs through A in mid-1955, he did not impose both of these constraints when estimating $1/\beta_2$. Interestingly, this exercise (IIC) gives a much higher estimate of $1/\beta_2$, and this estimate is lower for the later period than for Okun's sample period. With OLS, the sample period differences are not nearly significant for IIC and not quite significant for IIA and IIB. The differences between up and down quarters are statistically significant for all three sample periods.

Method III

From the model,

$$\left(\frac{100 - U}{100 - 4}\right) = \left(\frac{A}{P}\right)^{\beta_3}$$

$$P = P_0 e^{r\tau} .$$

Okun derives his third estimation equation

$$\log(100 - U) = \log[96 P_0^{-\beta_3}] + \beta_3 \log A - \beta_3 r$$

$$= \alpha_3 + \beta_3 \log A + \gamma_3$$

for translation into Okun's Law, this can be rewritten as

$$\frac{P}{A} = \left(\frac{100 - 4}{100 - U}\right)^{1/\beta_3} = \left(1 + \frac{4-U}{96}\right)^{-1/\beta_3}$$

$$= 1 + - \frac{1}{\beta_3}\left(\frac{4-U}{96}\right) + \frac{\left(-\frac{1}{\beta_3}\right)\left(-\frac{1}{\beta_3} - 1\right)}{2}\left(\frac{4-U}{96}\right)^2 + \ldots$$

since $\left|\frac{4-U}{96}\right| < 1$ for $U < 100$. Thus, the first two terms of this expansion give Okun's Law

$$\frac{P}{A} = 1 + \frac{1}{96\beta_3}(U-4) .$$

I've consequently reported my estimates in Tables I and II for Okun's third equation in the form $1/96\beta_3$. If we were to account for the higher order terms in the expansion, these estimates should be raised slightly.

In contrast to the first two methods, the OLS estimated parameter declines in the later period (and by a statistically significant amount). The differences for $\Delta A \gtrless 0$ are somewhat perverse and statistically significant within the later period, within the period as a whole, and between the early and later period. The Auto-Inst estimates are again lower than the other estimates, and lower for the recent period than for Okun's period.

Method IV (a direct estimation)

If Okun's second method is modified as follows

$$U = \alpha_4 + \beta_4 \left(\frac{P-A}{A} \right) .$$

Then the estimated parameters can be directly (rather than approximately) transformed into Okun's Law

$$\frac{P}{A} = 1 + \frac{1}{\beta_4}(U - \alpha_4) .$$

Using

$$P = P_0 e^{rt}$$

I again tried various values of r and again settled upon $r = .00925$ for all three sample periods. Tables I and II report my estimates (under the label IVA) of $1/\beta_4$ with P_0 fixed so that $P = A$ in mid-1955; and (as IVB) $1/(\beta_4 + 4 - \alpha_4)$, which would be the estimate of $1/\beta_4$ if α_4 were fixed at 4. Raising α_4 again lowers P_0.

Again we have evidence of the OLS and expansionary OLS coefficient increasing in the recent period with the Auto-Inst estimate lower and declining. Interestingly, using Okun's sample period and constraining $\alpha_4 = 4$ gives an estimate of $1/\beta_4$ identical to Okun's subjective summary of his three methods. However, when α_4 is constrained to be 4.0 and P is constrained to equal A in mid-1955, we obtain (reported as IVC) a much larger estimate of $1/\beta$, with the later estimate slightly lower than the early one. For IVC, I again used r = .00925 ; the sum of squared residuals were minimized for the three periods by .00975, .00925, and .00925. (Using .00975 for the early period would have lowered $(\widehat{1/\beta})$ from .0416 to .0411.)

The estimated differences between the early and later period are statistically significant for IVA and IVB and not nearly significant for IVC; all of the $\Delta A \gtrless 0$ differences are significant at the 5% level.

III

As explained above, I decided to concentrate on the accuracy of Okun's estimation equations in predicting the unemployment rate. As an initial test, I compared the observed unemployment rate with 95% prediction intervals based on OLS estimates of each of the Okun's three relations and my direct estimation (here labeled IV). These intervals are based on the assumption that the disturbances are normally distributed with the same variance in and out of sample. For convenience, the prediction intervals were calculated so as to be symmetric about the forecast value of the dependent variable in each estimated relation.

The equations were first estimated for the period 1947-1960 with confidence intervals calculated for this period and for the post-sample period 1961-1973; the equations were then reestimated using the 1961-1973 data with intervals calculated for this period and for the pre-sample period 1947-1960. Table III displays the average width of the various prediction intervals and the number of times that these intervals do not contain the observed value of the unemployment rate. (Method IIO is Okun's version of Method II, using 1953-1960 data with a 3-1/2% annual rate of growth of potential output. In addition to its 16 post-sample errors, 8 of 23 intervals for the pre-sample period 1947-1952 did not contain the actual U .)

The prediction intervals are wide, allowing a margin of error of about 1 point either way in forecasting the unemployment rate. This caution seemed to be confirmed by the out-of-sample experience, as (ignoring IIO) 74 out of a total of 642 out-of-sample intervals did not contain the observed U . However, only 7 of these 74 errors occur when equations estimated with Okun's data are used to forecast the subsequent quarters. The remaining 67 errors arise from reestimating the equations with the recent data and then attempting to predict during Okun's period; the preponderance of these errors arise from method III and from attempts to forecast the late 1940's.

The second test that I considered measured the accuracy of the point forecasts of U for all estimation techniques both in and out of sample. For comparative purposes, three simple naive models were also used. Table IV displays the root mean squared errors (RMSE's) when the models are estimated using Okun's data and then allowed to mechanically forecast the subsequent quarters, and when the models are estimated using the recent data and then allowed to backcast over Okun's sample period.

The unemployment rate seems to have been considerably more stable and predictable in the period 1961-73 than during the earlier years that Okun examined. All of the models are more successful in forecasting both in and out of sample during this more recent period than during the more distant period; and 15 of the 17 models forecast more accurately in 1961-73 out-of-sample than in 1947-1960 in-sample. The only model that forecasted poorly in this recent period was Okun's reported version of the second relation (IIO), which had an out-of-sample RMSE of 1.067 for 1961-73.

Based on out-of-sample rankings for 1961-73, the naive models are slightly superior to model I, followed by model III and then models II and IV. For out-of-sample forecasts for 1947-60, model I is superior, with the naive models and models II and IV doing about equally well; model III does poorly in this period. Generally, the most troublesome periods were: the low level of U in 1947.IV-1949.I; the steep rise in 1949.II and 1949.III, and again in 1954; why a lower level did not prevail in 1955 and 1956; and the high level in 1972 and 1973. With respect to estimation technique, the in-sample ranking is generally A-I, ΔA, OLS while the out-of-sample ranking is usually ΔA, OLS, A-I.

Overall, the equations' forecasting prowess is neither exceptional nor worthless, but rather roughly comparable to (or perhaps slightly better than) that of most macro models when compared to naive models for 1 quarter ahead forecasts. With a longer horizon, I would expect the naive models to lose much of their luster.

As a final forecasting experiment, I examined the implied time paths of potential output. Okun's methods I and III do not explicitly invoke this concept, but do contain implicit estimates of how rapidly actual GNP would have to grow in order to hold unemployment constant. Okun's Law implies that this will be equal to the rate of growth of potential output, since holding U constant here requires a constant P/A . These implicit estimates of $\Delta P/P_{-1}$ are displayed in Table IV.

For Okun's second method and my direct estimation (IV), I explicitly assumed a quarterly rate of growth of potential output of .00929. Although the estimates are not uniform, methods I and III generally imply a slightly higher growth rate than this. The Council of Economic Advisors has used an annual exponential growth rate for P of 4.3% for 1947 through 1951, 3.5% for 1952 through 1962, 3.75% for 1963 through 1965, and 4% since 1966. This corresponds to a constant quarterly growth rate from 1947 through 1973 of .00961.

An alternative way of approaching the problem is to allow Okun's Law to predict a series of values for P given the observed values of A and U. This would correspond to repeatedly using Okun's Law as a rule of thumb for calculating the level of aggregate demand necessary for reaching full employment. To analyze the properties of this procedure, we can consider for example estimation equation IV:

$$U = \alpha_4 + \beta_4\left(\frac{P-A}{A}\right) + \epsilon_4$$

where $P = P_0e^{rt}$. If α_4 and β_4 are the estimated parameters, then $\epsilon_4 = U - \hat{U}$. Transforming this relation,

$$P = A\left[1 + \frac{1}{\beta_4}(U - \alpha_4)\right] - \frac{A}{\beta_4}\epsilon_4 \ .$$

Thus, if we use

$$\hat{P} = A\left[1 + \frac{1}{\beta_4}(U - \alpha_4)\right]$$

then

$$\hat{P} = P_0e^{rt} + \frac{A}{\beta_4}(U - \hat{U}) \ .$$

Thus, our rule of thumb calculations will wiggle about a smooth exponential path in accord with the estimation equation's errors in forecasting U. If for example unemployment is larger than expected, then $\hat{P}$ will be above the trend path, since (given A) it requires a larger P to explain a higher U. Notice also that if ϵ_4 has a constant variance, then the variance of $\hat{P}$ around the trend line will increase as A increases. The other estimated relations give similar though more complicated sources of fluctuations in the rule of thumb potential output series.

Graph I displays the logarithms of A , a .00925 trend line running through A in mid-1955, and P as predicted by Okun's Law. This rule of thumb P has been below the trend line for much of the period, and would be so even more frequently were we to follow the CEA in raising the rate of growth of the trend line. Presumably, this tendency is due to the use of $U_f = 4.0$ rather than the somewhat lower estimated values. Instead of lowering U_f , we could get a better fit by lowering the trend line or by raising Okun's coefficient, .032.

IV

Graph II displays the actual unemployment rate and the rate predicted by Okun's Law when potential output is assumed to follow a 3.7% quarterly trend line through actual GNP in mid-1955. Okun's Law has clearly been a very durable rule of thumb, in that it has not gone far astray during 13 years of quarterly out-of-sample forecasts. The other side of the coin is that its prediction intervals are very wide and that its forecasts have been no better than those of the most naive models. In this section, I will give my interpretation of the evidence presented in this paper, and offer my suggestions as to how Okun's Law might be fruitfully modified.

Okun's Law can be written as

$$\hat{P} = A\left[1 + \frac{1}{\beta}(U - \alpha)\right]$$

or, for predicting unemployment, as

$$\hat{U} = \alpha + \beta\left(\frac{P-A}{A}\right)$$

where P might be some variant of

$$P = P_0 e^{rt} .$$

We could follow Okun's procedure of constraining α to be 4.0 and estimating $1/\beta$ from a subjective averaging of estimation methods I, II, and III. For this purpose, I would place little weight on method III using 1961-73 data, due to its relatively poor performance in the RMSE and confidence interval tests. While the application of methods I and II to the recent data strongly suggest an upward revision of Okun's coefficient $(1/\beta)$, the previous close agreement between the estimates has largely evaporated. On this basis, I would cautiously lean toward a value of .035 or .036 for the postwar period as a whole, which may be an unimportant modification for a rough rule of thumb.

There has also been a dramatic decrease in the Auto-Inst estimates of $1/\beta$, which were already considerably lower than the OLS estimates. This divergence could be explained by the increased stability of GNP; however, if the Auto-Inst estimates are taken seriously, then a remaining puzzle is the question of why these estimates have declined so dramatically. Fluctuations in GNP have also apparently had more impact on unemployment during contractions than during expansions, and this difference is more evident in the more recent data. Part of the rise in the OLS estimates may consequently be attributed to the larger number of contractions during Okun's sample period. While the Auto-Inst estimates may be of structural interest, they are of limited relevance to a rule of thumb. Similarly, the ΔA estimates are useful in reducing forecast errors but would seemingly overcomplicate Okun's Law.

My direct estimation (IVA and IVB) suggests an upward revision of $1/\beta$ based upon the recent data. Moreover, constraining both α to equal 4.0 and the trend P to run through A in mid-1955 (as with IVC) indicates that a major improvement in the accuracy of unemployment forecasts could be obtained by raising Okun's coefficient as high as .040.

In terms of using Okun's Law to forecast potential output, lowering α would raise the Okun's Law forecasts while lowering P_0 would lower the trend projections. Either of these modifications would counter the tendency of the rule of thumb to lie below the trend. Increasing $1/\beta$ would raise Okun's forecasts when U is greater than 4.0 and lower them when U is below 4.0. This would be generally helpful since U has usually been above 4.0, but would aggravate the poor rule of thumb forecasts for 1972 and 1973. Despite these recent errors, my own preference is to stick with $\alpha = 4.0$ and the trend P and to raise $1/\beta$. This would substantially improve the Okun's Law unemployment forecasts based upon trend P, and frequently improve the Okun's Law forecasts of P based upon the unemployment rate. If the rule of thumb and trend forecasts disagree, I would lean towards the trend projections.

Another alternative is to follow the CEA in raising the rate of growth of the trend line. Although this has improved the tracking ability of Okun's Law, I have not been convinced by an independent rationale. Table VI displays the RMSE's when forecasting unemployment directly from Okun's Law for a variety of values of $1/\beta$ and for $r = .00925$, $r = .00950$ and the CEA potential output path. The RMSE's for the 1947-1973 period as a whole are minimized by values of $1/\beta$ of .0404 for $r = .00925$, .0443 for $r = .00950$, and .0352 for the CEA potential output path. If α

is to be kept at 4.0 and the potential output path is to run through actual GNP in mid-1955, then it is apparent that $1/\beta$ should be raised. The remaining options are to use a single trend line or to follow the CEA in using a segmented trend. The latter course would allow the modification of $1/\beta$ to be minor, while a single trend line would seemingly compel a major upward revision of this parameter.

TABLE I

Confidence Intervals for OLS Estimates
($1/\hat{\beta}$ corresponds to Okun's coefficient, .032)

	Method	$\hat{\beta}$	$\hat{\sigma}_{\hat{\beta}}$	$\frac{1}{\hat{\beta} + 2\hat{\sigma}_{\hat{\beta}}}$	$\frac{1}{\hat{\beta}}$	$\frac{1}{\hat{\beta} - 2\hat{\sigma}_{\hat{\beta}}}$
1947.II-1960.IV	I	29.7813	3.227	.0276	.0336	.0429
	IIA	33.077	2.331	.0265	.0303	.0352
	IIB	32.595	2.254	.0270	.0307	.0356
	IIC	25.791	1.887	.0338	.0388	.0454
	III	31.226	2.375	.0278	.0323	.0378
	IVA	30.719	2.133	.0286	.0326	.0378
	IVB	31.195	2.212	.0281	.0320	.0374
	IVC	24.054	1.736	.0363	.0416	.0486
1961.I-1973.IV	I	15.012	3.101	.0471	.0666	.1135
	IIA	30.092	2.083	.0292	.0332	.0386
	IIB	29.931	2.010	.0295	.0334	.0386
	IIC	27.406	1.356	.0332	.0365	.0405
	III	34.711	1.976	.0259	.0288	.0325
	IVA	27.618	1.935	.0318	.0362	.0421
	IVB	27.755	2.009	.0315	.0360	.0421
	IVC	25.513	1.266	.0357	.0392	.0435
1947.II-1973.IV	I	25.905	2.333	.0327	.0386	.0471
	IIA	31.893	1.607	.0285	.0314	.0349
	IIB	31.556	1.552	.0289	.0317	.0351
	IIC	26.561	1.174	.0346	.0376	.0413
	III	31.000	1.468	.0295	.0323	.0356
	IVA	29.458	1.482	.0308	.0339	.0377
	IVB	29.780	1.538	.0304	.0336	.0374
	IVC	24.748	1.085	.0371	.0404	.0443

TABLE II

Auto-Inst and $\Delta A \gtrless 0$ Estimates
corresponding to Okun's coefficient, .032

		1947.II-1960.IV	1961.I-1973.IV	1947.II-1973.IV
Auto-Inst	I	.0150	.0176	.0150
	IIA	.0256	.0132	.0221
	IIB	.0260	.0134	.0224
	III	.0250	.0170	.0232
	IVA	.0278	.0135	.0239
OLS, $\Delta A > 0$	I	.0344	.0763	.0435
	IIA	.0292	.0334	.0308
	IIB	.0296	.0336	.0311
	III	.0311	.0281	.0319
	IVA	.0312	.0362	.0331
	IVB	.0308	.0361	.0328
OLS, $\Delta A < 0$	I	.0302	.0227	.0271
	IIA	.0284	.0279	.0282
	IIB	.2091	.0286	.0289
	III	.0297	.0287	.0294
	IVA	.0312	.0315	.0312
	IVB	.0304	.0308	.0305

TABLE III

Prediction Intervals for U

		I	IIA,B	IIC	IIO	III	IVA,B	IVC
In-Sample	Average Width	1.5	2.4	2.7	1.8	2.3	2.3	2.6
(1947-1960)	No. Outside	3	0	0	0	0	0	0
Post-Sample	Average Width	1.5	2.4	2.7	1.9	2.6	2.3	2.6
(1961-1973)	No. Outside	2	2	0	16	0	3	0
In-Sample	Average Width	0.8	1.8	1.8	---	1.5	1.8	1.8
(1961-1973)	No. Outside	3	1	0	---	2	1	0
Pre-Sample	Average Width	0.9	1.8	1.8	---	1.8	1.8	1.8
(1947-1960)	No. Outside	12	5	10	---	25	7	8

TABLE IV

RMSE's

	Forecasting U		Backcasting U	
	In-Sample 1947-60	Post-Sample 1961-73	Pre-Sample 1947-60	In-Sample 1961-73
$\hat{U} = U_{-1}$	.570	.250	.570	.250
$\Delta U = \Delta U_{-1}$	.709	.230	.709	.230
$\hat{U} = \sum_{i=1}^{8} \alpha_i U_{-i}$	.453	.217	.489	.179
I (OLS)	.352	.246	.418	.204
I (A-I)	.435	.377	.521	.186
IIA,B (OLS)	.561	.501	.610	.437
IIA,B (ΔA)	.543	.454	.583	.405
IIA,B (A-I)	.490	.519	.595	.194
IIC (OLS)	.655	.455	.659	.449
III (OLS)	.550	.432	1.098	.353
III (ΔA)	.535	.417	1.152	.290
III (A-I)	.489	.448	.894	.175
IVA,B (OLS)	.555	.503	.609	.441
IVA,B (ΔA)	.535	.464	.581	.410
IVA,B (A-I)	.490	.512	.600	.194
IVC (OLS)	.648	.456	.652	.450

TABLE V

Implicit Estimates of $\Delta A/A_{-1}$ Required to Hold U Constant

		1947.II-1960.IV	1961.I-1973.IV	1947.II-1973.IV
OLS	I	.01003	.00854	.00988
	III	.00955	.00990	.00949
Auto-Inst	I	.00909	.01020	.00966
	III	.00903	.01001	.00941
OLS, $\Delta A > 0$	I	.00991	.00777	.00921
	III	.00953	.00997	.00949
OLS, $\Delta A < 0$	I	.00787	.00105	.00449
	III	.00934	.00970	.00928

I : $-\alpha_1/100\beta_1$

III : $e^{-\gamma_3/\beta_3} - 1$

TABLE VI

RMSE's for Okun's Law

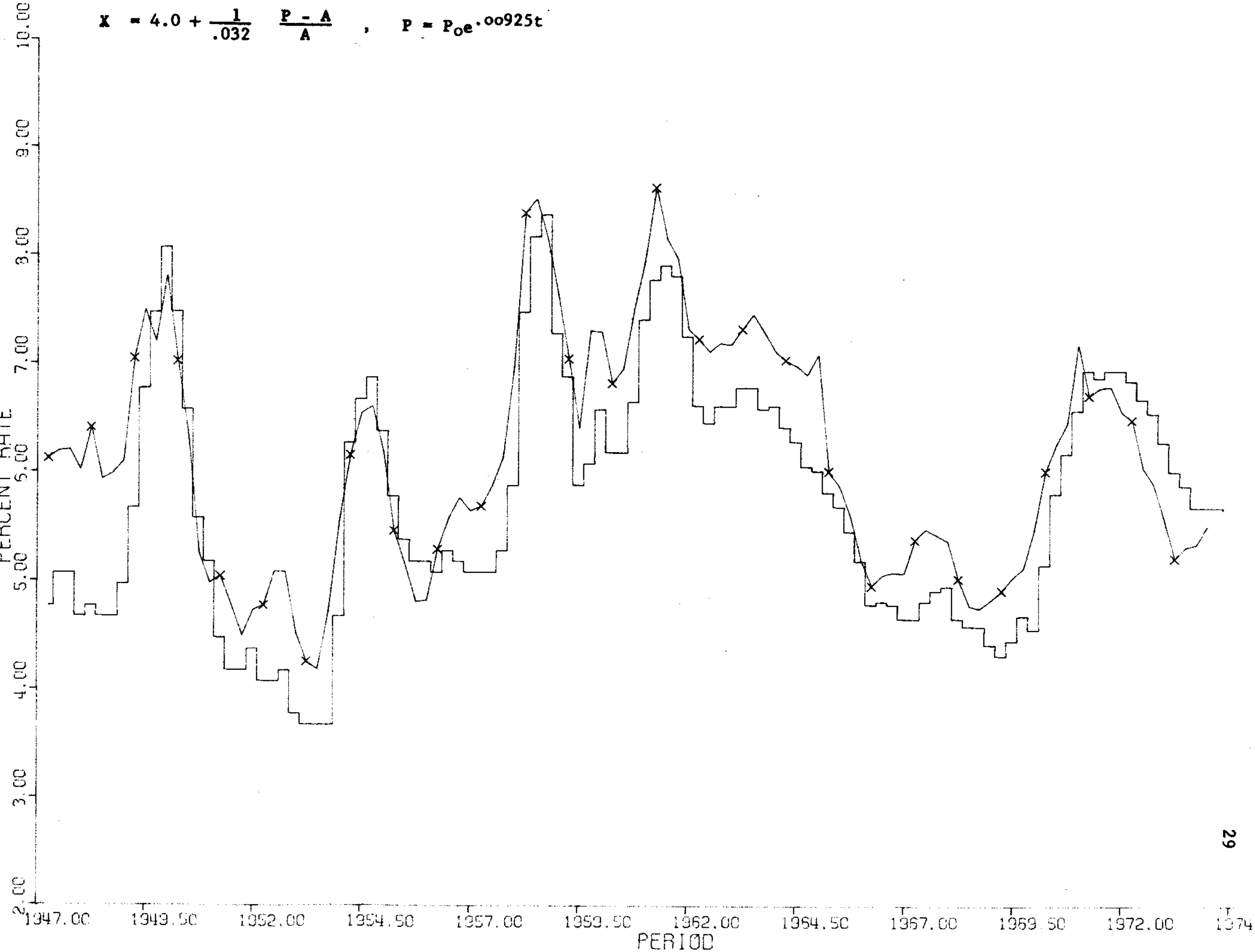
$$\hat{U} = 4.0 + \beta\left(\frac{P-A}{A}\right), \quad P = P_0 e^{rt}$$

	1947-1960			1961-1973		
1/β	r = .00925	r = .00950	CEA r	r = .00925	r = .00950	CEA r
.029	.852	.817	.683	.633	.926	.397
.030	.816	.780	.660	.595	.869	.373
.031	.785	.749	.642	.561	.818	.355
.032	.759	.722	.627	.533	.772	.343
.033	.736	.699	.616	.509	.730	.337
.034	.717	.680	.609	.490	.693	.336
.035	.710	.664	.604	.475	.659	.340
.036	.688	.652	.601	.464	.630	.346
.037	.678	.642	.601	.456	.603	.356
.038	.670	.635	.603	.451	.581	.367
.039	.665	.629	.605	.450	.561	.380
.040	.661	.626	.610	.450	.545	.394
.041	.659	.625	.614	.453	.531	.408
.042	.658	.625	.621	.458	.520	.423
.043	.658	.626	.628	.463	.512	.439
.044	.660	.628	.635	.471	.506	.454
.045	.663	.632	.643	.479	.501	.470
.046	.666	.636	.651	.487	.499	.484
.047	.670	.640	.659	.497	.498	.499
.048	.675	.646	.668	.506	.498	.514
.049	.680	.652	.676	.516	.500	.528
.050	.685	.658	.685	.527	.503	.542
.051	.691	.664	.694	.537	.507	.555

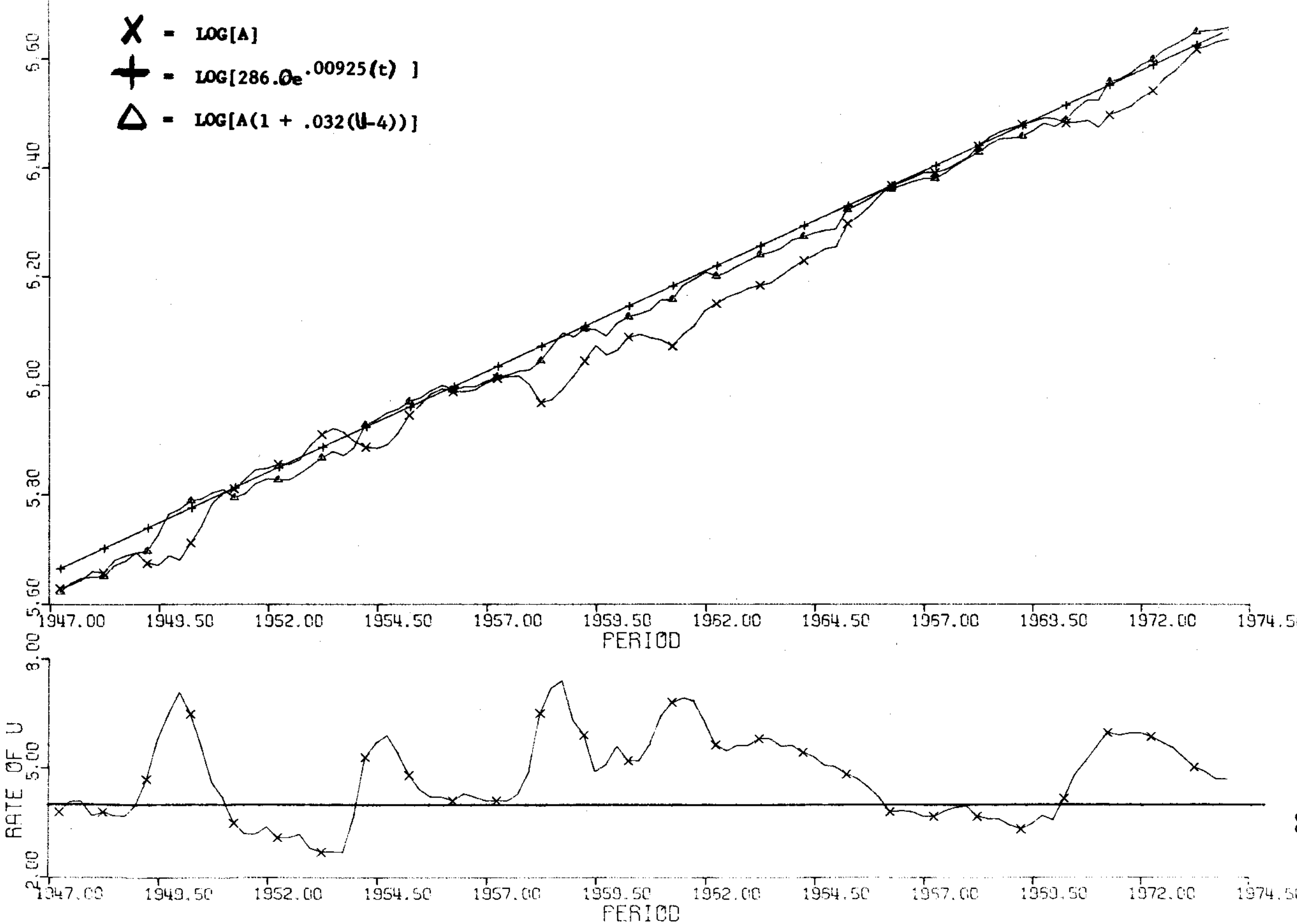
GRAPH II

ACTUAL U

$$X = 4.0 + \frac{1}{.032}\ \frac{P - A}{A}\ , \quad P = P_o e^{.00925t}$$



GRAPH I


X = LOG[A]
+ = LOG[286.0e^.00925(t)]
△ = LOG[A(1 + .032(U-4))]
5.60
5.80
6.00
6.20
6.40
6.60
6.80
1947.00
1949.50
1952.00
1954.50
1957.00
1959.50
1962.00
1964.50
1967.00
1969.50
1972.00
1974.50
PERIOD
RATE OF U
2.00
5.00
8.00

REFERENCES

Coen, R., and B. Hickman, "Constrained Joint Estimation of Factor Demand and Production Functions," The Review of Economics and Statistics, 52 (August, 1970), 287-300.

Duesenberry, J.; G. Fromm; L. Klein; and E. Kuh, The Brookings Quarterly Econometric Model of the United States (Rand McNally, 1965).

__________, The Brookings Model: Some Further Results (Rand McNally, 1959).

Fair, R., The Short-Run Demand for Workers and Hours (North-Holland, 1969).

________, "The Estimation of Simultaneous Equation Models with Lagged Endogenous Variables and First Order Serially Correlated Errors," Econometrica, 38 (May, 1970), 507-516.

________, A Short-Run Forecasting Model of the United States Economy (Heath-Lexington, 1971).

Nadiri, M. and S. Rosen, "Interrelated Factor Demand Functions," American Economic Review (September, 1969), 457-71.

Nerlove, M., "Notes on the Production and Derived Demand Relations Included in Macro-Econometric Models," International Economic Review (June, 1967), 223-42.

Okun, Arthur, "Potential GNP: Its Measurement and Significance," Proceedings of the Business and Economic Statistics Section of the American Statistical Association, 1962.

Perry, George, "Labor Force Structure, Potential Outputs and Productivity," Brookings Papers on Economic Activity (3: 1970), 533-578.

Smith, Warren, Macroeconomics (Irwin, 1970).

Solow, R., "Technical Progress, Capital Formation, and Economic Growth," American Economic Review (May, 1962), 76-96.

Thurow, L. C., and L. D. Taylor, "The Interaction between the Actual and Potential Rates of Growth," Review of Economics and Statistics (November, 1966), 351-60.