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ON THE ROLE OF NUMBERS AND INFORMATION IN COMPETITION

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ON THE ROLE OF NUMBERS AND INFORMATION IN COMPETITION*

by

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1. ON THE GAP BETWEEN THE VERBAL DESCRIPTION OF THE COMPETITIVE MARKET AND MATHEMATICAL ECONOMICS

In the verbal descriptions of price in a competitive market the role of many competitors in bringing about the existence of a competitive price system is critical. Yet in the purported mathematization of these descriptions the role of the presence of many competitors disappears. The results of Arrow and Debreu[1] are utterly independent of the number of competitors assumed to be in the market.

The mathematical economist who may reserve his rigour for theorem proving rather than for assumption testing may argue "well when there are many small competitors in a market, then effectively each is so insignificant that he can be treated as a passive pricetaker." It is precisely this piece of legerdemain that has resulted in a general equilibrium price theory that is unable to distinguish between a competitive and a centralized economy using a price system. Furthermore by getting rid of numbers the way they did, the propounders of the general equilibrium theory managed to maintain if not enlarge the split between

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oligopoly theory and general equilibrium theory. The general equilibrium model cannot be easily enlarged to include mixed markets with oligopolistic and competitive sectors existing in the same economy.

2. HOW TO TAKE NUMBERS INTO ACCOUNT: REPLICATION

My first interest in trying to formulate a clearly well defined model of the role of numbers in a market came about through reading the basic work of Cournot[6] on competition among firms in a market and Mathematical Psychics by Edgeworth on the recontract of parties to a bargain.[8] In both instances it was shown that as numbers increase a competitive price would emerge. However what do we mean by "as numbers increase"? Do we mean more, but different people in the market or do we mean more of the same type of people--doppelgangers or replicas?

Edgeworth elected to consider closely related but not identical individuals:

> it may appear that the quantity of final settlements is diminished as the number of competitors is increased. To facilitate conception let us assume that the field consists of two Xs, not equally, but nearly equally, natured; and of two Ys, similarly related.[8]

Boehm Bawerk in his discussion of a horsetrading market had price emerge as the number of traders increase, but he too introduced different traders.[4] Shapley and Shubik have considered the emergence of price in the Boehm Bawerk market in terms of a game theoretic analysis.[21] Boehm Bawerk's analysis is closer to that of Edgeworth than Cournot in the

sense that the former two were dealing with closed economic systems, i.e. with systems in which both sides of any market were considered as containing active economic agents. This contrasts with the open model of Cournot where the only agents of interest are the firms. The consumers are given by an aggregate demand function.

Cournot, in contrast with the other work noted was clear in his description of a replication process.

> To make the abstract idea of monopoly comprehensible, we imagined one spring and one proprietor. Let us now imagine two proprietors with two springs of which the qualities are identical, and which, on account of their similar positions, supply the same market in competition.[6]

The work that is related to that of Cournot in the sense that large group competition was modeled as a noncooperative game, is that of Chamberlin.[5*] When Chamberlin discusses monopolistic competition however each firm is differentiated from the other, albeit symmetrically. Chamberlin, like Cournot, studied an open system. When he talked about the entry of new competitors he did not make it clear how the differentiation among the large set of products related to the smaller.[25]

In all of the above discussions the intuitive idea of a limiting process for the study of competition was present. However even in Edgeworth, attention was not paid to the details of definition of related economic models with different numbers of economic agents.

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*Cournot used quantity of an identical product as the independent variable. Chamberlin used the price of differentiated products and also included entry, but the solution concepts were mathematically the same.

Around 1952 in the context of the two different models of open oligopoly and bilateral monopoly Shapley and Shubik discussed and formulated the method of replication player types.[17, 24, 25]

From the very start a choice had to be made between duplicating economic agents or fractionating agents. One method would lead naturally towards a framework where work would always be with large finite sets of agents and where although between successive stages the power of the individual might be attenuating it nevertheless would at any stage be there. The other method would lead naturally towards considering a measure theory approach where a powerless trader could be considered as having a measure of zero. In the context of political models of voting Shapley suggested this for "oceanic games"[18] where the individually powerless mass of small voters could be considered as an "ocean" or continuum of voters. Considerably later Aumann adapted this approach to economic models.[2]

The reasons for opting for the replication method when dealing with the oligopoly models of Cournot, Bertrand[3] and Edgeworth[9] had to do with the modeling of the firm as an institution. A simple example will help to illustrate this point.

Consider a single firm, a monopolist with a classical "U-shaped" average cost curve as shown in Figure 1. The mere drawing of the average cost curve of this variety is a method of modeling a host of subtle institutional assumptions concerning the firm. It is an economist's dream and an accountant's and operations researcher's nightmare. In the story that is usually told to justify this shape, real world considerations

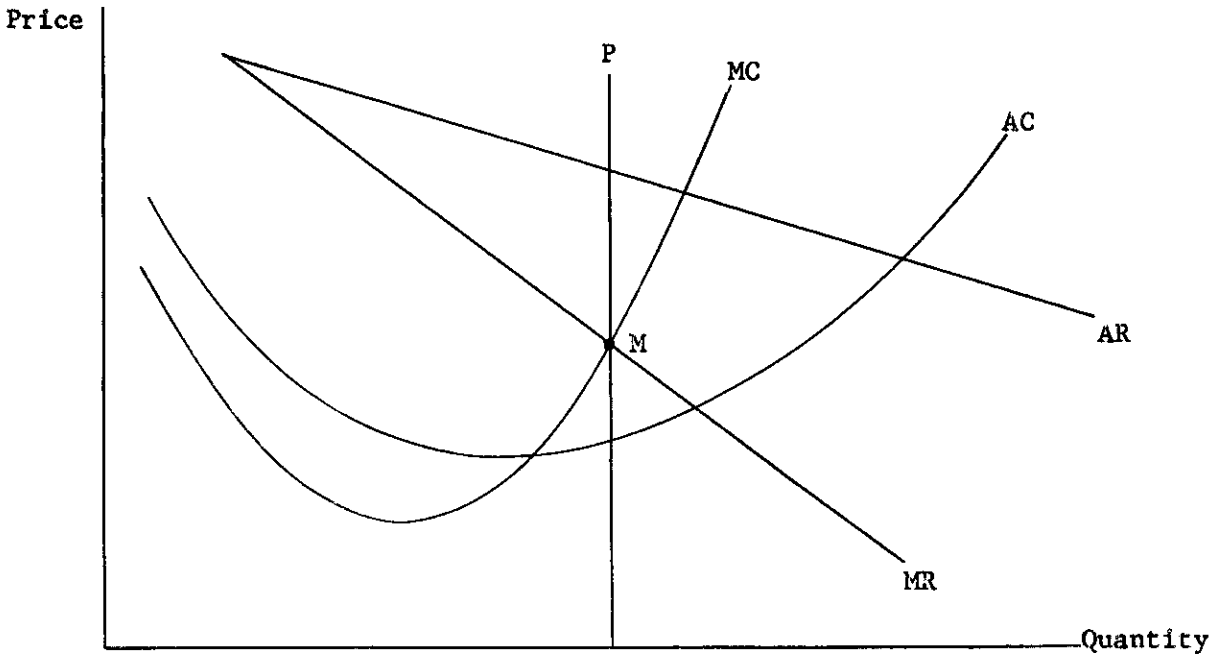

FIGURE 1

such as capacity constraints, limits on management, organization, internal information costs and controls, minor indivisibilities, overheads and all the host of factors which define a firm as an institution are included. This contrasts starkly with the neat production function that is homogeneous of order 1. For the single product firm with given input prices and no overheads such as plant, management or other forms of organization this would yield a constant average cost function.

The difference between the model of the firm as an institution and as a noninstitutional operator of a constant returns input-output formula is precisely that between a restaurant and many simple recipes. As a good approximation a steak dinner for two requires twice as many ingredients as for one. Usually however the requirements on the kitchen and chef differ in a nonlinear manner.

Returning to Figure 1, we can calculate the monopoly price that the firm should charge to maximize net revenue. It is indicated by the point P , where the line MR is the marginal revenue curve and the line AR is the demand. Suppose we now wish to answer the question "what is the effect of more competition on this price?" Our problem is how to make this question sufficiently precise that when we have answered it we know what we have answered.

Let us be even more specific, suppose we wish to contrast the power exercised by a duopoly with that of a monopoly. Intuitively it would be nice to cut the monopoly in half, but how? What do the two new average cost curves look like when we try to saw AC in half?

We might try to introduce another monopolistic firm that is identical with the original, but then we will have so much overcapacity in the market that the question we can answer is not really the one we have in mind. Let us try to operationalize the question in a different way.

Suppose a single firm operates in a market. Now suppose that the market were doubled in size, i.e. at any price twice as much could be sold as could be sold previously. Now suppose that in the new market there were two firms each identical with the single monopoly firm that existed in the smaller market. It is easy to see that if the two acted as one* then the monopoly price will be as before and the individual

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*We also need some conditions forbidding monopolistic discrimination against market segments. This point has been discussed elsewhere.[24]

revenues and sales of the two firms will be each the same as the monopolist's in the smaller market.

Now if we apply a solution concept other than monopoly, for example the Cournot noncooperative behavior, or a variant such as those of Bertrand, Edgeworth or Chamberlin then we may obtain some insight into the question: "How is monopoly power weakened when we contrast the gains of the identical firm operating in a market in which the size of its absolute share is constant, but its percentage share decreased as 1/n ?"

Using this method of replication it is possible to show that for a host of somewhat different models involving price or quantity as the strategic variable and including product differentiation, then the noncooperative solution (which is Nash's mathematical generalization[11] of Cournot's original solution) shows an approach to the competitive level as numbers increase. This is sketched in Figure 2.

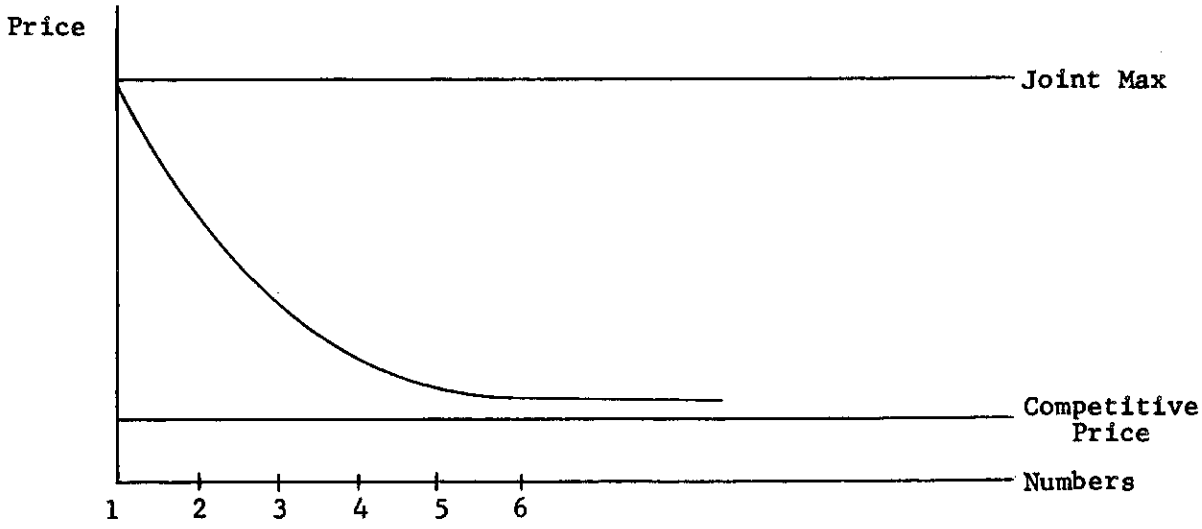

FIGURE 2

It is possible to renormalize the overall size so that we replace the firms by shrunken images of themselves (with the point of minimum average cost for each firm being at a level of output of $1/n$ in the n firm market). However the replication appeared to be intuitively more acceptable.

If the firms had linear costs or no costs then fractionation might appear to be the easiest. This was the case with Cournot's market.

In devising a method to study competition in a closed trading market the natural point of departure was Edgeworth's work. Shapley and Shubik originally elected to push in the direction of sidepayment models because we believed that they offered mathematical tractability and that the insights and results obtained from the analysis of these models would lead to insights, conjectures and methods to handle no sidepayment models.

Essentially what is meant by the existence of a sidepayment medium is that the utility function for each individual contains at least one common commodity for which that component of each individual's utility function is linear and separable. If there are $m+1$ commodities where the last is a transferable utility or a sidepayment medium then the preferences of an individual i can be represented by a utility function of the form

$$U_i(x_1, x_2, \ldots, x_m, x_{m+1}) = \varphi_i(x_1, x_2, \ldots, x_m) + \lambda_i x_{m+1} .$$

Furthermore it is assumed that there exists enough of the $m+1^{st}$ commodity and that it is sufficiently distributed that the Pareto optimal

surface when seen in the n dimensional utility space will be flat, i.e. it will be a hyperplane.

The mathematical methods needed to describe the powers of coalitions of traders are much simpler with sidepayments than without. Figures 3a and 3b show this in a 3-person market.

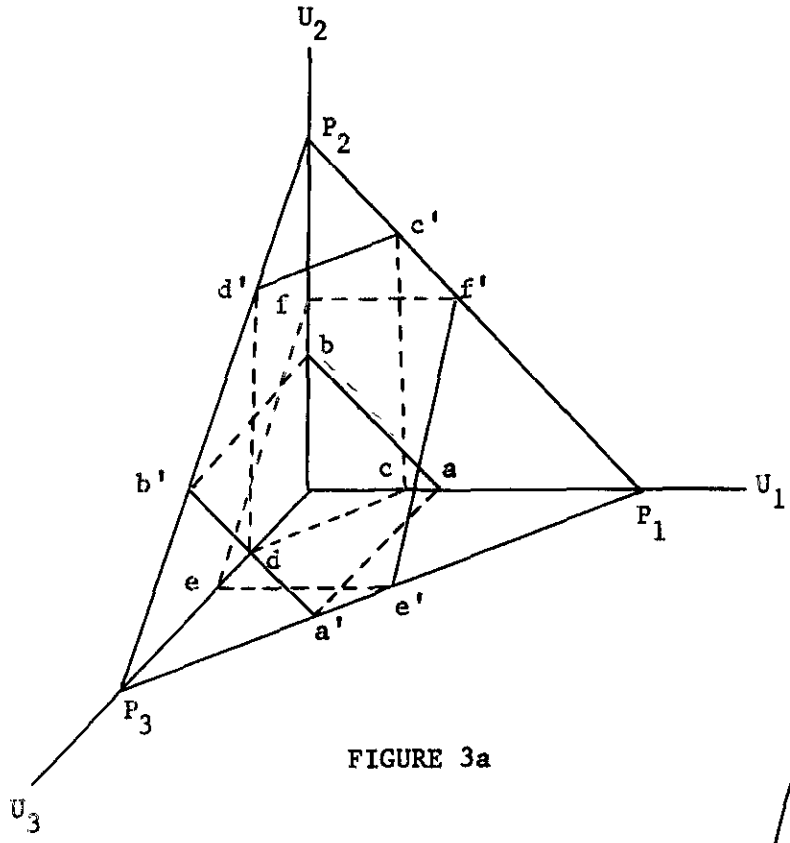

FIGURE 3a

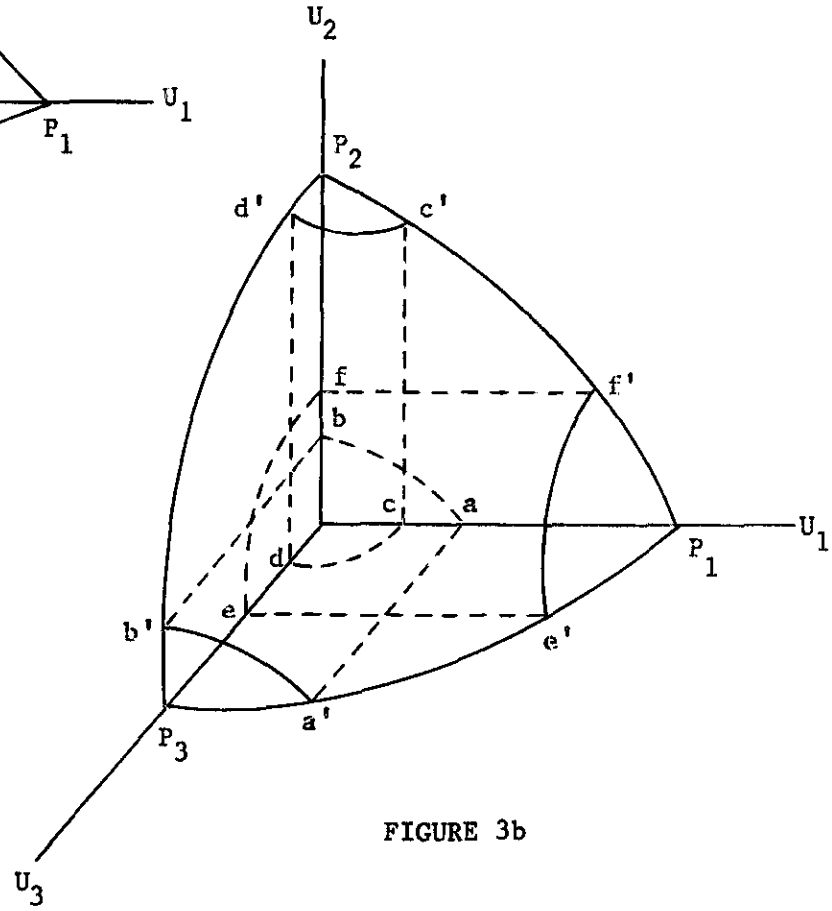

FIGURE 3b

With sidepayments the overall Pareto optimal surface is flat as is shown by the plane $P_1P_2P_3$. Without sidepayments it is curved as is shown by the surface $P_1P_2P_3$ in Figure 3b. Similarly the two dimensional Pareto optimal surfaces which show how well pairs of traders can do if they trade by themselves are in the two cases respectively lines or curves. These are shown by ab (for traders 1 and 2), cd (for 1 and 3) and ef (for 2 and 3).

3. THE CORE AND OTHER COOPERATIVE SOLUTIONS

3.1. The Replication Results

The core of a market game is that set of imputations which are undominated by any coalition; i.e. those divisions of goods such that no subcoalition can achieve more by trading separately. In terms of Figures 3a and 3b the core is the set of imputations bounded by $a'b'd'c'f'e'$.

Edgeworth's model of the shrinking of the contract curve was put forth as a model of perfect competition.[8]

> There is free communication throughout a normal competitive field. You might suppose the constituent individuals collected at a point, or connected by telephones--an ideal supposition, but sufficiently approximate to existence or tendency for the purposes of abstract science.
>
> A perfect field of competition professes in addition certain properties peculiarly favourable to mathematical calculation; namely, a certain indefinite multiplicity and dividedness, analogous to that infinity and infinitesimality

which facilitate so large a portion of Mathematical Physics (consider the theory of Atoms, and all applications of the Differential Calculus). The conditions of a perfect field are four; the first pair referrible[1] to the heading multiplicity or continuity, the second to dividedness or fluidity.

I. Any individual is free to recontract with any out of an indefinite number, e.g., in the last example there are an indefinite number of Xs and similarly of Ys.

II. Any individual is free to contract (at the same time) with an indefinite number; e.g., any X (and similarly Y) may deal with any number of Ys. This condition combined with the first appears to involve the indefinite divisibility of each article of contract (if any X deal with an indefinite number of Ys he must give each an indefinitely small portion of x); which might be erected into a separate condition.

III. Any individual is free to recontract with another independently of, without the consent being required of, any third party, e.g., there is among the Ys (and similarly among the Xs) no combination or precontract between two or more contractors that none of them will recontract without the consent of all. Any Y then may accept the offer of any X irrespectively of other Ys.

IV. Any individual is free to contract with another independently of a third party; e.g., in simple exchange each contract is between two only, but secus in the entangled contract described in the example (p. 17), where it may be a condition of production that there should be three at least to each bargain.

There will be observed a certain similarity between the relation of the first to the second condition, and that of the third to the fourth. The failure of the first involves the failure of the second, but not vice versa; and the third and fourth are similarly related.

A settlement is a contract which cannot be varied with the consent of all the parties to it.

A final settlement is a settlement which cannot be varied by recontract within the field of competition.

Contract is indeterminate when there are an indefinite number of final settlements.

He thus suggested recontracting and free communication. Furthermore although the method of analysis of the range of contract was clearly by means of replication, as can be seen from the quotation above he had in mind a continuous distribution of traders.

When I first considered Edgeworth's model in terms of the theory of games, the core had just been suggested as a solution concept by Gillies.[10] I observed the connection between Edgeworth's approach to the emergence of price and the core. However my prime interest was based on two considerations. The first was that the general equilibrium models of Walras[32] and subsequently Arrow and Debreu[1] did not in their mathematical formulations make explicit use of the numbers of competitors whereas Edgeworth's model and the core did.

The second point involved stability. It appeared to me that the approach of studying the stability of a competitive equilibrium point in economics by a displacement and measurement of excess supply and demand together with an adjustment mechanism depending on excess supply and demand was too slavish an imitation of the virtual displacement model in physics and was not necessarily the best model for economic stability. The core on the other hand could be viewed as a set of points which are combinatorically or coalitionally stable and this type of stability is different from the former and more socio-economically oriented.

It was the approach to stability that prompted me to propose the conjecture that the core of a general no sidepayment trading game would "shrink" under replication to Scarf immediately after he had presented

a paper on an example of global instability of the competitive equilibrium[14] at Columbia University.

The limited results of Edgeworth[8] and Shubik[26] and the more general results of Scarf[15] all using replication for trading games with a finite number of traders established that the core would shrink and that the competitive equilibrium points are always within the core.

3.2. The Measure Theory Results

Aumann has offered a different mathematical approach treating small traders as an "ocean" or continuum.[2] This certainly appears to catch the underlying implications of both Walras and Edgeworth.

The result of Aumann is that the core and competitive equilibria coincide. This is a much stronger statement than the statements concerning the shrinking of the core.

It might be thought that the Aumann result provides the appropriate logical underpinnings for the concept of a price system in the competitive market. It clearly does not. The fact that given Aumann's analysis two extremely different concepts of market organization coincide should lead us to suspect that they must be each telling only a partial story.

3.3. Why the Core was the Wrong Model of Competition

The Walrasian, Arrow-Debreu models of competition present a mechanistic maximizing price-taking nonstrategic model of economic man. The strategic power of the individual is removed by assumption and coalitions are ruled out *a priori*.

The core presents a combinatoric view of the market. All coalitions are simultaneously coalitions-in-being. This solution concept provides implicitly a high information state, costless communication, static, institution-free model of the economy. Unlike the competitive equilibrium model the core can immediately accomodate one form of market imperfection. If a group of traders (or a single trader) has a finite measure a mixed market with some competitive and some oligopolistic components can be defined.

In spite of the greater flexibility of the core solution when information and production are considered new difficulties appear.

From Adam Smith onward the verbal description of the competitive market price has clearly been of a group of competitors each with individual freedom acting independently in an environment with low levels of information and communication. The core implicitly models high levels of information and communication.* As is noted in Section 4 below the model of Cournot appropriately modified provides an alternative to both of the approaches noted above. Furthermore it is a low information and communication strategic model.

When one tries to model production, immediately difficulties are encountered in modeling either decreasing or increasing returns to scale. Are production processes individually or jointly owned? Without going

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*The situation is even worse with the competitive equilibrium model which is not suited to handle nonsymmetric information conditions, i.e. where individuals have different amounts of information concerning the economy. See for instance Radner.[13]

into detail at this point, the crux of the difficulty appears to be that the production process essentially requires at least a rudimentary dynamic treatment. Even at the simplest level it requires a framework in which elementary institutions called firms play a distinguished role.

The core makes use of the coalitional form representation of the economy[19] whereas a model that adequately portrays production calls for the strategic form[19] and one that adequately portrays information conditions calls for the extensive form.[19] Scarf has managed to provide, as he himself points out a not completely satisfactory extension of the core model with production.[16] The difficulty is with the model not the mathematics.

3.4. Other Cooperative Solutions

Using the method of replication Shapley and Shubik have been able to show that several other cooperative solutions which differ considerably in concept from either the competitive equilibrium or the core also "approach" the competitive equilibrium as the number of replications is large. These solutions include the value[20] and the bargaining set.[22]

Unfortunately these solutions have all of the same weaknesses encountered with the core. They are noninstitutional static, high information models, which although they may be of considerable interest in and of themselves, are not necessarily the appropriate models for the study of the competitive process.

4. REPLICATION WITH STRATEGIC AND EXTENSIVE FORM MODELS

4.1. Realism and Process

Walras may be regarded as providing the mathematical structure for the general equilibrium model of the economy, Edgeworth provided the basis for a cooperative game model of the general equilibrium exchange economy.

In contrast with the above, Cournot provided a mathematical model for competition in a partial equilibrium or open economic system.

In Cournot's model the individuals do not communicate, debate and recontract--they act independently. Cournot shows how the competitive price emerges as the number of competitors increases.[6] As his model was selected with zero costs of production replication or the division of firms are equivalent.

His model was presented in strategic form, i.e. a payoff for each firm is specified as:

$$\Pi_i = D_i f(D_1 + \ldots + D_n) .$$

From the context of the model one could construct a single move extensive form model to indicate that all firms select their levels of production in ignorance of each others' moves.

The criticisms of the Cournot model are primarily of two types: (1) the realism of the description of market moves (in particular the selection of production as the strategic variable) and (2) the treatment of information and dynamics.

Various writers have argued that the Cournot model could be made more "realistic" by considering product differentiation, product variation, price as the strategic variable or price and production simultaneously as strategic variables. These are criticisms concerning relevance and the empirical problems in the modeling of markets.

I believe that the importance of strategic variables varies from market to market as a function of technological and institutional details. The selection of the most important variables cannot be done a priori and gives rise to a task in applied economics.

4.2. Institutions and Information

Unlike the core the Cournot model can be converted into a dynamic model virtually directly. Instead of considering a single move model we might consider a series of periods with moves each period and various information conditions.

Different markets and institutions use various processes thus there are sealed bids, auctions, Dutch auctions, two-sided matching procedures used in some stock markets, elaborate distribution and retail systems and so forth. They reflect information differences and evaluation procedures. Buying antiques, houses or bonds differ in the amounts of inspection, evaluation and calculation required of the buyers.

Although Cournot did not concern himself with closed noncooperative models of an exchange economy it is possible to construct such models. Shapley and Shubik[23] and Shubik[28, 30] have built such models and have shown

that under replication the noncooperative price approaches the competitive price. It is my belief that these models are far closer to being the appropriate mathematical models of the competitive price system than either the competitive equilibrium or the core.

4.3. Behavioral Solutions

The multiperiod process models of markets call for a complete specification of all positions that the system can attain, as well as a complete specification of the information states. Given the model one can examine it for the existence of noncooperative equilibria; however for games in extensive form there are several types of noncooperative solutions which can be specified and furthermore they melt imperceptibly into behavioral solutions. This is discussed in more detail elsewhere.[29] Here the three major classes of solutions are noted:

(1) Noncooperative Equilibria with State Strategies: This class contains the Cournot equilibrium, other noncooperative oligopoly models and the competitive equilibrium as a special case.

(2) Noncooperative Equilibria with Historical Strategies: This class includes equilibria whose stability depends not merely upon the current state of the system, but the history of the process leading to the current state.

(3) Behavioral Solutions: The emphasis is changed from a search for equilibrium to a specification and study of process. Equilibrium may be reached and may be consistent with the noncooperative solution, but the focus is on process and equilibrium considerations are incidental.*

*See for instance Nelson and Winter.[12]

5. FURTHER MODELING PROBLEMS

5.1. Money and Credit

In trying to model a closed economy as a game in strategic form problems are encountered in defining strategies and payoffs for the individuals. In particular because the system must be defined for all positions of disequilibrium as well as equilibrium if individuals move independently there arise many occasions for which the books do not balance. Unless we postulate a world of perfect trust we need to introduce money and credit to help to "decouple" individual decisions and to simplify market structures. A simple pair of examples shows this.

We first consider a world with two types of traders trading in two commodities; where each type owns one of the commodities. Next we consider trade in more than two commodities.

Suppose traders of Type 1 each have an initial bundle of $(A,0)$ and traders of Type 2 have $(0,B)$. Let prices be quoted in terms of the first commodity. Thus a strategy for trader i of Type 1 might be (p_i, d_i) where p_i is the price he offers for the second commodity and the amount he demands. Similarly trader j of Type 2 has as his strategy (q_j, s_j). The price he asks for the second commodity and the amount he offers. Assume that some market mechanism produces a final price.[30]

It is reasonable to impose the conditions that $p_i d_i \leq A$ and $s_j \leq B$. There is no need for a money or credit.

Suppose now that there are $k+1$ commodities and traders of Type 1

have $(A_1, A_2, \ldots, A_{k+1})$ and Type 2 have $(B_1, B_2, \ldots, B_{k+1})$. For purposes of illustration assume that traders of Type 1 want to be buyers of the first m commodities and sellers of the rest, and for Type 2 it is vice versa.

We might simplify and organize the markets in such a way that instead of having $k(k+1)/2$ markets for every pair of goods the $k+1^{st}$ good is used as money and there are k markets, each for one good and the money.

If we impose the condition on traders of Type 1 that $\sum_{k=1}^{m} p_i^k d_i^k \leq A_{k+1}$ this would be tantamount to granting them no credit for the goods they are selling. This is far too stringent a financial restriction and would result in nonoptimal trade. If we used the limit $\sum_{k=1}^{m} p_i^k d_i^k \leq A_{k+1} + \sum_{r=m+1}^{k} p_i^r s_i^r$ this would be extremely liberal as it amounts to believing that an individual will sell everything he offers at the price he names.

In order to completely define the model we need to postulate a bank or referee who will extend "commercial credit" for the period of trade.

5.2. Bankruptcy and Negative Claims

If credit is granted then there is the possibility that an individual is unable to pay back that which he owes. In order to cover this eventuality, bankruptcy laws must be specified.

A reasonable way to model insolvency and bankruptcy conditions is to introduce credit as a "paper commodity" which unlike real commodities can take on positive and negative values. Whereas with real commodities it is not particularly natural or meaningful to consider "minus a ton of wheat," most financial instruments appear on two balance sheets with opposite signs. If we consider the financial instrument as a special form of good the extension into a negative orthant is natural and we can thereby extend our indifference maps to reflect insolvency.

Figure 4 shows a simple case of an economy with one commodity, one commodity money and a credit instrument. If we assume that "the bank"

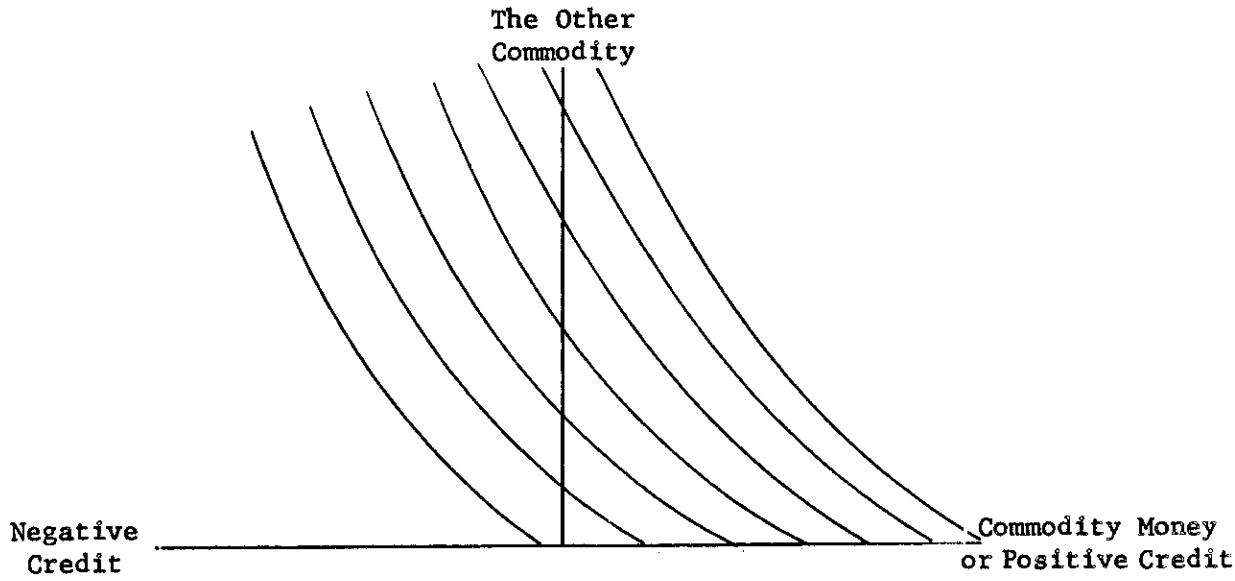

FIGURE 4

or the system pays out any creditor whatever his full claim is in the commodity money then we need only two dimensions, as a positive claim on paper would be the same as ownership of the commodity money. The

positions which involve ending up in debt will have the trader's indifference curves shaped by his preferences for the outcomes determined by insolvency and bankruptcy. Shapley and Shubik are investigating this type of extension of the indifference map in considering credit.

In a modern economy financial instruments are as real as bread. They provide the means for defining subtle distinctions in ownership rights and responsibilities. Mass market, low information dynamic models which include goods, services and financial instruments appear to me to offer a far better potential for the understanding of the formation of the price system than do either the competitive equilibrium or core models.

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