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TOWARDS KEYNESIAN MICRO-DYNAMICS OF

PRICE, WAGE, SALES AND EMPLOYMENT

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TOWARDS KEYNESIAN MICRO-DYNAMICS OF

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by

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1. Introduction and Summary

The purpose of this paper is to construct a simple short-run model

of an individual firm which operates in a non-Walrasian disequilibrium

economy in order to provide microeconomic foundations of Keynesian dynamic

theory of unemployment and inflation.[1]

In the next section we shall develop our basic view of the non-

Walrasian disequilibrium economy. It will turn out that our picture of

the economy drawn as a basis for Keynesian disequilibrium analysis is akin

to that of the theory of monopolistic or imperfect competition. It is

indeed surprising to notice that both so-called Keynesian revolution and

so-called monopolistic competition revolution in 1930's did not go as far

[1]Elsewhere we have developed another version of the Keynesian micro-dynamic
theory of price, wage and employment determinations. K. Iwai, "The Firm
in Uncertain Markets and Its Price, Wage and Employment Adjustments,"
Review of Economic Studies, forthcoming (April 1974).

as their originators had expected and the ancien régime of neoclassical

competitive equilibrium theory has succeeded in counter-revolution.[2]

However, it is all the more surprising to find out that these two theories

have seldom been tried to be synthesized.[3] One of the intentions of the

present paper is to show that some elements of the theory of monopolistic

competition are indispensable microeconomic basis for the Keynesian dis-

equilibrium analysis.

In Section 3 we shall set up the basic framework of the model of

an individual firm producing a single product by employing a single variable

factor, labor. Essentially, the model is composed of an entrepreneur's

subjective product demand and labor supply schedules he faces in the markets

and a production function which summarizes the firm's production activity.

In Section 4, we shall explore the structure of optimal sales and employ-

ment policies for our entrepreneur under the assumption that he is a myopic

decision-maker.[4] It will be shown that his decisions on sales and employ-

ment are constrained either by the realized supply in the labor market,

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[2] Some rehabilitation efforts have been made for both the theories recently. For the Keynesian economics, see R. Clower, "The Keynesian Counter Revo-lution: A Theoretical Appraisal," in Hahn and Breckling (eds.), The Theory of Interest Rates (New York: Macmillan, 1965), and A. Leijonhufvud, On Keynesian Economics and the Economics of Keynes (London: Oxford University Press, 1968). For the monopolistic competition theory, see R. Kuenne (ed.), Monopolistic Competition Theory (New York: John Wiley, 1967).

[3] A notable exception is, of course, Kalecki's various works. Cf. M. Kalecki, Theory of Economic Dynamics, Revised Second Edition (London: George Allen and Unwin, 1965).

[4] The assumption of myopic decision-makings will be relaxed in Appendix 2.

by the realized demand in the product market, or by the firm's economic
productive capacity. When either the realized product demand or the economic
productive capacity is insufficient, we shall observe that some workers
are involuntarily unemployed in the sense that they are refused to be
hired even if they are ready to accept the current money wage. In the
next Section 5, we shall try to relate our micro theory of involuntary
unemployment to that of Keynes. The involuntary unemployment, we shall
claim, is the price that workers are forced to pay for the entrepreneur's
mistakes due to his imperfect information about his market environment.

In Section 6 we shall determine optimal product price and money
wage. The optimal conditions require our myopic entrepreneur to adopt
the monopolistic-cum-monopsonistic pricing rule and to plan to clear both
the product market and the labor market while anticipating the economic
productive capacity to be underutilized. Of course, no plans can be
free from errors in a non-Walrasian economy, and, ex post, the firm's
activity will be constrained either by the labor supply, by the product
demand or by its economic capacity.

The optimal price and money wage policies are governed by the
entrepreneur's short-term expectations about the product demand and labor
supply schedules he is facing, with reference to his long-term expecta-
tions embodied in the existing capital equipment. But these short-term
expectations are somehow formed by his experiences in the past markets.
As time goes on, he acquires new observations and revises his short-term
expectations according to them. But changes in expectations, in turn,
urge him to adjust both product price and money wage if changing them

incurs no costs. Therefore, the theory of price and wage adjustments by a single firm is nothing but a dynamic system of simultaneous equations composed of the optimal product price and money wage equations which state how price and wage are dependent upon the entrepreneur's short-term expectations and of his expectation-formation equations which specify how he revises his short-term expectations responding to new observations he obtains in the current market.

In Section 7 a plausible specification of our basic model will enable us to derive simple price and wage adjustment equations. They state that proportional increases in both price and money wage are linear functions of a realized rate of excess demand in the product market, a realized rate of excess supply in the labor market, a deviation of capacity utilization from the normal rate, a rate of change in expected aggregate price, a rate of change in expected aggregate wage and a growth rate of real capital stock. It is clear that all these explanatory variables but the last one can be regarded as measures of the entrepreneur's errors in his short-term expectations variously revealed in the current markets. These adjustment equations are a generalization and a choice-theoretic derivation of the well-known law of supply and demand. However, while the conventional law is an impersonal one pretending to be a pseudo-behavioral equation of a fictional market participant--Walrasian auctioneer or Invisible Hand, our more complicated law is deduced as true behavioral equations of a Visible Hand--the firm--which is actively participating in market transactions in the non-Walrasian economy. However, in contrast with the Walrasian tâtonnement model of price change, our adjustment

equations imply that changes in price and wage will be followed by changes
in employment and sales, which may, in turn, generate involuntary unemploy-
ment and/or involuntary unfilled-orders whose burdens are borne by workers
and customers. Therefore, it seems more appropriate to interpret our
new law of supply and demand as a microeconomic characterization of the
so-called (short-run) Phillips curve relations.[5]

Before embarking on detail analysis of our model, we must emphasize
here that our theory is a partial disequilibrium theory in the sense that
it concerns only with an individual firm's short-run behaviors in dis-
equilibrium markets. However, each firm constitutes only a minute part
of the whole economy which is composed of many firms, many households,
and many other economic institutions. The dynamic workings of the economy
as a whole should be analyzed by taking full account of dynamic inter-
actions among its basic economic agents. The present paper is no more
than a preliminary step towards the general dynamic theory of the whole
economy out of equilibrium, whose foundation was laid out by Keynes more
than thirty-five years ago.[6]

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[5]A. W. Phillips, "The Relation Between Unemployment and the Rate of Change
of Money Wage Rates in the United Kingdom, 1861-1957," Economica, XXVI
(November 1958), and R. G. Lipsey, "The Relation Between Unemployment
and the Rate of Change of Money Wage Rates in the United Kingdom, 1862-
1957," Economica, XXVII (February 1960).

[6]J. M. Keynes, The General Theory of Employment, Interest and Money
(London: Macmillan, 1936).

2. On the Non-Walrasian Economy

We start our analysis by banishing the Walras' Demon--market auctioneer --from our picture of the economy.[7] Once we have abandoned the well-told fable of Walras that there exists an auctioneer in a market, who quotes a market price, signals it to all the market participants free of charge, calculates the market demand and supply, and adjusts the price according to excess demand or supply until the market is finally cleared, we can no longer rely comfortably on the basic assumptions of the conventional neoclassical equilibrium theory. If the price is ever to be changed, somebody in the economy who is actively participating in market transactions must take over the process of price determination. The so-called impersonal law of supply and demand must be somehow personalized.

We picture the economy consisting of many firms and households; firms are selling products and buying factors; households are selling factors, especially their labor-services, to firms and buying back desired commodities from them. However, since no one in the economy supplies complete information about possible trading opportunities open to market

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[7] See Arrow's pioneering work on this problem. K. Arrow, "Toward a Theory of Price Adjustment," in Abramovitz, et al., The Allocation of Economic Resources (Stanford: Stanford University Press, 1959). Recent explorations into this problem are vast. See, for example, A. Leijonhufvud, op.cit.; E. Phelps, et al., Microeconomic Foundations of Employment and Inflation Theory (New York: W. W. Norton, 1970); W. Nordhaus, "Recent Developments in Price Dynamics," in Eckstein (ed.), The Econometrics of Price Determination (Washington, D.C.: Board of Governors of the Federal Reserve System and Social Science Research Council, 1972); M. Rothschild, "Models of Market Organization with Imperfect Information: A Survey," Journal of Political Economy, forthcoming; and the other contributions cited therein.

participants, each buyer must search for sellers and each seller, in turn,
must search for buyers. Relevant information must be acquired by each
market participant's own search requiring his own economic resources.
Costs required to obtain perfect information concerning all the possible
trading opportunities--location of customers or suppliers, their quoting
prices, characteristics of commodities, rivals' reactions, etc.....--are,
of course, prohibitive. Hence, each transactor has no coince but to be
content with imperfect information in deciding whether to trade with such-
and-such a person or firm at such-and-such a price. The price _may_ vary
from one seller to another, from one buyer to another and from one trade
to another.[8] In this economy all the market participants share, more
or less, some responsibility of the determination of exchange prices
which is monopolized by a fictional market auctioneer in the Walrasian
equilibrium theory.

In such a monopolistic-monopsonistic economy, exchange prices may
be settled by bilateral negotiations between a seller and a buyer, both
exerting their monopolistic and monopsonistic influences directly on the
process of price determination. However, the price negotiation itself is
a costly economic activity. It requires both the seller and the buyer
to have detailed information about the other; it is frustrating and time-

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[8]G. Stigler, "The Economics of Information," _Journal of Political Economy_
(June 1961). It is often misunderstood that the dispersion of prices
is a necessary condition for the existence of monopolistic or monopsonistic
powers among market participants in the market for a homogeneous commodity.
But it is almost tautological to point out that so long as search is costly
there is no way for an individual trader to ascertain whether prices are,
in fact, uniform or dispersed. Moreover, if the price dispersion is per-
ceived to be smaller, then it becomes less worthwhile to keep searching
and traders may choose to be less informed of the actual price distribution
than before, which may, in turn, enhance the monopolistic power of price-
setters. Of course, in order to settle this issue, we need a model of the
market as a whole. See Rothschild, _op.cit._

consuming and its outcomes are uncertain. If a seller must deal with
large number of prospective customers, or if a buyer must buy from many
sellers, the costs incurred in each price negotiation may be absurdly
high relative to the profit to be gained from a resulting transaction.
Then the trader may find it less costly to abandon the opportunities of
price negotiations and adopt the policy of <u>uniform, take-it-or-leave-it
pricing</u>. By this we mean that the seller or the buyer announces a single
price for his selling product or desired commodity and asks his prospective
buyers or sellers to trade at this price if they find it advantageous to
do so or else leave it. This trader can save the costs of troublesome
price negotiations at the expense of extra profits that could be gained
if discriminatory pricing were made possible by negotiations. It should
be emphasized that the uniform price thus unilaterally announced is not
only the mere figure of an exchange ratio between the commodity in question
and the means of payment (money) but also functioning as a conveyor of
partial information concerning prospective trading opportunities obtainable
from this trader. Clearly, the larger the number of his dealings, the
more effective can the uniform price function as an information signal.
It is like a public good. Moreover, for the price to work as an effective
signal it must be announced by the firm <u>prior to</u> actual transactions and
fixed for a while to enhance its reliability.

If one side of the market adopts the uniform, take-it-or-leave-it
pricing policy, traders in the other side of the market have little choice
but to take the unilaterally imposed price as a given parameter. Thus,
even though the traders in the clustered side of the market are potential

monopolists or monopsonists, they are forced to behave as if they were price-takers; they can influence the determination of prices only indirectly through their search for the most advantageous trading partners but not directly.

In any case, as K. Arrow forcefully points out, the immediate location of price or wage settings depends essentially on the structure of the market in question, even in the case of the well-organized market for a homogeneous commodity.[9] However, in the capitalist economy, the firm is **by definition** a large economic agent relative to a household, which hires not a small number of workers and usually sells its products to not a small number of customers. It often differentiates its products from the rivals' by slightly changing physical characteristics, by offering special services to the customers, by exploiting the priviledges allowed by patents and trademarks, by relying on its reputations, or through differences in geographical location. Therefore, it is usually the firm as a seller of its own products that monopolizes the control on the price and asks its prospective customers to take this price or else leave it.[10]

The situation in labor markets can be easily understood if we reinterpret workers as buyers of the money-wage and the firm as its seller; the means of payments for their trades are, of course, various kinds of labor-services.

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[9] See K. J. Arrow, op.cit.

[10] In fact, it is almost impossible to discuss the problem of search costs without imagining the market of some differentiated products. If the commodity is homogeneous and all the shops are located at one spot, then why is search necessary? This is exactly the structure of markets in which the Walrasian paradigm works.

The firm can differentiate its money wage by geographical location, by special priviledges to its facilities, by types of works, etc., even though it is selling the most homogeneous commodity--money--to workers. Therefore, in the markets for unorganized labors it is again the firm that adopts the policy of uniform, take-it-or-leave-it pricing, so long as it employs workers and is not employed by them.

The economy which we shall examine in the present paper is the one in which every firm is a monopolist for its own products and a monopsonist for its workers and adopts the uniform, take-it-or-leave-it pricing policy in both the markets. This picture of the economy is akin to that of the theory of monopolistic competition initiated by J. Robinson and E. H. Chamberlain among others.[11] This is certainly a bad description for some producers' goods markets and for many organized labor markets, in which prices and/or wages are settled by bilateral or multilateral negotiations. Such oligopolistic price and wage determinations require a careful game-theoretic analysis, which is beyond the scope of the present paper.

3. The Basic Framework of the Model of a Firm

Let us consider a firm which produces a single product by employing a single variable factor, labor-service, in the non-Walrasian economy.[12] At the beginning of period, say t, the firm announces its product price

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[11] E. Chamberlain, The Theory of Monopolistic Competition, 8th Edition (Cambridge; MA: Harvard University Press, 1962); J. Robinson, The Economics of Imperfect Competition (London: Macmillan, 1933). Their partial equilibrium framework was developed in the direction of general equilibrium by R. Triffin, Monopolistic Competition and General Equilibrium Theory (Cambridge, MA: Harvard University Press, 1940).

[12] We can easily extend our model to the case where the firm produces multiple products by employing many variable factors.

p_t and its money wage w_t and starts selling its product and recruiting workers. Then, customers attracted by this product price send orders to its sales department and workers who want to work at the announced money wage apply for their jobs to its recruiting department.

The firm is both a monopolist for its product and a monopsonist for its workers. The product demand schedule and the labor supply schedule which its entrepreneur faces in the current markets are usually downward-sloping with respect to the current product price and upward-sloping with respect to the current money wage, respectively. However, an individual firm can be neither a perfect monopolist nor a perfect monopsonist. Customers' decisions whether to send orders to its sales department and prospective workers' decisions whether to apply for their jobs to its recruiting department are influenced not only by its product price and money wage but also by numerous other factors that are beyond the direct control of an individual firm. The latters may include such variables as the spectrum of rivals' current prices and money wages, sizes and distributions of the aggregate income and the aggregate labor supply, the extent of unemployment pool, weather, etc... Some of them are, in principle, observable in the markets by the end of period t ; but when the entrepreneur announces p_t and w_t they are unknown parameters to be regarded as random variables. The product demand and labor supply schedules may also depend on such time-series data as prices and money wages, labor turn-over rates, queues of unfilled orders, etc. of this firm as well as its rivals up to period $t-1$.

However, that the demand for the firm's product and the labor
supply to the firm are influenced by the variables listed above should
not be confused with the statement that the entrepreneur actually takes
account of all these variables in making his decisions. On the contrary,
in our non-Walrasian economy, he cannot know how all these variables
interdependently affecting product demand and labor supply. In other
words, his subjective demand and supply schedules may differ from the
true or objective demand and supply schedules. And, of course, it is
the subjective schedules that are only relevant to the entrepreneur's
short-run decisions in disequilibrium markets. We must pay attention
to the objective schedules only when we shall examine the workings of
the whole economy consisting of many interdependent monopolistic and
monopsonistic firms.

Let us assume that the entrepreneur's <u>subjective product demand
function</u> and the <u>subjective labor supply function</u> can be respectively
expressed as follows:

(1) $$x_t = X(p_t, a_t) ,$$

(2) $$\ell_t = L(w_t, b_t) ,$$

where a_t is a vector of random variables which the entrepreneur believes
affecting the product demand in period t, b_t is a vector of random
variables which he believes affecting the labor supply in period t,
x_t is the amount of orders he expects to receive in period t, and
ℓ_t is the number of workers expected to apply for the jobs in period t.

Both x_t and ℓ_t are regarded as random variables at the beginning of period t .

We suppose that both (1) and (2) satisfy the following conditions.

(3a) $X(P,a) > 0$ and $\dfrac{\partial X(p,a)}{\partial p} = X_p < 0$ for $0 \leqq p < \bar{\bar{p}}$, $X(p,a) = 0$

for $p \geqq \bar{\bar{p}}$, where $0 < \bar{\bar{p}} \leqq +\infty$,

(3b) $L(w,b) = 0$ for $0 \leqq w \leqq \underline{\underline{w}}$, $L(w,b) > 0$ and $\dfrac{\partial L(w,b)}{\partial w} = L_w > 0$

for $w > \underline{\underline{w}}$, where $0 \leqq \underline{\underline{w}} < +\infty$,

(4a) $\dfrac{\partial^2 \{pX(p,a)\}}{\partial p^2} = 2X_p = pX_{pp} > 0$ for $0 \leqq p < \bar{\bar{p}}$,

(4b) $\dfrac{\partial^2 \{wL(w,b)\}}{\partial w^2} = 2L_w + wL_{ww} > 0$ for $w > \underline{\underline{w}}$,

(5a) $\lim\limits_{p \to \bar{\bar{p}}} pX(p,a) \geqq 0$, $\lim\limits_{p \to 0} pX(p,a) < +\infty$,

(5b) $\lim\limits_{w \to \underline{\underline{w}}} wL(w,b) = 0$, $\lim\limits_{w \to \infty} wL(w,b) = +\infty$.

The conditions (3a) and (3b) imply, among other things, that the entrepreneur believes that his firm is monopolistic in the current product market and monopsonistic in the current labor market. The conditions (4a) and (4b) are standard assumptions in the theories of monopoly and monopsony; namely, they are respectively equivalent to assuming that the marginal revenue curve is downward-sloping and that the marginal

factor-cost curve is upward-sloping in Marshallian quantity-price diagrams. The remaining conditions in (5) are imposed so as to take care of the behaviors of (1) and (2) at the end points.

By the end of period t, the entrepreneur observes the actual amount of orders, $\bar{x}_t$, he received during period t and the actual number of workers, $\bar{\ell}_t$, willing to work at the announced money wage w_t. Now he can hire $\bar{\ell}_t$ workers if he wishes and he can accept $\bar{x}_t$ orders if he has sufficient outputs at hand. However, this does not necessarily mean that the entrepreneur actuarially hires all the willing workers and accepts all the received orders. Indeed, they only impose the following market constraints on his ex post policies on sales and employment.

$$(6) \qquad\qquad s_t \leq \bar{x}_t \; ,$$

$$(7) \qquad\qquad n_t \leq \bar{\ell}_t \; ,$$

where s_t is the sales or the actual amount of orders he has accepted during period t and n_t is the actual number of workers he has hired during period t.

It goes without saying that the entrepreneur cannot pursue both the sales policy and the employment policy independently. They are interconnected through the firm's production activity. Let us assume that its production activity can be summarized by a production function:

$$(8) \qquad\qquad q_t = F(n_t, k_t) \; ,$$

where k_t is an index of real capital endowed at the beginning of period

t in the forms of machines, buildings, technical-knowhows, managerial

skills, intermediate products, etc., and q_t is the level of outputs

producible from n_t workers with the help of the existing real capital

k_t . We shall assume that labor-service is an indispensable productive

factor and its marginal product is positive.

$$(9) \qquad F(0,k) = 0 \quad \text{for} \quad k \geqq 0 \; , \quad \frac{\partial F(n,k)}{\partial n} = F_n > 0 \; .$$

We shall also suppose that the production function (8) is twice-differentiable,

strictly concave and satisfying the boundary conditions specified below.

$$(10) \qquad \frac{\partial^2 F(n,k)}{\partial n^2} = F_{nn} < 0 \; , \quad \lim_{n \to 0} F_n(n,k) = +\infty \; , \quad \lim_{n \to \infty} F_n(n,k) = 0 \; .$$

But these strong conditions are imposed solely for an expository simplicity.

Most of our subsequent arguments will remain intact even if we drop the

conditions (10) and allow weakly concave or linear or even mildly increasing

returns production functions. The so-called U shaped marginal cost case

which presupposes an increasing S shaped production function can be

also dealt with within our framework. Let I_t be the level of inventory

inherited from the previous period and Φ_t is the level of inventory

the entrepreneur plans to carry over to the next period. Then, by the

end of period t he can supply $q_t + I_t - \Phi_t$ in the product market.

In other words, he is entitled to accept the above amount of orders without

embarrassing his clients. Thus, we have

(11)
$$s_t \leq F(n_t, k_t) + I_t - \phi_t .$$

The argument given above implicitly presupposed that the production period is negligible. However, if we reinterpret s_t as the number of products which the entrepreneur has promised to his customers to be able to deliver in near future, we may dispense with this rather restrictive assumption in establishing (11). According to this interpretation, actual delivery of products will be made in some future period after the production has been completed, while the sales contracts between our entrepreneur and his customers are signed in the present period.[13] Of course, this implicitly assumes the absence of technological uncertainties in the production activity.

4. The Optimal Sales and Employment Policies

Let us now turn to the examination of the entrepreneur's optimal policies on product price, money wage, sales and employment. It is clear that he cannot make all of these decisions simultaneously. In fact, both product price and money wage must be determined and announced at the beginning of period t, while the full determination of both employment and sales must wait until the close of the period when he obtains full knowledge of the actual product demand and labor supply that have been induced by his price and wage and revealed in the product and labor

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[13]Of course, this is not the only possible modeling of the relation between production and sales. In our previous paper cited in footnote 1, we considered another model in which the entrepreneur can sell products only after their production has been completed and they are actually in his hands. It seems that both versions are practiced in the real economy.

markets. Therefore, the information structure on which his sales and employment decisions are based are different from and newer than the one on which the price and wage decisions are based. In order to take account of this evolutionary information structure, we shall reverse chronological order of the entrepreneur's decisions and examine his sales and employment policies first.

Let us assume for a while that the entrepreneur is a myopic maximizer who ignores all the dynamic interactions between his decisions in this period and his economic activities in the future periods. This assumption has at least three implications. Firstly, price and money wage in period t are decided without taking into consideration of their effects on future product demand and labor supply. In other words, he neglects both the learning processes of his prospective customers and workers who are searching in the markets based upon their past experiences and also ignores oligopolistic interdependence with his rivals. Secondly, since his world ends at the close of the present period, he has no intention to carry over inventories beyond the current period; namely, we have $\Phi_t = 0$ in (11). In the following, however, we shall allow the possibility that the firm has somehow inherited some inventory $I_t \geqq 0$ from the past periods. Thirdly, we treat the index of real capital k_t as an exogenously determined parameter. In Appendix 2 we shall relax this myopic assumption and briefly discuss the firm's optimal inventory adjustment, although we shall stick to the assumption of the exogeneity of real capital throughout the present paper.

The single-period profit is written as

(12) $$r_t = P_t s_t - w_t n_t \ .$$

It involves no random element at the end of period t . Therefore, in the determination of s_t and n_t , we do not have to worry about the entrepreneur's attitude towards risk. Thus, the problem imposed on him at the end of period t is to find s_t and n_t that maximizes the single-period profit, (12), subject to the two market constraints (6) and (7) and to the single production constraint (11).

Define the Lagrangean function as:

(13) $$P_t s_t - w_t n_t - \lambda(s_t - \bar{x}_t) - \mu(n_t - \bar{\ell}_t) - \nu\{s_t - F(n_t, k_t) - I_t\} \ ,$$

where $\lambda \geq 0$, $\mu \geq 0$ and $\nu \geq 0$ are the Lagrangean multipliers associated with three inequalities, (6), (7) and (11). The Kuhn-Tucker conditions to this constrained maximization problem are given by

$$P_t - \lambda - \nu \leq 0 \ , \quad s_t \geq 0 \ , \quad (P_t - \lambda - \nu)s_t = 0 \ ,$$

$$\nu F_n(n_t, k_t) - (w_t + \mu) \leq 0 \ , \quad n_t \geq 0 \ , \quad \{\nu F_n(n_t, k_t) - (w_t + \mu)\}n_t = 0 \ ,$$

$$s_t \leq \bar{x}_t \ , \quad \lambda \geq 0 \ , \quad (s_t - \bar{x}_t)\lambda = 0 \ ,$$

$$n_t \leq \bar{\ell}_t \ , \quad \mu \geq 0 \ , \quad (n_t - \bar{\ell}_t)\mu = 0 \ ,$$

$$s_t \leq F(n_t, k_t) + I_t \ , \quad \nu \geq 0 \ , \quad \{s_t - F(n_t, k_t) - I_t\}\nu = 0 \ .$$

Because of the strict-concavity of the production function, these conditions

uniquely determine the optimal sales level s_t^* and the optimal employ-ment level n_t^* .

A close inspection of the Kuhn-Tucker conditions will lead us to realize that s_t^* and n_t^* can be characterized by the following switching rules.[14]

$$(14) \qquad n_t^* = \min[\bar{\ell}_t, \ G(\bar{x}_t - I_t, \ k_t), \ N_t^*] \ ,$$

$$(15) \qquad s_t^* = F(n_t^*, \ k_t) + I_t = \min[F(\bar{\ell}_t, \ k_t) + I_t, \ \bar{x}_t, \ Q_t^* + I_t] \ .$$

In the above formulae, $G(x,k)$ is defined as the inverse function of $F(n,k)$ with respect to n , i.e., $F(G(x,k), \ k) = x$, with the under-standing that $G(x,k) = 0$ when $x \leqq 0$, and N_t^* and Q_t^* are defined as the level of employment and the level of output at which the so-called competitive marginal productivity principle holds; i.e., they are defined respectively as follows,

$$(16) \qquad P_t F_n(N_t^*, \ k_t) = w_t \ ,$$

$$(17) \qquad Q_t^* = F(N_t^*, \ k_t) \ .$$

By (10) both N_t^* and Q_t^* are uniquely determined so long as neither P_t nor w_t is zero. It is also easy to see from (16) and (17) that they are continuous functions of a predetermined price-wage ratio P_t/w_t and a given index of real capital k_t ;[15] i.e.,

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[14] Equations (14) and (15) may be regarded as a microeconomic justification of rather ad hoc macro employment and output policies adopted by Solow and Stiglitz, and Barro and Grossman. R. Solow and J. Stiglitz, "Output, Employment and Wages in the Short Run," Quarterly Journal of Economics, LXXXII (November 1968); R. Barro and H. Grossman, "A General Disequilibrium Model of Income and Employment," American Economic Review, LXI (March 1971).

$$N_t^* = N^*(p_t/w_t, \, k_t) \, ,$$

$$Q_t^* = Q^*(p_t/w_t, \, k_t) \, ,$$

where both N_t^* and Q_t^* are increasing in p_t/w_y , Q_t^* is increasing in k_t , and N_t^* is increasing in k_t if $\partial^2 F(n,k)/\partial n \partial k = F_{nk} < 0$ and decreasing in k_t if $F_{nk} > 0$.

We shall call Q_t^* and N_t^* the economic productive capacity and the capacity employment, respectively. At the capacity employment level our entrepreneur is, in fact, maximizing the single-period profit (12) if he were not to be bound by a market constraint, (6) or (7). It will not pay for him to hire more than N_t^* workers and expand the production beyond Q_t^* , even if he can, for it would only reduce his profit. The economic capacity Q_t^* should not be confused with the so-called physical capacity for the firm's production. Our Q_t^* depends not only on the real capital stock k_t endowed in the firm but also on the price-wage ratio p_t/w_t set by the entrepreneur at the beginning of the period.

We are now in a position of interpreting our optimal employment and sales policies given by (14) and (15). They state that our entrepreneur's sales-cum-employment activity is bound either by the realized supply in the labor market, by the realized demand in the product market, or by the economic productive capacity and the associated capacity employment.

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[15] If the production function (8) exhibits constant returns to labor input n , i.e., if $y = cn$, $c > 0$, then $Q^* = N^* = +\infty$ for $p \geq cw$ and $Q^* = N^* = 0$ for $p < cw$. If it is subject to increasing returns to scale, they also become unbinding at least for a certain domain of the values of p and w . In the case of S-shaped production function, they are discontinuous functions of p/w .

When the labor supply $\bar{\ell}_t$ turns out to be the smallest component in the right-hand-side of equation (14), the labor market condition becomes a constraint on our entrepreneur's activity. In this case of labor shortage, he must hire all the workers willing to work at the announced money wage w_t, while he refuses to accept excess orders $\bar{x}_t - F(\bar{\ell}_t, k_t) - I_t$ received in the product market, thus leaving some customers frustrated, and also fails to utilize the economic capacity fully; i.e., $F(\bar{\ell}_t, k_t)/Q_t^* < 1$.

When the demand for product $\bar{x}_t$ turns out to be the smallest component in the right-hand-side of equation (15), the product market condition restricts our entrepreneur's activity. In this deficient demand case, he must accept all the orders $\bar{x}_t$ received from his customers, while he must refuse to hire some excessive workers $\bar{\ell}_t - G(\bar{x}_t - I_t, k_t)$, even though they are eager to accept the announced money wage w_t. He must also lower the capacity utilization rate $(\bar{x}_t - I_t)/Q_t^*$ below unity. This case might be regarded as a microeconomic characterization of the <u>Keynesian involuntary unemployment</u> caused by the deficiency of effective demand in the product market. Finally, the entrepreneur may find that the economic capacity Q_t^* and the associated capacity employment N_t^* are binding his activity. Then, he hires only N_t^* workers in the labor market and accepts only $Q_t^* + I_t$ orders in the product market. In other words, we observe that the involuntary unemployment, $\bar{\ell}_t - N_t^*$, and the unfilled orders, $\bar{x}_t - Q_t^* - I_t$, coexist in this case of insufficient economic capacity.

5. On Involuntary Unemployment

In our very simple model of an individual firm, we could grasp two different but closely related sources of involuntary unemployment. The one lies in the deficiency of effective demand in the product market and the other is due to the insufficiency of its economic capacity of production. In either case some workers are involuntarily unemployed in the sense that their offers are rejected by their employer, even if they are willing to accept the money wage announced by him and would work in his firm if they were offered jobs there.

One of the definitions of involuntary unemployment given by Keynes is that the marginal disutility of employment falls short of the marginal product of labor. Keynes maintains that "the mere existence of an insufficiency of effective demand may, and often will, bring the increase of employment to a standstill before a level of full employment has been reached"[16] and that "the insufficiency of effective demand will inhibit the process of production in spite of the fact that the marginal product of labor still exceeds in value the marginal disutility of employment."[16] In other words, "the equality of the real wage to the marginal disutility of employment presupposed by the second postulate [of the classical theory of employment], realistically interpreted, corresponds to the absence of 'involuntary' unemployment."[16] Since Keynes has never denied the first postulate of the classical theory that "the [real] wage is equal to the marginal product of labor,"[16] he uses these two concepts, the real wage and the marginal product of labor, interchangeably.

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[16]All these quotations are from J. M. Keynes, op.cit., pp. 30, 31, 15 and 5, respectively.

However, by allowing the possibility that the product market condition today directly affects the entrepreneur's employment policy today, our theory of involuntary unemployment denies not only the second postulate as Keynes did but also the first postulate of the classical employment theory which he retained. That is to say, in our model the actual employment n_t^* may not be equal to the firm's capacity employment level $N_t^* = N^*(p_t/w_t, k_t)$ at which level of employment the real wage, w_t/p_t, equals the marginal product of labor, $F_n(N_t^*, k_t)$. When the product demand $\bar{x}_t$ is deficient, the real wage may diverge from both the marginal product of labor and the marginal disutility of employment. In other words, both the first and the second postulates of the classical theory are violated. (The marginal disutility of employment in our model can be interpreted as being congealed into the true labor supply schedule which our entrepreneur observes at the end of the period in the labor market.) This deficiet product demand case seems to correspond to the so-called Keynesian involuntary unemployment due to the insufficiency of effective demand. However, our model points out another possible source of involuntary unemployment, which is not directly dependent upon the current state of effective demand. Namely, when the firm's economic capacity $N_t^* = N^*(p_t/w_t, k_t)$ is insufficient the entrepreneur must curtail his employment below the full-employment level. The classical first postulate is retained, while the second one is violated in this case. This is attributable either to the wrong price-wage ratio predetermined at the beginning of the period or to the underaccumulation of real capital in this firm. Since "past expectations, which have not yet worked themselves

out, are embodied in the today's capital equipment with reference to which
the entrepreneur has to make today's decisions,"[17] this is nothing but
a manifestation of our entrepreneur's wrong decisions in the past based
upon incorrect long-term expectations.

Essentially, these involuntary unemployment phenomena are created
by the fact that, in an economy where no Walrasian auctioneer takes charge
of price and wage adjustments, each individual firm must commit itself
to the determination of product price and money wage before it starts
recruiting workers and selling products. Since price and money wage
are chief conveyors of information concerning possible transactions
opportunities to prospective customers and workers, they must be announced
before the customers and workers start searching in the markets and at
least temporarily fixed to facilitate their searches. Therefore, the
entrepreneur's price and wage decisions have to be guided by his expec-
tations of his trading partners' trading preferences which are summarized
in the forms of subjective product demand and labor supply functions.
But expectations are expectations; there is no guarantee that they will
be fulfilled in the actual markets without the omniscient auctioneer.
If the entrepreneur has underestimated the supply of labor relative to
the actual product demand and the capacity employment level, some workers
must be involuntarily unemployed. The involuntary unemployment is, there-
fore, the price that workers are forced to pay for the errors made by

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[17] Keynes, op.cit., p. 50.

entrepreneur because of the latter's imperfect information about his market environment.[18]

Needless to say, the fundamental concern of Keynes in his General Theory is why can the effective demand be persistently deficient and how does the involuntary unemployment persist in the labor market. This is essentially a dynamic macroeconomic problem, in which the problem of volatile fixed investments plays the central role. It seems, however, that the precise microeconomic understanding of how the involuntary un-employment is generated by the deficiency of effective demand or by other reasons is a prerequisite for the more ambitious investigation of the phenomenon of persistent unemployment as a dynamic macroeconomic problem.

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[18] Therefore, our theory of microeconomic involuntary unemployment is a rebuttal against the recently current view that unemployment is essentially self-employment in information collection for the better job prospects. See, for example, A. Alchian, "Information Costs, Pricing and Resource Unemployment," pp. 27-52, in Phelps, op.cit. Keynes would have included this kind of unemployment in the category of "frictional" unemployment in the sense that it is compatible with the classical second postulate whose validity he adamantly denied. The second postulate that the real wage equals the marginal utility of employment is another way of saying that the actual employment is always on the labor supply schedules derived from workers' ex ante utility maximization. However, our theory suggests that the frictional unemployment due to differential search costs between while employed and while unemployed is not the only kind of unemployment workers experience in the non-Walrasian economy. There exist ample possi-bilities that workers are refused to be hired by the entrepreneur even though the current money wage is perfectly agreeable to their ex ante utility maximization. We do not deny that some kind of unemployment is consistent with the workers' ex ante utility maximization and that they are important, but unless workers can control both their supply and their own employment it seems wide of the mark to construct the theory of unemployment based solely on their individualistic utility maximization behaviors.

Note in passing that, as a counterpart of the involuntary unemployment, we also observe in the product market that some orders sent by the customers are sometimes declined by the entrepreneur. These orders are involuntarily unfilled either because the labor market is stringent or because the firm's economic capacity of production is insufficient. We can examine the phenomena of involuntary unfilled-orders in exactly the same way as the phenomena of involuntary unemployment.

6. The Optimal Product Price and Money Wage Policies

So far we have discussed how the entrepreneur makes his decisions on employment, production and sales at the end of period t when he already determined and fixed price and money wage. Our next problem is to examine how he determines these two price variables at the beginning of the period, although this is chronologically a preceding problem.

At the beginning of period t , our entrepreneur has no choice but to regard the product demand and the labor supply which will be revealed by the close of the period as random variables depending upon vectors of random parameters, a_t and b_t . So, when viewed at the beginning of period t , the optimal employment and sales policies, (14) and (15), derived in Section 4, should be reinterpreted as being conditional on the random parameters, a_t and b_t , yet to be realized. We can then rewrite them as stochastic functions,

$$(20) \quad n_t^* = \text{Min}[L(w_t, b_t), \ G(X(p_t, a_t) - I_t, \ k_t), \ N^*(p_t/w_t, \ k_t)] \ ,$$

$$(21) \quad s_t^* = \text{Min}[F(L(w_t, b_t), \ k_t) + I_t, \ X(p_t, a_t), \ Q^*(p_t/w_t, \ k_t) + I_t] \ .$$

Then, uncertain profit in period t calculated at the beginning can be expressed as

$$(22) \qquad r_t = p_t \cdot s_t^* - w_t \cdot n_t^* .$$

If we adopt the expected utility hypothesis, we can determine the optimal product price and money wage by maximizing $E\{u(r_t)|\Pi_t(a_t, b_t)\}$, where $u(\cdot)$ is a von Neumann-Morgenstern utility and $E\{\cdot|\Pi(a,b)\}$ is the expectation operator defined relative to our entrepreneur's subjective joint probability distribution of a_t and b_t formed at the beginning of period t and denoted by $\Pi_t(a,b)$.

However, this requires a very messy calculation. In the present paper, we shall instead assume that our entrepreneur has a poor computational ability and his decisions on product price and money wage are solely based upon his point-expectations of a_t and b_t , to be denoted by $\hat{a}_t$ and $\hat{b}_t$ respectively.[19] Under this simplifying assumption, the entrepreneur can obtain the optimal product price p_t^* and the optimal money wage w_t^* simply by maximizing his expected profit, $\hat{r}_t$, defined below, independent of the functional form of $u(\cdot)$

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[19]This is a drastic simplification. But even if we examine the more general stochastic maximization case we will obtain qualitatively similar results to ours. In fact, our previous paper (Iwai, op.cit.) handled the case where the entrepreneur maximizes the expected linear utility of the profit relative to his subjective probabilities and compared the optimal price and wage policies under uncertainty with those obtained under the degenerate one adopted here, though the model itself is different.

$$(23) \quad \hat{r}_t = p_t \, \text{Min}[F(L(w_t, \hat{b}_t), k_t) + I_t, \, X(p_t, \hat{a}_t), \, Q^*(p_t/w_t, k_t) + I_t]$$

$$- w_t \, \text{Min}[L(w_t, \hat{b}_t), \, G(X(p_t, \hat{a}_t) - I_t, k_t), \, N^*(p_t/w_t, k_t)] \; .$$

We shall show in Appendix 1 that p_t^* and w_t^* are uniquely determined by the following two conditions, if $I_t < X(0, \hat{a}_t)$.[20]

$$(24) \qquad X(p_t^*, \hat{a}_t) = F(L(w_t^*, \hat{b}_t), k_t) + I_t \; ,$$

$$(25) \qquad MR(p_t^*, \hat{a}_t)F_n(L(w_t^*, \hat{b}_t), k_t) = ME(w_t^*, \hat{b}_t) \; ,$$

where $MR(p, \hat{a})$ represents the expected marginal (unconstrained) revenue: $p\{1 + X(p, \hat{a})/pX_p(p, \hat{a})\}$ and $ME(w, \hat{b})$ the expected marginal (unconstrained) expenditure: $w\{1 + L(w, \hat{b})/wL_w(w, \hat{b})\}$.

The first optimal condition states that our entrepreneur must equalize the expected product demand with the expected full-employment product supply. However, since he can manipulate both p_t^* and w_t^* simultaneously, there still remains one degree of freedom. The second optimal condition maintains that he uses up this remaining freedom by adopting the so-called monopolistic-cum-monopsonistic pricing rule, by which we mean that he equalizes the expected marginal revenue product, $MR(p, \hat{a})F_n(L, k)$, with the expected marginal expenditure on wage, $ME(w, \hat{b})$.

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[20]If $I_t \geq X(0, \hat{a}_t)$, w_t^* is any wage between 0 and $\underline{w}$ and p_t^* is determined by $X(p_t^*, \hat{a}_t) = I_t$.

Let us define λ_t^* and ξ_t^* by

$$(26) \qquad \lambda_t^* = L(w_t^*, \hat{b}_t) \ ,$$

$$(27) \qquad \xi_t^* = X(p_t^*, \hat{a}_t) \ .$$

We can interpret λ_t^* as the <u>expected</u> <u>labor</u> <u>supply</u> and ξ_t^* as the <u>expected</u> <u>product</u> <u>demand</u>. Then, the second optimal condition (25) can be rewritten as

$$(28) \qquad MR(p_t^*, \hat{a}_t)F_n(\lambda_t^*, k_t) = ME(w_t^*, \hat{b}_t) \ .$$

Comparing this with the definition of the capacity employment N_t^* given by (16), and noting that $MR(p_t^*, \hat{a}_t)/ME(w_t^*, \hat{b}_t)$ is strictly smaller than p_t^*/w_t^* by (3) and also noting that $F_n(n,k)$ is decreasing in n by (10), we can obtain inequalities:

$$(29) \qquad \begin{aligned} \lambda_t^* &= N^*(MR_t/ME_t, \ k_t) < N^*(p_t^*/w_t^*, \ k_t) = N_t^* \ , \\ \xi_t^* &= F(\lambda_t^*, \ k_t) + I_t < F(N_t^*, \ k_t) + I_t = Q_t^* + I_t \ . \end{aligned}$$

While equation (24) indicates that the entrepreneur expects that both his product and labor markets will be cleared if his expectations $\hat{a}_t$ and $\hat{b}_t$ turn out to be correct, the above inequalities assert that he predicts that he will underutilize the firm's economic capacity if his expectations are fulfilled. It should be noted, however, that this remark applies only in an <u>ex ante</u> sense. No expectations can be free from errors, and <u>ex post</u> anything can happen in the markets. At the end of period t

his actual activity may turn out to be constrained either by the insuf-
ficiency of the economic capacity, by the shortage of the labor supply
or by the deficiency of the product demand, as was extensively discussed
in Section 4.

7. <u>The Dynamic Theory of Price and Wage Adjustments</u>

In the previous section, we discussed the structure of the optimal
product price and money wage policies for our entrepreneur. If we solve
the two optimal conditions, (24) and (25), we can express p_t^* and w_t^*
as functions of point-expectations of random parameters, $\hat{a}_t$ and $\hat{b}_t$,
a given index of real capital, k_t , and a given level of initial inventory,
I_t .

(30) $p_t^* = \pi(\hat{a}_t, \hat{b}_t, k_t, I_t)$, $w_t^* = \omega(\hat{a}_t, \hat{b}_t, k_t, I_t)$.

The functional forms of the above policy functions are dependent upon
specific functional forms of the basic structural equations; namely, the
subjective product demand function (1), the subjective labor supply function
(2) and the (objective) production function (8). Although we can proceed
comparative statics of p_t^* and w_t^* by mechanical use of calculus, we
rather prefer this to be left for interested readers.

Since I_t and k_t are treated as exogenous parameters in our
myopic model, the entrepreneur's decisions on product price and money
wage are governed by his short-term expectations concerning the current
market environment which are expressed in the forms of point-expectations
$\hat{a}_t$ and $\hat{b}_t$. If his expectations are rigid, i.e., if $\hat{a}_t$ and $\hat{b}_t$ are

believed to be fixed, then equations (29) and (30) are all what can be said about his price and wage determinations.

However, our entrepreneur is making his decisions in a non-Walrasian disequilibrium economy in which nobody has perfect information and every-body is searching for the most advantageous trading partners. We have supposed that he gave up differentiating his customers and workers and adopted the uniform, take-it-or-leave-it price and wage policy. This does not mean, however, that he stops searching in the markets. On the contrary, it only means that his searching activity now consists in forming the best point-expectations of the random parameters in his subjective product demand and labor supply functions. In general, he has little choice but to predict them from his own experiences in the past markets. According to the theory of rational expectation, a rational entrepreneur has a certain econometric model, however crude it might be, of the dynamic workings of the economy which he has been participating in.[21] He stores a set of time-series data of observable variables which, he believes, systematically influence the random parameters a_t and b_t within the framework of his own econometric model. He calculates their forecasts $\hat{a}_t$ and $\hat{b}_t$ so as to optimize a certain criterion, for instance, the minimum mean square error of predictions. In the present paper, however, we shall not get involved into detailed discussions on how our entrepreneur

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[21]See J. Muth, "Rational Expectation and the Theory of Price Movements," Econometrica, Vol. 29 (July 1961); and M. Nerlove, "Distributed Lags and Unobserved Components in Economic Time Series," in Fellner, et al., Ten Economic Studies in the Tradition of Irving Fisher (New York: John Wiley, 1967).

can rationally obtain the best point-expectations $\hat{a}_t$ and $\hat{b}_t$.[22] Here,

we simply state that they are often expressed as weighted averages of

time-series observations of relevant variables that appear in his econo-

metric model.

As time goes on our entrepreneur acquires new observations from

his market activities and revises his point-expectations in the light

of new information. We shall suppose in this paper that there are no

institutional pressures against wage and price changes and that other

costs of changing them accrued to the firm are also negligible. Then,

revisions of expectations will necessarily lead its entrepreneur to adjust

its announced product price and money wage according to the optimal price

and wage equations (29) and (30). Thus, dynamics of price and wage deter-

mination by a single firm under no adjustment costs is completely char-

acterized by a system of dynamic equations composed of the optimal price

and wage equations which relates price and wage to expectations of relevant

parameters and of the expectation-formation equations which specify how

the entrepreneur revises his expectations in accordance with new obser-

vations acquired in the current markets.

In order to illustrate this, let us consider the following simple

specification of the model. Assume that the entrepreneur's subjective

product demand function (1), his subjective labor supply function (2)

and the firm's production function (8) are all of the constant-elasticity

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[22]Iwai, op.cit., developed a theory of optimal predictions under the
hypothesis of rational expectation using some of the results in Nerlove,
op.cit.

type, which can be expressed as:

$$(31) \qquad x_t = \left(\frac{p_t^*}{P_t}\right)^{-\eta} y_t \ , \quad \eta > 1 \ ,$$

$$(32) \qquad \ell_t = \left(\frac{w_t^*}{W_t}\right)^{\epsilon} z_t \ , \quad 0 < \epsilon < 1 \ ,$$

$$(33) \qquad q_t = n_t^{\gamma} k_t^{\delta} \ , \quad 0 < \gamma < (1+\epsilon)\eta/\epsilon(\eta-1) \ , \quad \delta > 0 \ .$$

P_t and W_t are aggregate product price and money wage (or some indices of price and wage distributions) of the economy (or the industry) as a whole, and y_t summarizes all the real forces in the economy, such as aggregate real income, which influence the demand for this firm's products, and z_t is an amalgam of all the real variables that affect workers' total labor supply. Equations (31) and (32) state that our entrepreneur believes that customers and workers are free from money illusions and, other things being equal, concerned only with relative price, p_t^*/P_t , and relative wage, w_t^*/W_t . Demand and supply elasticities, η and ϵ , are assumed to be known and invariant over time.[23] We shall assume that P_t and W_t

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[23] When a consumer's preferences are representable by an addilog indirect utility function: $\psi(p,M) = \Sigma_i \alpha_i (M/p_i)^{\beta_i}$, where p_i is the price of ith commodity, M his fixed money income, and α_i and β_i positive constant, his demand function can be expressed as:
$x_i = \alpha_i \beta_i M^{\beta_i - 1} p_i^{-\beta_i - 1} / \Sigma_j \alpha_j \beta_j M^{\beta_j - 1} p_j^{-\beta_j}$. Cf. H. S. Houthakker, "Additive Preferences," _Econometrica_, Vol. 28 (April 1960). Let $\eta = \beta_i + 1$, $P = (\Sigma_j (\alpha_j \beta_j / \Sigma_k \alpha_k \beta_k) M^{(\beta_j - \beta_i)} p_j^{-\beta_j})^{1/\beta_i}$ and $y = (\alpha_i \beta_i / \Sigma_k \alpha_k \beta_k) M/P$, then

are unknown random variables at the beginning of period t but are observable in the markets by the end of period t ; whereas y_t and z_t will be assumed to be unobservable both <u>ex ante</u> and <u>ex post</u>.[24] As for the Cobb-Douglas production function (33), note that it allows not only decreasing returns but also increasing returns to labor input, at least to some degree. Its factor elasticities, γ and δ , are again assumed to be invariant over time and known.

It can be easily shown that the optimal price and wage equations given by (30) can be now expressed by the following log-linear forms, if $I_t = 0$:

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the above demand function becomes equivalent to the one given by (31), although the aggregate price P requires a fairly complicated aggregation procedure. In the special case of CES indirect utility function where $\beta_i = \eta-1$ for all i , the aggregate price can be simply written as

$$P = (\Sigma_j (\alpha_j / \Sigma_k \alpha_k) p_j^{-(\eta-1)})^{-1/(\eta-1)}$$ independently of money income M . A

similar justification for the form of the labor supply function (32) may be possible. It should be noted that we can incorporate workers' leisure-labor choices, etc. into (32) by making the variables z_t as a function of aggregate real wage W_t/P_t .

[24] If the entrepreneur can observe some of the real variables affecting y_t and z_t by the end of period t , then they should be further decomposed into observable and unobservable parts. For the sake of simplicity the present paper will not seek this generalization.

(34) $\log p_t^* = -c_0 + c_1 \log \hat{P}_t + (c_1/\eta)\log \hat{y}_t + (1 - c_1)\log \hat{W}_t$

$$- (1 - c_1)/\epsilon \log \hat{z}_t - c_2 \log k_t \ ,$$

(35) $\log w_t^* = d_0 + d_1 \log \hat{P}_t + (d_1/\eta)\log \hat{y}_t + (1 - d_1)\log \hat{W}_t$

$$- (1 - d_1)/\epsilon \log \hat{z}_t + d_2 \log k_t \ ,$$

where $\hat{P}_t$, $\hat{W}_t$, $\hat{y}_t$ and $\hat{z}_t$ are point-expectations of P_t , W_t , y_t and z_t , respectively; constant coefficients c_i and d_i are all positive and defined by $c_0 = \sigma\epsilon\gamma \log\{\eta(1+\epsilon)/\gamma\epsilon(\eta-1)\}$, $d_0 = \sigma\eta \log\{\eta(1+\epsilon)/\gamma\epsilon(\eta-1)\}$, $c_1 = \sigma\{\eta + \epsilon\eta(1-\gamma)\} < 1$, $d_1 = \sigma\eta < 1$, $c_2 = \sigma\delta(1+\epsilon)$ and $d_2 = \sigma\delta(\eta-1)$, where $1/\sigma = \eta + \epsilon\gamma + \eta\epsilon(1-\gamma) > 0$.

Let us transform these price and wage equations into time-difference forms:

(36) $\Delta \log p_t^* = c_1\Delta \log \hat{P}_t + (1 - c_1)\Delta \log \hat{W}_t + (c_1/\eta)\Delta \log \hat{y}_t$

$$- (1 - c_1)/\epsilon\Delta \log \hat{z}_t - c_2\Delta \log k_t \ ,$$

(37) $\Delta \log w_t^* = d_1\Delta \log \hat{P}_t + (1 - d_1)\Delta \log \hat{W}_t + (d_1/\eta)\Delta \log \hat{y}_t$

$$- (1 - d_1)/\epsilon\Delta \log \hat{z}_t + d_2\Delta \log k_t \ ,$$

where Δ is the time-difference operator defined by $\Delta \log p_t^* \equiv \log p_{t+1}^* - \log p_t^* = \log(p_{t+1}^*/p_t^*) \cong (p_{t+1}^* - p_t^*)/p_t^*$ and so forth. In words, both the rates of changes in price and money wage are linear functions of the rates of changes in expected aggregate price, expected aggregate money wage, expectations of two real random variables and real

capital stock. Adjustments of price and wage are thus seen to hinge
crucially upon how the entrepreneur forms the expectations of the four
point-expectations, $\hat{P}_t$, $\hat{W}_t$, $\hat{y}_t$ and $\hat{z}_t$ in the right-hand-side of
equations (36) and (37).

As an illustrative example, let us assume that our entrepreneur
forms these expectations employing the so-called multiplicative Koyck-
lag formulae and, accordingly, revises their levels based upon the following
multiplicative adaptive expectation equations.

(38)
$$\Delta \log \hat{P}_t = \mu(\log \bar{P}_t - \log \hat{P}_t) ,$$

(39)
$$\Delta \log \hat{W}_t = \nu(\log \bar{W}_t - \log \hat{W}_t) ,$$

(40)
$$\Delta \log \hat{y}_t = \alpha(\log \bar{y}_t - \log \hat{y}_t) ,$$

(41)
$$\Delta \log \hat{z}_t = \beta(\log \bar{z}_t - \log \hat{z}_t) ,$$

where adaptive coefficients, μ , ν , α and β are numbers between
zero and one, and $\bar{P}_t$, $\bar{W}_t$, $\bar{y}_t$ and $\bar{z}_t$ are the realized values of
P_t , W_t , y_t and z_t in period t . While $\bar{P}_t$ and $\bar{W}_t$ are obser-
vable in the markets at least by the end of period t , $\bar{y}_t$ and $\bar{z}_t$
are unobservable. Instead, our entrepreneur can only observe the actual
product demand $\bar{x}_t$ and the actual labor supply $\bar{\ell}_t$ in his own markets,
which are partly determined by $\bar{y}_t$ and $\bar{z}_t$ but also partly influenced
by p_t^*/P_t and w_t^*/W_t . Therefore, the values of $\bar{y}_t$ and $\bar{z}_t$ must be
indirectly inferred from other variables whose observations are available

to him. If the entrepreneur believes that the functional forms of his subjective product demand and labor supply schedules, (31) and (32), are invariant over time, he can solve them for y_t and z_t and compute their realized values as log-linear functions of the observations at hand, $\bar{x}_t$, $\bar{\ell}_t$, $\bar{P}_t$, $\bar{W}_t$, p_t^* and w_t^* .

(42)
$$\log \bar{y}_t = \log \bar{x}_t + \eta \log p_t^* - \eta \log \bar{P}_t ,$$

(43)
$$\log \bar{z}_t = \log \bar{\ell}_t - \epsilon \log w_t^* + \epsilon \log \bar{W}_t .$$

On the other hand, similar log-linear expressions can be obtained for $\hat{y}_t$ and $\hat{z}_t$ simply by rewriting the definitions of ξ_t^* and λ_t^* given by (26) and (27) under the constant-elasticity assumptions.

(44)
$$\log \hat{y}_t = \log \xi_t^* + \eta \log p_t^* - \eta \log \hat{P}_t ,$$

(45)
$$\log \hat{z}_t = \log \lambda_t^* - \epsilon \log w_t^* + \epsilon \log \hat{W}_t .$$

Substituting (38), (39), (42)-(45) into (40) and (41), we can express the adaptive expectation equations of $\hat{y}_t$ and $\hat{z}_t$ in the following way.

(46)
$$\Delta \log \hat{y}_t = \alpha \log(\bar{x}_t/\xi_t^*) - \eta\alpha \log(\bar{P}_t/\hat{P}_t)$$
$$= \alpha \log(\bar{x}_t/\xi_t^*) - (\eta\alpha/\mu)\Delta \log \hat{P}_t ,$$

(49)
$$\Delta \log \hat{z}_t = \beta \log(\bar{\ell}_t/\lambda_t^*) + \epsilon\beta \log(\bar{W}_t/\hat{W}_t)$$
$$= \beta \log(\bar{\ell}_t/\lambda_t^*) + (\epsilon\beta/\nu)\Delta \log \hat{W}_t .$$

The term $\log(\bar{x}_t/\xi_t^*) \cong (\bar{x}_t - \xi_t^*)/\xi_t^*$ is a proportional rate of discrepancy between realized and expected product demand. It is a measure of the entrepreneur's forecasting error of the demand in the product market. However, since ξ_t^* would be the level of output which would be supplied by him if all of his expectations were fulfilled in the current markets, $\log(\bar{x}_t/\xi_t^*)$ might be regarded as an index of "excess demand" pressure in the current product market. By the same token, the proportional rate of discrepancy between actual and expected labor supply, $\log(\bar{\ell}_t/\lambda_t^*) \cong (\bar{\ell}_t - \lambda_t^*)/\lambda_t^*$, might be regarded as an index of "excess supply" pressure in the current labor market. Our entrepreneur revises his expectations of real random variables in his subjective demand and supply schedules when he has found excess demand or excess supply, other things being equal. However, other things may not be equal. In fact, equations (46) and (47) maintain that changes in $\hat{y}$ and $\hat{z}$ are also made by his realization of the forecasting errors in the aggregate price and wage, which, in turn, leads to the revision of their expectations according to (38) and (39). In other words, the excess demand and excess supply in the current markets must be decomposed into two different components--the one consisting of the forecasting error of the real variables and the other being that of the aggregate price (or nominal) variables in the economy.

Combining the adaptive expectation equations, (38), (39), (46) and (47) with equations (36) and (37), we can obtain the following price and wage adjustment equations.

$$(48) \quad \Delta \log p_t^* = (c_1\alpha/\eta)\log(\bar{x}_t/\xi_t^*) - (1-c_1)(\beta/\epsilon)\log(\bar{l}_t/\lambda_t^*) + c_1(1-\alpha/\mu)\Delta \log \hat{P}_t$$
$$+ (1-c_1)(1-\beta/\nu)\Delta \log \hat{W}_t - c_2\Delta \log k_t ,$$

$$(49) \quad \Delta \log w_t^* = (d_1\alpha/\eta)\log(\bar{x}_t/\xi_t^*) - (1-d_1)(\beta/\epsilon)\log(\bar{l}_t/\lambda_t^*)$$
$$+ d_1(1-\alpha/\mu)\Delta \log \hat{P}_t + (1-d_1)(1-\beta/\nu)\Delta \log \hat{W}_t + d_2\Delta\log k_t ,$$

with $\Delta \log \hat{P}_t = \mu(\log \bar{P}_t - \log \hat{P}_t)$ and $\Delta \log \hat{W}_t = \nu(\log \bar{W}_t - \log \hat{W}_t)$.

If we, as an econometrician, possesses time-series data of the variables appearing in the above adjustment equations, we can, in principle, estimate their constant coefficients, which themselves are simple functions of the constant structural parameters, η , ϵ , γ , δ , μ , ν , α and β . If we could somehow estimate the values of these structural parameters, these price and wage adjustment equations can be used not only for predicting the entrepreneur's price and wage policies in disequilibrium markets but also for detecting his perceived market environment summarized by η and ϵ , his expectation-formation structure represented by μ , ν , α and β , and his available technology specified by γ and δ . Unfortunately, the indices of excess demand and excess supply, $\log(\bar{x}_t/\xi_t^*)$ and $\log(\bar{l}_t/\lambda_t^*)$, in equations (48) and (49) are not "revealed" to us in the markets. They exist only in the mind of our entrepreneur. (This is also true for $\Delta \log \hat{P}_t$ and $\Delta \log \hat{W}_t$.) However, after some manipulation they can be transformed into observable variables in the following manner.

It can be easily shown that under the constant-elasticity assumption the expected product demand ξ_t^* is a constant fraction of the economic

capacity Q_t^* , i.e.,

(50) $$\xi_t^* = \theta Q_t^* ,$$

where $\theta = \{(\eta-1)\epsilon\gamma/(1+\epsilon)\eta\}^{\gamma/(1-\gamma)}$ is smaller than unity. This constant

fraction θ is the rate of capacity utilization which is optimal in an

ex ante sense; when the entrepreneur's expectations turn out to be correct,

he will utilize the economic capacity at the rate θ . Thus, it will be

called the normal capacity utilization rate. Decomposing $\log(\bar{x}_t/\xi_t^*)$

and $\log(\bar{\ell}_t/\lambda_t^*)$, substituting (50) and rearranging terms, we obtain:

(51) $$\log(\bar{x}_t/\xi_t^*) = \log(\bar{x}_t/s_t^*) + \log\{(s_t^*/Q_t^*)/\theta\} ,$$

(52) $$\log(\bar{\ell}_t/\lambda_t^*) = \log(\bar{\ell}_t/n_t^*) + (1/\gamma)\log\{(s_t^*/Q_t^*)/\theta\} ,$$

where s_t^* and n_t^* are the optimal (and actually chosen) levels of sales

and employment. Therefore, $\log(\bar{x}_t/s_t^*) \cong (\bar{x}_t - s_t^*)/s_t^*$ is the ratio of

unfilled orders to the actual sales, and $\log(\bar{\ell}_t/n_t^*) \cong (\bar{\ell}_t - n_t^*)/n_t^*$

is the ratio of involuntary unemployment to the actual employment. The

former is nothing but the observable rate of excess product demand and

the latter the observable rate of excess labor supply. They are revealed

to us in the markets. The remaining term $\log\{(s_t^*/Q_t^*)/\theta\} \cong \{(s_t^*/Q_t^*) - \theta\}/\theta$

is the proportional rate of deviation of the actual capacity utilization

rate from the normal rate. In practice, this might be approximated by

a variation in capacity utilization rate around its average computed over

some relevant time periods.

We are now in a position of presenting our price and wage adjustment equations. Substituting (51) and (52) into (48) and (49), we finally obtain

(53) $\Delta \log p_t^* = (c_1 \alpha/\eta) \log(\bar{x}_t/s_t^*) - (1 - c_1)(\beta/\epsilon) \log(\bar{l}_t/n_t^*)$

$$+ c_3 \log\{(s_t^*/Q_t^*)/\theta\} - c_2 \Delta \log k_t + c_1(1 - \alpha/\mu)\Delta \log \hat{P}_t$$

$$+ (1 - c_1)(1 - \beta/\nu)\Delta \log \hat{W}_t \ ,$$

(54) $\Delta \log w_t^* = (d_1 \alpha/\eta) \log(\bar{x}_t/s_t^*) - (1 - d_1)(\beta/\epsilon) \log(\bar{l}_t/n_t^*)$

$$+ d_3 \log\{(s_t^*/Q_t^*)/\theta\} + d_2 \Delta \log k_t + d_1(1 - \alpha/\mu)\Delta \log \hat{P}_t$$

$$+ (1 - d_1)(1 - \beta/\nu)\Delta \log \hat{W}_t \ ,$$

with $\Delta \log \hat{P}_t = \mu \log(\bar{P}_t/\hat{P}_t)$ and $\Delta \log \hat{W}_t = \nu \log(\bar{W}_t/\hat{W}_t)$. (The coefficients of the third terms are defined by $c_3 = \sigma\{\epsilon\alpha(1-\gamma) + (\alpha-\beta)\}$ and $d_3 = \sigma\{\epsilon\beta(\gamma/1-1) + (\alpha-\beta)\}$. Their signs are a little bit ambiguous; but so long as the values of α and β are close we can suppose they are both positive. In the following we shall assume that this is indeed the case.)

Our price and wage adjustment equations say that, other things being equal, the existence of excess demand in the product market will work to increase both product price and money wage and that the existence of excess supply in the labor market will reduce both of them. Moreover, when the economic capacity is overutilized relative to its normal operation rate (if other things are equal) the price will be raised and the money wage will be lowered in order to restore the normal capacity utilization

rate; and conversely for the case of underutilization. The productivity

growth due to capital accumulation tends to lower the price and raise

the money wage. Thus, we have succeeded in establishing the personalized

law of demand and supply. The so-called Walrasian law of demand and supply

is mathematically formulated as $dp/dt = H(\text{Excess-Demand})$, $H(0) = 0$

and $H'(\cdot) > 0$.[25] This ad hoc law is introduced into the Walrasian

equilibrium analysis as a pseudo-behavioral equation of a fictional market

auctioneer or Invisible Hand in a market out of equilibrium. In contrast

with this well-told fable, our price and wage adjustment equations are

deduced choice-theoretically as true behavioral equations of an individual

firm--a Visible Hand--in the economy where no deus ex machina is in sight.

In spite of their formal similarity, the economic implications of

our model of price and wage adjustments are very different from the

Walrasian tâtonnement model of price adjustment.[26] In our model, changes

in price and wage are followed by changes in sales and employment policies.

So long as the entrepreneur's expectations about his market environment

are subject to errors, the burden of their adjustments is partly borne

by workers and customers in the form of involuntary unemployment and

unfilled-orders. In this sense it seems more appropriate to interpret

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[25] See, for example, P. A. Samuelson, Foundations of Economic Analysis (Cambridge, MA: Harvard University Press, 1947).

[26] We should also point out that there exists a spill-over effect between the product market and the labor market in our price and wage adjustment equations. These two markets are interconnected through the firm's production activity.

our price and wage adjustment equations as a microeconomic characteriza-
tion of the so-called <u>Phillips</u> <u>curve</u> <u>relations</u>.[27]

In addition to the variables representing market demand and supply
pressures, $\log(\bar{x}_t/\xi_t^*)$, $\log(\bar{l}_t/\lambda_t^*)$ and $\log\{(s_t^*/Q_t^*)/\theta\}$, and the
variable representing productivity change, $\Delta \log k_t$, adjustments of
price and wage are also correlated with the entrepreneur's revisions
of his expectations of aggregate price and aggregate wage, $\Delta \log \hat{P}_t$,
and $\Delta \log \hat{W}_t$. However, how they are correlated is somewhat ambiguous.
It hinges upon whether the entrepreneur adapts his expectations of P_t
and W_t to their realized values more promptly than his adaptation of
the expectations of y_t and z_t to their realized values; namely, it
depends on whether $\mu \gtreqless \alpha$ and $\nu \gtreqless \beta$. If $\mu < \alpha$ and $\nu < \beta$, we
will have a paradoxical situation in which upward (downward) revisions
of $\hat{P}_t$ and $\hat{W}_t$ will be followed by downward (upward) changes in p_t^*
and w_t^* . Of course, since P_t and W_t are observable random variables
whereas y_t and z_t are not, it is plausible for us to expect that the
formers are more quickly adjusted than the latters. However, even if
we suppose $\mu > \alpha$ and $\nu > \beta$, the sum of the coefficients of $\Delta \log \hat{P}_t$
and $\Delta \log \hat{W}_t$, $1 - c_1\alpha/\mu - (1 - c_1)\beta/\nu$, falls short of unity, so long
as α and β are both non-zero. In other words, our micro Phillips
relations appear to involve <u>dynamic</u> <u>money</u> <u>illusion</u>, even though our
entrepreneur is free from any form of money illusion at least in a static
sense. This apparent money illusion is due to the fact that when he observes

[27]See Nordhaus <u>op.cit</u>. for the survey of recent developments in both
theoretical and empirical studies of the Phillips relations.

excess demand or excess supply in the market it must be attributed not
only to his forecasting error of real variables in the economy but also
to his failure to anticipate aggregate price or wage correctly, as is
shown in equations (46) and (47). Although it is tempting here to make
a general comment on the controversy between proponents of long-run
Phillips relation and those of natural unemployment rate, we shall not
go into this in the present paper. The problem concerning the existence
of long-run Phillips relation is essentially a macroeconomic problem
requiring a careful model-building of the economy as a whole,[28] which goes
far beyond the scope of this article.

Our discussions of the micro Phillips relations have been so far
based upon several naive assumptions concerning the entrepreneur's expec-
tation formations. In particular, the assumption that he predicts the
levels of the random variables according to the Koyck-lag formulae pre-
supposes a very simplistic stationary property about the underlying
stochastic processes generating their dynamic movements. The more realistic
model of micro Phillips relation undoubtedly necessitates the more ela-
borate formulation of the expectation formation, which, in turn, presupposes
a much more complicated econometric model being in the mind of our entre-
preneur. In fact, we can supply unlimited number of alternative price
and wage adjustment equations by introducing various distributed or other
lag formulae. However, the theory of expectation formation is the most
mysterious area in economic science; and, no matter how complicated can
we make their formulations, our main theme remains the same. We shall not,
therefore, pursue this problem further in this already long article.

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[28]For example, from the macroeconomic viewpoint there may exist some cor-
relations between $\log(\bar{x}_t/s_t^*)$ and $\Delta \log \hat{P}_t$ and between $\log(\bar{I}_t/\Uparrow_t^*)$ and
$\Delta \log \hat{W}_t$, which may disturb the Phillips relations expressed in (53) and
(54) in the long-run.

APPENDIX 1

The Determination of the Optimal Price and Wage Policies

The purpose of this appendix is to derive the optimal conditions

for p_t^* and w_t^* given by (24) and (25) in the text. The maximand of

our problem is the expected profit function (23). Since this is not a

concave function of p_t and w_t, we must carefully examine the phase-

diagram of its gradient vectors to find out the optimal solution.

It is easy to see that the non-negative orthant of p-w plane

is partitioned into three regions, A , B and C . (See the diagram

below.) They are respectively defined by $A = \{(p,w)\,|\,Q^*(p/w) \leq X(p)$

and $F(L(w)) + I\}$, $B = \{(p,w)\,|\,X(p) \leq Q^*(p/w)$ and $F(L(w)) + I\}$ and
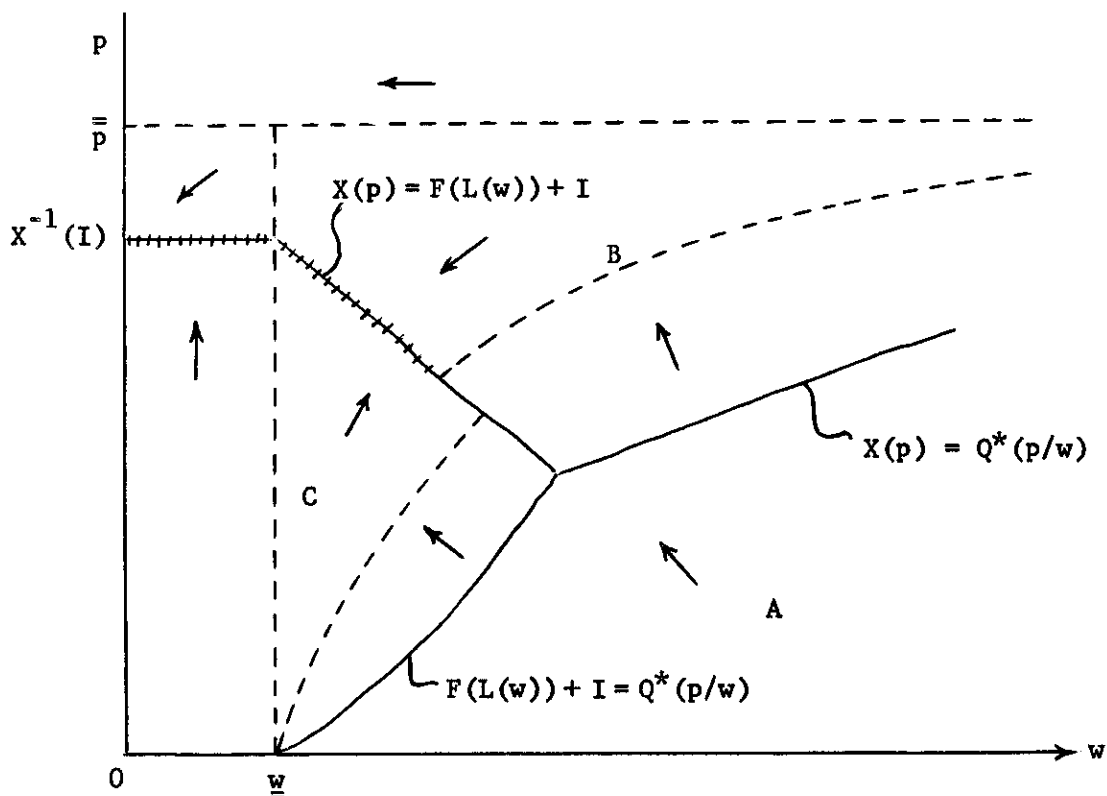
$C = \{(p,w) | F(L(w)) + I \leqq Q^*(p/w) \text{ and } X(p)\}$. (In the following we shall

suppress the parameters, k_t , $\hat{a}_t$ and $\hat{b}_t$ and also time-subscript t

to simplify the notation.) Omitting tedious calculations, positive direc-

tions of normal vectors of the iso-profit contours are indicated by arrows

in the diagram. They point in the directions of higher expected profit.

It is clear that no maximum can attain in the region A , because all

the arrows there point in the south-west direction. Regions B and C

are further divided into three subregions according to the directions of

the arrows. However, it is also clear from the diagram that we can find

no maximum in the interiors of these two regions. Instead we can quickly

conclude that the maximum point which corresponds to a stationary point

of gradient flow vectors is located somewhere along the line of the boundary

between the regions B and C . This boundary line is characterized by

$$X(p) = F(L(w)) + I ,$$

which is nothing but the first optimal condition (24). Along this line

the expected profit function (23) can be expressed as

$$\hat{r} = X^{-1}(F(L(w)) + I)\{F(L(w)) + I\} - wL(w) ,$$

where $X^{-1}(x)$ is the inverse function of $x = X(p)$ defined over

$0 \leqq x \leqq X(0)$. Since this is now a function of a single variable w ,

its optimal solution is easy to obtain. We shall assume that $0 \leqq I < X(0)$.

Then, in view of (3), (4), (5), (9) and (10) the above function attains

the unique maximum at the point defined by the following monopolistic-

cum-monopsonistic pricing rule, if $MR(X^{-1}(I)) > 0$:

$$MR(X^{-1}(F(L(w)) + I))F_n(L(w)) = ME(w) .$$

But if $MR(X^{-1}(I)) \leqq 0$ it is obviously maximized at any w between 0 and $\underline{w}$.

APPENDIX 2

Far-Sighted Policy and Optimum Inventory Adjustment

In this appendix, we shall briefly discuss how our proceding analysis must be modified if we drop the assumption that our entrepreneur is concerned only with a single-period profit in his decision-makings. Suppose now that he extends his planning horizon to more than two but still finite periods and maximizes the discounted sum of expected profits:

$$\sum_{t=0}^{T} \alpha^t \hat{r}_t ,$$

where $2 \leqq T < +\infty$ is a given planning horizon and $\alpha > 0$ is a given discount factor. Then, following the standard backward-induction procedure in the finite-horizon dynamic programming, he can solve this dynamic maximization problem backward from $t = T$ to $t = 0$. In the final period of the planning horizon, however, the entrepreneur is necessarily a myopic optimizer; his world will end at the close of that period. His optimal policies in the final period are exactly the same as the ones we discussed in the text. Thus, we can skip $t = T$ and immediately enter into the analysis of the optimal policies in period $T-1$.

Let us calculate the maximum expected profit in period T by substituting the optimal price and wage policies given by (30) into (23). It can be expressed as a function of $\hat{a}_T$, $\hat{b}_T$, k_T and I_T ; i.e.,

$$\text{max:} \quad \hat{r}_T = R(\hat{a}_T, \hat{b}_T, k_T, I_T) \ .$$

The point-expectations, $\hat{a}_T$ and $\hat{b}_T$, and an index of real capital k_t are uncontrollable by our entrepreneur. However, at the end of period $T-1$, he can decide how much inventory to be carried over to the next period. Thus, if we neglect inventory depreciation, we have

$$I_T = \Phi_{T-1} \ ,$$

where Φ_{T-1} is the level of intended inventory accumulation determined at the end of period $T-1$. (There exists no unintended inventory in this far-sighted model.) In view of (24) and (25) it is easy to show that

$$(A.1) \quad \begin{cases} \partial R/\partial I = R_I(\hat{a}, \hat{b}, k, I) = MR(p^*, \hat{a}) > 0 \quad \text{when} \quad 0 \leqq I < \tilde{I} \\ \qquad\qquad = MR(X^{-1}(I, \hat{a}), \hat{a}) \leqq 0 \quad \text{when} \quad \tilde{I} \leqq I < X(0, \hat{a}) \ , \\ \partial^2 R/\partial I^2 = R_{II}(\hat{a}, \hat{b}, k, I) < 0 \quad \text{for all} \quad 0 \leqq I < X(0, \hat{a}) \ , \end{cases}$$

where $\tilde{I}$ is defined by $MR(X^{-1}(\tilde{I}, \hat{a}), \hat{a}) \equiv 0$ and $p = X^{-1}(x, a)$ is the inverse function of $x = X(p, a)$ in respect to p .

Now the far-sighted entrepreneur's maximization problem at the end of period $T-1$ can be formulated as

Max: $\alpha R(\hat{a}_T, \hat{b}_T, k_T, \Phi_{T-1}) + p_{T-1}s_{T-1} - w_{T-1}n_{T-1}$

s.t.: $s_{T-1} \leq \bar{x}_{T-1}, \quad n_{T-1} \leq \bar{\ell}_{T-1}, \quad s_{T-1} \leq F(n_{T-1}, k_{T-1}) + I_{T-1} - \Phi_{T-1}$,

where his control variables are $s_{T-1} \geq 0$, $n_{T-1} \geq 0$ and $\Phi_{T-1} \geq 0$. (In the following we shall occasionally omit the parameters, $\hat{a}_T$, $\hat{b}_T$, k_T and k_{T-1} and the time subscript T-1 to simplify our exposition.) The Kuhn-Tucker theorem implies that the optimal solution is determined by the following set of complementary slackness conditions.

$$p - \lambda - \nu \leq 0 , \quad s \geq 0 , \quad (p - \lambda - \nu)s = 0 ;$$

$$\nu F_n(n) - w - \mu \leq 0 , \quad n \geq 0 , \quad \{\nu F_n(n) - w - \mu\}n = 0 ;$$

$$\alpha R_I(\Phi) - \nu \leq 0 , \quad \Phi \geq 0 , \quad \{\alpha R_I(\Phi) - \nu\}\Phi = 0 ;$$

$$s \leq \bar{x} , \quad \lambda \geq 0 , \quad (s-\bar{x})\lambda = 0 ; \quad n \leq \bar{\ell} , \quad \mu \geq 0 , \quad (n-\bar{\ell})\mu = 0 ;$$

$$s \leq F(n) + I - \Phi , \quad \nu \geq 0 , \quad \{s - F(n) - I + \Phi\}\nu = 0 ;$$

where λ , μ and ν are Lagrangean multipliers.

Careful examinations of these Kuhn-Tucker conditions enable us to conclude that the structure of the optimal sales, employment and inventory policies in period T-1 are completely characterized by the following switching rule:

$$\begin{cases} s^* = \min[\bar{x}, \max\{F(\bar{\ell}) + I - \varphi(p), 0\}, \max\{Q^* + I - \varphi(p), 0\}] , \\[2mm] n^* = \min[\bar{\ell}, G(\bar{x} - I + \emptyset(w, \bar{x} - I)), \max\{N^*, G(-I + \emptyset(w, -I))\}] , \\[2mm] \Phi^* = F(n^*) + I - s^* , \end{cases}$$

where Q^* and N^* are economic productive capacity and capacity employ-ment which are defined by (16) and (17), $n = G(q)$ is the inverse func-tion of $q = F(n)$ with $G(q) = 0$ when $q \leq 0$. The new functions $\varphi(p)$ and $\emptyset(w,\chi)$ are respectively defined by

$$\alpha R_I(\varphi(p)) \equiv p \quad \text{when} \quad \alpha R_I(0) > p, \quad \text{and} \quad \varphi(p) \equiv 0 \quad \text{otherwise};$$

$$w \equiv \alpha F_n(G(\chi + \emptyset(w,\chi)))R_I(\emptyset(w,\chi)) \quad \text{when} \quad w > \alpha F_n(G(\chi))R_I(0) \quad \text{and} \quad \chi > \tilde{I},$$

$$\emptyset(w,\chi) \equiv 0 \quad \text{when} \quad w \leq \alpha F_n(G(\chi))R_I(0) \quad \text{and} \quad \chi > \tilde{I}, \quad \text{and}$$

$$\emptyset(w,\chi) \equiv \tilde{I} \quad \text{when} \quad \chi \leq \tilde{I}.$$

In view of (A-1) both functions are unambiguously defined. Note that $\varphi(p)$ is also dependent upon $\hat{a}_T$, $\hat{b}_T$, k_T and $\emptyset(w,\chi)$ upon $\hat{a}_T$, $\hat{b}_T$, k_T and k_{T-1}. These far-sighted optimal policies are similar to the myopic optimal policies discussed in the text. However, in con-trast with the latter there exist basically four possible combinations of the optimal sales, employment and inventory policies. They are:

$$
\begin{cases}
\text{(i)} \quad n^* = \max[N^*, \; G(-I + \emptyset(w,-I))], \quad s^* = \max[Q^* - I - \varphi(p), \; 0] \\
\qquad \text{and} \quad \Phi^* = \min[\varphi(p), \; \emptyset(w,-I)], \\[2ex]
\text{(ii)} \quad n^* = \overline{\ell}, \quad s^* = \max[F(\overline{\ell}) + I - \varphi(p), \; 0] \quad \text{and} \quad \Phi^* = \varphi(p), \\[2ex]
\text{(iii)} \quad n^* = G(\overline{x} - I + \emptyset(w, \overline{x} - I)), \; s^* = \overline{x} \quad \text{and} \quad \Phi^* = \emptyset(w, \overline{x} - I), \quad \text{and} \\[2ex]
\text{(iv)} \quad n^* = \overline{\ell}, \quad s^* = \overline{x} \quad \text{and} \quad \Phi^* = F(\overline{\ell}) + I - \overline{x}.
\end{cases}
$$

(i), (ii) and (iii) correspond to the three possible cases in the myopic

sales and employment policies except that the entrepreneur now decides

how much inventory to carry over to the next period. As before, they

state that his activity is constrained either by the economic capacity

for production, by the labor market condition or by the product market

condition. The level of intended inventory is set at $\min[\varphi(p), \phi(w, s^* - I)]$.

However, the last case (iv) is new. It says that our far-sighted entre-

preneur hires all the workers and at the same time accepts all the orders

when the realized product demand and the realized labor supply turn out

to be in a non-degenerate zone characterized by $F(\bar{l}) + I - \varphi(p) \geqq \bar{x}$

$\geqq \max[F(\bar{l}) + I - \varphi(w/F_n(\bar{l})), 0]$. In this case, the level of inventory

can be profitably adjusted at the end of the period so as to clear both

the labor and product markets. In other words, an introduction of the

intended inventory adjustment into our model somewhat lessens the friction

of disequilibrium markets revealed in the form of involuntary unemploy-

ment and involuntary unfilled-orders.

At the beginning of period $T-1$, our entrepreneur must decide

the optimal price p^*_{T-1} and the optimal wage w^*_{T-1} . However, different

from the myopic case his price and wage decisions in period $T-1$ influence

not only his activity in the same period but also his activity in the

next period. Firstly, they will change the optimal inventory policy

at the end of the period, which will, in turn, alter the product supply

condition in the next period. Secondly, they may directly influence the

product demand and labor supply schedules in the next period. Taking

account of these intertemporal dependences and maximizing the discounted

sum of the expected profits in period T and $T-1$, we can obtain the optimal conditions for p^*_{T-1} and w^*_{T-1} . In the special case where product demand and labor supply schedules in period T are believed to be independent of p_{T-1} and w_{T-1} they can be expressed as:

$$MR(p^*_{T-1}, \hat{a}_{T-1})F_n(L(w^*_{T-1}, \hat{b}_{T-1}), k_{T-1}) = ME(w^*_{T-1}, \hat{b}_{T-1}) ,$$

$$\alpha R_I(\hat{a}_T, \hat{b}_T, k_T, \hat{\phi}^*_{T-1}) = MR(p^*_{T-1}, \hat{a}_{T-1}) ,$$

where $\hat{\phi}^*_{T-1} = F(L(w^*_{T-1}, \hat{b}_{T-1}), k_{T-1}) + I_{T-1} - X(p^*_{T-1}, \hat{a}_{T-1})$. Here, we have ignored the possibility of corner solutions. The first optimal condition is the same monopolistic-cum-monopsonistic pricing rule as (25). The second one states that the entrepreneur sets p^*_{T-1} and w^*_{T-1} so as to bring an equality between the discounted marginal contribution of the intended inventory to the expected profit in period T and the expected marginal revenue in period $T-1$. We should also point out that if his expectations turns out to be correct the product demand and labor supply are realized in the zone (iv) in which labor and product markets are simultaneously cleared and the economic productive capacity is underutilized. However, in the general case where demand and supply schedules in period T are directly affected by p_{T-1} and w_{T-1} , we can no longer get any transparent optimal conditions. Suffice it to say here that both p^*_{T-1} and w^*_{T-1} are again expressible as functions of k_{T-1} , k_T , I_{T-1} and the point expectations of the random parameters in demand and supply schedules in period $T-1$ as well as period T .

Repeating similar analyses given above for period T-2 , period

T-3, ..., up to the initial planning period, we can, in principle,

characterize the whole structure of the dynamic optimal policies of

prices, wages, sales, employments and inventories for our far-sighted

entrepreneur. If we again introduce certain expectation-formation equa-

tions of the random parameters in his subjective demand and supply

schedules in the future as well as present periods, we can develop the

fully dynamic theory of price and wage adjustments in the non-Walrasian,

non-myopic disequilibrium economy.