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**A THEORY OF MONEY AND FINANCIAL INSTITUTIONS**

**PART XV**

**A TRADING MODEL TO AVOID TATONNEMENT METAPHYSICS**

**Martin Shubik**

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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART XV

A TRADING MODEL TO AVOID TATONNEMENT METAPHYSICS\*

by

Martin Shubik

1. INTRODUCTION

When Mr. Jones sells 100 shares of General Motors at 43 at 10 a.m. and the stock goes to 47 by 2 p.m. the friendly kindly auctioneer does not step in and run a Walrasian tatonnement telling Mr. Jones that his sale at 10 a.m. was only "for fun." A week or so later when he gets his check for somewhere around \$4,100 and some change and he finds that General Motors has gone to 62 he has plenty of time to reflect as to when to cover his short position.

When one is trying to construct a static theory of distribution and production with no interest in process or control we can abstract out of the actual process of price formation. When trying to write rules for the operation of a stock market, trying to auction off cattle or used cars or trying to price groceries or dresses this is not the case.

This article is devoted to examining some of the problems involved in specifying an explicit trading mechanism which enables us to consider price formation in a non-tatonnement context. In other words where offers

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\*The research described in this paper was undertaken by a grant from the Office of Naval Research.

to buy or sell which are accepted are contracted regardless of what happens to the rest of the market. The traders live with and pay for their errors.

A simple two-sided market is examined. It is sufficiently simple that it can be examined experimentally. It is my hope to examine solutions to far more general structures than the one built here and to examine the market behavior of individuals in experiments with specific market structures. However these are beyond the scope of this particular article.

## 2. A TWO-SIDED MARKET

There is already a large literature on auctions and bidding where the market is specifically run by an auctioneer.<sup>1</sup> This includes models of Dutch auctions and other variations. Vickery,<sup>10</sup> Griesmer and Shubik<sup>3</sup> and others have considered various conditions on information. Details concerning knowledge, sequencing of moves, resolution of ties and so forth must all be specified in the model of the market.

There are a host of different institutions and market mechanisms for moving goods at a price in a modern economy. Some involve the actual transportation of the goods to a single point of sale where buyers and sellers actually meet. Others have goods stored in warehouses and buyers and sellers dealing through brokers virtually anonymously exchanging ownership claims.

To the outsider the ongoing market appears to be a parallel process with a price given mysteriously by the instantaneous matching of supply and demand. The individual trader sees a sequential process where he makes or fails to make an advantageous trade. Both views may be consistent if individual behavior is independent, and is aggregated appropriately.

## 2.1. Bidding

Consider two types of traders trading in two commodities. A trader type is specified by a utility function and an initial endowment. Suppose, for example, there were:  $m$  traders of Type 1 each with a utility function  $\phi(x,y)$  and an initial endowment  $(0,M)$ . Similarly there are  $n$  traders of Type 2 each with a utility function  $\psi(x,y)$  and an initial endowment  $(N,0)$ .

A strategy by a trader  $i$  of Type 1 is to specify a pair  $(p_i, q_i)$  which is an offer to buy  $q_i$  units or less of the first commodity at a price  $p_i$  or less. Where in this simple example the price may be regarded as an exchange rate between the two commodities and  $p_i q_i \leq M$ .

A strategy by a trader  $j$  of Type 2 is to specify a pair  $(r_j, s_j)$  which is an offer to sell up to  $s_j \leq N$  units of the first commodity at a price  $r_j$  or more.

## 2.2. The Market Mechanism

### Moves and Information

In many markets the information advantages of the insiders may be both real and important. A peek at the specialist's book can be of great value, as can be preliminary information on corporate gain or disaster. Price formation and change in a modern mass enterprise economy is heavily a process of evaluation as well as mere exchange.

As a first approximation the aspects of evaluation are ignored in the sense that the utility functions are given and we assume that they are known by all. We also assume away the insider information advantage.

by considering a single market to which each individual may submit a bid in ignorance of what the others are bidding. Thus although from the viewpoint of the clerk recording the bids they are being entered sequentially, those bidders whose names are being entered on the books at approximately the same time have operationally moved simultaneously.

We first consider the one-period market; after which we can generalize to the multiperiod cases. All bids are submitted to the "specialist." He arranges them sequentially in terms of prices bid and prices offered. A tie-breaking rule for bids or offers at the same price is needed. Say they randomize to determine the order of execution.

Convention 1. Possibly the simplest price formation mechanism is as follows. The specialist draws the supply and demand schedules, as is shown in Figure 1.

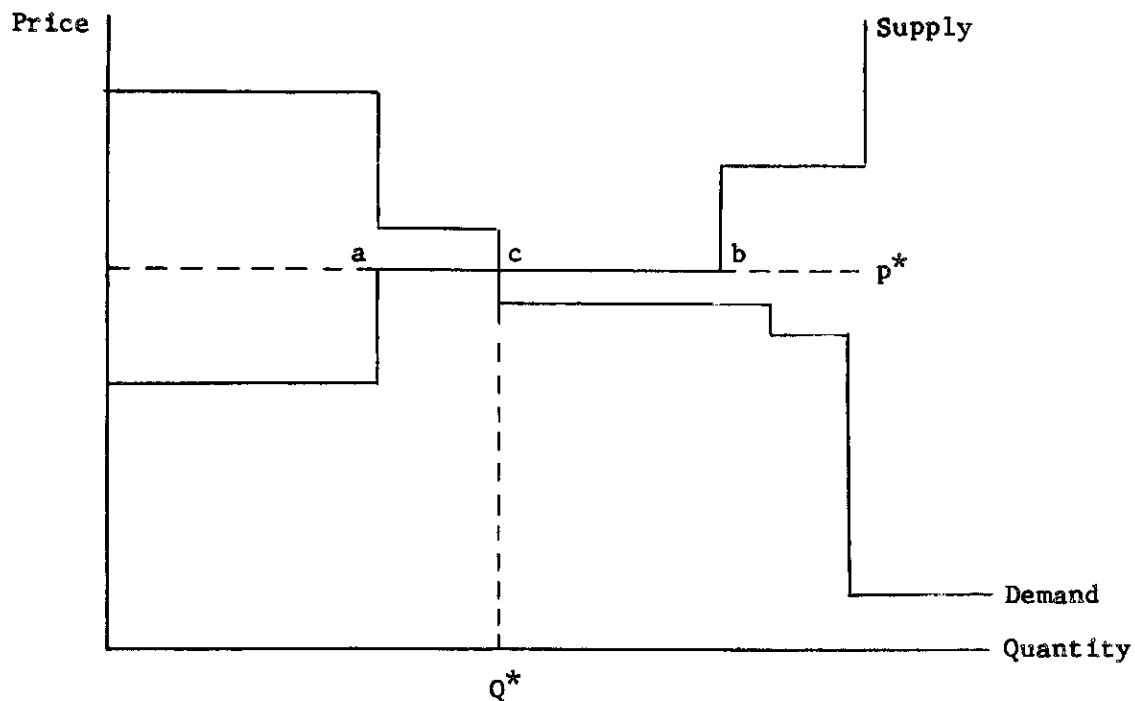


FIGURE 1

He then announces the price at which supply equals demand and executes all trades that can be executed at that price. This may involve partial execution of some order as is indicated in Figure 1 where there is excess supply at the price  $p^*$ .

Convention 2. Another trading convention might be to try to maximize the money value of trade. This would have the specialist execute trades at a series of different prices. If he were working on a percentage based on monetary volume of trade rather than physical volume of trade this would be better for the specialist.

Convention 3. A different trading convention is to have random matching until no further trade can take place. In all instances the payoff to an individual who fails to trade during the period is the value of his initial bundle. No transactions costs have yet been introduced.

### 2.3. The Noncooperative Equilibria

We select Convention 1 with a randomization rule for breaking ties and a fulfillment of partial orders.

The payoffs to traders of Type 1 are:

$$\begin{aligned}
 (1) \quad \Pi_1^i &= \varphi(0, M) && \text{if } p^* > p_i \\
 &= \varphi(q_i, M - p^*q_i) && p^* \leq p_i \text{ and no rationing} \\
 &= \varphi(k_i, M - p^*k_i) && p^* \leq p_i \text{ and the trader is rationed.}
 \end{aligned}$$

There is a similar expression for traders of Type 2.

The game is not yet fully defined. We need a rule to specify what happens when the supply and demand schedules do not intersect.

An example of this is shown in Figure 2.

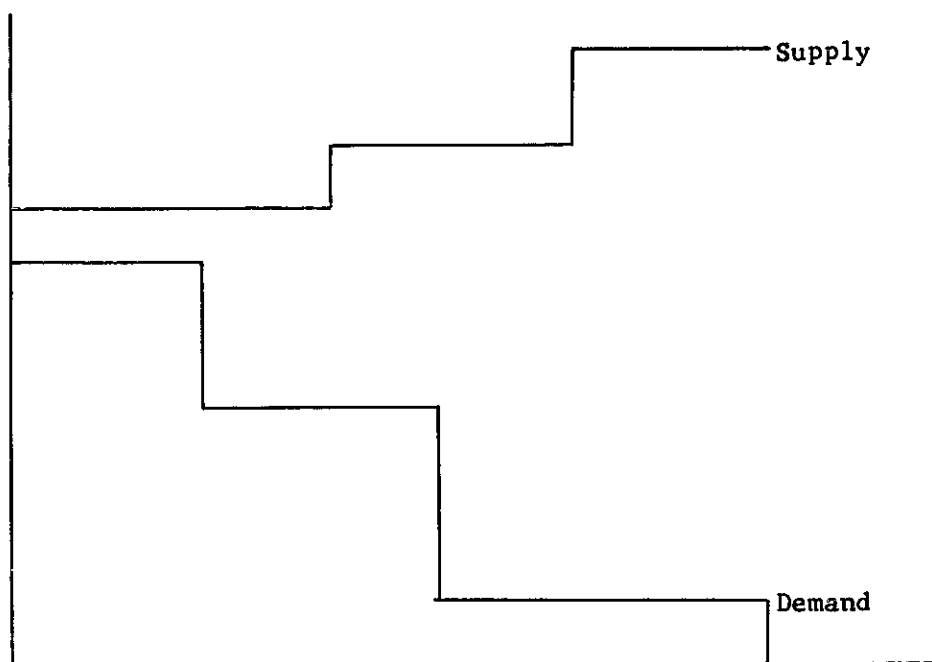


FIGURE 2

The simplest rule is that the market is inactive and no trade takes place.

If we wish to consider a dynamic model then we can charge all traders an aborted trading time cost and consider the next market period.

In terms of a stockmarket model we can introduce a specific role for the specialist in the model by giving him an inventory and have him required to "make a market" while maximizing his own gain or minimizing his loss in doing so. For simplicity in the static model we assume no market.



Consider the simple case where the indifference curves of the traders are continuous and strictly convex.

Theorem 1(a). The pure strategy equilibrium points of this game include the competitive equilibria together with a no trade equilibrium, when  $n \geq 2$  and  $m \geq 2$ .

Theorem 1(b). The market price at a noncooperative equilibrium must be within  $\epsilon(k)$  of a competitive equilibrium where  $\epsilon(k) \rightarrow 0$  as  $k \rightarrow \infty$  where  $k$  is the  $k^{\text{th}}$  replication of the original market.\*

Proof

(A) The No-Trade Equilibrium

If all bidders of Type 1 set their demands at  $c$  and all bidders of Type 2 set their offers at  $c'$  no market will form but no individual will be motivated to change. These points are the ends of the Edgeworth contract curve, in Figure 3. The prices are the slopes  $O_c$  and  $O_{c'}$ .

It is evident that this no trade point will in general be unstable. Any pair of traders, one of each type could improve jointly. Hence although it is technically in equilibrium against disturbance by a single player, once one player has departed others will follow.

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\*The method of replication was originally used by Cournot and adopted next by Shubik. Edgeworth gave a verbal description of replication but was purposely somewhat vague.

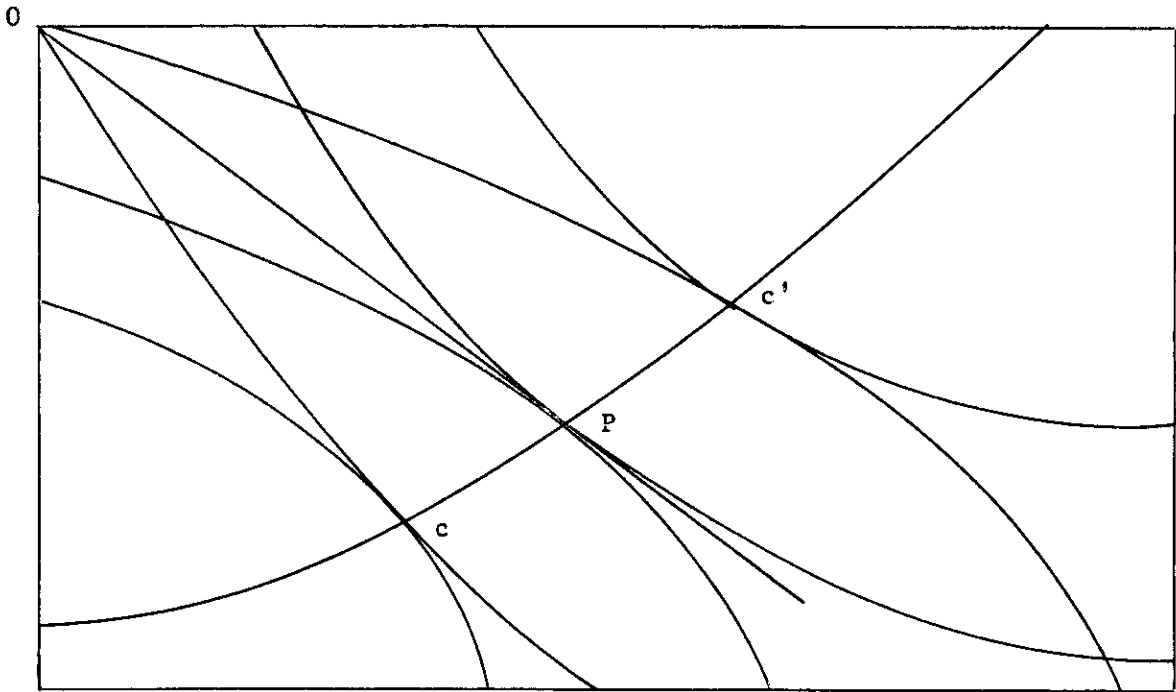


FIGURE 3

**(B) The Competitive Equilibria as Noncooperative Equilibria**

If all traders announce  $p^* = p_e$ , a competitive equilibrium price and all offer individually optimal amounts at that price say  $s_e$  and  $d_e$  then  $ns_e = md_e$  and the competitive equilibrium is also a non-cooperative equilibrium.

If at least one trader offers to buy at a price  $p^*$  or lower while the others offer to buy at prices  $p_1, p_j, \dots$  or lower where these prices are greater than  $p^*$ ; and if at least one trader offers to sell at a price  $p^*$  or higher, while the others offer to sell at prices  $p_a, p_b, \dots$  or higher where these prices are lower than  $p^*$  and if all supply and demand respectively  $s_e$  and  $d_e$ ; then if  $p^* = p_e$  this

is a noncooperative equilibrium with price and sales the same as the competitive equilibrium. Figure 4 shows the type of demand and supply arising from such a strategy. However here  $p^* = p_e$  and  $d'$  and  $s'$  would coincide.

(C) The Noncooperative Equilibrium with Trade

(i) If any traders are potentially active at an equilibrium, all traders are. Consider the ranges of  $p^*$ . Let  $p_c$  and  $p_c'$  be the prices associated with the rays  $oc$  and  $oc'$ ; for  $p^* > p_c$  and  $p^* < p_c'$ , no equilibrium exists as one side or the other would prefer no trade to trade. If  $p_c > p^* > p_c'$ , then there exists a trading range at which all traders can improve over their initial position.

Consider a trader who is out of the market at a suggested equilibrium with trade. There is some trader of the same type who is trading and who is obtaining a payoff at least equal to or better than his no trade payoff. Thus the excluded trader will improve by matching an active trader.

(ii) At an equilibrium with all traders active, all obtain the same price  $p^*$ , hence any extramarginal trader of a single type will offer the same amount for sale as any other player of that type.

From (i) the equilibrium supply and demand schedules must be of the form shown in Figure 4. No demander will offer less than  $p^*$  or supplier ask for more. As there must be a finite supply and demand at  $p^*$  any extramarginal trader will be motivated to change his supply or demand quantity if it differs from  $s^*$  or  $d^*$  which are the price parameter supply and demand at  $p^*$ .

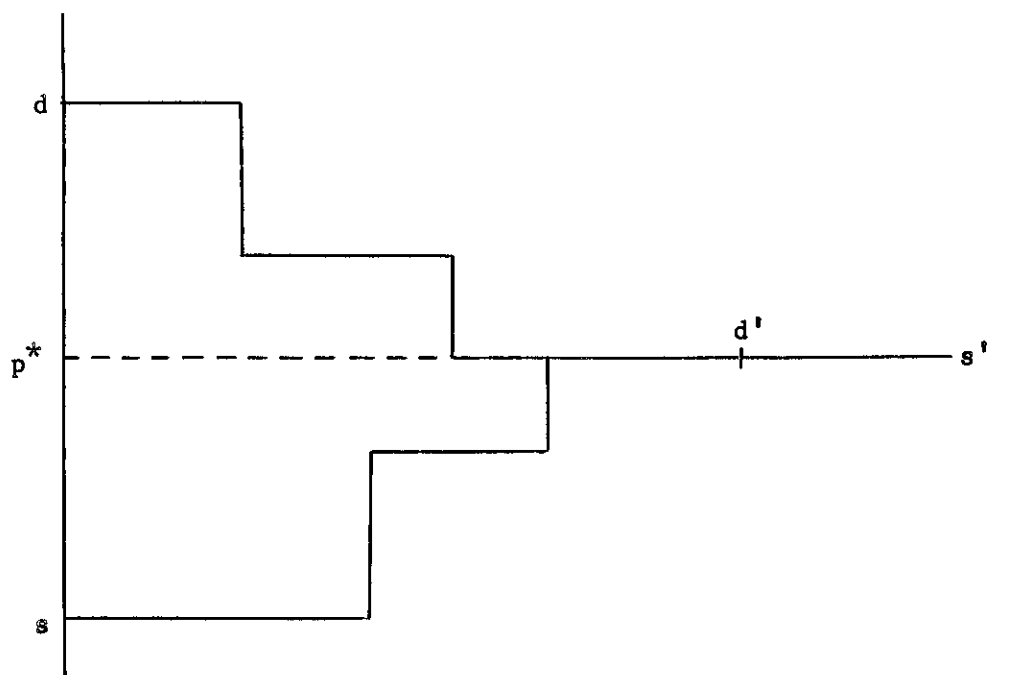


FIGURE 4

(iii) At  $p^*$  if there is excess supply or demand there can be only one trader of the type with the excess who charges  $p^*$ .

If there were two or more, instead of randomizing to determine who is rationed, a trader can change his bid to  $p^* - \epsilon$  or  $p^* + \epsilon$ . This will not change the trading price but as the trader is now served ahead of those bidding  $p^*$  rather than rationed he obtains a higher payoff.

(iv) Either there is one trader on the other side of the market charging  $p^*$  or there are several all offering or demanding  $s^*$  or  $d^*$ .

A single trader will supply or demand  $s^*$  or  $d^*$  or less. If there is more than one buyer facing excess supply at  $p^*$  and he has

demanded less than  $d^*$  he can increase his payoff by enlarging his demand up to  $d^*$ . If he were to wipe out excess supply before attaining  $d^*$  then we are in a position of excess demand and case (iii) applies. A similar condition applies to more than one seller facing excess demand.

(v) The greatest amount of excess supply of the second good there can be at a market equilibrium is  $N$ ; and excess demand for the second good of  $M/p_c$ , (this is the individual holding of the second good times the lowest possible price for the first good that can lead to an equilibrium). This follows immediately from (iii).

(vi) For a fixed  $p^*$  the excess supply or demand in the  $k^{\text{th}}$  replication of the market is at least  $k$  times as large, hence for some  $k$  it will exceed  $M$  or  $M/p_c$ . Thus if excess demand or supply are to remain within these limits then  $p^*$  must approach  $p_e$ .

### 3. A MULTICOMMODITY WORLD WITH MANY MARKETS

The model in Section 2 does not generalize directly when many commodities are considered. The reasons for this failure to generalize are precisely those noted by Shapley and Shubik<sup>6</sup> and Shubik.<sup>7</sup> A commodity must be distinguished to be used as a money in a set of  $g$  markets in a world with  $g+1$  commodities or  $g$  commodities and a money. Otherwise credit conditions must be modeled.

Conjecture. In a world with  $g+1$  commodities with the last commodity used as a money; if all individuals have a sufficient supply of this money then there is a noncooperative game in which individuals buy by

naming a price and a quantity of the good they intend to buy and sell by naming a price and quantity of the good they intend to sell; such that the competitive equilibria are noncooperative equilibria and the equivalent conditions of Theorem 1 hold.

#### 4. THE MULTIPERIOD MODEL

##### 4.1. Experimentation and Behavior

This model lends itself to simple straightforward experimentation. Two groups of individuals are each given payoff functions and an initial endowment. For example traders of Type 1 have  $(0, M)$  and payoffs

$$(2) \quad \pi_1^i = ax - bx^2 + M - py$$

and traders of Type 2 have  $(N, 0)$  and payoffs

$$\pi_2^i = py + c(N-y) .$$

The players are actually paid according to their scores. Traders of Type 1 write down two numbers which they hand to the referee. They are  $(p^i, d^i)$ ; a price offered for the first commodity and the quantity bid for. Traders of Type 2 name  $(q^j, s^j)$  their price and quantity offered. The referee announces the prices and trades.

It is unlikely that with ordinary mortals an equilibrium will be reached on the first play. Some will fail to make sales and will be penalized accordingly.\*

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\*A model related to this one has been used for experimentation by V. Smith.<sup>9</sup>

A sequential game can be defined simply as this market held  $T$  times with the same initial conditions each time, i.e. no inventory carry-over.

In order to fully define the sequential game we must specify information conditions. A reasonable set of conditions are that individuals are told the market price and total sales, as is shown by  $Q^*$  in Figure 1.

Whether the individuals approach the competitive equilibrium in the multistage model depends upon a mixture of economic, strategic, informational and psychological factors. Unlike in a tatonnement model, each step at which a poor move is made is paid for in score.

Although it is trivially easy to show that the repeated competitive equilibrium is an equilibrium, by working a backward induction or merely observing that in its simplest form each period is independent economically from the other. This gives us no inkling of process. It may be conjectured that if errors are costly individuals will try to avoid them faster than otherwise.

#### 4.2. Dynamic Market Models

Suppose that the market described in Section 2 were to meet  $T$  times and the payoffs were

$$(3) \quad \Pi_1^i = \sum_{t=1}^T \varphi(x_t^i, y_t^i) \quad \text{to Trader } i \text{ of Type 1}$$

$$\Pi_2^j = \sum_{t=1}^T \psi(x_t^j, y_t^j) \quad \text{to Trader } j \text{ of Type 2.}$$

Suppose that each individual were supplied each period with  $M$  units of commodity 2 to traders of Type 1 and  $N$  units of commodity 1 to traders of Type 2.

The commodities last only one period. At this point a modeling decision must be made. If an individual tries to sell a perishable, but fails can he use it himself or is it too late? For example, if I send ripe tomatoes to market I may take back rotten tomatoes. Had I not offered them for sale I would at least have their use. On the other hand less ripe tomatoes would return usable. In the second instance error is less costly than in the first.

A comparison of the payoffs for one period of a "returned in good condition" with the "returned rotten" case is given in (1) and (4) below:

$$\begin{aligned}
 (4) \quad \Pi_1^i &= \varphi(0, M - p^*q_1) && \text{if } p^* > p_1 \\
 &= \varphi(q_1, M - p^*q_1) && p^* \leq p_1 \text{ and no rationing} \\
 &= \varphi(k_1, M - p^*q_1) && p^* \leq p_1 \text{ and the trader is rationed.}
 \end{aligned}$$

Another modification with an empirical basis is to charge for new decisions. Thus if the same decision is used in the next period no charge is levied; if a new decision is used this costs some amount  $w$ .

All of the above modifications give absolutely the same state stationary equilibrium points hence the models offer no particularly interesting distinctions except when studied in disequilibrium.

Both the economic and psychological interest comes in seeing if



the speed of adjustment will vary in the different models even if the eventual equilibrium is the same.

#### 4.3. Uncertainty and Evaluation

In the economic world how much do we know about the tastes and actions of others? In general we have some aggregate information such as Australians drink more beer than Americans, or most Americans eat bread and so forth. When amateurs go to an antique auction they usually scarcely know what an item is worth to the others and even themselves. Thus three forms of uncertainty all related to the evaluation of things and behavior must be considered when studying economic adjustment processes they are:

- (1) the forecasting of the behavior of others;
- (2) the estimating of the preferences of others;
- and (3) the estimating of your own preferences.

The first type of uncertainty is strategic and can be approached from a game theoretic or a behavioral point of view. Either way avoids the sterile non-strategic non-behavioral discussions of "perfect foresight."

As a first approximation the other two can be modeled by introducing prior distributions on the payoffs of others and the individual economic agent.

## 5. A DYNAMIC PROCESS WITH FINANCIAL INSTITUTIONS

### 5.1. The Price-Quantity Duopoly

In previous work Shubik<sup>8</sup> and Levitan and Shubik<sup>4</sup> have considered a particular piece of esoterica for economic theorists known as the price quantity oligopoly model. In our attempts to formulate the model we described an open one-sided market which appeared to give rise to phenomena such as the Edgeworth cycle<sup>2</sup> which has always seemed to be unreasonable and an artifact of incomplete or insufficiently complex modeling.

In this paper the basic model is a closed two-sided pair of price-quantity oligopolies. The trick in formulation was to describe a simple explicit and reasonably plausible market mechanism which was completely well-defined.<sup>5</sup>

An interesting and important question in economic dynamics and the design of institutions is what classes of mechanisms or institutions yield the competitive equilibrium as a noncooperative equilibrium. Furthermore how do they differ in terms of adjustment processes.

### 5.2. On Money, Financial Institutions and Markets

Most markets are in general, two-sided and one side pays the other mainly in money or credit. Money is the great decentralizer. A monetary and financial system provide the controls for many decentralized economic processes.

As was noted in Section 3 when three or more commodities are being traded then the full importance of being able to use one to serve as a

money in a set of two-sided markets becomes paramount. However the timing of activities in many decentralized markets immediately causes cash flow and credit problems. Thus the financial components of an economy appear as an integral part of the guidance mechanism whenever the system is not in equilibrium.

### 5.3. The Process Model of Mass Markets

The general equilibrium model is essentially not fully defined for disequilibrium states. The noncooperative game model is defined for all system states. It is a natural basis from which both strategic and behavioral models of an economy can be formulated.

The noncooperative game model provides a means to prove strategic weakness of individuals in low information state mass economies. Neither the low information and communication nor the presence of masses of individuals is made explicit in the mathematical treatment of competitive equilibrium. The strategic weakness of individuals is assumed and they become automatons. In the model presented here individuals have preferences but the usual supply and demand curves of microeconomics do not exist--what exists are bids and offers and a way for resolving them into prices.

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