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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART XIII

TRADE WITH SPOT MARKETS, FLAT MONEY AND INTERNAL BANKING

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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART XIII

TRADE WITH SPOT MARKETS, FIAT MONEY AND INTERNAL BANKING*

by

Martin Shubik

1. ON BANKING

In this paper we continue the exploration of extremely simple models of markets with monetary components. In the last two papers we examined how a simple economy might run utilizing only a fiat money with and without trade in ownership claims.^{12,13} In the models investigated all markets were spot markets and there was no banking and no uncertainty. It appears that hoarding is sufficient to adjust relative prices and if all consumer goods are produced from producer goods then a trade in ownership certificates provides sufficient backing that Pareto optimal trades can be obtained.

2. ECONOMIC MODELS OF TRADE WITHOUT BANKING

The structure of trade and credit calls for us to distinguish a host of different market models where the reasons for the differentiation appear to depend upon a number of variables which are frequently omitted from economic models or are present only implicitly in the models.

Among the more important factors governing the nature of markets and trade are the number of commodities being traded, the number of markets and the number of traders. Interwoven with these are the levels

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of communication among the traders, the degree of trust they have for each other; their knowledge of the markets and the moves of others in the markets; their ability to evaluate, to obtain information and to process the information.

Paramount in the evolution of financial institutions are factors involving risk and uncertainty and the different evaluations of the evolution of future processes made by independent economic agents.

This paper is devoted to a first rudimentary consideration of a banking system and although it will be argued that in a world without uncertainty a monetary system without a banking system might be sufficient for trade without trust;* the additional host of problems which appear with the introduction of uncertainty are not dealt with at this point.

Simple examples are used to illustrate problems, difficulties and solutions. The emphasis throughout is upon explicit detailed microeconomic model building. This entails specifying precisely what information conditions are assumed, how trade is presumed to take place, what the banking mechanism is, how money passes between banks and traders, what are the payoffs at every position of disequilibrium and all the other rules which may be necessary to completely define and specify the process of trade.

Draconian simplification are made in the belief that many (if not most) of the major problems in designing financial control processes for a mass economy are encountered in a one or two commodity world and that it is desirable to separate out economic problems as much as possible.

*Lack of trust and uncertainty are not necessarily related. Individual A may be perfectly certain that B is untrustworthy.

A brief review of several different models of a closed economic system is now presented as a preliminary to the building of models involving money and banking.

Model 1. Barter with no credit or futures markets

The classical model of Edgeworth illustrating bilateral monopoly⁴ (even though it was originally cast in terms of a wage bargain) can and has been used to illustrate one-shot barter trade for two or more commodities. It can be generalized as a many person many commodity one period cooperative trade where considerations of process and dynamics are avoided. This has been done in the work on the core.^{3,5,9}

Model 2. Barter with Futures Markets and Total Trust

This is the important model of the economy presented originally by Walras¹⁸ and elegantly developed by Debreu.² This model is also static, nonstrategic and implicitly simplistic in its treatment of information conditions.

The device of dating future commodities as different commodities provided an interpretation in terms of a multiperiod model with complete trust and one could also read into it trade in a bookkeeping or unit of account money. Both of these features appear in the interpretation of the single budget constraint on the trading possibilities of each individual. The cash flow constraints which play a clear and important role in the decentralized imperfect trust monetary economies we live in, play no role in this model.

Model 3. Trade with a Commodity Money and No Credit

The models suggested by Shubik,¹⁰ and Shapley and Shubik⁷ are based upon a noncooperative strategic version of trade originally implicit in Cournot.¹ However an essential modification in modeling which involved treating one commodity in trade as strategically different from the others was called for in order to be able to deal with this version in a closed economic system.

The noncooperative model of trade with a commodity money (and implicitly no trust) does not necessarily lead to an efficient price system unless there is (according to the appropriate definition) "enough" commodity money in the economy.

Model 4. Trade with a Fiat Money and No Credit

The other extreme from the competitive equilibrium world with all futures markets available for trade is the monetary noncooperative world where all trades are for money and no futures markets exist.

If the money is fiat money we are often confronted with the question "why do people trust fiat money?" In my opinion this is a misleading question as it contains a generally false premise. Most people who are capable of thinking about the properties of money do not put their trust in the paper or the money itself but in the rules of the game. If they have any trust or faith at all it is in the laws, regulations, tax benefits or real goods backing behind the paper. They do not even need that level of trust if the system is designed to coerce them to trade in money and the real costs of alternatives are too high to the individual.

The reasons for the use of fiat money have been discussed elsewhere,¹¹ here we will assume that the use of fiat money is given as an operating rule of the system. Shubik¹² and Shubik and Whitt¹⁶ have considered some simple noncooperative models of trade through time using fiat money without futures markets or credit. They were able to show that sometimes, but not always is it possible to obtain Pareto optimal trades.

The mechanism for adjusting prices in a world with a fixed money supply involves hoarding. Hoarding may be regarded as an act which is equivalent to borrowing and repaying. This is easy to see by considering two individuals at an auction where A wants the goods offered on the first round somewhat more than B and viceversa on the second round. B "lends to A" by staying out of the first market (i.e. hoarding) and A repays B by staying out of the second market (i.e. hoarding).

Coordinated hoarding involves trust. B has to have faith that A will honor his commitment to hoard. Unless there is trust or an internal enforcement mechanism B cannot be guaranteed that A will "repay his loan" by hoarding in the second market.

Model 5. Trade with Two Types of Paper but No Credit

If the consumer goods in an economy are produced from producer goods which last more than one period the possibility for collaboration without trust is increased. The producer goods may either be posted as bond or given in escrow until certain conditions have been fulfilled or ownership paper may be issued against them and this paper may be sold in part or in toto.

If there are no escrow agencies and loan facilities an individual may be able to achieve his goals by selling and buying back his ownership claims rather than by posting them as security.

It is important to note the difference between the two types of paper in use in this economy:

- (1) Fiat money is in fixed supply and must be used to carry out all transactions. It can be used to buy consumer goods, ownership claims to producer goods or it can be hoarded.
- (2) Ownership claims are entitlements to the money proceeds from their sale and the money proceeds from the sale of the goods produced by the producer goods. They are in fact equity shares in the production capacity of the society. They cannot be used to buy goods directly.

Shubik¹³ has provided an example of where trade with fiat money without banking is not optimal if there is no trade in ownership claims, but becomes optimal if trade in ownership claims is allowed. It is conjectured that in a world without uncertainty the existence of producer goods which produce the total supply of consumer goods provides enough backing for ownership paper that Pareto optimality can be achieved without trust.

3. ECONOMIC MODELS OF TRADE WITH BANKING

The introduction of banking poses several extremely different problems. They include the following:

- (a) the rules governing the creation and destruction of money.
- (b) The rules concerning the enforcement of contract and the conditions to be imposed if contract conditions cannot be met.
- (c) The role of banks as public institutions. Is there such a thing as a "neutral banking policy"?
- (d) The interrelationship between banking and uncertainty.

In this paper an attempt is made to deal with aspects of the first two problems. The other two are, in many ways, more important and more complex. However they will be dealt with separately; although a few comments on them now follow.

It appears as if early banking was strongly associated with aspects of trust and uncertainty. These in turn were based on considerations of information processing and communication. The honest well-known merchant becomes a source of credit because he is a focal point of communication and the information contained in the knowledge of his words and deeds is easy to decode.

Both the physical and informational transactions costs involved in handling money make a banking system an extremely useful device. This is especially so when a central government needs to tax or subsidize.

3.1. Dynamics, Disequilibrium and Contract Fulfillment

When we view a market as a strategic process it is necessary to specify the payoffs associated with all feasible outcomes or states that the system can reach. These states may include some outcomes that are clearly a disaster to all.

If we do not assume omniscience then we must allow for the possibility that ex ante individual behavior which appeared to be utterly reasonable could lead to ex post madness. Thus even though the bank's expectations may be extremely low that certain states will be reached it must nevertheless know how to handle the consequences if they are reached.

The above view of a market as a strategic process implies that in order to completely specify the process all positions of disequilibrium as well as all positions of equilibrium must be evaluated.

When ownership claims or goods are sold without fraud or concealment, beyond the transaction no future contract exists hence if, for example an ownership claim which at one time appears to all to be of considerable worth turns out to be worthless at a later date the buyer is out of luck. But no failure to fulfill contract has been encountered.

If individual A contracts to pay individual B eleven dollars next year in return for ten dollars now, even though A may be worth millions at this time there may be some state of the system that can be reached where A cannot pay B.

When we permit borrowing, the system dynamics may force us to specify features of the rules which correspond to the real world phenomena of:

- (a) insolvency,
- and (b) bankruptcy.*

These have implicit within them the rules for seizing and disposing of entailed assets of debtors.

It must be noted that the above features are needed even without the presence of exogenous uncertainty. Strategic uncertainty is enough to make them needed. It may well be that under certain patterns of behavior of the system which maintain it in or near an equilibrium, there is never a need to call on any of these extra rules. No one ever is insolvent or goes bankrupt. This may be so--however this must be deduced from the system behavior, not imposed by assuming away the problem.

When we have mass behavior it is likely that certain regularities will emerge which enable us to deduce that the system will almost always reach states we deem to be "reasonable." There nevertheless may still be a small set of utterly unreasonable states that can only be reached with extremely small probabilities which nevertheless could be attained.

3.2. Banking, Money and Other Paper

In the economic models without credit or futures but with money the following items may play a role:

- (a) consumer goods,
- (b) producer goods,
- (c) ownership paper "backed" by the producer goods,
- (d) fiat money obeying a law of conservation.

*This is not always needed if an extremely conservative banking policy is adopted, see 5.3.

Internal Banking

When internal banking is introduced, i.e. individuals may lend each other money (but not extend other credit), the law of conservation of "present" money is preserved however a new commodity is introduced. The new commodity consists of:

- (e) contracts in future money.

When internal banking is considered and when there are few individuals it is not completely unreasonable to consider bilateral loan arrangements. A lends B money and B has a contract to pay A alone at some point in the future.

When there is a mass market with thousands of economic agents it is more reasonable to consider an institution which cuts down on information processing and communication and is known to all agents. Thus we may have a bank which can neither create nor destroy present money but acts as a market clearing mechanism between current money and contracts in future money. The existence of an "inside bank" creates at least four new distinctions in the monetary system.

- (a) Depositors or "savers" are defined (and saving is different from hoarding);

and (b) Borrowers are defined.

In addition to insolvency and bankruptcy we must now specify the rules which cover:

- (c) bank failure or insolvency,
- and (d) seniority of creditors.

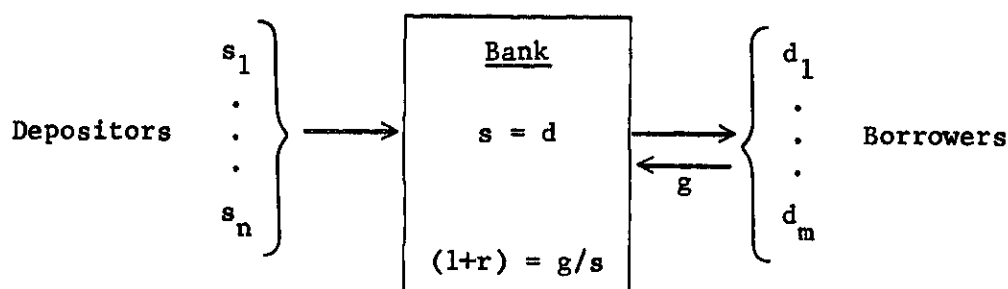


FIGURE 1

From Figure 1 we can see the reason for the new features. The bank aggregates savings obtaining the amount s from n depositors. It is required to lend no more than s . If we let supply and demand equalize via price then the borrowers will be offering in toto an amount g in future money for the present money. If this bank is merely a market mechanism and not a monopolistic profit maximizer then the ex ante price of money will be set at $(1+r) = g/s$. Thus borrower j will pay g_j for a loan of $\frac{g_j}{g} s$.

Suppose borrower j is unable to pay the bank the g_j for which he has contracted. The bank will then be unable to pay all of its depositors. A rule to cover the bank's failure to meet its commitments even though caused by the failure of a borrower's commitment is now called for. If assets are not sufficient to fully pay all claims a seniority of creditor rule is also called for.

External Banking

By external banking I mean a bank that can violate the laws of conservation of present money. The system is able to vary the supply of present money in the hands of all economic agents (leaving aside the banks).

We have fiat money not obeying the laws of conservation. Although for some purposes it might be convenient to distinguish two types of present or current fiat money. They are:

- (a) Specie money obeying a law of conservation,*
- and (b) bank money not obeying a law of conservation.

3.3. Loan Length and Rollover Conditions

Given that dealings in future money are called for it is necessary to specify the rules concerning the mechanism for making loans. This includes the term of the loan and the renewal conditions. It appears that the reasons for considering a varied term structure for loans are related heavily to uncertainty and the length of time that various projects take. As this paper is confined to the most elementary of banking it seems reasonable to limit loans to one period.

In order to analyze or avoid the effects of the type of chain-letter unboundedness of borrowing where individuals borrow increasing amounts every period from new sources to finance old debts we must be explicit on the rules for refinancing. For example one might require that an individual must either pay back his loans before he borrows more or that he merely has to cover the servicing of the loan, or neither. In a world without uncertainty in an equilibrium state I conjecture that many of the institutional differences in loan procedures do not effect the final equilibrium. However it is highly likely that they do effect the behavior of the system in disequilibrium.

*The government and its financial operation are excluded hence we may consider the amount of specie as fixed.

4. A PRELIMINARY TO BANKING

Several simple examples are now examined in detail to show the effects of numbers of competitors, length of horizon, and money on trade. In Section 5 we start to deal with internal banking.

The examples are designed to illustrate the differences with a minimum of calculation. It should be of interest to note that clearly new and complex phenomena appear even in the extremely simple models examined. We must check to see if these are artifacts or special properties of the examples. A sensitivity analysis must be considered and we must at least investigate the possibility that by adding greater complexity to the model (such as many consumer goods and an array of producer goods with different lives) that the same problems are not encountered.

4.1. The Basic Single Consumer Good Economy

Consider an economy where each individual i has a utility function of the form

$$(1) \quad U_i = \sum_{t=0}^{\infty} \beta_i^t \varphi(q_{t+1}^i)$$

where φ is assumed to be concave and β_i is the "natural discount" factor of trader i , q_t^i is the amount of the consumer good obtained by i at time t . There is one consumer good which cannot be stored. We may use two alternative conventions for introducing its supply.

Ownership claims

Convention 1. "Manna from heaven": at the start of period t an amount A_t of the consumer good appears in a central warehouse where it is sold for money. The money is distributed to all of the n individuals in the economy in proportion to the ownership claim of each. There are a series of numbers: $\alpha_t^1, \alpha_t^2, \dots, \alpha_t^n$ where $\sum_{j=1}^n \alpha_t^j = 1$ which represent the ownership claim of the i^{th} individual to the proceeds from the sale of the consumer good during the t^{th} period.

Convention 2. There exists an array of producer goods which each period reproduce themselves and produce an amount A_t of the consumer good. There is an initial distribution $\alpha_1^1, \alpha_1^2, \dots, \alpha_1^n$ of ownership claims to the producer goods, where $\sum_{j=1}^n \alpha_1^j = 1$. The ownership claims may be sold and entitle the holder to the income derived from selling the consumer good produced by his share of the producer good plus the income arising from selling his claim whenever he sells his ownership rights.

Money Holdings

Without banking, or with only internal banking we assume that there exists a given supply of money M (without any loss of generality we may set $M = 1$) which is held initially by the traders. Trader i holds

γ^i where $\sum_{j=1}^n \gamma^j = 1$. At subsequent periods the money holding of trader

i is given by s_t^i which is calculated from the cash flow conditions.

$$s_1^i = \gamma^i.$$

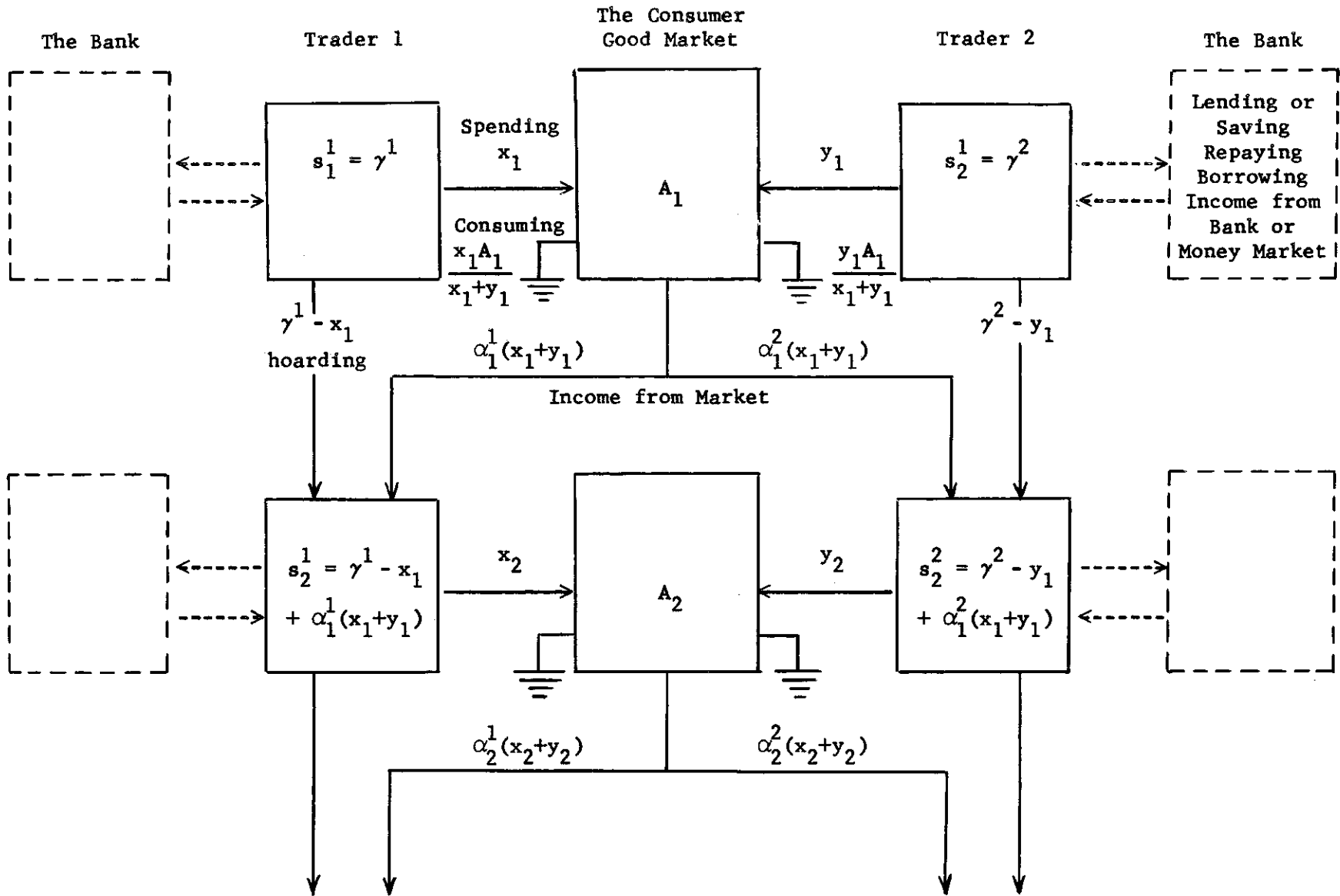


FIGURE 2

Figure 2 shows a two trader market. The actual spending, hoarding, consuming and income patterns are indicated explicitly. The bank sector is only indicated with no explicit details supplied. This is because further modeling concerning information and sequencing of market and financial moves is required; these are supplied in Section 5 below.

4.2. The Competitive Equilibrium

A relatively simple model serves to illustrate the differences between the noncooperative economy with spot markets with and without banking and the same economy treated by general equilibrium theory.

Consider two types of traders, each type has the same utility function of the following form:

$$(1) \quad \text{Type 1} \quad U_1 = \epsilon \sum_{t=0}^{\infty} \beta^{2t} q_{2t+1}^1 + \sum_{t=0}^{\infty} \beta^{2t+1} q_{2t+2}^1, \quad 1 > \beta > \epsilon$$

$$\text{and} \quad \text{Type 2} \quad U_2 = \sum_{t=0}^{\infty} \beta^{2t} q_{2t+1}^2 + \epsilon \sum_{t=0}^{\infty} \beta^{2t+1} q_{2t+2}^2 .$$

A trader of Type 1 owns α of the manna during odd periods and $1-\alpha$ during even periods. A trader of Type 2 owns $1-\alpha$ during the odd periods and α during the even periods. The manna is an exogenous input at each period. The traders are informed in advance of what the inputs are to be. They are in toto:

$$A, B, A, B, A, B, \dots \quad \text{where } A, B > 0 .$$

In terms of 4.1 the ownership conditions are given by:

$$(2) \quad \alpha_1^1 = \alpha_{2t+1}^1 = \alpha \quad \alpha_1^2 = \alpha_{2t+1}^2 = 1-\alpha \quad \text{for } t = 0, 1, \dots$$

$$(3) \quad \alpha_2^1 = \alpha_{2t}^1 = 1-\alpha \quad \alpha_2^2 = \alpha_{2t}^2 = \alpha \quad \text{for } t = 2, 3, \dots$$

A competitive equilibrium exists for this market with

$$(4) \quad p_t = \beta^t \quad t = 0, 1, \dots$$

The endowments of traders of Type 1 are evaluated at

$$(5) \quad \pi_1 = \frac{\alpha A}{1 - \beta^2} + \frac{(1-\alpha)}{(1 - \beta^2)} \beta B$$

and for traders of Type 2 their endowments are

$$(6) \quad \pi_2 = \frac{(1-\alpha)A}{1 - \beta^2} + \frac{\alpha \beta B}{(1 - \beta^2)} .$$

On the Meaning of This Example

Before the consumption pattern of the traders is specified it must be noted that there are several ranges of solutions and that these ranges are determined by the relative values of the parameters in this model. Before further limitations are placed on these values the reasons for selecting this example and the meaning of the different features are given.

Fluctuation in Taste

The introduction of the factor ϵ in the utility function in the odd periods for traders of Type 1 and in the even periods for traders of Type 2 provides for a fluctuation in taste which calls for intertemporal trade with Type 1 preferring consumption in even periods and Type 2 in odd periods. This feature will be sufficient to show why under certain circumstances (to be specified) a noncooperative model of trade fails to achieve Pareto optimality.

Fluctuations in Income

The alternation of ownership claims introduces a certain amount of complication in the cash flow conditions of the traders. This will provide an important contrast when we consider the possibility of trade in ownership claims.

Variability in the Amount of Goods Available

The reason for having the amounts of goods vary periodically from A to B is to produce the simplest model to differentiate the aspects of internal and external banking. It will be shown that when $A = B$ internal banking can handle the lending needs of this simple economy with a fixed money supply. When $A \neq B$ a variable money supply may be called for and an outside banking mechanism must be built.*

*The alternative to not having a change in money supply may be to have periods when all members of society hoard.

In the remainder of this paper further limitations may be made on the parameters in order to illustrate different effects and to keep calculation to a minimum.

Suppose

$$(7) \quad \frac{\beta B}{1 - \beta^2} \leq \frac{\alpha A}{1 - \beta^2} + \frac{(1-\alpha)}{1 - \beta^2} \beta B ,$$

then traders of Type 1 have enough wealth to buy all of the output on even periods plus some of the output on odd periods. In which case the competitive equilibrium distribution will be for traders of Types 1 and 2.

$$(8) \quad \alpha(A - \beta B), \quad B, \quad \alpha(A - \beta B), \quad B, \quad \dots$$

$$\text{and} \quad A - \alpha(A - \beta B), \quad 0, \quad A - \alpha(A - \beta B), \quad 0, \quad \dots$$

where we assume $A \geq \beta B$.

4.3. The Noncooperative Market with Money, No Banking or Credit and No Trade in Ownership Claims

In order to completely define the noncooperative model we have to specify several extra features. They are the initial distribution of money in the system, the information conditions, structure of moves and payoffs in all states of the system.

We assume the initial distribution of money to be given by:

$$(9) \quad s_1^1 = \gamma \quad \text{and} \quad s_1^2 = 1 - \gamma .$$

The markets are modeled as a sequence of simultaneous move games where each trader decides upon how much money to bid for the consumer good. They are then completely informed about the state of the system and after the receipt of their income they bid again in the next period.* This process is illustrated in Figure 3.

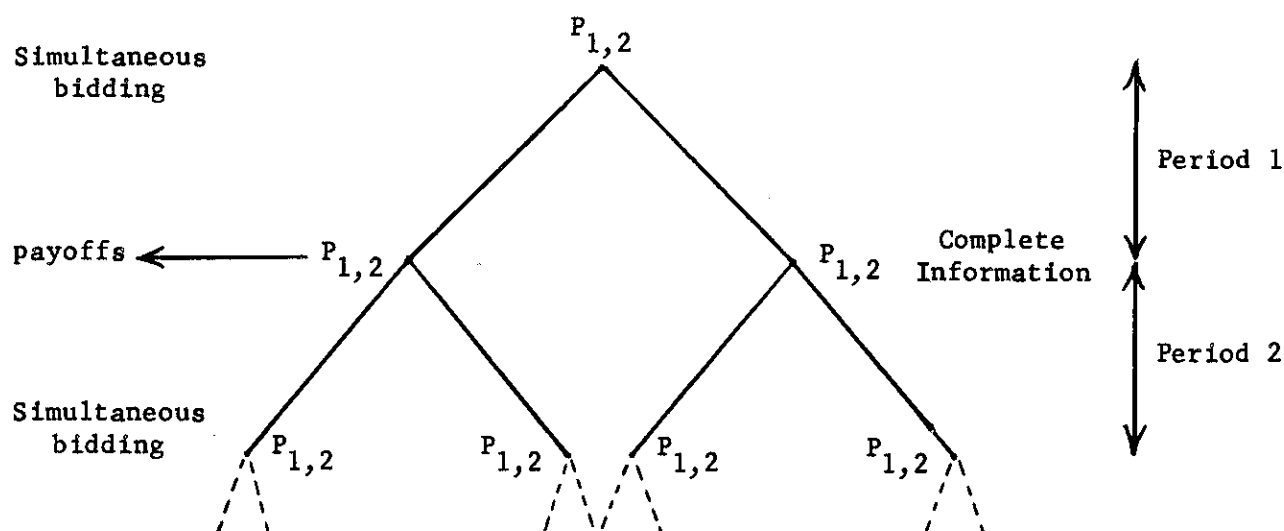


FIGURE 3

Let x_t be the amount bid by the trader of Type 1 during period t , and y_t the amount bid by the trader of Type 2. If there is an amount A_t to be sold then they obtain respectively

$$(10) \quad q_t^1 = \frac{x_t A_t}{x_t + y_t} \quad \text{and} \quad q_t^2 = \frac{y_t A_t}{x_t + y_t} \quad \text{when } x_t, y_t > 0$$

and the price of the commodity is given by:

*It is conjectured that a much more parsimonious model where individuals know only their own income, the aggregate income figures and aggregate supply will have the same state equilibrium points.

$$(11) \quad p_t = (x_t + y_t)/A_t .$$

When $x_t = y_t = 0$ the values in (10) are undefined. There are several conventions we could use to define them. The simplest is that $q_t^1 = q_t^2 = 0$ this has the interpretation that if the consumer good is not sold at the warehouse it rots or is destroyed before it can be sent to its owner.

The Finite Two Trader Market $\Gamma(\gamma, 1-\gamma, 1, 1, 2)$

The notation $\Gamma(\gamma, 1-\gamma, m, n, T)$ stands for the noncooperative game with two types of traders where each trader of the first type has an initial amount of money γ , and traders of the second have $1-\gamma$. There are m traders of the first type and n of the second. The game lasts for T time periods. We assume $0 < \gamma < 1$.

In the game with a finite horizon as money is worthless after the last period we may assume that all traders spend everything during the last period. A backward induction may then be applied to calculate the state equilibria. The two trader two period game has payoffs as follows:

$$(12) \quad \Pi_1 = \frac{e x_1 A}{x_1 + y_1} + B\{\gamma - x_1 + \alpha(x_1 + y_1)\}B$$

and

$$(13) \quad \Pi_2 = \frac{y_1 A}{x_1 + y_1} + e\beta\{1 - \gamma - y_1 + (1-\alpha)(x_1 + y_1)\}B, \quad \text{where } x_1 \leq \gamma \text{ and } y_1 \leq 1-\gamma .$$

Even at this level of simplicity we can see that Pareto optimal trade is not obtained in this noncooperative market. This follows immediately from (8). In order to achieve a consumption pattern with no consumption by traders of Type 2 during the second period their income would have to be zero, which could only occur if $x_1 = y_1 = 0$, but these strategies are not in equilibrium. This example has been discussed in detail elsewhere.¹²

The fact that there were only two traders is not where the source of the difficulty lies. The difficulty lies in their being no way to force traders of Type 2 to hoard during the second period.

There are two ways which do not involve banking which may restore Pareto optimality here. One involves considering the infinite horizon and the other calls for allowing the sale of ownership claims.

The first method suffices to patch up this example but will not work in general for reasons explained after the example now given.

Consider the market where $\alpha = 1/2$ and $A = B$. We may now demonstrate the existence of an initial money distribution γ to Type 1 and $1-\gamma$ to Type 2 such that the noncooperative equilibrium gives the same real goods distribution as the competitive equilibrium, but has hoarding always present.

The distribution of real goods called for by the competitive equilibrium in this instance is:

$$\begin{array}{l} \frac{A}{2}(1-\beta), \quad A, \quad \frac{A}{2}(1-\beta), \quad A, \quad \dots \\ \frac{A}{2}(1+\beta), \quad 0, \quad \frac{A}{2}(1+\beta), \quad 0, \quad \dots \end{array}$$

If we give traders of Type 1 an amount of money

$$(14) \quad \gamma = \frac{\beta}{1+2\beta} \quad \text{and Type 2} \quad 1-\gamma = \frac{1+\beta}{1+2\beta}$$

we may observe that the following strategies are in equilibrium and yield the same distribution as the competitive market. Table 1 presents the accounts (for $\beta \geq 1/2$).

TABLE 1

	<u>Type 1</u>	<u>Type 2</u>
<u>Period 1</u>		
Assets	$\frac{\beta}{1+2\beta}$	$\frac{1+\beta}{1+2\beta}$
Hoarding	$\frac{2\beta-1}{1+2\beta}$	--- (Case distinction if $\beta < 1/2$)
Spending	$\frac{1-\beta}{1+2\beta}$	$\frac{1+\beta}{1+2\beta}$
Spot price	$\frac{2}{(1+2\beta)A}$	
<u>Period 2</u>		
Income	$\frac{1}{(1+2\beta)}$	$\frac{1}{(1+2\beta)}$
Assets	$\frac{2\beta}{(1+2\beta)}$	$\frac{1}{(1+2\beta)}$
Hoarding	---	$\frac{1}{(1+2\beta)}$
Spending	$\frac{2\beta}{(1+2\beta)}$	---
Spot price	$\frac{2\beta}{(1+2\beta)A}$	
<u>Period 3</u>		
Income	$\frac{\beta}{(1+2\beta)}$	$\frac{\beta}{1+2\beta}$
Assets	$\frac{\beta}{1+2\beta}$	$\frac{1+\beta}{1+2\beta}$

A similar example has been discussed in detail elsewhere.¹² It is important to note that this equilibrium does not depend on the number of traders beyond there being at least two of each type.* The number of traders does effect the speed of adjustment to equilibrium.¹⁶

The reason why this infinite economy with hoarding works in this case whereas the finite economy does not work** is because in the finite economy at the last period there is no support to make players of any type hoard. In this cyclical example as long as there is enough of a future then support is supplied. In this example "enough" is two periods. However it is possible to select a model for which beyond some specific date from then on one set of traders would be required to hoard if Pareto optimality is to be achieved. This is not possible, hence the mere introduction of the infinite horizon does not appear to be sufficient to support all Pareto optimal competitive equilibria as noncooperative equilibria. The simplest counterexample is provided by an economy with two types of traders $A_t = A$

and $U_1 = \sum_{t=0}^{\infty} \beta_1^t q_{t+1}^1$ and $U_2 = \sum_{t=0}^{\infty} \beta_2^t q_{t+1}^2$ where $\beta_1 \neq \beta_2$. This has been discussed in detail elsewhere.¹⁶

*In order to have a trader of Type 1 spend everything in period 2 there must be at least one other trader of Type 1 in the market "to keep him honest."

**I.e. gives the same real results as the general equilibrium.

4.4. The Noncooperative Model with Money, No Banking or Credit but Trade in Ownership Claims

In a previous paper¹³ it has been shown that we could introduce trade in ownership claims. Thus we assign at time $t = 1$ the $\alpha_1^1, \alpha_1^2, \dots, \alpha_1^n$ which represent the initial ownership claims of the individuals to the productive plant of the economy, but from then on future claims are determined by trade in the current claims (in a more complex model this would include the building of capital goods and their depreciation).

It is conjectured that without exogenous uncertainty if all consumer goods are produced by the productive resources which back the ownership claims then trade in these claims will provide a sufficient backing for any competitive equilibrium distribution to be obtained as a noncooperative equilibrium in a market with money and ownership claims* but no banking.

5. INTERNAL BANKING

The term inside or internal banking refers to the market in which there is conservation of present money. Loanable funds are generated from one group of traders and borrowed by another group. Net borrowing and lending is always zero. As has already been noted in 3.2 we must distinguish between bilateral borrowing and lending where a specific individual A lends to another individual B and the first more "banklike" type of

*An extremely simple model of ownership paper is used here. In particular there is only one type of paper and its quantity is conserved from period to period. In a fully general model with uncertainty there would be an array of this paper with the possibility to create or destroy different types.

arrangements where a group of depositors "save" and the bank institution lends to many borrowers. This has been illustrated in Figure 1.

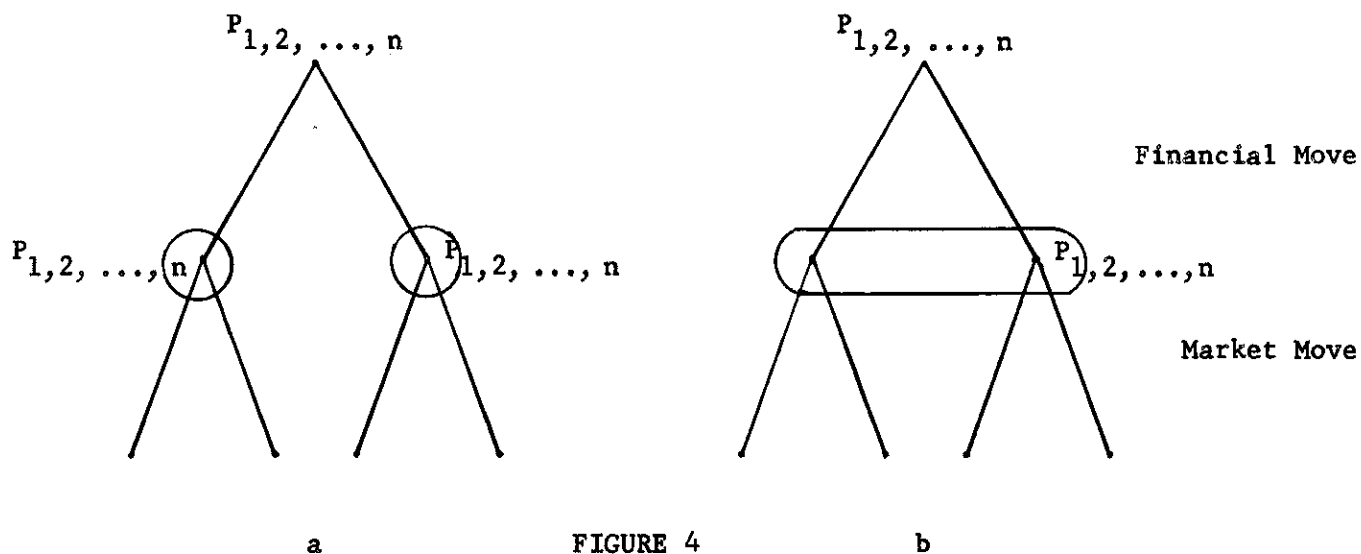
It is my conjecture that in a noncooperative world without uncertainty, banking (together with the appropriate laws needed to define the procedures for dealing with contracts under all circumstances) is a sufficient but not a necessary device for achieving optimal noncooperative equilibria.

One of the simplest examples we can consider is the one already given in 4.3. We consider this model, modified to include internal banking at this point.

5.1. Financial and Market Moves

In order to completely define the market with banking we must specify the sequencing of moves involving borrowing and lending and buying in the market and the information conditions which prevail. It is argued elsewhere.¹⁴ that the correct way to model trade in a mass market calls for considering a random sequencing of moves which, in many circumstances, as numbers become large may be reasonably approximated by the model with simultaneous moves.

Figure 4 shows the two possibilities we consider here. They are illustrated for n traders where the symbol $P_{1,2, \dots, n}$ indicates that n traders move simultaneously, hence without information on each others' immediate actions. Figure 4a indicates that they all take their financial move simultaneously. This involves two parts. First a settlement of accounts outstanding. This may be regarded as automatic rather than strategic. Loans are repaid with interest (for formal simplicity this includes the



returning of deposits which then may be redeposited immediately). If there are default problems they are taken care of at this point. (We return to discuss this later.) The second part of the financial activity is strategic. Individuals put up money to be loaned or offer future money in return for current money. In Figure 4a it is assumed that before market purchases take place all individuals are informed of the financial moves.

In Figure 4b it is assumed that each trader makes both his financial and market move in ignorance of all moves of others.*

The difference between the two formulations is that in a T period market the first model calls for $2T$ maximizations in sequence, whereas in the second a sequence of T simultaneous two variable maximizations is called for. The first is considered here.

*Actually Figure 4b is not quite correct in the sense that when an individual makes his market move he knows what he borrowed from the bank and this is not really shown in this diagram.

Loan Conditions

u_t^i = the amount of money offered for loan by trader i at time t

v_t^i = the amount of future money offered in return for a loan by
trader i at time t

$$(15) \quad u_t = \sum_{j=1}^{2n} u_t^j \quad \text{is total money for loan}$$

$$(16) \quad v_t = \sum_{k=1}^{2n} v_t^k \quad \text{is total future money to be repaid.}$$

An individual i lending will be repaid the amount:

$$u_t^i \left(\frac{v_t}{u_t} \right),$$

and an individual j borrowing will obtain

$$v_t^j \left(\frac{u_t}{v_t} \right).$$

The rate of interest or the "price of money" is given by

$$\left(\frac{v}{u} \right) - 1.$$

Spending

x_t^i = the amount of money spent by a trader i of Type 1 at time t

y_t^j = the amount of money spent by a trader j of Type 2 at time t

Initial Money and Ownership Claims

We assume that a trader of Type 1 has γ units of money and a trader of Type 2 has $1-\gamma$ units. We eliminate any cycle in ownership claims. A trader of Type 1 has an ownership claim of α and Type 2 of $1-\alpha$.

Using the notation above and limiting α and γ so that $0 \leq \alpha \leq 1$ and $0 \leq \gamma \leq 1$ we note that we now have two types of paper in the economy both of which have the property that the system is homogeneous of order zero with respect to the initial supply of either form of paper. The first type of paper is fiat money and the second type can be regarded as non-negotiable* stock certificates. In this model we keep the restriction that no market for shares exists however as has been noted in 4.4 we may consider the other condition.

If $\gamma = \alpha$ an individual's money holdings are in direct proportion to his ownership claims. If $\gamma > \alpha$ then traders of Type 1 are at an advantage.

5.2. The Two Period Oligopolistic Market

A study of the two period market for a variable number of traders is given. Then a special example of the infinite market with many traders is considered. The two period market has the undesirable and unrealistic feature to it that money is worthless after the second period. This

*Restrictions to negotiability by the owner in actuality frequently exist for insider or other forms of "tainted stock" or for stock held as security against loans.

immediately implies that the no money will be offered on loan during the last period hence if the loan market is active at all it must be during the first period.

In spite of the undesirable feature of the two period market, that, combined with the infinite market special case can be used to illustrate most of the important qualitative features of the model.

Even with as simple a model as the one proposed, the calculations are tedious and there are several case distinctions. The procedure adopted will be to assume certain properties of the model, use them to obtain a solution and justify their use either immediately or after the solution has been obtained.

Assumption 1. There exists a symmetric solution, in the sense that there exists a perfect equilibrium point for which traders of the same type receive the same treatment.*

Assumption 2. The rate of interest in equilibrium cannot be negative, hence $v/u \geq 1$; because otherwise lenders would hoard rather than lend.

Assumption 3. For $\gamma \geq \alpha$ and $\epsilon_A < \beta_B$ traders of Type 1 either lend, hoard or spend everything. They do not borrow. Traders of Type 2 borrow and spend or only spend.

Using Assumption 3 we can write down the payoffs as follows:

For traders of Type 1

*The existence of nonsymmetric equilibria and the general problem of uniqueness is not considered at this point.

$$(17) \quad \pi_1^i = \frac{ex^i An}{x+y} + \frac{\beta\{\gamma - x^i - u^i + \alpha(x+y) + u^i(v/u)\}Bn}{n},$$

and for Type 2

$$(18) \quad \pi_2^j = \frac{y^j An}{x+y} + \frac{\beta\{1 - \gamma - y^j - v^j + (1-\alpha)(x+y) - v^j(u/v)\}Bn}{n}.$$

A term n appears in (17) and (18). This is introduced so that we have a natural way for comparing markets with different numbers of competitors. The per capita amount of goods and money is assumed to be constant in the economies of different size. Thus changes observed in the solution may be attributed to the effect of competition. This method for making a comparison between markets of different size has been discussed elsewhere.⁸ Here, for a market with $2n$ traders there are n units of money, of ownership certificates, nA units of the first period good and nB of the second.

Before we are in a position to solve this model we must discuss the problems posed by bankruptcy and how they are dealt with here.

5.3. Conservation Laws, Insolvency, Bankruptcy and Credit Restriction

To talk about the demand and supply for money without being completely explicit about conditions imposed on the quantity of present money and future money is to discuss an ill defined model. In a world with fiat money and with one period loans there exists two types of financial paper; the money (or money now) and the promissory notes to pay current money one period hence (or money tomorrow). In this model

we have implicitly assumed that the notes are nonnegotiable. In other words the bank does not factor or otherwise sell its claims.

By the simple expedient of issuing n units of fiat money and permitting only inside banking, conservation of the amount of present money is achieved. The act of borrowing however creates new paper. What bounds must there be put on this new paper?

The claims or "money tomorrow" will, in one period hence, have to be exchanged for "money now" at that time. The rules of the model permit no rollover of fancy versions of checkkiting. However as there is a conservation condition on the amount of fiat money, if the face value of the promissory notes is greater than the amount of money in the economy, at least one debtor will not be to honor his debts and will be insolvent. In market terms the conditions for "a corner" are present.

In order to handle the insolvency we need to specify the recourses available to the creditor. An extremely natural resource is to have the creditor garnish the future income of the debtor until the debt is paid. Another somewhat harsher rule is to have the creditor take possession of the ownership claim of the debtor, or sell the ownership claim of the debtor.

An individual is insolvent when he is unable to meet current claims even though he may have sufficient assets which, if liquidated would cover the claims. Bankruptcy proceedings may arise if an insolvent cannot obtain the appropriate accommodation from his creditors. The cash flow and liquidation features of the debtor's assets are critical to the way an insolvency must be handled.

With internal banking there are three case distinctions to be made. Suppose that the i^{th} individual in an n person society has γ_t^i units of money at time t and a constant ownership claim of α^i to future income.

Even though the bank is treated as a mechanism there are nevertheless three parties not two to the matching of deposits and borrowing. They are the depositors or savers; the bank acting as the lender; and the borrowers.

We must be extremely precise in specifying the supply of deposits; the bank loan policy and the demand for loans. It appears that the number of insolvencies in the society will depend directly upon the bank's loan policy--the more liberal, the more likely are insolvencies. Furthermore the control of the number of insolvencies depends directly upon the amount of information the bank has on consumer spending. The existence of credit constraint policies which depend in a flexible manner on the amount of information will be dealt with elsewhere.¹⁵ Here we limit ourselves to three credit constraint policies, any one of which is announced as a "rule of the game."

The credit restrictions act as limits on what the borrowers can bid in terms of future money. If a borrower is permitted to bid too much he will undoubtedly be unable to repay his debt and will become insolvent. If insolvency is to be avoided the size of bidding for loans must be constrained.

Case 1: Extremely Conservative Banking*

An individual may incur indebtedness up to $(\alpha^i + \gamma_t^i)/n$. This is the largest amount that he will be in a position to pay back regardless of what happens in the economy. Thus with this banking rule the largest extension of credit will be $2/n$ of the supply of current money. However in this system there is no possibility for default.

Case 2: Secured Banking

In this simple economy we have individuals hold ownership claims to the real resources, which entitle them not directly to the real resources but to the money income derived from their sale. Without exogenous uncertainty it is reasonable to believe that the banker will attempt to assess the expected income of the traders as reflected in their ownership claims. There is still uncertainty present in the form of strategic uncertainty, hence the banker is not sure of the projected incomes of the traders.

A natural limit to bidding for a loan is provided by requiring that no individual can offer for a loan a larger percentage of the money supply than his percentage claim on the ownership paper supply.** As all assets are monetized every period this is tantamount to the bank accepting the ownership claim as the backing for its loan.

*The importance of this case and the desirability of analyzing it was pointed out by Ward Whitt. Even though lending approaches zero as n grows, the case distinction can be made and is important when traders are few.

**This is an upper bound to his offer to pay, not on the loan size in the sense that with internal banking the funds for lending may be highly restricted.

A bankruptcy law which appears to be relatively straightforward is that the bank takes over the ownership paper and the income derived therefrom until its claims are satisfied. This is still not fully defined but will be discussed further elsewhere.¹⁵

It appears that the credit limit convention would hold equally well for a world with many consumer commodities and many different ownership claims. It also appears to be generalizable as a credit rule including exogenous uncertainty.

Case 3: "Unsound" Banking

"Unsound" banking involves lending unconstrained by expectations concerning ability to repay.

The three cases can be summed up as follows

- (1) Maximum individual indebtedness $\leq (\alpha^i + \gamma_t^i)/n$
- (2) Maximum individual indebtedness $\leq \alpha^i$
- (3) Maximum individual indebtedness unbounded by credit constraint.

Conditions on the Demand for Loans

We may interpret the three cases above from the demand rather than supply side. The supply is clearly bounded by the resources of the bank depositors. They could contribute more or less than the bank is prepared to lend.

We can however interpret the three cases above as providing different upper bounds on bids that will be seriously entertained by the bank. A would-be borrower is not permitted to exceed these bounds in his offers of future money for whatever loans he can get because it is clear to the bank that if he does so he cannot meet the safety criterion.

In particular in Case 3 if no constraints are in force the total amount of promissory notes which can be created is unbounded. If the total were merely to exceed 1 this would guarantee that some trader would have to be unable to pay his loans.

Our second conservation law would have been violated. If present money is conserved then future money or promissory notes created can never exceed present money without default becoming a certainty.

5.4. Solutions to the Two-Period Market

The calculations here call for the solving of x^1 and y^j in (17) and (18) in terms of u^1 and v^j . It follows that if Type 2 borrows:

$$(19) \quad y^j = 1 - \gamma + v^j \frac{u}{v} .$$

The interest rate will not be less than zero at equilibrium or the lenders would prefer to hoard. If it is greater than or equal to zero the borrower will borrow for immediate use.*

Differentiating (17) by x^1 we obtain

$$(20) \quad \epsilon A n \left\{ \frac{x + y - x^1}{(x+y)^2} \right\} = \left(1 - \frac{\alpha}{n} \right) \beta B .$$

Set $K = [(1 - \alpha/n)\beta B]/\epsilon A$ and rewrite (20) as

*This is not totally true under all loan rationing conditions. A borrower might borrow to hoard to prevent another borrower from obtaining funds.

$$(21) \quad K(x+y)^2 - n(x+y) + nx^i = 0 .$$

Summing over $i = 1, \dots, n$ and dividing by n we obtain:

$$(22) \quad K(x+y)^2 - n(x+y) + x = 0$$

and similarly summing over $j = 1, \dots, n$ in (19) we obtain

$$(23) \quad y = n(1-\gamma) + u$$

hence from (22) we obtain

$$(24) \quad x = \frac{n-1}{2K} - [u + n(1-\gamma)] + \frac{1}{2K} \sqrt{(n-1)^2 + 4[u + n(1-\gamma)]K}$$

or given $x^i = x/n$

$$(25) \quad x^i = \frac{1}{2K} \left(\frac{n-1}{n} \right) - \frac{u}{n} - (1-\gamma) + \frac{1}{2Kn} \sqrt{(n-1)^2 + 4[u + n(1-\gamma)]K} .$$

We may now substitute the values of x^i and y^j in (17) and (18) and differentiate these by u^i and v^j respectively then solve for u^i and v^j . There is a proliferation of cases and the calculations are extremely tedious. The purpose here, however for using this example is to explore its qualitative properties and to demonstrate the existence of various phenomena. Hence a complete solution may not be necessary at this point. In a previous paper Shapley and Shubik have explored a somewhat simpler model in detail.²³ The qualitative results obtained from there relevant to oligopolistic competition with banking will be summarized in 5.5, however, here we start by considering just the conditions for no hoarding.

Internal Banking and No Hoarding

Under what conditions could this system operate without hoarding? Let us assume that there are some circumstances that an internal banking system would be active without hoarding. If this were the case we could immediately specify the following:

$$(26) \quad x^i = \gamma - u^i$$

$$(27) \quad y^j = 1 - \gamma + v^j \frac{u}{v}$$

$$(28) \quad \frac{\partial x^i}{\partial u^i} = -1 \quad \text{and} \quad \frac{\partial x^i}{\partial v^j} = 0$$

$$(29) \quad \frac{\partial y^j}{\partial v^j} = \frac{u}{v} \left(\frac{n-1}{n} \right) \quad \frac{\partial y^j}{\partial u^i} = \frac{1}{n} .$$

Solving for a special example, set $\alpha = 0$, $\gamma = 1$, $\epsilon = 1$. This limits us to an economy with identical tastes but cyclical ownership. We obtain for large n

$$(30) \quad s = \frac{v}{u} = \frac{A}{\beta B} \quad \text{where} \quad A \geq \beta B .$$

The calculations to establish this are lengthy and are given elsewhere in joint work by Shapley and Shubik.⁶

We have:

$$(31) \quad u = \frac{A}{\beta B + A} \quad \text{and} \quad v = \frac{\beta B}{\beta B + A} .$$

5.5. So What?

To some extent it may appear that we have belabored the obvious. There are several results obtained here that are well known. In particular:

- (1) An internal banking system as sketched here is a "100% reserves" banking system related to the banking suggestions of Henry Simons.¹⁷
- (2) From (30) if $A = B$ then in a stationary state economy the money rate of interest is directly related to the "natural discount" if such a discount exists. Here $\rho = (1-\beta)/\beta$. This is a well known and highly misleading result as different individuals may have different preferences with no discernable time discount yet a money interest rate will be defined.

There are some less obvious conclusions which can be drawn from even as simple a model as that presented in 5.4.

We have solved for a steady state mass market solution with active internal banking and no hoarding. In doing so for the simple example above the parameter γ was set as $\gamma = 1$. The system for any other value of γ would have at least some individual at a boundary and the solution obtained from (32) and (33) would not be valid. In order to understand the dynamics of adjustment we need to consider the behavior of the system for arbitrary γ and for many time periods. It is conjectured that the speed of adjustment towards a steady state equilibrium is directly related to the numbers of traders in the market.

It is important to stress that in this model the money rate of interest is deduced from the system of bidding for "now money" in terms of promissory notes. The conservation law exists on the current money supply which in turn supplies a bounding condition on the creation of promissory notes or "money tomorrow" contracts. From the solution given in (33) and (34) we see that the "conservative banking" criterion given in 5.3 is not sufficient whereas the "secured banking" criterion is. Furthermore it is easy to check that the equilibrium point suggested by (33) and (34) is an equilibrium under "secured banking" without any insolvency and hence no need to consider the details of bankruptcy. If we consider "unsound banking" then if an individual borrower wishes to write promissory notes for more money than there is in the economy someone must become insolvent and in the test for equilibrium we must specify bankruptcy.

The model presented here has an explicit role for numbers of competitors. This contrasts with the mechanistic role assigned by general equilibrium theory and the cooperative role in core theory. For example by setting $s = 1$ we can calculate for what size n the banking system will become active (see Shapley and Shubik).⁶

By solving the system without the "no hoarding" constraint, regions can be obtained where all of the following phenomena are encountered: (1) Active banking, (2) inactive banking, (3) hoarding simultaneously with banking for finite numbers, and (4) hoarding combined with inactive banking for an indefinitely large number of traders.⁶

5.6. The Infinite Horizon

Consider the special example illustrated in 5.4 where $\alpha = 0$, $\gamma = 1$, $\epsilon = 1$. It is easy to see that the solution for the two period model can be extended immediately to the infinite horizon. The hoarding solution and internal banking solution are presented side by side.

It can be seen that each of these expenditure patterns define an equilibrium point in the infinite noncooperative markets the first with no banking facilities available and the second with internal banking.

TABLE 2

<u>Period 1</u>	<u>Hoarding</u>			<u>Internal Banking</u>	
	<u>Type 1</u>	<u>Type 2</u>		<u>Type 1</u>	<u>Type 2</u>
Assets	$\frac{1+\beta}{1+\beta+\beta^2}$	$\frac{\beta^2}{1+\beta+\beta^2}$	Assets	1	0
Hoard	$\frac{1}{1+\beta+\beta^2}$	---	Lend	$\frac{\beta}{1+\beta}$	→
Spend	$\frac{\beta}{1+\beta+\beta^2}$	$\frac{\beta^2}{1+\beta+\beta^2}$	Spend	$\frac{1}{1+\beta}$	$\frac{\beta}{1+\beta}$
<u>Period 2</u>					
Income	0	$\frac{\beta+\beta^2}{1+\beta+\beta^2}$	Market Income	0	1
Assets	$\frac{1}{1+\beta+\beta^2}$	$\frac{\beta+\beta^2}{1+\beta+\beta^2}$	Financial Income	$\frac{1}{1+\beta}$	←
Hoard	---	$\frac{\beta^2}{1+\beta+\beta^2}$	Assets	$\frac{1}{1+\beta}$	$\frac{\beta}{1+\beta}$
Spend	$\frac{1}{1+\beta+\beta^2}$	$\frac{\beta}{1+\beta+\beta^2}$	Spend	$\frac{1}{1+\beta}$	$\frac{\beta}{1+\beta}$
<u>Period 3</u>					
Income	$\frac{1+\beta}{1+\beta+\beta^2}$	0	Market Income	1	0
Assets	$\frac{1+\beta}{1+\beta+\beta^2}$	$\frac{\beta^2}{1+\beta+\beta^2}$	Financial Income	0	0
			Assets	1	0

Inactive Internal Banking

Consider the special example where $\alpha = 1/2$, $\gamma = 1/2$, $\epsilon = 1$ and $\beta B > A$. Here all individuals have the same tastes and claims. There are more goods available on the even periods than the odd periods.

It is easy to see that with internal banking there will be no active banking because in the even periods all members of society would like to borrow but as net borrowing must be zero this cannot be achieved, hence the price system is adjusted by hoarding as is shown in Table 3.

TABLE 3

<u>Period 1</u>	<u>Type 1</u>	<u>Type 2</u>
Assets	$\frac{1}{2}$	$\frac{1}{2}$
Hoard	$\frac{1}{2}(1 - A/\beta B)$	$\frac{1}{2}(1 - A/\beta B)$
Spend	$\frac{1}{2}(A/\beta B)$	$\frac{1}{2}(A/\beta B)$
<u>Period 2</u>		
Income	$\frac{1}{2}(A/\beta B)$	$\frac{1}{2}(A/\beta B)$
Assets	$\frac{1}{2}$	$\frac{1}{2}$
Spend	$\frac{1}{2}$	$\frac{1}{2}$

If there were an outside bank in a position to create new current money it is clear that we could replace this dynamic economy with hoarding and an unused internal bank system with one that would not hoard but would use the money creating bank.

6. CONCLUDING REMARKS

Hoarding is "banking-through-the-looking glass." Coordinated hoarding is the equivalent of lending, borrowing and repaying. One cannot always achieve Pareto optimality by coordinated hoarding because there may exist no mechanism to enforce agreements to hoard.

Given enforceable contracts and no commodities futures markets it appears that an economy without exogenous uncertainty can be run with an efficient price system, spot markets and fiat money in fixed supply by any of the following three methods (or mixtures thereof):

- (1) Hoarding with contracts,¹²
- (2) A market in ownership paper claims,¹³
- (3) Internal banking.

Unless one wishes to make explicit assumptions concerning special information processing or control, banking does not appear to be necessary for economic efficiency in equilibrium. It may however speed adjustment from positions of disequilibrium.

Although banking may not appear to be necessary it can be seen that if it is available to the competitors it may be used as an alternative to hoarding.

The unwieldiness of the mathematical model of internal banking and its limitations lead us to pass it by for a mathematically simpler and economically more relevant model of external banking; i.e. banking in which current money can be created and destroyed so that it is possible for the net indebtedness of all traders to be nonzero.

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