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INPUT QUALITY, OBSOLESCENCE AND UNEMPLOYMENT

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by

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In a number of growth models of the Solow-vintage variety (Solow, 1959) which assume capital vintages and homogeneous labor, one finds scrappage of capital (retirement of capital before it is worn out) such as in Matthews (1964). Akerlof (1969) considered the "dual" of such models: a model in which different qualities of labor are available but capital is homogeneous. He showed that in such a model "scrappage of labor" could occur. Workers of lower skill levels are thus left unemployed. Since in fact both capital and labor are of varying quality (see Stephens, 1967, 1971), it is interesting to consider the questions of capital and labor scrappage in the context of a model in which both capital and labor are of differing quality. We shall see that the Solow capital scrappage and Akerlof labor unemployment explanations cannot in general be combined, as one might have expected they could, to explain simultaneous capital scrappage and unemployment of labor.

It will be sufficient for our purpose here to follow the practices of the appropriate models referred to above and assume that differences of

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labor quality may stem from many causes, but that differences in capital quality are due to the capital having been constructed at different times, thus embodying different levels of technology.

Let the production function be given by $Q_{v,\alpha} = F[v, \alpha, L_{v,\alpha}, K_{v,\alpha}]$ expressing the output obtained by vintage v capital being worked with grade α labor as a function F of the number of laborers of grade α working with vintage v capital, the amount of vintage v capital being worked by grade α labor, the construction date v of the vintage, and the grade α of the labor. The production function F is assumed to be homogeneous of degree one in K and L and have diminishing marginal returns. The partial derivatives of F with respect to α and v are assumed to be strictly positive given $L_{v,\alpha} > 0$ and $K_{v,\alpha} > 0$, and the marginal products of capital and labor are assumed to be positive when the other factor is present. It is further assumed that $F(v, \alpha, 0, 1) = F(v, \alpha, 1, 0) = 0$ and that any combination of capital vintage and labor grade yields positive output.

The basic mechanism whereby scrapping of capital may occur in capital-vintage models is that the available supply of homogeneous labor is spread over the various vintages of capital so as to maximize output, thus making the marginal product of labor the same for production with each vintage of capital. If some capital is sufficiently unproductive that no amount of labor assigned to it will achieve this marginal productivity, that capital is scrapped. Since the differences in capital quality are due to differences in when they were constructed it will be the oldest capital--presumably used when it was new--that is unused, hence the term scrapping is particularly appropriate and the scrapping may be described as due to obsolescence.

In Akerlof's model the available homogeneous capital is spread over the various qualities of labor so as to maximize output. The marginal product of capital is thus the same whatever quality of labor works it. Low quality labor will thus be unemployed if no amount of capital assigned to it would achieve this marginal productivity.

In our model we cannot speak of spreading capital over labor or labor over capital since both are heterogeneous; the allocation process will be much more complex than that. But assume capital and labor are so allocated as to maximize output: more formally, so as to maximize

$$\int_v \int_\alpha F[v, \alpha, L_{v,\alpha}, K_{v,\alpha}] d\alpha dv$$

subject to

$$\int_v L_{v,\alpha} dv \leq L_\alpha$$

for all α and

$$\int_\alpha K_{v,\alpha} d\alpha \leq K_v$$

for all v , where the inequalities represent the constraints that no more of any vintage of capital or any grade of labor may be employed than is available.

Can capital scrappage occur in this model? Consider an initial situation in which capital and labor are fully employed and introduce one additional unit of capital of vintage v_0 . In order for this unit of capital to be employed, some labor must be found to work with it which will

have a higher marginal product by doing so than in its present use. Unless the production function is such that

$$\frac{\partial F(v_0, \alpha, L_{v_0, \alpha} - 1)}{\partial L_{v_0, \alpha}}$$

approaches ∞ as $L_{v_0, \alpha} \rightarrow 0$ for some α , it is easy to see that there may be no such labor, and so capital scrappage may occur.

It can be seen by an entirely similar argument that unemployment of low grade labor may also occur. However, the argument showing the possibility of capital scrappage depends crucially on the assumption that labor is fully employed, and vice versa. For if both capital and labor were unemployed, the unemployed capital and unemployed labor could be combined so as to produce positive output. Hence when output is maximized capital and labor cannot both be unemployed.¹

The model constructed here is a generalization of putty-putty capital vintage models and of Akerlof's heterogeneous labor model.² We have seen that in this model either capital scrappage or unemployment of labor may occur, but not both. How then do we explain the fact that unemployment of labor and scrappage of capital both occur in the real world? A number

¹In a model analogous to this one but with homogeneous capital and labor, still assuming positive marginal products and no indivisibility or fixed coefficients, both capital and labor would, of course, be fully employed when output is maximized (except in the extreme case where one factor is totally absent).

²Putty-clay capital vintage models are not special cases of this model since they do not have universal diminishing returns ex post. See Appendix.

possibilities suggest themselves, some primarily technological and some primarily institutional.³

1. It may be that for legal reasons (minimum wage laws, etc.) or for private institutional reasons (union policies, etc.) some workers cannot be paid a real wage as low as the marginal product they would have when working with the capital which would be assigned to them under the optimal allocation. This may be because an absolute wage that low is impossible, or because a wage so low relative to the wage of higher grade workers producing the same product is impossible.

2. Production may take place with some third factor in limited supply. In this case the more productive labor and capital may take over all of the third factor, leaving the less productive labor and capital unemployed. A good example of this would be that land being farmed with tractor and driver is then unavailable to the man walking behind a mule.

3. In the model of this paper, capital and labor differ as to quality, but the model is one-sector and the difference is one-dimensional. Suppose instead one constructed a model with two sectors, assuming that neither capital nor labor can be transferred from one sector to another once put in use. Then one might have scrappage of capital in one sector and unemployment of labor in the other.⁴

4. Another possibility arises from the fact, generally ignored in capital-

³The arguments given here are for the putty-putty case, but most could be modified for the other cases.

⁴For a similar situation under different assumptions, see Appendix.

vintage growth models, that capital may have a positive scrap value. Thus capital may be scrapped because its scrap value exceeds the present value of the returns from continuing to operate it.

5. So far this discussion has been in the neoclassical world where adequate aggregate demand is assumed. Inadequate aggregate demand can of course explain labor also becoming unemployed where the full output production situation calls for capital scrapping.

In conclusion, we see that while it is quite possible to offer explanations for simultaneous unemployment and scrapping of capital, it generally must occur for reasons quite different from those for one factor being discarded suggested by the capital-vintage models or the Akerlof model.

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APPENDIX

In the model in the body of this paper both capital and labor are assumed capable of working with different amounts of the other input (no fixed coefficients). This flexibility is common for labor-quality growth models (Akerlof, 1969; Dobell, 1969; Stephens, 1971). However, for capital vintages several approaches have appeared, distinguished by whether coefficients were variable both before and after construction (putty-putty), only before construction (putty-clay), or not at all (clay-clay). If we consider that labor too may be restricted to working with a fixed amount of capital once trained, but assume both capital and labor are malleable ex ante, we may classify the following cases (giving first the situation with regard to capital and then that with regard to labor and assuming one doesn't mind sounding like an outboard motor):

1. Putty-putty; putty-putty
2. Putty-clay; putty-putty
3. Putty-putty; putty-clay
4. Putty-clay; putty-clay

Case one is discussed in the body of the paper. In cases two and three, simultaneous scrapping of capital and unemployment of labor will be impossible even in a model like that of this paper since some of the available fixed coefficient input can be put to work with the available variable-coefficient input, at the appropriate ratio until one input is fully employed. Only in the peculiarly rigid case four does a really new possibility arise. There one might have capital and labor both left over but with fixed and incompatible coefficients. Adapting an example from Brems (1968, Ch. 16), one might have left over abacuses and men who can operate electronic desk calculators, but no electronic desk calculators or men who can operate abacuses.