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AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

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SOME FURTHER RESULTS ON THE MEASUREMENT OF INEQUALITY

Michael Rothschild and Joseph E. Stiglitz

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Some Further Results on the Measurement of Inequality*

by

Michael Rothschild
Harvard University and
Princeton University
Joseph E. Stiglitz
Yale University

1. INTRODUCTION

How should we measure inequality? This old question has received much attention[1] since the appearance of Atkinson's paper "On the Measurement of Inequality" [3]. Drawing on recent work[2] in the theory of choice under uncertainty, Atkinson showed that many seemingly different procedures for evaluating income distributions are equivalent. The major purpose of this note is to prove a stronger version of Atkinson's theorem.

Another purpose of Atkinson's paper was to argue that comparisons of income distributions are most intelligently made when the social welfare function implicit in the use of any single summary measure of income distribution

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[1]Among the recent contributions are those of Allingham [1], Dasgupta [6], Feldstein [7], Newberry [12], Schultz [14], Sen and Starrett [15], and Sheshinski [16]. For a somewhat different exploitation of the relationship between uncertainty and income distribution see Hamada [9] and [10].

[2]The papers in this area are those by Hadar and Russel [8], Hanoch and Levy [11], and ourselves [13].

is made explicit. Following Dalton [5], he restricted attention in his paper to additive social welfare functions. The stronger theorem of this paper shows that this restriction is not necessary. But it does still leave open the question of whether additivity is a property which ethically defensible social welfare functions must have. We discuss this in section II in the context of the debate between Newberry and Sheshinski about the reasonableness of the Gini coefficient as a measure of social welfare.

The social welfare functions in Atkinson's and most papers are proclaimed to be "individualistic." In the third section of the paper we discuss briefly the nature and meaning of this statement. We argue that in the one good economy which most discussions of income distribution assume, the requirement is equivalent to assuming that income is desired by all individuals and that there are no externalities. If there is more than one good, the implications are substantial. A consideration of them leads to a discussion of the index number problems inherent in any discussion of income distribution.

II. RANKING INCOME DISTRIBUTIONS

In this section we state and prove a stronger version of Atkinson's theorem on the equivalence of different methods of ranking income distributions. We claim little credit for the statement of the theorem. Sen and Starrett first suggested how Atkinson's theorem could be improved. We, and, independently, Das Gupta strengthened their results somewhat. The proof follows that of our earlier paper quite closely with one significant difference. The hard part of the proof of that paper was the extension of results obtained for discrete distributions to continuous distributions. Comparisons among income distributions are naturally confined to the discrete case[3]; this fact makes the present proof quite simple[4].

Suppose $X = (x_1, \ldots x_n)$ and $Y = (y_1, \ldots, y_n)$ are different ways of distributing the same amount of income[5] $(\Sigma x_i = \Sigma y_i)$ among n people. What do we mean by the statement: "X is a more equal distribution that Y?" Among the many possible answers, the following three are likely to command assent:

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[3]Another important difference between choice under uncertainty (evaluating probability distributions) and evaluating income distributions is that in the former case we naturally use additive (Von Neuman-Morgenstern) utility functions. It is not so clear that we should so restrict ourselves when comparing income distributions. This problem is discussed further below.

[4]Extension of these results to the continuous case along the lines of our previous paper or of the more sophisticated earlier statements of this result (see Strassen [17]), should present no difficulty.

[5]We consider rankings of distributions of different amounts of total income below.

1. <u>The Lorenz curve of X is inside that of Y</u> .

A common measure of the inequality of a particular distribution X , is the share of total income received by the poorest $\alpha \times 100\%$ of the population which we arite as $s(\alpha, X)$. As α goes from 0 to 1, $s(\alpha, X)$ traces out the Lorenz curve associated with X . The Lorenz curve of a distribution X is inside that of Y if

$$s(\alpha, X) \geq s(\alpha, Y) \quad \text{for all} \quad \alpha \epsilon [0, 1] \; . \tag{1}$$

If (1) holds we shall write $X \geq_L Y$. Note that if X and Y are ordered,

$$x_1 \leq x_2 \dots \leq x_n ; \; y_1 \leq y_2 \leq \dots \leq y_n \; , \tag{2}$$

(1) is equivalent to

$$\sum_{i=1}^{k} x_i \geq \sum_{i=1}^{k} y_i \quad , \qquad k=1, \dots n. \tag{3}$$

2. <u>All those who value equality prefer X to Y</u> .

Among the various possible ways of completely ranking income distributions, some exhibit a clear preference for equality and some do not. If all methods of ranking income distributions which valued equality chose X over Y then we would be prepared to say that X is more equal than Y . To implement this notion it is necessary to have a criterion for deciding whether or not a particular method of choosing among income distributions values equality or not. Finding such a criterion may seem at first glance more difficult than simply deciding which income distributions are more equal. Fortunately it is not; the following weak, and, we think, appealing, condition is sufficient for our purposes: an ordering of income distributions, is equality preferring if, whenever it is indifferent between two income distributions, it prefers a distribution which is a weighted average of both to either one alone. If we restrict our consideration, as we may do without loss, to orderings which are closed and which therefore may be represented by a social welfare function, then our criterion is that the social welfare function be quasi-concave. A further requirement is that all persons be treated similarly or that the social welfare function be symmetric. Thus we shall write $X \geq_W Y$ if

$$W(X) \geq W(Y)$$

for all real valued, quasi-concave functions from R^n to R^1 which satisfy

$$W(Z) = W(\pi(Z))$$

for, π any permutation of Z.

3. Y is obtained from X by transfers from the poor to the rich.

Suppose X is an ordered income distribution and $\hat{X}$ is another one obtained from X by transferring income d from individual k to the richer j. That is:

(i) $x_i = \hat{x}_i \qquad i \neq j, k$

(ii) $x_k - d = \hat{x}_k$

(iii) $x_j + d = \hat{x}_j$

(iv) $\frac{x^k - x^j}{2} \geq d \geq 0 \; ; \quad j > k$

Then we shall say $\hat{X}$ differs from X by a single regressive transfer (RT)[6]. Surely $\hat{X}$ is less equal than X. If Y can be obtained from X by a finite sequence of RT's then we shall write

$$X \geq_T Y .$$

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[6] Atkinson first pointed out that regressive transfers (first discussed by Dalton [5]) were similar to the mean preserving spreads of our earlier paper. Readers of that paper will observe that a particular level of RT, one which would result from an angle between individuals of the same income level, corresponds to the partial ordering $\leqq a$ of that paper.

We now show that for comparison of distributions of the same amount of income among the same number of people all three methods will lead to the same answer.

Theorem I.

If X and Y are distributions of the same amount of income, then the following statements are equivalent:

A. $X \geq_L Y$

B. $X \geq_T Y$

C. $X \geq_W Y$

Proof. The proof consists of demonstrating the chain of implications

$$A \Rightarrow B \Rightarrow C \Rightarrow A .$$

(i)

$$X \geq_L Y \Rightarrow X \geq_T Y .$$

Suppose $X \geq_L Y$, we show how to get Y from X by a sequence of RT's. Let k be the first integer such that $x_k \neq y_k$. Since $X \geq_L Y$, $x_k > y_k$ and we may obtain a new distribution, X(k) by an RT which lowers the income of the kth individual to y_k and raises the income the $k+1^{st}$ by the same amount. The distribution, X(k) has the properties:

$$x_i(k) = y_i, \; i \leq k$$

$$\sum_{i=1}^{k+1} x_i(k) = \sum_{i=1}^{k+1} x_i$$

$$x_j(k) = x_j, \; j > k+1$$

Thus $X(k) \geq_L Y$ and $X(k)$ agrees with Y in k places. We may use the same procedure to find an RT which when applied to $X(k)$ produces a distribution $X(k + j)$ which is $\geq_L Y$ and which agrees with Y in $k + j (\geq k+1)$ places. Continuing, we produce Y from X by a sequence of less than n RT's.

(ii) $$X \geq_T Y \Rightarrow X \geq_W Y$$

Since $X \geq_T Y$ means that Y can be obtained from X by a finite sequence of RT's it will suffice to show that a single RT does not increase the value of $W(x_1, \ldots, x_n)$ where W is quasi-concave and symmetric. We may suppose without loss of generality that Y is obtained from X by an RT affecting the income of only the first two persons. Thus,

$$y_1 = x_1 - d$$

$$y_2 = x_2 + d$$

$$y_i = x_i \qquad i \geq 3$$

$$d \geq 0 ,$$

Consider

$$q(t) = W(x_1 - t, x_2 + t, x_3 , \ldots, x_n)$$

Since W is quasi-concave so is $q(t)$. Because W is symmetric, $q(t) = q(t^*)$ where $t^* = x_1 - x_2 - t$. Thus $q(t)$ attains a maximum when $t = t^*$ i.e., $t = \frac{x_1 - x_2}{2} < 0$. For $t > (x_1 - x_2)/2$, $q(t)$ is decreasing. Thus,

$$w(x_1, \ldots, x_n) = q(0) \geq q(d) = W(y_1, \ldots y_n) .$$

(iii) $$X \geq_W Y \Rightarrow X \geq_L Y .$$

Consider

$$W^k(X) = \sum_{i=1}^{k} x_i .$$

$W^k(X)$ is symmetric[7], non-decreasing, and quasi-concave. Thus $W^k(X) \geq W^k(Y)$ or

$$\sum_{i=1}^{k} x_i \geq \sum_{i=1}^{k} y_i$$

This completes the proof of Theorem I.

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[7]Symmetry follows from the fact that the distributions X and Y are ordered. $W^k(X)$ is the total income received by the k poorest people under distribution X.

Remarks

1. Comparing distributions of different amounts of income.

It is not hard to define the partial orderings $\geq_L$, $\geq_T$, and $\geq_W$ so that they apply to distributions of different amounts of income. The criterion for $\geq_L$ given in (3) is unchanged. A distribution X is preferred to a distribution Y if the income received by the k poorest people is greater under X than under Y for all k. We write $X \geq_W Y$ if $W(X) \geq W(Y)$ whenever W is quasi-concave, symmetric, <u>and</u> monotone increasing. Define an inefficient regressive transfer, IRT, (the appropriateness of the symbolism will be apparent to all New Yorkers) as a redistribution of income in which a poor man gives up d while a richer man receives less than d. Then write $X \geq_T Y$ if Y could be obtained from X by a sequence of IRT's. With these definitions, Theorem I holds when applied to comparisons of distributions of different amounts of income. The proof given above goes through with trivial changes.

2. Other definitions of equality preferring orderings.

Atkinson originally suggested that social welfare functions should meet stronger criteria than those we have put forth, before they could classify as equality preferring. That is he suggested that a reasonable criterion for X to be considered more equal than Y was that

$$\sum_{i=1}^{n} U(x_i) \geq \sum_{i=1}^{n} U(y_i) \quad \text{for all concave } U . \tag{4}$$

If (4) holds, we shall write $X \geq_U Y$. Is $\geq_U$ a more appealing criterion than $\geq_W$? We would be inclined to say no. There seems no reason why a social welfare function should have to be additively separable for it to be equality preferring. However, arguments for additivity can be given (see below p. 12), and this may not seem a very persuasive answer to our query. Fortunately the answer does not matter because $\geq_W$ and $\geq_U$ are equivalent. Since the social welfare function in (4) is quasi-concave and symmetric, $X \geq_W Y$ implies $X \geq_U Y$. The reverse implication follows from Theorem I and Atkinson's proof that $\geq_U$ implies $\geq_L$. An independent proof may be constructed by using the fact that $m_z(x) = \min(x-z, 0)$ is concave to prove that $X \geq_U Y$ implies $X \geq_L Y$.[7a]

Sen and Starrett first suggested that it was not necessary to limit the scope of equality preferring orderings to additively separable ones. They suggested a weaker and more appealing criterion was that $W(X) \geq W(Y)$ for all social welfare functions which could be written

$$W(X) = S(U(x_1), \ldots, U(x_n)) \tag{5}$$

where S is symmetric and concave and U is concave. If this is true we shall write $X \geq_S Y$. Since the function W in (5) is quasi-concave, $X \geq_W Y$ implies $X \geq_S Y$. Furthermore $\geq_U$ is obviously a special case of $\geq_S$ so that $X \geq_S Y$ implies $X \geq_U Y$. Since $X \geq_U Y$ implies $X \geq_L Y$, which is equivalent to $X \geq_W Y$ by Theorem I, we have proved

<u>Theorem II</u>.

The following are equivalent:

A. $X \geq_W Y$

B. $X \geq_U Y$

C. $X \geq_S Y$.

[7a] Define $k_x(z)$ as the maximum value of k for which $x_k < z$. $X \geq_U Y \Longrightarrow \sum_{x_i \leq z} x_i - k_x(z)z \geq \sum_{y_i \leq z} y_i - k_y(z)z$. Now assume the Lorenz curves crossed, so

As we have stated it, Theorem II applies only to comparisons of distributions of the same total income. If all functions used to define $\geq_S$, $\geq_U$, and $\geq_W$ are additionally required to be monotone increasing then Theorem II still holds and may be used to make comparisons of different income distributions.

The questions naturally arises of whether this is the strongest theorem which can be proved. It is not. We would like to characterize the class of functions $W(\cdot)$ satisfying

$$X \geq_L Y \Rightarrow W(X) \geq W(Y) . \qquad (6)$$

It is trivial to show that if $W(\cdot)$ is continuous then (6) implies that it is monotonic and symmetric. It is, however, possible to construct $W(\cdot)$ satisfying (6) which are not quasi-concave. An example is the social welfare function with the indifference curves of Figure 1. Clearly that social welfare function is symmetric and monotone increasing. The requirement that every RT decrease welfare is equivalent to the requirement that lines perpendicular to the 45° line cut the indifference curves exactly twice. Since the line segment $\overline{\alpha\beta\gamma\delta}$ is drawn to be perpendicular to the 45° line this requirment is satisfied. Clearly it would not be difficult to construct a "smooth" $W(\cdot)$ satisfying (6) which was not quasi-concave. A useful characterization of the set of functions satisfying (6) eludes us.

[7a](continued)

$$\sum_{i=1}^{j} x_i > \sum_{i=1}^{j} y_i$$

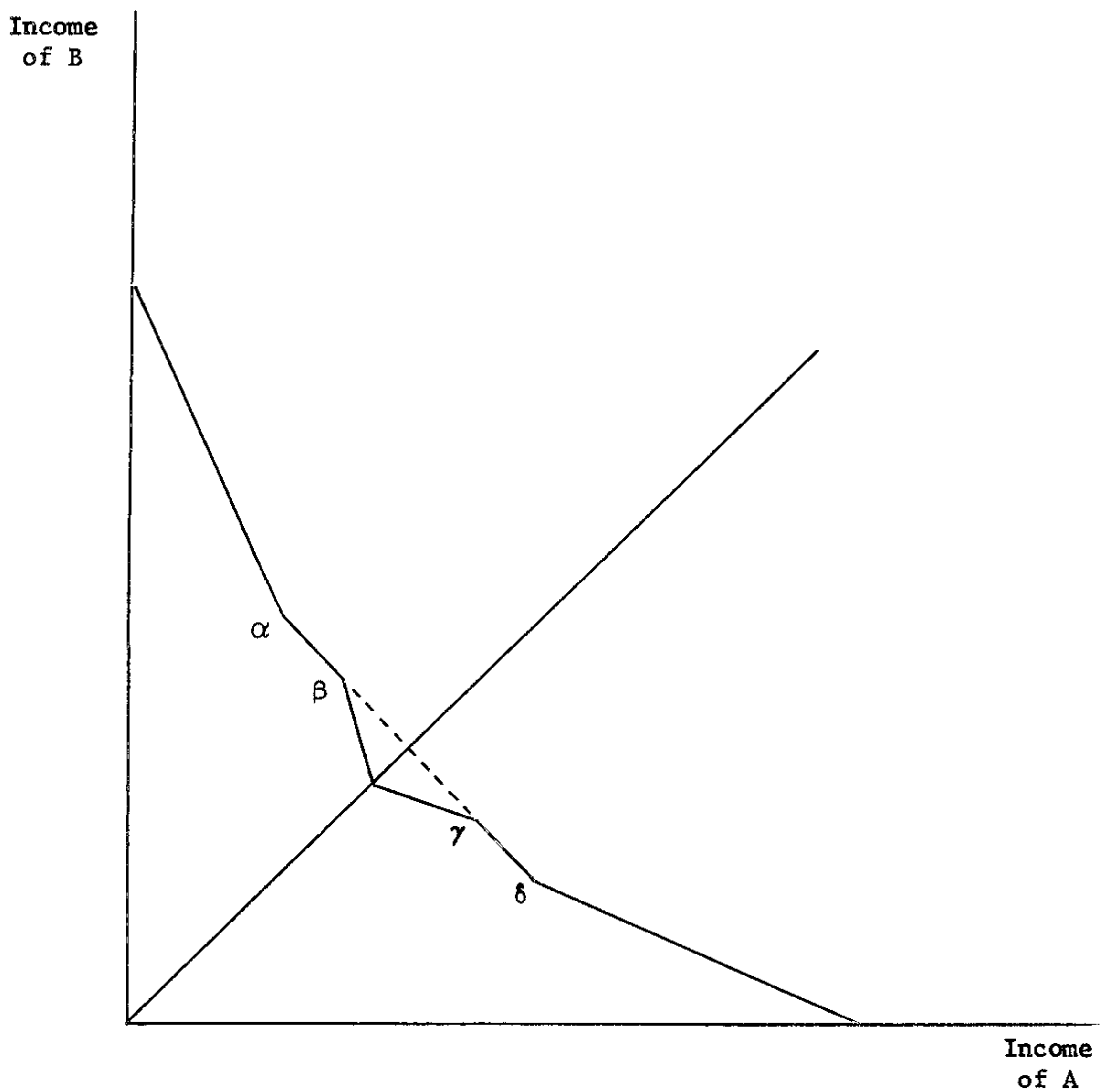
$$\sum_{i=1}^{j+1} x_i < \sum_{i=1}^{j+1} y_i$$

which implies that $x_{j+1} < y_{j+1}$. Then, for $z = x_{j+1}$

$$\sum_{x_i \leq z} x_i = \sum_{1}^{j+1} x_i < \sum_{i=1}^{j+1} y_i < \sum_{y_i \leq z} y_i < \sum_{y_i \leq z} y_i + z(k_x(z) - k_y(z))$$

a contradiction.

Figure 1



III. The Gini-Coefficient and Additivity

The status of additivity as a desideratum for a social welfare function is of some import since it bears directly on whether the Gini-coefficient is an acceptable measure of inequality. Newberry [12] proved that there was no additive social welfare function which ranked incomes in the same order as the Gini-coefficient. Sheshinski argued that "an additive welfare function has no particular significance" and exhibited a social welfare function which agreed with the Gini-coefficient in its ordering of distributions of the same amount of income[8].

We disagree with Sheshinski that additivity has no significance. Two quite separate interpretations of the meaning of additivity are available. First, drawing on the analogy between choice under uncertainty we can ask the following question: Given a complete transitive ordering $\geq$ over the distribution of income $(x_1, \ldots, x_n)$ among n persons what conditions must $\geq$ satisfy for it to be the case that:

(E) There exists a function U such that

$$(x_1, \ldots, x_n) \geq (y_1, \ldots, y_n) \text{ if } \Sigma U(x_i) \geq \Sigma U(y_i) \quad .$$

If we examine Arrow's [2] exposition of the expected utility theorem and make the appropriate translations we see that of the postulates which guarantee that (E) holds, the only one which a symmetric ordering, $\geq$, might not satisfy

[8] See Sheshinski [16]. For different arguments against the use of the Gini index see Atkinson [3] and Theil [18].

is what Arrow calls "dominance." In the language of income distribution the postulate means roughly the following: Suppose there were two islands, A and B. Of two distributions of income among the inhabitants of A, A_1 is preferred to A_2, or $A_1 \geq A_2$. Suppose the islands are now merged and B is any distribution of income among the inhabitants of island B, then

$$(A_1, B) \geq (A_2, B) \quad .$$

Secondly, if $W(x_1, \ldots, x_n)$ is a social welfare function; then there exist functions U^i $(i=1, \ldots, n)$ such that

$$W(x_1, \ldots, x_n) = \sum_i U^i (X_i)$$

if $W(\cdot)$ satisfies the well known Leontief separability conditions. These conditions have a perfectly clear meaning in this case: Consider a benevolent dictator distributing income among his subjects according to his social welfare function. Additional income becomes available; he discovers that he may either give an additional dollar to individual A or an additional p dollars to individual B. There is a p for which the dictator is indifferent between giving an additional income to A or B. If p is independent of the income of everyone in society but A and B then his social welfare function is additive.

Both these arguments show that additivity is a defensible, even a reasonable, property for a social welfare function to have. It is not, however, compelling. To prove a social welfare function morally indefensible, one needs a stronger case than its failure to exhibit additive separability.

The question then naturally arises of what ethical properties a social welfare function which ranks income distributions by their Gini-coefficient can and cannot have. We give part of an answer here. We show that no social welfare function which ranks income as does the Gini-coefficient is strictly quasi-concave. However if $G(X)$ is the Gini-coefficient of an income distribution X, the function which Sheshinski discussed,

$$W(X) = \sum_{i=1}^{n} x_i \, (1-G(X)) \, , \tag{7}$$

is concave and is diminished by any RT. Thus the Gini-coefficient can be part of an equality preferring social welfare function. Both these results follow from the fact that if X is ordered $(x_1 \leq x_2 \leq \ldots \leq x_n)$ and $T(X) = \sum_{i=1}^{n} x_i$ then

$$G(X) = (T(X)n)^{-1} \sum_{i=1}^{n} (2i-n-1)x_i \quad . \tag{8}$$

Thus, if X and Y are ordered distributions of the same income $(T(X) = T(Y))$, and if $G(X) = G(Y)$, then it follows from the linear form of (8) that

$$G(\lambda X + (1-\lambda)Y) = \lambda G(X) + (1-\lambda)G(Y) = G(X)$$

so that no function which ranks distributions of the same amount of income as does the Gini-coefficient is strictly quasi-concave.

However, we may use (8) to rewrite (7) as

$$W(X) = T(X)\,\frac{2n+1}{n} - \left(\sum_{i=1}^{n} 2ix_i\right)/n \qquad (9)$$

from which it is obvious that an RT diminishes $W(X)$. It is also easy to show that the function (9) is concave. To do so we must change notation slightly since we may no longer assume X and Y are ordered. Let $Z = \lambda X + (1-\lambda)Y$. Let $x(i)$ be the ith smallest member of the distribution X and define $y(i)$ and $z(i)$ similarly. Since $T(Z) = \lambda T(X) + (1-\lambda)T(Y)$ it is only necessary to show that

$$\sum_{i=1}^{n} 2iz(i) \leq \sum_{i=1}^{n} \lambda\, 2ix(i) + (1-\lambda)2iy(i)) . \qquad (10)$$

We may write $z(k)$ as $z(k) = \lambda x(\rho(k)) + (1-\lambda)y(\sigma(k))$ where ρ and σ are permutations of 1, 2, ..., n. Thus the LHS of (10) is equal to

$$\sum_{i=1}^{n} \lambda\, 2i\, x(\rho(i)) + (1-\lambda)2i\, y(\sigma(i)) \quad .$$

Clearly $\sum_{i=1}^{n} 2ix(i) \geq \sum_{i=1}^{n} 2ix(\rho(i))$

and

$$\sum_{i=1}^{n} 2i\, y(i) \geq \sum_{i=1}^{n} 2i\, y(\sigma(i))$$

for which (10) follows.

IV. Individualistic Social Welfare Functions

Economists often refer to "individualistic" social welfare functions. What restrictions are implicit in the term "individualistic?" There is no uniformity in usage, so the following represents one reasonable approach:

(A) No individual's utility depends on that of any other individual, or of his consumption of particular commodities.

(B) Social welfare depends simply on the levels of utility of the different individuals, and is an increasing function of each.

In the case of a single commodity economy of the kind we have been discussing so far, conditions (A) and (B) impose only one restriction on the set of functions W(X) that may be called individualistic, namely, $W_i > 0$. Since we are looking for increasing real valued functions V: $R^n \to R^1$ and U: $R^1 \to R^1$ such that

$$V(U(x_1), \ldots, U(x_n)) = W(x_1, \ldots, x_n)$$

if we set $V=W$ and define U by $V(x) = x$, the result is immediate.

In the case of a many commodity economy, the restriction has a great deal of content, for it is a statement about separability, which in terms of the Leontief conditions can be expressed as

$$\frac{\partial(\partial W/\partial C_i(k)/\partial W/\partial C_i(\ell))}{\partial C_j(m)} = 0 \quad \text{for} \quad j \neq i$$

where $C_i(k)$ is the i^{th} individual's consumption of the k^{th} commodity.

Furthermore the marginal rates of substitution between $C_j(\ell)$ and $C_j(k)$ for an individualistic social welfare function will be individual j's own marginal rate of substitution:

$$\frac{\partial W/\partial C_j(k)}{\partial W/\partial C_j(\ell)} = \frac{\partial U_j/\partial C_j(k)}{\partial U_j/\partial C_j(\ell)} \quad .$$

Even if the social welfare function should satisfy these restrictive conditions, we still encounter many difficulties trying to apply the results of the previous sections in a many commodity world. Suppose that all individuals have identical utility functions. Then if the social welfare function is individualistic we may write:

$$W(U(C_1(1), \ldots C_1(m)), \ldots, U(C_n(1), \ldots C_n(m)))$$

$$= W(V(x_1, P), \ldots V(x_n, P)$$

$$= T(X, P)$$

where $V(x,P)$ is the indirect utility function of all persons in society. We are certainly justified in concluding that if $X \geq_L Y$ then $T(X;P) \geq T(Y;P)$ but in general we will have difficulty making statements like:

$$X \geq_L Y \text{ implies } T(X;P) \geq T(Y;P^*) \quad .$$

Obviously, if we could get a measure of "real income" then we could make such inferences (but of course only between different distributions of real income; statements like $X \geq_L Y$ would have no meaning unless both X and Y were measured in real income or if prices were the same under both distributions.) The conditions under which such a true cost of living index exists are both well known and very restrictive; individuals' utility functions must be homothetic. A consequence of this argument is that individualistic social welfare functions which rank income as the Gini-coefficient does, even when prices change, exist only if all individuals' indifference curves are homothetic and "real income" is used to calculate the Gini-coefficient. It is hard to say what meaning can be ascribed to comparisons of Gini-coefficients calculated in economies with widely different relative price structures.

Far more serious problems arise when individuals differ. When individuals differ in tastes, a rise in the price of a commodity which is important for one person may not be of any concern to another. The distribution of money incomes will remain unchanged, but the distribution of real incomes may become more or less unequal. Again, we require that there exists a true cost of living index to convert money incomes to real incomes, which requires each individual to have a homothetic indifference curve. But even this does not resolve our difficulties. For what are we to mean by "equal levels of utility"? Assume all individuals had a homothetic indifference map, and we were willing to adopt a cardinal utility function homogeneous of degree one for each individual. This still leaves us with one parameter to choose, which specifies in effect at which set of relative prices individuals are said to have the same level of utility[9]. This problem is illustrated in Figure 2. Do individuals

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[9] A more satisfactory way to handle the problem of measuring equality in a multi-commodity world could possibly be constructed by using the multi-dimensional versions of the theorems on choosing probability distributions on which Theorems I and II are modeled. These theorems make it possible to talk about more and less unequal distributions of all commodities thus permitting unambiguous orderings of distributions of consumption goods independent of prices. See Blackwell and Girshick [4] and Strassen [17] for generalizations of the results of [13].

A and B have the same real income when prices are given by P_1 and A consumes $C_A(1)$ and B , $C_B(1)$ or when prices are given by P_2 and A consumes $C_A(2)$ and B , $C_B(2)$? The same sort of difficulty can arise when two individuals have the same shape indifference curves but may possibly vary in their ability to produce socially valued utility from the same amount of goods. However, the arbitrariness of scale seems much greater in the first case because the lack of symmetry between the individuals gives no clues as to how real income should be parameterized. When individuals have the same indifference curves it is natural to suppose they are the same in other respects. There is no such obvious solution to the problem presented by Figure 2.

Figure 2

The point of these remarks is to emphasize that Theorems I and II, although they help clarify the meaning of income inequality, do not resolve the index number problems inherent in comparing income distributions.

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