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BENEFIT-COST ANALYSIS AND TRADE POLICIES

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1. Introduction

1.1. Motivation

In this paper we address ourselves to two questions:

(1) In an open economy, in which the government is pursuing various trade and tax policies, and where some of the policy instruments are optimally chosen but the others are not, and in which, as a consequence, domestic price ratios differ from international price ratios, what should be the relationship between shadow prices in benefit-cost analysis, international prices and domestic prices?

(2) Under what conditions is it optimal to impose differential taxes on imports/exports from domestically produced commodities; and what is the optimal tariff structure in these circumstances?

The motivation behind these questions is probably transparent. Much of the recent discussion of benefit-cost analysis (in particular [12] and [22]) has been aimed at a detailed specification of the procedure that governments

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ought to follow in choosing shadow prices for project evaluation. At the same time no economy that we are aware of pursues a completely free trade policy.

This means that there are divergences between domestic and international prices. Which should be used for project evaluation? Or neither? Perhaps something in between?

Many developing countries rely heavily on tariffs and export duties for government revenue. The main justification in developing countries for the primary reliance on trade taxes as a source of revenue seems to be in its administrative convenience, although "infant industry" arguments are often given in the justification of particular tariffs.[1] Tariffs act, of course, as discriminating taxes, interfering with production efficiency, and where they can be replaced by more general taxes, there has been a presumption in favor of doing so. But this presumption is based on ignoring important constraints, e.g. the non-existence of lump sum taxes or 100% rent (profit) taxes. In more "realistic" situations, is it ever optimal to retain the differential treatment of domestically produced goods from foreign produced goods when it is feasible to impose more general taxes? How does the optimal structure of tariffs, i.e. the relative size of tariffs imposed on different commodities, depend on whether taxes are levied, for instance, on consumption as well.

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[1] The proportion of total tax revenue collected through border taxes is often very high in developing countries. For instance, customs and export duties constituted in 1961-1962 the following percentages of total tax revenue in the following African countries: Sierra Leone, 68%; Ghana, 76%; Nigeria, 72%; Uganda, 54%; Tanganyika, 49%; and Zanzibar, 90%. (See Due [10]). Profits of state controlled marketing boards ought also to be included in this total when the domestic price charged by the marketing board differs from the international price or when the policies of the marketing board (for exports) affects the international price.

The second set of questions is closely related to the recent literature on effective protective rates. (See, e.g. [6, 19].) The interest in this concept was presumably primarily motivated by asking the question; what structure of tariffs would not distort <u>relative</u> prices of different commodities and factors of production? But the converse of the well-known proposition that in general equilibrium it is only relative prices that are determined is that without changing relative prices, a tax system can raise no revenue. A more limited objective is to find the tariff system which will not distort relative commodity prices (but will in general change relative factor prices and prices of commodities relative to factors). Of course, a system which does not change relative commodity prices <u>if</u> relative factor prices are assumed not to change will, in general, change relative commodity prices, and much of the recent discussion in the literature seems directed at this point. But the more fundamental question is, why should we be interested in a tariff structure which leaves unchanged relative commodity prices, or even relative supplies of different commodities? We know that most taxation is distortionary. The relevant question is, if you must have distortions, what is the best set of distortions to have?

These are questions in the theory of the "second best," a subject of much discussion in recent years.[1] But even within this very large field of enquiry some broad strands of investigation seem to have emerged, of which it is relevant to mention two:

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[1]Although the first extensive discussion of "second best" welfare economics was that of Meade [13], the first real exercise in second best welfare economics was probably that of Ramsey [18].

(1) Those investigations that have been concerned with obtaining some general propositions about optimal public policy for economies where lump-sum taxation is feasible and in which the "first-best" can in principle be attained but in which distortions have been introduced by, for instance, unions.[1]

(2) Those investigations that have been concerned with establishing fairly detailed relationships between shadow prices in the public sector, and producer and consumer prices, when the divergences between producer and consumer prices are due to taxes which are optimally chosen subject to various institutional constraints.[2] Typical of the constraints that have been looked at we mention in particular the government's inability to impose lump sum taxation (as in [9] and [21]), and further constraints, such as (a) an over-all budget constraint on the public sector (as in [5] and [21]), (b) the government's inability to tax certain commodities ([13], [17] and [21]), and (c) the requirement of taxing different commodities at the same rate (as in [19]).

The present paper falls in the second of the two strands that we have just mentioned. Here we attempt to extend to economies that trade the analysis of some of the problems that we raised earlier in [21], and investigate a number of problems that are peculiar to open economies.

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[1] See Bhagwati [2] for a recent excellent discussion of this.

[2] The classic discussions on optimal taxation are Ramsey [18], Meade [13] and Boiteux [5]. Among the more recent contributions are Diamond and Mirrlees [9], Stiglitz and Dasgupta [21], Dasgupta and Stiglitz [8], and Mirrlees [15].

1.2. The Open Economy

Some years ago Pigou [16] attempted to extend to open economies the basic principles of optimal taxation that Ramsey had earlier described for economies that do not trade. Ramsey had, for instance, argued that in the case of independent demand and supply curves,[1] for small government revenue taxes ought to be proportional to the sum of the inverse of the demand and supply elasticities.[2] Since the supply elasticity of foreign commodities is presumed in general to be greater than that for domestically produced commodities, Ramsey's analysis implied that rather than imposing a surtax i.e. imposing a higher tax on imports than on domestically produced goods, one should tax imports at a lower rate. But Pigou did not answer the question of what should be done for commodities which are both domestically produced and imported. Should they receive differential treatment? Should the tax on imported cars be lower than that on cars produced at home?

There are, moreover, some important differences between taxation policy in closed economies and that in open economies, and it is not immediately apparent when proper account of this is taken whether Pigou's conclusions remain valid. Three differences come to mind:

1. For imported goods, the producer surplus which accrues to producers of commodities which are not in perfectly elastic supply accrues to nationals of other countries, and does not accordingly affect domestic welfare. For domestically produced goods it does.

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[1]That is, the demand and the supply for a commodity depends only on the price of that commodity (relative to, say, the price of labor).

[2]Some of the assumptions implicit in Ramsey's analysis, e.g. the non-taxation of profits, have recently been made explicit. See Stiglitz and Dasgupta [21].

2. Many countries face what is sometimes referred to as a "foreign exchange constraint." This implies that the earnings from export do not measure their social value, and the cost of imports does not measure the social cost. To what extent this should affect the rates at which different imports should be taxed, and whether this should result in governments' using in benefit-cost analysis prices that are different from those that rule at the border are questions that are still often debated on. In the study of development economics in fact two different constraints have been distinguished as meriting consideration. They are (a) a savings constraint (gap) and (b) a foreign exchange constraint (gap). To see what is involved consider an open economy in which there are no barriers to trade or to the flow of capital. If p_{it} is the international price of commodity i at time t, and z_{it} is the net import of commodity i at t, then trade balance requires only that

$$\sum_t \sum_i p_{it} z_{it} = 0 . \tag{1.1}$$

A "foreign exchange" constraint imposes an additional constraint: in any particular year, t, for instance, the deficit may not be larger than so much, say ε_t. That is[1]

$$\sum_i p_{it} z_{it} \leq \varepsilon_t . \tag{1.2}$$

[1]Perhaps a more realistic way of writing the constraint is that the cumulative deficit at time t be less than some number, $\hat{\varepsilon}_t$:

$$\sum_{\tau \leq t} \sum_i p_{i\tau} z_{i\tau} \leq \hat{\varepsilon}_t .$$

This complicates the mathematics but does not affect the qualitative results given below in Section 7.

This is equivalent to saying that the country cannot borrow more than e_t from abroad in any given year. The foreign exchange constraint (1.2) is binding if and only if the country would like to borrow more than it can, which in turn implies that there is a constraint on the supply of investible funds (in other words, a savings constraint). If as the analysis seems to suggest, these two constraints are in fact closely related, it is pertinent to ask whether it is fair to argue that the foreign exchange constraint does not affect relative prices used in benefit-cost analysis while the savings constraint does.[1]

3. Although most governments do use excise and specific commodity taxes on domestic consumption to a limited extent, they are usually justified in terms of distributional objectives (e.g. taxing luxuries, like perfumes) or in terms of merit wants (e.g. cigarettes and tobacco), externalities, or as benefit taxes (e.g. gasoline). On the other hand almost all countries impose extensive import duties at varying rates, and both the commodities taxed at the border and the rates at which they are levied have probably less to do with welfare considerations than with the strength of various pressure groups. Thus, the parable of a government (as in say [9] and [2]) that levies distortionary taxes that have been optimally chosen (although subject to various administrative constraints) is likely to be even a less plausible story of the planning apparatus in the case of an open economy than in the closed economy.

[1] See Little and Mirrlees [12] for a discussion of this.

To put the matter differently, the conventional planning literature treats the "government" as if it were a single unit, the central planning agency being the basic coordinating body. It simultaneously decides on taxes, tariffs and investment projects. Although this assumption has its merits--it allows one to pose a number of questions in their most pristine form--it is doubtful whether it makes for a good theory of government action.[1] Suppose that agency A is required to take a decision. It may well be advised to assume a non-optimal decision of agency B as a fact of life. It may not only be the case that A would wish to assume that B's non-optimal decision is unchanging. There may be situations where A might know that B will respond in a non-optimal way to A's decision. To take an example, suppose that project X requires equipment E as an input which it is optimal to import (since domestic production of E is too costly). If E is in fact imported then assume X is a desirable project. But the government project evaluator may know that if X in undertaken there will be powerful pressure groups at work forcing the departments responsible for levying tariffs to raise the tariff on E so as to enable private domestic producers to produce more of it domestically to meet the augmented demand for it. What should then be the decision with regard to X ?

It does not seem apparent to us, at least, whether, in a closely interdependent economy a non-optimal tariff on one commodity should affect the tariffs and shadow prices of other commodities.

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[1] See also Sen [20].

Nevertheless it has been argued that tariffs whether optimally chosen or not should be neglected for the purpose of benefit-cost analysis.[1] One of the purposes of this paper is to investigate the kinds of circumstances under which this argument is correct.

In order to bring out the principles involved as clearly as possible we consider in the following sections a sequence of models, in each of which the powers of the planner are somewhat more circumscribed than in the previous. In Section 2 the only constraint on the government is the impossibility of lump-sum taxation. The government can impose 100% profit taxes, and taxes on all production, consumption, and trade. In Section 3 we then consider the consequences of the government not being able to impose 100% profit taxes. In Section 4 we further restrict the government in allowing it to impose only trade taxes. In Section 5 some of the trade taxes are assumed given and quotas are assumed to be imposed on some commodities. But the government must still take decisions concerning the choice of investment projects and the rates of tariffs on the remaining commodities. In Section 6 the consequences of trade policy partially dependent on project selection are explored. In Section 7 the consequences of a foreign exchange constraint are considered, while in Section 8 those of a budgetary constraint on the government are investigated. In Section 9 we indicate briefly how distributional considerations may be integrated with our analysis. Finally in Section 10 we summarize the major conclusions of the paper in terms of

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[1] See in particular, Little and Mirrlees [12].

a set of rules.[1]

Before setting out, we note three assumptions which will limit the generality of our conclusions:

(1) We shall assume that every commodity is either tradeable at a fixed international price or not tradeable at all.[2] This not only simplifies the mathematics, but also allows us to separate the tax revenue-welfare effects of concern here with the terms of trade effects, which have been the focus of the optimal tariff literature so far.

(2) We shall assume that the only "distortions" in the economy are those introduced by government tax policy; otherwise, prices of factors and goods are competitively determined. Accordingly, there is, for instance, no unemployment. The presence of other distortions, e.g. unemployment undoubtedly will introduce discrepancies between market prices and shadow prices, whether there are tariff distortions or not.

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[1]There has been a good deal of discussion (besides that in Pigou cited above) on optimal tariff and tax policies for open economies. These fall primarily into three categories: (1) those which consider economies in which the terms of trade are affected by trade policy; e.g. Johnson [11]; (2) those which consider distortions which the trade/tax policy is supposed to alleviate, e.g. Bhagwati and Ramaswami [3]; and (3) those which compare the welfare costs of tariffs and other tax-subsidy schemes, e.g. Mieszkowski [14] and Bhagwati, Ramaswami and Srinivasan [4].

The two authors who have come closest to discussing the issues on tariff policy presented here are Meade [13] and Ramaswami and Srinivasan [17]. The model considered in [17] may be viewed as a special case of that presented here, for Ramaswami and Srinivasan are concerned exclusively with an economy in which only border taxes can be levied (a circumstance we discuss in Section 4). Moreover they do not allow for the possibility of public production. In particular, their conclusion that inputs used in export production must be free of duty is shown to hold for all intermediates when profits can be taxed at 100% (see Section 4.2). Their conclusion that duties need not be uniform in a second best revenue tariff is reestablished here (section 4.1) and the precise formulae for the optimum tariffs are given.

(3) For most of the analysis, we shall assume all individuals are identical. This means that we will not investigate the implications for tariffs and shadow prices of the interaction between savings and income distribution.

All of these raise important problems which we hope to pursue elsewhere.

2. The Fully Controlled Economy

In this section we consider an economy in which there are no foreign exchange constraints, and no trade and domestic distortions apart from those which the government deliberately imposes. The only constraint imposed upon the government is the unfeasibility of its employing lump sum taxes. The government's problem is to decide what taxes to impose on domestic producers, what tariffs to levy on imports (both intermediate and final commodities), what excise taxes to set on domestic consumption, and which projects to undertake. It wishes to do this in such a way that all markets clear, that trade balances, and so that welfare is maximized.

2.1. Notation

We introduce the following notation:

c_i consumption of the i^{th} commodity (for factors supplied $c_i < 0$).[1]

[2]We assume, in other words that the country in question is sufficiently small so as to have no influence on international prices. Although this may be true for some commodities, for numerous commodities (e.g. coffee) there are international quotas. For other commodities there are restrictions and regulations that effectively limit the quantity that can be exported (e.g. beef), and for other commodities, in particular manufactures, access to foreign markets may be limited by monopolistic practices of the few major marketing orginizations. Moreover, the elasticity of demand for most commodities is larger in the long run than the short run. On the importing side, the price taking assumption is undoubtedly far more realistic.

[1]These commodities may be time dated, so the results are valid for the intertemporal case as well as the static case.

y_i production of commodity i by domestic private industry (for factors used by private industry $y_i < 0$).

y_i^j production of commodity i by the j^{th} producer.

q_i consumption price of commodity i.

p_i producer price of commodity i. When different producers are set different prices for the same commodity, the price is denoted as p_i^j.

z_i net import of commodity i. If it is a net export then $z_i < 0$.

x_i net output of commodity i in the public sector. If it is a net input then $x_i < 0$.

t_i tax on the import of commodity i.

Γ_i tax on producers of commodity i. (When different producer faces different taxes, the tax is denoted by Γ_i^j.)

τ_i difference between the domestic producer price and the international price for commodity i.

Where there is no risk of ambiguity we shall denote a vector, say (c_i), simply by c. The scalar product of two vectors, say q and c will be denoted by $q \cdot c$. Occasionally we shall denote by $[\eta]$ a square matrix with typical element η_{ik}.

Now clearly not all commodities are tradeable. Nor are all commodities consumable. We shall denote by A the set of <u>all</u> commodities in the economy; by T the set of all <u>tradeable</u> commodities; by N the set of all <u>non-tradeable</u> commodities; by D the set of all <u>domestically produced</u> commodities (some of which still might be traded, of course); by I the set of all <u>intermediate</u> goods; and by C the set of all <u>consumption</u> goods. Thus $A = C \cup I = T \cup N$, $C \cap I = \emptyset$, $T \cap N = \emptyset$, $N \subset D$. Our numeraire good will be indexed as $i = 0$. We shall throughout suppose that it is at the same time traded, domestically produced and consumed. That is $i = 0 \epsilon$ $C \cap T \cap D$. Our units are chosen so that the international price of every traded commodity is unity.

2.2. Basic Relations

There are a few simple relations existing between the variables defined in Section 2.1 which we set forth here. To begin with, the commodity balance equations are:

$$c_i = y_i + x_i + z_i \tag{2.1}$$

where it is understood that

$$\begin{aligned} z_i &= 0 \qquad && i \in N \\ c_i &= 0 && i \in I \\ y_i &= 0 && i \notin D \\ x_i &= 0 && \end{aligned}$$

The total output y_i from domestic private firms is

$$y_i = \sum_j y_i^j \qquad i \in D\,. \tag{2.2}$$

Now domestic consumption prices equal international prices plus the taxes on traded goods. We then have

$$q_i = p_i + \Gamma_i = 1 + t_i \qquad i \in C \cap T \cap D \tag{2.3a}$$

$$q_i = p_i + \Gamma_i \qquad i \in C \cap N \tag{2.3b}$$

$$q_i = 1 + t_i \qquad i \in C \cap T \text{ and } i \notin D \tag{2.3c}$$

$$p_i + \Gamma_i = 1 + t_i \qquad i \in T \cap D \cap I \tag{2.3d}$$

In other words,

$$\Gamma_i - t_i = \tau_i = 1 - p_i \qquad i \in T \cap D \,. \tag{2.3e}$$

Thus $t_i > 0$ for a tariff on an import or a subsidy on an export

$t_i < 0$ for an export tax

$\tau_i > 0$ for a production tax <u>in excess of the tariff</u>

$-t_i < \tau_i < 0$ for a production tax that is less than the tariff.

If $\tau_i = -t_i$, domestic production is neither taxed nor subsidized $(q_i = p_i)$.

2.3. <u>Consumer Behavior</u>

Throughout the discussion (except for the discussion in Section 9) we shall suppose that all individuals are identical. We shall, therefore, talk of the "representative" individual. He is assumed to maximize his utility,

$$\text{maximize } U(c) \tag{2.4}$$

subject to his budget constraint

$$q \cdot c = M \tag{2.5}$$

where M is his income. If there are no lump sum taxes or subsidies, then M is just equal to profits (pure rents) after tax. If π^j denotes the net profit of the j^{th} firm, and τ_π denotes the tax rate on profits, then

$$M = \sum_j (1 - \tau_\pi)\pi^j = (1 - \tau_\pi)\pi \quad \text{where} \quad \pi = \sum_j \pi^j \ . \tag{2.6}$$

The solution to (2.4) may be written as

$$V(q, M) = \max \cdot U \tag{2.7}$$

V is known as the consumer's indirect utility function.

2.4. Behavior of Firm

There are m different private firms in the economy. For simplicity we assume that all firms face strictly concave production functions.[1]

$$F^j(y^j) = 0 \qquad j = 1, \ldots, m \ . \tag{2.8}$$

The j^{th} firm maximizes profits at the given price vector p^j :

$$\text{maximize} \quad p^j \cdot y^j \qquad \text{subject to} \quad F^j(y^j) = 0 \ . \tag{2.9}$$

The solution to (2.9) is denoted by $\pi^j(p^j)$. Moreover, in equilibrium it will be the case that

[1]As Dasgupta and Stiglitz [8] have shown, there is no important loss of generality by assuming strict concavity. See [8] and Mirrlees [15] for a discussion of some possible anomolies.

$$p_i^j/p_k^j = \frac{\partial f^j/\partial y_i^j}{\partial f^j/\partial y_k^j} \qquad i,\ k \in D\ . \tag{2.9a}$$

These equations can be solved for the supply of commodities (demands for factors) as a function of p^j :[1]

$$y_k^j = y_k^j(p^j)\ . \tag{2.9b}$$

Indeed, it is a well known result that

$$\frac{\partial \pi^j}{\partial p_k^j} = y_k^j\ . \tag{2.9c}$$

One other fact will be of use: Differentiating the production constraint, we obtain

$$\Sigma\ \frac{\partial f^j}{\partial y_i^j}\ \frac{\partial y_i^j}{\partial p_k^j} = \frac{\partial f^j/\partial y_0^j}{p_0}\ \Sigma\ p_i\ \frac{\partial y_i^j}{\partial p_k^j} = 0\ . \tag{2.9d}$$

2.5. Trade Balance

The condition for the balance of trade may be written as

$$B = \sum_{k \in T} z_k = c_k - y_k - x_k\ . \tag{2.10}$$

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[1] Note that if we had a constant returns to scale function, then we could solve for relative factor intensities, but not for total factor demands without knowing quantities produced. See, however, [8].

Thus, we can view B as a function of q , p , and x :

$$\frac{\partial B}{\partial q_i} = \sum_{k\epsilon T} \frac{\partial c_k}{\partial q_i} \tag{2.10a}$$

$$\frac{\partial B}{\partial p_i} = y_i \sum_{k\epsilon T} \frac{\partial c_k}{\partial \pi} - \sum_{k\epsilon T} \frac{\partial y_k}{\partial p_i} \tag{2.10b}$$

$$\frac{\partial B}{\partial x_k} = -1 \tag{2.10c}$$

2.6. The Government

The government's production function we shall denote by

$$G(x) = 0 \ . \tag{2.11}$$

As equation (2.11) indicates, we are tacitly supposing that the public sector is concerned with the transformation of the same set of goods as the domestic private sector is. We assume this simply for notational ease.[1]

[1] We are supposing, then, that there are no public goods in this economy. If we were to assume that the public sector produces public goods as well we would obtain one additional set of first order conditions to the set of conditions obtained below. This additional set would dictate the optimum supply of public goods. For a discussion of the implications of such a condition see Diamond and Mirrlees [9] and Stiglitz and Dasgupta [21]. Our concern here is with obtaining shadow prices of private goods; and it is trivial to show that the results derived here apply without modification to the case where public goods are also produced by the government. For this reason we ignore public goods here.

Similarly there may be several production units in the public sector. But the result derived earlier in Boiteux [5], Diamond and Mirrlees [9] and Stiglitz and Dasgupta [21], that the public sector should always be productively efficient remains valid. To keep the notation simple we have therefore amalgamated all public sector production units into one single unit.

We shall suppose that the government controls trade and production, but only indirectly through tax and tariff policies. The problem before the planning agency is straightforward: it wishes to maximize the welfare V of the representative consumer subject to the private sector production possibility curves (2.8); the public sector production possibility curve (2.11); the balance of payments condition (2.10); and the market clearing equations for the non-traded goods. The government's controls are direct government production, tariffs, consumption, production and profits taxes. But it is easy to establish (using (2.3)) that controlling taxes and tariffs is essentially equivalent to controlling producer and consumer prices and the latter are conceptually and analytically easier to use. Similarly, although controlling p, q, and x determines z, so the only constraint on the traded goods is the balance of payments constraint (i.e. we do not need to impose separate market clearing constraints on each of the traded commodities), it is convenient to formulate the problem as if z were control variables, and impose the additional market clearing equation for each of the traded goods.[1] Finally, we note (see [21]) that optimality

[1]In other words, the problem

$$\max_{\{x,p,q\}} V(q, \pi(1-\tau_\pi)) + \lambda\left(\sum_{k\in T}(x_k + y_k - c_k)\right) + \sum_{k\in N}\rho_k(x_k + y_k - c_k)$$
$$+ \mu G + \sum_{j=1}^{m}\psi_j f^j(y^j)$$

and the problem

$$\max_{\{x,p,q,z\}} V(q, \pi(1-\tau_\pi)) - \lambda\sum z_k + \sum_{k\in A}\rho_k(x_k + y_k - c_k) + \mu G + \sum_{j=1}^{m}\psi_j f^j(y^j)$$

are equivalent.

requires the government to set $\tau_\pi = 1$, provided the government requires some resources to be raised by distortionary taxes. In view of the fact that the analysis is trivial in those cases where the government does not need to impose distortionary taxes to raise its revenue we set $\tau_\pi = 1$ whenever it is assumed to be feasible. For the remainder of this section we suppose that it is.

As is conventional, we normalize by setting $q_0 = 1$ and $p_0^j = 1$ for all j.[1]

2.7. Optimal Public Policy

We now form the Lagrangean of the problem:[2]

$$\mathcal{L} = V(q, 0) - \lambda B + \mu G + \sum_{j=1}^{m} \psi_j f^j(y^j) + \sum_{k \in A} \rho_k \left(\sum_{j=1}^{m} y_k^j + x_k + z_k - c_k \right) \quad (2.12)$$

We now obtain the first order conditions and write them as:

$$\frac{\partial V}{\partial q_i} - \sum_{k \in C} \rho_k \frac{\partial c_k}{\partial q_i} = 0 \qquad i \in C \text{ and } i \neq 0 . \quad (2.13)$$

$$\sum_{k \in D} \left(\psi_j \frac{\partial f^j}{\partial y_k^j} + \rho_k \right) \frac{\partial y_k^j}{\partial p_i^j} = 0 \qquad i \in D \text{ and } i \neq 0 , \quad j = 1, \ldots, m \quad (2.14)$$

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[1] For a discussion of these normalizations, see Dasgupta and Stiglitz [ ].

[2] We recall the convention adopted in discussing (2.1) that it is understood that $z_i = 0$, $i \in N$, $c_i = 0$, $i \in I$, $g_i = 0$, $i \notin D$, $x_i = 0$, $i \notin D$.

$$\mu \frac{\partial G}{\partial x_i} + \rho_i = 0 \qquad i \in D \tag{2.15}$$

and

$$-\lambda + \rho_i = 0 \qquad i \in T \,. \tag{2.16}$$

From equations (2.14)-(2.16) we obtain the familiar result:[1]

$$\frac{\partial G/\partial x_i}{\partial G/\partial x_0} = \frac{\rho_i}{\rho_0} = \frac{\partial f^i/\partial y_i^j}{\partial f^j/\partial y_0^j} = p_i^j = 1 \qquad i \in T \cap D\,, \quad j = 1,\ \ldots,\ m \tag{2.17a}$$

and

$$\frac{\partial G/\partial x_i}{\partial G/\partial x_0} = \frac{\partial f^i/\partial y_i^j}{\partial f^j/\partial y_0^j} = \frac{\rho_i}{\lambda} = p_i \qquad i \in N \cap D \,. \tag{2.17b}$$

Equations (2.17a) and (2.17b) describe the desirability of overall production efficiency for the economy. All production units in the economy ought to use international prices in selecting their optimum techniques, and there ought to be no taxes or tariffs on intermediate goods. (Cf. [12], [22].)

It is also apparent that in this case, Pigou's conclusion, that domestic production ought to be taxed at a higher rate than imports, is incorrect. No differentiation between domestic and foreign production should be made.

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[1] The second equality follows for observing that, for given j, we have as many equations of the form (2.14) as commodities; since we thus have a system of n homogenous equations in n unknowns, we require

$$\psi_j \frac{\partial f^j}{\partial y_k^j} + y_k = 0 \,.$$

See [8] for a more detailed discussion.

Turning to equation (2.13) it is now simple to show that it reduces to:

$$\sum_{k \in T \cap C} \frac{t_k}{c_i}\left(\frac{\partial c_i}{\partial q_k}\right)_{\bar{U}} + \sum_{k \in N \cap C} \frac{\Gamma_k}{c_i}\left(\frac{\partial c_i}{\partial q_k}\right)_{\bar{U}} \equiv - \sum_{k \in T \cap C} \frac{t_k}{q_k} \eta^d_{ik} - \sum_{k \in N \cap C} \frac{\Gamma_k}{q_k} \eta^d_{ik} = -\theta \qquad i \in C \qquad (2.18)$$

where

$$\theta \equiv 1 - \frac{1}{\lambda}\frac{\partial V}{\partial M} - \sum_{k \in T \cap C} t_k \frac{\partial c_k}{\partial M} - \sum_{k \in N \cap C} \Gamma_k \frac{\partial c_k}{\partial M}$$

and

$$\eta^d_{ij} \equiv -\left(\frac{\partial \ln c_i}{\partial \ln q_j}\right)_{\bar{U}} .$$

It follows from equation (2.18) that for small taxes the consumption of all commodities is reduced by the same amount (from what it would have been had producer prices been charged).[1]

3. Limited Profit Tax on Private Firms

3.1. General Considerations

For a number of reasons no government imposes a 100% tax on profits (rents). This requires a good deal of modification in the analysis. Now

[1] Note the difference between this result and the underlying notions of the effective tariff literature. There, the focus is on a tariff structure which reduces production of each commodity by an equal percentage from the pre-tax situation and the analysis often ignores the change in factor prices of non-traded inputs; here, we reduce consumption by an equal percentage from what it would have been with the new producer prices; for traded goods, the only change in production results from the change in prices of non-traded inputs.

profits enter directly into the utility function, and changes in profits affect the demands for various goods. For simplicity we consider the polar case where no tax on profits can be imposed. The intermediate cases are trivial to work out.

The Lagrangean now takes the form

$$V(q, \sum_{j=1}^{m} \pi^j(p^j)) - \lambda \sum_{k\in T} z_k + \mu G(x) + \sum_{j=1}^{m} \psi_j f^j(y^j) + \sum_{k\in A} \rho_k (\sum_{j=1}^{m} y_k^j + x_k + z_k - c_k) . \tag{3.1}$$

The first order conditions now read as:

$$\frac{\partial V}{\partial q_i} - \sum_{k\in C} \rho_k \frac{\partial c_k}{\partial q_i} = 0 \quad i \in C , \quad i \neq 0 \tag{3.2}$$

$$\frac{\partial V}{\partial M} \frac{\partial \pi^j}{\partial p_i^j} + \sum_{k\in D} \left(\psi_j \frac{\partial f^j}{\partial y_k^j} + \rho_k \right) \frac{\partial y_k^i}{\partial p_i^j} - \sum_{k\in C} \rho_k \frac{\partial c_k}{\partial M} \frac{\partial \pi^j}{\partial p_i^j} = 0 \qquad j = 1, \ldots, m \quad i \in D \text{ and } i \neq 0 \tag{3.3}$$

$$\mu \frac{\partial G}{\partial x_i} + \rho_i = 0 \qquad i \in D \tag{3.4}$$

$$-\lambda + \rho_i = 0 \qquad i \in T . \tag{3.5}$$

3.2. All Commodities Tradeable

Consider for simplicity the case where all goods are tradeable. Using equation (3.5) we can write equation (3.2) as

$$\frac{\partial V}{\partial q_i} - \lambda \sum_{k\in C} \frac{\partial c_k}{\partial q_i} = 0 \qquad i \in C \text{ and } i \neq 0 . \tag{3.2a}$$

Equation (3.2a) has an immediate interpretation. By raising tariffs (consumer prices) we improve the balance of payments, by discouraging imports. The "cost" of this is of course a decrease in utility--higher prices for consumer goods makes one worse off. There is, therefore, a trade-off between "trade surplus" and welfare. Equation (3.2a) reflects the fact that the marginal rate of substitution between the two should be the same regardless of which price we vary; i.e. the loss in welfare per unit gain in the balance of payments surplus should at the margin be the same for all commodity prices.[1] Turning to equation (3.3), by using (2.9c) and the fact that

$$\sum_k \frac{\partial f^j}{\partial y_k^j} \frac{\partial y_k^j}{\partial p_i^j} \equiv 0 \qquad \text{all} \quad i \, ,$$

we obtain

$$y_i^j \frac{\partial V}{\partial M} = \lambda \left(y_i^j \sum_{k \in C} \frac{\partial c_k}{\partial M} - \sum_{k \in D} \frac{\partial y_k^j}{\partial p_i^j} \right). \tag{3.3a}$$

Equation (3.3a) has an interpretation similar to (3.2a). An increase in the producer price of an output increases profits and makes the consumer better off. It has two effects on the balance of payments: by increasing the supply of the commodity it is improved, but by increasing the demand (as a result of the higher income from the higher profits) it is deteriorated. Equation (3.3a) reflects the fact that at the optimal point the latter effect on the balance of payments must dominate the former, and indeed, the marginal rate of substitution of gains in welfare and the worsening of the balance of payments must be the same, regardless of which producer price is varied.

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[1] Exactly the same reasoning applies to exports. An export tax lowers the domestic price, increases welfare, and because it increases consumption of the given commodity, makes the balance of payments situation worse.

Equations (3.4) and (3.5) have the following similar interpretation: the marginal rate of transformation between two commodities in public production must be equal to their relative marginal effects on the balance of payments at fixed producer and consumer prices. Thus $\partial B/\partial x_i = 1$; the government project evaluator should use international prices in selecting public sector projects.

To see more precisely what equations (3.2a) and (3.3a) imply for the structure of tariffs and taxes we begin by noting that equation (3.2a) reduces to the form

$$\sum_{k\in C} \frac{t_k}{c_i}\left(\frac{\partial c_i}{\partial q_k}\right)_{\overline{U}} = -\theta < 0 \quad \text{where} \quad i \in C\ . \tag{3.6}$$

This is the familiar formula for the optimal tax structure obtained earlier as equation (2.18). It implies that taxes (tariffs) ought to be such that consumption of all commodities is reduced by the same percentage along the compensated demand curve from what it would have been had international prices been charged.

To see what equation (3.3a) implies we note that from it we can derive the following relationship:[1]

[1]Substituting (3.2a) into (3.3a), and using the fact that

$$\frac{\partial V}{\partial q_\ell} = -c_\ell \frac{\partial V}{\partial M}$$

we obtain

$$-\frac{1}{c_\ell}\Sigma\frac{\partial c_k}{\partial q_\ell} = \Sigma\frac{\partial c_k}{\partial M} - \frac{1}{y_i^j}\Sigma\frac{\partial y_k^j}{\partial p_i^j}\ .$$

Using the Slutsky equation, we obtain

$$\sum_{k\epsilon D} \frac{\tau_k^j}{y_i^j} \frac{\partial y_i^j}{\partial p_k^j} = - \sum_{k\epsilon C} \frac{t_k}{c_\ell} \left(\frac{\partial c_\ell}{\partial q_k} \right)_{\overline{U}} = \theta > 0 \,, \quad j = 1, \ldots, m \,, \quad i \epsilon D \text{ and } \ell \epsilon C \,. \quad (3.8)$$

Equation (3.8) implies that the percentage change in the j^{th} firm's output of the i^{th} commodity or input of the i^{th} factor should be the same for all firms, commodities, and factors. Notice that we have included for the possibility of intermediate goods. Therefore, as far as producer taxation is concerned one observes that intermediate goods are treated in no way differently from final consumer goods.[1]

$$\frac{1}{c_\ell} \Sigma \left(\frac{\partial c_k}{\partial q_\ell} \right)_{\overline{U}} = \Sigma \frac{\partial y_k^j}{\partial p_i^j} \frac{1}{y_i^j} \qquad (3.7)$$

Finally, since

$$\sum_{k\epsilon D} p_k^j \frac{\partial y_k^j}{\partial p_i^j} = \sum_{k\epsilon D} q_k \left(\frac{\partial c_k}{\partial q_\ell} \right)_{\overline{U}} = 0 \quad \text{when} \quad \ell \epsilon C \quad \text{and} \quad i \epsilon D$$

and

$$\frac{\partial y_k^j}{\partial p_i^j} = \frac{\partial y_i^j}{\partial p_k^j} \,, \quad i, k \epsilon D$$

$$\frac{\partial c_k}{\partial q_\ell} = \frac{\partial c_\ell}{\partial q_k} \,, \quad \ell, k \epsilon C$$

and using (2.3), (3.7) reduces to (3.8)

[1]Our general formulation allows us to tax imports of intermediate goods at a different rate than domestic production of intermediate goods, and this in fact will in general be desirable, since there will be different elasticities associated with each.

To see more clearly what equation (3.8) implies for the structure of producer taxes consider the case where we treat all the private producers identically, and hence can aggregate them together. Define

$$\hat{\tau}_k \equiv \frac{\tau_k}{1 - \tau_k} = \frac{\tau_k}{p_k} \quad \text{and} \quad \eta^s_{ik} \equiv \frac{p_k}{y_i} \frac{\partial y_i}{\partial p_k} .$$

Then equation (3.8) can be written in the matrix form

$$\hat{\tau} \cdot [\eta^s] = \theta E \ ; \quad \text{or} \quad \hat{\tau} = \theta E \cdot [\eta^s]^{-1} \tag{3.9}$$

where $E = (1, \ldots, 1)$, a vector with unity everywhere.

Equation (3.9) provides a simple expression for calculating the optimal structure of producer taxes.

Consider, for instance, the case of a production process taking an intermediate good into a final good, by the process

$$y_f = h(y^f_I)$$

where y^f_I is the input of the intermediate good I in the production of the final good f . Then

$$\frac{p_I}{p_f} = h'(y^f_I)$$

$$\frac{d \ln y_f}{d \ln p_f} = - \frac{\partial \ln y_f}{\partial \ln p_I} = \left(-1 \Big/ \frac{h'' y^f_I}{h'} \right) \frac{y^f_I p_I}{y_f p_f} .$$

Hence

$$(\hat{\tau}_f - \hat{\tau}_I)\left(-1\Big/\frac{h''y_I}{h'}\right)\frac{p_I y_I^f}{p_f y_f} = \theta$$

or

$$\hat{\tau}_f - \hat{\tau}_I = \frac{\theta}{\eta_f^s\left(1 - \frac{\pi}{p_f y_f}\right)} . \tag{3.10}$$

The intermediate good should be taxed at a lower rate than the final good; the amount by which it should varies inversely with the elasticity of supply and positively with the ratio of profits to value of output. Notice that only in the limiting case of infinite supply elasticities (constant returns to scale) is it optimal to impose no tax on domestic production.

(3.10) gives relative tax rates; to obtain absolute tax rates, assume y_I is produced by the numeraire alone. Then

$$\hat{\tau}_I = \frac{\theta}{\frac{\partial \ln y_I}{\partial \ln p_I}}$$

there is a production tax on y_I .

Not too much emphasis should be placed on these results; a tax-tariff structure affects only relative prices, so whether there is a tax or subsidy on a particular commodity depends on what we choose as our numeraire, i.e. the price of a given commodity will rise relative to that of some other commodities, fall relative to still others. A natural choice of numeraire for a small country with a principal export crop is that export crop. Then by definition, $\hat{\tau}_f = 0$, $\tau_I = \theta/\eta_f^s(1 - \pi/p_f y_f) < 0$, i.e. a production sudsidy (relative to the tariff on the good) is imposed on the intermediate good.

It follows that Pigou's conjecture that the tax on the output of a given commodity by a domestic producer ought to be higher than the tax on the import of the same commodity, is not, in fact, a well-formulated question; whether the statement is true depends on the choice of numeraire. The question may be reformulated, if

$$\frac{q_i}{q_0} > 1$$

is

$$\frac{p_i}{p_0} < 1 \;?$$

Consider a change of numeraire to j : $q_j/q_0 > 1$, $p_j/p_0 < 1$. Then the sign of $(q_i/q_j)-1$ and $(p_i/p_j)-1$ are ambiguous. In particular, it is possible for $q_i/q_j > 1$ while $p_i/p_j > 1$.

In many developing countries the government authorizes construction of projects in the private sector as well as directly making investments in public projects. Much of benefit-cost analysis is directed to an evaluation of these "private" projects. One of the questions which usually arises in the evaluation of these projects is the planned tax treatment of the given private firm. Assume no restrictions are imposed on the government in its tax treatment (except the levying of profit taxes). Then, from (3.1), whether the k^{th} small project increases welfare depends on the sign of

$$\frac{\partial V}{\partial M}\pi^k + \lambda\{\Sigma\Delta y^k - \Sigma\Delta c^k\}$$

where Δc^k is the induced increase in consumption. The first term is the profits, evaluated at domestic prices, the second term is the effect of the project on the balance of payments. Thus profits in domestic prices must exceed the total foreign exchange cost of the project evaluated at the shadow price of foreign exchange.

This may be reformulated: since $\Sigma q_i \Delta c_i^k = \pi^k$,

$$\begin{aligned}\Sigma\Delta y^k - \Delta c^k &= \Sigma\Delta y^k - \pi^k + \Sigma t_i \Delta c_i^k \\ &= \Sigma\tau_i^k \Delta y_i^k + \Sigma t_i \Delta c_i^k \equiv R^k\end{aligned}$$

where R^k is the government revenue from the project. Thus, we require

$$\pi^k + \frac{\lambda}{\partial V/\partial M} R^k \geq 0 .$$

Profits at domestic prices, plus government revenues, evaluated at $\frac{\lambda}{\partial V/\partial M} > 1$, must be greater than zero. This makes clear that the criteria sometimes advocated

$$\pi^k + R^k \geq 0 ,$$

profits plus government revenue or, $\Sigma\Delta y^k > 0$, profits at international prices, be nonnegative are overly restrictive. Profits and government revenue should not be evaluated at the same shadow profits.

Still a third way of expressing our criterion is that

$$\Sigma\hat{p}_i \Delta y_i^k \geq \frac{\lambda}{\frac{\partial V}{\partial M} + \lambda} \Sigma\Delta c_i^k$$

where

$$\hat{p}_i = \frac{\frac{\partial V}{\partial M} p_i + \lambda}{\frac{\partial V}{\partial M} + \lambda} .$$

Profits, evaluated at a weighted average of domestic and international prices, must be greater than $\frac{1}{1 + \frac{\partial V}{\partial M}/\lambda}$ times the change in consumption, evaluated at international prices.

3.3. Non-Tradeables

When there are non-tradeables we must include in our calculations the change in consumption and production of these commodities. As equations (3.4) and (3.5) make clear, non-tradeables are to be evaluated in terms of their marginal foreign exchange costs. That is

$$p_i = \lambda \frac{\partial G/\partial x_i}{\partial G/\partial x_k} \quad k \in T \text{ and } k \in N . \tag{3.11}$$

The shadow price of a non-tradeable (in utility numeraire) is therefore, equal to the value (at international prices) of the foreign exchange which would be generated were one unit less of it produced domestically, and instead more units of the tradeables were produced, valued at the shadow price of foreign exchange (in utility numeraire).

The rest of the analysis proceeds easily. One obtains from equations (3.2) and (3.3) tax formulae which are similar to equations (3.6) and (3.8), except now, for the non-tradeables

$$t_i = q_i - \frac{\rho_i}{\lambda} \qquad i \in N \cap C \tag{3.12a}$$

$$\tau_i = \frac{\rho_i}{\lambda} - p_i \qquad i \in N \tag{3.12b}$$

which give the difference between the consumer price and the shadow price (equation (3.12a)) and the difference between the shadow price and the producer price (equation (3.12b)). If we now suppose that there is a single producer of a given non-traded good (or that the producers of the good can be aggregated) and also assume that the demand and supply curves are independent of other prices we find that the tax levied on a unit of a non-traded good is

$$q_i - p_i = t_i + \tau_i = \theta\left[\frac{1}{\eta^d_{ii}} + \frac{1}{\eta^s_{ii}}\right] \qquad i \in N \cap C\ . \tag{3.13}$$

The formula given in (3.13) is a generalization of the result in Ramsey [18].[1]

[1] It would perhaps have been more natural to choose as our numeraire labor, a non-traded "good." Then, assume we have two methods of obtaining a commodity, one by producing it with labor, the other by importing. This is the case that Pigou probably had in mind, and a simple recasting of our results yields the conclusion that a surtax is imposed on the domestic production of the commodity.

A simple diagrammatic analysis may make this result more convincing. Consider the special case where there are independent demand and supply curves (i.e. demand and supply depend only on the price of the commodity) and assume there is constant marginal utility of leisure so that we can ignore income effects. (See Figure 1.) The demand and supply curves initially intersect at an output of ${}_0y$ and a price of ${}_0p$. A tax of t is imposed. If the slope of the demand curve is $-d$, and that of the supply curve is s, then the reduction in output is

$$\Delta y = \frac{t}{d+s} .$$

The loss of consumer plus producer surplus is

$$S = 1/2\ t\Delta y = \frac{1/2\ t^2}{d+s}$$

and the government revenue is

$$R = t({}_0y - \Delta y) = t\,{}_0y - \frac{t^2}{d+s} .$$

Clearly, optimality requires the marginal loss of consumer plus producer surplus per unit gain in government revenue to be the same regardless of the commodity being taxed: That is

$$\frac{dS}{dR} = \frac{dS/dt}{dR/dt} = \frac{t/d+s}{{}_0y - 2t/d+s} = \theta$$

or

$$\frac{t(1+2\theta)}{d+s} = {}_0y\theta$$

or

$$t = \frac{\theta}{1+2\theta}(d+s)\,{}_0y$$

or

$$\frac{t}{{}_0p} = \frac{\theta}{1+2\theta}\left(\frac{1}{\eta^s} + \frac{1}{\eta^d}\right) .$$

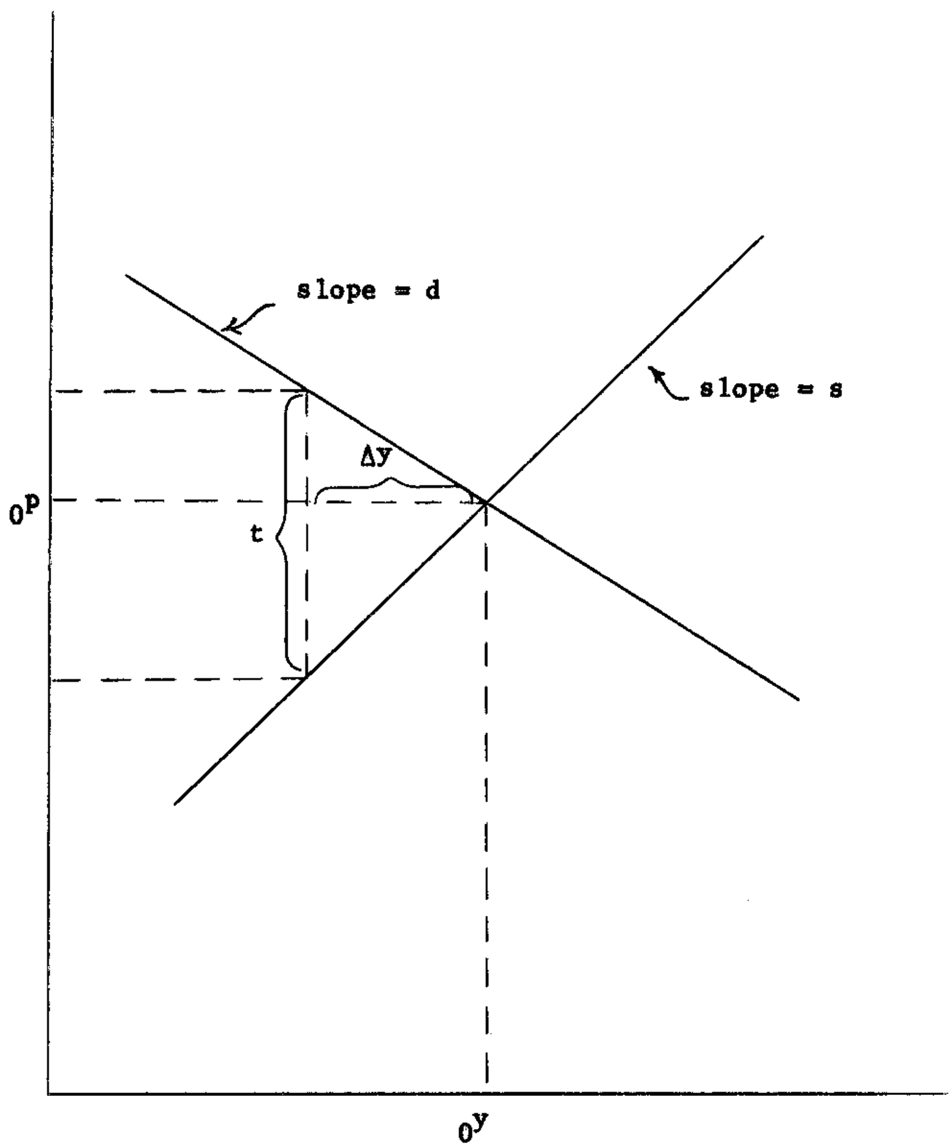

FIGURE 1

3.4. Foreign Firms

In the foregoing analysis we assumed that profits accrued to the nationals of the country in which production occurred. In most developing countries a signficant part of the profits is repatriated to some foreign country. Consider then the case where all profits of a given firm are repatriated. These profits do not affect the welfare of the representative consumer, and indeed contribute to the deterioration of the balance of payments situation. In this situation, the balance of payments condition reads as:

$$\sum_{k\epsilon T} z_k + \sum_{j\epsilon F} \pi^j = 0$$

where F is the set of foreign firms. Suppose again, for ease of exposition, that all goods are tradeable. For $j \epsilon F$ we note that the first order condition (3.3) can be written as

$$-\lambda \frac{\partial \pi^j}{\partial p_i^j} + \sum_{k\epsilon D} \left(\phi_j \frac{\partial F^j}{\partial y_k^j} + \rho_k \right) \frac{\partial y_k^j}{\partial p_i^j} = 0 \,, \quad j \epsilon F \text{ and } \begin{matrix} i \epsilon D \\ i \neq 0 \end{matrix} \;. \tag{3.14}$$

On using equations (3.4), (3.5), and (2.9a), in equation (3.14) one obtains

$$-\lambda y_i^j + \phi_j \frac{\partial F^j}{\partial y_0^j} \sum_{k\epsilon D} p_k^j \frac{\partial y_k^j}{\partial p_i^j} + \lambda \sum_{k\epsilon D} \frac{\partial y_k^j}{\partial p_i^j} = 0$$

or using (2.9d)

$$\sum_{k\epsilon D} \frac{\partial y_k^j}{\partial p_i^j} - y_i^j = 0 \qquad i \epsilon D \text{ and } j \epsilon F \,.$$

An increase in the producer price has two effects: it increases domestic production, thus reducing the need for imports and improving the balance of payments, and it increases profits, which worsens the balance of payments (when repatriated). Optimality requires at the margin that first offset each other.

Again using (2.9d) and (2.3), we obtain

$$\sum_{k\epsilon D} \frac{\tau_k^j}{y_i^j} \frac{\partial y_i^j}{\partial p_k^j} = 1 \qquad i \epsilon D \quad \text{and} \quad j \epsilon F . \tag{3.15}$$

Writing $\hat{\tau}_k^j = \dfrac{\tau_k^j}{p_k^j}$ and $\eta_{ik}^{sj} = \dfrac{\partial y_i^j}{\partial p_k^j} \dfrac{p_k^j}{y_i^j}$ one obtains from equation (3.15) the formula

$$\hat{\tau}^j = E \cdot [\eta^{sj}]^{-1} . \tag{3.16}$$

One should observe that as before, industries which have more inelastic supply curves should be taxed at a higher rate. More interesting, and more important is the fact that equation (3.16) implies that the tax rates are independent of the government's need for revenue. (This is in contrast with the case of domestically owned firms, where the tax rates depend on θ .) That is to say, the tax rates on the commodities produced by foreign owned firms are determined completely by balance of payments considerations. This results in the tax rates depending only on the properties of the production function of the industry.

The cost benefit criterion for accepting projects of foreign owned corporations must similarly be modified. The criterion should be simply whether

$$\lambda(-\pi^k + \Sigma\Delta y^k) = \lambda(\Sigma(1 - p_i)\Delta y_i^k) = \lambda(\Sigma\tau_i \Delta y_i^k) > 0$$

i.e. whether it contributes net tax revenue. (This assumes that the project will not affect prices facing other firms.)

4. Inability of Taxing Domestic Production

4.1. No Profit Taxation

We now modify the model somewhat to assume that the government cannot impose producer taxes. We suppose that the only taxes that are feasible are trade taxes. Domestic producers then face the same price vector as does the consumer. To keep the notation simple we suppose initially that all goods are tradeable, consumable and domestically produced. That is, $A = D = T = C$. We can again formulate the problem as if the controls at the disposal of the government are the domestic price vector q, the vector of public production, x, and the vector of trade, z.[1] (Note that we must have $q = p = 1+t$.) Using the Lagrangean expression (3.1) we now obtain the first order conditions as

$$\frac{dV}{dq_i} + \sum_{j=1}^{m} \sum_{k \in A} \left(\frac{\partial F^j}{\partial y_k^j} \psi_j + \rho_k \right) \frac{\partial y_k^j}{\partial q_i} - \sum_{k \in C} \rho_k \frac{dc_k}{dq_i} = 0 \qquad i \in A \text{ and } i \neq 0 \quad (4.1)$$

[1] The actual controls are tariffs, t, and government production x. See above, p. 18.

$$\mu \frac{\partial G}{\partial x_i} + \beta_i = 0 \qquad i \in A \tag{4.2}$$

$$-\lambda + \rho_i = 0 \qquad i \in A \tag{4.3}$$

where dV/dq_i and dc_k/dq_i are to be regarded as total derivatives.[1] Now, equations (4.2) and (4.3) together imply once again that international prices ought to be used in benefit cost analysis of public sector projects.[2]

Using this fact we can re-write equation (4.1) as

$$\frac{dV}{dq_i} + \lambda \sum_{k \in A} \frac{\partial y_k}{\partial q_i} - \lambda \sum_{k \in A} \frac{dc_k}{dq_i} = 0 \qquad i \in A \text{ and } i \neq 0 \tag{4.1a}$$

where $y_k = \sum_{j=1}^{m} y_k^j$.

The interpretation of equation (4.1a) is as before: the marginal cost in welfare per unit gain in the balance of payments should be the same regardless of which price (tariff) is varied. Now, since the government can be viewed as controlling both trade and public production, and since the shadow prices in the public sector are equal to the international prices we can treat trade flow and public production indifferently. Writing $e_k = c_k - y_k = x_k + z_k$ as the "excess demand" for commodity k we obtain from equation (4.1a) the fact that

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[1]That is,

$$\frac{dV}{dq_i} = \frac{\partial V}{\partial q_i} + \frac{\partial \pi}{\partial q_i} \frac{\partial V}{\partial M}$$

$$\frac{dc_k}{dq_i} = \frac{\partial c_k}{\partial q_i} + \frac{\partial \pi}{\partial q_i} \frac{\partial c_k}{\partial M}$$

[2]The evaluation of projects in the private sector follows as in the previous section.

$$\sum_{k\epsilon A} \frac{t_k}{e_i}\left(\frac{\partial e_i}{\partial q_k}\right)_{\overline{U}} = -\theta \text{ ,}^{1} \tag{4.4}$$

where

$$\theta = 1 - \frac{\partial V/\partial M}{\lambda} - \sum_{k\epsilon A} t_k \frac{\partial c_k}{\partial M} .$$

The percentage reduction in excess demand should be the same for all commodities.

Now clearly

---

[1] Equation (4.1a) can be re-written as:

$$\frac{\partial V}{\partial q_i} + y_i \frac{\partial V}{\partial M} + \lambda \sum_{k\epsilon A} \frac{\partial y_k}{\partial q_i} - \lambda \sum_{k} \frac{\partial c_k}{\partial q_i} - \lambda y_i \sum_{k} \frac{\partial c_k}{\partial M} = 0 \qquad i \epsilon A$$

or $$-(x_i + z_i) \frac{\partial V}{\partial M} + \lambda \sum_{k\epsilon A} (q_k - t_k) \frac{\partial y_k}{\partial q_i} - \lambda \sum_{k\epsilon A} (q_k - t_k) \frac{\partial c_k}{\partial q_i} - \lambda y_i \Sigma \frac{\partial c_k}{\partial M} = 0$$

or $$-\lambda \sum_{k\epsilon A} t_k\left(\frac{\partial y_k}{\partial q_i} - \left(\frac{\partial c_k}{\partial q_i}\right)_{\overline{U}}\right) = (x_i + z_i) \frac{\partial V}{\partial M} - \lambda c_i + \lambda \sum_{k\epsilon A} t_k c_i \frac{\partial c_k}{\partial M} + \lambda y_i \sum_{k\epsilon A} (q_k - t_k) \frac{\partial c_k}{\partial M}$$

$$= (x_i + z_i)\left(\frac{\partial V}{\partial M} + \lambda \sum_{k\epsilon A} t_k \frac{\partial c_k}{\partial M} - \lambda\right)$$

or $$\sum_{k\epsilon A} \frac{t_k}{e_i}\left(\frac{\partial e_i}{\partial q_k}\right)_{\overline{U}} = \sum_{k\epsilon A} t_k \frac{\partial c_k}{\partial M} - 1 + \frac{1}{\lambda} \frac{\partial V}{\partial M} \equiv -\theta .$$

$$-\left(\frac{\partial \ln e_i}{\partial \ln q_k}\right)_{\overline{U}} = \eta^d_{ik} + \frac{y_i}{e_i}\left(\eta^s_{ik} + \eta^d_{ik}\right) .$$

Therefore, equation (4.4) can be re-written (on writing $\hat{t}_i = t_i/q_i = t_i/1 + t_i$) as (where I is the identity matrix)

$$\hat{t}\cdot\left[[\eta^d] + \left[\frac{y}{e}\right][I][\eta^s + \eta^d]\right] = \theta E. \tag{4.5}$$

In equation (4.5) [y/e] denotes a vector an element of which is (y_k/e_k) . We should contrast equation (4.5) with the results of Section 3.2, where we found

$$\left.\begin{aligned}\hat{t}\cdot\eta^d &= \theta\\ \hat{\tau}\cdot\eta^s &= \theta\end{aligned}\right\} \tag{3.8}$$

We can re-express equation (4.5) to obtain the optimal tariff structure as:

$$\hat{t} = \theta E\cdot[\eta^e]^{-1} = \theta E\cdot\left[[\eta^d] + [y/e]\cdot[\eta^s + \eta^d]\right]^{-1} \tag{4.6}$$

where

$$\eta^e_{ik} = -\left(\frac{\partial \ln e_i}{\partial \ln q_k}\right)_{\overline{U}} \quad \text{and} \quad E \equiv (1,\ 1,\ 1,\ \ldots,\ 1) .$$

In the case of independent demand and supply curves, equation (4.6) reduces readily to

$$\hat{t}_i = \frac{1}{\eta^e_{ii}} = \frac{\theta}{\eta^d_{ii} + \frac{y_i}{e_i}\left(\eta^d_{ii} + \eta^s_{ii}\right)} \qquad i \in A . \tag{4.7}$$

The formula given in (4.7) expresses the fact that at the optimum the tax rate (as a percentage of the consumer price) is inversely proportional to the elasticity of excess demand. Notice also that equation (4.7) implies an export tax for exports whose net supply curve is positively sloped, and a subsidy for exports whose net supply curve is backward bending. It is clear then that duties need not be uniform in this second best revenue tariff.[1]

We turn now to some other cases. For instance, if $y_i = 0$, which implies that the commodity does not enter into domestic production, then the tariff is identical to that described in Section 3.2.

For intermediate goods $y_i = -e_i$ so the tariffs/taxes are chosen to make the producers prices the same as discussed in Section 3.2. Note that this implies a tariff structure with different rates on different commodities, but such that the production of all intermediate goods is increased the same percentage. If it is an imported intermediate good, imports are reduced by the same percentage.

But not even this simplicity of structure obtains if firms are foreign owned. For foreign owned firms equation (4.4) must be modified, to read[2]

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[1]See Ramaswami and Srinivasan [17].

[2]Equation (4.1a) becomes

$$\frac{\partial V}{\partial q_i} + \lambda\Sigma\left(\frac{\partial y_k}{\partial q_i} - \frac{\partial c_k}{\partial q_i}\right) - \lambda y_i = 0$$

or

$$-c_i \frac{\partial V}{\partial M} - \lambda\Sigma(q_k - t_k)\left(\frac{\partial e_k}{\partial q_i}\right) - \lambda y_i = 0$$

$$\lambda\Sigma t_k \left.\frac{\partial e_k}{\partial q_i}\right|_{\bar{U}} = \lambda\left[c_i\sum_k\left(\frac{\partial c_k}{\partial M}\right)t_k - c_i + y_i\right] + c_i\frac{\partial V}{\partial M}$$

$$= \lambda\left[-e_i + e_i\Sigma\frac{\partial c_k}{\partial M}t_k + y_i\Sigma t_k\frac{\partial c_k}{\partial M}\right] + (e_i + y_i)\frac{\partial V}{\partial M}\,.$$

$$\sum_{k\epsilon A} \frac{t_k}{e_i}\left(\frac{\partial e_i}{\partial q_k}\right)_{\overline{U}} = -\theta + \frac{y_i}{e_i}\left(\frac{\partial V}{\partial M}\cdot\frac{1}{\lambda} + \Sigma\, t_k \frac{\partial c_k}{\partial M}\right) . \qquad (4.8)$$

It follows that the relative reduction in excess demand is greater than for domestically owned firms (as we would expect) and is greater, the smaller excess demand is to total private production, if $y/e_i < 0$. Just the opposite holds if $y/e_i > 0$.

4.2. Possibility of Taxing Profits at 100%

There is one further case that deserves brief mention. We now suppose that the government can (and therefore, does) set the tax on pure profits at 100%. Then, (4.1) becomes[1]

$$\sum_{k} \frac{t_k}{e_i}\left(\frac{\partial e_i}{\partial q_k}\right)_{\overline{U}} = -\theta \frac{c_i}{e_i} . \qquad (4.9)$$

This implies that the relative reduction in excess demand is simply inversely proportional to the ratio of "excess demand" to consumption. In particular, in the case of intermediates used to produce a non-consumed export good, note that no tariffs should be imposed on imports. (Cf [12].) Thus the conclusion of Ramaswami and

[1] The introduction of non-tradeables does not change these results. Letting

$$\hat{p} \equiv \frac{\rho_k}{\lambda} - p_k \qquad \epsilon\ D \cap N$$

be the difference between the shadow price and the private producer price of a non-tradeable, (4.9) can be rewritten

$$\sum_{k\epsilon T} t_k\left(\frac{\partial e_i}{\partial q_k}\right)_{\overline{U}} + \sum_{k\epsilon N} \hat{p}_k\left(\frac{\partial e_i}{\partial q_k}\right) = -\theta c_i .$$

Srinivasan [17], that "inputs used in export production must be free of duty" is only partially correct: if the tax authorities can only impose import duties and 100% profits taxes, then only if the good is also not consumed should it be exempt from duties. If it can impose 100% profits taxes and consumption-production taxes as well as tariffs, no intermediate, whether used for producing exports or import substitutes should be taxed. If profits are not taxed at 100%, and if the same tax rate must be imposed on domestic production as on imported intermediates, then even though the good is not consumed, a tariff-production tax is imposed.

5. Benefit-Cost Analysis with Given Taxes

5.1. All Taxes Fixed

We now consider the case where the government project evaluator needs to take the tariff and tax structure as *given* and *fixed*. The question is: what shadow prices ought he to use in project evaluation? Notice that the number of controls in the planning exercise is now drastically reduced. The Lagrangean of the planning problem is still expression (3.1), but now the set of controls to be chosen are the vector of public production, x, and the volumes of trade, z.

If all goods were traded, fixing all taxes would fix all consumer and producer prices. Since our focus in this section is explicitly on public production, it is convenient to assume there is a single public good; resource savings from greater efficiency in the public sector can be used to increase the expenditure on the public good. Letting g denote the public good, our Lagrangean can now be formulated:[1]

$$\mathcal{L} = V(q,g) - \lambda \Sigma z + \Sigma \;_k(x + z + y - c) + \mu G(x,g) \;.$$

The first order conditions now read as

$$V_g + \mu G_g - \Sigma \rho_k \frac{\partial c_k}{\partial g} \qquad (5.1)$$

$$-\lambda + \rho_k = 0 \qquad k \in D \qquad (5.2)$$

$$\rho_k + \mu \frac{\partial G}{\partial x_k} = 0 \qquad k \in T \;. \qquad (5.3)$$

From equations (5.2) and (5.3) we note that for traded goods the marginal rates of transformation in the <u>public</u> sector must equal the international price ratios, unity. That is, the project evaluator ought to use international prices in project evaluation. Notice that we have not assumed that the fixed tax rates are in any sense optimal. Turning to non-tradeables we note a result similar to that obtained in Section 3.3: The shadow price of a non-traded good is its marginal foreign exchange cost.

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[1]We assume for notational simplicity in this formulation that the public good is not traded, and all other goods are. If there are non traded goods, fixing the production of consumption taxes does not fix the consumer price; that depends on x , the quantity of the good produced by the government. Letting

$$y_i = y_i(q_i - \Gamma_i) \qquad i \in C \cap N$$

we obtain the following additional condition

$$V_{q_i} + \Sigma \rho_k \left(\frac{\partial y_k}{\partial p_i} - \frac{\partial c_k}{\partial q_i} \right) = 0 \;.$$

(It is obvious that this set of results continue to hold if profits can be taxed up to a 100%.)

Turning to the "evaluation" of a private domestically owned project, the analysis for arbitrary but fixed tariffs and taxes is identical to that of Section 3 for optimally chosen tariffs and taxes. There, it will be recalled, we established that a project is worth undertaking (i.e. worth giving a license to) if one of the following equivalent conditions were satisfied:

(a) The profits--evaluated at domestic market prices--are greater than the net foreign exchange cost times the shadow price of foreign exchange.

(b) Profits evaluated at domestic prices plus the induced government revenues (from taxes and tariffs), evaluated at the shadow price of foreign exchange, must be positive.

(c) Profits evaluated as a weighted average of domestic and international prices must be greater than $\frac{1}{1 + \partial V/\partial M/\lambda}$ times the induced change in consumption evaluated at international prices.

If only tariffs are imposed, two further ways of expressing the criterion may be formulated.

(d) A private project is worth giving a license to it

$$\sum_i \Delta y_i^j + \sum_i t_i \Delta y_i^j \geq \frac{\lambda}{\partial V/\partial M} \left(\sum_i \Delta c_i - \sum_i \Delta y_i^j\right) \tag{5.3}$$

where $\sum_i \Delta y_i^j$ are profits evaluated at international prices; $\sum_i t_i \Delta y_i^j$ are "direct" government revenues. Equation (5.3) implies that the government ought to give a license to a private project if profits, evaluated at international prices, plus the government's (direct) tax revenue is greater than the foreign exchange cost (evaluated at the shadow price of foreign exchange).[1]

(e) Alternatively, we can write

$$\frac{1}{\lambda/\frac{\partial V}{\partial M}}\sum_i t_i \Delta y_i^j + \frac{1+\lambda/\frac{\partial V}{\partial M}}{\lambda/\frac{\partial V}{\partial M}}\sum_i \Delta y_i^j \geq \Sigma \Delta c_i \ . \qquad (5.4)$$

That is, the "weighted sum" of government "direct" tax revenue and profits must be greater than the value of the change in consumption (all evaluated at international prices).

As we noted earlier, for a foreign firm, where all profits are repatriated, and hence for which there are no direct consumption and utility effects the criterion is remarkably simple:

$$\sum_i t_i \Delta y_i^j \geq 0 \qquad j \in F \ .$$

The project must yield a net revenue to the government.

5.2. Some Taxes Controllable

How does the fact that some tariff rates are unalterable affect the choice of the remaining tariff rates? Assume for one set of commodities, say R , q_i can be chosen by the government but that for the remaining goods they are given as fixed. Assume for notational ease that all goods

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[1] It is important to observe, that throughout, the technique the firm is following is chosen on the basis of domestic prices. We are, therefore, not evaluating projects that would have been undertaken had international prices had been charged. Rather, we are evaluating at international prices projects that are actually undertaken.

are tradeable; i.e. $A = T$. It is then trivial to confirm, on repeating the analysis of Section 3.1 that

$$\sum_{k\epsilon R} t_k \left(\frac{\partial c_i}{\partial q_k}\right)_{\bar{U}} + \sum_{k\notin R} t_k \left(\frac{\partial c_i}{\partial q_k}\right)_{\bar{U}} = -c_i\theta \qquad i \in R \,. \tag{5.5}$$

It follows that the consumption of all freely taxable goods should be reduced by the same percentage. This implies that if demand elasticities are assumed independent the tax rates on those commodities for which we can choose are unaffected. More generally, if elasticities of demand are not independent we have

$$\hat{t}_R \cdot \eta_R^d + \hat{t}_{R'} \cdot \eta_{R'}^d = -\theta E$$

or

$$\hat{t}_R = -\theta\ E - t_{R'} \cdot \eta_{R'}^d] \cdot [\eta_R^d]^{-1}$$

where $R' \equiv A-R$ and where

$$\hat{t}_R \equiv \left(\frac{t_i}{p_i}\right) \qquad i \in R$$

$$\hat{t}_{R'} \equiv \left(\frac{t_i}{p_i}\right) \qquad i \in R'$$

and

$$\eta_R^d = \{\eta_{ik}^d\} \qquad i,\ k \in R$$

$$\eta_{R'}^d = \{\eta_{ik}^d\} \qquad i \notin R \text{ and } k \in R \,.$$

5.3. Quotas

It is not surprising that the presence of quotas alters both the tariff structure of the remaining commodities and the shadow prices to be used in the public sector for those commodities on which quotas have been imposed. To the basic Lagrangean (3.1) we now add an extra term to obtain

$$\mathcal{L} = V(q, \sum_j \pi^j(p^j)) - \lambda \sum_{k \in T} z_k + \mu G(x) + \sum_{j=1}^{m} \psi_j F^j(y^j) + \sum_{k \in A} \rho_k (\sum_j y_k^j + x_k + z_k - c_k)$$
$$+ \gamma(\bar{z}_s - z_s)$$

where $\bar{z}_s$ is the quota imposed on commodity s. It is immediate that the first-order conditions (3.4) and (3.5) continue to hold here for all $i \in D$ and $i \neq s$. If s enters into domestic production then the first order conditions pertaining to its shadow price in the public sector read

$$\mu \frac{\partial G}{\partial x_s} + \rho_s = 0 \tag{5.6}$$

$$-\lambda + \rho_s - \gamma = 0 \quad \text{where} \quad \gamma \geq 0 \quad \text{and} \quad \gamma(\bar{z}_s - z_s) = 0 \,, \tag{5.7}$$

from which we obtain the fact that

$$\frac{\partial G/\partial x_s}{\partial G/\partial x_c} = 1 + \frac{\gamma}{\lambda} \geq 1 \,.$$

Generally speaking then, the shadow price of s in the public sector is equal to unity only when the quota does not bite. Otherwise it is greater than unity. Turning to the structure of optimum taxes we note simply that both the tariffs and the production taxes are affected. For instance, if we assume that tariffs may be chosen on all commodities and that s is consumable, then the equation (corresponding to equation (3.6)) yielding the structure of taxes on consumer goods reads as

$$\sum_{k \varepsilon C} \frac{t_k}{c_i}\left(\frac{\partial c_i}{\partial q_k}\right)_{\overline{U}} - \frac{\gamma_s}{\lambda c_i}\left(\frac{\partial c_i}{\partial q_s}\right)_{\overline{U}} = \theta \qquad i \varepsilon C . \tag{5.8}$$

In the case of independent demand curves we notice from equation (5.8) that the tariff of only commodity s is altered, and it is set so as to capture the profits which would otherwise accrue to the importers of s. Complements of s have their consumption reduced more proportionately than is the consumption of substitutes. That is to say the tax structure ought to be designed so as to encourage the consumption of substitutes for the commodity on which a quota has been imposed. This is clearly what intuition suggests.

6. Trade Policy Dependent on Project Selection

Quite often a decision maker is cognizant of the fact that the presence of a particular project in the country is likely to give rise to changes in the tariff policy. In particular there is a long experience of newly established industries requesting and obtaining protection, even when on "purely economic" grounds this protection is questionable. To attempt to capture this kind of situation we postulate for simplicity of notation that for some commodity ℓ the tariff t_ℓ is an increasing function of the level of imports of ℓ. That is $t_\ell = t_\ell(z_\ell)$, $t'_\ell > 0$. Assume also for simplicity of notation that the model is otherwise the same as that in Section 3. The drastic assumption that we make here, as the reader will immediately recognize, is the assuming away of any game theoretic problems that this kind of tariff "response" obviously has built within it. We are thus assuming that the tariff on ℓ responds passively to the level of imports of ℓ. It is then simple to see that the equations corresponding to equations (3.4) and (3.5) read here as

$$\mu \frac{\partial G}{\partial x_i} + \rho_i = 0 \qquad i \in D \tag{6.1}$$

$$-\lambda + \rho_i = 0 \qquad i \neq \ell \quad \text{and} \quad i \in T \tag{6.2}$$

$$\frac{\partial V}{\partial q_\ell}\frac{\partial t_\ell}{\partial z_\ell} - \lambda + \rho_\ell - \sum_{k \in C} \rho_k \frac{\partial c_k}{\partial q_\ell}\frac{\partial t_\ell}{\partial z_\ell} = 0 \, . \tag{6.3}$$

It follows that for all tradeable goods except ℓ, shadow prices are their international prices. So far as commodity ℓ is concerned, we can re-express equation (6.3) as

$$\rho_\ell = \lambda + \frac{\partial t_\ell}{\partial z_\ell}\left(c_\ell \frac{\partial V}{\partial M} + \sum_{k \in C} \rho_k \frac{\partial c_k}{\partial q_\ell}\right) . \tag{6.4}$$

From equation (6.4) we can readily verify that the shadow price of commodity ℓ is[1]

[1] $$\frac{\frac{\partial V}{\partial M}}{\lambda} + \Sigma \frac{\rho_k}{\lambda}\frac{\partial c_k}{\partial q_\ell}\frac{1}{c_\ell} = \frac{\frac{\partial V}{\partial M}}{\lambda} + \frac{1}{c_\ell}\Sigma\left(\frac{\rho_k}{\lambda} - q_k\right)\frac{\partial c_k}{\partial q_\ell} - 1$$
$$= \frac{\partial V/\partial M}{\lambda} - \frac{1}{c_\ell}\Sigma t_k \frac{\partial c_k}{\partial q_\ell} - 1 + \left(\frac{\rho_\ell}{\lambda} - 1\right)\frac{\partial c_\ell}{\partial q_\ell}\frac{1}{c_\ell}$$
$$= \frac{\partial V/\partial M}{\lambda} + \Sigma t_k \frac{\partial c_k}{\partial M} - 1 - \frac{1}{c_\ell}\Sigma\left(\frac{\partial c_\ell}{\partial q_k}\right)_{\overline{U}} t_k + \left(\frac{\rho_\ell}{\lambda} - 1\right)\frac{\partial c_\ell}{\partial q_\ell}\frac{1}{c_\ell} .$$

$$1 - c_\ell \frac{\partial t_\ell}{\partial z_\ell}\left(\theta + \sum_{i \varepsilon C} \frac{t_i}{c_\ell}\left(\frac{\partial c_\ell}{\partial q_i}\right)_{\bar{U}}\right) \Big/ 1 - \frac{\partial c_\ell}{\partial q_\ell}\frac{1}{c_\ell} \, . \qquad (6.5)$$

It follows from expression (6.5) that the shadow price of commodity ℓ is below or above the international price of ℓ depending on whether the given tax structure reduces its consumption by less or more than is optimal. In particular, if demand curves are independent it depends simply on whether

$$\frac{t_\ell}{1 + t_\ell} \gtrless \frac{t_\ell^*}{1 + t_\ell^*} \equiv \frac{\theta}{\eta_{ii}^d} \, .$$

More generally, it is simple to demonstrate (see Dasgupta and Stiglitz [7]) that for tradeable commodities, if the tariff on some commodity k depends on the level of imports of some other commodity k' as well as on, say, the government's net production of some commodity, k'', then the shadow prices of <u>only</u> k' and k'' differ from their international prices. It follows from this that the fact that the tariff on commodity k responds unoptimally to the import or the public production of some other commodity k' is no argument for setting the shadow price k (or for that matter any commodity other than k') different from its international price. We have found it surprising that this should be so in such an interdependent system as we have been considering.

If demand curves are interdependent, then the tariffs on other goods will, however, be affected by the constraint on the tariff on commodity ℓ. Just as in the case of quotas, substitutes for commodity ℓ will have their production reduced by less than complements.

7. Foreign Exchange Constraint

Although the most natural interpretation of the models presented thus far is in terms of the conventional models of static international trade, there is no reason why the different commodities could not be treated as well as dated commodities. As we noted earlier in Section 1.2, a foreign exchange constraint may be interpreted as a limitation on the amount a country can borrow in any period. To make the interpretation clearer, we let c_{it}, y_{it}, x_{it}, z_{it} be the consumption, net output in the private sector, net output in the public sector, and net imports of commodity i at time t. We assume that all relative international prices (within each period) are constant, so that we normalize all at unity. Let r_t be the international rate of interest at t.[1] Then the trade balance condition can be written as

$$\sum_t \frac{\sum_i z_{it}}{1 + r_t} = 0 . \tag{7.1}$$

Borrowing constraint at some year t' reads as

$$\sum_i z_{it'} \leq \varepsilon_{t'} . \tag{7.2}$$

The basic Lagrangean (3.1) now has an extra term and it reads as

$$V(q, \sum_j \pi^j(p^j)) = \lambda \sum_t \frac{\sum_k z_{kt}}{1+r_t} + \mu G(x) + \sum_{j=1}^m \psi_j F^j(y^j) + \sum_t \sum_k \rho_{kt}\Big(\sum_{j=1}^m y^j_{kt} + x_{kt} + z_{kt} - c_{kt}\Big)$$
$$+ \sum_{t'} \gamma_{t'}(\varepsilon_{t'} - \sum_k z_{kt'}) \tag{7.3}$$

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[1]That is, $1/1+r_t$ is the price today of a promise to deliver 1 dollar of foreign exchange at time t.

where $\gamma_{t'} \geq 0$ and $\gamma_{t'}(e_{t'} - \sum_k z_{kt'}) = 0$. The first order conditions, corresponding to equations (3.4) and (3.5), now read as:

$$\mu \frac{\partial G}{\partial x_{it}} + \rho_{it} = 0 \tag{7.4}$$

$$\rho_{it} - \frac{\lambda}{1 + r_t} = 0 \qquad t \neq t', \quad i \in T \tag{7.5}$$

$$\rho_{it'} - \gamma_{t'} - \frac{\lambda}{1 + r_{t'}} = 0 \qquad i \in T . \tag{7.6}$$

Equations (7.4)-(7.6) yield the important implication that within any period international prices ought to be used in the evaluation of public sector projects. But if the constraint on foreign exchange is binding at t', the rate of interest (the rate of discount) at t' should not be $r_{t'}$; rather it should be somewhat higher than that.[1] On the other hand, for non-traded goods the shadow price is simply the value at international prices of the foreign exchange that could have been generated if the production of the non-traded good had been decreased by a unit and the production of traded goods increased.

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[1]

$$\frac{\frac{\partial G}{\partial x_{it}}}{\frac{\partial G}{\partial x_{jt}}} = \frac{\rho_{it}}{\rho_{jt}} = 1$$

$$\frac{\frac{\partial G}{\partial x_{it}}}{\frac{\partial G}{\partial x_{it'}}} = \frac{\rho_{it}}{\rho_{it'}} = \frac{\frac{\lambda}{1 + r_t}}{\frac{\lambda}{1 + r_{t'}} + \gamma_{t'}} .$$

If profits are taxed at 100% then it is easy to confirm that the private sector ought to face the same set of prices as the public sector. That is to say, at the optimum the economy is productively efficient. With less than 100% profit tax this is no longer true, and indeed the rate of discount that should be used for different non-traded goods may well be different.

Turning to the structure of taxes, we note once again that consumption of commodities in periods which are substitutes for consumption in periods in which the constraint is binding is reduced by less than the consumption in periods that are complements. That is to say, if the contraint is binding for initial years the tariff structure should be designed so as to encourage savings in the initial periods (e.g. by having higher taxes on complements of leisure in order to encourage work in initial periods) and a gradual lowering of tariffs over time (the substitution effect encouraging the postponement of consumption). We do not reproduce the formal argument here, since it is pretty straightfoward.

8. Budget Constraint

In this section we consider a constraint on the government budget deficit in any given year. For a closed economy this constraint was looked at originally by Boiteux [5], and his analysis was extended in Stiglitz and Dasgupta [21].

The constraint may be written as[1]

[1] In writing (8.1), we have implicitly assumed that $D = C$; for intermediate goods, we have to use production prices. This does not change the results at all.

$$\sum_{i \in D} q_{it} x_{it} \geq b_t \ . \tag{8.1}$$

If δ_t is the dual associated with the constraint (8.1) then we have, on writing $\hat{\lambda}_t \equiv \lambda / \prod_{n=1}^{t} (1 + r_n)$, the simple result

$$\frac{\partial G/\partial x_{it}}{\partial G/\partial x_{kt}} = \frac{\hat{\lambda}_t + \delta_t q_{it}}{\hat{\lambda}_t + \delta_t q_{kt}} \qquad i, k \in D \cap T \ . \tag{8.2}$$

Without loss of generality, let $q_{it} > q_{kt}$. Then

$$\frac{q_{it}}{q_{kt}} > \frac{\partial G/\partial x_{it}}{\partial G/\partial x_{kt}} > 1 \ . \tag{8.3}$$

The shadow prices to be used in the public sector lie between the international prices and the domestic prices, regardless of whether the domestic prices are determined by optimal or non-optimal tariffs and taxes. Similarly,

$$\frac{\partial G/\partial x_{it}}{\partial G/\partial x_{it+1}} = \frac{\hat{\lambda}_t + \delta_t q_{it}}{(\hat{\lambda}_t/1 + r_{t+1}) + \delta_{t+1} q_{it+1}} \ . \tag{8.4}$$

If we were to assume $q_{it} = q_{it+1}$ then

$$\frac{\partial G/\partial x_{it}}{\partial G/\partial x_{it+1}} \gtrless (1 + r_{t+1}) \quad \text{as} \quad \frac{\delta_{t+1}}{\delta_t} \lessgtr \frac{1}{1 + r_{t+1}} \ . \tag{8.5}$$

Condition (8.5) gives a qualitative rule for the choice of the rate of discount in the public sector.[1]

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[1]It is, perhaps, of some interest to note that if profits were taxed at 100% and the only constraint facing the government were the budget constraint (8.1), the optimum tax structure would be such that the prices of tradeable goods faced by the <u>private</u> sector would be unity. That is to say, at the optimum $\tau_i = 0$ for all i. (See [7].)

9. Distributional Objectives

Thus far we have been assuming the existence of the representative individual whose welfare the government is maximizing. It is often argued, with extremely good reason, that considerations of income distribution ought to weigh heavily both in the determination of the structure of taxes and tariffs and in the selection of investment projects in the public sector. Assume that the government is interested in maximizing the individualistic social welfare function of the form

$$W = W(U_1, \ldots, U_r)$$

where there are r non-identical individuals in the economy. It is then easy to show that corresponding to W there is an indirect social welfare function

$$V = V(q, M_1, \ldots, M_r) \qquad (9.1)$$

where, if β_j^k is the share of the j^{th} firm owned by individual k ,

$$M_k = \sum_j \beta_j^k \pi^j \ .$$

Then, in the analysis of Sections 1 to 8 we simply replace $V(q, (1-\tau_\pi)\pi \cdot E)$ by the function defined in expression (9.1). It is then easy to show (see [7]) that in this instance, even though the structure of optimal taxes and tariffs is effected by distributional considerations, the basic qualitative propositions concerning public investment criteria (i.e. the nature of the shadow prices to be used) remain unchanged. That is to say the government will still use international prices to evaluate public projects in those cases where when we ignored distributional objectives it was optimal to do so.[1]

10. Conclusions

We now summarize the basic results obtained in this paper in the form of a few simple rules.

10.1. Public Investment Criteria

Rule 1. The public sector ought to use a uniform set of shadow prices in all its projects.[2]

Rule 2. The shadow price of a tradeable commodity is its international price unless (a) there is a government budgetary constraint; (b) there is a foreign exchange constraint; (c) there is a quota on that commodity; or (d) the level of net imports (or net public production) of that particular commodity influences unoptimally some control (e.g. some tariff) at the disposal of the government.[3]

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[1]For a fairly extensive discussion of the implications of distributional objectives for the structure of taxation in a closed economy, see Atkinson and Stiglitz [1].

[2]This does not, of course, pertain to projects at different locations in an economy where there are substantial transport costs. As in usual general equilibrium analysis, one would then wish to expand the commodity space to account for locational differences.

[3]There is a sense in which (c) may be viewed as a special case of (d). A quota is like a discontinuous tariff imposition.

Rule 3. When there is a government budgetary constraint the shadow price of tradeable lies between the world price and its domestic price.

Rule 4. When there is a foreign exchange constraint the relative prices of commodities within each period are equal to the world price ratios; but the rate of interest used in project evaluation is not equal to the world rate of interest.

Rule 5. When there is a quota on a particular commodity its shadow price should be greater than its international price, if the quota is binding.

Rule 6. When the net level of imports of a commodity influences its tariff level its shadow price should be greater or less than its international price depending on whether as a consequence of the tariff response the consumption of the commodity is reduced (from what it would have been had international prices been charged) or increased.

Rule 7. Except under those exceptions noted in Rule 2, non-tradeables ought to be valued at their "foreign exchange" equivalent. That is the value of the foreign exchange that would be earned if one less unit of the given non-tradeable were produced, and the resources diverted to the production of tradeables.

In short, there is a presumption for using international prices in project evaluation unless one of the specific exceptions noted in Rule 2 occur.[1]

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[1] Here we certainly do not wish to pronounce on whether the exceptions (a) to (d) in Rule 2 are an exception or the rule in economies one knows. Our purpose in this paper has been solely to bring out the kinds of reasons one would wish to bring to bear when discussing whether or not international prices are appropriate in benefit-cost analysis.

What is particularly useful about these results is that they show the project evaluator what to do even in a "second best" (optimally chosen tariffs and taxes) or "third best" (non-optimally chosen tariffs and taxes). They show that even when there are quotas (or similar restrictions) although one does not use international prices for the commodities on which there is a quota, for other commodities one does.

10.2. Tariffs and Taxes

All the following rules are predicated on the assumption that lump sum taxation is unfeasible, so that the government has to resort to distortionary taxes to raise revenue. These rules are not dependent on the assumption of a representative consumer. (Although our analysis employed this assumption, a careful reading of the proofs show that it nowhere entered the arguments for any of these results, or any of the results cited above.)[1]

Rule 1. In a centrally controlled economy, or in an economy where 100% profit taxes may be levied, and in which both consumption, trade, and production taxes may be imposed no taxes (either on trade or on domestic production) should be levied on intermediates. Only general consumption taxes ought to be imposed.

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[1] If we assume an individualistic social welfare function

$$W = W(U^1, \ldots, U^r) = W(V^1(q), \ldots, V^r(q)) = W(q) ,$$

the analysis procedes exactly as in the text, except $\partial W/\partial q_i \neq c_i$.

Rule 2. Under the same circumstances as in Rule 1, if the only taxes which can be levied are trade taxes, then the output of intermediate goods should not be changed from what it would be at international prices. Goods which are used both as inputs into production and as consumption goods should be taxed (if it is impossible to treat the same good differently according to use).

Rule 3. In an economy where profits are not completely taxed away, both consumption and trade taxes should be employed. Imported intermediate goods should not be taxed if they can be treated differently from the (same) domestically produced intermediate goods; otherwise they should be.

Rule 4. If profits are not taxed at 100%, and firms are foreign owned then the production tax ought to be such as to reduce the output of all commodities by the same percentage; the tax is independent of the desire of the government for tax revenue. (See Equation (3.16).)

The following rules provide the detailed form of the tax and tariff structure (for small taxes) under the assumption of a "representative" consumer. The exact formulae that are valid regardless of the size of the revenue are given by the equations in parentheses in the text.

Rule 5. If there is no budget constraint, no foreign exchange constraint, no quotas, and no commodity whose tariff responds unoptimally to the level of import or public production; and if it is feasible to impose both consumption and production taxes,[1] then the tariffs

[1]The same qualifications apply to Rules 6 and 7.

and export duties should be such that the consumption of all commodities is reduced (along the compensated demand curve) by the same percentage from what it would have been had international prices been charged (equations (2.18) and (3.6)).

Rule 6. If profits are not taxed at all, and all firms are domestically owned, then there should be a tax on domestic production such that the output of all commodities is reduced exactly by the same percentage as that of the consumption of all commodities due to the trade taxes.[1] (See equation (3.8).)

Rule 7. If only trade can be taxed, and there is no profit tax, then the trade taxes ought to be chosen so as to reduce "excess demand" of each commodity by the same percentage (again along the compensated demand curve) from what it would have been had international prices been charged. (Equation (4.4).) If firms are foreign owned the relative reduction in excess demand should be smaller the smaller is the ratio of excess demand is to domestic private production (equation (4.8). If there is 100% profit taxation then the reduction in excess demand should be inversely proportional to the ratio of excess demand to consumption[2] (equation (4.8a).

[1]As we noted earlier, intermediate goods are treated just like final goods. Recall that we are treating outputs and inputs symmetrically.

[2]In particular, this means that output of intermediate goods is not reduced at all.

Rule 8. If some tariffs (export duties) are fixed, then the remaining duties ought to be chosen with a view to seeing that the consumption of commodities for which trade taxes can be varied is reduced by the same percentage (along the compensated demand curve) from what it would have been had international prices for all commodities been charged.[1]

Rule 9. If there is a quota on commodity k, or if the level of tariff on k is an increasing function of the net import of k, then the tariffs are chosen so as to ensure that the consumption of complements of k is reduced, by more than is the consumption of substitutes.

The rules for the evaluation of projects in the private sector are considerably more complicated. Typical of the kind of results available is the following; for the case where trade taxes are given and profits are untaxed, the profits, valued at domestic prices must be greater than the foreign exchange cost (valued at international prices) times the shadow price of foreign exchange in terms of domestic consumption.

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[1]Rules 8 and 9 are predicated on there being both consumption (trade) and production taxes for the commodities in question. Moreover we assume that profits are not taxed.

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