

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

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COWLES FOUNDATION DISCUSSION PAPER NO. 339

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ON THE IRRELEVANCE OF CORPORATE FINANCIAL POLICY

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April 6, 1972

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1. Introduction

This paper extends the argument of Modigliani and Miller [6] and Stiglitz [8], that the financial policy of the firm is of no consequence, to a multi-period model. In doing so, we are able to consider a much wider class of financial policies: not only does the firm choose a debt equity ratio, but it also selects a dividend-retention ratio, a maturity structure of debt, and it may even decide on holdings of assets (securities) in other firms. We wish to show, in the context of a general equilibrium model, that none of these policies has an effect on the valuation of the firm, under certain seemingly weak circumstances. Whether these assumptions are "realistic" or not is a question of some debate, about which we shall have a few words to say later. But by clarifying the assumptions, we hope at least to focus the discussion on the relevant issues.

*This is a revised version of a lecture delivered at a conference at Hakone, June 25-26, 1970. I am very much indebted to H. Uzawa, who organized the conference, and to the other participants for the helpful comments and discussion. The research described here was supported by the Guggenheim Foundation, the National Science Foundation, and the Ford Foundation.

Like all workers in the area of corporate finance, I owe a great intellectual debt to M. Miller and F. Modigliani; in addition, I have benefited greatly over the years from helpful discussions with them on the issues discussed here. I am also indebted to R. Merton and P. Kerbel for their helpful discussions.

The question of the effect of firm financial policy on the valuation of the firm is obviously of central concern to students of corporate finance; if the conditions under which the "irrelevance" theorems obtain are deemed realistic, it robs them of much of their stock in trade. But the question of the irrelevance of financial policy is of far greater significance.

We can divide the decisions of the firm into four groups:

- (a) How should the firm finance its investment?
- (b) How should the firm distribute its revenue?
- (c) How much should the firm invest?
- (d) Which projects should the firm undertake (or what techniques of production should the firm employ)?

The first two decisions of the firm are the financial decisions of the firm, the latter two the real decisions. The theory of corporate finance focuses on the financial decisions. The two financial decisions are closely related (see below), and so are the two real decisions. What is not obvious is the relationship between the real decisions and the financial decisions. An answer to this question requires an analysis of the relationship between corporations and the household sector of the economy, and to further our understanding of this relationship is a primary object of this paper. If the hypothesis that the financial policy of the firm makes no difference to firm's market valuation is correct, it also means that if firms maximize their market value, the real decisions are the only decisions that count, and the financial decisions have no bearing on them. In particular, it means that analyses of the real sector based on "flow of funds analysis" --and conclusions such as that of Kaldor [4] that because the flow of funds

from the household sector to the corporate sector is very small, the decisions of households with respect to savings are of relatively little significance in the determination of the equilibrium of the economy--are not likely to give us much insight into what is really going on: at best they provide us with some spurious correlations.¹ Moreover, if the maturity structure of debt is of no consequence, it casts some doubt about the validity of the partial equilibrium models attempting to relate the maturity structure to the term structure of interest rates. (See, for instance [7].)

In the literature, two different but closely related propositions have been confused: they both assert that the financial policy of the firm has no effect on its valuation. One asserts, however, that the individual is indifferent to alternative financial policies, in particular to debt equity ratios, and hence there is no determinate debt equity ratio for the economy as a whole. That is to say, any change in the financial policy of the firm can be completely offset by the actions of the stockholders (and indeed will be offset in the new general equilibrium situation).

The other asserts that the individual may not be indifferent to alternative financial policies, that there may be for instance a determinate debt equity ratio for the economy as a whole, but the financial policy of any particular firm makes no difference. One asserts, in other words, the irrelevance of the financial structure for the entire economy, and therefore

¹E.g. the correlation between retained earnings and investment does not provide an explanation of the determination of the level of investment. See, for instance, [5].

of the particular firm, the other only asserts the irrelevance of the financial structure of an individual firm. Clearly the former proposition is a much stronger one than the latter.¹ We are concerned here with both kinds of propositions.

The paper will proceed as follows. In Section 2, we set up the basic model. In Section 3, we prove our fundamental theorem on the irrelevance of the firm's financial policy from the point of view of any individual. Section 4 will comment briefly on the assumptions we have made and their limitations. Section 5 will show that financial policy need not be of concern to any particular firm, even if it is of concern to individuals, under much weaker conditions than those used to demonstrate the earlier proposition.

2. The Basic Model

The various financial decisions of the firm are clearly very closely related. One of the interests in a multi-period model is to explore these relationships. A decision to increase the amount to be distributed as dividends means, if the firm were to leave its investment decision unchanged, that it would have to raise additional revenue to pay for the planned investment. If it raises more by issuing bonds, the amount left over for distribution next period will be decreased, and hence either retained

¹The Modigliani-Miller theorem [6] was really of the latter type: they show that if there were two or more firms of the same risk class (the same pattern of returns across the states of nature) then the debt-equity ratio of any particular firm was indeterminate. Stiglitz's theorem [8] was of the former type.

earnings or dividends next period must be reduced. If it raises the revenue by issuing shares, it means the amount distributed to each shareholder next period (if retained earnings were unchanged next period) would be reduced. Thus, the interrelations among all the decisions are complex and any decision today may have ramifications for many periods into the future.¹

For expositional simplicity, we shall use a "one commodity" model;² each period there is a single commodity input and a single commodity output (dollars or yen). We shall look at the consequences of alternative financial plans on the firm's market valuation, given a "real plan" of the firm. A real plan is characterized by a statement of the investment level and choice of technique in each period contingent on the state of nature (the set of events that have occurred up to that time). Thus, given the real plan, we know the level of profits in each period, depending of course, on the state of nature. We let

¹The importance of these relationships has often been missed by even as astute students of the theory of corporate finance as Baumol and Malkiel [1] and Modigliani and Miller [6]. In discussing the impact of taxation on the optimal financial policy of the firm they observe that increasing the debt decreases the tax liability of the firm and hence increases its value. For an ongoing firm, with a given investment policy, increasing the debt equity ratio implies that the firm retains less of its earnings and thus the capital gains will be smaller.

²The analysis for the multi-commodity model is identical, except now a new set of financial decisions becomes available to the firm: it can denominate its bonds in terms of money or in terms of some other commodity, or some composite commodity (e.g. the cost of living index). Indeed, certain decisions which may appear to be "real" are in fact financial: when relative prices are uncertain firms must decide on whether to buy futures (or hold inventories) of inputs or sell futures (or hold inventories) of outputs. In short, all such "hedging" decisions have (under the assumptions below) no effect on the market value of the firm.

$I_i(t, \theta(t), k)$ = the level of investment of the i^{th} firm at time t , if the state of nature at that date is $\theta(t)$, under plan k .

$X_i(t, \theta(t), k)$ = the output of the i^{th} firm or gross profit at time t , if the state of nature at that date is $\theta(t)$ under plan k .^{1,2}

Firms have available to them a large number of alternative ways of financing their investment:

(a) It can finance its investment with retained earnings or by issuing new securities.

(b) If it issues new securities, it can use a large number of different financial instruments: common stock, bonds, preferred stock, convertible bonds, etc. Each of these financial instruments carries with it different contractual rights with respect to the distribution of the gross profits of the firm, and the part the owner of those instruments can play in the decision making of the firm. For instance, bonds yield a fixed sum in every state of nature except when the firm goes bankrupt, in which case the proceeds of the firm are divided among the bondholders. On the other hand, except when there is the distinct possibility of the firm not being able to meet its debt obligations, in general the bondholders have no voting

¹No loss of generality is had by interpreting I_i and X_i as vectors of inputs and outputs. In the proofs, we then need to replace I_i by $\underline{v} \cdot \underline{I}_i$ and X_i by $\underline{v} \cdot \underline{X}_i$, where \underline{v} is the vector of relative prices (at time t , in state $\theta(t)$).

²Since in our simplification, there are no other inputs, the output of the firm and its gross profit (before paying interest on debt, etc.) are identical. The modification required when there are other inputs are straightforward.

rights in the management of the firm. The return to a common stock, on the other hand, is variable--except when the firm goes bankrupt, in which case it gets nothing. A shareholder is entitled to receive a proportionate share of the dividends of the firm. The dividends, of course, depend not only on the real policy of the firm but on the particular financial policy chosen, so to know the stream of returns, the shareholder must know both the real and the financial decisions of the firm. On the other hand, if our argument that financial policy is irrelevant is correct, then although changes in dividend policy effect the pattern of returns received by any single share of the firm, the individual is indifferent to these changes.

The shareholder (like the bondholder) can of course sell his shares at any date and receive what he can for them. Finally, ownership of shares generally gives one a proportionate vote in the stockholders meeting (although some firms also issue shares which do not have voting rights).

In the ensuing analysis, we shall assume for simplicity that there are only two classes of financial instruments, bonds and common shares.

(c) If it decides to issue bonds, it must decide on what maturity--one year, two year, etc.--and what the coupon rate will be. For simplicity we shall assume that bonds carry no coupons. Thus, a t period bond is a promise to pay in t periods 1 dollar. When it is issued, it obviously sells at a discount.¹ We let

¹There are of course still other financial decisions, including ownership claims on other firms, what numeraire to denominate bonds in, etc. All of these could be included in our model--at some expense in notational complexity.

$p(t, \tau, \theta(t))$ be the price at time t in state $\theta(t)$ of a bond which promises to pay 1 dollar at time τ .

If there is uncertainty, the individual will not know what the price of such a bond will be in future periods except that, if there is no bankruptcy,

$$p(t, t, \theta) = 1 \text{ for all } \theta, t.$$

The relationships among the various financial decisions are expressed by the two accounting identities: Total investment must be equal to the value of the change in outstanding bonds plus the value of the change in outstanding shares plus retained earnings:

$$(1) \quad I_1(t, \theta, k) = \sum_{\tau=t+1}^{\infty} p(t, \tau, \theta)(B_1(t, \tau, \theta(t)) - B_1(t-1, \tau, \theta(t-1))) \\ + q_1(t, \theta)(S_1(t, \theta(t)) - S_1(t-1, \theta(t-1))) + RE_1(t, \theta(t))$$

where $B_1(t, \tau, \theta(t))$ ¹ is the number of bonds outstanding at the end of period t in state θ with maturity at time τ

$q_1(t, \theta)$ is the price of a share of the i^{th} firm at time t in state θ

$S_1(t, \theta)$ is the number of shares outstanding at the end of the period

$RE_1(t, \theta(t))$ is retained earnings.

There is, of course, no natural unit for shares, so it is just as simple to define

$$E_1^+(t, \theta) = q_1(t, \theta)S_1(t, \theta(t))$$

as the value of the shares outstanding at the end of the period, while

$$E_i^-(t, \theta) = q_i(t, \theta) S_i(t-1, \theta(t-1))$$

as the value of the shares outstanding at the beginning of the period.

Thus $E_i^+(t, \theta) - E_i^-(t, \theta)$ is the value of the change in the number of shares outstanding resulting from issuing new shares during the t^{th} period; this should not be confused with $E_i^-(t+1, \theta) - E_i^+(t, \theta)$, which is the change in the value of the shares outstanding from the t^{th} to the $t+1^{\text{st}}$ period. The latter is the capital gain (or loss) on existing shares.

The second accounting identity states that total income in state θ at time t must be equal to the income distributed (to bondholders and to shareholders) plus that retained by the firm.²

$$(2) \quad X_i(t, \theta, k) = B_i(t-1, t, \theta(t-1)) + D_i(t, \theta(t)) + RE_i(t, \theta(t))$$

where $D_i(t, \theta(t))$ are the dividend payments to stockholders on record at the beginning of the period, i.e. each share receives³

$$\frac{D(t, \theta)}{S(t-1, \theta(t-1))}$$

or the dividend per dollar invested at time $t-1$ is

$$\frac{D(t, \theta)}{E_i^+(t-1, \theta(t-1))} .$$

¹We could of course make the financial plan be part of the total set of plans of the firm, denoted by k . But since we shall argue that the financial plan is of no consequence, the k is omitted. q_i is also a function of k , but for notational simplicity we suppress the k .

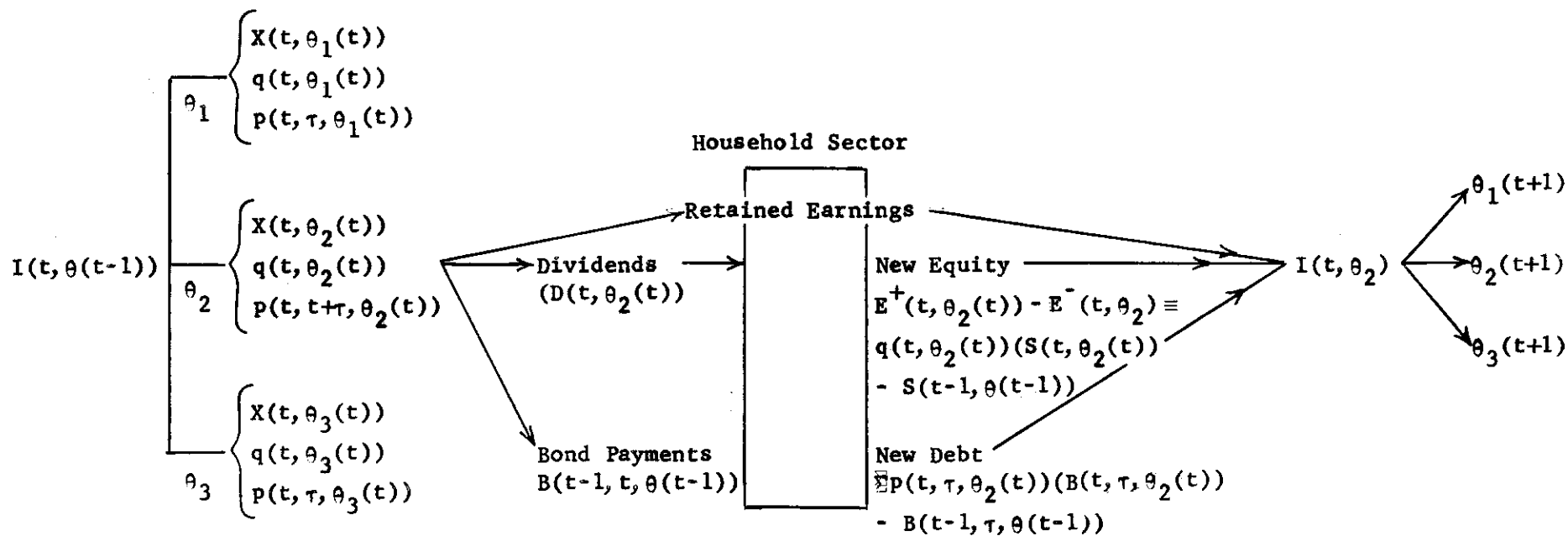
²Recall that we are assuming for notational simplicity that there are no coupons on bonds; thus bondholders only receive income from the firm upon maturation of bonds.

³Here and elsewhere, where there is no ambiguity, we omit the subscript i .

We illustrate in Figure 1 a flow of funds diagram for this economy over time. (Because the flow of funds occurs over time, the diagram is not made circular.) It should be noted that we have drawn the line for retained earnings through the household sector with a dotted line: the fact that the earnings do not pass through their hands directly does not necessarily mean that the household sector does not include (in some sense) such retained earnings in their income.

The diagram also served to clarify the timing implicit in our analysis: Let us break into the diagram at a point where the firm has just made its "new" investment decision having raised the requisite capital. The output (profits) next period (which depends not only on investment in the period just ended, but on investment in all preceding periods, as well as the specification of the environmental path for these preceding periods) is unknown; we await the specification of the "environment" for time t , e.g. the rainfall, temperature, etc. These are then announced, i.e. $\theta(t)$ is then given. This means that the set of possible outcomes for $t+1$ and beyond has immediately been substantially narrowed. In Figure 1 we can, for instance, now completely ignore all but one of the environmental paths passing through t .

Given the new information embodied in the announcement of the state $\theta(t)$, the value of the shares and the prices of bonds are determined. In particular, the value of the equity of the firm now is $E_1^-(t, \theta(t))$. Moreover, at this point, for the particular plan we have denoted by k , we know exactly what the firm plans to do this period: we know its investment, $I_1(t, \theta, k)$, how much dividends it plans to give out, how much



Investment	State of nature announced with associated	Profits and prices of securities (shares and bonds)	Distribution of profits to shareholders (dividends) and bondholders; remaining profits retained	New Capital Raised (issuing shares and debt) which, together with retained earnings finance	New Investment
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FIGURE 1

it plans to retain, how many bonds and of what maturity it plans to issue, how many new shares it plans to issue, etc. We still don't know, of course, what its investment will be in the future; for this we await further information. But we do assume that we do know what the firm will do in each contingency.¹

In the theorem we shall present in the next section, two concepts play a crucial role; one is the value of the firm, the other is bankruptcy.

The total value of the firm is the present value of its outstanding bonds plus the value of its equity:

$$(3a) \quad V_i^-(t, \theta(t)) = E_i^-(t, \theta(t)) + \sum_{\tau=t+1}^{\infty} p(t, \tau, \theta(t)) B_i(t-1, \tau, \theta(t-1))$$

$$(3b) \quad V_i^+(t, \theta(t)) = E_i^+(t, \theta(t)) + \sum_{\tau=t+1}^{\infty} p(t, \tau, \theta(t)) B_i(t, \tau, \theta(t)) .$$

Bankruptcy is somewhat more difficult to define. The basic notion is, of course, that the firm is unable to meet its debt obligations. In the two period model discussed in [8], a firm is bankrupt whenever the profits are less than the nominal claims of bondholders.

¹The assumption that the raising of new capital follows (in each period) the distribution of the profits and the bond payments is made simply for expositional convenience. In fact, these two operations may be thought of as occurring simultaneously. The important assumption is that the same price of a security prevails at the beginning of the period as at the end. Since once the state of nature is announced, everything that will occur during that period is known, this is not an unreasonable assumption; alternatively, if we think of the distribution of returns and the raising of new capital as occurring simultaneously, this is clearly the appropriate assumption.

$$X_i < (1+\hat{r})B_i$$

where \hat{r} is the nominal rate of interest on the bond. If r^* is the nominal rate of interest on a perfectly safe bond, and $X_{\min} = \min X(\theta)$ is the minimum profit in any state of nature, the probability of bankruptcy is zero provided

$$B_i \leq \frac{X_i \min}{1 + r^*}$$

while it is positive if

$$B_i > \frac{X_i \min}{1 + r^*} .$$

See Figure 2. The analogous statement here would be

$$(4) \quad X_i(I_i, \theta(t)) < B_i(t-1, t, \theta(t-1)) .$$

But this will not do. For firms always have the option, if their returns in a particular period in a particular state are low, of borrowing more or issuing new shares to meet these debt obligations. Indeed, this is exactly what they would normally do, provided their future prospects of returns are sufficiently good, i.e. the condition (4) is at best a statement about very short term liquidity, not about the solvency of the given firm. In a multi-period model with a terminal date, in the last period, the condition for bankruptcy is given by (4), since the firm cannot (by assumption) issue new shares or borrow further. But there is no reason to restrict ourselves to a finite period model.

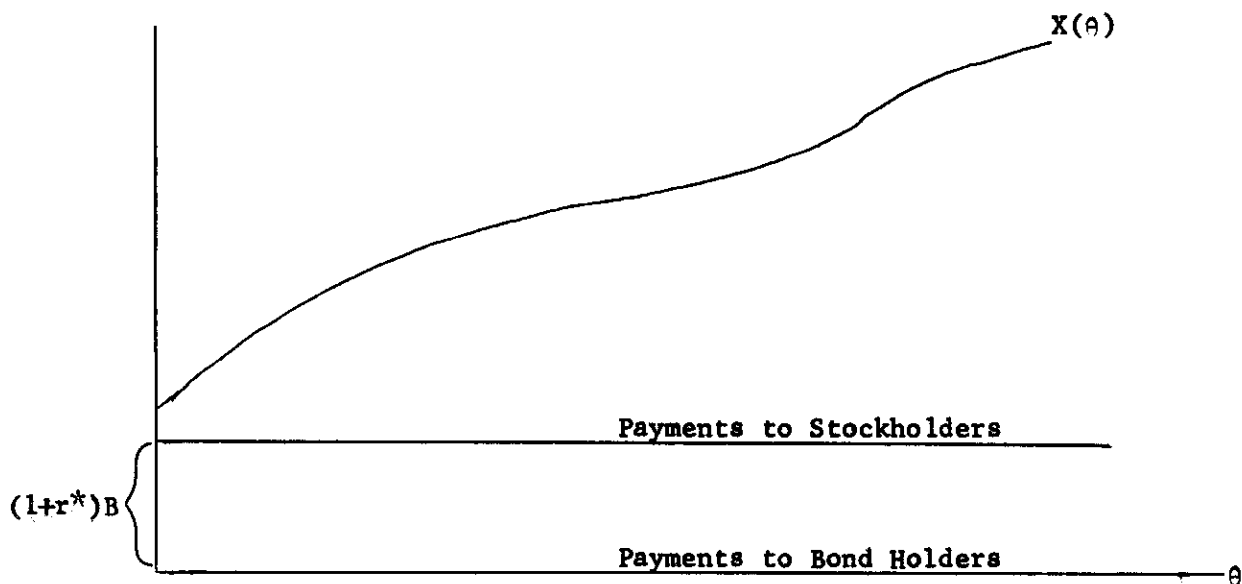


FIGURE 2a. Distribution of Returns: No Bankruptcy

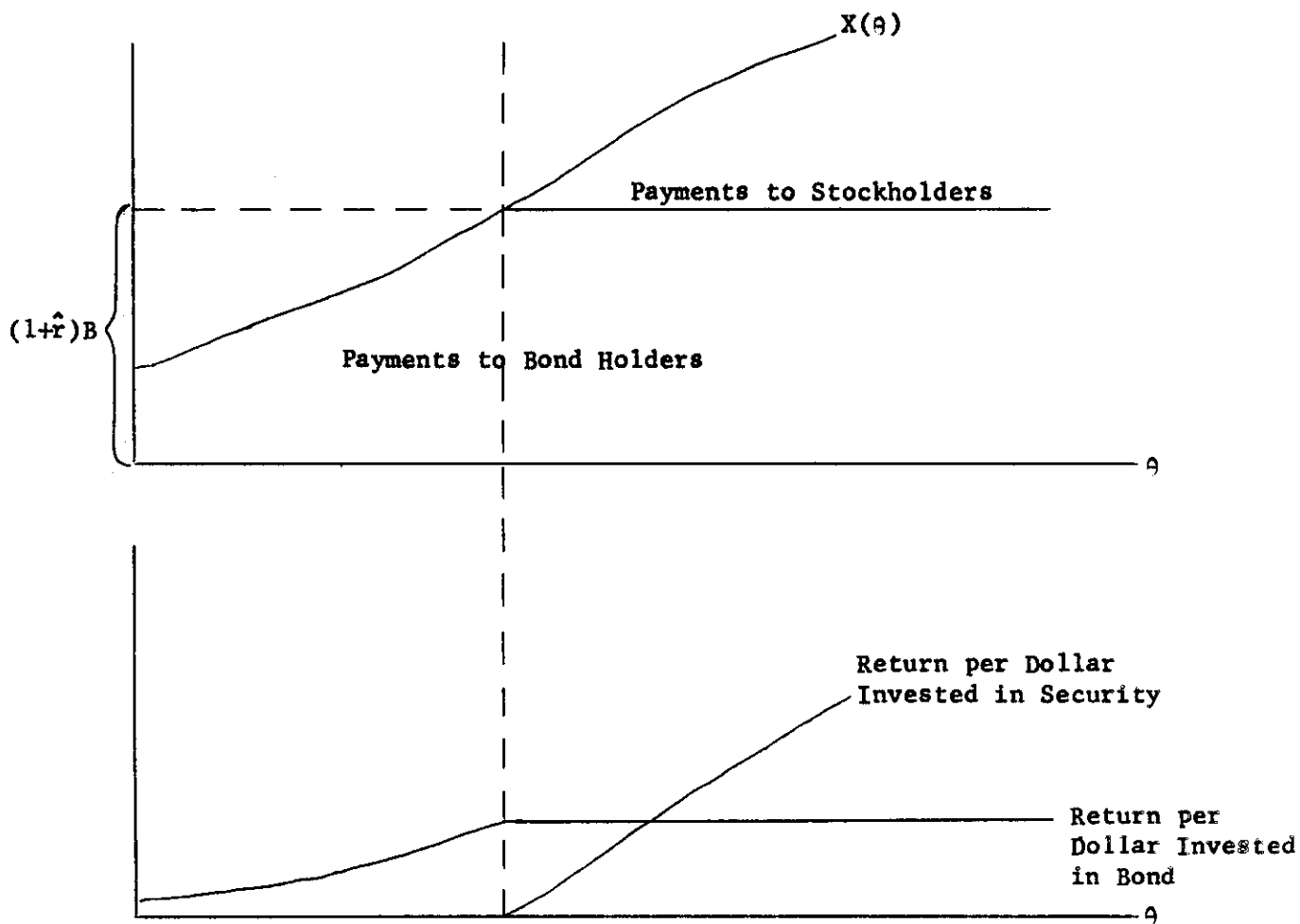


FIGURE 2b. Distribution of Returns: Bankruptcy

Clearly, what we mean by bankruptcy is that at some date, in some state of nature, the value of the maturing bonds of a firm are less than their face value

$$p(t, \tau, \theta(t)) < 1$$

for some t and $\theta(t)$. This is equivalent to saying that at that date and in that state of nature the value of the equity of the firm is zero (or negative if there is not limited liability)¹

$$(5) \quad E_i^-(t, \theta(t)) \leq 0$$

or

$$(5') \quad V_i^-(t, \theta(t)) \leq \sum_{\tau} B_i(t-1, \tau, \theta(t-1))p(t, \tau, \theta(t)) .$$

The fact that the value of the equity of the firm in some state of nature at some date in the future is zero does not mean that the value of the equity will be zero today; if there is some chance that the firm will not go bankrupt, clearly the value will be positive. But it does mean that bonds issued with maturities at the date of potential bankruptcy or beyond are risky securities, i.e. their terminal value is uncertain, and clearly the price of these bonds will not be the same as the price of a bond whose terminal value is certain. A change in the financial policy which results in a change of bankruptcy, i.e. in a chance that the firm will not be able to meet its debt obligations, thus changes the prices of the bonds the firm issues, and

¹Clearly, if the price of a share is zero, the firm cannot use more equity to pay off the debt holders.

it is the invariance of the price of the bonds which will be crucial in the argument of the next section.

3. The Basic Theorem

We are now prepared to state and prove our central

Theorem 1.

- (a) Assume there is no bankruptcy of any firm in any state of nature.
- (b) Assume that there is a perfect market for perfectly safe bonds of all maturities. (By perfectly safe, we mean that the amount that they pay upon maturity is known for certain; the price of these different maturities at all other dates is highly variable.)¹
- (c) All firms have already made their real decisions (i.e. the value of k for each i is given.)
- (d) Assume there is a general equilibrium, with all markets clearing, characterized by a given price in each state of nature at each time for each maturity bond and each firm having a given valuation in each state of nature and at each time t and a given financial policy (i.e. a specification of debt-equity ratio, retention ratio, maturity structure of bonds).

¹From a consumption point of view, it cannot be said which maturity is the "safer" bond, which is the riskier, i.e. it cannot be said the long term bonds are riskier than short term bonds. See Stiglitz [9]. What we shall show here is that from the point of view of the firm, the maturity structure is irrelevant.

Then, there is another general equilibrium solution where any firm (or group of firms) have changed any (or all) of its financial policies, but in which the value of the firm and the price of all maturities of all bonds are unchanged (for all periods and states of nature).

The argument of the proof is simple. We shall show that if the value of the firm and the price of all maturities of all bonds are unchanged (for all periods and states of nature) then the set of consumption possibilities available to any individual is unchanged. Since the set of consumption possibilities is unchanged, the individual will choose the same consumption path (i.e. the same plan of consumption over time, which is clearly a contingent plan, depending on which states of nature occur.) To do this, he must adopt a certain investment-portfolio allocation plan (i.e. a plan of how he will allocate his portfolio in each state of nature at each date). Finally, we show that if the set of investment-portfolio allocation plans originally adopted by the different individuals in the economy (that is, before the firm changed its financial policy) was an equilibrium, so that at each date and in each state of nature markets cleared, then the set of new investment-portfolio allocation plans also constitutes an equilibrium.

The proof is inductive in structure and for convenience is broken into two parts.

1. Let us denote his wealth at the end of period t in state θ in the original situation as $w^{j+}(t, \theta(t))$. Let $C(t, \theta(t))$ denote his consumption in period t in state $\theta(t)$. Let us denote the corresponding variables in the new situation (i.e. where the firm pursues some alternative financial policy) by \hat{w}^{j+} , \hat{C}^j , etc. We shall now show that

(i) if the value of firm and the price of all bonds is identical in the two situations, and (ii) if his wealth at time $t-1$ and his consumption at time t are identical,

$$(6a) \quad w^{j+}(t-1, \theta(t-1)) = \hat{w}^{j+}(t-1, \theta(t-1))$$

$$(6b) \quad C^j(t, \theta(t)) = \hat{C}^j(t, \theta(t))$$

then the opportunity set of wealth at the end of period t is identical in the two situations.

His terminal wealth is held either in the form of bonds or stocks, so

$$(7) \quad w^{j+}(t, \theta(t)) = \sum_i E_i^{j+}(t, \theta(t)) + \sum_{\tau} p(t, \tau, \theta(t)) B^j(t, \tau, \theta(t)),$$

where E_i^{j+} is the value of shares in the i^{th} firm owned by the j^{th} individual (at the end of the period).

$B^j(t, \tau, \theta(t))$ is the j^{th} individual's ownership at time t in state $\theta(t)$ of bonds maturing at time τ .

Let

$$(8) \quad \alpha_i^j(t, \theta(t)) = \frac{E_i^{j+}}{E_i}$$

the proportion of the shares of the i^{th} firm owned by the j^{th} individual in state θ at time t . From (8) and (6), we have

$$(9) \quad w^{j+} = \sum_i \alpha_i^j E_i + \sum_{\tau} p B^j$$

Now, recalling the definition of the value of the firm, V_1^+ , (equation (3b)) we obtain

$$(10) \quad w^{j+} = \sum_i \alpha_i^j V_1^+ + \sum_{\tau} p(B^j - \sum_i \alpha_i^j B_i)$$

Between the end of period t and the end of period $t-1$ three things will happen: he will receive dividends; he will purchase consumption goods; and the value of his bonds and shares will have changed. Thus his terminal wealth at time t can be written

$$(11) \quad w^{j+}(t, \theta(t)) = \sum_i \alpha_i^j(t-1, \theta(t-1))(E_1^-(t, \theta(t)) + D_1(t, \theta)) \\ + \sum_{\tau} p(t, \tau, \theta(t))B^j(t-1, \tau, \theta(t-1)) - C^j(t, \theta(t)) .$$

But from equations (1)-(3) we obtain

$$(12) \quad D_1(t, \theta(t)) = X_1(t, \theta(t)) - I_1(t, \theta(t)) + \sum_{\tau} p(t, \tau, \theta(t))(B_1(t, \tau, \theta(t)) \\ - B_1(t-1, \tau, \theta(t-1))) + E_1^+(t, \theta(t)) - E_1^-(t, \theta(t)) .$$

Hence, using (5), we have¹

$$(13) \quad D_1 = X_1 - I_1 + V_1^+ - V_1^- .$$

Substituting (13) into (11), we obtain

¹Here as elsewhere, we write D_1 for $D_1(t, \theta(t))$, etc. where there is no ambiguity as a result.

$$\begin{aligned}
(14) \quad w^{j+}(t, \theta(t)) &= \sum_i \alpha_i^j(t-1, \theta(t-1)) [v_i^{j+}(t, \theta(t)) + x_i(t, \theta(t)) - I_i(t, \theta(t))] \\
&\quad + \sum_{\tau} p(B^j(t-1, \tau, \theta(t-1))) - \sum_i \alpha_i^j(t-1, \theta(t-1)) B_i^j(t-1, \tau, \theta(t-1)) \\
&\quad - C^j(t, \theta(t)) .
\end{aligned}$$

Consider any feasible portfolio allocation, i.e. a set of $\{\alpha_i^j, B^j\}$ satisfying (10). Now consider the situation where firms are pursuing some alternative financial policy. Then the allocation

$$(15a) \quad \hat{\alpha}_i^j = \alpha_i^j$$

$$(15b) \quad \hat{B}^j = B^j + \sum_i \alpha_i^j (\hat{B}_i - B_i)$$

is also feasible if $v_i^{j+} = \hat{v}_i^{j+}$, $p = \hat{p}$, $w^{j+}(t-1, \theta(t-1)) = \hat{w}^{j+}(t-1, \theta(t-1))$; for then

$$\begin{aligned}
\hat{w}^{j+}(t-1, \theta(t-1)) &= \sum_i \hat{\alpha}_i^j \hat{v}_i^{j+} + \sum_{\tau} p(\hat{B}^j) - \sum_i \hat{\alpha}_i^j \hat{B}_i \\
&= \sum_i \alpha_i^j v_i^{j+} + \sum_{\tau} p(B^j) - \sum_i \alpha_i^j B_i \\
&= w^{j+}(t-1, \theta(t-1)) .
\end{aligned}$$

But from (14), this implies that for all $\theta(t)$, $\hat{w}^{j+}(t, \theta(t)) = w^{j+}(t, \theta(t))$ if $C^j(t, \theta(t)) = \hat{C}^j(t, \theta(t))$ and $v_i^{j+}(t, \theta(t)) = \hat{v}_i^{j+}(t, \theta(t))$.

It immediately follows that, under the given assumptions about unchanged prices and valuations, the set of feasible consumption paths in

the two situations are identical.¹ Hence if a particular consumption path, $\{C^{j*}(t, \theta(t))\}$ is preferred to all other consumption paths in the original situation, then it is preferred in the new situation.

2. If along the path $\{C^{j*}(t, \theta)\}$, all markets cleared at each point of time (for each state of nature) in the original situation, they also do in the new situation.

Market clearing in the securities markets requires

$$(16) \quad \sum_j E_i^j(t, \theta) = E_i(t, \theta) \quad \text{all } i, \theta, t$$

in the original situation, and

$$(16') \quad \sum_j \hat{E}_i^j(t, \theta) = \hat{E}_i(t, \theta)$$

in the new situation. But if markets cleared in the original situation

$$\sum_j E_i^j(t, \theta) = \sum_j \alpha_i^j E_i(t, \theta) = E_i(t, \theta)$$

or

$$(17) \quad \sum_j \alpha_i^j(t, \theta) = 1 \quad \text{for all } i, t, \theta.$$

But since $\alpha_i^j = \hat{\alpha}_i^j$ all securities markets must clear in the new situation.

Similarly, market clearing in the bond markets requires

¹In particular, since the original path and the new one are identical over the period before the contemplated change in financial policy begins, if the values of the firm and prices are the same on the two paths, the values of w^j are identical on the two paths for those dates. It is convenient to think of the contemplated "change" in financial policy as beginning at time 1, so that $w^j(0, \theta(0)) = \hat{w}^j(0, \theta(0))$.

$$(18) \quad \sum_i B_i(t, \tau, \theta) = \sum_j B^j(t, \tau, \theta) \quad \text{all } t, \tau, \theta.$$

To show that in the new situation

$$\hat{B}_i(t, \tau, \theta) = \sum_j \hat{B}^j(t, \tau, \theta)$$

we recall (15), from which it follows that (using the fact that the securities markets are also clearing)

$$\begin{aligned} (19) \quad \sum_j \hat{B}^j(t, \tau, \theta) &= \sum_j B^j(t, \tau, \theta) + \sum_j \sum_i \alpha_i^j (\hat{B}_i - B_i) \\ &= \sum_j B^j - \sum_i B_i + \sum_i \hat{B}_i \\ &= \sum_i \hat{B}_i. \end{aligned} \quad \text{Q.E.D.}$$

4. Comments on the Theorem and its Proof

There are four kinds of comments which we have to make. In 4.1 we provide an intuitive interpretation of the theorem. In 4.2 we point out how much weaker the assumptions employed in our analysis are than those used in previous proofs. In 4.3 we discuss briefly the limitations on the proof, and how critical they are for the general validity of the theorem. In Section 4.4 we discuss the competitive forces at work to eliminate the "inefficiency" resulting from the resource allocation to financial management.

4.1. Intuitive Interpretation

The basic argument of the theorem, is that individuals can exactly "undo" any financial policy undertaken by the firm. Let us consider verbally

what actions of the individuals are required to offset various actions by the firm. Assume the firm decreases its dividend pay out ratio. This means that it has more retained earnings, so, if the two basic financial accounting identities are to be satisfied, either it must borrow less (perhaps it even lends) or issue fewer new shares. To make up for the loss in dividends, i.e. to keep the same consumption path, individuals buy fewer new shares in the firm or buy fewer new bonds. Assume the firm simply issued fewer shares. In one case, the value of the equity grew because of issuing new shares, in the other case, the value of the equity grew because of retained earnings. From the point of view of the stockholders, the two are perfectly equivalent. This change in dividend pay out ratio thus leaves the debt equity ratio unchanged. On the other hand, if the firm decreases the number of bonds issued, it will lead to a lower debt equity ratio. Then individual borrow on their own account. One can think of it as if the individual takes the proceeds of the loan to purchase the increased equity in the firm (since the two are exactly equal; this is only a convenient way of looking at it; since all funds are fungible, there is no real connection between the two). The increased borrowing by individuals exactly offsets the decreased borrowing by firms so markets continue to clear. Similarly, if the firm decides to issue more three year bonds and fewer five year bonds, the individual can undertake exactly offsetting actions in his own portfolio.

4.2. On the Generality of the Theorem

(a) Risk classes, Arrow-Debreu securities, mean-variance analysis. It should be emphasized that in our proof there do not have to be two firms which are otherwise identical; our argument does not require the existence of risk classes, as many states of nature as securities, or the assumption that returns can be described by means and variances, assumptions which have been crucial in other proofs of the more limited theorem on the irrelevance of debt equity ratios.

(b) Competitiveness of capital market. No assumption about the competitiveness of the capital market has been made; the only assumption is that there be no discriminatory pricing, i.e. the price paid by one individual (firm) for a bond (or share) be the same as for all other individuals. But the market rate of interest--and hence the interest rate paid by a firm--may be affected by the amount of capital it raises from the market.

(c) Rationality of consumers. The only restriction on individual behavior is that given a set of feasible consumption paths, he always selects the same consumption path. Thus, the individual may maximize his discounted expected utility, but no such restrictive assumption is required for our result to obtain.

(d) "Control" of firm. Even if the individual does care about his "political power" ("control") within the firm, which he may if the real decisions of the firm depend on the stockholders, if the role of each stockholder in decision making is simply a function of the proportion of the total share he owns, the financial policy makes no difference, since "political power" of any shareholder is identical in the two situations.

One might have argued that a smaller equity base would make a "take-over" more likely; but under the assumption of no bankruptcy, this would not be true, since the group taking over the firm could borrow on the strength of the equity in the firm as collateral; if in the low equity situation, the group taking over could raise the requisite capital for a takeover, they would have no problem doing so in the high equity situation.¹

(e) Source of uncertainty. No assumption about the source of uncertainty is required.²

(f) Multiplicity of equilibrium. Our theorem is a theorem about market equilibria. It states that there are an infinite number of general equilibrium solutions of the economy all of which are identical in all respects except for the financial policies of firms and the value of bonds and shares (separately) held by individuals (although the proportions of the shares of each firm owned by any individual are the same). There may of course be more than one general equilibrium solution to the economy at any given set of financial policies. As usual, very strong conditions would be required to ensure uniqueness. But what our theorem does assure us is that if there are two (or three or...) at a given set of debt equity ratios,

¹For a more extended discussion of the relationship between debt equity ratios, bankruptcy, and takeovers, see [10].

²In particular the distinction between technological uncertainty and price uncertainty, which played such an important role in Diamond's [3] analysis, is of no consequence here. It should also be noted that Diamond's assertion that his results do not depend on the no bankruptcy assumption is incorrect.

then there are two (or three or...) at any other set of debt equity ratios. We have nothing to say about the important question of which one of these will be in fact chosen.

(g) Differing expectations. The argument of our proof does not require that individuals have the same expectations. The only agreement in expectations that is required is that the firm will not go bankrupt in any state of nature. (See below for a discussion of this assumption.)

(h) Market clearing. The particular path we have described in the above analysis is an equilibrium path, i.e. one where individuals make plans all of which are consistent with one another, i.e. they are market clearing. In fact, the only thing required for our analysis is market clearing at time 0. In making their portfolio-consumption decision for time 0, the individual must have expectations of prices and firm valuations at all future dates and states. These may not, of course, be realized; at each successive date expectations are then revised. It is important to the analysis that these revisions depend on "real events" not on the financial structure of the firm. (See below.)

4.3. Limitations on the Theorem

There are three critical limitations on the theorem.

(a) Independence of expectations from financial policy. Our analysis requires that these expectations be unchanged as the firm changes its announced financial policy for the future.

If it should turn out that these expectations are a function of the financial policy of the firm, then in fact the financial policy of the firm will affect its valuation this period. The expectations that financial policy will affect market valuation are, at least in this very rough sense, fulfilled. But note that our argument for equilibrium paths shows that there is no reason that these expectations ought to change.

(b) Individual borrowing an imperfect substitute for firm borrowing. Perhaps the major objection to the proposition that the firm's financial policy is irrelevant is that individual borrowing is not a perfect substitute for firm borrowing. There are four principle reasons for this: (i) higher interest rates faced by individual borrowing than for corporate borrowing; (ii) limitations on the amount that individuals can borrow from the market; (iii) transactions costs; (iv) special tax provisions (differential treatment of capital gains and deductability of interest payments for the corporate income tax). I have discussed these limitations in greater detail elsewhere [8]. Here I wish only to make a few observations. First, the higher nominal interest rates individuals pay, and the quantitative restrictions on their borrowings are primarily a reflection of the higher probability of default on the part of individuals. They are, in other words, a particular manifestation of the general problems that the chance of bankruptcy brings to the analysis. Second, the first three place restrictions on the set of financial policies among which the individual is indifferent, but there is no reason to believe that these restrictions are very severe, they may mean that firms cannot have all equity policies, but individuals will still be indifferent among a wide set of debt equity ratios. The point is that,

if for instance firms were to decrease their debt equity ratio, the analysis does not require that individuals borrow from the market to purchase the additional shares issued by the firm; it only requires that they decrease their holdings of bonds. Hence so long as the total debt held by the different firms in the individual's portfolio is sufficiently large that the individual is a net lender rather than a net borrower, the individual is indifferent. This places a lower bound on the "average debt equity ratio" of the firms in the individual's portfolio (although not on the debt equity ratio of any single firm). This constraint may become an important constraint if at those debt equity ratios there is a finite probability of bankruptcy; that is, if it is only in conjunction with the bankruptcy constraint that this constraint becomes significant.

Thirdly, it does not place restrictions on the debt equity policy to be pursued by any particular firm, only the set of debt equity policies that groups of firms can follow; i.e. even if the constraint is binding; in general one firm can increase its debt equity ratio when another firm decreases its debt equity ratio. One cannot speak of an "optimal debt equity" or optimal retention ratio.

(c) Bankruptcy. In our judgment, the most restrictive assumption is that of no bankruptcy.

The careful reader may have wondered where the restriction of no bankruptcy was used in the proof. Because of limited liability laws, it is clear (as we noted before) that

$$E_i \geq 0 .$$

If the firm issues a sufficiently large number of bonds so that in some state of nature at some date

$$\sum pB_i > V_i$$

for V_i to be the same as in the original (reference) situation E_i would have to be negative. But this is impossible.

The assumption was not only critical to our proof, but we would argue, critical to the general validity of the theorem. To put it one way, it is not reasonable to assume that the price of bonds for which there is a positive probability of default at maturation would be the same as a perfectly safe bond. One might argue that the decline in the nominal value of bonds is compensated for by an equivalent increase in the value of equity, and under certain circumstances--the existence of as many securities as states of nature or the mean-variance model with homogeneous expectations --this is true. But in the more general case, bankruptcy changes the opportunity set facing a given individual, so that the value of the firm is changed; not only is the financial policy of importance, but no separation between the financial and real decisions is possible.¹

¹These remarks should serve to clarify the difference between our theorem, both its meaning and its proof, from that of Modigliani and Miller [6]. They both assert that the financial policy is of no consequence. But Modigliani and Miller made use of risk classes in their proof, the use of which seemed to imply objective rather than subjective probability distributions over the possible outcomes. The mechanism which ensured that the debt equity ratio made no difference for the value of the firm was individual arbitraging among different firms in the same risk class. Such arbitrage does not play any role in our analysis. Moreover, their argument was based on partial equilibrium analysis rather than general equilibrium analysis, it was not clear from their analysis whether the theorem held only for competitive markets, and how the possibility of firm bankruptcy affected their results was not clear. The basic insight of the MM analysis, that individual leverage could substitute for firm borrowing remains the basis of our argument.

4.4. Competitive Forces to Eliminate "Waste" of Resources on "Financial Management"

One might ask, if financial policy were really of no importance, why do firms waste resources on "money managers"; shouldn't competitive forces lead all firms to ignore financial policy, since worrying about it costs resources, and can't increase the market valuation of the firm, clearly firms that spend resources on financial management have lower profits to distribute to their stock holders than firms who don't. There are five answers to this:

(i) We have ignored some important considerations, in particular, taxes, which do make it profitable to worry about financial structures. Does this mean that we believe that in the absence of taxation financial managements would wither away? Not necessarily, or only very slowly, as the remaining points argue.

(ii) We have already argued that if individuals believe that financial policy affects firm valuation, then it will, and the firm that ignores the popular "prejudices" may do worse than one which takes them into account. There may have been no rational reason for tulip bulbs to rise as they did in the tulip bulb mania, but since they were rising, at least in the short run, one could make a "profit" by investing in them. (See below.)

(iii) Moreover, this relationship between firms financial policy and expectations about profits may not be as "irrational" as the above analysis suggests. Changes in financial policy may be an important signal for the "real prospects" of the firm. This would not be the case in our model, because in it there is no such thing as a liquidity crisis; but in the real world bankruptcy may be important, and the fact that banks and other lending

institutions are unwilling to lend the firm money (so for instance forcing a reduction in dividends to meet the liquidity requirements) may be a signal that those who know more about the prospects of the firm than the relatively uninformed shareholder are not sanguine about the prospects of the firm.¹

(iv) There is, moreover, no reason that in the short run the different valuations lead to any inconsistencies or more generally that there are any forces leading individuals to reformulate their expectations so that valuations are independent of financial policies. Even if we have two firms which are identical in every real respect (that is, they belong to the same risk class, in the terminology of Modigliani and Miller [6]) there is not necessarily any method by which individuals can arbitrage (over any short or medium run² period). To see this in the extreme case, we need only consider the situation where profits minus investment are known for certain and the firm issues no new shares. Then dividends for say firm 1 may be written (in a continuous time formulation)

¹ Indeed, one might argue that this signalling effect of financial policy is one of its more important functions. If firms never issued dividends, simply retaining earnings (even in the form of bond purchases) then it might be possible for firms to postpone letting shareholders know when they are in "bad straits" even longer than they do at the present. This may provide part of the explanation of why, in spite of strong tax advantages to not issuing dividends, firms continue to do it.

² This qualification is imposed because, under certain circumstances, it can be shown that if different financial policies are pursued, with the firms having different valuations and equal returns to the individual, then, in finite time, the relative valuations must become infinite. But finite, in this context, may be very long indeed. Such differences in valuations are (at least mathematically) very similar to the speculative booms (or depressions) which often seem to characterize price movements on the stock market. For a general discussion of these problems in a slightly different context see Shell and Stiglitz [7].

$$D_1(t) = X_1(t) - I_1(t) - rB_1(t) + \dot{B}_1$$

where all bonds are assumed to be short term bonds, earning an instantaneous rate of return of $r(t)$. Assume there are two firms which are identical except that one issues no bonds at all, i.e.

$$D_2(t) = X_1(t) - I_1(t) .$$

The total rate of return from owning shares in either company is simply the sum of dividends and capital gains, which we require to be the same for both.

$$\rho = \frac{\dot{E}_1}{E_1} + \frac{D_1}{E_1} = \frac{\dot{V}_1}{E_1} + \frac{X-I}{E_1} - \frac{rB_1}{E_1} = \frac{\dot{V}_2}{V_2} + \frac{X-I}{V_2}$$

or

$$(20) \quad \frac{\dot{V}_1}{V_1} - \frac{\dot{V}_2}{V_2} = -\rho \frac{B_1}{V_1} + \frac{rB_1}{V_1} - \frac{(X-I)}{V_1} + \frac{(X-I)}{V_2} .$$

It is clear that if $\rho = r$, and $V_1 = V_2$ initially, then the two firms have the same value forever. But note that (20) can also be satisfied with $\rho = r$ and $V_1 \neq V_2$, in which case there will be cumulative changes in the ratio of the valuations. Eventually, these will probably lead to one of the firms having an unusually low or unusually high earnings-valuation ratio, and this will probably lead to a revaluation of the firm. But there is no reason (without perfect futures market), as we have argued in detail elsewhere, that this might not go on for a long time.

(v) Finally, we note that the resources "wasted" on financial management may be relatively minor (relative, say, to total profits of the firm) and hence the "competitive forces" to eliminate this inefficiency may operate with relatively little strength.

5. Irrelevance of Financial Policy of Any Particular Firm

The above proposition established the irrelevance of the financial structure of the economy as a whole. The crucial assumption employed was that of no bankruptcy. We can remove this assumption and prove a weaker theorem about the irrelevance of the financial policy of any particular firm.

Theorem 2. Assume there is a general equilibrium solution for the economy, which is characterized by a given market rate of interest (on safe bonds) by a given nominal rate of interest on the risky bonds of each of the firms which faces a chance of bankruptcy, and by each firm having a given market valuation and a financial policy (dividend-retention ratio, maturity structure of debt, etc.) and in which a given fraction of the shares of the firm are owned by the i^{th} individual.

Let any firm (or any group of firms) change its financial policy. If financial intermediaries may be established costlessly, then there exists a new general equilibrium solution for the economy with the same market rate of interest, in which every firm has exactly the same market valuation as before, and in which the proportion of each firm's shares owned by the i^{th} individual, either directly or indirectly through intermediaries, is exactly the same as before.

The argument is simple; since the argument for changes in debt equity ratio is perfectly analogous to changes in other financial policies, we shall focus our remarks on the debt equity ratio. Assume in the initial equilibrium, there were no financial intermediary purchasing bonds and shares of the given firm. The firm changes its debt equity ratio. A financial intermediary is created which reconstitutes the firm, i.e. purchases all of its bonds and shares, then issues bonds and shares in exactly the same ratio as in the original situation. The opportunity set facing the individual is completely unchanged, and hence the market valuations, rates of interest, etc. are completely unchanged.

One might argue however, that the opportunity set has been changed, because in principle the individual can buy bonds and shares in the firm directly as well as through the intermediary. Thus, if there is a probability of bankruptcy, his opportunity set is larger now than it was before. This may lead to an increase in the demand for the securities (bonds and stocks) of the given firm, so that the new situation is not an equilibrium one. There will be a new general equilibrium situation, with the value of bonds and equities of the firm greater than before. But this would imply that the original situation could not have been an equilibrium. For a financial intermediary could have purchased and issued the same fraction of the bonds and stocks of the given firm, thus obtaining a given fraction of the income of the firm in every state of nature, and then issued bonds and shares in the ratio of the debt equity ratio of the "new" situation. The organizers of the intermediary would have then made a pure profit for themselves, equal to the difference between the value of the firm in the two situations.

The point of the argument is the following:

If, corresponding to a given set of real and financial decisions of the other firms, the financial decision of the firm does make a difference, free entry of financial intermediaries will ensure that a set of financial securities will be marketed which maximizes the value of the firm regardless of the debt-equity ratio of the firm.

Assume the original debt equity ratios of the firm is d , and the k^{th} financial intermediary purchases α^k of the bonds and equity of the firm and issues a debt equity ratio of d^k . $1 - \sum \alpha^k$ is the proportion of the firm purchased directly by individuals (not through intermediaries). Now assume the firm changes its debt equity ratio to d' . Then all intermediaries except the one for which $d^k = d'$ are unaffected. It now issues a debt equity ratio of d , and purchases $1 - \sum \alpha^k$ of the bonds and equity of the firm.

If one took the assumption of costless creation of intermediaries seriously, there is no reason to suppose that the process of proliferation of intermediaries would stop short of creating as many securities as states of nature; in which case, not only is the financial structure of any individual firm of no consequence, the financial structure of the economy is irrelevant.¹ The fact that we do not see such a proliferation of financial intermediaries suggests either that they are unnecessary (either Pareto Optimality can be obtained with a limited number of such intermediaries² or that the condition

¹That is, all financial structures that provide as many securities as states of nature are equivalent.

²As in those circumstances in which the portfolio separation theorem is valid. See [2].

of Theorem 1, p. 14 are satisfied so that the financial structure is of no consequence) or that there are significant transactions costs relative to the gains to be had by the creation of such intermediaries.¹

6. Concluding Comments

We have established the irrelevance of financial policy under a fairly general set of conditions. Three classes of limitations were noted to our first theorem: (a) expectations of real returns dependent on firm financial policy; (b) individual borrowing not a perfect substitute for firm borrowing; (c) bankruptcy. The absence of transactions costs was the critical limitation in the second theorem. Whether these limitations are important in practical applications is a moot question. But whether they are or are not, the theoretical importance of the theorem is not diminished: an understanding of it is to corporate finance as an understanding of the frictionless surface is to the understanding of the physics of motion. For some practical problems, friction can be ignored in a first approximation; in others it cannot. But even when it cannot, an understanding of what would happen in the absence of friction is essential. The empirical testing of the model is another matter: in physics we can attempt to approximate a frictionless surface, and observe motions under those conditions; to do the analogous thing here, would require us to create a world without, for instance, transactions costs,

¹Included in "transactions costs" are the cost of obtaining information about different securities. If there is a finite probability of bankruptcy, purchasers of bonds have to evaluate the riskiness of the bonds. Thus not only does bankruptcy result in transactions costs when it occurs, but the potentiality of bankruptcy results in transactions costs at the time the bonds are sold.

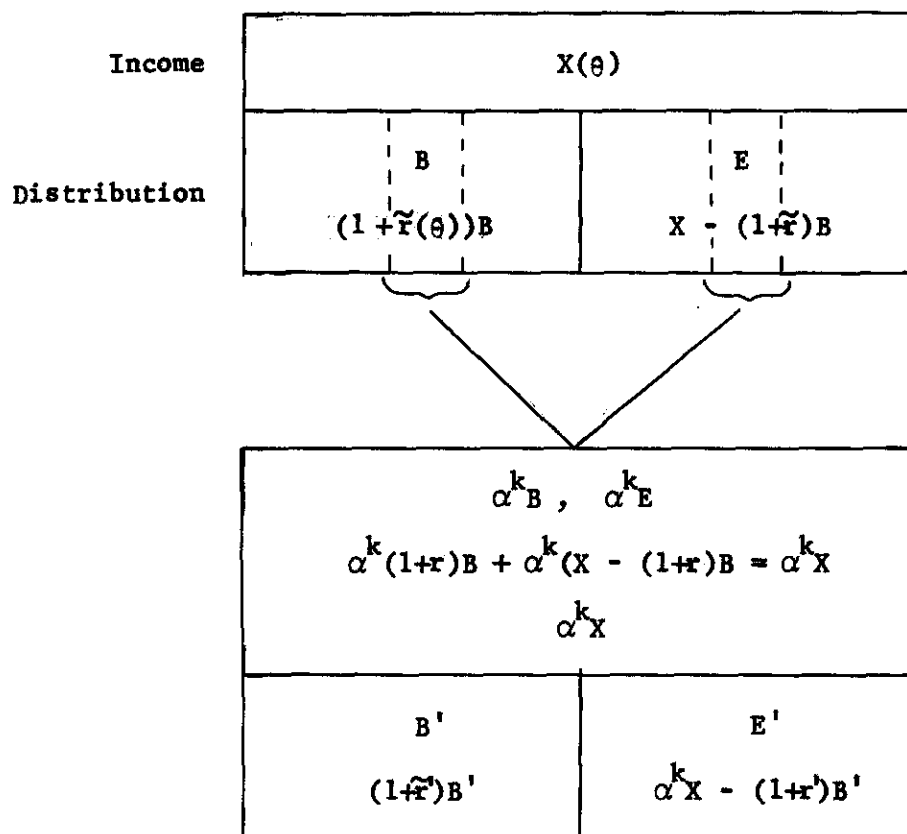


FIGURE 2

Operation of Financial Intermediary

and tax distortions, and see if firms ignored their financial structure. Thus, of course it is essentially impossible to do. Fortunately, the issue is not whether under those circumstances the financial policy would be irrelevant--most of us would agree that it would be--the issue, how significant are the "limitations" and in what way do they affect corporate financial policy. The tests performed so far--such as examining the value (per unit scale) of firms thought to be essentially identical except for their debt equity ratio--do not discriminate between worlds in which "Theorem 1" is valid, those in which "Theorem 2" is valid but not Theorem 1, or worlds in which neither Theorems 1 or 2 are valid (in which financial policy is important, but in which value maximizing firms have selected the set of financial policies which maximize the firm's valuation). What is required is a greater understanding of the implications of these limitations and more refined tests to discriminate among the alternative hypotheses.

REFERENCES

- [1] Baumol, W. and B.G. Malkiel. "The Firm's Optimal Debt-Equity Combination and the Cost of Capital," Quarterly Journal of Economics (November, 1967), pp. 547-578.
- [2] Cass, D. and J.E. Stiglitz. "The Structure of Preferences and Returns and Separability in Portfolio Allocation: A Contribution to the Pure Theory of Mutual Funds," Journal of Economic Theory (September, 1970).
- [3] Diamond, P. "The Role of a Stock Market in a General Equilibrium Model with Technological Uncertainty," American Economic Review (September, 1967), 57, pp. 759-776.
- [4] Kaldor, W. "Marginal Productivity and the Macro-Economic Theories of Distribution," Review of Economic Studies (1966), 33.
- [5] Meyer, J. and E. Kuh. The Investment Decision, An Empirical Study. Cambridge: Harvard University Press, 1957.
- [6] Modigliani, F. and M.H. Miller. "The Cost of Capital, Corporation Finance, and the Theory of Investment," American Economic Review, 48 (June, 1958), pp. 261-297.
- [7] Shell, K. and J.E. Stiglitz. "The Allocation of Investment in a Dynamic Economy," Quarterly Journal of Economics (1967).
- [8] Stiglitz, J.E. "A Re-Examination of the Modigliani-Miller Theorem," American Economic Review (December, 1969).
- [9] _____. "A Consumption Oriented Theory of the Demand for Financial Assets and the Term Structure of Interest Rates," Review of Economic Studies (July, 1970).
- [10] _____. "On Some Aspects of the Pure Theory of Corporate Finance, Bankruptcies and Takeovers," Bell Journal of Economics and Management Science, Fall, 1972.
- [11] _____. "Taxation, Corporate Financial Policy, and the Cost of Capital," Cowles Foundation, mimeo, 1972.