

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

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COWLES FOUNDATION DISCUSSION PAPER NO. 338

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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART V

THE RATE OF INTEREST ON FIAT MONEY IN A CLOSED ECONOMY

Martin Shubik

April 5, 1972

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THE RATE OF INTEREST ON FIAT MONEY IN A CLOSED ECONOMY*

by

Martin Shubik

***This research was supported by the Office of Naval Research. The research was also partially supported by a grant from the Ford Foundation.**

ABSTRACT

In a well controlled monetary economy with no uncertainty and a money market, money is not merely a veil, it is a cocoon. There are no idle cash balances* in a competitive monetary economy with a money market without uncertainty. The presence of uncertainty calls for the holding of cash.

In a trading economy with perfect foresight, without taxes, but with traders having positive time discounts the rate of interest on paper money is positive.

With uncertainty, fiat money is a form of insurance or generalized futures contract.

The mathematical differences among static general equilibrium theory, a theory of money in a static general equilibrium context with perfect foresight and an evolutionary system are essentially the differences among maximization subject to equalities; convex programming and dynamic programming.

*They are needed for current transactions or are loaned.

TABLE OF CONTENTS

Section	Page
1 SOME PROBLEMS IN MODELING.....	1
2 ON THE ROLE OF A MONEY MARKET IN GENERAL EQUILIBRIUM.....	2
2.1. The Barter or Nonmonetary "Trust" Model.....	3
2.2. The Market Model with Trade in Money.....	7
2.3. Steady State Equilibrium with Money but No Money Market.....	10
2.4. The Unending Market.....	15
2.5. Perfect Foresight and a Static Model.....	16
3 ON THE ROLE OF MONEY AND A MONEY MARKET IN GENERAL EQUILIBRIUM.....	17
3.1. Modeling the Money Market: The Bank.....	19
3.2. The Money Supply and the "Float".....	20
3.3. The Rate of Interest and the Government.....	21
3.4. The Limits on Borrowing: The Mortgage Problem.....	22
3.5. Steady State Equilibrium with a Money Market.....	23
3.6. The Unending Market.....	27
3.7. The Issuing of Money and the Volume of Trade.....	27
3.8. A More General Model: Inventories and the Float....	31
3.9. The Next Step.....	34
4 DISCUSSION: MATHEMATICAL INSTITUTIONSL ECONOMICS.....	35
4.1. Information Conditions and Expectations.....	35
4.2. Uncertainty and Other Complications.....	39
4.3. The Market as a Noncooperative Game.....	40
4.4. G.N.P., the Supply of Money and Multiplicity of Equilibria.....	43
4.5. Inflation, Deflation and Monetary Issue.....	45
4.6. Concluding Remarks.....	46
REFERENCES.....	49
APPENDIX.....	50

1. SOME PROBLEMS IN MODELING

In the previous two papers the role of commodity money and fiat money in the running of a market economy has been examined. In both cases our attention was confined to the single period or a steady state economy. No attention was given to the effect of differences in time discounts or in the differences in the pattern of ownership of resources, or the need for a money market. For example, one might have important cyclical patterns in the availability of agricultural crops for sale.

In this paper an attempt is made to isolate yet another aspect of the monetary mechanism. In particular, an economy with a money market is considered. The affects of uncertainty are not studied in this paper. A natural question for us to ask is "at what level of complexity is it necessary to introduce a money market?" It might appear that cyclicity in the availability of different resources might be a sufficient reason for a money market. It might be that uncertainty makes the money market a necessity.

This article concentrates on examining the implications of some simple models of a closed economy with several trading periods and with a changing ownership pattern of resources.

The approach adopted is that of studying the behavior exhibited by a static maximization model and then noting the role for an appropriately constructed set of dynamic programs. A further study calls for the investigation of a noncooperative game model of the market.

The major emphasis is on the construction of a set of relatively simple examples which are then analyzed. No general theorems are proved. However, several conclusions may be drawn and new conjectures are made based upon the analysis of the examples.

2. ON THE ROLE OF A MONEY MARKET IN GENERAL EQUILIBRIUM

Consider a two sided economy of indefinite duration with the traders having a natural time preference and a fluctuating endowment as is noted below:

Traders of type 1 have $A, 0, A, 0, A, 0, \dots$, and
traders of type 2 have $0, A, 0, A, 0, A, \dots$.

Let each trader have the same bounded utility function of the form:

$$(1) \quad U = \sum_{t=1}^{\infty} \rho^{t-1} u(x_t^i)$$

where x_t^i is the amount of the (single) commodity consumed in period t by trader i . It is assumed that the commodity cannot be stored. It appears as "manna" at the start of each period.

We wish to consider an infinite decentralized market run by a price system. Suppose that prices

$$p_1, p_2, p_3, \dots, p_t, \dots$$

are announced. The worth of the endowments of a trader of each type can

be evaluated. Those of type 1 have a worth of $A/1 - \rho^2$ and type 2 of $\rho A/1 - \rho^2$ "at market." It can be seen immediately that at the announced prices, each period the traders will share the real resource in the proportions of $A/1+\rho$ to $\rho A/1+\rho$. The final total utility for each is given by:

$$(2) \quad U_1 = \frac{\varphi\left(\frac{A}{1+\rho}\right)}{1-\rho} \quad \text{and} \quad U_2 = \frac{\varphi\left(\frac{\rho A}{1+\rho}\right)}{1-\rho}$$

on the assumption that φ is convex.

2.1. The Barter or Nonmonetary "Trust" Model

In the above description, although prices were specified no role has been assigned to a monetary mechanism. In particular if there were only two traders we could conceive of a "trust" or credit mode of operation as shown in Figure 1. Here during the first period the trader of the first

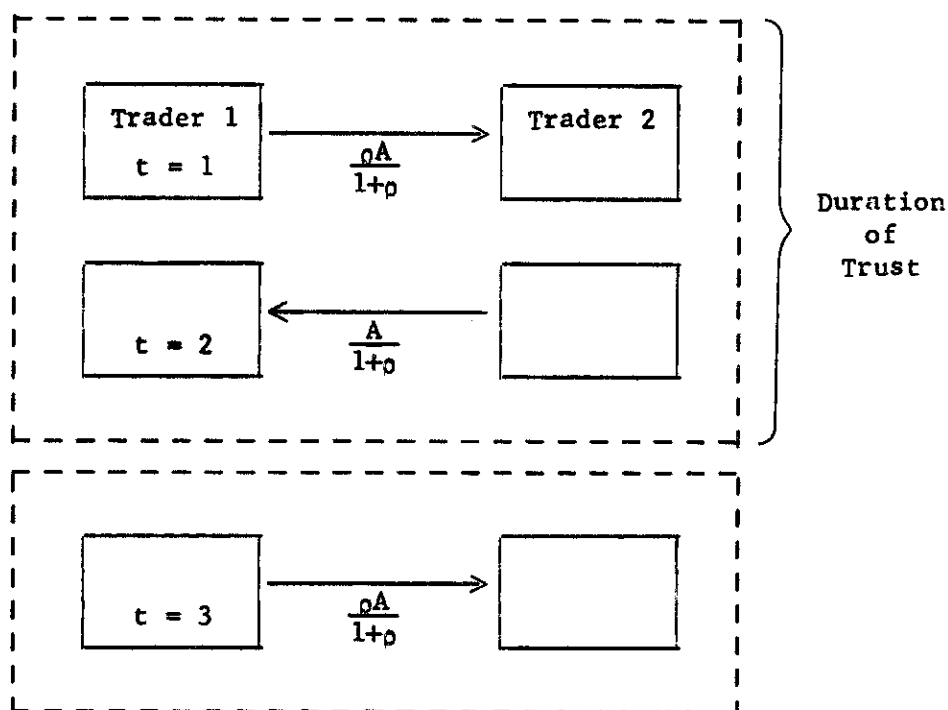


FIGURE 1

type gives the other $\rho A/(1+\rho)$ units. During the second period the trader of the second type gives the first $A/(1+\rho)$ units and so forth. Each iteration involves the extension of credit or trust for the duration of a period, as is indicated by the one way flow of resources in Figure 1.*

Confining ourselves to the simple two-period cycle example described above we may examine the competitive equilibrium diagrammatically by looking at a two-period trade as though it were a bilateral monopoly exchange in two goods where the endowments are $(A, 0)$ and $(0, A)$ respectively and the utility functions are:

$$(3) \quad U_1 = \varphi(x) + \rho\varphi(y) \quad \text{and} \quad U_2 = \varphi(A-x) + \rho\varphi(A-y) .$$

In Figure 2 the Pareto optimal surface goes from B_1 to B_2 . The contract curve is C_1C_2 where TC_1 and TC_2 are given by:

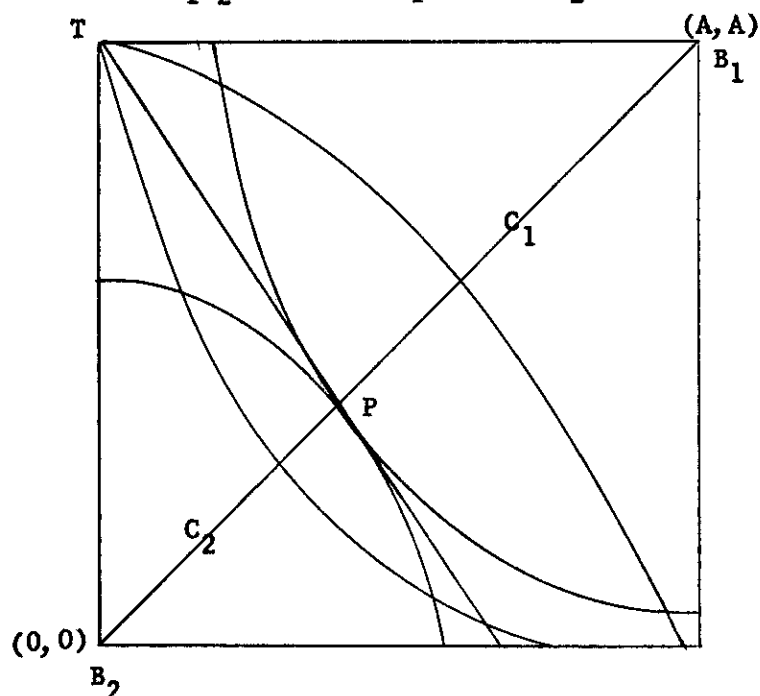


FIGURE 2

*This might be called a "quasi-cooperative solution." Cooperation for a time, followed by recontracting.

$$(4) \quad \varphi(x) + \rho\varphi(y) = \varphi(A) \quad \text{and} \quad \varphi(x) + \rho\varphi(y) = \rho\varphi(A) .$$

The general equilibrium price or the efficient price (or the competitive market price) is shown as P , and is determined from solving the constrained maximization of the utility functions in (3). In this simple example it is easy to observe that we may go directly from the two-stage market to the infinite stage by considering a series of two-stage markets. This immediately gives (2).

Although the process described above might describe trade for two or few, it is hard to conceive of a market involving thousands or millions of traders, even with the aid of high speed computers carrying out a large volume of trade on pure trust. The nature of the system would call for as anonymous and detail-free an operation as possible.

A system that offers considerable possibilities in facilitating trade is one that uses fiat money as a transferable symbol of trust. This enables strangers or virtually anonymous parties to deal with each other with a minimal need for information especially of the variety called for to establish credit.

On Markets

Before we discuss a market with fiat money, several distinctions must be made. They involve the modeling of economic processes. We must distinguish among

A "spot" or true barter market

Barter with limited credit or trust

A nonmonetary price system without futures trading

A nonmonetary price system with futures trading.

In Figure 1, barter with limited trust was illustrated. It was discussed and presented as an evolving system. The only prices that mattered were between any two periods $2t-1$ and $2t$. The "books were balanced" every two periods and in that sense the system decouples.

In a true barter market the books must balance at every trade. A price system without futures trading suggests a mass market but no trust. It is suggestive of commodity money trading in gold or bars of salt...etc, where the supply of the commodity money just supports its "eating price".*

A nonmonetary price system with futures trading implies a highly sophisticated mass economy with well developed institutions, trust and accounting. This is a far cry from a barter economy. It really is close to "an accounting money" economy. The specification of the rules of trade with the paper generated by the writing of futures contracts is critical.

On Future Prices

In our investigation it is important to distinguish among:

- a given future price without a futures market;
- a given future price with a futures market;
- an expected futures price;

and no prices.

The first two are relevant to models with perfect foresight. The third is relevant to an evolving or truly dynamic model in which individuals are regarded as being constrained to act as price-takers. And the last

*See M. Shubik, Part III.

is relevant to the market considered as a quasi-cooperative game (as shown in Figure 1).

In the last instance if there were only two individuals trading through time, they might have expectations concerning each other's strategies or bargaining power, but it is hardly useful to call such expectations "price expectations." If there were many individuals in the market then the core [1] defined appropriately for the repeated two period game might provide a rationale to discuss the emergence of a current price and future prices.

2.2. The Market Model with Trade in Money

Rather than consider the face-to-face trade described above, the model is modified to require that trade is carried out in markets using money. As a first approximation, all trade is monetized. This amounts to assuming that all goods are deposited at a central market from which they are sold for money. Their owners must buy back any supplies they need. After or before trade (which we may assume clears the market) the owners obtain as income their shares of the receipts.

The introduction of money trading does two things to the economic system immediately:

- (1) It introduces an externality in the form of the market in which each individual must trade.
- (2) It creates a cash flow and timing problem and a float.

A problem in understanding the effect of money is to describe the conditions under which a market economy trading in money provides the same allocation of real goods as does a nonmonetary price system.

There are many fine details involved in setting up a monetary system and many further details if a money market which permits futures contracts in money, i.e., lending and borrowing, is to be considered. Several of these details do not appear to be important if the economy is regarded as an automatically functioning price system. They become of critical importance if the economy is viewed as a noncooperative game. In particular rules and limits on lending and the bankruptcy laws play a critical role in the latter instance. These are to be discussed in a subsequent investigation of the limiting properties of a dynamic n -person noncooperative game.

Confining our attention to the functioning of money and the need for a money market in a price system economy which uses money we discuss the conditions needed to make such a system function.

In particular we begin by introducing money, but excluding a money market. Our assumptions are that all individuals must trade in money and use an anonymous market. All transactions are in cash. It is necessary to specify the sequence of trade and the cash flow in detail. Obviously in an actual economy there will be many variants of trading each of which may imply a slight difference in transactions needs and variations in trust and custom. One simple process is as illustrated in Figure 3. Specifically it is assumed that all goods are initially "sent to the market." Traders of both types then buy from the market, paying cash. This is shown by the pairs of lines each period representing goods and money flows in each direction.

At the end of each period (after the goods have been sold for cash in transactions that are regarded as virtually simultaneous), the market

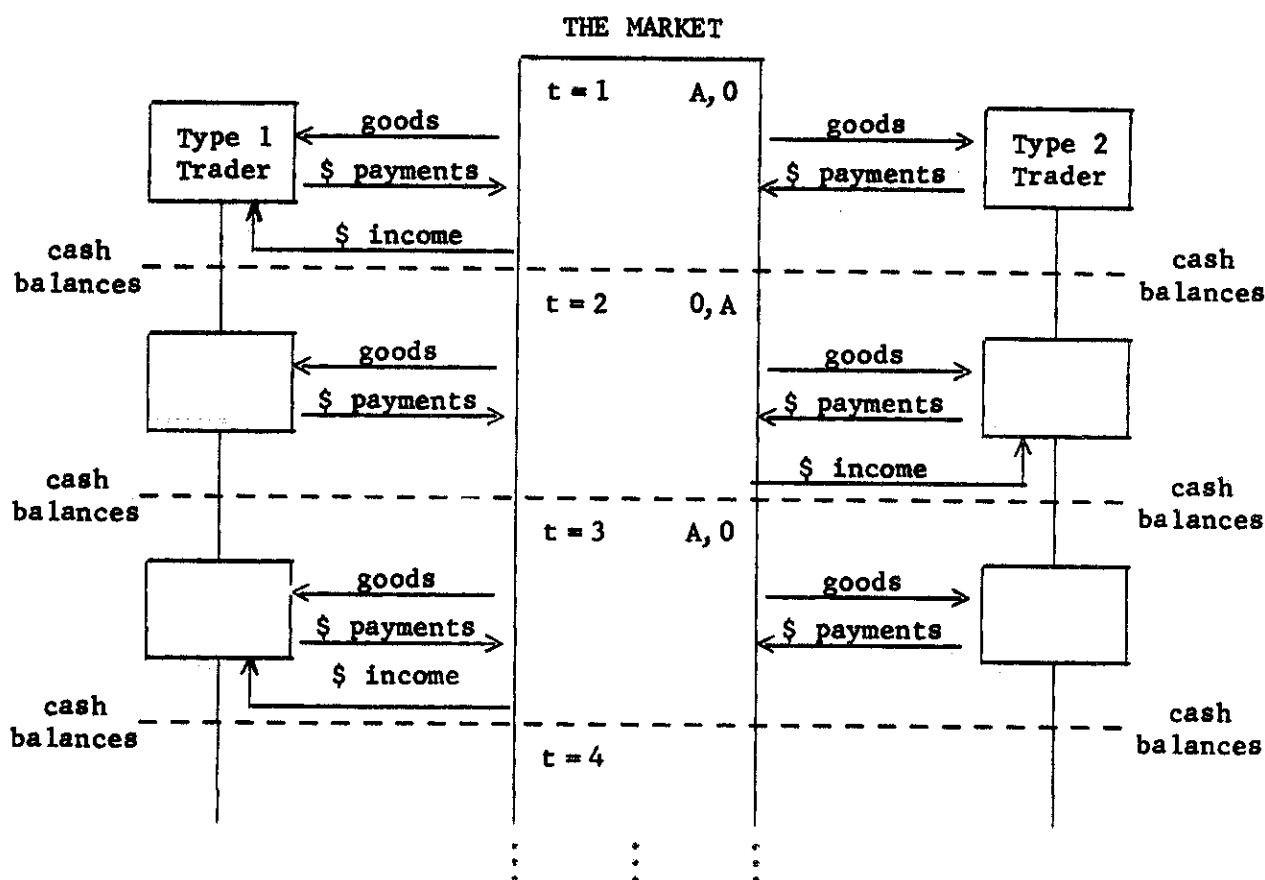


FIGURE 3

settles the ownership accounts by remitting incomes in cash to the owners of the goods. In this simple model, only cash or specie is considered. There are no checks or other financial paper in the system.

The structure given in Figure 3, implies that an individual needs to hold cash to buy his own goods. This rule conforms with some marketing practices where the goods are consigned and the owner paid after their sale. Here where the price system is being examined without money markets we make this assumption. We must also specify how money is issued initially. In

the study of the money market and banking great care must be taken in specifying the mechanism of new issue and cash flow to avoid the accidental creation of loanable funds.

A comparison of Figures 1, 2, and 3 shows the importance of money and the markets as a systems interlinkage device or an externality. In Figures 1 and 2 one can decouple the system into two stage segments. In Figure 3 money and the markets form an infinitely linked system. Such a linkage could be forged by real durable assets, but the point here is that money suffices to provide this linkage.

2.3. Steady State Equilibrium with Money but no Money Market

If there is no market for borrowing or lending money, but trades must be carried out in money then each trader must hold idle cash balances to "tide him over" periods when he has little or no income. Reverting to our investigation of the simple model given at the start of Section 2 we observe that there are many solutions which give the same real allocation as the nonmonetary model. They are however, intuitively unsatisfactory and involve increasing hordes of money as the price system deflates.

For simplicity we consider a two stage market with utility functions as in (3). Payments must be made in money. The government issues amounts M and N to traders of the first and second type respectively. The money must be returned to the issuing agency at the end of the second period. This repayment is a device for looking at the market of finite length. Further discussion of the use of this device is given in the investigation of the market of indeterminate length in 2.4.

Suppose that amounts of trading chips or money M and N are given to the traders at the start of the first period. They must be returned at the end of the second period.

Suppose that the prices in the first and second period are p_1 and p_2 respectively. Let the purchases of a trader of the first type be x_1 and x_2 , and of a trader of the second type y_1 and y_2 . The following constraints on trade hold:

$$(5) \quad M_0 = M \qquad N_0 = N \qquad (\text{Initial Cash Balances})$$

$$(6) \quad 0 \leq x_1 \leq M_0/p_1 \qquad 0 \leq y_1 \leq N_0/p_1 \qquad (\text{Budget Constraint})$$

$$(7) \quad M_1 = M_0 - p_1 x_1 + p_1 A \qquad N_1 = N_0 - p_1 y_1 \qquad (\text{Cash Balances})$$

$$(8) \quad 0 \leq x_2 \leq M_1/p_2 \qquad 0 \leq y_2 \leq N_1/p_2 \qquad (\text{Budget Constraint})$$

$$(9) \quad M_2 = M_1 - p_2 x_2 \qquad N_2 = N_1 - p_2 y_2 + p_2 A \qquad (\text{Cash Balances})$$

$$(10) \quad M_2 \geq M \qquad N_2 \geq N \qquad (\text{Closing the books})$$

We can replace (10) by equalities. This enables us to express x_2 and y_2 in terms of x_1 and y_1 thus the maximization problem faced by the traders can be expressed as:

$$(11) \quad \text{Max}_{x_1} \left\{ \varphi(x_1) + \rho \left(\frac{p_1}{p_2} (A - x_1) \right) \right\}$$

and

$$(12) \quad \text{Max}_{y_1} \left\{ \varphi(y_1) + \rho \left(A - \frac{p_1 y_1}{p_2} \right) \right\}$$

where conditions (5) and (10) must be satisfied.

The market conditions call for equations (13) to be satisfied.

$$(13) \quad x_1 + y_1 = A \quad \text{and} \quad x_2 + y_2 = A \quad (\text{market balance})$$

In this one commodity case the values for x_1 , x_2 , y_1 and y_2 can be determined and are:

$$(14) \quad x_1 = \frac{A}{1+p} \quad , \quad y_1 = \frac{p_2 A}{p_1(1+p)} \quad , \quad x_2 = \frac{p_1}{p_2} \left(\frac{\rho A}{1+p} \right) \quad \text{and} \quad y_2 = \frac{\rho A}{1+p} \quad .$$

From (7) and (14) we obtain:

$$(15) \quad p_2 = \rho p_1 \quad .$$

From (6) and (9) we obtain

$$(16) \quad p_2 \leq \frac{N(1+p)}{A} \quad .$$

Figure 4 shows conditions (15) and (16). This shows that there will always be a sufficiently low price level indicated by the sector OB such that the nonmonetary prices will not be interfered with by the amounts of fiat money distributed for trading purposes. This result is quite general for trade in many commodities and for any finite number of time periods. It merely utilizes the property that the price system is homogeneous of order zero.

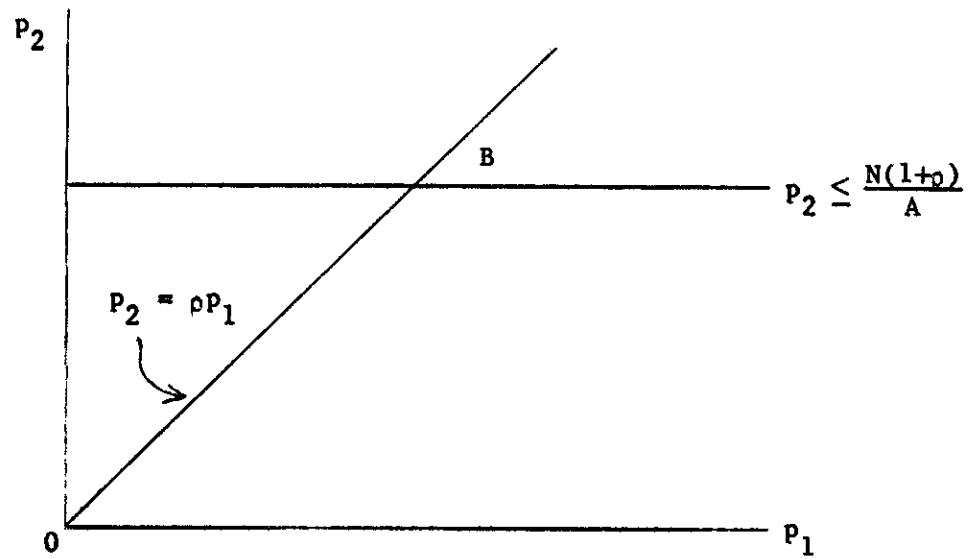


FIGURE 4

Money in the above situation is a veil, but an extremely loosely fitting veil because if the announced prices are on OB but not at B then everyone will hold inactive balances all of the time until they are returned at the end of the last period.

If instead of considering two time periods we had selected $2T$ periods the existence of a set of prices compatible with the nonmonetary model amounts to asking if the following conditions can be satisfied:

Do there exist prices $p_1, p_2, p_3, \dots, p_{2T}$ such that:

$$(17) \quad v_1 = \max_{x_t} \sum_{t=1}^{2T} \rho^{t-1} \phi(x_t) \quad \text{subject to conditions (19) to (22)}$$

and

$$(18) \quad v_2 = \max_{y_t} \sum_{t=1}^{2T} \rho^{t-1} \phi(y_t) \quad \text{subject to conditions (19) to (22)}$$

can be simultaneously satisfied together with the market conditions (23).

The conditions on the traders are:

	<u>For Trader 1</u>	<u>For Trader 2</u>	
(19)	$M_0 = M$	$N_0 = N$	(Initial Cash Balances)
(20)	$0 \leq x_t \leq M_{t-1}/p_t$	$y_t \leq N_{t-1}/p_t$	(Budget Constraint)
	$M_t = M_{t-1} - p_t x_t$	for t even $N_t = N_{t-1} + p_t(A - y_t)$	
(21)			(Cash Balances)
	$M_t = M_{t-1} + p_t(A - x_t)$	for t odd $N_t = N_{t-1} - p_t y_t$	
(22)	$M_{2T} = M$	and $N_{2T} = N$	(Closing the Books)

The market balance conditions are given by:

$$(23) \quad x_t + y_t = A \quad \text{for } t = 1, 2, \dots, 2T.$$

The answer is yes if $M > 0$, $N > 0$ and the price system is:

$$(24) \quad p_t = \rho p_{t-1} \quad \text{for } t \geq 2,$$

if p_1 is chosen to be small enough. As is shown in Figure 4, given that (24) is satisfied any sufficiently small p_1 will provide an adequate price system.

For any finite horizon we note that there is monetary price system which produces the steady state results of the barter system of 2.1. But this system, if viewed dynamically is not in a steady state itself. The price level is continuously shrinking and unused "chips" are piling up in the traders accounts to be returned at the end of the sequence of markets.

The initial distribution of money only influences the range of selection of the first price.

2.4. The Unending Market

If we assume that the system described by (17) to (23) runs indefinitely this in effect eliminates the conditions (22) from the constraints on the traders thereby adding an extra degree of freedom to each.

We observe that the solution given by (24) holds (with a two period steady state flow of real goods as described by (14)) for any finite market. The need to return the chips at the end of the market forces individuals to keep them.

When the system is unending there does not appear to be a constraint that forces the holding of the extra chips in the deflating economy (with price level at p^{t-1} by period t). The equilibrium is destroyed. The accounts are thrown out of balance as the money need never be returned.

One formal device for preserving the equilibrium with the exponentially falling prices is to remove the excess money exponentially at the same rate from the system.* This can be looked upon as a tax of ρ on money holdings each period, or a negative interest rate on idle balances much in the spirit of Silvio Gessel.

Although we may thereby save the equilibrium for the infinite market, this device does not appear to have much intuitive or socio-psychological appeal. Are there other devices which appear to be more reasonable?

*This calls for the replacing of equations (21) by equations involving ρ on the right hand side.

Before we consider other possibilities, the conclusions from the investigation of this simple model are summarized.

(1) A price system exists for any finite length economy using fiat money which must be returned at the end. The initial distribution may cause an upper bound to be placed upon p_1 but has no other effect.

(2) No steady state price system exists, even though the amount of real resources are the same each period.

(3) No equilibrium exists for the infinite length economy, unless the excess money can be removed from the system in an appropriate manner.

2.5. Perfect Foresight and a Static Model

The maximization problem studied in 2.3 is completely static. The dynamic aspects of it can be, at best, described as "conversational." The system does not evolve as can be seen from the peculiar information conditions which may be described as "perfect foresight." These amount to stating that all future prices are known in advance. No mechanism for generating price expectations in an evolutionary manner is given in the system. This point is discussed further in 4.1.

The individual is presented with an opportunity to trade in a finite (or infinite) set of interlinked markets. He is confronted with a maximization problem with perfect information and some constraints linking one market to another which must be satisfied sequentially.

3. ON THE ROLE OF MONEY AND A MONEY MARKET IN GENERAL EQUILIBRIUM

In 2.3 it was seen that a money market for borrowing and lending is not necessary for the functioning of a finite length monetary economy with certainty. However, the system appeared to require considerable amounts of idle balances and a specific time linked ratio in prices.

We may introduce a money market into the same physical economy to see if it would be used and how the outcome differs from the market without borrowing and lending. Figure 5 which is a modification of Figure 3 shows the flow diagram for our simple example.

Several features of Figure 5 must be noted. Because there is the possibility of borrowing it may no longer be necessary that all traders start with positive amounts of money.

In contrast with Figure 3 the lines indicating the forwarding of cash balances have been left out. This was only for convenience. They should be in. However, if the interest rate were zero then the forwarding of cash balances in this model is the equivalent of lending 1-period money at zero interest.

Given that the money market as well as the goods market is two-sided we have not drawn the borrowing and lending conditions in full generality. A priori one cannot state who will be a lender and who will be a borrower at any time period.

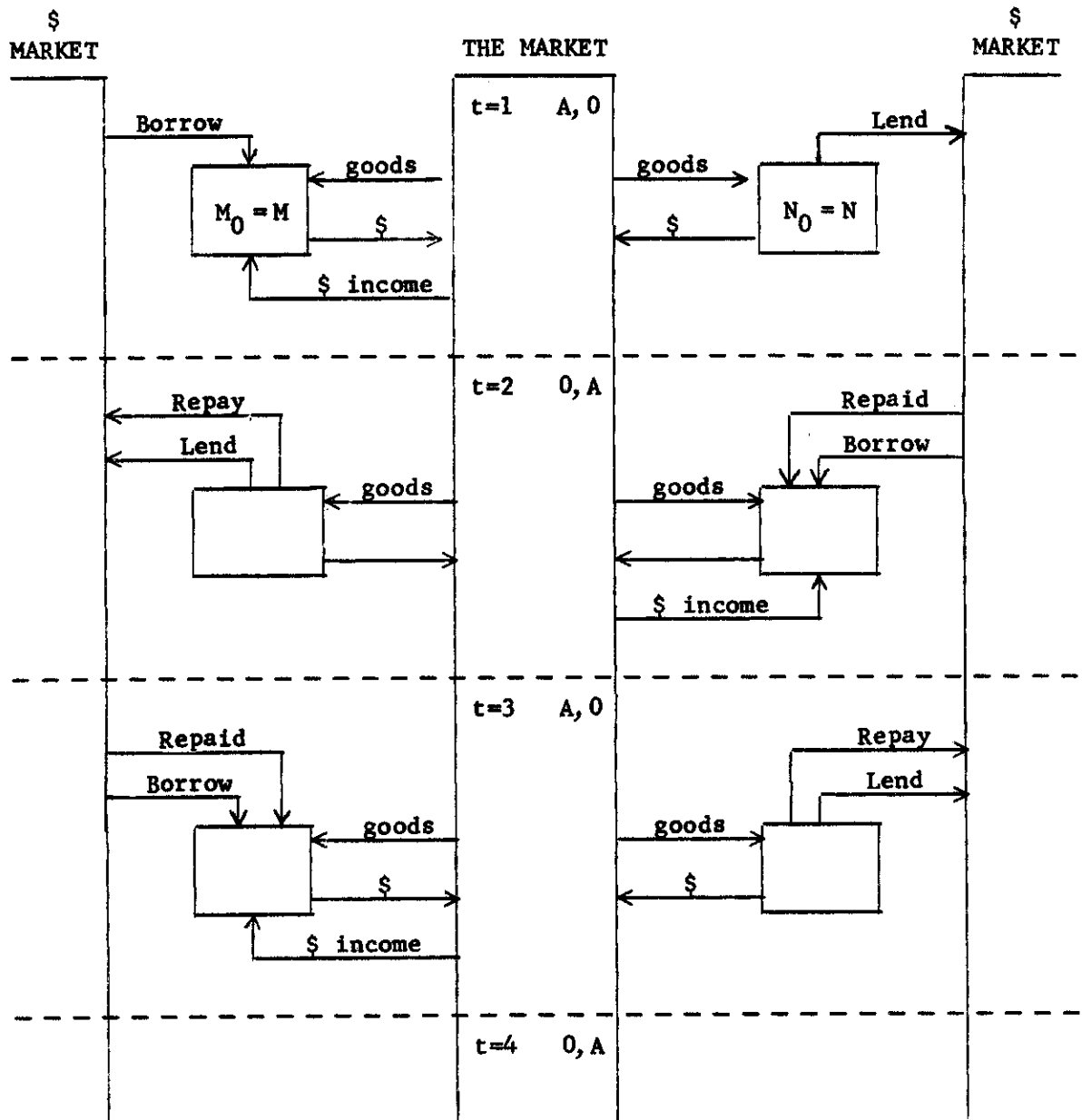


FIGURE 5

3.1. Modeling the Money Market: The Bank

Figure 5 implies a sequencing of operations which is not necessarily sacrosanct, but with money markets it is important to be specific. Undoubtedly the timing of loan settlements, borrowing, purchasing and receipt of income all effect the functioning of a monetary system. Different combinations will call for different cash requirements.

For specificity in this model let us assume that there is a short term money market where loans can be repaid or renewed. The sequence of trade that is analyzed is as follows:

There is a settlement or renewal of outstanding loans.

Further borrowing and lending takes place. Goods are purchased. The market pays an owner for this period's income.

The renewal of loans permits a one-period "roll-over"* of indebtedness at possibly different rates. The importance of a loan period limit comes in immediately in the market of indefinite length. Loans must be appropriately bounded even in a finite market.

In order to define the system completely the nature of the money market must be specified. For clarity an extremely simple "banking system" is suggested. There is a neutral mechanical banking system which stands prepared to act as a broker for lenders and borrowers at no cost. It "brokers" money at the same rate in either direction.

The bank is assumed to be "conservative" in the sense that it is only a broker or a market. Net loans and deposits each period must be zero.

*A debtor who does not actually pay may exchange one short term debt for another.

3.2. The Money Supply and the "Float"

In the system investigated in Section 2 it was necessary to set the money supply exogenously. This bounded (but did not fix) the price level. In this model it is still necessary to set the amount of money. Furthermore, even in this extremely simple model, care must be taken to distinguish between loans made in accounting money or in specie lest "money" be accidentally created by the loan procedure.

The creation of extra loanable funds can come about by using the "float." The float in this instance is a measure of the decoupling or temporary disequilibrium in the system. It is a measure of how much money is "in transit" through the market.

Suppose that the market were to pay the owner of goods for sale at the start of the period, before it had sold the goods. This income is available to the owner of the goods to lend and obtain interest during the period. The market is then financing the float and needs an inventory of money to cover the time between its payment to the owner and its receipts from the purchaser.

If we forced all trades through the market but paid the owner of goods at the end of trade we would then force him to borrow to finance purchases of his own goods. This penalizes the owner of the goods.

A way in which we can specify how currency is to be issued is to imagine the following arrangement. Initially the traders are provided amounts at random. These may be regarded as payments for previous goods and services. Alternatively we may require that these sums must be paid back at the end of trade hence they are merely an interest free loan to finance

the float. The government may also issue an amount of specie or "blue chips" to the market (the market might be a government supervised institution, such as those which exist in many countries). The government can use the market as a means for distributing further amounts of money and thereby enables the market to costlessly finance the float. At the end of the first period the market is able to pay the owners of resources their incomes in full.

We may regard all financial activities as taking place using specie. At every transaction including lending and borrowing payments are made in the trading chips. Deposits and loans are nontransferable contracts that are binding between the bank and the individual. In other words, beyond the specie no other financial paper can be monetized. In this system the size of the money supply is well defined. One counts the number of units of specie held by the traders and unissued but authorized for issue held in the bank.

3.3. The Rate of Interest and the Government

This system, unlike that described in Section 2 has a money market hence a new series of prices r_1, r_2, r_3, \dots , the price of money at any period, must be determined. By adding the money market the freedom of the system has been increased. Given a specified amount of money are prices and the rate of interest determined or has the freedom of the system been increased to the extent that more variables can be specified exogenously?

Who sets the interest rate? Do we regard it as a control variable of the government in an evolutionary system or does the inner logic of the system force its choice? At this point we limit our attention to the static

"perfect foresight" model where the series of prices p_1, p_2, \dots and interest rates r_1, r_2, \dots are assumed to be given.

3.4. The Limits on Borrowing: The Mortgage Problem

Consider a market lasting $2T$ periods. What are the limits (if any) that must be placed upon the size of borrowing at any period of time? Making use of perfect foresight a banker can always calculate the future income of any individual and hence the limit on his ability to pay back any loan.

Lloyd Shapley has observed that this is a well known problem and is equivalent to calculating the largest mortgage than an individual can service.

Suppose that at time $t = \tau$ an individual applies for a loan of size B_τ . The largest amount that he can borrow and successfully pay back is given by basing our calculations on the assumption that all future income is devoted to servicing the loan.

Consider the second trader with endowment of $0, A, 0, A, \dots$. Let B_τ be the amount of a loan borrowed at time τ and let $B_{\tau, \tau+k}$ be the amount outstanding k periods later, after the payment of that period.

Suppose that τ is odd then:

$$(25) \quad B_\tau = B_{\tau, \tau}$$

$$(26) \quad B_{\tau, \tau+1} = (1 + r_\tau)B_\tau - p_{\tau+1}A$$

$$(27) \quad B_{\tau, \tau+2k+1} = (1 + r_{\tau+2k-1})(1 + r_{\tau+2k})B_{\tau, \tau+2k-1} - p_{\tau+2k+1}A$$

and

$$(28) \quad B_{\tau, 2T} = 0.$$

The last condition indicates that the loan is completely liquidated by the last payment. In the infinite case this condition is removed. Similar expressions are obtained for τ even.

Letting D_{τ} be the amount borrowed by a trader of the first type, expressions similar to (25) to (28) are obtained.

In an evolving system without perfect foresight future income is only an expectation hence bankruptcy and other lending laws must be specified.*

3.5. Steady State Equilibrium with a Money Market

The existence of the money market changes the boundary conditions specified in (19) to (23). Suppose that the market is to run for $2T$ periods. The new maximization problem may be expressed as:

Do there exist prices p_1, p_2, \dots, p_{2T} and interest rates r_1, r_2, \dots, r_{2T} such that:

$$(29) \quad V_1 = \max_{t=1}^{2T} \sum_{t=1}^{2T} \rho^{t-1} u(x_t) \quad \text{subject to conditions (31) to (38)}$$

and

$$(30) \quad V_2 = \max_{t=1}^{2T} \sum_{t=1}^{2T} \rho^{t-1} u(y_t) \quad \text{subject to conditions (31) to (38)}$$

*This point is fully developed in Part VI.

which can be simultaneously satisfied together with the market and financial market conditions (39) and (40).

Let w_t and v_t be the amounts borrowed or loaned by traders of Type 1 and 2 respectively during the financial move in period t . If they are negative this signifies a loan, positive implies borrowing.

We assume that initially the traders have M and N units of money which must be returned at the end of the markets.

The conditions on the traders are:

<u>For Trader 1</u>	<u>For Trader 2</u>	
(31) $M_0 = M$	$N_0 = N$	(Cash balance)
(32) $-M_{t-1} \leq w_t \leq D_t$	$-N_{t-1} \leq v_t \leq B_t$	(Loan limits)
(33) $M'_{t-1} = M_{t-1} - w_{t-1} + w_t - r_{t-1}w_{t-1}$		(Post money)
	$N'_{t-1} = N_{t-1} - v_{t-1} + v_t - r_{t-1}v_{t-1}$	(market balances)
(34) $0 \leq x_t \leq M'_{t-1}/p_t$	$0 \leq y_t \leq N'_{t-1}/p_t$	(Budget constraint) (on purchases)
(35) $M_t = M'_t - p_t x_t$ for t even	$N_t = N'_t - p_t y_t + p_{t+1}A$	
		(Cash balances)
$M_t = M'_{t-1} - p_t x_t + p_{t+1}A$ for t odd	$N_t = N'_t - p_t y_t$	
for $1 \leq t \leq 2T - 1$.		

The market ends as follows: Trading closes at the end of period $2T$.

The money market remains open during the start of period $2T + 1$ for the purpose of settling debts and closing the books.

$$(36) \quad M_{2T} = M'_{2T-1} - p_t x_t \quad N_{2T} = N'_t - p_t y_t$$

$$(37) \quad M'_{2T} = M_{2T} - (1+r_{2T})w_{2T} \quad N'_{2T} = N_{2T} - (1+r_{2T})v_{2T} \quad (\text{Final settlement})$$

$$(38) \quad M'_{2T} = M_0 \quad N'_{2T} = N_0 \quad (\text{Closing the books})$$

The goods market conditions are given by:

$$(39) \quad x_t + y_t = A \quad \text{for } t = 1, 2, \dots, 2T.$$

The money market conditions are given by:

$$(40) \quad w_t + v_t = 0 \quad \text{for } t = 1, 2, \dots, 2T.$$

This system has more degrees of freedom than the monetary economy without a money market hence we may suspect more rather than fewer solutions. In particular this system should contain a solution related to the more constrained system of 2.3. In the context of this system that amounts to: given that $M > 0$ and $N > 0$

$$P_t = \phi P_{t-1}$$

r_t is undefined and no borrowing takes place.

If say $N = 0$ then there are two solutions. The first is given by:

$$(41) \quad P_t = \phi P_{t-1} \quad \text{and} \quad r_t = 0 \quad \text{for } t = 1, 2, 3, \dots, 2T.$$

A zero rate of interest is consistent with the borrowing and lending of $p_t \phi A / (1+\phi)$ each time period. We see that in this instance the amount of borrowing decreases at the rate of ϕ per period.

Idle balances grow throughout the process, but in the finite market they do not disturb the equilibrium as the balances are being kept to be returned at the end when the books must balance.

We may check for another solution. Is there a steady state solution such that not only is the distribution of physical goods in steady state but prices and the rate of interest are also in a steady state?

It is relatively simple to check that the following:

$$(42) \quad p_t = p_{t-1} \quad \text{and} \quad r_{t-1} = (1-\phi)/\phi \quad \text{for } t = 2, 3, \dots, 2T$$

$$(43) \quad x_t = \frac{A}{1+\phi} \quad \text{and} \quad y_t = \frac{\phi A}{1+\phi}$$

and

$$(44) \quad w_t = -v_t = \frac{p_1 \phi A}{1+\phi} \quad \text{for } t \text{ odd} \\ = 0 \quad \text{for } t \text{ even}$$

where

$$(45) \quad p_1 = M/A .$$

Both solutions are indicated in Figure 6.

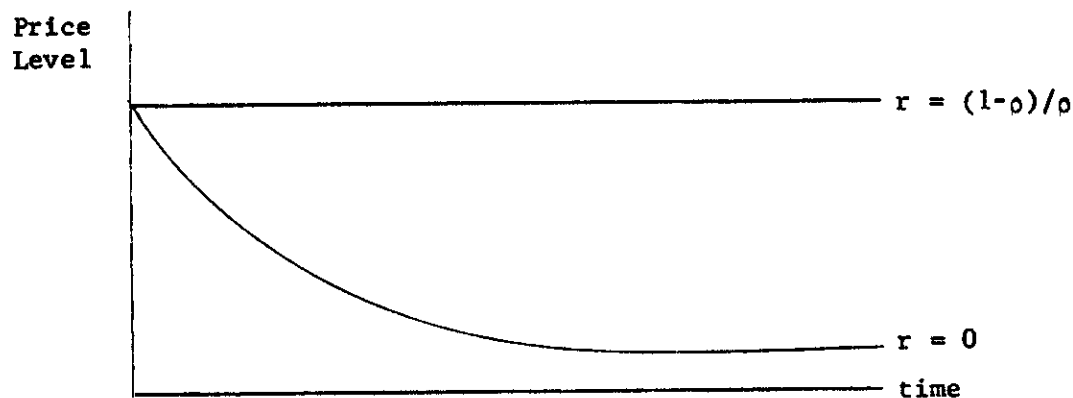


FIGURE 6

At first glance it might appear that all intervening interest rates in the range $0 < r < (1-\rho)/\rho$ will have associated (deflationary) price systems that will provide a solution to the market. If we have fixed the amount of money in the whole system this is not the case. For any intermediate rate the lenders are always willing to lend all of their idle balances, however the borrowers do not need to borrow this amount hence the money market cannot be in equilibrium.

3.6. The Unending Market

When the rate of interest is less than $(1-\rho)/\rho$ the deflation removes the constraints of (36) to (38) and as was encountered in 2.4 the equilibrium is destroyed.

When $r = (1-\rho)/\rho$ all of the money is active each period and the equilibrium is maintained.

For the market of indefinite duration without uncertainty it appears more accurate to describe money as a cocoon rather than as a veil. It is never idle or hoarded but is always actively used for current purchases or is on loan.

3.7. The Issuing of Money and the Volume of Trade

In Sections 2 and 3 an implicit assumption about the issue of money has been made. We have assumed that a certain amount of specie (or bills, or chips) is introduced into the economy at the start. The method of introduction is either by handing them out to traders on the conditions they are paid back after a finite time or taxed away. Or the money could be issued in payment for goods at the first period via the market mechanism.

One way in which money may be set in circulation is by paying government employees with newly printed bills.

There is the possibility that the amount of money in circulation may vary with government policy each period, i.e., it can be regarded as a control variable.

In the previous example a fluctuation in ownership of resources was introduced in order to construct the simplest model in which trust or money

is needed. Each period, however in this one commodity world the amount of commodity available to be traded is the same. Furthermore inventories were ruled out. Thus a constant amount of money is sufficient to support a constant price level. What happens if the quantity of real resources fluctuates from period to period? The following simple example serves to illustrate the problem. Suppose that the endowments of a trader of type 1 and type 2 are respectively:

$$A, 0, B, 0, A, 0, B, 0, \dots$$

and

$$0, A, 0, B, 0, A, 0, B, \dots$$

It can be seen that for a fixed amount of money the prices must fluctuate. Indeed, if no money is hoarded then prices must alternate every two periods from M/A to M/B . However, if $\rho^2 B > A$ for any set of $r_t \geq 0$ the system cannot be in equilibrium.

Given $\rho^2 B > A$ it will pay the traders during periods $4t - 3$ and $4t - 2$ to carry money forward to the periods of "cheap prices" in $4t - 1$ and $4t$. A zero rate of interest in the first two periods is sufficient to have money carried forward. A positive rate merely aggravates the process.

Either the amount of money in the system must be sufficiently increased during periods $4t - 1$ and $4t$ or we must consider negative rates of interest.

If negative rates of interest are to be considered several definitional problems must be overcome. Specifically we must distinguish the treatments of lenders and borrowers, the treatment of money in the money market and

money hoarded or held in idle balances and the conditions on loan limitations, and the rate of velocity.

In this model velocity is assumed to be less than or equal to 1.* Velocity is less than 1 if some traders hoard money; where by hoarding we mean that the money is neither used for the purchase of goods, nor is it loaned during the period.

If we wish to discourage the carrying forward of money it is not sufficient to have a negative rate on lending, it must also apply to hoarding. Furthermore, borrowers cannot be presented with a bonus in the form of a positive incentive to unbounded borrowing.

In order to patch up the negative rate of interest it is no longer recognizable as what we choose to call "an interest rate," it is a tax. Thus the interest rate as we know it is not sufficient to preserve an equilibrium with a fixed money supply. An alternative approach is to vary the money supply.

If the supervising of the mass markets in a monetary economy where millions of individuals trade using symbols of trust is a role of government, it has several quite natural control variables at its disposal.

In a monetary economy with a neutral government, it has as control variables:

- (1) The rate of interest
- (2) Taxes and Subsidies
- (3) Other methods of issuing new currency
- (4) Partial reserve banking control.

*A discussion of why this is a reasonable first approximation in general has been given elsewhere [2].

An economically neutral government in this system may also control one price, say the wages of the most prevalent type of civil servant. This both pegs the price system and may even provide an extra way for circulating new currency.

Taxation by an economically neutral government can be used to mop up potentially idle or disequilibrating balances. Thus in the example above if a government wished to maintain an equilibrium price level in face of the fluctuation of real resources, it could do it by the issue of more money at the start of every period $4t - 1$ and a tax to remove the money at the start of every period $4t + 1$. A more sophisticated way of getting money into and out of the economy is by using reserve ratios at the banks.* This however now creates two types of money; the currency originally issued by the government and the credit created at the banks.

It appears that in an economy without uncertainty the rate of interest is not adequate as a device for preserving the distribution of resources called for by a related nonmonetary economy.

A positive rate of interest however does guarantee that cash will not be hoarded. Varying the money supply takes care of the volume of trade without destroying the equilibrium as it controls the price level in the sense that the passive availability of "new money" does not impose a constraint on the movement of prices called for by the "real economy." A fixed supply of money does impose a constraint that may destroy an equilibrium price system.

We may still consider a fixed amount of money if we count both active cash and "sterilized" or passive issuing power of the bank; i.e. loans it is permitted to make when trade requirements call for them.

*A model of reserve banking has been discussed elsewhere^{7/}.

3.8. A More General Model: Inventories and the Float

The approach adopted in this paper has been to show by means of extremely simple examples some necessary conditions for the functioning of a monetary system and a money market.

A large step towards generalizing this analysis can be taken by considering a model with goods which can be inventoried. Once more an extremely simple model may serve to illustrate a new phenomenon.

As our interest is focussed on monetary phenomena and not upon production we may assume the simplest of "production functions" to represent the inventory process. Specifically we may assume that an individual can carry forward any good for any length of time at no cost whatsoever, or at an inventory cost related to the amount at hand.

If we wish to include an inventory cost we must be careful to state the physical units in which it is to be measured. The simplest type of cost is like that of radioactive decay or exponential depreciation. At the end of a period an amount η of any unit remains where $0 \leq \eta \leq 1$.

Suppose there are n traders and T time periods. At each period t a trader i is given a supply of A_t^i of new goods. They can be consumed or carried forward as inventory yielding ηA_{t+1}^i of next period's goods.

Suppose that each trader i wishes to maximize $\phi_i(x_1^i, x_2^i, \dots, x_T^i)$. Let $x_t = \sum_{i=1}^n x_t^i$ and $A_t = \sum_{i=1}^n A_t^i$. Then the difference between a market without inventory possibilities and with can be expressed in terms of the inequalities

$$(46) \quad x_t \leq A_t \quad \text{for } t = 1, 2, \dots, T \quad \text{no inventories}$$

and

$$(47) \quad \sum_{k=1}^t x_k \leq \sum_{k=1}^t \eta^{k-1} A_k \quad \text{for } t = 1, 2, \dots, T \quad \text{inventories.}$$

The inequalities (47) contain (46) as a more restrictive special case hence it is trivial to observe that in the latter case the traders are at least as well off as in the former.

When we turn to study the implications of inventories for the monetary system and the float, a new phenomenon related to the financing of inventories and the size of the float appears. In equilibrium an individual may be indifferent between inventorying a good for consumption several periods later or selling it using the money obtained and buying back the same good when needed. An example illustrates this. Consider an economy with two traders with utility functions

$$\begin{aligned} \varphi_1(x_1^1, x_2^1, x_3^1) &= x_2^1 \\ \varphi_2(x_1^2, x_2^2, x_3^2) &= x_1^2 + x_3^2. \end{aligned}$$

Their endowments, added exogenously at the start of the three periods are 0, 0, A/η^2 and A/η , 0, 0 respectively. Inventories depreciate at the rate of η .

There are three solutions to this trading problem in terms of variants of a competitive market. They are respectively:

- (1) The nonmonetary competitive equilibrium with $p_1 = 1$, $p_2 = 1/\eta$, $p_3 = \eta$ and no interest rate defined. The final distribution is $\varphi_1(0, A, 0)$ and $\varphi_2(0, 0, A/\eta^2)$.

When goods must be paid for in cash then not only must the final budget constraint be satisfied as in the nonmonetary competitive equilibrium but each period cash flow conditions must be met. These are the conditions noted in equations (31) to (38) in 3.5.

In terms of the simple example under consideration, for Trader 2 the following must be satisfied if the same allocation as in the non-monetary economy is to exist

$$(1 + r_2)p_1 \frac{A}{\eta} \geq p_3 \frac{A}{\eta} ;$$

and for Trader 1

$$(1 + r_2)(1 + r_3)p_2 A \geq \frac{A}{\eta} p_3 .$$

These are satisfied by: $p_1 = 1$, $p_2 = 1/\eta$, $p_3 = \eta$ and $r_1 = r_2 = r_3 = 0$ and they are also satisfied by:

$$p_1 = p_2 = p_3 = 1 \text{ and } r_1 = 0 , \quad r_2 = r_3 = \frac{1-\eta}{\eta} .$$

We note that the second trader wants to sell his inventory during the first period to avoid inventory costs and to gain loanable funds. The first trader for opposite reasons does not want to buy in the first period. By a device involving a distinction between time and demand deposits this can be made into an equilibrium.

If we postulate that the market is a merchant banking device which buys, sells and inventories goods and which can issue money against goods

and make loans, then the bookkeeping balances out. An example of the detailed accounting is tedious to follow and is given in the Appendix.

3.9. The Next Step

The above analysis has suggested a money rate of interest to support a constant price level in a stationary economy. This may be deemed to be consistent with the writings of the Austrian School.

Apart from the need to generalize for different time preferences for different individuals and for other interlinkages in the utility functions. It is also necessary to consider the role of production.

In a monetary economy production (including the act of carrying inventories) enters in two important ways. It provides an extra control variable so that individuals may select among alternative streams of goods and services. But it also provides a supply of assets which have an array of important properties such as durability, immovability and ease of identification which enables a society to issue a variety of paper against these items where the equity of the owner of the paper is protected by the laws of the society. Thus once more (as with fiat money) trust is depersonalized. An equipment trust note, or an assignat is backed by an ownership lien on a railroad car or land, to be enforced by law rather than on trust between specific individuals.

The two roles of assets and production are discussed in a subsequent paper.

It should be noted however, that adding a control variable to the decision power of the individual may weaken the control ability of a government.

4. DISCUSSION: MATHEMATICAL INSTITUTIONAL ECONOMICS

The use of a monetary system is associated with mass market phenomena. The acceptance by the individual of money as a means for trading implies his acceptance or acquiescence to an institutional superstructure capable of issuing and controlling money.

A basically symmetric nonmonetary market, by the introduction of money becomes a control system where key monetary variables fall naturally into the hands of a government. Leaving aside public finance for public or joint goods and services there may be other reasons for the use of the control variables.

4.1. Information Conditions and Expectations

The whole concept of the Walrasian price system is plagued with an intuitively unacceptable set of requirements for information. Furthermore, even when multi-period models are considered they are treated in an essentially static manner (see 2.5).

It is my belief that the techniques employed in the study of games in extensive form present the natural tool to understand the implications of information conditions in multistage economics. This point is discussed further in 4.3.

The existence conditions for prices, interest rates and optimal distribution of resources have been examined in a static manner even when considering an infinite process. The question posed has been: "does there exist a series of prices p_1, p_2, p_3, \dots and r_1, r_2, r_3, \dots consistent

with a set of one-person maximization programs?"

This question concerning existence is not the same as asking are there processes in which prices, rates of interest, etc. evolve as the process unfolds. When we pose such a question, in trying to answer it, it becomes necessary to try to bridge the gap between a utilitarian, microeconomic nonadaptive approach and a macroeconomic behavioristic approach.

In particular, we may replace the question concerning the existence of two infinite series: p_1, p_2, p_3, \dots and r_1, r_2, r_3, \dots with a question such as: "Do there exist expectation functions, $p_t = E_1(p_{t-1}, p_{t-2}, \dots, p_1, r_{t-1}, r_t, \dots, r_1)$ and $r_t = E_2(p_{t-1}, \dots, p_1, r_{t-1}, \dots, r_1)$ such that if an individual follows a short term maximization policy based on his current state and his expectations, prices and interest rates and the economy will evolve consistently with all expectations?"

If we make the expectations functions extremely complex the answer to the question will be affirmative and useless. Too many degrees of freedom will have been introduced and the problem posed becomes a sterile exercise in curve fitting.

A more useful exercise than the above is to consider extremely simple expectation functions. Furthermore, at this point the economist must check with the psychologist and social-psychologist for whatever understanding there is of the formation of expectations. He must also use institutional knowledge to cut down the set of possible information conditions.

The control of the price level or of the rate of change of the price level may not be necessary for the existence of a monetary market that

distributes real resources in the same manner as a nonmonetary market. It may however, be extremely important if instead of regarding an infinite series of prices as given they are assumed to be generated sequentially by a simple expectations function. For example a constant price level or a fixed rate of inflation may be much easier to adjust to than one that fluctuates.

Thus if we regard the monetary competitive market as an evolving process, government control of the money supply and the rate of interest must have the goal of keeping the appropriate relationship between expectations and actuality. We do not know how to model "open mouth operations," but it is at this point that their effect is relevant.

Formally the next mathematical step in getting rid of "perfect foresight" can be taken in two ways. We can go to a full game theoretic formulation which calls for the generation of prices as a result of the play up to that point in the process. This was the procedure adopted in a previous paper [2]. Or we can stay with a maximization model introducing a behavioral component. This component takes the form of an "expectations" or learning function concerning future prices and rates of interest.

For example consider the expectations functions (without learning): "I expect the rate of interest to be the same as last period and inflation to continue at the same speed."

We face an index number and aggregation problem in trying to define inflation. However, in the one commodity world discussed in Sections 2 and 3 there is no difficulty in defining inflation or deflation. It is an intertemporal change in the price level, which here is specified by a single

price each period. Thus the expectations functions at time t can be described as:

$$(48) \quad p_{t+k} = (1+\eta)^k p_t \quad \text{for } k = 1, 2, 3, \dots$$

where

$$(49) \quad \eta_t = (p_t - p_{t-1})/p_{t-1} \quad \text{for } t = 2, 3, \dots$$

and η_1 must be specified in the initial conditions.

$$(50) \quad r_{t+k} = r_t \quad \text{for } k = 1, 2, 3, \dots$$

and r_1 must be specified in the initial conditions.

Given the expectations conditions we may now ask the question concerning allocation as follows: "Given that the individual traders know what their future real resources will be and given that their expectations are as indicated by (48) to (50), will this system evolve in such a manner that it allocates real resources in the same way as a nonmonetary system with perfect foresight?"

In this instance because of the extreme simplicity of the example in Sections 2 and 3 the answer is yes. In general however, the question is tantamount to asking if a set of simultaneous dynamic programs have solutions which evolve in the same manner as the solutions to a set of infinite but basically static convex programming problems. Although I conjecture that for broad and important classes of expectations the answer is yes the question needs careful specification and the answer is open.

4.2. Uncertainty and Other Complications

All of the analysis in this and previous papers has been based on economies without exogenous uncertainty present. Leaving aside the enormous effect that exogenous uncertainty must have upon expectations, other extremely important complications appear. In particular how is individual risk to be handled? What are the minimal types of insurance companies and futures markets needed?

At the first level albeit not intuitively satisfactory the use of fiat money itself provides a rudimentary form of a mechanism for trading under uncertainty. In order to see this we need only consider two types of traders in essentially the same situation as in Section 2.2. with the modification that a chance move each period determines who owns the real assets that period. In such a model the individuals may hold cash balances for a "pre-cautionary motive" in case their incomes fail.

The need for and structure of insurance, investment banking, governmental insurance and futures calls for a separate discussion.

In a previous paper [2] the importance of information conditions was stressed. In terms of uncertainty it is important to note that there are at least four extremely different problems which must be dealt with.

They are:

- (1) Simple exogenous uncertainty.
- (2) Game theoretic uncertainty.
- (3) An information structure without positions of complete information.
- (4) "Internal behavioristic" uncertainty.

Examples of the first are crop failure, floods, winning at Roulette, etc. The second is strategic uncertainty and refers to lack of knowledge of the strategies of others. This type of uncertainty is not relevant to a mechanistic or general equilibrium model of economic affairs.

The third type of uncertainty comes about when the structure of the process makes it fundamentally impossible for any participant within the process to know exactly to where it has evolved [3].

The last type of uncertainty concerns "the rules of the game" and is a matter of modeling. For example, is the assumption reasonable, that in economic affairs individuals more or less know what they want and behave (in some weak sense) more or less rationally?

4.3. The Market as a Noncooperative Game

The investigation of the functioning of the economy in this paper has been in terms of a mechanistic trading model. It is my belief that a more satisfactory treatment calls for the study of the market as a dynamic noncooperative (or possible "quasi-cooperative") game.

Static economic theory and game theory deal with three types of solutions. They are:

- (1) Cooperative solutions
- (2) mechanistic solutions

and

- (3) noncooperative solutions.

The cooperative solutions are primarily concerned with distribution, the achievement of Pareto optimality is usually assumed as a matter of course. The nature of the bargaining, the type of communication, the whole of the settlement process are all abstracted out of consideration. In other words cooperative solutions such as Edgeworth's contract curve or Zeuthen's bargaining point, even if they are accompanied with a verbal description of process are essentially non-institutional.

The mechanistic or competitive equilibrium price system solution (referred to as mechanistic because in the mathematical formulation the strategic freedom is removed from each individual--he is not permitted to do anything but "the right thing"), is also non-institutional. In the verbal discussions a "tatonnement process" may be described but it does not appear in the mathematics of general equilibrium [4].

The noncooperative solution, as exemplified by Cournot, is essentially concerned with the details of strategy. If it is applied to a multistage market in order to even define the strategies it is necessary to be absolutely specific concerning every detail of the process. Nothing takes place beyond the game. Every nuance must be specified. Slight changes in information patterns may make an enormous difference to the process.

The complete specification of structure and process needed for a non-cooperative game is tantamount to stating that much institutional detail must be specified. How banks pay checks, how money enters the economy; details of cash flows; what happens if an individual cannot pay his debts; must all be described.

Obviously someone looking for a general economic theory wishes to avoid being overwhelmed by detail of little or no relevance. But to search for the minimal institutional forms that reflect cash flows, market clearing, money issue, banking, etc., is different from trying to avoid the specification of institutional forms in the belief that such an act removes one's work from the roles of pure and general theory.

When we go from statics to dynamics the clearly specified distinction between cooperative and noncooperative solutions begin to blur. We need to consider "quasi-cooperation" where the means for cooperation over a limited period are built into the process. In economics many of these means tend to be in the form of financial institutions backed by the law and government. They are banks, escrow agents, insurance companies, legal contracts, bankruptcy laws, loan conditions, etc.

The modeling and analysis of a monetary economy calls for the development of a mathematical institutional economics. Any fruitful attempt at developing a dynamics will probably need more attention paid to the institutions or "carriers" of the process.

In this paper by using the competitive equilibrium formulation, although some attention has been given to institutional detail we have avoided the important problems concerning the specification of information conditions and bankruptcy laws. Both of these are necessary for the construction of a noncooperative model of the same economy.

4.4. G.N.P., the Supply of Money and Multiplicity of Equilibria

Although it has been noted that the use of a commodity money may limit the number of competitive equilibria [5], it does not necessarily select a single equilibrium. Furthermore, this gain in resolution is lost when a fiat money is used. A fiat money issue is consistent with any equilibrium point in the nonmonetary system.

It might be that a system with complex production and inventory interlinkages might call for different growth patterns and interest rates to be associated with the different equilibria. It might also be the case that the introduction of risk into an economic system using money casts light on the problem of uniqueness.*

Until the problem concerning uniqueness is resolved we have no way of resolving preferences for one price system or another. Although it is worth noting that if we fix the price of money so that the price of the first commodity in the first period is one monetary unit then there will be a price system (in general unique, except for degeneracy such as extreme symmetry) for which the amount of money (relative to the other price systems) required is minimal. We might prefer to use a welfare criterion or a minimize cash flow criterion to select a unique equilibrium.

*Especially considering that if one is willing to accept a von Neumann-Morgenstern approach to risk [6] added conditions are given to determine choice between points on the Pareto optimal surface.

From the above observations we may conclude that the comparison of the GNP at one point in time with the GNP at alternative price systems associated with other equilibria at that same point in time has no welfare connotations.

4.5. Inflation, Deflation and Monetary Issue

An adequate discussion of inflation and deflation calls for a game theoretic formulation and for a discussion of the neverending market. These enable us to provide a better model to account for a dynamics of price change given money flowing into or out of the economy.

Inflation and deflation are not symmetric when viewed as decision processes.* The individual economic agents will call for more money in an inflationary system, hence the banks or new money issuing agencies can be passive. In a deflationary system however, the individuals do not wish to shrink their money supply. If the real resource equilibrium is to be maintained it requires action initiated by the monetary authorities in terms of taxation.

*This can be shown from a simple dynamic game formulation of the markets. An example has been presented in a previous paper [8].

4.6. Concluding Remarks

The addition of a monetary mechanism to the general equilibrium model serves as an immediate link between microeconomics and macroeconomics. It provides a natural means for introducing institutions into a general equilibrium context. The difference between a static and an evolutionary model (both of which can be defined for the same economic background) is crucial in distinguishing the gap between the microeconomic and macroeconomic approaches.

The macroeconomic approach is evolutionary. As such, great stress must be laid upon expectations and control. These are intimately related with information processes and institutions. The microeconomic approach avoids these problems.

A game theoretic approach is closer to the macroeconomic model than the microeconomics. This is true in the sense that dynamic game models, such as that for a repeated noncooperative game, may use behavior strategies and are basically evolutionary. The prices and interest rates emerge during the course of play. They are not data.

The game theoretic approach has the additional advantages that it stresses the roles of the state of information and the importance of numbers of competitors in markets. A full understanding of the monetary mechanism calls for three different types of limit procedures to be applied simultaneously. They are increasing the number of competitors in all markets indefinitely; increasing the length of the functioning of the market indefinitely and specifying how information and communication conditions change as numbers increase.

A new mathematical development is called for in the form of dynamic games. These appear to be naturally formulated in terms of simultaneous dynamic programs employing state strategies.

A contemplation of the nature of monetary dynamic economic systems leads one to conjecture that disequilibrium may be the prevalent state of the system. With an introduction of even a small amount of exogenous uncertainty this may be true all of the time.

Even without uncertainty and taxation there can be a positive rate of interest on paper money in as simple as closed an economy as that presented in Sections 2 and 3.

The rate of interest cannot be used alone to control an equilibrium price level in an economy with perfect foresight. The money supply must be adjusted as well.

It is conjectured that all of the observations concerning the rate of interest are quite general and that it should be possible to replace these somewhat informal remarks with relatively general theorems.

The major difficulty in reconciling micro- and macroeconomic systems comes in finding natural ways to loosen the coupling in the general equilibrium system. Money is a natural strategic decoupling device.

The role of information provides a critical link between micro and macroeconomics. A formal study of the implication of the aggregation of information and the nature of noncooperative equilibria in markets with aggregated information, is called for.

Institutions play a key role in the functioning of a loosely coupled economic system. They also serve to provide government with a natural set of points of entry into the broad control of an otherwise competitive economy.

There are many different facets to the role of money in a modern economy. It is my belief that in spite of the relative "unreality" of simple models, the most fruitful approach is to investigate many of these different features separately prior to trying to construct a "the theory of money."

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APPENDIX

The accounting for trade and cash flow for the example in 3.8 requires three sets of books, one for each trader and one for the merchant bank. Each period is split into three moves. They are respectively:

- (1) The financial move,
- (2) the market move, and
- (3) market settlement.

During each move we must distinguish between money on loan and money cash or in the float and goods in trade or inventory and goods for consumption. The transition rules between states are as follows:

- (1) Money in cash or float during any period remains in float and earns no interest.
- (2) Between the end of one period and the start of the next money may be transferred from float to loanable funds and vice versa.
- (3) Money in the loan market must remain there for a full period. It earns a rate of interest.
- (4) The merchant bank may issue money against goods it purchases. It may also lend money and will accept deposits (really time deposits for one whole period) at the going rate of interest.
- (5) Goods for consumption at the end of a period disappear by the start of the next.
- (6) Goods for trade or inventory at the end of a period are transported to the start of the next period diminished by the inventory costs.

In the table below we also present three overall financial statistics. They are total money in circulation, total time deposits and total loans.

Trader 1				Trader 2				"Market" or Merchant Bank			Total \$ in Circulation	Total Time Deposits	Total Loans
Money on Loan	Cash or Float	Goods in Trade	Goods in Cons	\$L	\$F	G_T	G_B	\$L	\$F	G_T			
		A/η									---	---	---
	A/η								$-A/\eta$	A/η	A/η	---	---
	A/η								$-A/\eta$	A/η	A/η	---	---
A/η				$-A$	A			$-A/\eta + A$		A	A	A/η	A
A/η				$-A$		A		$-A/\eta + A$	A^*		---	A/η	A
A/η				$-A$			A	$-A/\eta + A$	A^*		---	A/η	A
	A/η^2			$-A/\eta$		A/η^2		A/η			A/η^2	---	A/η
		A/η^2		$-A/\eta$	A/η^2			A/η			A/η^2	---	A/η
			A/η^2	$-A/\eta$	A/η^2			A/η			A/η^2	---	A/η
---	---	---	---	$-A/\eta^2$	A/η^2			A/η^2			A/η^2	---	A/η^2

* There is an accounting discrepancy in this column which hinges upon how the bank's issue and withdrawal of currency is to be accounted for. If we permit the bank to pay itself interest during the same period as it sells an asset it can then finance inventories and the books balance the A in column $\$F$ in period 2 may be replaced by A/η in column $\$L$ of the bank.