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ALTERNATIVE THEORIES OF WAGE DETERMINATION AND UNEMPLOYMENT IN L.D.C.'s:

I. THE LABOR TURN-OVER MODEL

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1. Introduction

The problem of unemployment and underemployment in L.D.C.'s has long been a central concern of development economics. More recently, the discussion has focussed on unemployment and underemployment in the urban sector. The common diagnosis of the source of urban unemployment, particularly in economies such as those in East Africa where there does not seem to be "surplus labor" in the agricultural sector, i.e. there appears to be a fairly compe-tivite market for labor in agriculture paying a positive wage,[1] is that there is a large wage differential between the urban and rural sectors which encourages migration into the urban sectors. And, finally, there seems to be a consensus that the remedies for this--if it is impossible to in fact lower the urban wage to the level in the urban sector—are (a) a wage subsidy, to encourage private employers to hire more laborers (use more labor intensive techniques and (b) the use of a shadow price of labor for projects in the government sector which is lower than the market wage in the urban sector.

Although economists have advised governments all over the world to undertake these measures, they have based these policy prescriptions on partial equilibrium models which have not traced out the full implications of these policies; in particular, they have failed to take into account

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[1]Indeed, in certain seasons for certain crops there are complaints of a labor shortage.

(a) the determination of the unemployment rate or level in the economy and (b) the determination of wages in the urban sector. The possible implications of these failures may easily be seen. If the number of people in the urban sector is fixed, then hiring one person from the urban unemployed has a zero social cost, and the unemployment rate will be reduced. If the unemployment rate in the urban sector is unchanged, at U, hiring one more person will lead to approximately U more unemployed people, so (1+U) people have joined the urban unemployed. What the social cost is then depends on what they were doing before: if they were unemployed in the agricultural sector, then of course the social cost is still zero. But if they were receiving the agricultural wage, and this was equal to their marginal product, then there is a significant social cost in excess of the rural wage.[1] In between, there is the possibility that the number unemployed in the urban sector is constant, so that hiring one individual from the unemployment pool results in an in migration to the urban sector of one person. If he was employed in the agricultural sector, the opportunity cost is then just the rural marginal product of a laborer. Evidently, those who advocate using something like the rural wage as the shadow price of labor in the urban sector are making this assumption; but on no basis do they justify this assumption.

Similarly, one has to have a theory of the determination of the urban wage to know whether a wage subsidy will be "shifted." Evidently, most of the advocates of wage subsidies not only believe that the urban wage is rigid downward, but that is also rigid upward; yet even non-competitive bargaining theories might suggest that some of the wage subsidy is absorbed by the workers in the form of higher nominal wages. If this is the case, a wage subsidy might exacerbate the unemployment problem by increasing the nominal differential between urban and rural wages.

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[1]Others have advocated using a shadow wage in excess of the rural wage because of the distributional implications of alternative employment policies; i.e. since workers consume a larger fraction of their income, increasing urban employment will reduce savings. Since the shadow price of savings is in excess of that for consumption, it is optimal to hire workers to a point where their marginal product is somewhere between the urban and rural wage. In this paper we focus solely on the implications of wage and employment policy for the static equilibrium.

What is called for then is a simple general equilibrium model of a less developed country, explaining simultaneously the determination of wage differentials, urban unemployment, the allocation of labor between the urban and rural sectors, etc. But it is now commonly believed that a model which is appropriate to one L.D.C. may not be appropriate to another. Hence what is required is not a "single model" but a "family of models" and this I intend to provide in a series of related papers.[1] There are four main dimensions in which these models may differ from one another.

(1) The allocation of labor between the urban and rural sectors. Do workers migrate when the expected urban wage is equal to the average product in the rural sector or when it is equal to the marginal product? (or when it is equal to something else?) The answer to this question depends on the pattern of land ownership, in particular, on whether when individuals migrate from the rural sector they loose all claims on the rents from the land. This question I have discussed elsewhere in some detail.[2]

(2) The determination of urban wages. The fundamental question here is: why are firms willing to pay higher wages than the opportunity cost of labor in the rural sector?[3]

(3) The availability of opportunities for self-employment in the urban sector while looking for wage employment, and the presence of an artisan class in the rural sector. In most models, we focus on a simple two sector economy, ignoring these possibly important "service" sectors.

[1]Because this paper was written while I was at the Institute of Development Studies in Nairobi, when there is a choice of alternative assumptions, the consequences of which do not appear to affect the central point of the analysis, I have chosen the assumption which appears most appropriate to the African context. Thus for instance, we use below a model of an open economy and assume the rural wage relevant for the analysis of migration is equal to the marginal product (not the average product) of labor in the rural sector.

[2]See J.E. Stiglitz, "Rural-Urban Migration, Surplus Labor, and the Relationship between Urban and Rural Wages," E. African Econ. Rev., December 1969.

[3]Some of the wage differential is undoubtedly due to differences in costs of living; some too may be explained by non-pecuniary advantages of rural areas (although this may be offset to some extent by the non-pecuniary advantages of the urban areas--the "bright lights"); but the fact that must be explained is that the wage differential paid seems larger than is necessary to induce the required migration. In the following discussion, we shall ignore the cost of living differential and any non-pecuniary advantages or disadvantages of urban living.

They have some peculiar characteristics; e.g. because of free entry, workers receive their average product in these sectors, which may be significantly different from the social marginal product. The presence of these sectors may have important implications for patterns of migration; movement out of agricultural production in the rural sector decreases demand for the local artisan, and hence there is an induced migration of artisans. The availability of service jobs in the urban sector makes the "cost" of job search less than it otherwise would be.

(4) The effects of education on production and labor allocation. Recent attention has been focused on the problem of unemployment of the skilled (educated). To what extent is the labor market dichotomized by education (skill) levels? To what extent does education increase skills, to what extent is it used as a screening, or sorting device to identify skills and abilities, and to what extent is it used by employers as an essentially arbitrary criteria for selecting among a large number of applicants for a position.[1]

In this paper and in two sequels we focus on the process of wage determination;[2] in particular on the implications of alternative theories of wage determination for wage and employment policies.

We discuss four approaches to wage determination in the urban sector:

(1) The institutional (rigid wage) model
(2) The labor turnover model
(3) The efficiency wage model
(4) The "Cambridge" model

The first approach differs markedly from the other three, which provide an explicit endogenous theory of urban wages. The institutional approach stresses the role of unions and government in wage setting in L.D.C.'s, although there

[1]The latter view is taken in an extremely interesting paper by G. Fields, "Private and Social Returns to Investment in Education in Kenya," U. Michigan, 1972. He calls this model one of "bumping": the educated bump the less educated from their jobs.

[2]Accordingly, we simplify our analysis by (a) assuming all labor is homogeneous and (b) ignoring the urban self-employed sector and the rural artisan sector. Our qualitative results are not affected by these simplifications.

is not agreement even among the "institutionalists" on the relative importance of unions and the government in creating the wage differentials.[1] The other three theories may be described briefly--and in much oversimplified terms--as follows: the labor turnover model says that turnover costs (hiring and training) are greater in the urban sector than in the rural, the turnover rate is a decreasing function of the wage rate (in the urban sector relative to the rural sector) and that therefore it pays each competitive firm in the urban sector to offer more than the rural wage. The efficiency wage model says that the efficiency of a worker is an increasing function of the wage rate, and therefore, at least in a range, the cost per unit of labor services decreases with the wage rate. The third assumes a financial dichotomy between the urban and rural sectors. Savings in the urban (monetized) sector depends on the distribution of income (real wages). The macroeconomic "Savings Equals Investment" equilibrium condition determines the urban wage.

In our view, the "institutionalist" approach is unsatisfactory in several respects. Most importantly, it fails to provide an economic answer to most of the relevant economic questions: in any given country, it cannot explain the size of the wage differential, and in comparing wage differentials across countries or over time, it must appeal to differences in union "strength." But how is union strength ("power") to be measured, apart from the wage differentials that can be extracted from employers. And even if an independent measure of union "power" was obtained, one would still wish to know what were the economic factors which led union strength to be greater at one place or time than at another.

Secondly, wage differentials and the problem of urban unemployed to which they give rise were apparent long before unions or government became important in wage setting, as the extensive discussions of the "reserve

[1] That is, although wages in the union sector and in the government sector are higher than in the non-unionized urban sector, which are in turn higher than wages in the rural sector, is the size of the wage differential basically determined by union strength, with the government setting wages according to the pattern in the unionized urban sector, or are wage levels basically determined by the wages set by the government, with unions and unionized sectors following the pattern determined there?

army of the unemployed" in the nineteenth century economics literature seems to indicate. Lastly, it is true even today that in the non-unionized industrial sector wages are considerably above those in the rural sector.

None of these pieces of "evidence" against the "institutional" hypothesis is by itself convincing. The non-unionized sector may pay almost as high a wage as the unionized sector in order to avoid unionization. But it is equally true that the arguments for the institutional hypothesis are not convincing: is it not possible that firms (sectors) which are easily unionized are those which would otherwise have paid higher wages anyway? Accordingly, they put up less resistance to unionization.

Of the three "endogenous" theories the first is the one to which we ascribe the greatest plausibility at least for the labor markets in Eastern Africa; Elkan,[1] in particular, has stressed the importance of the high rates of labor turnover in determining the structure of the labor market in Uganda. The second is perhaps the most extensively discussed model in the literature, though remarkably almost always in a partial equilibrium context; its policy implications have never been fully spelled out. It may have more relevance to the economies of South Asia. The third is an extension to the problems of L.D.C's of the kind of growth and distribution model that the "Cambridge economists," Kaldor in particular, have developed over the last fifteen years. I, myself, do not think it to be a particularly important part of the explanation of wage determination.

It should be emphasized however that although for reasons of expositional convenience, we shall present the different endogenous theories as alternative explanations of the wage differential, they are in no way mutually exclusive; certain policy conclusions do, however, depend on the relative importance ascribed to the different explanations. Nor, in fact, are the endogenous theories to be presented necessarily inconsistent with institutional considerations playing an important role in wage determination.

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[1] Walter Elkan, Migrants and Proletarians, Oxford University Press, 1960, and W. Elkan, An African Labour Force, East African Studies, No. 7, Kampala, 1955.

The fact that even in developed economies, economists cannot resolve the question of the relative importance of unions in wage determination should make us not very sanguine about the possibilities of resolving these questions in L.D.C.'s.

Our position here is that the endogenous forces for wage differentials which we shall describe are at least a part of the process of wage determination, that they may explain, in part, some of the success of workers, unionized and non-unionized, in obtaining higher wages that they would have obtained in the rural sectors.

In this paper, we shall be solely concerned with the labor turnover model. After presenting the basic model in Section 2, we shall describe the competitive equilibrium in Section 3. In Section 4 we shall then contrast the competitive equilibrium with the wage and employment policy which the government would pursue if it controlled directly the urban sector. We shall call this the "optimal" allocation. But it should be emphasized that it is a second best optimum: the government is not allowed to directly control migration from the rural to the urban sector.[1] Sections 5 to 7 consider government policies for economies in which most of the production in the urban sector is in the private sector. Section 5 examines the often recommended policy of wage subsidies. Section 6 examines wage and employment policies of the government in a mixed economy while Section 7 considers the consequences of an urban income tax. Section 8 looks briefly and in a very heuristic manner at the movements over time in the unemployment rate, urban rural wage differentials, etc. engendered by our model.

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[1]A number of governments have attempted, unsuccessfully, to control migration directly by requiring all residents in urban areas to have work permits or pay taxes. When individuals are found without employment in the urban sector, they are returned to their native villages but they return shortly to the city. It does not appear possible to control migration directly without taking oppressive measures.

2. The Basic Model

There are six basic pieces in the construction of a general equilibrium model of an L.D.C.: (a) wage policy of the individual firm in the urban sector; (b) the determination of "equilibrium" wage policies in the urban labor market; (c) the employment policy of the single firm; (d) from (b) and (c) we derive the demand function for labor in the urban sector; (e) the determination of allocation of labor between the rural and urban sector (the supply of labor to the urban sector); and (f) the determination of the relation between the nominal wage in the urban sector and the "expected wage."

(a) Wage Policy of the Individual Firm. Since the crucial way in which our model differs from earlier models of dual economies is in the determination of the wage rate in the urban sector, we begin our analysis with a discussion of firm behavior in the urban sector. The representative firm produces output Q_u by means of a set of production processes which can be described by a production function of the form

$$Q_u = F(K_u, L_u)$$

with

$$F_{LL} < 0 .$$

(Throughout, we shall use the subscript u to denote variables pertaining to the urban sector, and the subscript r to refer to variables pertaining to the rural sector.)

Since our point here is the analysis of equilibrium positions, and not the dynamics by which they are reached, we shall assume the capital stock is given.[1] The firm wishes to maximize its profits; to do this it must minimize its labor costs. Labor costs consist of wage payments plus

[1] For the same reason, it makes no difference to our short run equilibrium analysis whether F is constant returns to scale.

It should be emphasized that the longer run implications of the various policies discussed below, e.g. a wage subsidy, may be quite different from the short run implications. Not only may such policies affect the level of savings (a point that has already received extensive attention in the literature) but also the intersectoral allocation of capital.

training and hiring costs.[0] Included in the latter is the lost output (broken machines, etc.) which inevitably results when a new employee is hired. The training-hiring costs per worker, T, are assumed for simplicity to be constant. Thus total training costs are a function of the rate of turnover of employees. The rate of turnover of employees is a function, in turn, of the wage paid by the firm, w_u, in comparison with (*a*) wages paid by other firms in the urban sector and (*b*) wages paid in the rural sector, w_r. It is also a function of the rate of unemployment, U. If q is the quit rate, i.e., the percentage of the labor force quitting at any time,

$$q = \underline{q}(w_u/\bar{w}_u, \; w_u/w_r, \; U) \tag{2.1}$$

where $\bar{w}_u$ is the average wage paid by all firms in the urban sector.[1] We assume that

$$\underline{q}_1 < 0$$

$$\underline{q}_2 < 0$$

$$\underline{q}_3 < 0 \; .$$

The average period of employment is increased by (the quit rate is reduced by) an increase in the wage paid by this firm relative to other firms and relative to the rural sector, and by an increase in the unemployment rate.

There are three sources of labor turnover (besides death and old-age retirements): individuals quit to take other jobs in the urban sector,

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[1] More accurately, the quit rate is a function of the wage paid by this firm and the entire distribution of wages paid by other firms in the urban sector. Since in our model, we are assuming all firms in the urban sector are identical, in equilibrium, all firms pay the same wage, and so the distribution is the improper distribution at $\bar{w}_u$. Nonetheless, individual firms may contemplate deviating the wage they pay from that of the rest of the sector. (For a more extensive discussion of the theory of labor turnover and the structure of wages, see J. Stiglitz, "A Note on the Theory of Labor Turnover and the Optimum Wage Structure," mimeo, 1971.) We assume in this paper that the only means of affecting the turnover rate is to lower the wage. There are other instruments available, e.g. seniority pay but these are sufficiently weak still to leave a significant amount of turnover to be affected by wage levels.

[0] These are assumed to be specific rather than general training costs.

individuals quite to return to the rural sector,[1] and individuals quit to seek other jobs in the urban sector, in the meantime joining the unemployment pool. The three arguments of our labor turnover function take account of these three alternative sources of labor turnover. In addition, the unemployment rate enters in still two further ways:

(a) When an individual is hired by a new firm, there is some probability that it will turn out that he is unsuitable for the job (so will be fired) or that he will dislike the job (or the personnel associated with it) and therefore seek to find still another job. The ease with which this is accomplished depends on the unemployment rate.

(b) Much has been written in recent years on rural-urban migration. In the African context, there is also an important urban-rural migration.[2] Individuals leave the urban sector to return to their rural homes. They may subsequently return to the urban sector, after a period of work or leisure in the rural districts. The ease with which they can find employment when they return to the urban sector depends on the unemployment rate, and hence the attractiveness of leaving the urban sector for a respite in the rural depends on the unemployment rate. To take the extreme case, if there were no unemployment whenever they grew "tired" of the urban sector, they would quit, for they would know that as soon as they want an urban job once again, they could acquire one. The unemployment rate acts to discourage this rural-urban-rural remigration.

The firm's total labor costs then are

$$w_u L_u + qTL_u \equiv w_u^* L_u \tag{2.3}$$

where w_u^* is the total labor cost per employee. For a given L_u, the firm seeks to minimize the cost per employee; it takes the unemployment rate and the wage rate of other firms as given, and chooses w_u to minimize $w_u + qT$.

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[1]We assume, in other words, that the substitution effect of a higher urban-rural wage differential in discouraging labor turnover is more important than any possible "income effect." If this is not the case, then $q_2 > 0$ (e.g. if individuals come to the urban sector to accumulate a <u>fixed</u> amount of savings). Whether $q_2 \geq 0$ does not affect our qualitative results, as the reader can check for himself.

[2]This has been particularly emphasized in the work of Elkan, <u>op.cit</u>.

This yields the first order condition[1]

$$1 + T\left(\frac{q_1}{\bar{w}_u} + \frac{q_2}{w_r}\right) = 0 \; . \qquad (2.4)$$

The marginal savings in turnover costs must be equal to the extra wage costs.

In Figure 2.1 we have depicted graphically the firm's choice of a wage rate for fixed unemployment rate U and given average wage rate in the urban sector, $\bar{w}_u$. We have plotted two different quit rate functions, corresponding to different $\bar{w}_u$. The firm chooses a "quit-rate-wage" combination at a point such as E where the quit rate function is tangent to an iso-labor cost curve $w_u + Tq$.

(b) The Urban Wage Equation. In the previous subsection, we described how each firm chose its wage to minimize labor costs given the unemployment rate, the rural wage, and the wage paid by all other firms. Since we are assuming that all firms are identical, equilibrium in the urban labor market requires all firms pay the same wage

$$w_u = \bar{w}_u \; . \qquad (2.5)$$

To see what this entails, observe in Figure 2.2 that when $\bar{w}_u$ is increased, the wage paid by the representative firm increases. In Figure 2.2 we have plotted the value of w_u chosen by the representative firm at different values of $\bar{w}_u$ (i.e. the curve ww in Figure 2.2 corresponds to the curve EE' in Figure 2.1). As we have drawn it, ww has a positive slope less than unity. The point where w crosses the 45° line is the equilibrium urban wage, given the rural wage and the unemployment rate.

[1]The second order condition requires

$$\frac{q_{11}}{\bar{w}_u^2} + 2\,\frac{q_{12}}{\bar{w}_u w_r} + \frac{q_{22}}{w_r^2} > 0 \; .$$

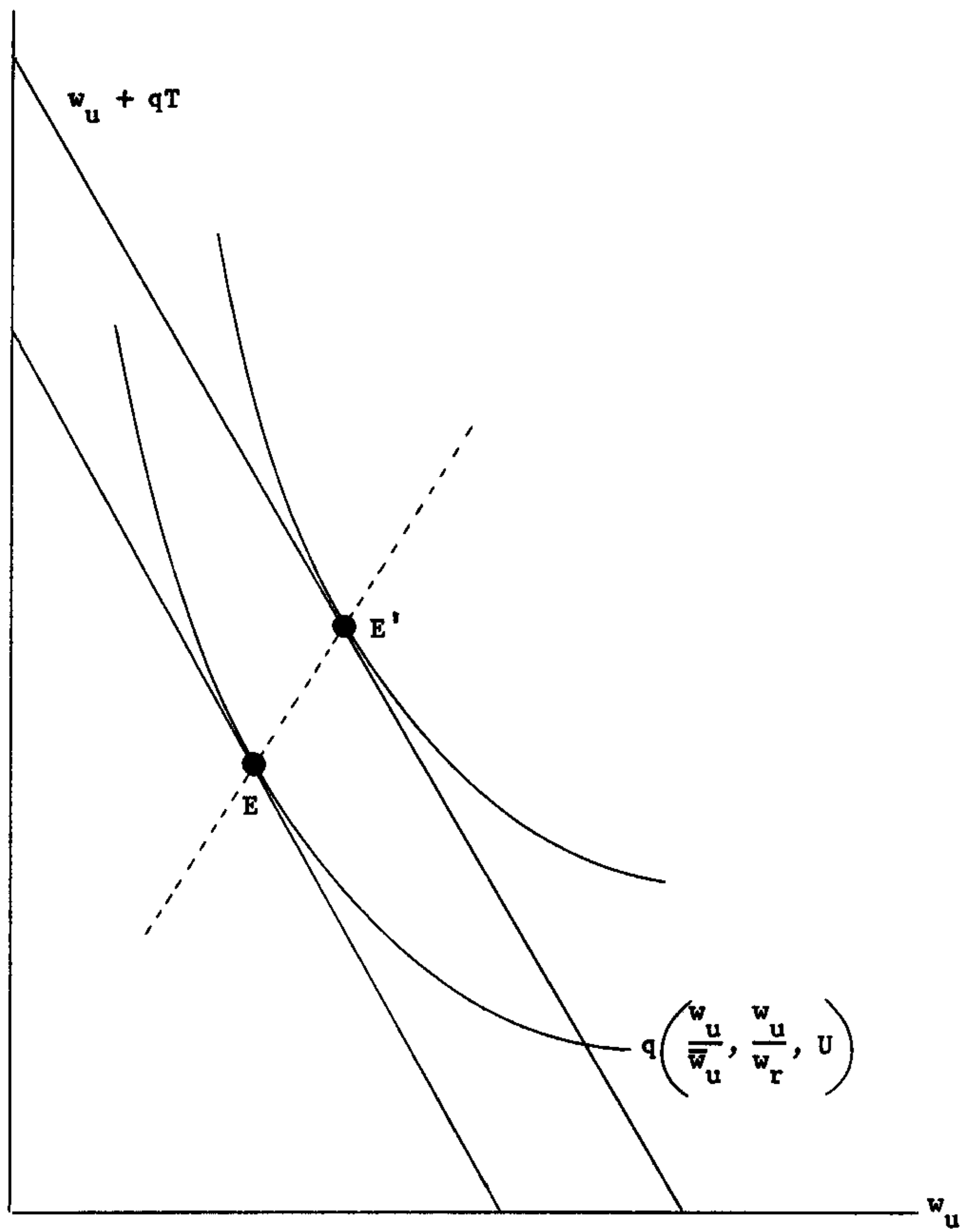

FIGURE 2.1

Firm's Choice of Optimal Wage Rate

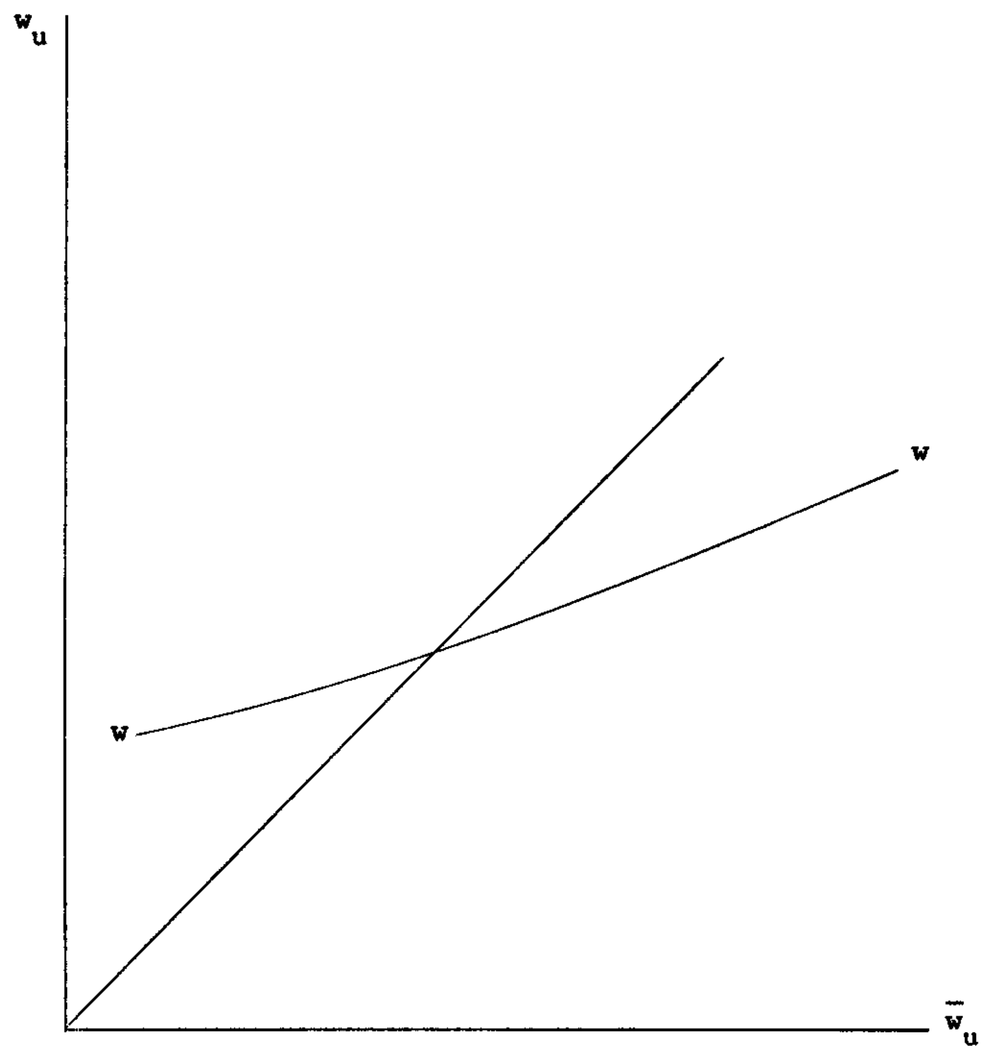

FIGURE 2.2

Determination of Equilibrium Urban Wage
(given w_r and U)

Formally, the solution to (2.4) may be written

$$w_u = w_u(\bar{w}_u, U, w_r) . \tag{2.6}$$

Substituting (2.5) into (2.6) we can solve for w_u as a function of the parameters facing the urban sector, U and w_r

$$w_u = \phi(U, w_r) . \tag{2.7}$$

Normally, we would expect that $\phi_U < 0$, an increase in the unemployment rate reduces the equilibrium relative wage in the urban sector. This can be seen in Figure 2.3. An increase in the unemployment rate reduces, for each $\bar{w}_u$, the w_u chosen by the representative firm. The quit rate function moves "downward" in a reasonably uniform fashion, so firms spend less on direct labor costs (wages) as well as on training.

Similarly, we would normally expect that an increase in w_r increases the wage paid in the urban sector, but less than proportionality.

These normal "presumptions" are described by the following restrictions on ϕ :[1]

$$\phi_U < 0 \tag{2.7a}$$

$$1 > \frac{d \ln \phi}{d \ln w_r} > 0 . \tag{2.7b}$$

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[1] To see the relationship between these restrictions on ϕ and the restrictions on q, we observe that

$$\frac{dw_u}{dU} = -\left(\frac{q_{13}}{w_u} + \frac{q_{23}}{w_r}\right) \Bigg/ \frac{q_{12}}{w_u w_r} + \frac{q_{22}}{w_r^2} - \frac{q_1}{w_u^2} \tag{2.7a'}$$

It should be clear that the model of wage determination we have presented is essentially one of monopolistic competition in the urban labor market. This can perhaps best be seen in Figure 2.4. Equilibrium requires $w_u = \bar{w}_u$. Hence, as the urban wage rate changes, the only effect on quit rates is from the increase in urban-rural wage differentials. This generates the curve we have labelled QQ. Each firm, on the other hand, believes it can take

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$$\frac{dw_u}{dw_r} = \frac{\dfrac{q_{12}}{w_r^2} + q_{22}\dfrac{w_u}{w_r^3} + \dfrac{q_2}{w_r^2}}{\dfrac{q_{12}}{w_u w_r} + \dfrac{q_{22}}{w_r^2} - \dfrac{q_1}{w_u^2}}$$

$$= \frac{w_u}{w_r}\left[1 + \frac{\dfrac{q_1}{w_u^2} + \dfrac{q_2}{w_u w_r}}{\dfrac{q_{12}}{w_u w_r} + \dfrac{q_{22}}{w_r^2} - \dfrac{q_1}{w_u^2}}\right] . \qquad (2.7b')$$

The numerator of (2.7a') will be positive, if, at higher unemployment rates, quit rates respond less to wage differentials, both within the urban sector and between the urban and rural sectors. The denominator will normally be positive. If an increase in the urban-rural wage differential decreases the sensitivity of quit rates to intra-urban wage differentials, $q_{12} > 0$. If q is convex, $q_{22} > 0$. Similarly, the denominator of (2.7b') is normally positive. In that case, it is clear that

$$\frac{d \ln w_u}{d \ln w_r} < 1 .$$

To argue that it is positive is more complicated. There are two factors at work. We argued before that q_{12} and q_{22} are normally positive, an increase in the rural wage decreases the sensitivity of quit rates to intra-urban wage differentials and to urban rural wage differentials. But $q_2 < 0$, but this effect we suggest is dominated by the other two terms in the normal case.

These conditions may be expressed in still another way: if both "direct labor costs" and training costs are nonregressive factors, in the sense that a change in U, w_r, or $\bar{w}_u$ which shifts the quit rate function out results in both new training costs and higher direct costs, then the numerator of (2.7a'), the numerator of (2.7b'), and the denominator of both are, respectively, positive.

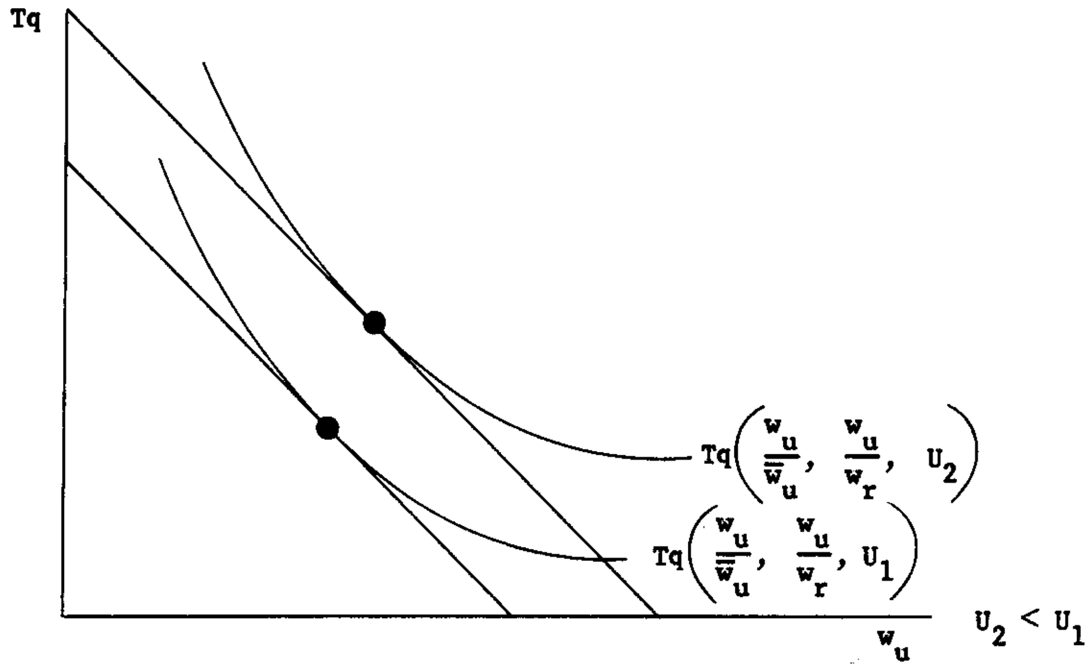

FIGURE 2.3(a)

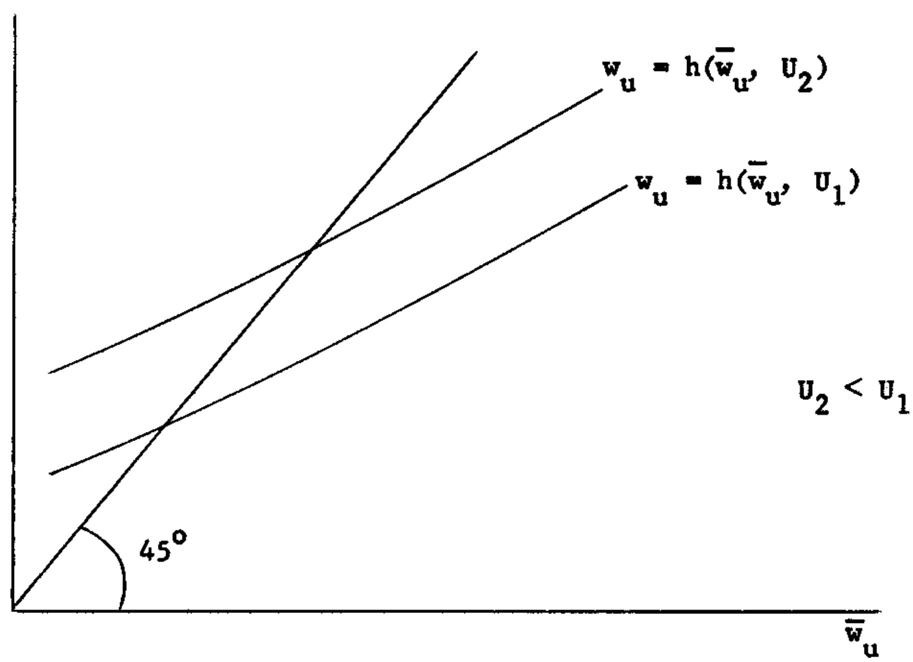

FIGURE 2.3(b)

Effect of Increased Unemployment Rate on Urban Wage

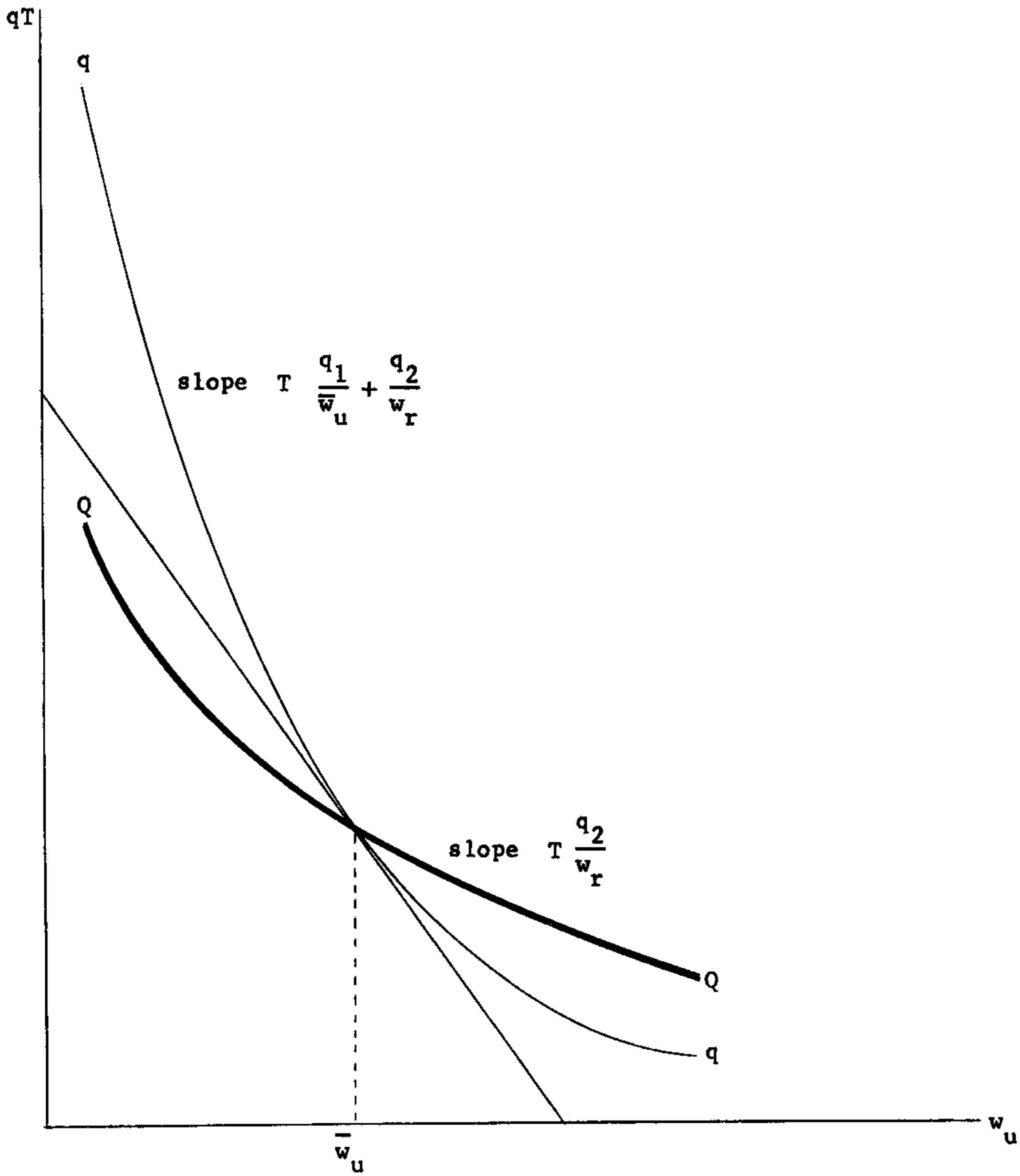

FIGURE 2.4

Competitive Choice of Urban Wage Rate
(Unemployment rate given)

some competitive advantage of the other firms by raising its wage relative to theirs. The quit function it perceives is steeper than QQ . (It is important to remember that this is still partial equilibrium analysis; throughout, w_r and U are assumed given.) The equilibrium is at a point where the qq curve intersects the QQ curve and has a slope of unity.

(c) Employment Policy of the Firm. The firm chooses its employment level to maximize its profits. Letting the price of output be unity, then it[1]

$$\max F(K_u, L_u) - w_u L_u - TqL_u = F - w^* L_u \qquad (2.8)$$

so that

$$F_L = w_u + Tq = w^* \qquad (2.9)$$

the marginal productivity of labor is equal to the wage plus training costs. Because $F_{LL} < 0$ this can be inverted to solve for the demand for labor by the representative firm, as a function of w^* :

$$L_u = L_u^d(w^*; K_u) \qquad (2.10)$$

where $dL_u/dw^* = 1/F_{LL} < 0$.

(d) Employment Function for the Urban Sector. Since w_u^* is a function of the nominal wage paid by the firm and the quit rate, and both of these in turn are simply functions of the rural wage and the unemployment rate, we can obtain (by substituting (2.7) into (2.3))

$$w^* = w_u + qT = \phi(U, w_r) + Tq\left(1, \frac{\phi(U, w_r)}{w_r}, U\right) = w^*(U, w_r) . \qquad (2.11)$$

[1]In principle, the firm is interested in maximizing the discounted cash flow, where presumably the discount rate is equal to the rate of interest. The requisite modifications in (2.9) are straightforward; their main effect is simply to complicate the algebra.

It is clear that normally increasing the unemployment rate must decrease labor costs, since at the same wage, quit rates will be reduced, and so labor costs are reduced; a fortiori when firms adjust their wages to minimize labor costs, labor costs are reduced.[1]

$$\frac{\partial w^*}{\partial U} < 0 \ . \tag{2.11a}$$

Similarly, increasing the rural wage increases labor costs and training costs:

$$\frac{\partial w^*}{\partial w_r} = Tq_2 \left(\frac{\phi_{w_r}}{w_r} - \frac{\phi}{w_r^2} \right) + \phi_{w_r} > 0 \tag{2.11b}$$

(using (2.7b)).

Finally, substituting (2.11) into (2.10), we obtain the demand for labor as a function of U and w_r

$$L_u = L_u^d(w_u^*(U, \ w_r)); \ K_u) \tag{2.12}$$

$$\frac{\partial L_u}{\partial w_r} < 0 \qquad \frac{\partial L_u}{\partial U} > 0 \ .$$

(e) <u>The Rural Sector and Urban-Rural Labor Allocation</u>. Elsewhere,[2] we have described how, at a given expected urban wage, the labor force allocates itself between the urban and rural sector. The allocation depends

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[1]
$$\frac{dw^*}{dU} = \left(1 + \frac{q_2 T}{w_r} \right) \phi_U + Tq_3$$
$$= T\left(q_3 - \frac{q_1}{w_u} \phi_U \right) < 0$$

(using (2.7a)).

[2] J.E. Stiglitz, "Rural-Urban Migration, Surplus Labor, and the Relationship between Urban and Rural Wages," <u>E. Africa Econ. Rev.</u>, December 1969.

on, among other things, the pattern of land ownership (whether owned privately or communally) and arrangements for "sharing" among members of a community. We argued there that in the African context, probably the most reasonable assumptions[1] yielded the result that labor allocated itself so that the marginal productivity of labor in the rural sector equalled the expected urban wage.[2] Other circumstances were delineated[3] where labor allocated itself so that the average productivity of labor in the rural sector equalled the expected urban wage. For most of the analysis, it makes little difference whether we work with the marginal or average product hypothesis. We shall accordingly assume that the expected wage in the urban sector equals the marginal product in the rural.

The production function in the rural sector is described by

$$Q_r = G(K_r, L_r, A)$$

where A stands for Arable Land. Again, for our short run analysis, we shall assume that K_r and A are fixed.

There has been some controversy over whether an "open" economy or "closed" economy model is more appropriate to various developing countries. There seems to be some consensus that at least for most smaller African

[1] E.g. if workers in the rural sector get paid (earn) their marginal product, and if they leave the rural sector, they keep their land there and so still earn "rents" on it. Indeed the common pattern in the African situation is that some members of the family remain in the rural sector, working the land, and some migrate (temporarily) to the urban sector. Thus in deciding whether to migrate or not, they need only look at their marginal product in the rural sector in comparison with their expected income in the urban sector.

[2] We ignore considerations of risk aversion, which although undoubtedly of importance, do not change the qualitative picture being presented here.

[3] In particular, this required the absence of a landless rural laboring class receiving as a wage its marginal product, and communal ownership of land, with the further stipulation that when individuals left the rural area they no longer received any returns from the land. These assumptions are clearly not satisfied in most African economies.

countries, the open economy model is more appropriate, and hence in the subsequent analysis we shall pursue its implications. However, the analysis may easily be modified for the closed economy situation.

This assumption enables us to choose our units so that the price of output in both the rural and urban sectors is (constant at) unity.

Then we shall denote the value of the marginal productivity of labor in the rural sector by w_r :

$$w_r = G_{L_r} \tag{2.13}$$

and our rural-urban equilibrium labor allocation condition can be written

$$w_u^e = G_{L_r} = w_r \ . \tag{2.14}$$

(f) "Nominal" and "Expected" Urban Wages and the Unemployment Rate. The expected wage in the urban sector differs from the nominal wage because some part of the time that the individual is in the urban sector he will probably be unemployed. The length of time unemployed depends on the model of hiring hypothesized. The two simplest are[1]

(i) a random selection from the unemployment pool, which leads to a Poisson distribution of the period of unemployment;

(ii) a queue model, in which the individuals are hired in order of time of arrival in the urban sector.

Both are extreme cases of a more general model where the probability of being hired depends on the length of time in the unemployment pool. Initially, it takes some time to make contacts with potential employers and with individuals who can make contacts with employers. The longer one is in the city, the more extensive the network of contacts is, and hence

[1] Harris and Todaro have considered still a different model in which individuals go to the hiring hall every day; the probability of being selected for work is just $1-U$, so the expected wage is $w_u(1-U)$. Remarkably enough, this is exactly the result yielded by the two models below when there is no growth. See J.E. Harris and M. Todaro, "Migration, Unemployment and Development: A Two-Sector Analysis," AER, 1970.

the greater the probability of being hired (in any interval of time). On the other hand, employers may feel hesitant to hire someone who has been unemployed for an extensive period of time. He may have lost the requisite "work habits" and there may be some reason that other employers have turned this individual down that the employer in question may not know about. This leads eventually to a decrease in the probability of being hired in any given interval of time.

In any case the expected wage is then a function of the nominal wage in the urban sector, the expected duration of unemployment, and the total anticipated time on the job (in the urban sector):

$$w_u^e = \frac{w_u D}{D + t_u} = \frac{w_u}{1 + t_u q} \tag{2.15}$$

where D is the expected duration on the job $(= 1/q)$ and t_u is the expected duration of unemployment.

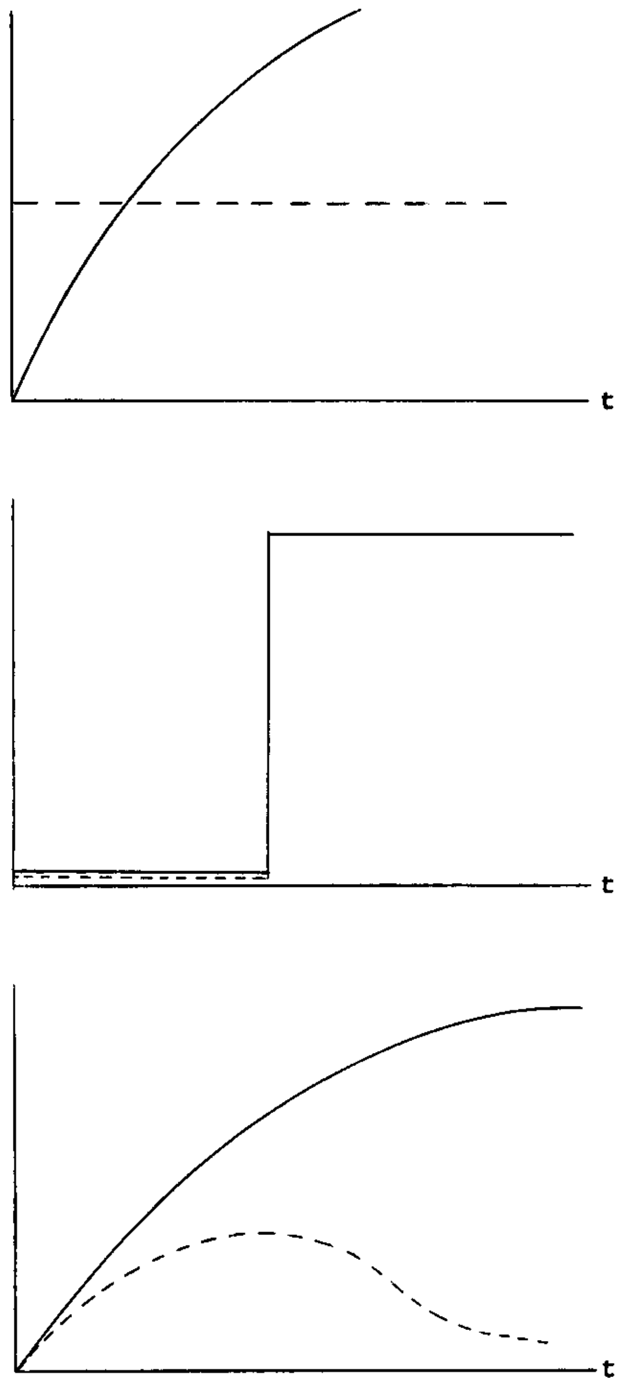
To see what qt_u depends on, consider first the "Poisson model"; the probability of being hired in any period is equal to the number of "hires" divided by the number of job seekers; the number of hires is equal to the number of new jobs being created, $g_u L_u$, where g_u is the rate of growth of jobs, plus the number of replacements arising from quits, qL_u.[1]

The number of job seekers is equal to the number unemployed. Let N_u be the total urban labor force, so

$$\frac{L_u}{N_u} = 1-U$$

and the remainder unemployed is just UN_u. Thus the probability of being

[1] It may have been noted that here as in (2.15) we use the same symbols for "duration in the urban sector" as we used earlier for "duration on the job." Although this makes sense in the context of our simplified model, since in the equilibrium all firms are identical and pay the same wage, so there is in equilibrium no intra-firm movement of labor in the urban sector, it should be emphasized that this is not an essential assumption in the analysis. We could have written (2.15) replacing q with $\hat{q}$ where $\hat{q}$ is the rate of leaving the urban sector, and the expression we derive below (2.18) would be unaffected.



(a) Poisson Model

(b) Queue Model

(c) More General Model

- - - probability of being hired in any interval of time

——— percentage of individuals arriving in urban sector employed by time t .

FIGURE 2.5

Alternative Hiring Patterns

hired in any small interval dt is just

$$\frac{L_u(g_u + q)}{UN_u}\,dt = \frac{(g_u + q)(1-U)dt}{U} \tag{2.16}$$

from which it immediately follows that

$$t_u = \frac{U}{(1-U)(g_u + q)} . \tag{2.17}$$

Substituting into (2.15) we obtain

$$w_u^e = \frac{w_u(1-U)}{1 - \dfrac{g_u q}{q + g_u}} \tag{2.18}$$

$$w_u^e = w_u(1-U) \quad \text{when} \quad g_u = 0 . \tag{2.18'}$$

If, alternatively, we consider the queue model, we obtain almost identical expressions. The flow of individuals into the urban sector equals (in a static equilibrium) the flow out: qL_u . The size of the unemployment pool is equal to the flow into it times the average duration of unemployment:

$$qL_u t_u = UN_u$$

or

$$t_u = \left(\frac{U}{1-U}\right)\frac{1}{q} . \tag{2.19}$$

Again substituting into (2.15) we obtain (2.18').[1] For simplicity, in the

---

[1]If the urban sector is growing at the rate g_u , we have

$$qL_u \int_{-t_u}^{0} e^{g_u t} = UN_u$$

or

$$\frac{q}{g_u}\left(1 - e^{-g_u t_u}\right) = \frac{U}{1-U}$$

When substituted into (2.15) this yields an expression slightly different from (2.18).

subsequent analysis we shall let $g_u = 0$. The analysis may easily be modified for the case $g_u > 0$.

Substituting (2.18') into (2.14), we obtain as our labor market equilibrium condition

$$G_L = w_r = w_u(1-U) \tag{2.19}$$

or

$$\frac{w_u}{w_r} = \frac{1}{1-U} .$$

One objection may be made to (2.19): it probably predicts too large unemployment rates for the observed magnitudes of wage differentials. For instance, the ratio of real wages in the two sectors are often of the order of magnitude 1.5 to 2,[1] so that the unemployment rate should be (according to (2.19)) of the order of magnitude of 33% to 50%. The difference may be accounted for by several factors, some of which we have already noted: (a) risk aversion: the uncertainty of obtaining a job undoubtedly deters a large number of individuals from coming to the urban centers, and leads to the actual level of unemployment corresponding to any given urban-rural wage ratio being smaller than predicted by equation (2.19). (b) Similarly, lack of liquidity (imperfect capital markets) results in individuals being unable to stay in the urban centers for extended periods of time if they do not get a job. (c) We have implicitly assumed that the individual has to be in the urban center in order to seek an urban job; in fact, individuals in the rural sector have contacts in the urban centers, and although their probability of getting an urban job is undoubtedly significantly enhanced by being in the urban center, it probably is still not worth their while to be away from the rural sector at peak demand times (harvest and planting) in the rural sector. (d) The exact form of (2.19) depended on one of the extreme hiring models (the queue or the Poisson model); other hiring patterns would yield different equations, although other models may actually lead to higher unemployment rates for given wage rate ratios (as, for example, if there is a growing labor force in the urban sector). (e) Transportation costs also discourage migration. (f) There may be relative non-pecuniary advantages of living in the rural districts, which also will discourage migration.[2]

[1] These probably exaggerate real wage differentials among unskilled laborers; relative prices differ markedly, so there is a serious index number problem.

[2] This is offset to some extent by the greater variability of agricultural income.

It is reasonable to assume, however, that in any case the equilibrium level of unemployment is a function of the ratio of urban to rural wages. Thus, we write the following generalization of (2.19)[1]

$$\frac{w_u}{w_r} = \varphi\left(\frac{1}{1-U}\right) \qquad \varphi' > 0 \ . \tag{2.20}$$

3. The Market Equilibrium

The equilibrium of the economy is described by the wage determination equations for the urban and rural sectors, the labor allocation-migration equation, and the equilibrium condition

$$L_r + N_u = L_r + \frac{L_u}{1-U} = L \tag{2.21}$$

where L is the total labor force: workers are either in the rural sector (L_r), employed in the urban sector $\left[(1-U)\,\frac{L_u}{1-U} = L_u\right]$ or unemployed $\frac{UL_u}{1-U}$.

To see graphically the solution, we rewrite the first order condition (2.5), making use of the equilibrium conditions (2.5) and (2.20)

$$\begin{aligned} w_u &= T\left(q_1\left(1, \varphi\left(\frac{1}{1-U}\right), U\right) + q_2\left(1, \varphi\left(\frac{1}{1-U}\right), U\right)\varphi\left(\frac{1}{1-U}\right)\right) \qquad (2.22) \\ &= h(U) \ . \end{aligned}$$

---

[1] To see this, we return to (2.15). The duration in the urban sector is an increasing function of the unemployment rate and a decreasing function of the hiring rate. In a static economy, the rate of hiring, as we have noted, is just equal to the quit rate, and from the earlier analysis, the quit rate is a function (in equilibrium) of the urban wage ratio and the unemployment rate. Thus, we have

$$\frac{w_u}{w_r} = 1 + q\left(1, \frac{w_u}{w_r}, U\right)t_u\left(U, q\left(1, \frac{w_u}{w_r}, U\right)\right)$$

which can be solved for w_u/w_r as a function of U.

Under the hypothesis introduced above in Equations (2.7a) and (2.7b) it is easy to establish that[1]

$$h'(U) < 0 \ . \qquad (2.23)$$

Then, from (2.20) we have

$$w_r = \frac{w_u}{\varphi(1/1-U)} = \frac{h(U)}{\varphi(1/1-U)} \qquad (2.24)$$

and from (2.3)

$$w^* = h(U) + Tq(1, \varphi(1/1-U), U) = w^*(U) \qquad (2.25)$$

$$\frac{dw^*}{dU} < 0 \qquad (2.25a)$$

the rural wage and total unit labors costs in the urban sector may both be simply written as functions of U . For each value of U , then, we can calculate the demand for labor (using (2.10)). Since labor costs move inversely with respect to U , as U increases, demand for labor in the rural sector increases, urban employment increases, and urban unemployment increases. Equilibrium is the point where (2.21) is satisfied. See Figure 2.6.

Figure 2.6 may be used to give us some quick comparative static results: (a) An increase in g_u , which results in a lower value of w_u/w_r corresponding to any given U (or any other change having the same

[1]For most of the ensuing analysis, it is the restriction embodied in (2.23) and not the restrictions embodied in (2.7) that is crucial. The relationship between the two can easily be seen by taking the derivative of (2.22) with respect to U and comparing the resulting expression with (2.7a') and (2.7b') derived above.

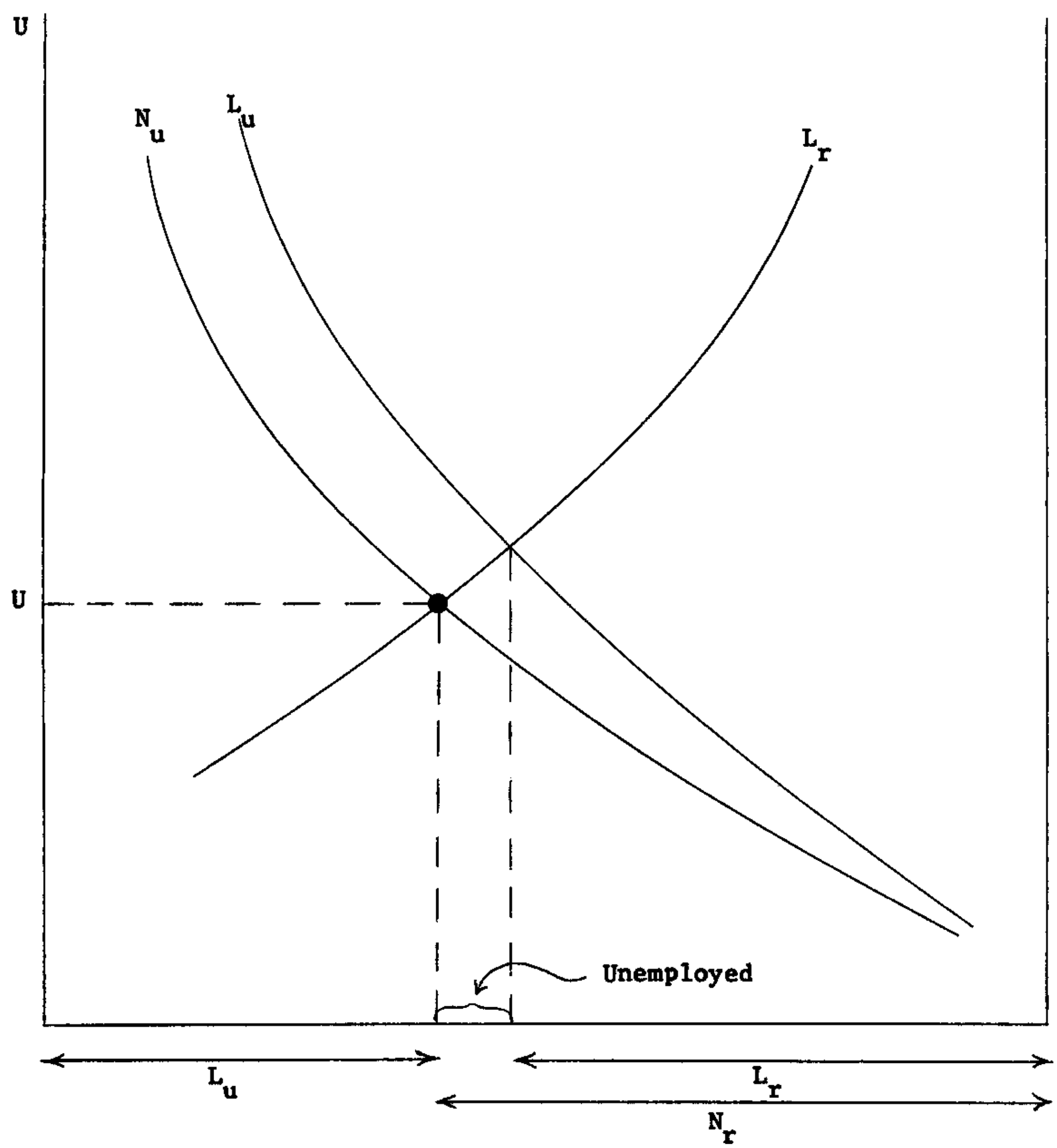


FIGURE 2.6

Market Equilibrium

effect on φ) raises the unemployment rate.[1] (See Figure 2.7a.) (b) A labor augmenting invention in the rural sector lowers the unemployment rate and the level of urban employment. (See Figure 2.7b.)[2] (c) A labor augmenting invention in the urban sector lowers[3] the unemployment rate and the level of rural employment (raises the rural wage). (d) The effects of an increase in the capital stock in the rural and urban sector are the same as those discussed in (b) and (c).[4]

---

[1]Let $\varphi[(1/1-U), \alpha] = w_u/w_r$ where α is a shift parameter. Let $\varphi_\alpha < 0$. Then

$$\left.\frac{\partial w_u}{\partial \alpha}\right|_{\bar{U}} = -T(q_{12} + q_{22}\varphi + q_2)\varphi_\alpha < 0$$

under conditions given in footnote 1, p.12. Similarly

$$\frac{\partial w^*}{\partial \alpha} = -T(q_{12} + q_{22}\varphi)\varphi_\alpha > 0$$

$$\frac{\partial w_r}{\partial \alpha} = \left(\frac{\partial w_u}{\partial \alpha}\right)_{\bar{U}} - \frac{h}{\varphi^2}\varphi_\alpha > 0 .$$

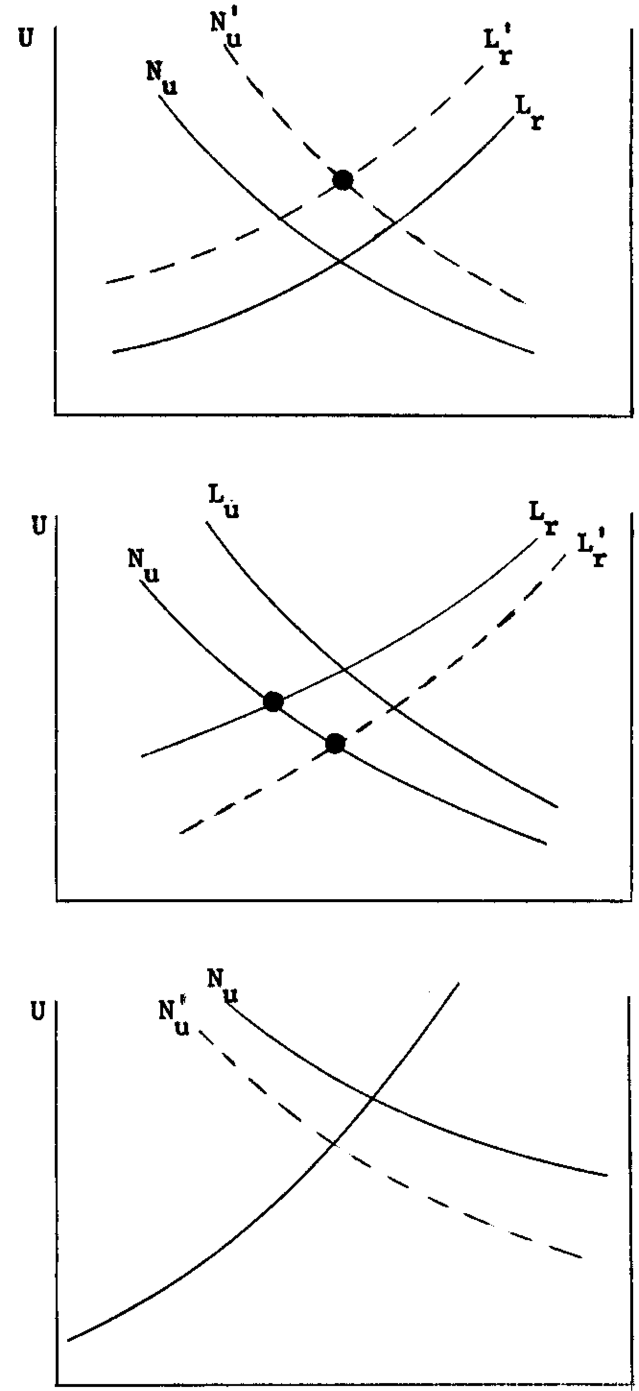
[2]Provided it is not "Pigou labor saving," i.e. provided the elasticity of substitution is sufficiently great.

[3]Under the same conditions given above, footnote 2.

[4]We can make use of the ϕ function derived earlier for an alternative way of showing the determination of the equilibrium. Consider first the case with w_r fixed (constant marginal product of labor in the rural sector). Substituting (2.8) into (2.20), we obtain

$$\frac{w_u}{w_r} = \frac{\phi(U, \bar{w}_r)}{\bar{w}_r} = \varphi\left(\frac{1}{1-U}\right) .$$

Since $\phi_U < 0$, $\varphi' > 0$ there is a unique equilibrium value of unemployment, as depicted in Figure 2.8. Knowing U , we can immediately solve for w_u/w_r . To determine the labor allocation between the urban and rural sector, we observe that w_u has a fixed relation to w_r , and $w^* = w_u + Tq\left(1, \left(\frac{1}{1-U}\right), U\right)$; L_u , employment in the urban sector can also be written as a declining function of w_r . But since U is given, the total number of individuals in the urban sector is simply $L_u/1-U$. See Figure 2.8(b).



(a) Increase in g_u

(b) Effects of Labor Augmenting Invention in Rural Sector

(c) Effects of Labor Augmenting Invention in Urban Sector

FIGURE 2.7

Comparative Statics

4. Optimal Allocation of Labor and Determination of Urban Wage Level

The preceding section provided an endogenous theory of the determination of an equilibrium level of unemployment and wage differential between the urban and rural sectors. How does this equilibrium compare with that which would be generated by a government attempting to maximize the value of national output but which cannot control migration directly?

The government's objective then is to maximize net national output

$$Q_u + Q_r - TL_u q \tag{2.26}$$

subject to the contraints

$$L_r + \frac{L_u}{1-U} = L \tag{2.21}$$

the labor allocation constraint, and

$$w_u/w_r = \varphi(1/1-U) \tag{2.20}$$

the free migration equilibrium condition (where as before $w_r = G_L$). Assume that the government directly controls the urban sector (but not the rural sector). Then it can "choose" L_u and w_u. But rather than maximize (2.26) subject to the constraints (2.20) and (2.21) with respect to these variables, let us assume the government controlled directly L_u and U, the unemploy-

---

In the more general case, for each value of w_r, we can solve for U. As w_r increases, $\frac{\varnothing}{w_r}$ decreases (i.e. w_u increases, but less than proportionately (equation (2.7)). Hence U decreases, as does w_u/w_r. Since for each value of w_r, we can calculate the equilibrium value of w_u and U, we can calculate the level of urban employment and unemployment. Since as w_r increases demand for rural labor decreases, and as w_u increases, demand for labor in the urban sector decreases and since U decreases, N_u decreases, there is a unique value of w_r for which (2.21) is satisfied. See Figure 2.9(b).

These diagrams can be used to show directly effects of various shifts in the functions. See, for instance, Figure 2.10.

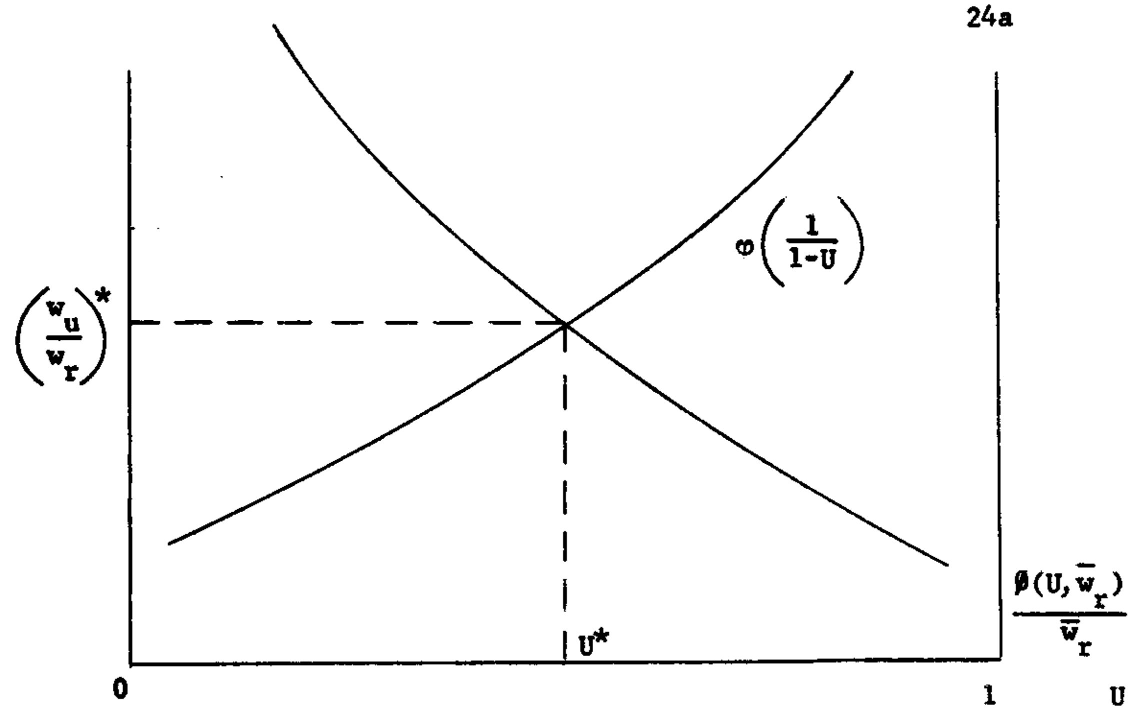


FIGURE 2.8(a)

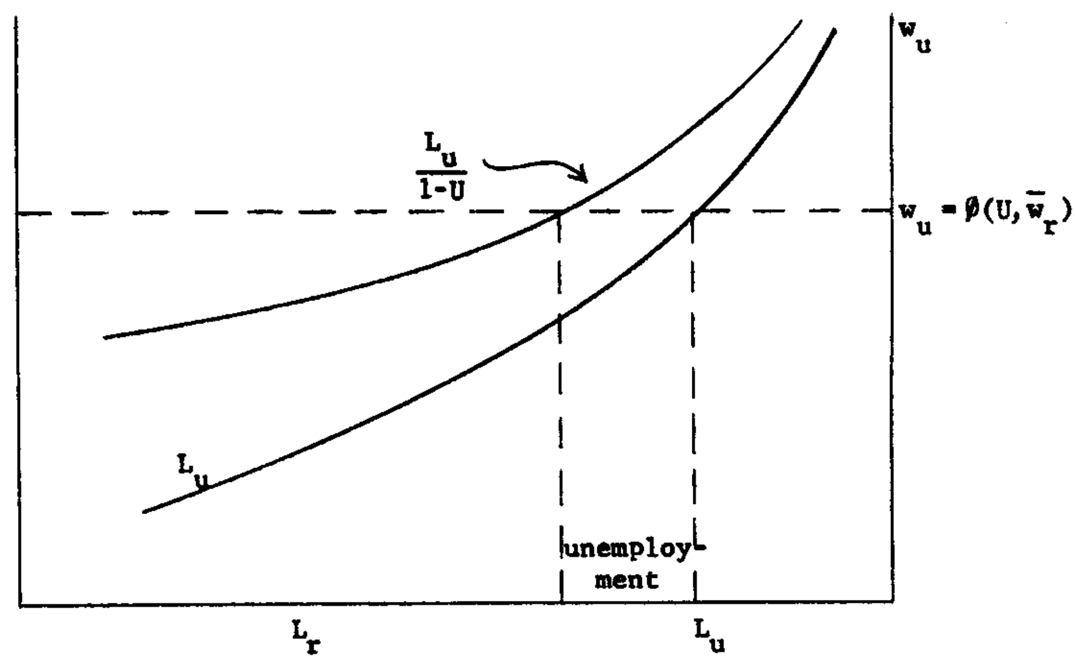
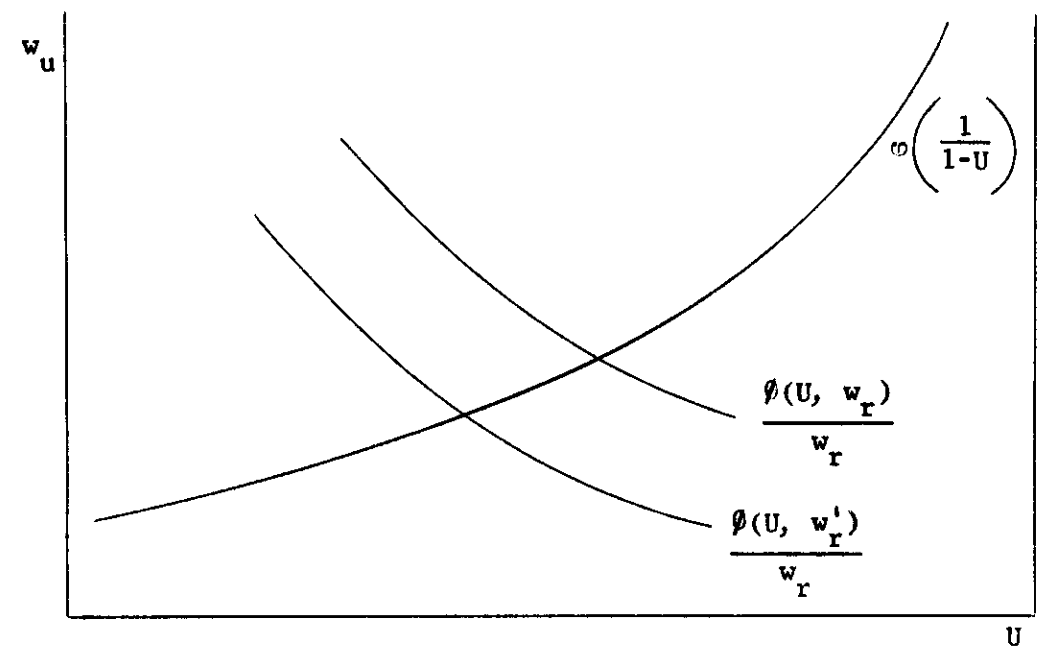
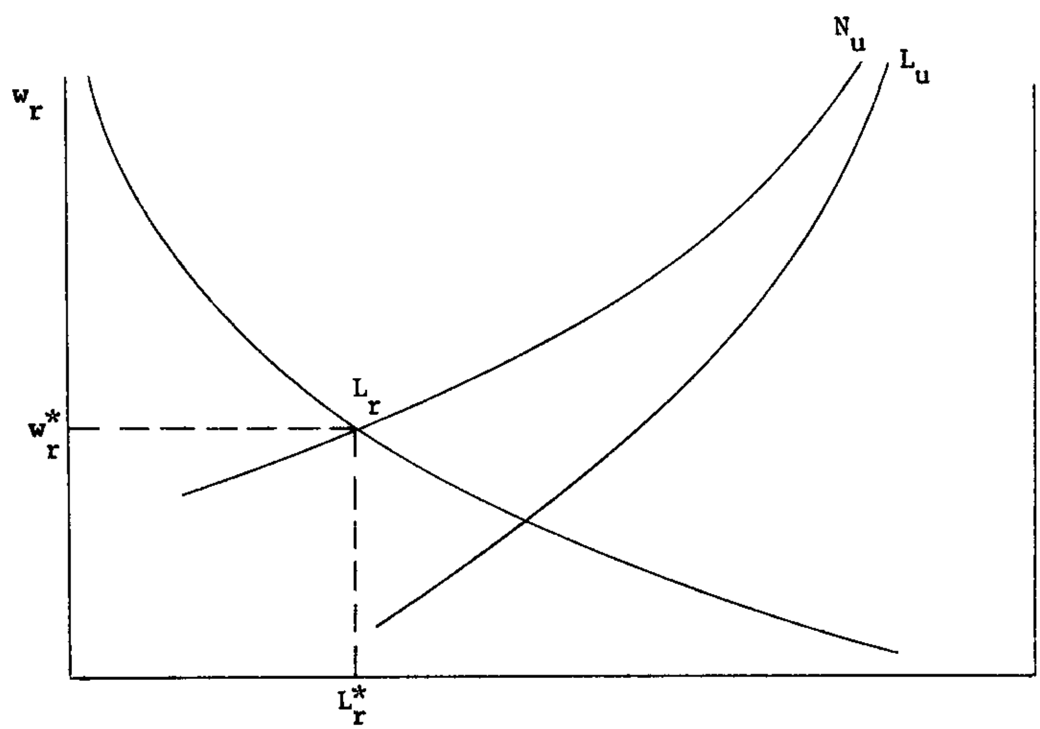


FIGURE 2.8(b)

Determination of Equilibrium: Constant w_r



(a)



(b)

FIGURE 2.9

Determination of Equilibrium

ment rate. By solving this problem, there will emerge an equilibrium w_u, which, if the government set the wage at that level, would generate the indicated level of unemployment.

Hence our problem may be reformulated (assuming K_u, K_r, and A are fixed).

$$\max_{\{U, L_u\}} G\left(L - \frac{L_u}{1-U}, K_r, A\right) + F(L_u, K_u) - Tq\left(1, \varphi\left(\frac{1}{1-U}\right), U\right) \quad \text{Lu} \quad (2.27)$$

which yields the first order conditions

$$\frac{-G_L}{1-U} + F_L - Tq = 0 \qquad \text{Lu} \qquad (2.28a)$$

$$\frac{-G_L L_u}{(1-U)^2} - T\left(\frac{q_2\varphi'}{(1-U)^2} + q_3\right)L_u = 0\ . \qquad (2.28b)$$

These may be simplified to read:

$$\frac{w_r}{(1-U)^2} + T\left(\frac{q_2\varphi'}{(1-U)^2} + q_3\right) = 0 \qquad (2.29a)$$

These etc. only if $\phi(\frac{1}{1-U}) = \frac{1}{1-U}$

$$\frac{w_r}{1-U} = \frac{G_L}{1-U} = \frac{w_u}{\phi(\frac{1}{1-U})(1-U)}$$

$$\frac{w_u}{\phi(\frac{1}{1-U})(1-U)} = F_L + Tq\ . \qquad (2.29b)$$

(2.29a) and (2.29b) should be compared with (2.4) and (2.9), the comparable equations for the competitive economy.

The first remarkable result is that (2.29b) shows that <u>the "shadow price of labor" in the urban sector is equal to the urban wage, even though there is unemployment</u>. The reason for this should be clear: at a fixed wage, when 100 additional workers are hired by the urban sector if the unemployment rate is say 5%, 105 workers leave the rural sector. The opportunity cost of labor is more than 5% below less than the urban wage, so ~~but~~ the induced unemployment ~~just~~ offset~~s~~ this. fails to offset

NO

$$\frac{w_u}{w_r} = \phi\left(\frac{1}{1-U}\right)$$

for given U, $w_u(1-U) > w_r$

$$\frac{w_u}{w_r} > \frac{1}{1-U}$$

$$\therefore \phi > \frac{1}{1-U}$$

$\phi(1-U) > 1$, so $w_u > F_L + Tq$

On the other hand, the firms are likely to make an incorrect wage-turnover rate decision. They make two mistakes in their calculations, which work in opposite directions. First, they take the unemployment rate as given. But when all firms increase the wage rate, it does affect the unemployment rate and hence lowers the turnover costs. This by itself would tend to lead to too low wages relative to the rural sector. Offsetting this is the fact that each firm believes it can get some competitive advantage relative to other firms in the urban sector in reducing labor turnover (due to movement of labor within the urban sector) by increasing its wage relative to them (i.e. q_1 was assumed to be negative). This effect is likely to be greater than the direct unemployment effect, and hence firms are likely to pay too high wages, leading to excess unemployment.

To see more clearly these differences, we rewrite (2.29a) as

$$w_u = -T\left(q_2\varphi\left(\frac{1}{1-U}\right)\varphi' + q_3\varphi(1-U)^2\right) = h^o(U) \tag{2.30}$$

which should be contrasted with (2.4)

$$w_u = -T\left(q_2\varphi\left(\frac{1}{1-U}\right) + q_1\right) = h^c(U) \ .$$

(Throughout the remainder of the paper we shall use superscript c to refer to functions, values of variables, etc. in the competitive market solution, and a superscript 0 for the optimal solution.) Hence, the value of w_u corresponding to any unemployment rate is larger or smaller in the optimum than in the market solution as

$$-(q_3\varphi(1-U)^2 - q_1 + q_2(\varphi' - 1)) \gtrless 0 \ . \tag{2.31}$$

Note that in the case of (2.19), $\varphi' = 1$, so this just reduces to

$$-(q_3\varphi(1-U)^2 - q_1) \gtrless 0 \ . \tag{2.31'}$$

Normally, we argued that the "interurban competition effect" q_1 was larger than the secondary effect of the change in unemployment on turnover; $-q_1 > -q_3\varphi(1-U)^2$, so w_u at any U is smaller in the optimal allocation. But the presumption that $h^o(U) < h^c(U)$ is weakened by the fact that φ' is normally greater than 1.[1]

Thus, it is not even assured that the urban wage (at any unemployment rate) is too high, as is usually asserted. If the L.H.S. of (2.31) is zero, then the urban wage is optimal. In the ensuing analysis, we shall follow the conventional presumption in calling the case where $h^c(U) > h^o(U)$ the "normal case."

The implications of this for the general equilibrium are easily set out. We first calculate

$$w^{*o} = w_u + Tq = h^o(U) + Tq\left(1,\ \varphi\left(\frac{1}{1-U}\right),\ U\right) = w^{*o}(U) \qquad (2.32)$$

and

$$w^{*o}(U) \gtrless w^{*c}(U) \quad \text{as} \quad h^o(U) \gtrless h^c(U) \quad \text{for any given } U .$$

Turning to Figure (2.11) we observe that since the optimal solution involves a lower w_u^* at every U, it involves a higher L_u (since (2.29b) and (2.9) are identical). Thus the N_u curve shifts to the left. Since w_u is lower at every U, w_r is also lower, and L_r greater, shifting the L_r curve to the right. The net effect is to lower the unemployment rate. Thus in the "normal case"

$$U^o < U^c . \qquad (2.33a)$$

---

[1] That is, if φ' is constant, we have argued that $w_u/w_r = k/1-U$ with $k > 1$, the wage rate ratio at any unemployment level is greater than (2.19) would have predicted.

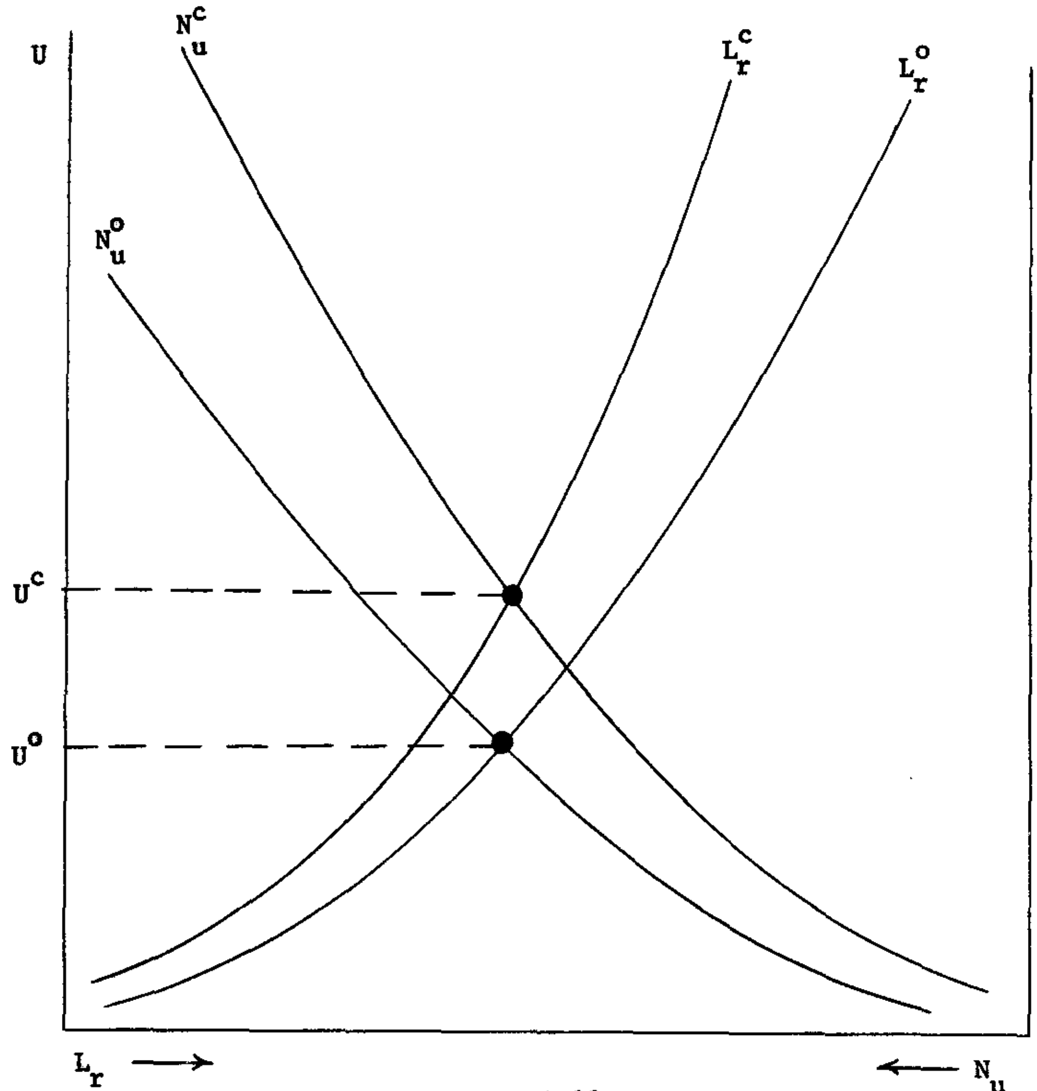


FIGURE 2.11

Comparison of Competition and Market Equilibria

What is not clear from Figure 2.11 is whether (a) rural employment or wages have increased or decreased, and (b) whether urban employment or wages have increased or decreased.

We can show that urban wages are lower in the optimal situation than in the competitive[1]

$$w_u^o < w_u^c . \tag{2.33b}$$

The other changes are of ambiguous sign. The direct effect of the "shift" of the wage curve, ($h^c(U)$ to $h^o(U)$) is to lower urban and rural wages and increase urban and rural employment in going from the competitive to the optimal allocation, but the secondary effect--the decrease in the unemployment rate --is in just the opposite direction. Moreover, it should be clear that if rural employment goes down (w_r rises) since U has fallen, w_u^* must fall. If rural wages fall, the more employment "absorbed" into the rural sector as a result, the more likely is w_u^* to rise (urban employment to fall).

Thus, certain polar cases are easy to analyze. Assume $L_r' = 0$, i.e. demand for labor in the rural sector is inelastic. Then $L_u/1-U$ is constant, so the lower U means a higher L_u, a lower w^*, and since training costs rise as U is lowered, a lower w_u. w^* is the more likely to rise (L_u fall) if the labor supply in the rural sector is very elastic (so any given change in w_r results in a large change in L_r, i.e. in $L_u/1-U$) and if L_u is small relative to L_r (so a given percentage change in L_r results in a large absolute and relative change in $L_u/1-U$). On the other hand, w^* is the less likely to rise if a given change in U results in a large change in w_u/w_r. If, for instance,

[1] Obviously, if $h^c(U) > h^o(U)$ the inequalities in (2.33a) and (2.33b) are reversed.

$$w_r = \frac{w_u(1-U)}{k} ,$$

a given change in $w_u(1-U)$ will have a smaller effect on w_r (and hence on L_r) if k is large.

For reasonable values of the parameters, it turns out that whether $w^{*o} \gtrless w^{*c}$ is ambiguous. For instance, if

$$k = 1 , \quad \frac{N_u}{L_r} = .1 , \quad \frac{L_r' w_r}{L_r} = 2 , \quad U = .2$$

then

$$w^{*c} < w^{*o}$$

while if all the other parameters are unchanged, but $k = 4/3$, $w^{*c} > w^{*o}$ similarly, if $w_u/w_r = 2$ when $U = .2$ so $k = 1.6$, then $w^{*c} > w^{*o}$ regardless of N_u/L_r or $L_r' w_r/L_r$. But if $k = 1.6$, $U = .3$, $-L_r' w_r/L_r = 2$, and $N_u/L_r = .1$, then $w^{*c} < w^{*o}$.

Just as it is the elasticity of demand for agricultural workers which determines what happens to urban workers, it is the elasticity of demand for urban workers that determines what happens to rural workers. If $L_u' = 0$ (the demand for urban workers is inelastic) then L_r must increase when U decreases, so w_r must fall. Unlike the previous case however, if $w_u/w_r = k/1-U$ with $k \geq 1$,[1]

$$L_r^o > L_r^c . \tag{2.33c}$$

---

[1]We are considering the consequences to the equilibrium of a shift in the $h(U)$ function; let h be a function of a shift parameter α : $h(U,\alpha)$, where $h(U,\alpha) = \alpha h^c + (1-\alpha)h^o$. Then equilibrium requires

An alternative way of seeing the difference between the optimal and competitive wage policies is an extension of the "monopolistic competition" diagram used earlier.

$$\frac{L_u(w^*(U,\alpha))}{1-U} + L_r\left(\frac{h(U,\alpha)}{\varphi(1/1-U)}\right) = L$$

$$\frac{dU}{d\alpha} = -\frac{L_u'\,\frac{\partial w^*}{\partial \alpha} + \frac{L_r'}{\varphi}\,\frac{\partial h}{\partial \alpha}}{L_u'\,\frac{\partial w^*}{\partial U} + L_r'\left(\frac{h_u}{\varphi} - \frac{h}{\varphi}\,\frac{\varphi'}{\varphi}\right) + \frac{L_u}{(1-U)^2}}$$

$$\frac{dw^*}{d\alpha} = \frac{\partial w^*}{\partial \alpha} + \frac{\partial w^*}{\partial U}\,\frac{\partial U}{\partial \alpha}$$

$$\sim \left[\frac{L_r'}{\varphi}\left(h_u - h\,\frac{\varphi'}{\varphi} - \frac{\partial w^*}{\partial U}\right) + \frac{L_u}{(1-U)^2}\right] h_\alpha$$

$$= \left[\frac{L_r'}{\varphi^2}\left(\frac{w_u^o}{(1-U)^2} - h\varphi'\right) + \frac{L_u}{(1-U)^2}\right] h_\alpha$$

$$\approx \frac{L_r'}{\varphi^2}\, h_\alpha w_u\left(\frac{1}{(1-U)^2} - k\right) + \frac{L_u h_\alpha}{(1-U)^2}$$

$$= \left[\frac{L_r'}{L_r}\,\frac{w_r}{k}\left(\frac{1}{(1-U)^2} - k\right) + \frac{N_u}{L_r}\right]\frac{L_r h_\alpha}{(1-U)}$$

Assume $k = 1$; $N_u/L_r = .1$, $(L_r'/L_r)w_r = -2$, $U = .2$. Then

$$\frac{dw^*}{d\alpha} \sim \left[.1 - 2 \times \frac{9}{16}\right] < 0$$

If $w_u/w_r = 2$ when $U = .2$, so $k = 2 \times .8 = 1.6$,

$$\frac{dw^*}{d\alpha} \sim \left[.1 + \frac{2 \times .0375}{2.16}\right] > 0$$

In Figure 2.12, we have drawn the training cost curve qT as a function of w_u, for given w_r. The slope of the curve perceived by the firm differs from the actual slope for the reasons discussed earlier in connection with (2.31). If (2.31) were equal to zero, the wage would be optimal. If, as we would expect, (2.31) is positive, the competitive wage is above the optimal level, resulting in a higher level of unemployment, less employment in the urban sector, and a lower rate of turnover than is optimal.

In Figure 2.12 we have also compared the competitive and optimum equilibria.[1]

$$\frac{dL_r}{d\alpha} = \frac{1}{1-U}\frac{dL_u}{d\alpha} + \frac{L_u}{(1-U)^2}\frac{dU}{d\alpha}$$

$$\sim \left(\frac{1}{1-U}\right)\frac{L_u'L_r'}{\varphi^2}\left[\frac{w_u^o}{(1-U)^2} - h\varphi'\right]h_\alpha - \frac{L_uL_\alpha}{(1-U)^2}\frac{L_r'}{\varphi}$$

If $\varphi = \frac{k}{1-U}$

$$\frac{dL_r}{d\alpha} \sim \left(\frac{w_u^o}{k} - w_u k^2\right)\frac{L_u'}{L_u} + 1 .$$

If $w_u^o \leq w_u$, at $k = 1$, $dL_r/d\alpha > 0$; but $\dfrac{\partial \frac{\partial L_r}{\partial\alpha}}{\partial k} > 0$, so $\dfrac{dL_r}{d\alpha} > 0$ for $k \geq 1$.

$$\frac{dw_u}{d\alpha} = h_\alpha + h_u\frac{dU}{d\alpha} \sim h_\alpha\left\{-L_u'\frac{w_u^o}{(1-U)^2\varphi} - L_r'\frac{h\varphi'}{\varphi^2} + \frac{L_u}{1-U}\right\}^2 > 0 .$$

($\sim$ denotes "is of the same sign as," $\approx$ denotes "approximately" (when $w_u^o - w_u^c$ is relatively small).)

[1]Figure 2.12 may easily be modified to take into account the fact that w_r will be different in the competitive and optimal allocations.

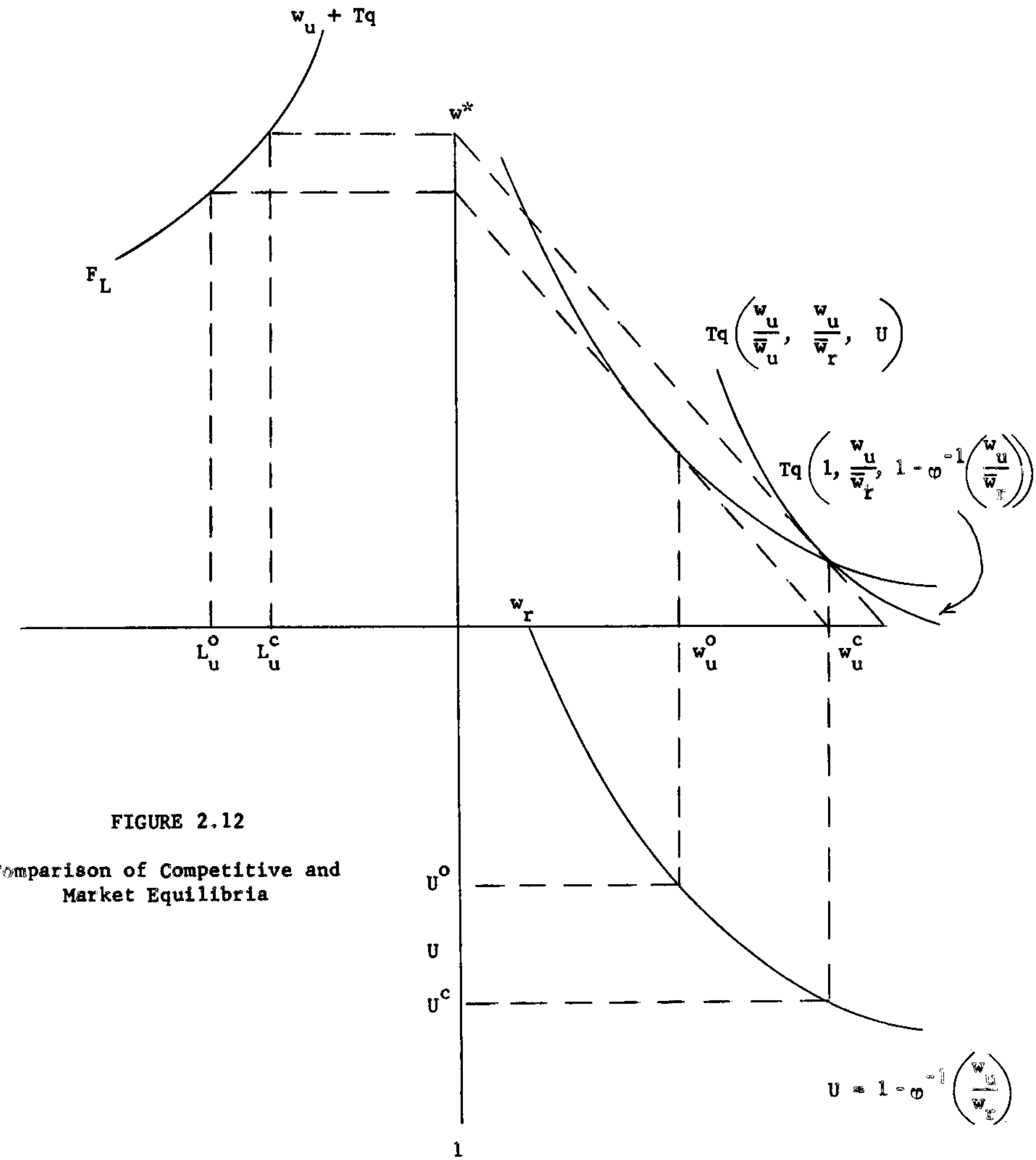


FIGURE 2.12

Comparison of Competitive and Market Equilibria

5. <u>Wage Subsidies</u>

The government can induce competitive firms to behave optimally by imposing a <u>hiring subsidy</u> (paid for by, say, a profits tax) at a rate

$$T\left(1 - \frac{(q_2\eta' + q_3(1-U)^2)\rho}{q_2 + q_1}\right)$$

where the variables are evaluated at the optimum (2.29). There are, of course, obvious practical difficulties to giving a hiring subsidy, particularly in African economies. An individual may have, for instance, several names, and he could be "hired" and "fired" successively by the same firm.

It is perhaps because of these practical difficulties most economists have advocated a wage subsidy rather than a hiring subsidy.

In discussing the consequences of a wage subsidy, we must specify (i) how the revenue for the wage subsidy is raised and (ii) what are the general equilibrium consequences both in the long run and in the short run of the wage subsidy-cum-tax system. It is in this respect that previous analyses recommending wage subsidies schemes have failed.

More specifically, previous arguments for wage subsidies have been less than convincing for the two reasons already noted in the introduction. First, they have failed to take into account the migration which would be induced into the urban sector as a result of the increased employment in the urban sector; this leads to increased unemployment (even at a fixed unemployment rate). Secondly, they have implicitly assumed that there is a fixed real wage which will be unaffected by the wage subsidy, i.e., there is no "shifting" of the wage subsidy to the employee. In our endogenous model of wage determination, we can always show that a wage subsidy leads to increased urban wages, and hence, not only does the number of unemployed individuals increase, but the unemployment rate in the urban sector actually increases as a result of the wage subsidy.

As a result of these two effects, a wage subsidy actually <u>reduces</u> G.N.P. Indeed, a wage tax is indicated.

In the discussion below, we shall assume that the tax revenue for the wage subsidy comes from a profits tax, which is not shifted at all. Thus the only consequences we need enquire into are those relating directly to the wage subsidy.

To see how a wage subsidy, say at the rate $(1-\tau)$, changes behavior, we observe that the first order condition (2.4) is now

$$1 - \tau + T\left(\frac{q_1}{w_u} + \frac{q_2}{w_r}\right) = 0 . \tag{2.4'}$$

or multiplying through by $w_u/1-\tau$, we obtain

$$w_u = -\frac{T}{1-\tau}\left(q_1 + q_2\frac{w_u}{w_r}\right) = \frac{h^c(U)}{1-\tau} . \tag{2.34}$$

Similarly

$$w_r = w_u\varphi(1/1-U) = \frac{h^c(U)\varphi(1/1-U)}{1-\tau} \tag{2.35}$$

$$\begin{aligned} w^* &= w_u(1-\tau) + Tq\left(1,\varphi\left(\frac{1}{1-U}\right)U\right) \\ &= h^c(U) + Tq\left(1,\varphi\left(\frac{1}{1-U}\right)U\right) = w^*(U) \end{aligned} \tag{2.36}$$

where, as before, $h^{c\prime} < 0$, $dw_r/dU < 0$, $w^{*\prime} < 0$.

Turning to Figure 2.13, we observe that changing τ does not affect the urban labor curves at all; but for each value of U, w_r increases when τ increases, and hence the demand for labor decreases. This results in a higher unemployment rate, a higher rural wage,[1] and more urban unemployment. Moreover, wage costs--both direct[2] and training costs--in the urban

---

[1]Since w_r is higher and U is higher, $w_u = w_r\varphi(1/1-U)$ must be higher.

[2]$w_u(1-\tau) = h^c(U)$.

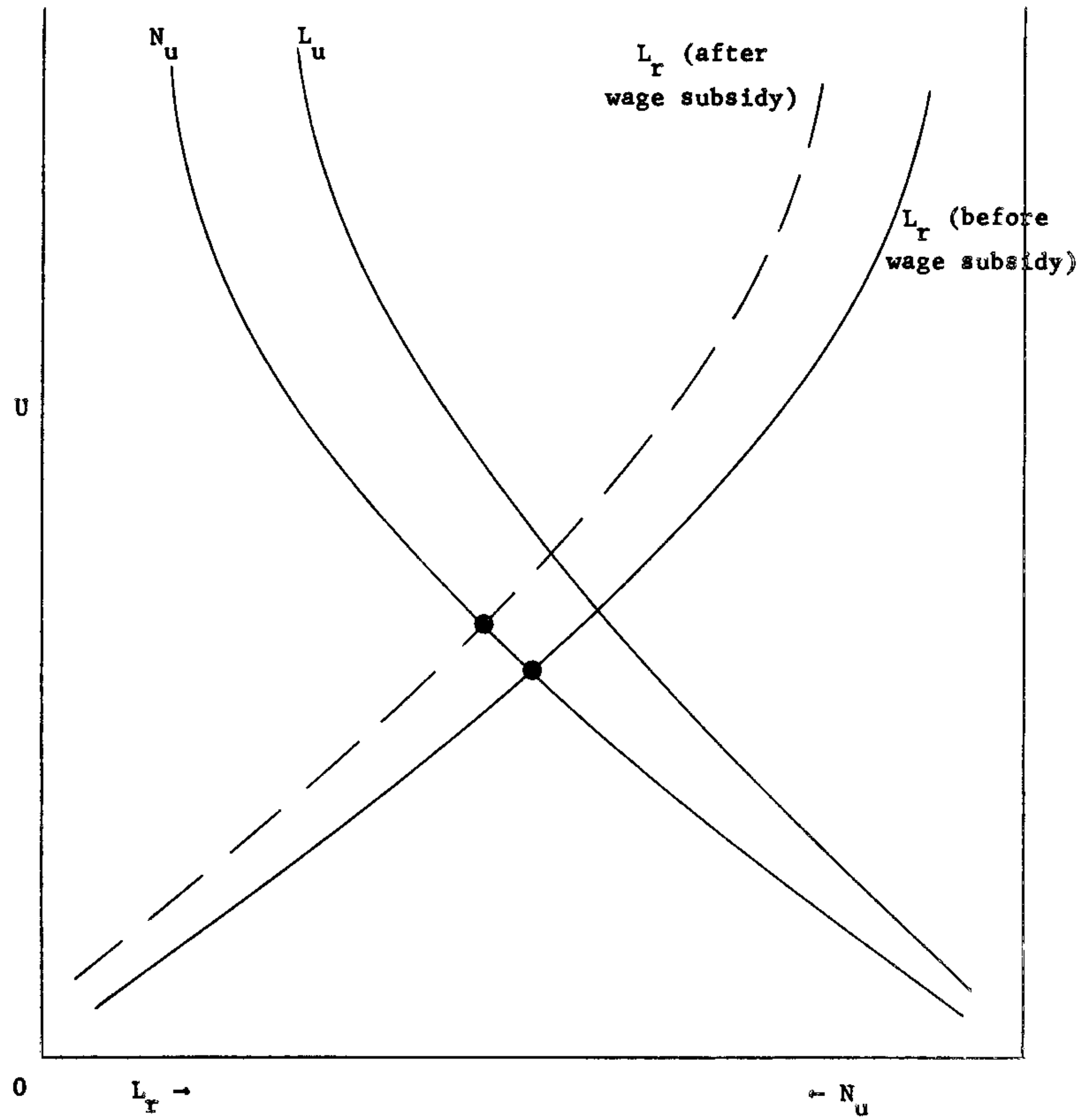


FIGURE 2.13

Effects of a Wage Subsidy

sector have decreased so urban employment has also increased.

Thus a wage subsidy does accomplish what its advocates claim--a higher employment rate--but it is accompanied by some deleterious side effects. In fact, these side effects are so strong that they imply that it is not desirable to have a wage subsidy. For at a fixed unemployment rate, the opportunity cost of hiring an extra urban worker is just his wage: for every extra urban worker hired, $1/1-U$ workers leave the rural sector, and the foregone output $G_L/1-U = w_u$, just the nominal urban wage. But now there is the additional effect of an increase in the unemployment rate, which results in additional losses in output, although this is partially --but only partially--offset by the reduced hiring/training costs from the reduced turnover rates. Thus the wage subsidy reduces national output.[1]

National output may be written

$$G\left(L - \frac{L_u}{1-U}\right) + F(L_u) - TqL_u .$$

The derivative of this with respect to τ is

$$\left(\frac{-G_L}{1-U} + F_L - Tq\right)\frac{dL_u}{d\tau} - (h^o(U) - h^c(U))L_u \frac{dU}{d\tau} < 0 \quad \text{for} \quad \tau > 0 .$$

Indeed, optimality requires

$$\tau^* = \frac{T\left(\frac{q_2\varphi'}{(1-U)^2} + q_3\right)L_u \frac{dU}{d\tau}}{w_u \frac{dL_u}{dw^*}\frac{dw^*}{d\tau}} = \frac{U\{h^c(U) - h^o(U)\}}{h^c(U)\eta_u^d \frac{\partial \ln w^*}{\partial \ln U}} < 0$$

where $\eta_u^d = d \ln L_u/d \ln w^*$. The fact that $\tau < 0$ implies that a wage tax, not a wage subsidy, is called for.

[1] Note that although there is this trade-off between urban employment and national output, there is no tradeoff between total employment and output.

The fact that $\tau < 0$ implies that a wage tax, not a wage subsidy, is called for.

Implicit in the above analysis is the assumption that training costs are fixed in "output" terms rather than in "labor" terms. More generally, we may write

$$w^* = w_u + (w_u\lambda + (1-\lambda))Tq$$

where for simplicity, we assume λ constant. With a wage subsidy, this becomes

$$w^* = w_u(1-\tau) + (w_u\lambda(1-\tau) + (1-\lambda))Tq$$

w_u is chosen so that

$$(1-\tau)\left[1 + \lambda Tq + w_u\lambda T \frac{dq}{dw_u} \right] + (1-\lambda)T \frac{dq}{dw_u} = 0$$

or

$$(1-\tau)[1 + \lambda(Tq - h(U))] - (1-\lambda) \frac{h(U)}{w_u} = 0$$

$$w_u = \frac{(1-\lambda)h}{(1-\tau)[1 + \lambda(Tq-h)]} \qquad \lambda < 1$$

$$w^* = \frac{(1-\lambda)h(1 + \lambda Tq)}{1 + \lambda(Tq-h)} + (1-\lambda)Tq = w^*(U)$$

so the analysis is unaffected provided $\lambda < 1$. In the polar case of $\lambda = 1$, we obtain:

$$\frac{dw^*}{dw_u} = (1-\tau)(1 + Tq - h(U))$$

which is just a function of U . Hence in equilibrium

$$h(U^*) = 1 + Tq(1, \varphi(1/1-U^*), U^*)$$

To find w_u, we solve the equations

$$w_r = (1-U^*)w_u$$

$$w_r = G_L(L_r)$$

$$w^* = (1-\tau)w_u(1 + Tq(1, \varphi(1/1-U^*), U^*) = F_L .$$

In this polar case, a wage subsidy does not change U ; it increases rural and urban wages, decreases rural employment, increases urban employment and unemployment and decreases national output.

Long Run Consequences of Wage Subsidies Financed by Profits Tax

So far, we have analyzed the short run consequences of the wage subsidy. The major long run consequences with which we need be concerned are those arising from the effects on savings.[1] If a larger proportion of profits are saved than of wages, the profits tax-wage subsidy will decrease savings on two accounts: First, we have noted that the average wage ($w_r = w_u^e$) has increased, so that total wage payments have increased. But we have also noted that output has decreased. Labor receives a larger proportion of a smaller "pie" and profits are unambiguously reduced. Thus, even if savings did not depend on the distribution of income, it would be reduced; a fortiori, when a larger percentage of profits are saved than wages. This decreased savings means in turn that the rate of creation of jobs in the future is also reduced.

Other Sources of Finance for Wage Subsidy

Other schemes for raising the revenue required for the wage subsidy besides a profits tax have also been suggested. One of the most widely discussed is a general sales tax. A general sales tax is, of course, simply equivalent to an income tax, i.e. a uniform tax on wages and profits. Such a tax clearly leaves the wage determination behavior in the urban sector unaffected, since it does not affect any of the relative wages (urban-rural or intra-urban). Thus, the short run consequences are identical to those described in the case of the profits tax; now however, the distributional impact is somewhat lessened, and hence the reduction in savings is smaller. If a tax on wage income only is imposed, then the net effect of the wage subsidy-cum-wage income tax is to reduce labor income:

$$\frac{dw_r(1-\tau)}{d\tau} < 0 .$$

Indeed the reduction in labor income is greater than the reduction in national output, so that profits are increased.

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[1] If the training costs were general rather than specific, there would be some advantages to increased employment in providing a more educated labor force.

$$\frac{d\{F - [w_u(1-\tau) + Tq]L_u\}}{d\tau} = -L_u \frac{dw^*}{d\tau} > 0 \ .$$

All of these tax-subsidy schemes have the same deleterious affects on output in the short run.

6. Wages and Shadow Price of Labor in the Public Sector

There are, of course, other policy instruments available to the government. First, even in mixed economies, a large part of the urban labor force is employed in the public sector. The government can decide (a) on the relative size of this public sector[1] and the choice of technique (labor intensity) of the public sector; (b) the location of public sector activities, i.e. in the rural or urban sector; and (c) the wages paid in both locations.

Secondly, it can use other tax instruments to discourage urban unemployment--e.g., urban income taxes--or to encourage the use of more labor intensive techniques--e.g. tariff reductions on selected machinery, etc.

The first set of questions is pursued in this section. The urban income tax is discussed briefly in the next.

In keeping within the framework of this paper, we assume that the capital stock in the government sector is given.[2] Hence the production alternatives facing the public sector may be described by the (short run) government production function:

$$G^g(L_u^g,\ L_r^g)$$

where L_u^g and L_r^g are government employees in the urban and rural sector. The government need not pay the same wage as the private sector, but may be constrained to pay the same wage in the urban and rural areas. The average

[1] We assume, however, that the government is restrained from taking direct control of the "private sector;" if it can do that, we are in the situation described in Section 4.

[2] If it were variable, it would simply give us two additional first order conditions. The results on optimal labor allocation and wage setting in the government sector are not affected by this assumption.

wage in the urban sector is now

$$\bar{w}_u = \frac{w_u L_u + w_u^g L_u^g}{L_u + L_u^g} \tag{6.1}$$

and we replace (2.20) by[1]

$$\frac{\bar{w}_u}{w_r} = \varphi\left(\frac{1}{1-U}\right). \tag{6.2}$$

The quit rate from the private sector should also now be a function of w_u/w_u^g. But if the government had the same training costs as the private sector and acted as a private firm (ignoring, for instance, its effects on the unemployment rate) then it would pay the same wage; but since the government should take these effects into consideration (and assuming that training costs per employee are no greater in this government sector than in the private) $w_u^g < w_u$ and there will not be labor turnover from the private to public (urban) sector. On the other hand, turnover rates in the urban government sector will depend on the urban wage:[2]

$$q_u^g = q^{g_u}\left(\frac{w_u^g}{w_u}, \frac{w_u^g}{w_r}, U\right).$$

The quit rate in the government rural sector will be assumed, for simplicity, to be fixed at q_r^g,[3] the result of which is that $w_r^g = w_r$.

---

[1]This assumes in effect that it is the agricultural workers who migrate, since $w_r \leq w_r^g$ (otherwise, the government cannot attract workers).

[2]These functions are meant simply to be a convenient simplification capturing the "first order effects" of a process which is clearly far more complicated; for instance, now that we have introduced a difference of wages in the urban sector, it is clearly possible for individuals to accept a government urban job and continue to seek employment in the private urban sector. This clearly has some effect on w_u^e, which (6.2) does not probably properly capture.

[3]We could let q_r^g be a function of relative wages as well; increasing w_r^g reduces labor turnover. If there is no government budgetary constraint this is the only effect, so w_r^g is raised to the point when the quit rate is unaffected by further increases in w_r^g.

Thus, the government wishes to

$$\max_{\{L_u^g, L_r^g, w_u^g\}} \quad G^g(L_u^g, L_r^g) + G\left(L - \frac{L_u + L_u^g}{1-U} - L_r^g\right) + F(L_u) - T_u q_u L_u$$
$$- T_u^g q_u^g L_u^g - T_r^g q_r^g L_r^g \; . \tag{6.3}$$

It controls directly L_u^g , L_r^g and w_u^g , and by changing these variables, it controls indirectly--and imperfectly-- w_u , L_u , L_r , and U :

$$w_u = \phi(w_r, U) \tag{6.4}$$

$$q_u = q_u\left(1, \frac{\phi(w_r, U)}{w_r}, U\right) \tag{6.5}$$

$$w_r = \bar{w}_u \varphi\left(\frac{1}{1-U}\right) \tag{6.6}$$

$$w_r = G_L \tag{6.7}$$

$$F_L - Tq_u = w_u \; . \tag{6.8}$$

We consider here only the case where w_r is constant.[1] Then the constraints (6.4) to (6.7) can be reformulated as:

$$w_u^* = \phi(\bar{w}_r, U) + Tq_u\left(1, \frac{\phi(\bar{w}_r, U)}{\bar{w}_r}, U\right) = w^*(U) \tag{6.9}$$

$$L_u = L_u^d(w^*(U)) \tag{6.10}$$

$$\bar{w}_u = w_r \varphi\left(\frac{1}{1-U}\right) \tag{6.11}$$

---

[1]Letting w_r be variable complicates the analysis without changing the basic qualitative results.

$$w_u^g = \frac{\bar{w}_u(L_u^d(w^*(U)) + L_u^g) - \emptyset(w_r, U)L_u^d(w^*(U))}{L_u^g} . \tag{6.12}$$

Thus, it is convenient to choose as our controls U, L_u^g and L_r^g. This will imply a particular value of w_u^g. The first order conditions are

$$-w_r\left(\frac{L_u + L_u^g}{(1-U)^2} - \frac{\frac{\partial L_u}{\partial w}\frac{\partial w^*}{\partial U}}{1-U}\right) - \left(T_u L_u \frac{dq_u}{dU} + T_u^g L_u^g \frac{dq_u^g}{dU}\right) = 0 \tag{6.13}$$

$$\frac{\partial G^g}{\partial L_u^g} - T_u^g q_u^g \quad - \frac{w_r}{1-U} - T_u^g L_u^g \left(\frac{q_{u1}^g}{w_u}\right) \frac{\partial w_u^g}{\partial L_u^g} = 0 \tag{6.14}$$

$$\frac{\partial G^g}{\partial L_r^g} - T_r^g q_r^g L_r^g - w_r = 0 . \tag{6.15}$$

The immediate implication of (6.15) is that the shadow price of labor in the rural public sector is the rural wage.

From (6.14), since

$$\frac{\partial w_u^g}{\partial L_u^g} = - \frac{(\bar{w}_u - w_u)L_u}{(L_u^g)^2} > 0 ,$$

if $w_u = w_r/1\text{-}U$ then the shadow price of labor in the urban sector is less than the urban wage. If $w_u = kw_r/1\text{-}U$ with $k > 1$, then the shadow price of labor in the urban sector is

$$\frac{\bar{w}_u}{k} + T_u^g L_u^g \frac{q_{u1}^g}{w_u} \frac{\partial w_u^g}{\partial L_u^g}$$

which is still less than the urban wage. Depending on the magnitude of k, the response of the quit rate to relative wage differentials, and the magnitude of the difference between government urban wages and private urban wages, the shadow price may be greater or less than the rural wage.[1]

In the formulation of our problem we have said nothing about the revenues for paying the government employees. For simplicity, we assume that G^g is a public good so no revenues are realized from its production, and the government faces a revenue constraint

$$w_u^g L_u^g + w_r^g L_r^g = \bar{B} .$$

This will have the effect of increasing the shadow price of labor in both the urban and rural sectors, and presumably decreasing the wages paid by the government in both sectors.

7. Urban Income Taxes[2]

A proposal to get more directly at the problems arising from urban-rural wage differential is to impose a tax on income in the urban sector only. The revenues from the tax may be used, for instance, to subsidize

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[1](6.13) gives us the optimal unemployment rate. If, as we would expect, an increase in the unemployment rate reduces turnover, then

$$\frac{1-U}{U} < \frac{1}{-\dfrac{L_u}{L_u + L_u^g} \eta_u^d \dfrac{d \ln w^*}{d \ln U}}$$

$$U > \frac{\dfrac{L_u}{L_u + L_u^g} \eta_u^d \left(- \dfrac{\partial \ln w^*}{\partial \ln U} \right)}{1 + \dfrac{L_u}{L_u + L_u^g} \eta_u^d \left(- \dfrac{\partial \ln w^*}{\partial \ln U} \right)}$$

Assume, for instance, $L_u = L_u^g$, $\eta_u^d = 1.5$, and that $(-\partial \ln w^*/\partial \ln U) = .2$ (so, e.g. an increase in the unemployment rate from 20 to 21% would reduce labor costs by 1%). Then

$$U > \frac{\frac{1}{2}\frac{3}{2}\frac{1}{5}}{1 + \frac{3}{20}} = \frac{3}{23}$$

i.e. there is still a fairly high unemployment rate.

[2]In this section and the remainder of the paper we shall ignore the government sector.

workers in the rural sector. Such a tax is always partially shifted, but the net result is always to decrease the unemployment rate. Again, the consequence of the decreased unemployment rate is to increase national output.

The wage determination equation in the urban sector is now derived from solving the problem

$$\min \left\{ w_u + Tq\left(\frac{w_u}{\bar{w}_u}, \frac{w_u \lambda}{w_r}, U \right) \right\} \tag{7.1}$$

where $\lambda < 1$ for an urban income tax. Thus,

$$1 = -T\left(\frac{q_1}{w_u} + \frac{q_2 \lambda}{w_r} \right) . \tag{7.2}$$

Since

$$\frac{\lambda w_u}{w_r} = \varphi\left(\frac{1}{1-U} \right) , \tag{7.3}$$

we obtain

$$w_u = h(U)$$

$$w_r = \frac{\lambda h(U)}{\varphi(1/1-U)}$$

$$w^* = w^*(U) .$$

A decrease in λ decreases the equilibrium unemployment rate, and hence increases w_u --the tax is at least partially shifted--and w^* . Hence <u>urban employment actually decreases</u>. But, at least for small taxes, and for k near unity the gain from the reduction in the unemployment rate more than offsets the loss from the reduction in the output of the urban sector (provided (2.31) is positive, as we would normally expect)

$$\frac{d\{Q_u - Tq + Q_r\}}{d\lambda} = \left\{ \left(F_L - Tq - \frac{G_L}{1-U} \right) \frac{dL_u}{dw^*} \frac{dw^*}{dU} - \frac{G_L L_u}{(1-U)^2} - T\left(\frac{q_2 \varphi'}{(1-U)^2} + q_3 \right) L_u \right\} \frac{dU}{d\lambda}$$

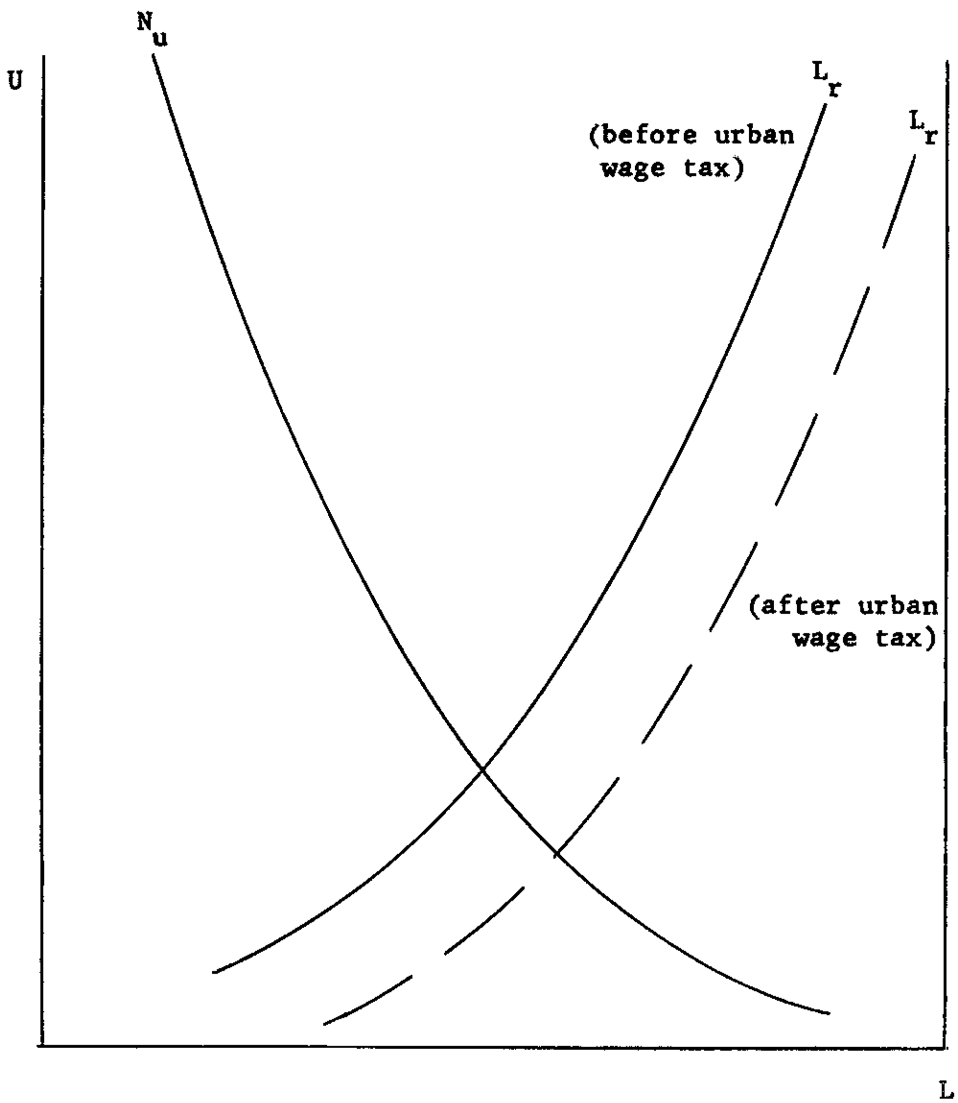

FIGURE 2.14

Effects of Urban Wage Tax

$$= \left\{ w_u \left(1 - \frac{\lambda}{\varphi(1-U)} \right) \frac{dL_u}{dw^*} \frac{dw^*}{dU} + L_u T \left(\frac{\lambda q_1}{\varphi(1-U)^2} - q_3 + \frac{(1-\varphi')q_2}{(1-U)^2} \right) \right\} \frac{dU}{d\lambda} < 0 \ ,$$

for λ and k near unity (where $k = w_u(1-U)/w_r$). The required tax is relatively small: if $k = 1$, and $\Delta w/w$ is the percentage discrepancy between the market wage and the optimal wage $T(q_1 - [q_3/\lambda\varphi])/w_u$ then

$$\frac{1-\lambda}{\lambda} = \frac{\frac{\Delta w}{w} \frac{U}{1-U}}{\frac{\partial \ln L_u}{\partial \ln w^*} \frac{\partial \ln w^*}{\partial \ln U}} \ .$$

Hence if $\Delta w/w = 25\%$, $U = .1$, $\partial \ln L_u / \partial \ln w^* = 1.5$, $\partial \ln w^* / \partial \ln U = .2$, then

$$\frac{1-\lambda}{\lambda} = \frac{\frac{1}{4} \times \frac{1}{9}}{1.5 \times .2} = .0926 \ .$$

If $k > 1$, the required tax is even smaller. If k is sufficiently large, a urban wage subsidy rather than a wage tax is called for. For instance, if $k = 1.6$, and all other parameters are as above,

$$\lambda = 1.47$$

i.e. a 47% urban wage subsidy is called for. If $U = .2$

$$\lambda = 1.3$$

i.e. a 30% urban wage subsidy.[1]

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[1]It should be emphasized although we have chosen what may be regarded as "reasonable" valued of the parameter, these are only meant to be illustrative.

8. Growth

Although most of this paper is directed at the policy implications of unemployment in l.d.c.'s when the unemployment rate and urban wage rate are endogenous variables, one use to which the model can be put is to attempt to explain changes in the unemployment rate and urban/rural wage differentials over time. Rather than present a formal growth model, it will prove more useful to isolate the various "forces" leading to higher and lower unemployment rates. There are essentially five changes which can occur over time.

(1) The rate of growth of employment in the urban sector may increase. This has the effect, at any given unemployment rate, of increasing the equilibrium urban-rural wage differentials.

(2) Training costs decrease, as the labor force becomes more adjusted to industrial work conditions. The equilibrium is then characterized by a lower level of unemployment and a lower urban-rural wage differential.

(3) Intra-urban competition for labor increases, leading firms to believe that they can get a greater advantage over their competitors by offering a higher wage. (q_1 increases). This tends to increase the wage offered at every level of unemployment, and has just the opposite effect to those described in (2).

(4) Because of capital accumulation and technical progress, the demand for labor at any given wage increases (in both the rural and urban sector, although probably at a faster rate in the latter than in the former). This tends to decrease the unemployment rate and urban-rural wage differentials.

(5) The increased population on a fixed land tends at any given level of technology and capital stock to decrease the proportion of the population which can be supported in the rural sector at any given wage rate as the population grows. This leads to increased unemployment and a higher urban-rural wage differential.

The first, third, and fifth tend to increase the urban-rural wage differentials, the second and fourth tend to decrease them. The net outcome depends on the relative strength of these different forces.

One might suspect that early in the stages of development, capital accumulation and technical progress, particularly in the rural sector, is going on at a sufficiently low rate that the ("proportionate") demand curve for labor in the rural sector is roughly constant. It takes some time before training costs start to decline, but the urban sector develops from one in which there is almost no competition to one in which there is more (although still limited) competition. The increase in the rate of expansion of urban jobs leads to an increasing unemployment rate and an increasing urban-rural wage differential. Eventually, the rate of growth of employment slackens off or even decreases, training costs begin to decline significantly, and capital accumulation and technical progress increase faster than the population growth rate. Thus, in the "second" stage of development, the urban unemployment rate declines and urban rural wage differentials become smaller.

9. Concluding Comments and Summary

This paper has provided a model in which the unemployment rate as well as the urban wage rate are endogenous variables. The model, we would argue, explains at least part of the urban rural wage differentials. Although we have focused on turnover costs as the "explanation" of why competitive firms are willing to pay more than is necessary simply to attract labor, other labor costs, such as absenteeism, and work "effort" are likely to depend on very similar considerations. The formulation of the model has enabled us to determine clearly the effects of alternative policies on national output, urban employment, the urban unemployment rate, etc.

The results run counter to much of the development folklore:

(a) Although the competitive wage is likely to be greater than the wage that the government would set if it controlled the urban sector directly (but could not directly control migration) the government will still set the wage at a level greater than the rural wage so there would be urban unemployment.

(b) Even though there is urban unemployment, the shadow price of labor is equal to the urban wage when the government controls directly the urban sector.

(c) A wage subsidy is not a good substitute for direct control of the urban sector.[1] The wage subsidy is always partially shifted and as a result a wage subsidy increases the unemployment rate and reduces national output.

(d) In a mixed economy, there is some presumption that the wage paid by the government in the urban sector will lie between the urban wage and the rural wage; the shadow price of labor in the rural sector is just the rural wage but in the urban sector it is less than the urban wage but may be greater or less than the rural wage.

(e) A tax on wage income in the urban sector is also always partially shifted, increases total labor costs, and decreases the unemployment rate. Usually, it also increases national output, but if $w_u(1-U)/w_r$ is large, just the opposite will occur.

(f) The model is consistent with a number of alternative "histories" of the economy, particularly with respect to wage differentials and unemployment rates. We have suggested the most likely "history" entails initially increasing unemployment rates followed eventually by decreasing unemployment rates. The model is consistent with increasing wage differentials occurring at the same time that the unemployment rate increases.

Some of these conclusions will need to be modified under alternative assumptions concerning the determination of the urban wage. Two such alternatives will be analyzed in the two sequels to this paper.

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[1] Within the limited bounds of our analysis; we are ignoring the dynamic effects, the advantages of individual entrepreneurship, etc. All of this is to say that although in our model direct controls are better than indirect controls, I would hardly use this as a basis for arguing that the government should control the urban sector directly.