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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART VII

MONEY, TRUST AND EQUILIBRIUM POINTS IN GAMES IN EXTENSIVE FORM

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by

Martin Shubik*

1. ON STATIC MODELS

The major part of microeconomic theory is static. This includes the elegant theorizing concerning the existence of a price system under relatively general conditions. There is a good reason for emphasis to have been placed on a static theory. It is far simpler than any dynamic theory promises to be.

The applications of the theory of games to the study of economic phenomena have been no exception to the rule calling for emphasis to be laid on statics. In the start of their book von Neumann and Morgenstern lay stress on the reasons for starting with a static theory.[1/]

Unfortunately an understanding of money and financial institutions, even at a relatively elementary level calls for the development of some

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dynamics. This is certainly indicated when one contrasts macroeconomic with microeconomic theorizing. It is argued here that a dynamic component needs to be added to microeconomic theorizing if we wish to progress in any depth in the study of money and financial institutions.

In these papers use is made of a game theoretic formulation of many of the basic features of economies with money and financial institutions. This formulation is particularly good in exposing the importance of strategic considerations, the role of trust, the role of information and the critical importance of special rules of the game concerning the enforcement of contract.

For these who are not at all acquainted with the basic concepts of the theory of games the following two sections will be somewhat elliptical, but they are reasonably self-contained. The basic background called for is given elsewhere.[2/]

1.1. Descriptions of an Economy

There are three completely different static descriptions of a game all of which are of direct applicability to an economy. They are:

(1) the extensive form,

(2) the normalized or strategic form,

and (3) the characteristic function or coalitional form.

A simple and relevant example is provided to illustrate all three of these descriptions. Consider two retailing firms in competition during one market period. We assume that during this period they each have time for two moves. These moves are respectively to buy from the wholesalers (or factories), i.e. to stock up and then to name a retail price at which each will sell to the customers.

Obviously the one period market described above is part of a continuous process. There are usually many market periods before the death or exit of a firm. Furthermore the move pattern is by no means as regular as might be implied by the description above. There are also many other moves such as the arranging of bank credit, the paying of wages and dividends, the setting of discounts and so forth.

A good microeconomist, after general results, may be inclined to adopt the attitude that much of the above represents fine and unnecessary institutional detail. Frequently he may be right. However this depends explicitly upon the question to be answered. In particular if he is interested in oligopoly theory or in finance then implicitly or explicitly he must be interested in dynamics. In these instances, that which might otherwise be regarded as "mere institutional fine detail" becomes of critical importance to the formulation of a model suitable for analysis.

Returning to our example with two firms each with two moves; in order to fully specify the model we must state the order of the moves and the information pattern. For simplicity we assume that the two firms each order stock simultaneously, they then name price simultaneously.

After the firms have ordered stock, but before they name prices do the firms find out what each has done? The pattern of information depends upon a host of institutional details. Two possibilities are illustrated. We assume first that each firm knows what the other has ordered prior to naming price; then we assume that neither knows what the other has ordered prior to naming price. There are also several nonsymmetric patterns of information which we do not examine.

For simplicity in the exposition of the example we assume that there are only two alternatives at each move to choose between. Call them "high" and "low." The first indicates a large stock order or high price, and the second, a small stock order or low price.

Figure 1 shows the extensive form of the market where each firm knows the stock order of the other prior to naming price. The branches show the low or high alternatives at each decision point. A move is the selection

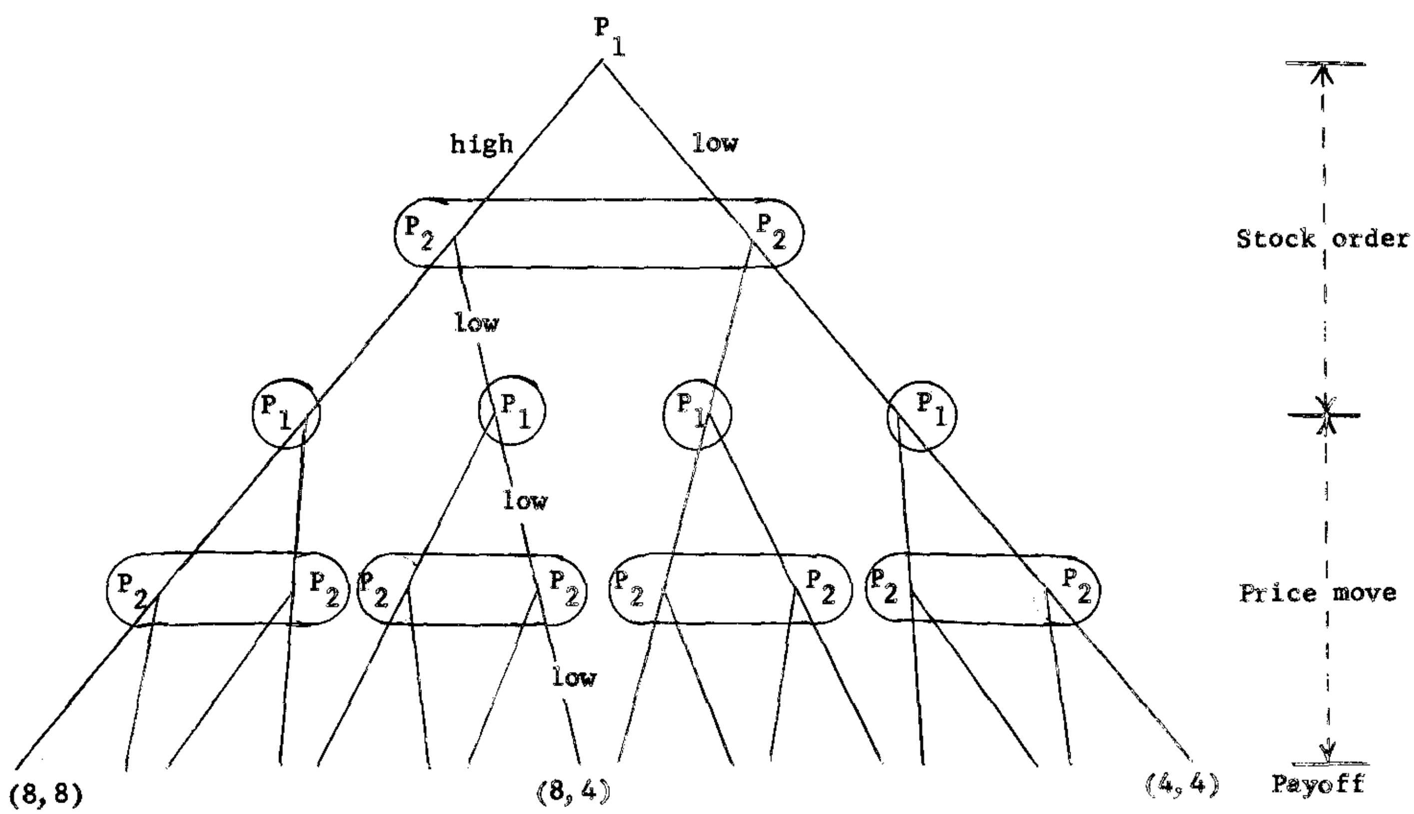

FIGURE 1

of an alternative at a decision point. The information sets are indicated by the curves enclosing one or two decision points. Each information set has a label P_1 or P_2 indicating that the move at those decision points belongs to the first or second firm. The meaning of two or more decision points lying in the same information set is that the firm being called upon to move cannot distinguish among these positions.

In many situations the important distinction between moves being made simultaneously or sequentially concerns the availability of information and not whether one move preceded another by a fraction of time. Thus if the first firm places its stock order ahead of the second, but the second is not informed until after he has placed his order, then strategically we could regard the firms as moving simultaneously. We could interchange the roles of the two firms and not change the strategic aspects of the situation.

In Figure 1 the first firm places a stock order. The second firm then places its stock order. The fact that the second firm does not know the move of the first is indicated by the information set which indicates that P_2 cannot distinguish which branch was selected by P_1. After P_2 has selected an alternative P_1 is called upon to name a price. The fact that he knows what the stock orders were is reflected in a condition of complete information which is manifested by the four one element information sets.

After both firms have selected a price the market ends and each receives his payoff. There are sixteen possibly different outcomes as can be seen at the terminal branches in Figure 1 and in the matrix in Table 1.

TABLE 1

		Firm 2			
		HH	HL	LH	LL
Firm 1	HH	8, 8	1, 20	8, 12	8, 4
	HL	20, 1	8, 8	8, 4	8, 4
	LH	12, 8	4, 8	12, 12	8, 4
	LL	4, 8	4, 8	4, 8	4, 4

A strategy by each firm is a complete plan of action which takes into account all known contingencies. Each firm has eight pure strategies. Each strategy can be regarded as a book of instructions that could be given to an agent. One such book that might be used by the first firm is given:

> "Select an H stock order. If Firm 2 selects an H order name an L price; if Firm 2 selects an L stock order name an H price."

This could be represented by (H; H, L; L, H) .

We could represent the game in <u>strategic form</u> by an 8x8 table similar to Table 1. In doing so we obliterate the details of the structure shown in Figure 1 and concentrate on the strategic question of what strategies should each firm select.

The third representation is used to stress distribution problems. It can be seen from Table 1 that the most that a firm can guarantee for itself unilaterally is 8. Together the firms can obtain 24. This information can be presented as follows:

TABLE 2

$$V(1) = 8 \quad , \quad V(2) = 8$$

$$V(1,2) = 24 \; .$$

This representation obliterates strategic detail in its focus on bargaining power in the consideration of distribution.

Returning to the extensive form, suppose that the firms did not find out about each others' stock order until they had also selected their prices. Figure 1 would have to be modified to reflect these changed information conditions. The new representation is shown in Figure 2.

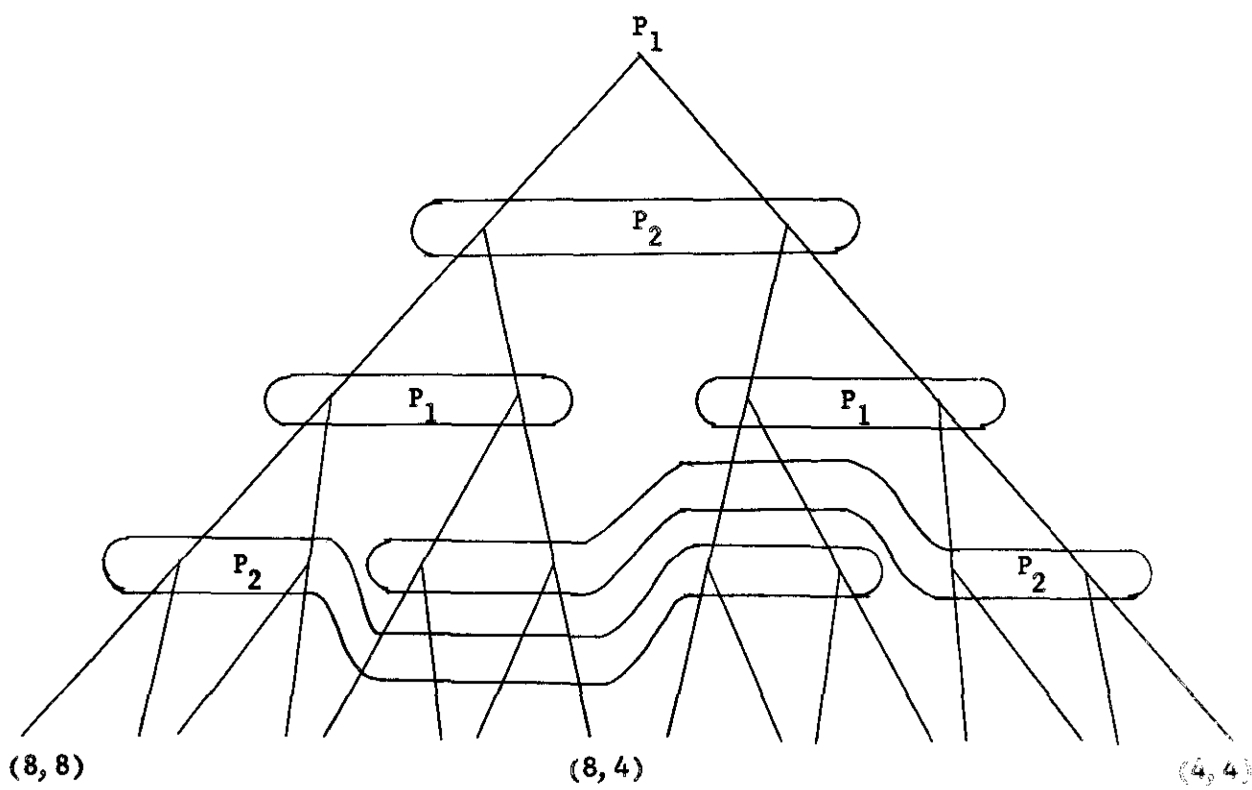

FIGURE 2

The strategic form for this game is given by Table 1. Each firm has four price strategies (H,H) , (H,L) , (L,H) or (L,L) . No contingent planning is called for as no information is received until it is too late to be of value.

In this game a strategy is merely the preselection of the choices at the two moves.

The third representation is precisely the same as it was for the previous game and is still given by Table 2. The function v is called the characteristic function of the game. Its arguments are the various coalitions that can form. An n-person game will have $2^n - 1$ nonempty different coalitions. This representation of a game is called the characteristic function form.

1.2. Static Solution Concepts

The different descriptions of a game may be regarded as presolutions. They may be models of an economy where each has been displayed from a different point of view. None of them tells us how the game will be played or the gains distributed.

The first representation lays stress on information and the fine structure of the process. The second stresses strategies and payoffs. The third emphasizes bargaining positions and the potential gains through cooperation.

If we regard a solution as a prescription or description of how to play then starting with one of these forms we may wish to add conditions describing how we expect choices to be made.

The strategic and characteristic function forms of a game may be regarded as aggregated models obtained from the extensive form. It is with these two representations that much of the work on solution concepts in game theory, has been done. In particular associated with the strategic form is the concept of the noncooperative equilibrium. Associated with the characteristic function form* are the core,[3/] value,[4/] bargaining set[5/] and several other cooperative solutions.

An n-person game in strategic form may be represented as follows:

Let each player i have a set of strategies S_i. Associated with each player is a payoff function which we denote by $P_i(s_1, s_2, \ldots, s_n)$ where $s_i \in S_i$ is a strategy belonging to player i.

A noncooperative equilibrium point[6/] is a set of strategies $(s_1^*, s_2^*, s_3^*, \ldots, s_n^*)$ such that:

$$\max_{s_i \in S_i} P_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*) \Longrightarrow s_i = s_i^*$$

for all $i = 1, 2, \ldots, n$.

The noncooperative equilibrium point is a generalization of the equilibrium suggested originally by Cournot.[8/] Two simple examples of noncooperative equilibria in 2x2 games are shown in Table 3. In the first case the equilibrium is at $(2,2)$ and in the second at $(1,1)$.

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*Without sidepayments the characteristic function must be replaced by a more general entity called a characterizing function.[7/] The technical details need not concern us here.

TABLE 3

	1	↓ 2
1	5, 5	-1, 8
→ 2	8, -1	0, 0

(a)

	↓ 1	2
→ 1	5, 5	2, 3
2	3, 2	0, 0

(b)

As our concern is with extending the concept of an equilibrium point to dynamics no discussion of the cooperative solution concepts is given beyond observing that if we base a solution on the characteristic function, all concern with bargaining and trust is glossed over in an implicit assumption that somehow or other contracts are arrived at and they are always honored by all parties for the duration of the game. The institutions or mechanisms for the enforcement of trust and the execution of contract appear only implicitly in the cooperative solutions.

2. STRATEGIES AND DYNAMICS

2.1. The Repeated Game

Consider a simple game in which each player has a single move with two alternatives. They make their choice simultaneously. Figure 3 illustrates such a game. It is easy to see that this game in extensive form has a strategic form given by the first matrix in Table 3.

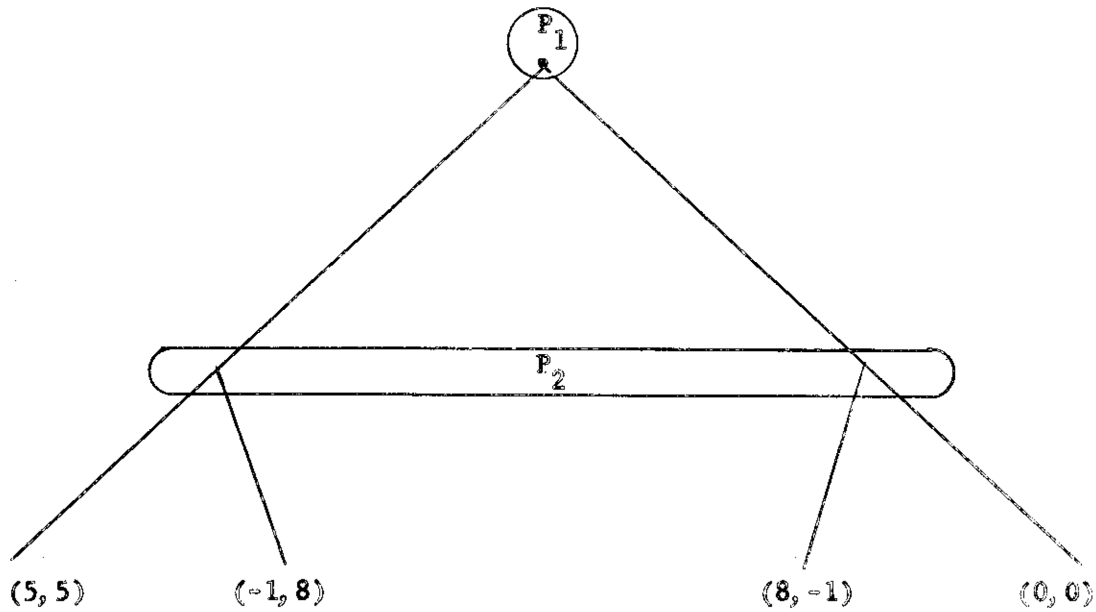

FIGURE 3

Suppose that this game is played twice. The extensive form of the repeated game would look like Figure 1. The difference being that we would need to note positional payoffs after the first play unless we adopt a convention that all payments are made at the end.

The strategic form of this game calls for an 8x8 matrix. If the game were repeated t times* the matrix would be of size $2^{2t-1} \times 2^{2t-1}$. We can see that for twenty or thirty iterations of a simple 2x2 matrix game the number of strategies ranges from the millions of millions upwards.

It is obvious that no individual playing a repeated matrix game searches through more than a minute portion of his strategies if he even thinks in terms of a long range program.

At least a part of the limitation on planning and information processing can be reflected by contrasting three types of strategy. They are:

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*Depending upon the definition of strategy this number could be higher.[9/] For this example it could be as high as $2^{(4^t-1)/3}$, this however includes highly redundant strategies, however when the information pattern is complex it is easier to count strategies with redundancy.

(1) A completely general strategy,

(2) a behavior strategy, and

(3) a state strategy.

A completely general strategy is a total plan of action decided upon before the start of play. Any finite matrix game will have a finite number of pure strategies. Each one may be regarded as a complete book of instructions as to how to play. In a game played in strategic form we could imagine each player giving a referee a book of instructions representing the strategy he has selected.

If a player gives the referee a single book we say that he is using a pure strategy. He might alternatively give the referee several books together with a probability distribution with instructions to randomize to select among them. Such a compounding of a pure strategies is known as a mixed strategy.

The possibility of using mixed strategies inflates the available strategies enormously. It can be shown that for certain classes of games this inflation is not completely necessary. If a game contains positions of complete information, at any position of complete information the overall game can be strategically decomposed in the sense that any randomization need not take place in advance of play but may take place at the start of any subgame.

A position of complete information is one at which all players know exactly to what position the game has progressed.

A strategy which employs at most local randomization (at a point of complete information) is called a behavior strategy. All pure strategies are behavior strategies. These strategies are a subset of all strategies. It has been shown that for games with positions of complete information there is no loss of efficiency in the use of behavior strategies.[10/]

We may limit strategies even further to state strategies. A state strategy is essentially non-historical. It depends only on the point or "state" to which the game has progressed, not on the history describing how the game reached this point. The easiest way to contemplate a state strategy is to think of a player in a game that lasts for t time periods as being an association of k agents where there is an agent for each state of the system. As the game progresses each agent who is reached during the play is called upon to "do his thing." He picks a strategy for his subgame (i.e. for that part of the game which carries the game into another identifiable state). As no history is used the number of state strategies is far fewer than behavior strategies.

In actual experimentation it appears that individuals do play period by period either without a strategy, or they devise a policy which is relatively parsimonious in the use of information. The policy may be in the form of a rule-of-thumb or a general principle of behavior. It is a behavior strategy which uses some, but not too much detailed history.

In mass markets it appears that, at least as a first approximation we might consider equilibria arising from the use of state strategies. When there are few individuals or when a code of behavior together with punishment for violators exists we may need to consider behavior strategies.

2.2. Strategies and Dynamic Equilibria

When we consider the game shown in Figure 3 played once it has a unique equilibrium point given by the strategy pair (2, 2) with the payoffs (0, 0) . This can be seen immediately from either the game tree or the matrix in Table 3a. Suppose that it is played twice. We could draw the tree which would look like Figure 1 but with different payoffs or we could look at the 8x8 matrix. In either case it can be shown that the only equilibrium outcome will be the one in which each player selects his second alternative each time. This behavior gives each a payoff of zero, whereas by cooperation they could achieve a joint gain of 5 per period.

It is well known that any finite series of repeated plays of this "Prisoners' dilemma" matrix game has only one equilibrium outcome* where the individuals obtain the payoff from the (2, 2) corner of the matrix.[11/]

Uniqueness is a desirable property for a solution. Unfortunately it is the exception rather than the rule. Suppose that we modify the game illustrated by Table 3a. We replace the 2x2 matrix by a 3x3 matrix as is shown in Tables 4a and 4b. By inspection we may observe that each of these

TABLE 4

	1	2
1	5, 5	-1, 8
2	8, -1	0, 0

(a)

	1	2	3
1	5, 5	-1, 8	-10, -10
2	8, -1	0, 0	-10, -10
3	-10, -10	-10, -10	-11, -11

(b)

*There is only one outcome but there may be many equilibrium points i.e. pairs of strategies which yield the same outcome.

games in their one-period version has a single equilibrium point at (2,2) with payoffs of (0,0) . This changes immediately when we consider two or more periods. If we allow the players in, say the two period version of the game shown in Table 4b to announce their strategies in advance then during the first period all of the four upper left cells of the matrix may belong to some equilibrium point. This is because the added row and column together with additional period (or periods) allow the players to specify strategies which contain within them a considerable element of threat.

When the size of a threat is large enough it may be used to stabilize even quite unsatisfactory situations. Here, for example a strategy by Player 1 might be as follows:

> "I am going to select 2 to begin with. If my competitor selects 1, I will select 2 on my second move; if he selects anything else I will select 3 on my second move."

Against this strategy it is optimal for Player 2 to choose 1 during the first period and absorb a loss of -1 and then select 2 in the second period. The loss entailed if the threat is carried out exceeds by far the loss from accepting an inferior position in the first period.

We note that in this two period game the threat of action in the second period is enough to enforce many equilibria. Suppose the same were repeated for t time periods. In order to enforce an equilibrium via threat a player cannot use a state strategy. The state strategy depends only on current position. A threat requires that the other side be informed of at least some of the contingent planning.

In the example given here only information concerning the next move is needed as the threat is so large that it has immediate effect. Suppose however that the matrix were as is shown in Table 5. If the first player

TABLE 5

	1	2	3
1	5, 5	-11, 18	-1, 0
2	18, -11	0, 0	-1, 0
3	0, -1	0, -1	-2, -2

were trying to enforce the playing of (1, 1) during the first period as part of an equilibrium he would have to threaten to punish the other for at least 13 periods. It is easy to construct examples where the period of repirsal against a violation must be of any length.

2.3. Communication, Threat and Strategies

A strategy is a general plan or rule describing how a player intends to play. A behavior strategy is a set of limited plans for playing a game in extensive form where each plan covers a segment of the game characterized by beginning and ending at positions of complete information; i.e. positions where the players information sets are characterized by one-element sets. A state strategy is a behavior strategy which does not use history.

Figure 1 shows a game in which the players could employ behavior or state strategies over the first and second periods. This does not hold true for Figure 2.

It is easy to observe that there may be games of infinite length in which there is never a point of complete information. Furthermore games of this type may arise quite naturally from as models of economic processes. One such model is illustrated in Figure 4. Suppose that two firms make secret investment plans. It is possible to keep the plan secret prior to starting the actual construction of the new plant. There is a two period time lag between the financial decision and the start of construction.

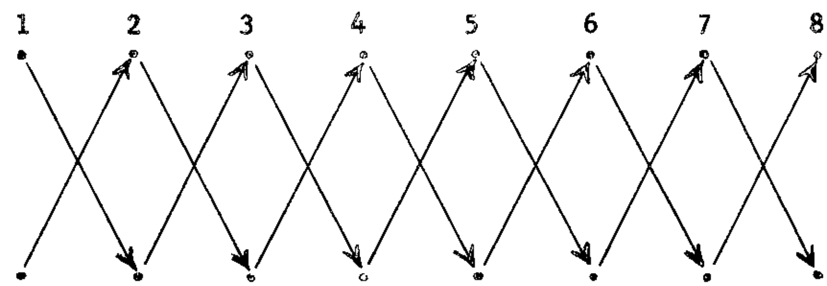

FIGURE 4

Figure 4 shows the period at which either player could decide upon ordering new plant then an arrow shows when the other will find out about the decision. The cross hatching present at each period indicates that at no period in the game does anyone know precisely the state of the system. A represen- tation of Figure 1 in this manner is shown in Figure 5. At the start of the third and fifth periods there are positions of complete information.

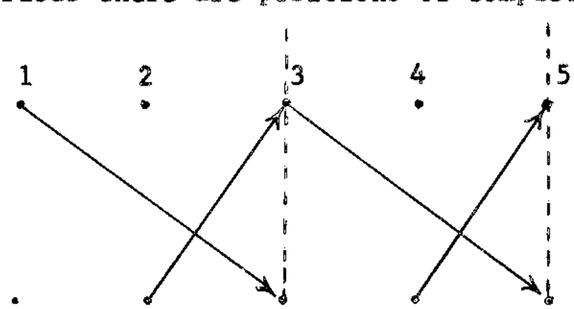

FIGURE 5

The state strategy manner of playing this game is to play it in blocks of two periods, i.e. to play it as though it were a series of repeated simple 2x2 matrix games where each is played independently.

3. ECONOMIC DYNAMICS, TRUST AND SELF-POLICING SYSTEMS

3.1. On General Solution Properties

A full discussion is needed of the general solution properties one might require for a dynamic equilibrium solution to any game. The recent, and as yet unpublished work of Harsanyi and Selten has been directed towards selecting a single equilibrium point as the solution to a game played in extensive form.

There is a less ambitious (and possibly less satisfactory) approach. That is to limit our attention to a very specific set of games and to make use of special properties of these games. This is the approach adopted here.

3.2. On Special Games and Solutions

Games representing economic phenomena in general and market games[12/] in particular are an extremely special class of games. Consider a closed economy where individuals employ quantity strategies in the markets (as in Parts III and IV).[13/] It was assumed in the earlier papers (and in general equilibrium theory) that after every period all individuals are completely informed about the state of the system.

Because of the special structure of a market game there is a natural way to modify the assumption concerning complete information. The economic goods being traded provide a natural metric for the aggregation of information. An individual trader selling, say corn, may not know the details of the strategies of thousands of other traders also selling corn. It is possible, however that he is informed of the aggregate or total sales of corn.

We may conceive of an n-person repeated market game played in extensive form where each individual sees only what he has done and the market averages. Thus there appears to be n parallel "two person" games where each individual views himself as playing against the aggregate market (for $n > 2$).

If we draw the game tree of this game we may observe that it is without points of complete information after it starts (for $n \geq 3$). This being the case neither behavior nor state* strategies can be used.

If, as a crude approximation, we degrade the information conditions even further by assuming that each agent is told the aggregate market figures but "forgets" the detail of what he did in particular last time then the dynamic game has a very simple description in terms of states, where after every period all agents are equally informed about the state that the system has entered and each may use a state strategy based on only this information.[14/]

In dynamic programming[15/] and many of the applications of control theory and differential games[16/] to the development of a theory of games a state description of the game and state strategies are invariably used.

*We may also call a state strategy a Markovian strategy as it is closely related to Markov processes.

In the discussion of fiat money[17/] the problem was implicitly formulated in terms of state strategies. Such a formulation appears to be a reasonable approximation for a mass market using only fiat money, possibly with a rudimentary banking system. It does not appear to be adequate for a full financial infrastructure where differentials in the information structure are of considerable importance.

3.3. The Aggregation of Information: Macro and Micro Economic Equilibria

The relationship between micro and macro economics appears clearly when the aggregation of information is considered. The utilization of economic aggregates by each individual amounts to describing the economy as a game with highly limited information. This contrasts with the other extreme implicit in the formulation of a multistage microeconomic general equilibrium model, where a game with perfect or nearly perfect information appears to be called for.

As information sets are aggregated the number of strategies available to the players are diminished. The number of noncooperative equilibria will shrink (or at worst remain the same). In general we do not have the guarantee that there is a game play which is in equilibrium under strategies resulting from all aggregations of information. For example consider the two games illustrated in Figure 6. They differ only in their information conditions. The first game has two equilibria which give rise to paths $1, 2$ or $2, 1$ and payoffs $(-1, 1)$ in each case. The second game has no pure strategy equilibrium point.

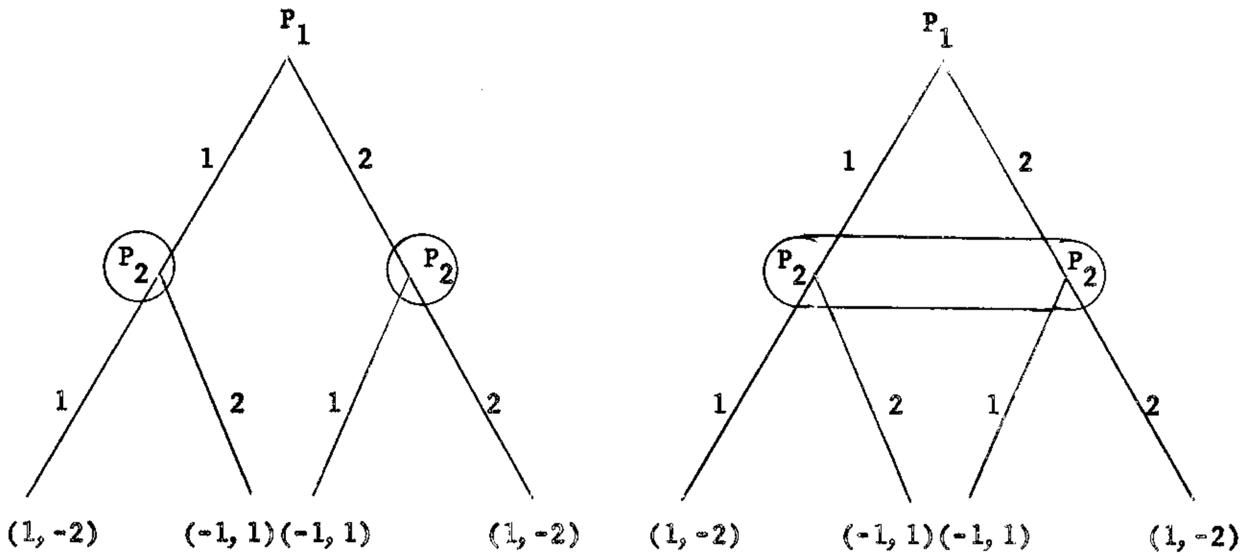

FIGURE 6

For quantity strategy games however we do have the property that there will be an equilibrium path through the game tree that will be generated as an equilibrium point for games with different levels of aggregation of information. In particular suppose that each period the same one-period quantity game is played, furthermore suppose that it has a steady state Cournot equilibrium outcome. This outcome will be attainable from equilibrium points with highly aggregated or disaggregated information.

The effect of information aggregation is to make threats less specific. If you do not know who were the high producers and who were the low producers, because all you know is aggregate production you cannot announce a threat based upon retaliation against a specific high producer.

In particular if, as a first approximation, we may represent an economy in which each period individuals decide upon quantities to buy or sell and in which bankruptcy is not particularly important. Then the game suggested in 3.2 where each individual knows his own actions and only macroeconomic aggregates of other actions will have an equilibrium path in common with the game with perfect information or the game in which each player at each period knows only the overall state of the system.

4. MONEY, FINANCIAL INSTITUTIONS, TRUST AND EQUILIBRIUM

4.1. Money and Generalized Trust

"In God we trust, all others pay cash."

A central problem in the development of a theory of financial institutions is the development of an adequate solution concept for a game played in extensive form. Such a solution must depend explicitly upon information conditions and must reflect differing degrees of trust and anonymity. The use of a fiat money (or a commodity money) goes a long way in mitigating the need for an elaborate system of trust in much of everyday trade.

In mass anonymous markets where the need to trust and to know people has been replaced by adherance to the use of a symbol of trust in the form of money whose use is enforced by the law, delicate information and threat structures are not particularly important. This indicates that as a reasonable approximation we may model mass market fiat money transactions as a dynamic noncooperative game where state strategies are employed and individuals operate upon highly aggregated information.

The above model contains implicitly many aspects of the savings in transactions costs obtained by using money. Many of these costs are associated with data processing.

4.2. Assets, Escrow and Specialized Trust

When we wish to consider the loan market, the information assumptions and the mass market assumptions may not provide an adequate model. For many loans the lender <u>is</u> interested in the history of the borrower. This means that policy is <u>not</u> independent of history and a state strategy description is not adequate.

When dealing with loans, real assets and a legal structure which enables these assets to be pledged, serve to minimize the need for trust and detailed information. Mass markets where loans are secured, such as the housing market and margin buying on the stock market, are examples of markets where information conditions need not be much higher than in cash transactions. A state strategy model where the state is somewhat more complicated (in the sense that it includes the state of real assets) may serve.

4.3. Insurance, Risk and Liability

In the literature on insurance, risk is divided into many categories. Among these categories a distinction is made between events caused by "nature," floods, fire, etc. and "moral hazard" caused by dishonesty, cheating, concealment of facts and so forth.

A game theoretic model of financial affairs calls for four major distinctions to be made among risks. They concern:

(1) Behavioral uncertainty,

(2) uncertainty in perception,

(3) exogenous uncertainty, and

(4) strategic uncertainty.

Behavioral uncertainty pertains to the modeling of human affairs. Do we assume that individuals comprehend and accept the laws? Are they assumed to be reasonably rational?

Uncertainty in perception involves both aspects of aggregation and coding. In the discussion in 3.3 the aggregation of information (which is a form of coding) was noted. It is important to stress that in a world in which information is not costless, even though complete information is available it may be more economic to operate on a sample of the information thereby consciously introducing uncertainty into a nonstochastic system.

Exogenous uncertainty pertains to events whose occurence is governed by "nature" rather than the players. One may wish to distinguish between "risk" and "uncertainty" as Knight,[18/] Shackle[19/] and others have done. However modern decision theorists such as Raiffa[20/] tend not to make this distinction.

Strategic uncertainty is game theoretic uncertainty. The players do not know what their competitors are going to do, even though the game itself may have no chance elements. Moral hazard may be modeled as a mixture of behavioral and strategic uncertainty. Given the rules we may expect the insured to maximize at the expense of the insurer. As long as they abide by the rules (here rules equal laws) this behavior can be covered by investigating a formal game theoretic model with the laws given. If the insured cheat by violating the rules (breaking the laws) then we must consider how to design a better model of behavior and a more basic or general set of rules which include within them the costs and conditions concerning the breaking of the law.

Many aspects of insurance call for more information to pass between the insurer and insured than passes in an ordinary money-goods transaction. This is especially true where moral hazard risks are great and some or all of the risk may be group specific. When this is so either one must abandon a model of behavior based upon a simple state description or use a relatively complex description of state to catch the differences. Attempts to categorize loan or policy risks by seven or eight features provide examples of the latter approach.

It appears that in general when mass markets are involved even for loans and insurance there will be a tendency for behavior to be based on states and not history. When markets are thin such as markets for large loans or special insurance the model of behavior called for appears to call a study of historical strategies. Thus when we try to study solutions for games in extensive form which represent economic processes, as numbers increase two, not one set of limiting processes need to be considered, they are the change in numbers (from few to many) and the change in information conditions. The latter may result from the former but this is not necessarily so. Figure 7 indicates the type of models that are, in my opinion, of prime importance in the study of financial economic dynamics.

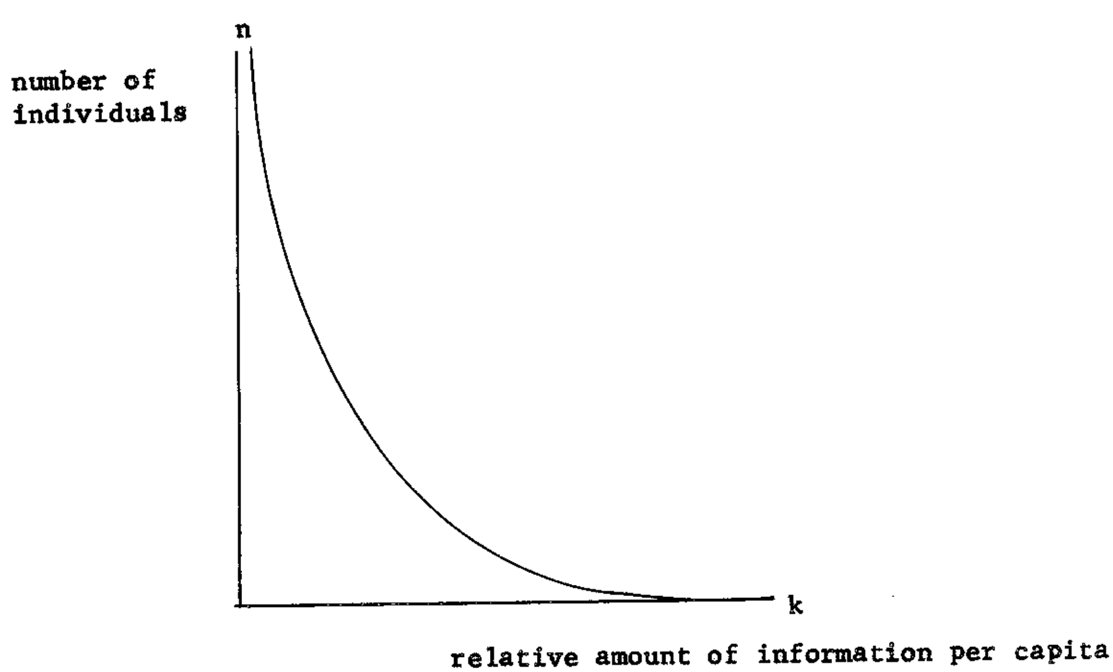

FIGURE 7

If we consider a game in extensive form, the largest amount of information available would be perfect information, i.e. when all information sets contain only one element. The least amount of information is where the game is played without any recall of previous acts, and the number of information sets is at a minimum.* We might crudely regard these conditions as providing a 1 and 0 on the scale of the amount of information per capita. If we replicate the number of players in a game the available information patterns will increase astronomically. If, however we assume that players only have available a limited amount of information such as averages, then the relative amount of information (relative to perfect information) fast becomes miniscule with the increase in numbers of players.

It is of interest to observe that in a world without insurance institutions which uses fiat money and is subject to random events, the fiat money in its role as legal tender takes on the role of a storage of wealth and hence provides its owners with a certain amount of insurance.

4.4. Financial Institutions, the Game and the Law

It has been suggested in this article that a microeconomic theory in which money plays a role calls for a dynamic solution concept. An attempt to consider equilibria in the context of a multistage market immediately calls for an examination of the role of information.

Tied in closely with dynamics and the state of information is the description of the nature of the various levels of trust and anonymity in trade.

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*In a subsequent paper a more specific discussion of the relative amount of information in a set of games will be given.

The existence of special institutions and special laws provides a context in which trade with a minimal amount of individual trust and personal information is needed.

We are confronted with two problems in modeling concerning the role of law and custom. Should we assume that laws and customs are to be modeled as rules of the game which are given and never broken? The other alternative is to model "metalaws" as rules. These include the laws and the penalties for violation. In economic affairs sometimes the breaking of contract, bankruptcy, concealment of information legally required and so forth do take place. Thus for some purposes the second alternative in describing the rules of the game is called for. It may be argued however that treating laws as rules may be a reasonable first approximation.

The second problem in modeling concerns why should individuals accept fiat money or the laws and customs of trade in the first place? This is far more a problem in economic history, anthropology and law than it is in economic theory. Once a system or set of laws and customs are in use, the use may provide for its own continuance. In other words once the system is in motion it provides for its own stability. The act of getting the system into motion might easily take place in many different ways by various historical accidents. The needs to avoid the costs of barter, information frictions, identification and evaluation problems all made the use of a money such as gold of extreme importance. The change from gold to gold certificates to pure fiat undoubtedly involved different forms of financial sophistication, public trust and force in various societies.

Given the existence of a fiat money and financial institutions, in a society even without exogenous uncertainty, but one in which there are costs attached to searching for trading partners, to gathering information, etc. then, if there are many individuals in the society, the costs of opting out of using the financial institutions are, in general too high. Occasionally barter trades take place and part of every economy bypasses the monetary sector, but in a modern mass market society these exceptions are small relative to the economy as a whole.

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