COWLES FOUNDATION DISCUSSION PAPER NO. 330

A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART IV

FIAT MONEY AND NONCOOPERATIVE EQUILIBRIUM IN A CLOSED ECONOMY

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February 7, 1972

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PREFACE

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Fiat money is a type of paper or symbol with which any individual may buy most things by law. It has virtually no intrinsic value but immediately assumes a trading value when its shortage* can prevent trades that would have been deemed profitable in a nonmonetary competitive equilibrium system.

This paper sketches an approach to a theory of fiat money by investigating the properties of a noncooperative dynamic trading game embedded within a closed economic system.

Among the conclusions are that inflation and deflation are not symmetric, and that it is not possible to define a noncooperative game involving borrowing without specifying "rules of borrowing" or a bankruptcy law.

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*I.e., when it is no longer a slack variable to everyone in the appropriate set of simultaneous dynamic programs.

1. COMMODITY MONEY

In a previous publication [1] the role of a commodity money in an economy where the individuals are regarded as players in a noncooperative game was discussed. The results of this investigation appeared to indicate that the distribution and quantity of commodity money is critical to the convergence of the noncooperative oligopolistic equilibrium to the competitive equilibrium as the number of traders in the market is increased. It was observed that too little money would lead to undertrading in the market.

In this paper an attempt is made to investigate the properties of a fiat money system. The analysis will depend heavily upon conceptualizing trading in the markets of a closed economy in terms of a noncooperative game given a cooperative act "outside of the game" to agree playing in the game using paper money as the means of payment.

Once the players have been hooked into a mass game of using fiat, it is reasonable to expect that under circumstances they will not be in a position to extricate themselves without cooperative action virtually at the level of society as a whole. Once fiat money is accepted the organization and communication costs involved in reverting to barter are so great that although the individual players may at some time feel that "the game is crooked" it is the only game in town and they have nowhere else to go. At least the costs of going anywhere else are exorbitant and will not be paid by individuals unless the economy is literally at the point

of breakdown; i.e., once a money economy exists it takes unified political or social behavior <u>outside</u> of the economic game to abandon it. Minor cheating such as swapping of services may take place, but on the whole a major breakdown is beyond economics and is in the domain of political or social behavior.

Many different initial causes involving politics, law, society and special institutions may explain the introduction and acceptance of paper money. Given that the money is in use and in common acceptance the explanation of the stability of its use may have little connection with the initial process for its introduction.

2. SOME MODELING CONSIDERATIONS

There are at least three reasons given for the holding of fiat money in an economic system. They are: (1) the transactions motive, (2) the speculative motive and (3) the precautionary motive [2]. The many roles of money are evidently closely interlinked with the institutional forms, habits and modes of communication of any society [3].

This paper is devoted only to exploring a very small bandwidth of the role of money, financial assets and financial institutions in a modern economy. Specifically, the goal is to examine a "neutral" fiat money economy. By the word neutral, I mean an economy in which the government or other fiat money issuing agencies are extremely constrained with respect to the action that they can take in the production of money. In particular, we rule out the government and the institutions which make up the financial

infrastructure of the economy as participants or players in the actual non-cooperative game. The rules for currency issue will be completely stated in advance "outside of the game."

Each of the three reasons given for the holding of fiat money, poses a set of modeling considerations in the construction of a formal model of a closed economy using a paper or a commodity money.

Transaction costs reflect a host of institutional details including habits, trust, the system of payments, and the state of communication. Population density, means of transportation for valuables or symbols of value, the degree of computerization of account processing, all play a basic role. Included in transaction costs are the inventory costs of money, [4] and costs of the paper work and communication processing.

The price paid for using the monetary medium must be initially* offset by its advantages as compared with alternative procedures of barter or face-to-face marketing by groups. The advantages of using some form of money are well known and there is both adequate evidence and many excellent articles on the subject that barter is too expensive and inefficient a way of doing business in a modern economy with millions of individuals engaged in trade [5]. This does not mean that barter should be ruled out, it does mean that as a first approximation, unless there are reasons to suggest that the monetary mechanism is breaking down then we can afford to ignore that sector of the economy which may be handled by barter.

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*Once in existence however, other organizational costs of getting out of the game may be too high to pay.

Even in an economy such as the United States, the value of the barter segment can be large. Not all goods and services are monetized. It may be conceptually difficult to decide when a decision is or ceases to be an economic decision. For example, the modeling of the services of housewives or for that matter, "do it yourself" house repairs is neither easy nor obvious.

Transaction costs and other costs attendant to accepting the use of fiat can be regarded as costs well worth spending for an important factor of production i.e., the ability to use impersonal markets.

The transactions motive for holding money does not necessarily involve uncertainty. The speculation and the precaution reasons for holding money are closely interlinked to the role of different types of uncertainty in the economic system. In the model presented here uncertainty in one sense will be ruled out.

There are two important ways in which uncertainty enters into a model. There are: (1) through exogenous random events such as a crop failure or the failure of an industrial process and (2) through the inability to predict the actions of a competitor. This latter type of uncertainty can be referred to as game theoretic or strategic uncertainty [6]. The speculative motive for holding money may be regarded as comprising both of these forms of uncertainty. There is the uncertainty of the financial manipulator playing a "money game" and there is also the uncertainty of the individual who predicts a crop failure.

The precautionary motive for holding money appears to be more closely related to uncertainty generated exogenously than uncertainty generated strategically.

In the model presented in Section 3, strategic uncertainty is present in the sense that all individuals are regarded as active independent players in a noncooperative game where trade is carried out by using fiat money. In a subsequent paper the important role of exogenous uncertainty or random events in the market will be considered. However, the approach adopted in this paper is to try to isolate one particular phenomenon that involves the use of fiat money in a closed economy through time, rather than to attempt to combine too many important features in the same model and thereby con-found the interpretation of the results.

In the noncooperative game [7] with money and uncertainty probabilistic outcomes in the economy must be evaluated. Furthermore, this addition of risk results in games in which it is reasonable for all individuals to hold fiat money. A new problem is posed in defining the meaning of a competitive equilibrium in a market with uncertainty. The Arrow-Debreu model is essentially a model of an economy with ordinal preferences [8]. In this formulation with noncooperative players, evaluating alternative futures by means of a von Neumann Morgenstern utility, the outcome will not be independent of transformations on this utility index. The role of uncertainty in a fiat money economy is not discussed further in this paper.

3. A NONCOOPERATIVE ECONOMY WITH FIAT MONEY

In this section the main thrust of the argument is illustrated by means of calculations of some examples. General proofs are not given. Undoubtedly there are many mathematical difficulties to be overcome before the generalizations suggested by the results obtained from the examples can

be established in full rigor and generality. Nevertheless it is my belief that not only is this a well-defined model, but the strategic and "open formulation" of the market as a noncooperative game permits the natural inclusion of differentiated players such as banks, insurance companies and government in a general equilibrium model. This is not done here, however, some observations on the need and the ways to do so are noted in Section 11.

A further point to note is that this type of model is especially suited to considering mixed modes of competition with oligopoly in one section and competition (reached as a limiting or attenuating behavior) in another section.

Our consideration is limited to a market with two types of traders. It is conjectured that the results hold for more types, however the mathematical difficulties are increased considerably.

The following notation is adopted. Let $B_{s,T}$ stand for a nonmonetary competitive economy of T time periods in length with s traders of each type. Thus $B_{\infty,\infty}$ stands for the limiting market structure with "many" traders of each type lasting for an indefinite time.

The expression $\Gamma(B_{s,T}, M_1, M_2)$ stands for the noncooperative "fiat money game" associated with the nonmonetary economy $B_{s,T}$. The M_1 and M_2 stand for the initial amounts of fiat money held by a trader of Type 1 or 2 respectively.

The simplest form of dynamic model together with a simple form of preferences are considered. A noncooperative fiat money game of T periods is assumed to be composed of T simultaneous move markets in sequence. The only interlink between two consecutive markets is money.

Money in these models is a set of Poker chips or tokens or pieces of paper which are intrinsically of no value but by the <u>rules</u> <u>of</u> <u>the</u> <u>game</u> must be used as a means of payment. If the game is of finite duration we expect that individuals will use all of the money they have on hand at the last period because after that it is literally worthless.

For simplicity we assume that each trader has a utility function which is the sum of his utility functions in any period. Suppose that all traders have the same shaped utility function each period. If there are m goods being traded in the economy then trader i at period t has a utility function of the form:

(1) $$\varphi_{i,t}(q_{1,t}^i, \; q_{2,t}^i \; \cdots \; q_{m,t}^i) = \rho^t \varphi(q_{1,t}^i, \; q_{2,t}^i \; \cdots \; q_{m,t}^i) \; .$$

The symbol ρ appearing in (1) is a "natural" time preference. When we deal with games of finite length we may wish to set $\rho = 1$. It is described here as a <u>property</u> of an individual's preferences. Conceivably we could have $\rho > 1$ however, here it is only introduced as an easy way in which the sum of the infinite series of terms comprising the total payoff of an individual in a game of indefinite length remains bounded. It is assumed that the payoff to trader i is given by:

(2) $$\Pi_i = \sum_{t=1}^T \rho^t \varphi_i(q_{1,t}^i, \; q_{2,t}^i, \; \cdots, \; q_{m,t}^i) \; .$$

It is further assumed that the φ_i are convex.

Each period it is assumed that each trader obtains a bundle of re-sources for trading and consuming during that period. There are <u>no</u> durables

other than money, and money does not enter utility function as it is intrinsically worthless.

Simplifying even further we assume that each period each trader obtains a bundle of resources the same as he obtained in the previous period (this assumption will be relaxed later in order to examine the role of the rate of interest). Trader i in period t obtains:

$$(3) \qquad A^i_t = (A^i_{1,t}, \ A^i_{2,t}, \ \ldots, \ A^i_{m,t}) = (A^i_1, \ A^i_2, \ \ldots, \ A^i_m) \ .$$

It is assumed as a rule of the game that _all_ goods are traded through the monetary system. Thus we may regard the rule as being that all goods are deposited at a central clearing house. Owners have to buy back their own goods, but the clearing house "keeps the books" and gives the owners all their receipts at the end of the trading period. By using this device* all trade is monetized and a "transaction cost" is implicitly introduced in the sense that the traders do not obtain the use of any of their assets directly which forces, for example, the farmer to hold extra cash to buy the milk he produces.

The model can now be formalized. A more general statement is given, then it is specialized.

3.1. The Money Game $\Gamma(B_{1,T}, \ M_1, \ M_2, \ \ldots, \ M_k)$

Let there be k types of traders, and one trader of each type. There are m goods. The game lasts for T periods. The information

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*This formulation was suggested by L.S. Shapley.

conditions and the nature of the strategies are indicated by Figure 1 for a game with two traders. Both traders first make an allocation of funds between present and future expenditures. Given initial money supplies of M and N the allocation problem is to select amounts x_1 and y_1 to spend. At the end of the first period the marketing board will send the traders the money obtained from the sale. Thus letting Z_t and W_t be the financial states at the start of period $t+1$, $Z_0 = M$, $W_0 = N$ and, as is indicated in Figure 1 the financial moves are to select $x_1 \leq Z_0$ and $y_1 \leq W_0$.

It is possibly easiest to imagine the traders as each consisting of a team of two agents. One sets the budget and the other makes the pur-chases each period. Once the budgets have been set for each purchaser they have a consumption allocation game to play where each purchasing agent attempts to maximize

$$(4) \qquad \varphi_i(q^i_{1,t}, \ q^i_{2,t}, \ \cdots, \ q^i_{m,t})$$

subject to

$$\sum_{j=1}^{m} P_{j,t} q^i_{j,t} = x^i_t \leq Z^i_{t-1} \ .$$

A strategy for the purchasing agent in his subgame is to allocate the money he has available to the purchase of different goods. Call this strategy

$$(5) \qquad d^i_t = (d^i_{1,t}, \ d^i_{2,t}, \ \cdots, \ d^i_{m,t}) \ .$$

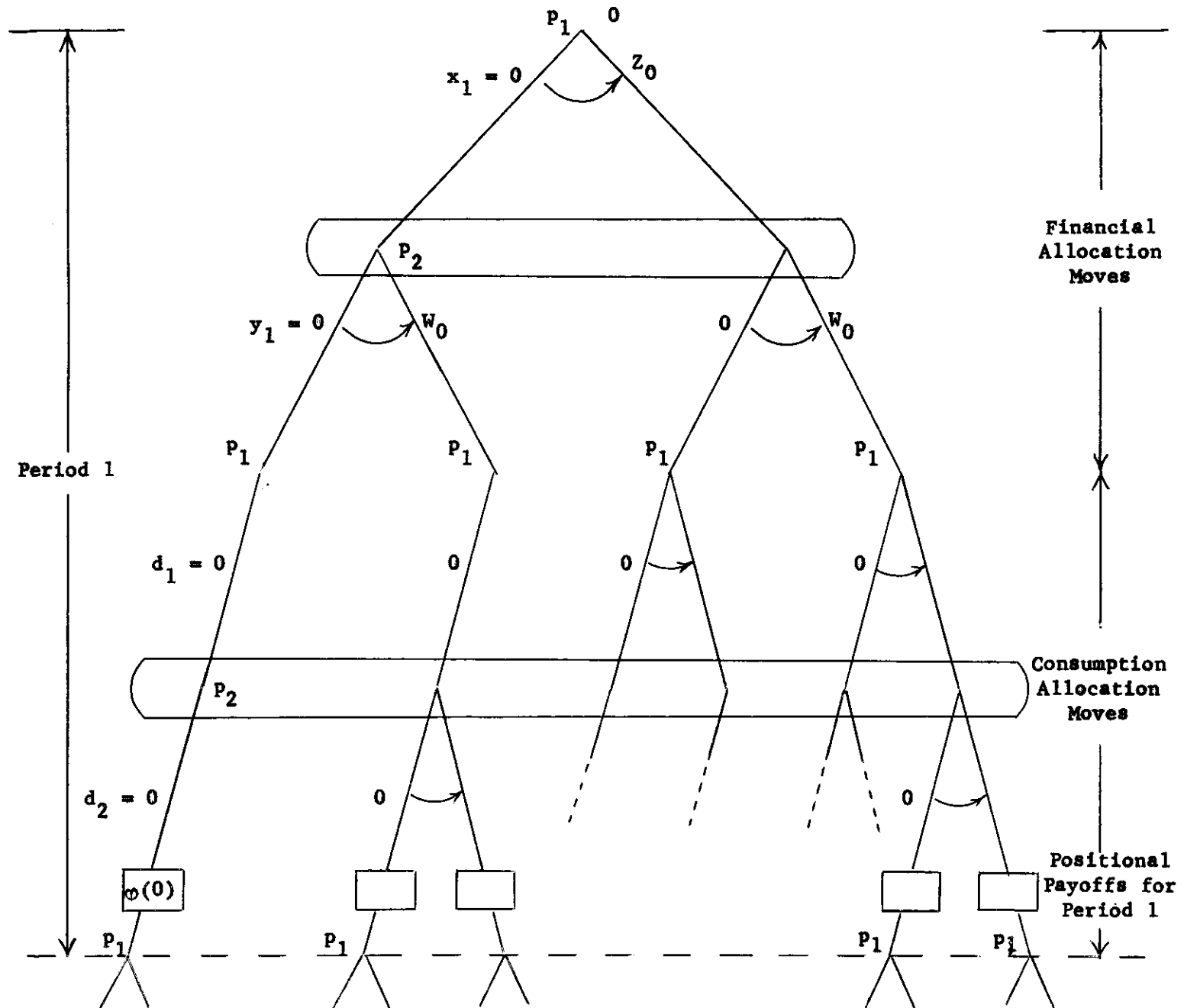FIGURE 1

Prices are determined by the amount of money "chasing" any particular good. Thus:

$$(6) \qquad P_{j,t} = \sum_{i=1}^{k} d_{j,t}^{i} \Big/ \sum_{i=1}^{k} A_{j,t}^{i} \ .$$

The amount of commodity j purchased by bidding $d_{j,t}^{i}$ can be expressed directly in terms of the strategic variables as:

$$(7) \qquad q_{j,t}^{i} = \frac{d_{j,t}^{i} \sum_{h=1}^{k} A_{j,t}^{h}}{\sum_{h=1}^{k} d_{j,t}^{h}} \ .$$

Equation (7) can be substituted directly into (4) we may maximize the k "positional payoffs" as a one period noncooperative game. This problem was discussed in a previous paper [9] and will be discussed further.* Here our interest is primarily upon the financial allocation problem rather than the single period budgetary allocation. This being the case we may consider that only one consumer commodity exists. At time t the endowment of a trader i of the commodity is A_t^i. Our "purchasing agent" now becomes unemployed because as soon as the financial allocation is made the consumption allocation is determined.**

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*This amounts to solving the game for state strategies, however one must justify the modeling of the game in terms of a state description.

**The simplification of looking at only one commodity is drastic, but it still permits us to consider some properties of the role of money in relative isolation. If two or more commodities are considered the allocation between expenditures and future income cannot be separated so easily. If there are only few individuals in the market their expenditures will have

The state variables z_t^i (when there are only two types we use z_t and W_t) are the overall monetary constraints

$$(8) \qquad z_{t+1}^i = z_t^i + \frac{A_t^i}{\sum\limits_{h=1}^{k} A_t^h} \sum\limits_{h=1}^{k} d_t^h - d_t^i \; .$$

We may now dispense with the symbol d_t^i and use x_t^i to stand for both the financial allocation to the purchasing agent and his action (when there are only two types we use x_t and y_t).

Letting $x_t = \sum\limits_{h=1}^{k} x_t^h$ and similarly for A_t, the game $\Gamma(B_{1,T}, M_1, M_2, \ldots, M_k)$ can be described as finding solutions to the set of simultaneous functional equations of the form:

$$(9) \qquad V_i(Z_{t-1}) = \max_{x_{t}^{i} \leq z_{t-1}^{i}} \left[\varphi_i\!\left(\frac{x_t^i A_t}{x_t} \right) + \rho V_i(Z_t) \right] \quad \text{for } i = 1, 2, \ldots, k \; ,$$

where $Z_t = (z_t^1, z_t^2, \ldots, z_t^k)$.

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an important influence on price and hence on the future income they will receive from their share of the sales receipts.

An even more basic problem to economic modeling concerns the change in the importance of usually implicit variables. In particular the nature of the information and communication processes.

One simplification which is consistent with many commodities is where the "financeer" sets λ^i, the value of money for his purchasing agent who then is instructed to maximize $\varphi_i(x_1, x_2, \ldots, x_n) + \lambda^i x_{n+1}$.

Given that we limit ourselves to one commodity the problems of multiple equilibria which are extremely difficult to cope with do not appear. This is of course both a weakness and a strength of the simplification. It enables us to study the implications of extremely simple models in detail, but it wipes out several important features of a better model of the economy.

3.2. Noncooperative Equilibria

At least for the finite stage game there are no additional conceptual difficulties in finding noncooperative equilibria. We may compute backwards from the Tth state knowing that in the final state all money will be spent.

Our problem is not with the existence of a noncooperative equilibrium point, but with the possibility of vast classes of equilibria. It is easy to immediately display a universal and unsatisfactory equilibrium point. That is the no trade point.*However, to say the least, it appears to be extremely unstable.

In the examples presented in subsequent sections it has been possible to select equilibria with interesting properties that relate to the competitive equilibria of the nonmonetary economy. It is conjectured that this will always be possible.

However if more general strategies are used there may well be hosts of noncooperative equilibrium points based upon threats and complex contingent statements. An adequate treatment of these would require discussing the level of communications and information in the economy. For example the assumption of complete information after the financial moves have been taken, as is indicated in Figure 1, implies that all savings intentions are known in detail by everyone.

Even at this level it is worth considering the formalization of the intuitive idea that one examines equilibria which depend on mass information such as figures on mass consumption and savings intentions.

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*It implies that no markets are active.

3.3. <u>The Money Market</u>

In the formulation given in 3.1 there is no money market. In order to discuss the money rate of interest it is necessary to specify it. This is done in the formal investigation of different time patterns of endowments in Part V.

Here we note that in order to do so the financial move must be split into two. First a decision must be made to borrow or lend. This is done by having the would-be borrowers specify how much of next period's money they will put up to bid for this period's money. Let the amount of next period's money bid for this period's be v_t and the amount offered be u_t. The price becomes:

$$(10) \qquad\qquad P_{m,t} = \frac{v_t}{u_t} \ .$$

A borrower obtains $\frac{v_t^i}{v_t} u_t$ immediately and pays v_t^i next period. The limitations on borrowing and the possibility of default must be covered. Further discussion of this is deferred to Section 9 and to Part V.

4. SOME EXAMPLES

4.1. <u>The Game</u> $\Gamma(B_{s,2}, M, N)$

We commence with a simple two stage example with s traders of each type. In this example the only difference between the traders is in their initial money holdings. The competitive equilibrium solution is trivial. In the nonmonetary economy no trade would take place and, the <u>pro forma</u>

prices interlinking the time periods are 1:1.

Using the simplification noted in the general description of the game we can regard the m real commodities as one commodity. Each trader is given a supply A of this commodity each period. Given two time periods, each wishes to maximize:

(11)
$$\varphi(q_1^i) + \varphi(q_2^i) \; .$$

Both for computational ease and clarity the discount factor ρ is omitted from (11). For example, suppose $\varphi = \log(q_t)$, then each trader i tries to maximize

(12)
$$\pi_i = \log \, (q_1^i) + \log \, (q_2^i) \; .$$

A strategy by a trader of the first type is to spend a certain amount of money x_1^i in the first period where $x_1^i \leq z_0^i = M$, and an amount x_2^i in the second period where $x_2^i \leq z_1^i$. Where the state variable z_t^i defines the amount of money available at the start of period t+1 to the ith trader of the first type.

A strategy by a trader j of the second type is to spend:

$$y_1^j \leq W_0^j = N$$

and

$$y_2^j \leq W_1^j \; .$$

As the markets are assumed to be cleared of the goods for sale the price for the consumer good during period t is given by:

$$(13) \qquad P_t = (\sum_{i=1}^{s} x_t^i + \sum_{j=1}^{s} y_s^j)/2A .$$

The quantity obtained by a trader i of the first type is given by:

$$(14) \qquad q_t^i = \frac{2Ax_t^i}{\sum_{h=1}^{s} x_t^h + \sum_{j=1}^{s} y_t^j}$$

and similarly for traders of the second type.

As there are only two time periods it is easy to see that:

$$(15) \qquad z_1^i = z_0^i - x_1^i + \left\{ \frac{\sum_{h=1}^{s} x_1^h + \sum_{j=1}^{s} y_1^j}{2s} \right\} .$$

As we have assumed that all traders have the same real endowments during each period the "clearing house" will take all of the money obtained during the period and send equal shares to all to be available for spending next time. Thus in terms of the strategic variable we may write the payoff for a player of the first type as:

$$(16) \qquad \Pi_i = \log \left\{ \frac{2sAx_1^i}{\sum_{h=1}^{s} x_1^h + \sum_{j=1}^{s} y_1^j} \right\} + \log \left\{ \frac{2sAx_2^i}{\sum_{h=1}^{s} x_2^h + \sum_{j=1}^{s} y_2^j} \right\}$$

and similarly for the second type. Given that this model is for only two periods and that money is worthless at the end we can simplify (16) considerably by observing that at the last period all will spend up to their budget constraints. Thus (16) can be rewritten as:

$$(17) \quad \Pi_i = \log \left\{ \frac{x^i}{\sum\limits_{h=1}^{s} x^h + \sum\limits_{j=1}^{s} y^j} \right\} + \log \left\{ \frac{M - x^i + \left\{ \frac{\sum\limits_{h=1}^{s} x^h + \sum\limits_{j=1}^{s} y^j}{2s} \right\}}{s(M+N)} \right\} + 2 \log 2sA .$$

The time subscripts have been dropped.

Assume that $M \geq N$. Given that the money is fiat, from the nature of the allocation game during each period we may deduce that the maximization process for the traders with less money will take place at a constraint. Thus we may set $y^j = N$.

We now wish to examine the optimal financial policy for players of the first type. It is shown that they will hold back funds in the first period if $M > N$. If $M = N$ they will spend all of their funds.

Maximizing (17) over x^i and setting $\sum\limits_{h=1}^{s} x^h = sx$ and $\sum\limits_{j=1}^{s} y^j = sy$ we obtain the condition

$$(18) \quad \frac{\partial \Pi_i}{\partial x^i} = \frac{s(x + y)}{x} \left\{ \frac{1}{s(x + y)} - \frac{x}{s^2(x + y)^2} \right\} = \frac{2 \left\{ 1 - \frac{1}{2s} \right\}}{(2M - x + y)} .$$

Setting $y = N$ in (18) we obtain

$$(19) \quad \frac{1}{x} \left\{ 1 - \frac{x}{s(x + N)} \right\} = \frac{2 \left\{ 1 - \frac{1}{2s} \right\}}{(2M + N - x)} .$$

Solving this for x yields a noncooperative equilibrium point of this game. Our interest is in the limit of this equilibrium as $s \to \infty$. The limiting noncooperative equilibrium is given by:

$$(20) \qquad \frac{1}{x} = \frac{2}{2M + N - x} \quad \text{or} \quad x = \frac{2M + N}{3} .$$

When $M = N$ then $x = M$ and the limiting noncooperative equilibrium yields the allocations of the nonmonetary market as can be seen by substituting $x^i = M$ in (16).

When $M \neq N$ the limiting noncooperative game does not give the competitive nonmonetary allocation. For example set $M = 7$ and $N = 1$ we obtain $x = 5$ and $y = 1$ with payoffs as shown in Table 1. This table shows also the effect of fewness of traders in the cases where $s = 1$.

TABLE 1

M	N	S	π_1	π_2	P_1	P_2
7	1	1	$2 \log\left(\frac{3A}{2}\right)$	$2 \log\left(\frac{A}{2}\right)$	2	1
7	1	∞	$\log\left(\frac{5A}{3}\right) + \log\left(\frac{5A}{4}\right)$	$\log\left(\frac{A}{3}\right) + \log\left(\frac{3A}{4}\right)$	3/2	1
M	M	1	$2 \log (A)$	$2 \log (A)$	1	1
M	M	∞	$2 \log (A)$	$2 \log (A)$	1	1
Non Monetary Competitive			$2 \log (A)$	$2 \log (A)$	1	1

We see that if the ratios of fiat money holdings are in proportion to the value of the real assets of the traders evaluated at a nonmonetary competitive equilibrium (here 1:1) then the noncooperative game immediately gives the competitive equilibrium.

If the ratios of holdings of fiat money are not as above then the outcome of trade in the two stage model is considerably different from competitive equilibrium. The limit noncooperative game as s → ∞ somewhat improves matters but there may not be a finite convergence to the competitive equilibrium.

It is conjectured that these results are quite general. Their meaning is relatively straightforward. If money is handed out in the ratio to the value of real assets at competitive equilibrium in this steady state noncooperative game, as the price system is homogeneous of order zero the traders immediately spends up to his holdings. This should not change with a natural discount rate. When the amount of money is not held in the "correct" proportions then there is an oligopolistic opportunity for someone to profit from the disequilibrium. Even in a two stage model the adjustment of money holdings starts to take place.

4.2. The Game $\Gamma(B_{2,T}, M, N)$ with a Natural Discount

We may write (21) to describe the maximization problem of the first trader with initial cash of M_0, in an otherwise completely symmetric steady state market lasting for T periods where there is a "natural time preference" of $0 \leq \rho < 1$.

$$(21) \qquad V_1(M_{t-1}, N_{t-1}) = \max_{x_t \leq M_{t-1}} \left[\varphi\left(\frac{x_t}{x_t + y_t} \right) + \rho V_1(M_t, N_t) \right].$$

An equation similar to (21) can be written for the other trader. We observe however that the traders each period play an allocation game that is a game of strict opposition. Their fortunes are negatively correlated (as they are on the one-dimensional Pareto optimal surface) during each period. If the ratios of holdings M_{t-1}/N_{t-1} differ sufficiently the optimal strategy for the poorer trader is to spend up to his limit which is N_{t-1}.

If $N_t = M_t$ we can observe from (21) that both traders must eventually spend all of their money at each period. Suppose that this were false and that they each had a (symmetric) policy which called for each to always hold a finite amount of cash. This idle balance held for ever becomes of arbitrarily small value. Hence given the strategy of one it will pay the other to spend some of his cash balances and the policies cannot be in equilibrium.

In one special sense we are now in a position to link the "value of money" with the natural discount rate. We may make a comparison between two economies, one with an extra "chip" given to some player, i.e., evaluate for some trader i

$$\frac{\partial r}{\partial M_i} (B_{S, T}, M_1, M_2, \ldots, M_n) .$$

As was noted in Section 3.3, in this model a money market has not yet been included. Thus the "value of money" noted above is <u>not</u> the value of a loan from one trader to another, but how much someone could make by "slipping an extra chip into the game." It is the marginal utility of money to any trader. This will depend among other things on the "natural discount" rate.

We can also examine how the marginal utility of money changes with changes in endowments of the traders and with the number of traders. From the basic equations of (9) it can be seen immediately that the marginal utility of money is a function of the game and of the type of solution concept applied to the game.

Furthermore it can be seen immediately from (9) that the imposition of trading conditions and the use of money amounts to placing an external economy on all participants. If the ratios of money holdings are "just right" and we impose the noncooperative equilibrium solution the externality apparently disappears. Nevertheless, it is present in an important way as can be seen if an attempt is made to model the same trading structure as a cooperative game [10]. The game is not a c-game [11], i.e., it cannot be adequately represented by a characteristic function unless the financial threats inherent in forcing all to trade through the market, can be adequately limited.*

For $N = M$ the distribution of real resources at a noncooperative equilibrium point is the same as at the competitive equilibrium for all $s \geq 1$ and $T \geq 1$.

The general exploration of the two-sided market calls for an understanding of the behavior of the system as $T \to \infty$, $s \to \infty$ and as the ownership of the real resources is varied. This amounts to investigating the solutions to the set of dynamic programs:

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*What appears to be happening is that in a very large market with many small players, the threat potential of any fixed group attenuates as the market size grows. Their power remains only in their ability to damage themselves.

$$(22) \quad V_1^i(M_{t-1}, N_{t-1}) = \max_{x_t^i \leq M_{t-1}^i} \left[\varphi\left(\frac{x_t^i}{\sum\limits_{i=1}^{s} x_t^i + \sum\limits_{j=1}^{s} y_t^j} \right) + \rho V_1^i(M_t, N_t) \right]$$

$$(23) \quad V_2^j(M_{t-1}, N_{t-1}) = \max_{y_t^i \leq N_{t-1}^j} \left[\varphi\left(\frac{y_t^j}{\sum\limits_{i=1}^{s} x_t^i + \sum\limits_{j=1}^{s} y_t^j} \right) + \rho V_2^j(M_t, N_t) \right]$$

where $M_t = (M_t^1, M_t^2, \ldots, M_t^s)$, $0 \leq \rho < 1$ and the initial conditions M_0, N_0 are given (where we assume that the initial holdings of any trader of the same type are the same).

Suppose that the traders of the first type together have individual holdings of α of the goods sold. Traders of the other type hold $(1-\alpha)$ of the goods. Given this information it is possible to link M_t^i and M_{t-1}^i.

$$(24) \quad M_t^i = M_{t-1}^i + \left(\frac{\alpha}{s} \right)\left(\sum_{i=1}^{s} x_t^i + \sum_{j=1}^{s} y_t^j \right) - x_t^i$$

and similarly:

$$(25) \quad N_t^j = N_{t-1}^j + \left(\frac{1-\alpha}{s} \right)\left(\sum_{i=1}^{s} x_t^i + \sum_{j=1}^{s} y_t^i \right) - y_t^j$$

where $0 \leq \alpha \leq 1$.

In this paper no attempt is made at examining this general system. It is conjectured however, that the behavior will be qualitatively similar to the special example examined.

Suppose that $\varphi(x) = x$ so that (21) becomes:

$$(26) \qquad V_1(M_{t-1}, N_{t-1}) = \max_{x_t \leq M_{t-1}} \left[\frac{x_t}{x_t + y_t} + \rho V_1(M_t, N_t) \right] .$$

For this example the game is essentially constant sum hence we may eliminate y_t as a variable by minimizing (26) with respect to y_t. This amounts to setting $y_t = N_{t-1}$.

If we consider the finite version of (26) we must specify the worth of the final stage. As money becomes completely worthless at the end of the game, both traders will use all that they have at the last trade hence:

$$(27) \qquad V_1(M_T, N_T) = \varphi\left(\frac{M_T}{M_T + N_T} \right) \quad \text{and} \quad V_2(M_T, N_T) = \varphi\left(\frac{N_T}{M_T + N_T} \right) .$$

The solution to equation[*] (26) is given by:

$$(28) \qquad V_1(M_{T-k}, N_{T-k}) = \frac{1 - \rho^{k+1}}{1 - \rho} - C_k(\rho)\left(\frac{N_{T-k}}{M_0 + N_0} \right)^{\frac{1}{k+1}}$$

where $C_k(\rho)$ is a function of ρ and T for each k which can be evaluated recursively.

When $T \to \infty$, $V_1(M_t, N_t)$ is essentially independent of the stage, but depends only on the amounts of money on hand. In this case the optimal policy for the first trader becomes:

$$(29) \qquad x_t = (1-\rho)M_{t-1} + \rho N_{t-1} .$$

---

[*]Solved by Mario Juncosa.

In this example we see that there is a rapid convergence to the competitive equilibrium proportions. However, in the process the individual with the extra money is able to remove some real resources from the market.

5. ON MACROECONOMIES AND MICROECONOMIES

When there are more than two types of traders and many commodities the possibilities for many equilibria and considerable instability in adjustment appear to proliferate.

It is at this point that there is a natural transition between microeconomic and macroeconomic modeling. The idea of each trader announcing an infinite series of bids and offers for equilibria can, to a limited extent, be formalized. However, having done so we know, that due to many different institutional costs this is not a reasonable model of process.

A series of models that are logically consistent with the game formulation call for aggregating information and representing behavior by state strategies. For example we might wish to aggregate the traders into two or three classes such as consumers and producers and to aggregate the bids and offers into aggregate savings, investment and consumption plans. The formulation of such models calls for institutional knowledge, especially with regard to the flow of information.

A full macroeconomic model for policy purposes requires that players other than traders or consumers and producers be introduced. A short list of the extra players includes various types of banks, tax authorities, the treasury and insurance companies.

A modern economic system does not work completely through markets. Bilateral "deals" are frequent. Each individual does not sell all of his time and then go into the market to buy back his leisure. This causes great difficulties in designing national income accounts [12]. These problems are reflected in the type of noncooperative game model described here and they indicate the nature of the difficulty in going between microeconomic and macroeconomic models. A market structure must be specified to describe how mass trading is institutionalized. Depending on this structure somewhat different strategic possibilities are available to the participants in the economy.

6. INFLATION AND DEFLATION

We may observe immediately from the models in Sections 3 and 4, that inflation and deflation are not fully symmetric processes. In this extremely simple model of an economy with a constant supply of real resources without consideration of assets and production, monetary adjustments are only inflationary. They are symmetric if the inflating agency or deflating agency is able to change the amount of money to all individuals in the same ratio. The reason for the lack of symmetry otherwise comes in the boundary conditions. This is illustrated by the example given by equation (26). At least one type of trader will be operating at a boundary at which he carries no cash. The adjustment process is at work in bringing the money holdings in line with the real goods holdings so that in the limit the economy is in the same equilibrium as a nonmonetary economy. The adjustment depends

very specifically upon how fast the individuals with cash holdings that
are in excess of the barter model price ratios decide to release their
money into the economy.

A specific example in terms of the model in Sections 3 and 4 can
serve to illustrate the process. Suppose that there were two types of in-
dividuals trading using money, and that furthermore the money had been ini-
tially handed out in proportion to the competitive equilibrium prices.
We now have a system which stays in equilibrium every period with the in-
dividuals holding no idle cash. They actually hold a small amount of cash
on the average over the period for transaction purposes but they do not
hoard cash. This is discussed in more detail in Section 7.

Suppose that the amount of money issued to players of each type is
M . Each trader will use all of his supply every period and he will buy
and sell to the extent that when he enters the next period he enters with
the same amount M available. In this model (as in most economies when
the government is not printing paper and pushing it into the economy at
ever increasing rates) the velocity of circulation is relatively stable
so that the price level is established by the quantity of money $2M$ and
the amount of goods available. We return to a discussion of velocity in
Section 8. As the velocity of money is fixed in equilibrium we have de-
termined the price level in each period.

We now compare two possible inflationary steps and two deflationary
steps. Suppose that an outside agency wishes to cut the price level in
half or wishes to double the price level. As the economy is already in
equilibrium there is one way of doing it which is quite simple. It could

recall half of the currency in a symmetric way, thereby reducing the holdings
of everyone to $\frac{M}{2}$ instead of M . Similarly for inflation it could print
twice as much currency and hand it out so that everybody went into the "game"
with initial holdings of 2M .

If the government pursued this policy of issuing or recalling paper
or for that matter of merely redefining the monetary unit, the price system
in this noncooperative game would react immediately to the increase or de-
crease in supply of paper. The individual traders have no particular faith
in the paper. They realize that from the viewpoint of consumption it is
a worthless object, however, they also realize that they are trapped in
this noncooperative game and have to play according to the rules.

It is worth noting that if the government issues too much paper enough
players may get fed up with the rules and change the game completely. In
this case, the limit of the monetary mechanism will be reached when the costs
of organizing a monetary reform or of going back to barter or of heading
to anarchy are perceived to be cheaper by a large enough group of players
than abiding by the rules. They must be willing to give up the convenient
convention of being participants in a large and relatively easy to run trading
noncooperative game.

Suppose that for taxation reasons the government did not wish to
treat individuals symmetrically and in the process of creating an inflation
or deflation it wanted the results of this to favor one group of traders.
For example, it could either take away 50 percent of the holdings of one
group in this symmetric market or it could add 100 percent to the holdings
of the other traders. This gives us two games which are compared below.

In the long run, they both lead to a change in the price level however, because of the nature of the boundary maximization problem the deflating economy will deflate further than originally planned and then reinflate as is shown in Figure 2. In the two-sided case when M/2 is removed from
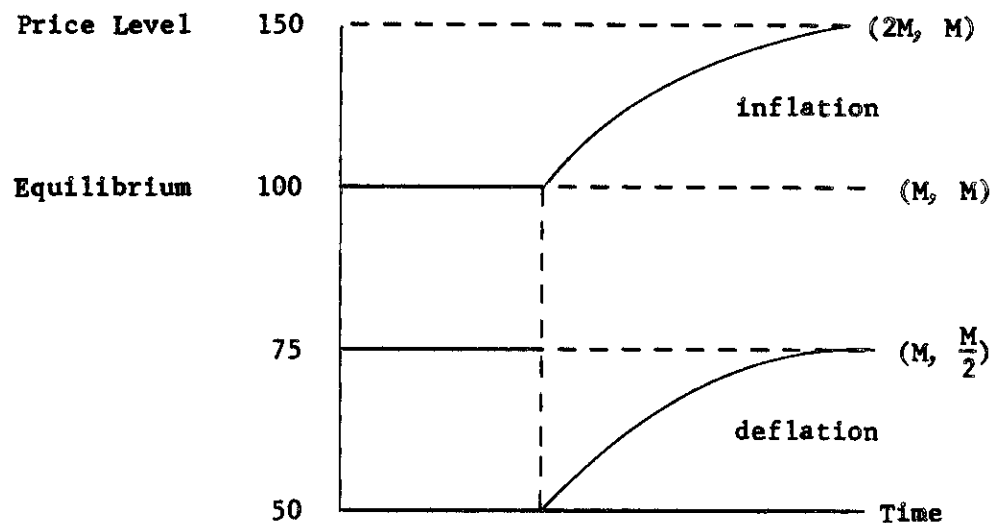
FIGURE 2

the money supply of one trader it may pay the other to restrict his spending even further. The correct level of deflation will be reached, but only after a steeper deflation followed by an inflation.

Another way of stating the above is that in economies with fiat monetary endowments such as (2M, M) and (M, M/2) are dynamically the same as the system is homogeneous of order zero. Hence, although the removal of money deflates the economy it "overshoots" and then builds up again.

7. TRANSACTION COSTS

There have been several investigations recently [13] of the role of transactions costs in a general equilibrium system.

In a noncooperative world with uncertainty, and with habit and faulty communications influencing the speed of payment and the cash flow position of the individual everyone may need to hold an inventory of money to facilitate transactions.

The expenses associated with large transactions are frequently not in proportion to the expenses incurred in small transactions. There is an increasing returns to scale effect that is present. The need to inventory money for trading purposes, and the cost advantages of large trades are two features of the monetary medium that are more closely related to its role as a factor of production rather than as a symbol of trust and exchange. They are properties that hold equally well for a commodity money as well as for a fiat money. They provide one further set of reasons for having individuals use some form of money.

As has been noted already it may take only some relatively weak reasons to initially hook individuals into using money, but once they are in this special noncooperative game it will take much stronger reasons for them to be able to get out.

The history of the spread of monetary mechanism provides many examples of different "games" [14]. Originally one might be trading in gold however, at some point it becomes feasible to issue gold certificates against the gold and to trade in these certificates. [15]. At some further point

it may be possible to formalize this into a set of laws governing trade,
at a still later point it may be possible to go from a gold that was re-
deemable only as a legal fiction (i.e., the redemption would not be possible
if everyone claimed their gold at the same time) to a situation in which the
redemption is no longer even fictionally possible. At each step in such
a process there may well be good solid production reasons for changing the
system. For example, paper is easier to carry and inventory than gold bullion.
Bigger transactions involving the exchange of paper can be made even easier
by introducing special forms of paper such as checks.

It is my belief that the role of transactions costs and their influence
on the monetary system comes from three extremely different sources. They
are:

(1) The savings in transaction costs made by using a commodity or
a paper money provide an inducement for a society to organize this type
of sophisticated trading. In general this provides a one way entry into
"the game" with exit possible only at the end of a war, a hyperinflation
or some other major societal upheaval.

(2) The ease of payment, simplification of paper work, abbreviation
of accounting procedures, are all aspects of the "productivity" of the money
mechanism resulting in improved trade. Understanding of these productive
roles is the same for money as for any other factor of production. When
set up costs or other mild nonconvexities appear, in any economy involving
production and exchange the functioning of a price system that is efficient
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set up costs or other mild nonconvexities appear, in any economy involving
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may be threatened.

(3) The third role in which the costs of transactions are extremely important is in information processing [16]. One of the important attractions of the modern mass market is that to a great extent it is anonymous. This is not true of barter. Anonymity is closely related with the aggregation of information, with the restriction on the availability of strategies which depend upon "too much informational detail" and with the treatment of information as a statistical aggregate property of the system. Even if the world were a world without uncertainty and with complete information available if there were costs attached to information processing, it would be more economical to replace the detailed structure by an aggregate statistical structure thereby introducing a degree of uncertainty. This appears to be a key interlink between the micro-economic and macro-economic models of human behavior.

With few exceptions virtually no attention has been given to the formal problems of integrating the information handling aspects of the market trading with money into a general equilibrium system.

In summary: The first aspect of transactions costs is related to "getting the players into the game"; the second aspect is not a property of fiat money alone, but can equally well be applied to commodity money and is a feature of a factor of production. The third aspect concerns the role of information and has scarcely been analyzed. There is every indication that the informational aspects of trading in modern markets are crucial to our understanding of monetary control and a price system [17].

8. THE VELOCITY OF CIRCULATION AND THE QUANTITY OF MONEY

Let Q be the amount of goods available in an economy and M the amount of money. Then is p is the price level we may write:

$$(30) \qquad\qquad p = \frac{M}{Q} \, .$$

This accounting relationship may be refined if we were to consider a time period in which, although the quantity of goods is held fixed, purchasing is irregularly sequential so that the same money may be used more than once. Thus the amount of purchases in the period is given by Mv where v is the average number of times a unit of money is used during the period. We may then have a relationship of the form

$$(31) \qquad\qquad p = \frac{Mv}{Q} \, .$$

In the model presented in Section 3, we implicitly made use of a modification of Equation (30) in the mechanism for determining price. Price was the outcome of a certain amount of money chasing a certain amount of goods. The nature of the formal model was such that the velocity of money was constrained to be no higher (but possibly less than) 1 per period. Thus, given the nature of the noncooperative game, equation (31) enables us to determine the price level once we understand that the amount of money being bid for all of the goods in society, even if the money is paper, may not necessarily be equal to the supply of money being held. This may be so even in an economy without exogenous uncertainty and without any trans-action frictions.

The formal model presented in Sections 3 and 4 is not a close model of "reality." In actuality trading during any accounting period is both sequential as well as in parallel. This being the case, it is possible for the velocity of money to be above 1 for the period under consideration.

The effects of change in velocity and sequential trading during a trading period can be broken into two broad categories. The first and most obvious concerns trading habit and institutional aspects of payment together with methods of communication and settlement. This includes the way some items are paid for in cash, other items are paid for by check and still other items are paid for periodically after a bill has been submitted and so forth.

The other aspects of velocity are tied in with the above however, they have a more explicit "game theoretic flavor." In particular, checkkiting, living off the "float" and not paying your tailor or doctor are specific acts influencing velocity. In a hyperinflation, resorting to barter and rushing to the nearest store as soon as you are paid provides an example of a deliberate act influencing velocity. It is my contention that the habit and custom aspects of velocity do not in general change quickly. They are determined not only by habit but also by institutions and technology. As such, the models presented in Sections 3 and 4 are consistent with modification to reflect variation in velocity arising from the secular changes in above sources. This is not the case for hyperinflation or for models of sharp financial practices where the aim is to live off minute differences in payment patterns.

In order to provide an adequate model of the last mentioned phenomena we would need to make explicit the nature of the trading patterns that take place in any economy.

The role of the quantity of money and the relative irrelevance of the velocity of circulation to the workings of the model presented in Sections 3 and 4 should not be construed as a critique in one way or the other of the classical or neoclassical money models which involve far different and richer models than the one presented here [18], [19].

In particular the important differences to stress is that with the model presented here we do not assume the existence of a price system, we deduce the price system. Here, neither money nor prices enter the utility function. To define prices prematurely or to put money or prices in the utility function of the individual would be self-defeating in the sense that it would amount to a circular argument where we begin by assuming that which we set out to prove. The way money and prices may enter into a "derived" utility function is determined by the solution concept used for that society.

9. A NONCOOPERATIVE GAME FOR THE MONEY MARKET

It is well known that the need for borrowing can come about in a world without uncertainty if there is a difference in time preferences among the traders and/or if there is a difference in the sequencing of the availability of resources. Thus, the farmer may need credit to tide him over the growing period for his crop. In Section 3.3 it was observed that the noncooperative model could be enlarged by adding a money market.

When one attempts to add a money market to a model of the economy
formulated as a noncooperative game, several new phenomena appear. They
concern the role of banking, the role of forward contracts and the role of
uncertainty.

One does not need statistical uncertainty or exogeneous uncertainty
to see the role of the rate of interest. For this reason it is easiest
to investigate the rate of interest excluding this type of uncertainty from
the model.

Banking presents the possibility of introducing a nonsymmetric player
into the game. An attempt has been made to do this elsewhere [20]. The
introduction of banking is obviously an attractive and desirable extension
of this type of modeling, especially when one wishes to consider policy
implications. However, at this level of abstraction the introduction of
a banking intermediary is an unnecessary complication.

There remains one other factor; that is the introduction of a forward
contract. In the model that has been presented in Sections 3 and 4, no
forward contract mechanism exists. If we were to introduce an alternation
in the ownership of resources or in preferences we may also wish to intro-
duce an impersonal money market as suggested in Section 3.3.

The money market introduces a qualitatively new phenomenon which
does not appear when we consider the system as a general equilibrium system
but does appear when we consider the system as a noncooperative game. The

buying and selling of money involves a futures contract. Money this time
period is sold for money to be delivered next time period. What are the
limits to be imposed on the strategy spaces of the players in this nonco-
operative game?

The limits on the actions of the players in the model outlined in
Sections 3 and 4 were explicit and simple. You could not bid more money
for goods you desired to buy than the amount of money you had on hand.
Thus there was no time interlinkage problem and no player was permitted
an "illegal strategy."

When players are promising to deliver future money this is not the
case; it is not axiomatic that the individual will be able to pay back the
money that he has borrowed when it is due at the start of the next period.

In order to well define the noncooperative game model involving a
money market it is necessary to describe rules for borrowing which include
the bankruptcy procedure.

The selection of an adequate set of rules for borrowing and for
bankruptcy is a by no means obvious procedure. The question to be asked
is "what borrowing rules and bankruptcy procedures are consistent with the
equilibrium points of the limiting noncooperative game containing the com-
petitive equilibria as a subset"?

The role of bankruptcy conditions is even more critical when more
complicated information conditions are postulated. They become absolutely
necessary for cutting the Gordian knot that presents itself in conjunction
with the definition of strategy. It is easy to conceive of a noncooperative
game in which the players are never completely informed about the state

that they are in. This can take place for example if there is a time lag
in the system which relays information about the moves of one set of players
to another set of players and vice versa. Because no player at any time
knows precisely at what state the system is, it is not possible to play
in such a game using behavior strategies. The analyses in Sections 3 and
4 depend explicitly upon being able to select equilibrium points which were
produced by having the traders use an optimal policy which depended only
upon the current state they were in.

10. TAXATION, GROSS NATIONAL PRODUCT AND WELFARE

In the noncooperative game model presented in Section 3, it is rela-
tively easy to introduce taxation and a public good. This can be done by
assuming a given exogeneous governmental policy which taxes the individuals
a certain amount at the start of each period and uses this money to bid in
the markets for the resource which is then transmuted into a public good.
This model is not investigated further at this time, however, it should
be noted that it reflects the interface between economic and political be-
havior. The model suggested above deals with the politics as given. The
individuals know how much they are to be taxed and they know how the govern-
ment intends to spend the money and they know what public good will be pro-
duced. In other words, the government is not a "live player" but a mechanism
which defines some of the aspects of a somewhat more complicated game of
strategy.

The use of fiat money at least in a steady state apparently enables
the society to achieve any one of the competitive equilibrium points for

the steady state model. At this level of modeling there does not appear
to be any natural way to distinguish between the choice of one competitive
equilibrium or another. Thus, any attempt to calculate aggregate GNP at
the different equilibrium points for welfare comparison purposes does not
appear to have any meaning in this model.

If there existed a commodity money in adequate supply which entered
into the utility functions of all individuals as a linear additive term
then the GNP calculation would have welfare significance.

11. CONCLUDING REMARKS

The approach to a theory of the operation of fiat money has been via
the limiting behavior of a noncooperative game. It is my belief that this
is a far more "natural approach" than assuming the existence of a competitive
equilibrium system [21].

The mathematical solution of even relatively simple one aggregate
commodity examples turns out to be relatively difficult. Even for a simple
iteration of a one period steady state economy, the problem of the non-
uniqueness of competitive equilibria and the nonuniqueness of noncooperative
equilibria remains when there are many commodities. Furthermore, when a
game theoretic definition of strategy is used, the existence of vast classes
of noncooperative equilibria is highly likely. Our attention has been limited
to a small subset of equilibrium points which can be obtained using policies
that only depend upon the individual's current position. It is observed
that when information conditions are complicated such policies may not exist.

This observation is closely related to the type of work done recently by Roy Radner [22].

It has not been possible to show that when there is more than one equilibrium point present that a displacement in a neighborhood will necessarily return the system to that equilibrium point in the noncooperative game. Herbert Scarf [23] and others have examples of instability in competitive equilibrium systems.

The major backing for many of the assertions made in this article is given by the structure of the model presented and by our ability to calculate at least a few simple examples. A calculation from an example does not constitute a general proof but in many instances it does serve to illustrate the existence or the nonexistence of some phenomenon that has been asserted. Thus, for example, even though the model is extremely simple the lack of symmetry between inflation and deflation has been illustrated. Furthermore, it has been shown that there do exist noncooperative games which contain among their limit equilibrium points the competitive equilibria of associated nonmonetary markets.

By the use of these extremely simple examples we have also been able to isolate the need for the introduction of bankruptcy rules into a model that has a money market. Furthermore, the anatomy of even a simple dynamic model helps to spell out the important role of information in defining a dynamic market process.

Tied in closely with the above approach is a belief in the extreme importance of dividing difficulties in attempting to analyze the role of money, banking, government finance and uncertainty and information in the context of a general equilibrium system.

The money, banking and taxation system of a modern state is a mass phenomenon. Microeconomic theory for the most part to date has been virtually unconcerned with the role of numbers [24]. The concern with numbers has been almost limited to oligopoly theory. The role of numbers of traders per se plays no major function in the usual mathematical presentation of general equilibrium theory. It has only been the recent introduction of the limiting process for game theoretic solutions (with the exception of Edgeworth) that has introduced the role of numbers in a general equilibrium system.

Macroeconomic theory in contrast to microeconomic theory has implicitly if not explicitly assumed that the equations represent aggregates of masses of individuals [25]. By constructing microeconomic models which permit us to "go to the limit" to study mass maximizing behavior it is my belief that we have the possibility of constructing a bridge between microeconomic theory and macroeconomic theory.

It is claimed in this article that such a bridge can be constructed by using the limiting properties of noncooperative game solutions and specializing the concepts of strategy information. The gap is large and we need many special models to study a host of extremely different difficulties that stand in the way of reconciling microeconomic theory with macroeconomic theory. For this reason a series of different problem areas are suggested below.

The simplest model involves the use of a commodity money in a non-cooperative game. This has been explored in Part III [26]. There still remains many mathematical problems concerning the relationship between the noncooperative equilibria and the competitive equilibria.

In this paper we have attempted to deal with the second simplest model. This involves the use of fiat money in an economy with no uncertainty no borrowing and positions of complete information. This can at least yield a "neutral money" in equilibrium, an adjustment process to equilibrium and nonsymmetry between inflation and deflation. Even at this level the difficulties in defining and analyzing the "dynamic game" are considerable.

The next complication involves the study of an economy with no uncertainty but with fluctuation in the holding of assets or with different time preferences. This will call for the specification of bankruptcy laws if a money market is introduced. This is the subject of investigation in a separate paper.

The functioning of fiat money in a system which has exogenous uncertainty, and positions of complete information is also a subject for separate investigation. This is closely related to insurance.

The introduction of a nonsymmetric player in the form of a bank produces a whole new set of models for the investigation of money and banking [27].

There are other types of financial institutions such as insurance companies and factors and the stock market which also need to be introduced and each presents a special problem [28], i.e., each defines a strategically different game.

The noncooperative solution is not necessarily the universal solution that should be tried in a general equilibrium model. It may well be desirable to have mixed models with different levels of cooperation reflected by no contract, implied contract and formal contract. In particular it is my belief

that it is at this level that much of the role of assets in a financial structure can be explained. Borrowing frequently requires that the borrower post a security in the form of a claim by the lender on some of the assets he owns. Thus, farm houses, factories, etc. are monetized in their relationship to the loan market [29].

The monetizing of the whole economy by having individuals buy back their own resources is probably a quite reasonable approximation for a modern economy except for labor and leisure. Labor requires a different treatment from the other commodities. One does not sell all 24 hours of time and then buy some of it back even as a crude approximation. Furthermore, as is well known, there are many other reasons such as problems in measuring quality of labor, learning, etc., that call for treating labor separately.

The difficulties in treating many commodities have been noted in Section 3. They remain for separate study.

The utility functions used in Section 3 were extremely special. A far more general approach to preferences over time is called for [30].

The models outlined have not included the role of production and growth [31]. Production and assets need to be considered even if one excludes the additional feature of assets as items which can be posted as bonds against loans.

The introduction of a government and taxation authorities introduces a new level of complexity. One can introduce the government exogenously with a well defined set of strategies given in advance or one might wish to conceive of the government as a player. The first alternative appears to be the simplest one to commence with and has the additional virtue that

that it separates the economic and political functions of the individual.

A theory of international finance calls for a new and completely different set of considerations. In the introduction of fiat money into the noncooperative game we modeled essentially one type of "blue chip". In order to understand international finance it becomes necessary to specify more than one type of chip and to specify the rules of manipulation of these different pieces of paper. Tied in with such rules may easily be special rules concerning commodities such as gold.

The above list of different aspects of monetary theory is given in the belief that it is possible to close the gap between micro and macro-economic theorizing in a perfectly consistent manner provided that we realize there is not a single problem to be solved but a host of problems. These can be first investigated in isolation, and then possibly be put together to reconcile microeconomic with macroeconomic theorizing in a manner that furthers application.

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