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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART I

THE GENERAL APPROACH ADOPTED

by

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Although this and most of the subsequent papers are of single authorship, I must acknowledge my considerable debt to Lloyd Shapley with whom I have collaborated for around 20 years. Our division of labor has been primarily between model building and careful generalization and rigorous mathematical proof. These papers concentrate on the first step in formalization and in obtaining well defined and viable even if somewhat special models of economies with money and financial institutions. The help rendered by Lloyd Shapley in conversations concerning even basic model building has been large.

It is my hope that at a later date these relatively simple (relative to the magnitude of the problem) models of money and financial institutions can be replaced by the appropriate generalizations together with proofs of general applicability.

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1. INTRODUCTION

This is the first in a series of papers on money and financial institutions in a general equilibrium context. The papers are written so that each stands independent of the others, although they are linked together by the subject matter and the methodology employed.

Specifically the approach adopted here can be regarded as an essay in mathematical institutional economies. It is argued that the development of the theory calls for a high degree of abstraction in the formulation of concepts such as financial strategy and the state of information. Yet at the same time the mechanisms through which decisions are transmitted cannot be abstracted out of the theory. Thus even though they may enter in a rudimentary form, institutions such as banks, insurance companies and markets must be modeled explicitly.

2. METHODOLOGICAL CONSIDERATIONS

In these papers mathematical model building is stressed. Considerable emphasis is laid upon the construction of consistent and complete mathematical models of monetary and financial processes. Yet at the same time few general theorems or mathematical proofs will be supplied. It is suggested, and discussed further below in 3 that the full mathematical investigation of this approach will take considerable effort primarily because the nature of the models calls for techniques which have not yet been sufficiently developed. In particular the two major gaps are (1) the lack of a sufficiently general, yet satisfactory equilibrium solution concept to games

played in extensive form. And (2) the difficulties encountered in solving simultaneous dynamic programs. It is argued in this and subsequent papers that both of these are of considerable importance to an adequate development of a theory of money and financial institutions.

Good model building in the study of economic processes is a matter of choosing among alternative levels of aggregation or disaggregation in order to lay stress upon some particular set of questions of interest. In these articles the stress is upon the monetary and financial system. This means that although problems concerning distribution, growth, production, oligopoly, public goods and other aspects of welfare are all relevant and intertwined with the monetary and financial aspects of the economy, the emphasis, in general, is laid upon the latter rather than the former. Questions such as how does financial policy influence growth have two aspects, one involves the details of financial policy, the other the development of the appropriate growth models and turnpike theorems that are applicable to describing nonmonetary growth processes.

2.1. Necessary Conditions for a Satisfactory Theory of Money and Financial Institutions

When we contemplate the role of money and financial institutions in a modern society there are many features that we can isolate that have to be accounted for by an adequate theory of money. Perhaps it is too early to talk about the theory of money, at this stage in the development of economic theory we are probably better off if we adopt a pluralistic approach and contemplate the construction of many small theories of money and financial

institutions prior to bringing them all together to constitute an integrated whole. A (not necessarily complete) list of features which must be reflected in the various theories of money and financial institutions is noted below.

Fiat money should not appear directly in the utility functions of the individuals. If one wishes to place it in the utility function and then engage in the study of general equilibrium systems an error in the form of circular reasoning will have been committed as the form of the utility functions will be determined as an outcome of the general system and cannot be regarded as a datum.

If we wish to study a partial equilibrium system, then the introduction of money into the utility functions of the individuals may be methodologically sound provided that it is explicitly assumed to be an approximation whose validity is confined to the domain of decision called for by the partial system. For example in the study of oligopolistic behavior among a few firms which may be large with respect to their market, but small with respect to the economy as a whole, it may be a reasonable assumption to treat fiat money as approximately the equivalent of "units of utility."

Commodity money should appear directly in the utility functions of the individuals, and under the appropriate circumstances of shortage, the dual roles of being both a commodity and a money should come into conflict.

Assets should not necessarily be within the utility functions of the individuals. The difference between steel plants and strawberries should be made clearly in the theory.

Labor should quite naturally play a role as a special commodity. Leaving aside the various sociological and physiological reasons for this

being so, there is an important market and financial reason for why this is so, it slavery is ruled out of our models. If an individual cannot sell himself, or be sold this immediately distinguishes the treatment of human capital in the market for productive assets.

The theory should explain the role of bankruptcy. Eventually it should also be able to cast some light on the question of the design of optimal bankruptcy laws.

Even in an economy with "perfect foresight" with a money market, loan limitation conditions should appear naturally. After all no banker should be willing to lend an individual more than he can possibly pay back even under the most favorable of circumstances. If we do not wish to be explicit about these limits, then we must be prepared to explain how earnings are unbounded in the models with which we deal.

A theory should be able to accomodate the phenomena of checkkiting and "living off the float." It also should be able to accomodate hyperinflation or to have within it an explanation of why and how the system breaks down under hyperinflation (the definition of what is meant by hyperinflation also poses some problems).

An adequate theory should distinguish institutional differences "naturally," i.e. in many cases they should appear parametrically. For example, slight variations in the sequence of payments or the cash flow pattern in a society should show up in the theory.

The theory should be explicit in the way in which velocity of circulation and variability in the order of trade influences the functioning of the system.

At least in one theory the role and importance of transactions costs must be covered.

An adequate theory should be able to explain the meaning and the role of "perfect foresight" and also should be able to get rid of the assumption of perfect foresight.

An adequate theory must be, at some point, genuinely dynamic and evolutionary, as contrasted with the virtual dynamics or extended static equilibrium of nonmonetary general equilibrium.

The basic role of information conditions must be reflected in an adequate theory. This involves the consideration of the roles of exogenous uncertainty, endogenous or strategic uncertainty; the explanation of the role of brokers and the other aspects of information processing services of a financial system. The adequate modeling of information conditions is extremely difficult as is evidenced by the work of Radner [4]. A basic link between microeconomics and macroeconomics lies in the aggregation of information for various decisionmaking units. The aggregation of information is central to the description of limited classes of dynamic (noncooperative or "quasi-cooperative") equilibrium points in the sense of game theory, as well as to the description of behavior equations for macroeconomics. A close look at the fine structure linking these two concepts provides a natural bridge between micro and macroeconomics.

In general in an adequate theory of money and financial institutions it should be easy to see precisely where the detailed nature of institutions and the law influence the workings of the system.

Last, but by no means least, if we are interested in investigating the functioning of free enterprise economies, then our theory should depend explicitly upon the role of the number of competitors in the various economic and financial activities.

2.2. The Many Roles of Money

Money is what money does. Money is part of the rules of the game. Money is many things. The conventional textbook definitions of money describe some of its functions. Money is used as:

A medium of exchange,

a store of value,

a standard of worth.

It helps to overcome the crudities of barter. It should also have certain functional properties associated with efficiency in transportation and information processing. Thus a money should be easy to handle, light, durable, easy to transport, easy to identify and hard to counterfeit.

An economy with a monetary system differs from an economy without a monetary system in the fundamental sense that it is governed by a different system of axioms. As there are many differences in monetary, credit and financial systems, there are many different economies which must be considered. We may wish to ask certain basic metatheoretical questions concerning these different economies, such as: "is there an optimal bankruptcy law?" Are there natural or basic forms of prototype banking, treasury and insurance institutions? Can we use a mathematical institutional approach to design such institutions?

Once we contemplate the questions raised above it should become clear that in any attempt to describe or define "money" we are probably better off in starting by distinguishing many different types of money. Einzig [2] devotes much discussion to how to distinguish between primitive monies and nonprimitive monies. It has been a favorite passtime of monetary theorists for several generations to argue about the definition of the money supply. In these papers I adopt a somewhat less ambitious goal. Many monies will be defined in terms of operations in specific systems. In terms of game theory, variations in rules concerning financial operations change the strategy spaces available to the participants in the economy and new properties of "moniness" can be explored.

A satisfactory theory of money and financial institutions should distinguish between fiat money, checks, gold certificates, warehouse receipts, I.O.U. notes and so forth in a nontrivial way. The need for such distinctions for the answering of many basic questions is not a minor correction to existing microeconomic theory. It is not a mere institutional dressing up or degrading of the elegant abstractions that make up general equilibrium analysis. It is a basically different general approach which stresses that if we hope to have a meaningful and possibly useful microeconomic theory which encompasses money and financial institutions then we cannot afford the luxury of throwing away the institutional baby in the mathematical bath water.

Money is:

An institutionalized symbol of trust, and

a (not perfect) substitute for insurance and futures contracts.

When considered strategically, some somewhat different properties of money appear. In game theory based models of the economy three critical properties of a money are of importance. They are:

money as a strategic decoupling device,

money as a criterion for distinguishing players, and

money as a means for sidepayments.

The introduction of a money into an economic system provides for an extra ~~degree of freedom~~ level of decoupling in the strategies of the players. Exchanges do not have to match in terms of goods alone, but in terms of goods and money. ~~This extra freedom amounts to a strategic decoupling of the acts of~~ purchase ~~and selling.~~ It also creates a role for cash flow in the economic maximization problem. The full importance of the role of money as a strategic decoupling device can be seen when one attempts to formulate a static model of ~~bilateral monopoly~~ trade as ~~a two person~~ an n-person symmetric noncooperative price strategy game. It can be done with ease with few markets when there is a money (commodity or fiat) present. Without a money, no natural symmetric model appears to exist ~~because there is really only one price in the system as either of the two commodities could be used for a numeraire.~~ except one with $k(k-1)/2$ bilateral markets to trade k commodities. ~~However the selection of a numeraire in such a model is tantamount to taking away the strategic freedom from the owner of those goods (this point is developed fully in Part III).~~

What an individual consumer can do with money or other financial paper is not the same as what a commercial bank can do and the strategies of a commercial bank differ considerably from the Federal Reserve Bank and the Treasury. Moniness in an economy appears as the consequences of extra rules on the game. These rules do not necessarily have to apply symmetrically

to all players. Distinguished groups of players such as bankers may be defined by rules which apply to them and not to others.

In much of game theory, use has been made of a special type of money or "utility pill" which appears as a separate and linear term in the utility functions of all members of the economy. As has already been noted in 2.1. above, for a partial equilibrium analysis this may be a good approximation. The use of money in a sufficiently small neighborhood might be approximated by transferable utility. In other models this assumption is not adequate and must be abandoned.

3. FOUR TYPES OF MATHEMATICAL MODELS

The development of a nonmonetary general equilibrium theory has been essentially the development of a static theory with perfect foresight (modified to some extent by the role of probability). There appear to be at least three ways to introduce monetary phenomena into a general equilibrium system. The first still leaves us with a static problem where perfect foresight plays an important role. The other two call for a dynamic and evolutionary treatment of the system. One of these corresponds to the macroeconomic or behavioristic approach and utilizes the role of expectations in an explicit manner. The other has its basis in game theory and makes use of the concept of a behavior strategy, or strategies which depend only upon the state that the system is in at any period of time.

3.1. Maximization Subject to a Budget Constraint Given Perfect Foresight

Suppose that there are n individuals in an economy; the economy lasts for T time periods and there are m_t commodities available during any period t. It is well known (see Debreu [1]) that this problem which apparently involves time, in the sense that there are T time periods, can be treated statically. Even if we consider an economy of infinite duration rather than one which lasts for only a finite number of time periods, provided that the appropriate items are bounded, the economic maximization problem can be treated statically.

In order to achieve a static treatment we make use of perfect foresight. If dealt with only verbally the assumption of perfect foresight can involve us quickly into a mire of ill-defined bad philosophical debate. If viewed mathematically it is quite simple. It amounts to the assumption that all individuals are initially supplied with information concerning the prices of all commodities during every period. Given this information an individual is able to calculate his income over the length of existence of the economy. He maximizes his welfare subject to one budget constraint.

The question of the existence of a competitive equilibrium, or an efficient price system in this market amounts to asking:
"Does there exist a set of prices p_{t,k_t} where $t = 1, 2, \ldots, T$ and $k_t = 1, 2, \ldots, m_t$ such that if each individual is informed of these prices (the assumption of perfect foresight) and of his endowments each will be able to maximize independently in such a manner that all budget constraints and market balances are satisfied and the outcome is Pareto optimal?"

Phrased in the above manner, the idea of perfect foresight in neither particularly strange nor dynamic. The mathematical question concerns the existence of an equilibrium (or a fixed point under certain transformations.) in a static or "one-shot" system; ~~The question concerns existence, not how do we get there~~. The existence problem has been solved successfully but it tells us little about the process, mechanism, dynamics and stability of price formation.

3.2. Convex Programs with Perfect Foresight

In the general equilibrium system without money the trick used to reduce what is apparently a dynamic problem to one that is basically static involves time-dating commodities so that 1971 wheat and 1972 wheat are regarded as two different commodities, each with a market that exists now in which they can be traded.

Uncertainty is handled in the same way. We merely formally declare an enormous inflation of all commodities so that "1972 wheat if there is a flood" and "1972 wheat if there is no flood" become different commodities and are traded in different markets. By this device we have the paradox of the assumption of perfect foresight coexisting with an assumption of (exogenous) uncertainty. The paradox is immediately resolved when the mathematical question is asked concerning the existence of an enormously enlarged set of prices, one for each physically defined commodity, at each time in each possible state at that time.

The assumption of the existence of markets in all future commodities under every possible state takes care of dynamics, uncertainty, (and the need for) insurance and money. In game theoretic terms it enables us to reduce a problem presented in extensive form, to a normalized or "strategic form" [5]. In doing

so the nuances of the information structure are destroyed in the sense that they are aggregated and the dynamics is essentially removed as all individuals are now left with a single move which is made simultaneously. They all declare a strategy.

The assumption of the simultaneous existence of all futures markets enables an individual to rid himself of any cash flow constraints and to consider only one nontemporal overall budget constraint.

By assuming the existence of some form of Poker chips or pieces of paper that we will call money, by eliminating futures markets except for a one period market in money, and by requiring that all trades be carried out using money the cash flow problem is reintroduced. A model having these specific properties is studied in Part V.

The mathematical problem posed by the monetary economy differs from the nonmonetary economy inasmuch as a whole host of intertemporal budget constraints must be satisfied. The one simple overall budget constraint of the nonmonetary economy must be replaced by a cascade of cash flow conditions.

We are still able to avoid having to consider a dynamic problem by making use of perfect foresight once more. The question concerning existence of an appropriate equilibrium is asked as follows:
"Does there exist a set of prices p_{t,k_t} where $t = 1, 2, \ldots, T$ and $k_t = 1, 2, \ldots, m_t$ and the price of money is set at 1 ; and a set of one period rates of interest r_t for $t = 1, 2, \ldots, T$ such that if each individual is informed of these prices and his endowments each will be able to maximize independently in such a manner that all budget constraints, loan constraints, market balances and money market balances are satisfied and the outcome is Pareto optimal?"

3.3. Dynamic Programs with Expectations

The embedding of monetary trading in the general equilibrium system while still leaving the model static and using perfect foresight does not appear to be in the spirit of what we would expect a reasonable theory of money to look like. We need to make the model explicitly dynamic. Before we turn to this question it should be stressed that although a static model with trading in money, perfect foresight and no transactions costs may appear to be highly "unrealistic," in trying to understand the behavior of money it is extremely important to be in a position to divide difficulties, lest the striving for premature realism confuse the analysis by the interaction of many different attributes of money and a financial system.

There is a natural way to convert the static general equilibrium system with money into a dynamic evolutionary system. It is the way that has been used in the study of macroeconomics and has been utilized in microeconomics in attempts to devise a behavioral theory of the firm. Essentially it boils down to replacing the assumption of perfect foresight by expectations. We may now regard each individual as composed of a set of agents, one agent for each time period. Each of these agents faces precisely the same problem as did the individual in 3.2. The difference between this model and the previous one, however shows up in the fact that at each time period new expectations are generated and a new maximization problem is posed. The system does not take the future as given, but period by period, generates its time path.

The arbitrary feature of this scheme (and indeed a basic arbitrary feature of macroeconomics) is how do we select the expectations functions?

We are called upon to add an item that is not neatly covered by the rubric of microeconomic maximizing man. Each agent at every time period acts in this way, but the forming of expectations calls for added assumptions. Methodological considerations call for the expectations formation to be relatively simple. If we make it too complex we introduce too many extra degrees of freedom. Nevertheless we must depend upon whatever psychoeconomic and psychological knowledge there is to justify the selection of a description of the generation of expectations.

The mathematical description of this system is a set of simultaneous dynamic programs each of which appears to be an independent program but which together must satisfy aggregate market and money market conditions each period. The question concerning the dynamics of the system can be phrased in a way to call for a comparison with 3.2.
"Given a specific method for generating expectations concerning future prices and interest rates does there exist a steady state solution to the resulting set of dynamic programs such that the market and money market conditions are satisfied each period and the system generates the same series of prices and interest rates which were given by the assumption of perfect foresight in 3.2.?"

A more detailed discussion of this system is given in Part V.

3.4. Dynamic Programs with Behavior Strategies

A different way in which the system can be made genuinely dynamic is via a game theoretic treatment. This calls for viewing the market as a game to be played in extensive form. This requirement immediately calls

for the specification of a solution concept for the game in extensive form. No broad generally accepted way for solving nonconstant sum games in extensive form exists at present. However one which can be reasonably well defined and applied with some success for certain classes of problems is a dynamic version of the noncooperative equilibrium point obtained as a result of all of the players using behavior strategies. This approach calls for the consideration of games with information conditions almost as ridiculously constrained as the usual information assumptions called for in general equilibrium theory. Specifically periodic positions of complete information are called for.

In the theory of games a distinction is made between cooperative and noncooperative solutions to a game. The first type of solution concentrates primarily upon the pattern of distribution at the expense of detail concerning information and strategy. The second type of solution concentrates on overall strategy. When games are considered in extensive form the distinction between the two types of solution begins to fade. Unfortunately little work has been done to date on the construction of satisfactory dynamic solutions. However it is my belief that the most fruitful candidate for such development is the noncooperative solution, which in a dynamic form should really lose the appelation noncooperative and possibly be called the "self-policing equilibrium solution." In subsequent papers the definition of the dynamic game theoretic solution is considered further.

The use of behavior strategies provides an alternative to the assumption of a mechanism for generating expectations. However the cost of utilizing this approach comes in one further level of mathematical complication. The

resulting dynamic programs are no longer in a single control variable, but reflect the important competitive features of a game of strategy. The individuals are expected to take into account explicitly the influence of the actions of others. An example of this type of simultaneous dynamic program game is provided in Part IV.

The treatment of an economy with money and financial institutions as a dynamic game has several important advantages. In particular it calls for the study of at least two very different types of limiting behavior. They are the study of the behavior of the system as the length of the market becomes infinite; and the study of the system as the number of participants becomes large. This means that it provides a vehicle for studying financial oligopolistic behavior.

4. MANY MODELS WITH MONEY AND FINANCIAL INSTITUTIONS

The underlying belief in the approach to the understanding of money and financial institutions suggested here is that it is vital to separate out difficulties and to construct models of possibly highly limited scope to study the various aspects of money and finance independently. This being the case, a substantial number of specific models are suggested some of which have been considered by Hahn [3], Starr [6] and others. And some of which are intended as subjects for investigation in this series of papers.

The simplest monetary model involves the use of a commodity money in a static noncooperative game embedded in a closed economy. This is considered in Part III.

The next level of complication calls for fiat money in an economy with no transactions costs, no uncertainty, no money markets or futures markets, but with oligopolistic uncertainty. Some simple cases are examined in Part IV.

Fluctuations in the supply of endowments, or differences in time preferences makes it necessary to consider some form of money market or other futures markets. This calls for the specification of loan conditions and the introduction of the rate of interest. This is done in Part V. If an economy with a money market or other futures contracts is considered as a dynamic game then it becomes necessary to specify bankruptcy laws. These are considered in Part VI.

The role of assets and production must be considered from two points of view: both as a device for providing a backing for financial paper and in terms of economic growth.

The role of a money in economic systems with transactions costs is a separate topic.

The functioning of a fiat money in a system which has exogenous uncertainty and positions of complete information is also a subject for separate investigation. Several different models can be formulated depending upon the correlation or noncorrelation of the risk, whether the risk is endogenous (game theoretic in nature) or exogenous and depending upon the institutions introduced into the model in order to cope with the effect of risk. These include fiat money, surety arrangements, private insurance companies, government insurance companies and futures markets.

A different set of models are needed to investigate the effects of markets which never contain positions with complete information. The work of Radner [4] has given some indication of the difficulties encountered. However his work has not been extended to consider the effects of money and financial institutions. It is my conjecture that such an extension would be a natural way to commence to investigate the economic role of brokers and investment banking. In these professions the differential in information processing and evaluation is critical.

Setting aside uncertainty or complex information patterns it is possible to produce a set of different yet relevant models by introducing a nonsymmetric player in the form of a banker. The rules imposed on his behavior may reflect various institutional forms such as reserve banking. Moniness under one banking system will be somewhat different from moniness under another system.

The introduction of a government with taxation authorities introduces a new level of complexity. One can consider the government as an exogenous agency with a specified set of strategies given in advance, or one might wish to consider the government as an active player.

The specification of the role of government calls for a consideration of the relationship between financial policy and public goods. It is easy to consider a monetary and financial system with a government playing a "neutral" or supervisory role, i.e. using taxation to assist in the running of a competitive monetary economy without necessarily supplying public goods with the proceeds from taxation.

A theory of international finance calls for a new and completely different set of considerations. In the consideration of fiat money in a single economy, essentially one type of Poker chip has to be introduced. In order to model international finance more than one type of chip has to be considered and the rules specifying the rights of the individuals to manipulate them must be given. This includes their relationship to gold and rules concerning the fixing of exchange rates.

The above partial list of monetary and financial models is given in the belief that it is possible to close the gap between micro and macroeconomic theorizing in a perfectly consistent manner provided that we realize that there is not a single problem to be solved but a host of closely related yet different problems. These can be first investigated in isolation, and then possibly can be joined together to reconcile microeconomic with macroeconomic theorizing in a manner that furthers application.

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