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TAXATION, RISK-TAKING, AND THE ALLOCATION OF INVESTMENT IN A COMPETITIVE ECONOMY

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by

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I. Introduction

1.1. Objectives of Paper

Two controversial questions in the theory of public finance have been: (a) What are the effects of alternative tax policies on risk taking? and (b) Should the government tax safe industries at a higher or lower rate than risky industries?

The earlier concensus that income taxation reduced risk taking was destroyed by the classic article of Domar and Musgrave [10], which argued that proportional taxation led to an increase in risk taking provided there were adequate loss-offset provisions. More recently, Mossin [14] and Stiglitz [19] have shown that the Domar-Musgrave conclusions are only valid under certain restrictive assumptions about individuals' attitudes towards risk taking, namely, that either (a) the risky asset is an inferior good, i.e. as wealth increases, less of it is demanded, or (b) as wealth increases,

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the proportion of wealth held in the safe asset increases but the absolute amount held in the risky asset also increases.[1]

But all of this has been limited entirely to the analysis of the demand side of the market. There have been few investigations of the effects of taxation on risk taking in the context of a general equilibrium model.[2] It is important to extend these partial equilibrium results to general equilibrium models for three primary reasons:

(a) The results from the demand and the supply side may be contradictory. This is illustrated by the following example, taken up in detail in Section 7. Under certain provisions of the tax code, namely that interest payments are exempted from corporate profits taxes, it can be shown that all firms that can, will increase their supply of bonds;[3] on the other hand, under suitable restrictions on the utility functions,[4] it can be shown that, at given prices, all individuals will decrease their demands for bonds.

(b) No account is usually taken in the partial equilibrium analysis of the proceeds of the tax. It will turn out that the effects of, say, an income tax depend on the assumptions one makes about whether the proceeds are thrown away, or used to provide commodities which are close substitutes to the other commodities on which the individual spends his income.

(c) It is important to know how the changes in demands for securities, i.e. in portfolio composition, lead to changes in the real investment of the economy.

The first purpose of our study then is to analyze the effect of taxation on risk taking in the context of a general equilibrium model.

The normative question of the optimal structure of taxes is, of course, a much more fundamental and interesting question. As we shall note, the specification of the objective function of the government is crucial in the analysis. The structure of optimal taxes in general equilibrium models has been the subject of several extensive recent studies, ([9, 25]) and a second objective of our study is to extend these results to the taxation of risky and safe industries.

1.2. Alternative Tax Structures

In this paper, we consider a number of alternative tax structures:

(a) We need to distinguish between taxation on investment in a given industry and on profits (output) in the given industry. Both are commonly employed, often in conjunction with one another. An investment credit is a negative tax on investment while a sales tax on machinery is a positive investment tax. The usual profits tax with depreciation allowances may be thought of as combining a positive tax on the profits and a negative tax on investment. Because profits are stochastic, the revenues generated by a profits tax are stochastic and changing the tax structure changes the probability distribution of the government's revenue (the patterns of returns across the states of nature).[5] This introduces a number of difficulties in evaluating alternative tax structures which are discussed in detail below.

(b) Different tax systems may impose different tax rates on different industries or may distinguish between the form in which the profits of the firm are distributed, e.g. payments to bondholders and shareholders may be treated differently.

(c) We must specify what is done with the revenues derived from the tax. We will consider in the subsequent discussion two polar cases: (i) the revenue from the taxation does not affect the individual's allocation decisions (e.g. it is thrown away), or (ii) the government uses the revenue to provide lump sum payments or a commodity which is a close substitute in consumption to the private good.

1.3. Plan of the Paper

After presenting the basic model in Section 2, Section 3 uses the simple mean-variance diagram to analyze the effects of profits taxation. Section 4 derives the optimal tax structure for investment taxes for the case where there are only two industries, while Section 5 discusses the equity considerations involved in differential treatment of safe and risky industries. Section 6 investigates briefly the implications of price uncertainty, while Section 7 discusses the effects of tax structures which differentiate between debt and equity. Finally, Section 8 summarizes the major conclusions of the paper.

2. The Basic Model[6]

(a) Technology. In our basic model, we shall assume that there is a single factor input and a single commodity output. The economy lives for only one period.[7] Firms differ not with respect to the goods they produce but with respect to the efficiency with which they convert the factor input into the commodity output in different states of nature. Thus different firms will yield different patterns of output across the states of nature, and increasing the input into any given firm increases its output in all states of nature by the same percentage.[8] If $X_i(I_i, \theta)$ is the output of the i^{th} firm in state θ, when the level of the input (which we shall call "investment") is I_i then[9]

$$X_i(I_i, \theta) = g_i(\theta)f_i(I_i) . \tag{2.1}$$

For most of the analysis, we shall, for simplicity, make the somewhat stronger assumption of stochastic constant returns to scale: doubling the level of investment doubles the level of output in each state of nature. The necessary modifications for the case of decreasing returns to scale are presented in footnotes. Thus, (2.1) takes on the form

$$X_i(I_i, \theta) = g_i(\theta)I_i . \tag{2.1'}$$

(b) Firm Behavior.

(i) Distribution of Profits. A firm issues only two kinds of securities, bonds (which are perfectly certain) and common stocks.[10] After paying back bondholders, the remaining "profits" of the firm are distributed to the shareholders in proportion to their individual shares in the firm: If B_i is the number of bonds issued by the i^{th} firm (one bond has a price of unity relative to the price of the factor input) and $r-1$ is the rate of interest, then total payments to bondholders (principal plus interest) is rB_i.[11] The total market value of the firm's shares will be denoted by E_i and hence the total market value of the firm can be denoted by

(2.2) $$V_i \equiv E_i + B_i$$

Thus, the return per dollar invested in the i^{th} firm's share is

(2.3') $$e_i \equiv \frac{X_i - rB_i}{E_i}$$

In the absence of taxation (which we have not yet introduced into the model), the Modigliani-Miller theorem [12, 21] assures us that the value will be independent of how the firm finances its investment, i.e. whether through bonds or equity. For simplicity in the subsequent analysis, except in those cases where the debt equity ratio does affect the valuation of the firm,[12] we shall assume that the risky firms issue no bonds. Then the return per dollar invested in the i^{th} firm can simply be written

(2.3) $$e_i(\theta) = X_i(I_i, \theta)/V_i$$

(ii) Investment. Of the total market value of the firm, I_i is issued (in bonds or shares) in exchange for the factor, so that the market value of the original shareholders' claims is[13]

(2.4) $$V_i - I_i$$

We shall assume then that each firm is a price taker, i.e. it assumes that its market value will increase in proportion to I_i,[14] and firms maximize the stock market value of the original shareholders. Hence, if $V_i > I_i$, the firm expands indefinitely, if $V_i < I_i$ it shuts down. Of course, as I_i changes, V_i/I_i will also change. Thus, competitive equilibrium will require $V_i = I_i$.[15]

Two firms with the same pattern of returns (i.e. for whom $X_i(I_i, \theta)/X_j(I_j, \theta)$ = constant, all θ, are said to be in the same risk class. Under the assumption of constant returns to scale,[16] it is unnecessary to distinguish among such firms. From now on, then, we shall aggregate all the firms in the same risk class together, and we shall denote by the subscript i the i^{th} "industry" (risk class).

One industry, to which we shall pay special attention in the subsequent analysis is the perfectly safe industry, and it will be denoted by a subscript S. Its shares are perfect substitutes for bonds.

(c) Individual Behavior. The j^{th} individual has two decisions to make: (i) How should he allocate his wealth among alternative investments? and (ii) How much of the factor I should he supply? Although in fact these decisions will be made simultaneously, we shall discuss them as if they were made independently.

(i) Assume the individual supplies I^j of the factor to the market. It is his initial wealth which he must allocate among alternative investment opportunities. If λ_i^j is the proportion of the individual's wealth allocated to the i^{th} security, then his terminal wealth (income) in state θ, $Y^j(\theta)$, is[17]

$$Y^j(\theta) = (\sum_i \lambda_i^j e_i + (1 - \sum_i \lambda_i^j) r) I^j \tag{2.11}$$

We assume that individuals may sell securities short (i.e. $\lambda_i^j \gtreqless 0$).

Individuals choose their portfolio (i.e. choose λ_i^j) in order to maximize their expected utility of terminal wealth[18]

(2.13) $$\max EU^j(Y^j(\theta)) , \quad U^{j'} > 0 , \quad U^{j''} < 0 .$$

Thus expected utility maximization leads to

(2.14) $$EU'^j e_i = EU'^j r$$

The average returns, weighted by marginal utilities, of all securities must be the same.[19] Since $U''^j < 0$ (2.14) is both a necessary and sufficient condition for an optimal allocation.[20, 21]

(ii) In the subsequent analysis, we shall make two alternative assumptions about the supply of the factor I . In the derivation of the optimal tax structure, we shall assume that the individual gets disutility from supplying the factor I (I may be thought of as labor). Thus he chooses I^j to maximize

(2.16) $$u^j = EU^j(Y^j(\theta)) + L^j(-I_j) \qquad L' > 0 , \quad L'' < 0$$

where $Y^j(\theta)$ is given by (2.12).[22] Hence utility maximization requires the marginal disutility of labor to equal the expected marginal utility of income:

(2.17) $$EU^{j'} e_i = L^{j'}$$

or

(2.18) $$EU^{j'} Y^j = I^j L^{j'}$$

Alternatively, where the gain in simplicity seems worth the loss in generality we shall assume that I^j is supplied inelastically.

(iii) Derivation of demand curves. From (2.14) we may derive demand curves for the securities of the different industries (risk classes). Since the total value of the i^{th} industry is V_i when its investment is I_i, i.e. it is producing I_i units of the pattern of returns across the states of nature defined by $g_i(\theta)$, it is natural to think of

$$p_i = V_i/I_i \tag{2.19}$$

as the price of a security of the i^{th} type. The j^{th} individual's demand for securities of the i^{th} firm, Z_i^j, may then be written as a function of the prices of the different securities[23]

$$Z_i^j = D_i^j(p_1, \ldots\ldots p_n) \tag{2.20}$$

Similarly, we can write the supply of the factor as a function of the same variables

$$I^j = I^j(p_1, \ldots p_n) \tag{2.21}$$

Because of the strong assumptions on differentiability imposed on our utility functions, it is easy to show that the demand functions for securities and supply function of the factor I are continuous in prices. (Continuity can be proved under much weaker conditions.)

(d) Market Equilibrium. From here, it is easy to describe the market equilibrium of this economy. Adding up the demand functions over the individuals, we obtain the aggregate demand functions,

(2.22) $$Z_i \equiv \sum_j Z_i^j = \sum_j D_i^j(p_1, \ldots p_n)$$

As we have already argued, competitive equilibrium requires

(2.23) $$p_i = 1 ,$$

i.e., $V_i = I_i$ for all firms. For demands to equal supply of each security,

(2.24) $$V_i = p_i Z_i$$

while for the factor market to clear,

(2.25) $$\sum_i I_i = \sum_j I^j$$

Substituting (2.19) into (2.24) we obtain

(2.26) $$I_i = Z_i(1, \ldots 1)$$

determining the pattern of allocation of investment. Walras' law ensures us that when all securities markets clear, the factor market clears [(2.25) is satisfied).[24]

3. Profits and Production Taxes

The purpose of this section is to review the partial equilibrium analysis of the effects on the allocation of investment of profits and production taxes, to show how, under certain circumstances, these results may be extended, with very little alteration, to the general equilibrium framework developed in the preceding section, and finally, to show how, under other circumstances, the partial equilibrium results must be modified when extended to the general equilibrium situation.

For expository convenience, we make two special assumptions, that the expected utility can be described completely by the mean and variance of the terminal wealth (income) and that the factor is inelastically supplied. These assumptions are not crucial to the ensuing analysis, but are only made so that we can employ the simple mean-variance diagrams; our results[25] may be readily extended to the more general case.

Subsection (a) considers the case where there are only two industries and the proceeds of the tax are used to provide goods which do not enter into the individual's utility function (for private goods) and hence do not affect his portfolio allocation. Subsection (b) considers, for the same case, the design of an optimal tax structure when the government also has attitudes towards risk which must be taken into account. Subsection (c) considers the situation where the government revenues are used to provide lump sum payments. The appendix to Section 3 extends the results to cases where there is more than one risky industry.

(a) Allocative Neutrality and the Effects of Proportional Taxation.

As we already indicated, we will employ in this section the assumption that expected utility can be written as a function of the mean, μ, and standard deviation, σ, of terminal wealth

$$\mu \equiv EY(\theta) \tag{3.1}$$

(3.2) $$\sigma^2 = E(Y(\theta) - \mu)^2$$

We thus assume that[26]

(3.3) $$EU(Y) = M(\mu, \sigma)$$

where

(3.4) $$M_1 \equiv \frac{\partial M}{\partial \mu} > 0 , \qquad M_2 \equiv \frac{\partial M}{\partial \sigma} < 0$$

There is positive marginal utility to increments in mean income and disutility to increments in standard deviation.

If the distribution of returns are jointly normally distributed, it can be shown that (3.3) will always hold, and that the indifference curves will be quasi-concave, provided only the $U'' < 0$. (Alternatively, (3.3) will hold independent of the probability distribution if U is quadratic.) The shape of the indifference curves will, of course, depend on the shape of U .

Because the only properties of the probability distribution of returns the individual is concerned with are the mean and variance, the risky asset may be completely described by

(3.5) $$Eg(\theta) \equiv \mu_R$$

(3.6) $$E(g - Eg)^2 \equiv \sigma_R^2$$

The safe industry's return will be denoted by r .

Because of constant returns to scale in both industries, in competitive equilibrium, the return per dollar invested by the firm must be equal to the return per dollar invested by the individual. Hence

$$Ee_R = \mu_R$$

$$E(e_R - \mu_R)^2 = \sigma_R^2$$

If the individual (society) invests a fraction a of its resources (I) in the risky industry, the mean and standard deviation of terminal income (wealth) is

(3.7a) $$\mu = I(a\mu_R + (1-a)r)$$

(3.7b) $$\sigma = Ia\sigma_R$$

The individual's and society's opportunity set may be depicted as in Figure 1, where R represents the point where all resources are invested in the risky industry and S the point where all resources are invested in the safe industry. The opportunity set, from (3.7), is clearly just the straight line between the points. If all individuals were identical the competitive equilibrium would then be depicted by the point E, at the tangency of the indifference curve with the opportunity set[27] i.e. where

(3.8) $$-M_2/M_1 = (\mu_R - r)/\sigma_R$$

We now consider the effects of the introduction of taxes. We shall discuss two kinds of taxes: Gross production taxes and profits taxes.

(i) Gross Production Taxes. In this case, t_S of the output of the safe industry and t_R of the output of the risky industry must be turned over to the government. Hence, the after tax return on an investment in the

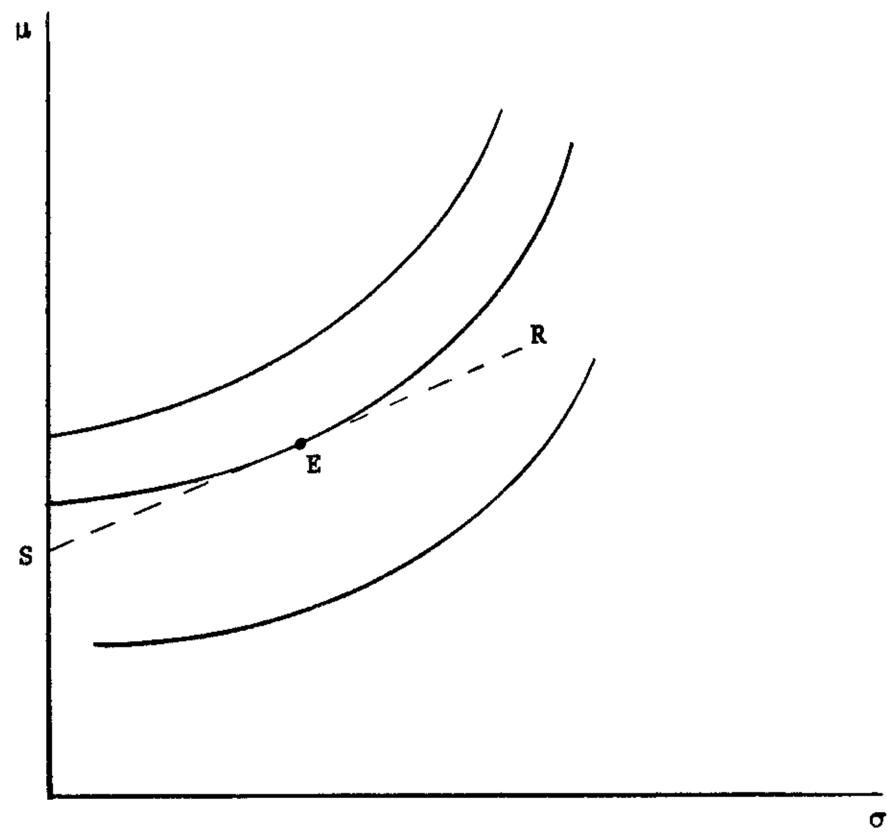

FIGURE 1

Determination of Equilibrium Allocation of Resources between Safe and Risky Industry: No Taxes

safe industry is

$$r' = (1 - t_S)X_S/I_S = r(1 - t_S) \tag{3.9}$$

and in the risky industry

$$(1 - t_R)X_R/I_R = g(\theta)(1 - t_R) \tag{3.10}$$

from which it follows that the after tax mean return in the risky industry is just

$$\mu_R' = (1 - t_R)\mu_R \ . \tag{3.11a}$$

Similarly

$$\sigma_R' = (1 - t_R)\sigma_R \tag{3.11b}$$

Thus, the after tax opportunity locus for the individual is given, as in Figure 2, by the straight line $R'S'$. Note that R' lies along the line from the origin through R; i.e. mean and standard deviation are reduced proportionately. If $t_R = t_S$, the new opportunity locus is parallel to the pre-tax locus. If a fraction $\underline{a}$ of the individual's portfolio is allocated to the risky investment, the before and after tax mean and standard deviation of terminal wealth (income) are given by

$$\mu = I(a\mu_R + (1-a)r) \qquad \mu' = I(a\mu_R(1 - t_R) + (1-a)r(1 - t_S))$$

$$\sigma = a\sigma_R I \qquad \sigma' = a\sigma_R(1 - t_R)I$$

Note that a is the ratio of the actual standard deviation to the total possible (both before and after tax). Thus $\underline{a}$ can always be calculated

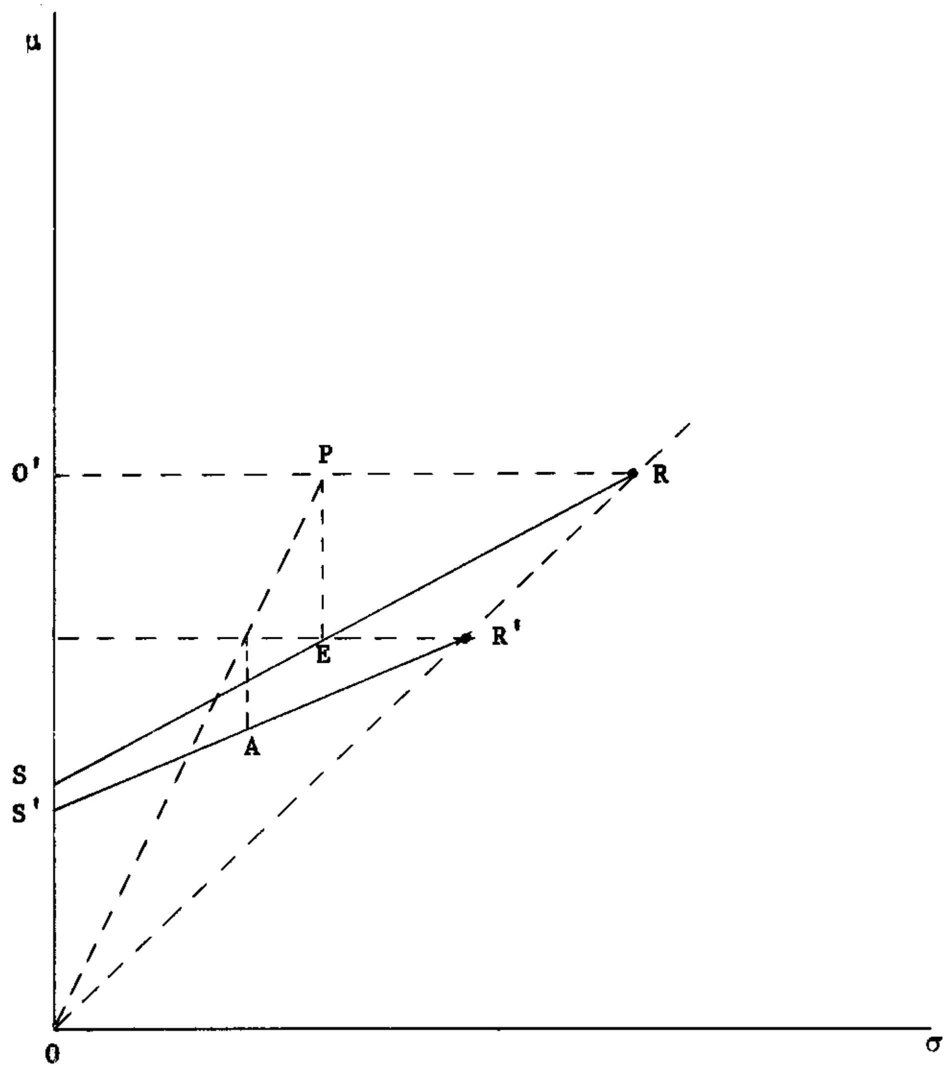

FIGURE 2

Production Tax: Determination of Point of Allocative Neutrality as follows. Draw a line through R parallel to the horizontal axis, and draw the projection of E on that line, P. Then a = O'P/O'R. (Figure 2)

From this, it is easy to describe the effect of a uniform production tax. It is immediate that if the original allocation is E, if the new allocation is at A, the tax is allocatively neutral, i.e. leads to the same proportion of resources invested in the two industries after tax as before tax; if it is to the left, it results in increased investment in the safe industry, if to the right, in the risky industry. But where the new equilibrium lies depends simply on the shape of the income consumption curve. We have already noted that a proportional tax simply shifts the opportunity locus down in parallel. Thus, if the income consumption curve bends upwards, there is an increase in investment in the risky industry and conversely if the income elasticity of the safe asset is less than unity.[28] (See Figure 3.)

The fact that a proportional tax increases risk taking when the income elasticity of demand for risky assets is less than unity does not mean that allocative neutrality requires in that case that a higher tax be placed on the risky industry--nor does the converse apply when income elasticity is greater than unity. We must also know the price elasticity of the demand for the safe asset. If an increase in tax (equivalent to an increase in price or decrease in return) for the safe asset leads to reduced demand for the safe asset, then allocative neutrality requires, when the income elasticity of demand for risky assets is less than unity, that the risky industry be taxed at a higher rate than the safe; but if the price elasticity is positive--as it may well be--just the opposite is required.

KEY: E = pre-tax equilibrium
E' = after-tax equilibrium
A = allocatively neutral investment allocation

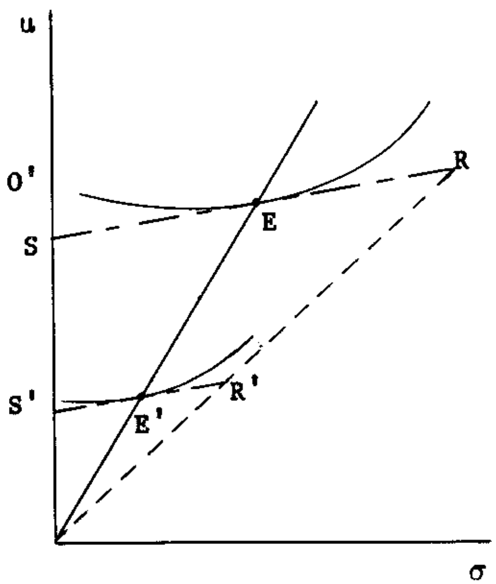

FIGURE 3a

Proportional Production Tax: Unitary Wealth Elasticity of Demand for Safe Asset (Constant Relative Risk Aversion) Investment Allocation Unchanged

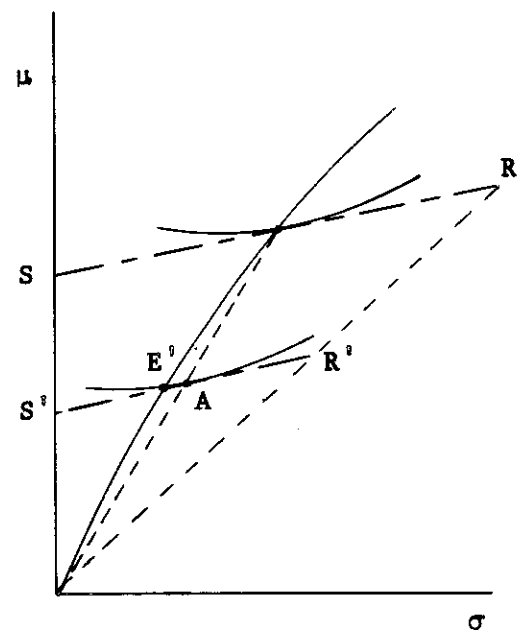

FIGURE 3b

Proportional Production Tax: Wealth Elasticity of Demand for Safe Asset Less than Unity (Decreasing Relative Risk Aversion) Investment in Risky Industry Increased

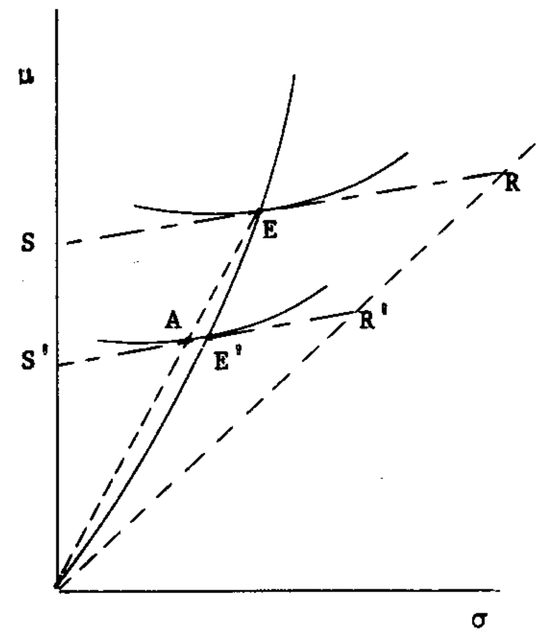

FIGURE 3c

Proportional Production Tax: Wealth Elasticity of Demand for Safe Asset Greater than Unity (Increasing Relative Risk Aversion) Investment in Safe Industry Increased

Note that although the substitution effect always leads to an increased standard deviation of income (and mean), it may or may not lead to an increased proportion of one's wealth in the risky asset. For the compensation required to maintain the individual at the same indifference curve means that the maximum possible standard deviation has also increased, and it is the relative increase in the two which determines whether the substitution effect leads to an increased or decreased demand for the risky asset relative to the safe. But the income effect may also lead to an increased or decreased demand of the risky asset relative to the safe, depending, as we have noted, on whether relative risk aversion is increasing or decreasing. Clearly then the net result is ambiguous. Figure 4 illustrates a case where the substitution effect leads to a decreased proportion of wealth allocated to the risky asset while the income effect leads to an increased proportion, the two effects exactly cancelling each other out. It is possible to show that sufficient conditions for the price elasticity to be negative are that (a) the income consumption curve is negatively sloped (which will be the case if absolute risk aversion is increasing) or (b) relative risk aversion is less than or equal to unity.[29]

Diagrammatically, the allocatively neutral tax t_S corresponding to the tax rate t_R on the risky industry is found by the intersection of the line $P'P'$ and the locus of tangencies of the indifference curves with the budget constraints rotated through the point R'.[30] (Figure 5)

(ii) Net Income (Profits) Taxes. Now the after tax return per dollar invested in the two industries is given by

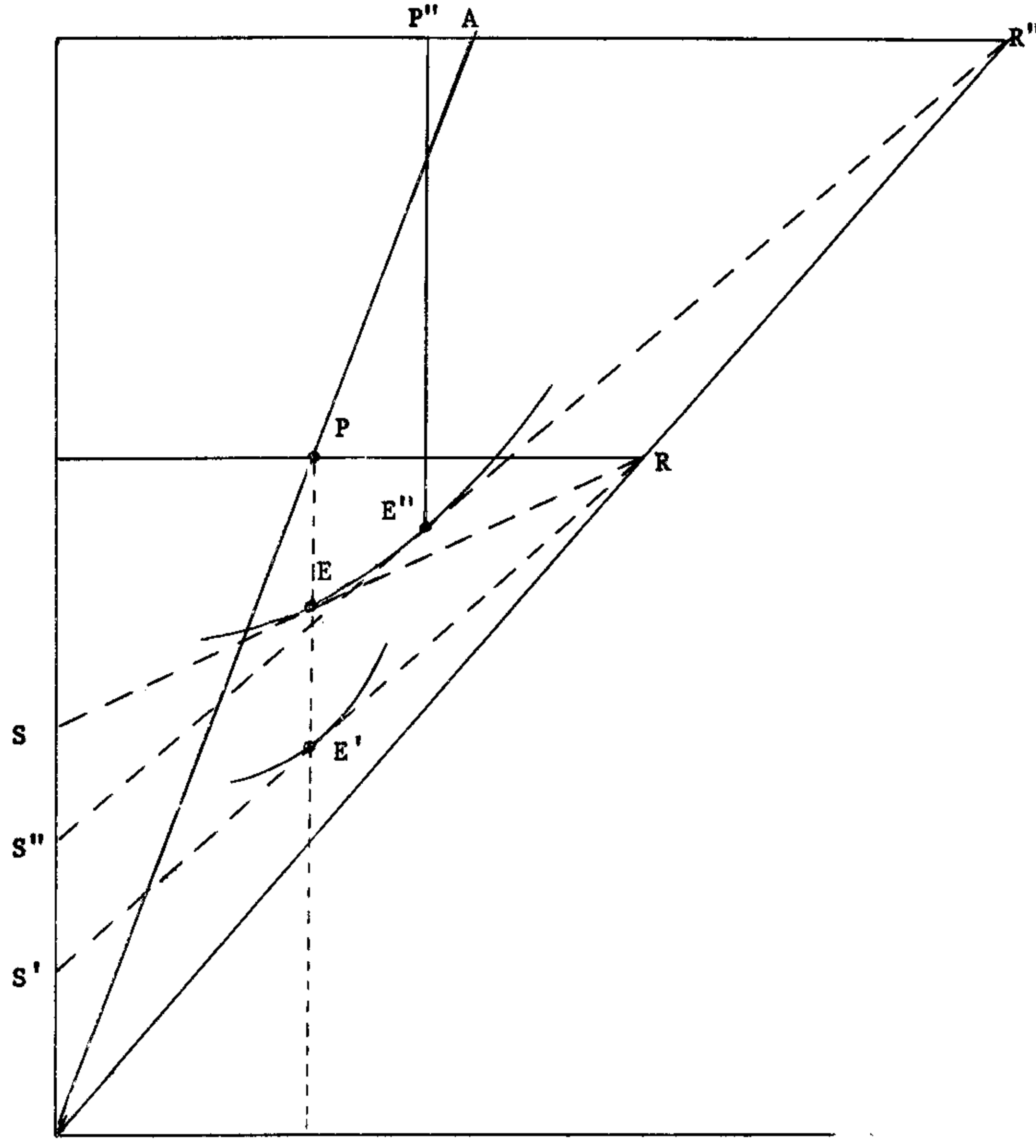

FIGURE 4

Substitution and Income Effects from an Increase in Tax in Safe Industry Alone

(S"R" is the budget constraint after the imposition of the tax on the safe industry and after the individual has been compensated to return him to the original indifference curve. S'R is the after tax (without compensation) budget constraint. E' lies directly below E , so the investment allocation is unchanged.)

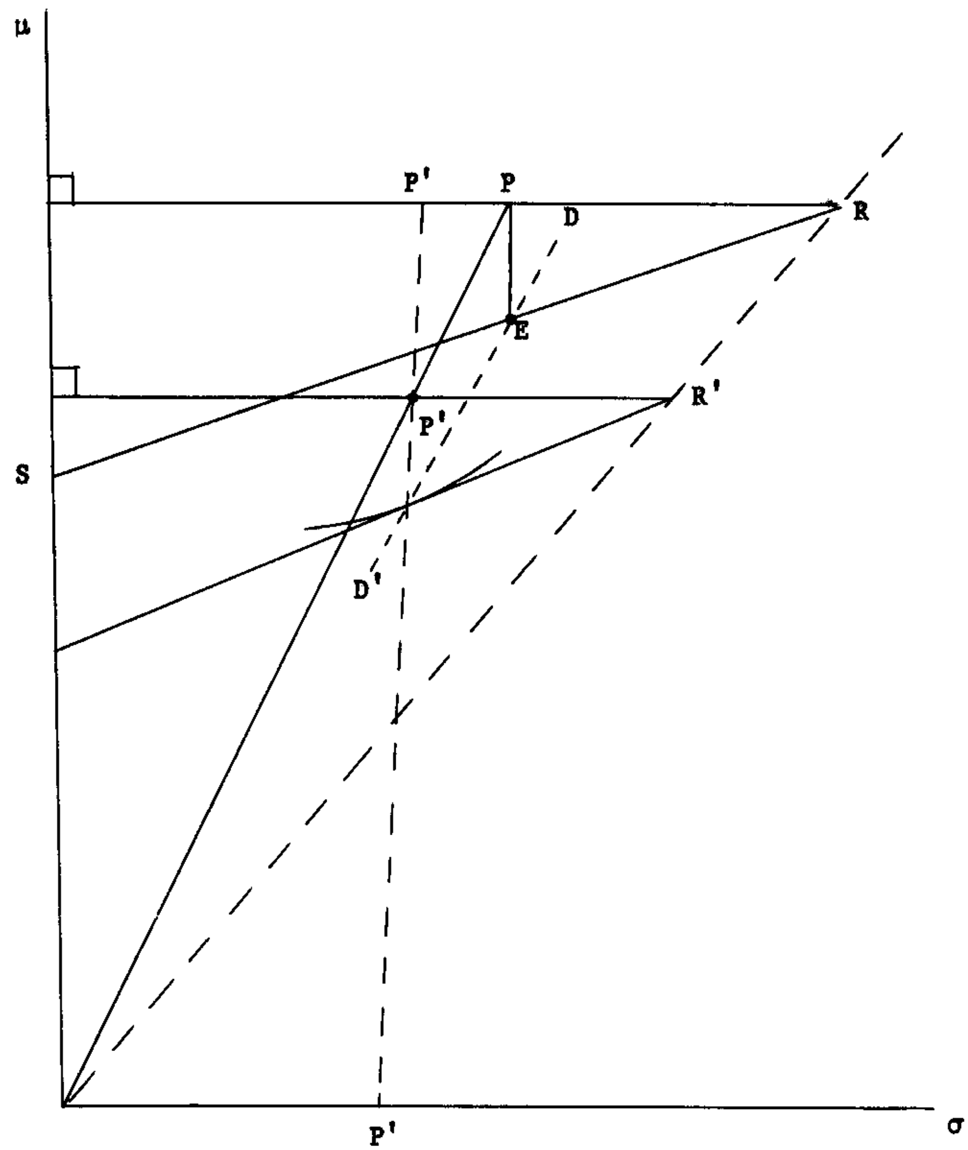

FIGURE 5

Calculation of the Allocatively Neutral Tax

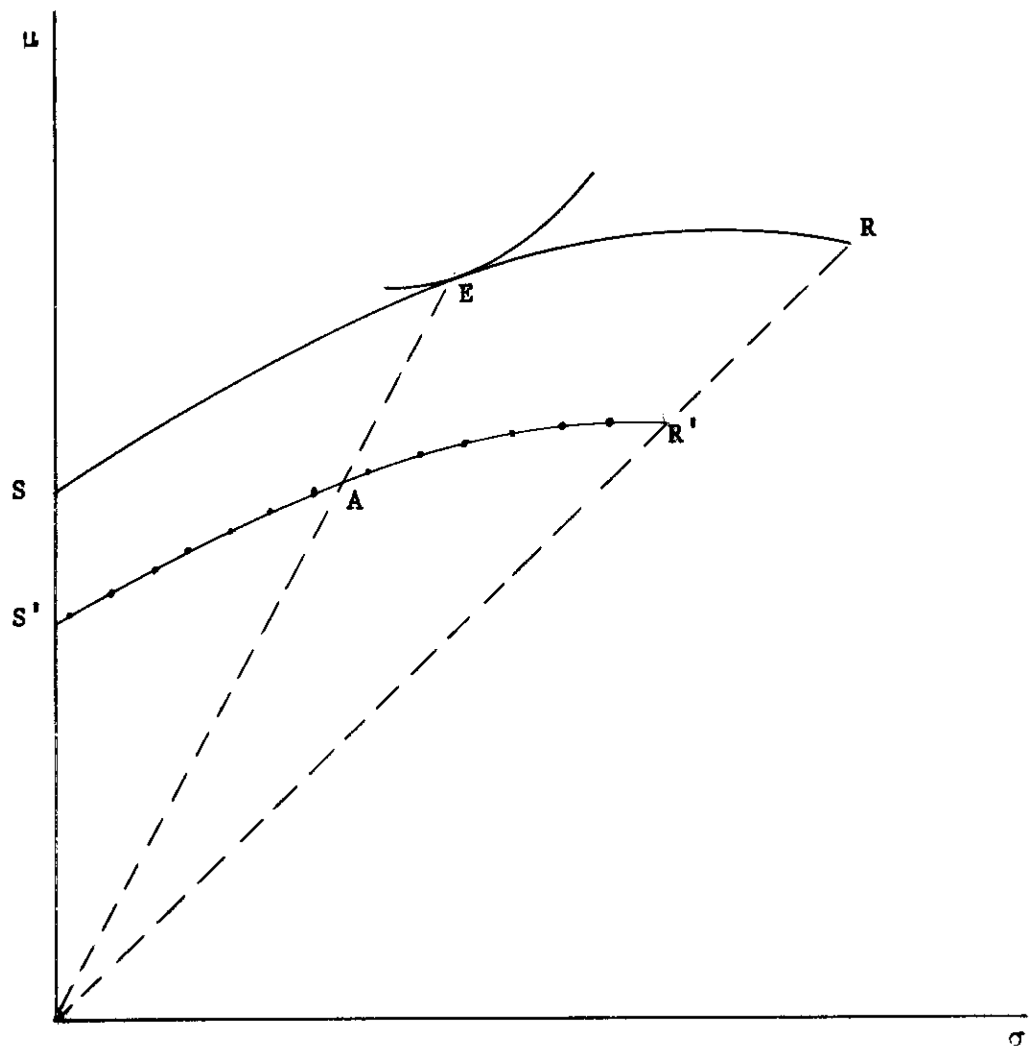

FIGURE 6

Proportional Production Tax: Non-Constant Returns to Scale

$$r' = (r-1)(1 - t_S) + 1$$

$$\mu_R' = (\mu_R - 1)(1 - t_R) + 1$$

$$\sigma_R' = \sigma_R(1 - t_R)$$

The after-tax point corresponding to R does not lie on the line RO but above it, as depicted in Figure 7, since the standard deviation is reduced in proportion to the tax, but the mean (gross) return is reduced less than in proportion. Indeed, if I denotes the terminal wealth of the individual if there were a zero rate of return on all assets, R' lies along the line IR. If a proportional tax is imposed in both industries at the same rate, i.e. $t_R = t_S$, then the opportunity locus is moved in parallel, since then the slope after tax equals that before tax:

$$\frac{\mu_R' - r'}{\sigma_R'} = \frac{\mu_R - r}{\sigma_R}$$

In the case of $t_R = t_S$, it is clear that the before tax income corresponding to the after tax point E' is A', i.e. the locus of constant allocation is a straight line through I. Hence, whether the profits tax results in increased or decreased allocation to the risky asset depends on whether the tangency of the pre-tax budget constraint lies to the left or right of A'. Notice that if risky assets are inferior, so that as individuals become wealthier, the variance of their income is reduced, the tax always results in increased risk taking. More generally if the income elasticity of risky assets is less than, or equal to, unity, there is increased investment in

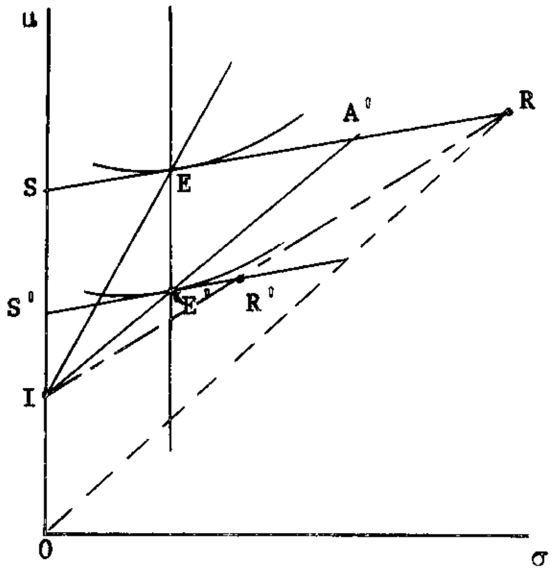

FIGURE 7a

Proportional Income Tax: Inelastic Demand for Risky Asset (Constant Absolute Risk Aversion): Increased Allocation of Investment to Risky Industry

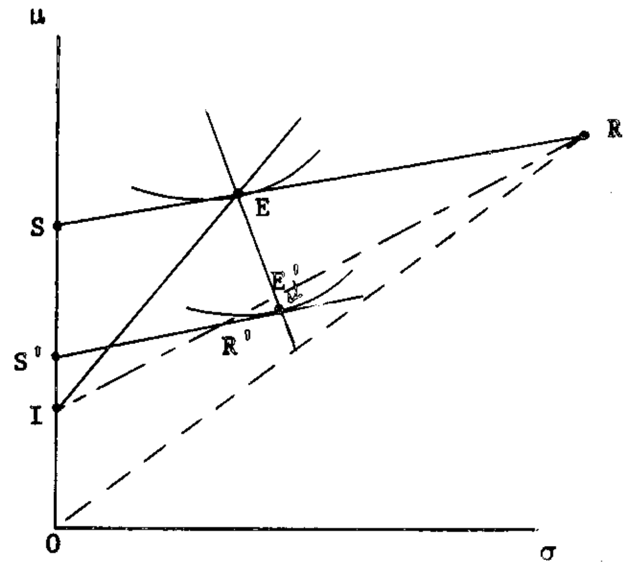

FIGURE 7b

Proportional Income Tax: Negative Wealth Elasticity of Demand for Risky Asset (Increasing Absolute Risk Aversion): Increased Allocation of Investment to Risky Industry

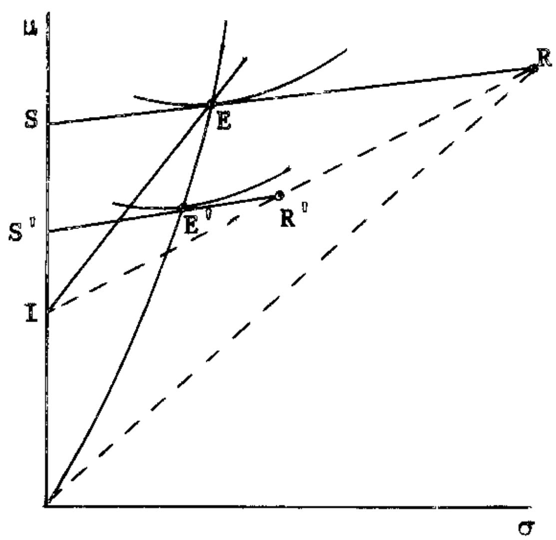

FIGURE 7c

Proportional Income Tax: Wealth Elasticity of Demand for Safe Asset Greater than Unity (Increasing Relative Risk Aversion): Increased Allocation of Investment to Risky Industry

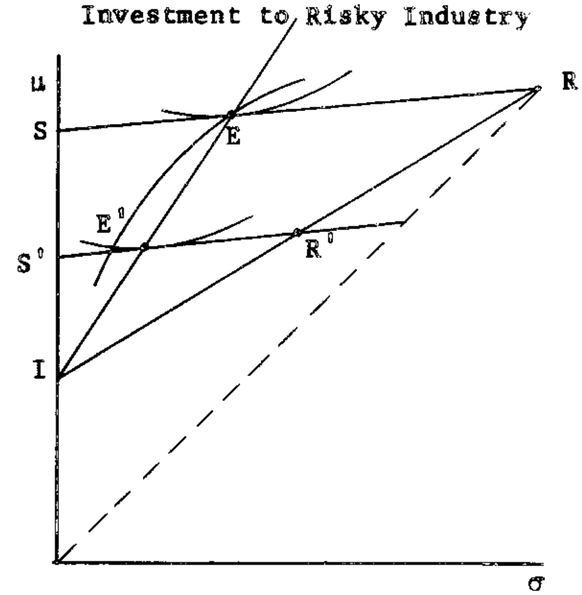

FIGURE 7d

Proportional Income Tax: Wealth Elasticity of Demand for Safe Industry Less than Unity; Allocation of Investment to Safe Industry May be Increased or Decreased

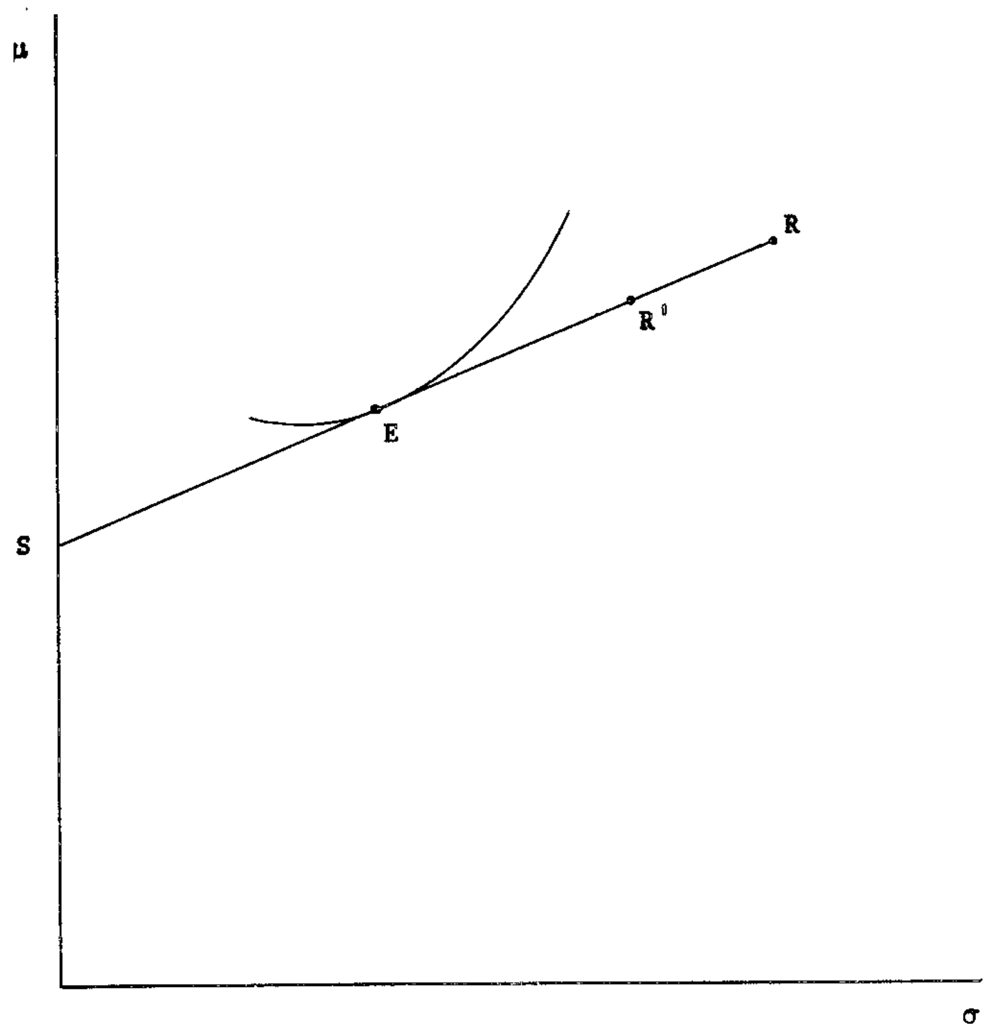

FIGURE 8

Proportional Income Tax: r-1 = 0
Investment in Risky Industry Increased

the risky industry. If, however, the income elasticity is sufficiently greater than unity, there will be decreased investment in the risky industry.[31]

Note that if the safe industry had a zero return (a pure storage activity) the after tax mean and standard deviation would be the same as before tax; to obtain this, the individual would, of course, have to increase his demand for the risky assets.[32]

The implications of these results for the design of an allocatively neutral tax are straightforward, and follow along the lines of the previous discussion.

(b) Optimal Production and Profits Taxes

When taxes are imposed on profits or production the conventional approaches to evaluating alternative tax structures, e.g. of comparing the loss of utility of equal revenue taxes, encounter difficulties, for now the revenue depends on the state of nature. Differing tax systems will change not only the distortions they impose on individuals, but also will change the pattern of returns across the states of nature: a tax on the safe asset only will yield a certain revenue to the government, while a tax on the risky asset will yield a variable revenue. One obvious criterion that has sometimes been suggested is comparing the loss of expected utility for equal expected revenue taxes, but this criterion assumes in effect that the government is risk neutral,[33] and unless it can be argued that that is (or ought to be) the case, this criterion is clearly unsatisfactory. Another criterion that has been sometimes suggested is allocative neutrality: What tax structure yields the same allocation (between safe and risky investments, between one risky industry and another, etc.) as the competitive pre-tax equilibrium allocation.[34] The preceding subsection investigated what this implies for relative tax rates, but it is important to realize that there is no presumption that from a social point of view one ought to have the same allocation after the tax has been imposed as pre-tax.[35]

This leads us to the suggestion that the government, like the individual, has a "utility function" of income, with attitudes towards risk. The government's attitudes towards risk may or may not be the same as those of the individuals in society or, more precisely, as individual attitudes towards risk in the consumption of private goods. One way of viewing this formulation of the problem is that the government provides a public good with the revenues from the taxation. Then the supply of these public goods will depend on the tax revenue. Assume the government is concerned with maximizing the representative individual's welfare. The individual's utility depends both on his consumption of private and public goods:

$$U = \Omega(Y, T) \tag{3.12}$$

where Y is the individual's after tax income, and T is equal to the supply of public goods which for simplicity we assume is just equal to the proceeds of the tax. For simplicity, let us assume that the utility function (3.12) is additive; i.e. $\Omega_{12} = 0$ and that the expected utility from private and from public goods can be written simply as a function of the mean and standard deviation of consumption of those goods. Then if the government imposes proportional production[36] taxes on the safe and risky industries at the rates t_S and t_R respectively, the level of social welfare is given by

$$(3.13)\quad M(r(1 - t_S)I_S + \mu_R(1 - t_R)(I - I_S),\ (I - I_S)(1 - t_R)\sigma_R)$$
$$+ G(t_s r I_S + t_R\mu_R(I - I_S),\ (I - I_S)t_R\sigma_R)$$

where G represents the expected utility from public goods; there is no reason that M and G ought to have the same shape: attitudes towards risk in the public good bear no necessary relationship to attitudes towards risk in private goods. It should be emphasized that this difference does not arise from Government's ignoring individual's attitudes towards risk, but from the fact that the government is concerned with individuals' preferences and individuals have different attitudes.

In short we are suggesting that the criterion for an optimal structure for a profits tax ought to be the maximization of social welfare, explicitly taking into account attitudes towards variability of government revenue.

Before discussing the optimal tax structure in the competitive economy, let us consider a controlled economy in which the government is constrained to specifying what proportion of the output of each firm will be allocated to each individual and to public goods, before production occurs. This is analogous to Diamond's second best constrained optimum [8]. The government wishes to maximize (3.13) with respect to t_S , t_R , and I_S . Thus,

$$(3.14a)\quad M_1(r(1 - t_S) - \mu_R(1 - t_R)) - M_2\sigma_2(1 - t_R) + G_1(t_S r - t_R\mu_R) - G_2 t_R\sigma_R = 0$$

$$(3.14b)\qquad -M_1 r I_S + G_1 r I_S = 0$$

$$(3.14c)\qquad -(M_1\mu_R I_R + M_2 I_R\sigma_R) + G_1\mu_R I_R + G_2\sigma_R I_R = 0$$

or

(3.15a) $$M_1 = G_1$$

(3.15b) $$M_2 = G_2$$

and

(3.15c) $$-M_1/M_2 = (\mu_R - r)/\sigma_R$$

Condition (a) says that the marginal utility of (mean) income will be the same in the government sector and in the private sector, and (b) that the marginal disutility of risk be the same. Condition (c) states that the adjusted coefficient of variation (taking the mean of the safe asset as the origin) equals the marginal rate of substitution between risk and average returns.

This equilibrium cannot in general be generated by a competitive economy with the government constrained to proportional taxes. To see this, observe for condition (c) to obtain in a competitive economy the marginal rate of substitution between mean and standard deviation must be equal to the before tax adjusted coefficient of variation, while in a competitive economy, it is equal to the after-tax adjusted coefficient of variation. For the two to be equal, the tax rate in both industries must be identical. In general, however, the solution to (3.15)(a-c) will not entail $t_R = t_S$. Hence we obtain the important result that if the government is restricted to imposing proportional taxes the level of social welfare obtainable under a centralized system of production and distribution is greater than

<u>that obtainable under a decentralized (competitive) system of production and distribution using</u>

Two questions remain:

1. What is the best that the government can do under a decentralized (competitive) system of production and distribution, assuming that the government is restricted to imposing proportional production taxes?

In this case, we can write the competitive allocation between the safe and risky investment simply as a function of the two tax rates, t_R and t_S. Thus, we wish to maximize

$$M(r(1 - t_S)I_S + \mu_R(1 - t_R)(I - I_S),\ (I - I_S)(1 - t_R)\sigma_R) + G(t_S r I_S + t_R\mu_R(I - I_S),\ (I - I_S)t_R\sigma_R) \tag{3.16}$$

We require,

$$-M_1 r I_S + G_1 r I_S + \{(t_S r - \mu_R t_R)G_1 - G_2\sigma_R t_R\} \frac{dI_S}{dt_S} = 0 \tag{3.17a}$$

$$-(M_1\mu_R I_R + M_2\sigma_R I_R) + G_1\mu_R I_R + G_2\sigma_R I_R + \{(t_S r - t_R\mu_R)G_1 - G_2\sigma_R t_R\} \frac{dI_S}{dt_R} = 0 \tag{3.17b}$$

We draw attention to two special cases:

(a) The government is risk neutral and the marginal utility of income is the same in the government and private utility functions: $G_2 = 0$ and $G_1 = M_1$. Then $t_S/t_R = \mu_R/r$. The taxes are inversely proportional to their mean return, i.e. the safe industry, since it yields a lower mean return, should be taxed at a higher rate than the risky industry.

Although a number of the lobbyists for special groups like oil and drugs have argued for some time that risky industries ought to be taxed at a lower tate, note the special conditions under which this obtains. This result is not surprising, since the social evaluation of risk is lower than the private, the social return to risky industries is higher than the private return and the government should accordingly encourage these industries.

(b) The disutility of risk to the government is identical to that for the private sector, and the marginal utility of income to the government is identical to that for the private sector: $M_1 = G_1$, $M_2 = G_2$. Under these circumstances, using (3.8) (3.17a) becomes

$$M_1\{r(t_S - t_R)\} \frac{dI_S}{dt_S} = 0$$

or

$$t_S = t_R$$

The risky and safe industries should be taxed at exactly the same rate. This in fact is close to the practice actually followed. It is important that the reader remember that because the goods supplied by tax revenues are not necessarily perfect substitutes for private goods, there is no necessary reason that the attitudes towards risk employed by the government in evaluating different patterns of tax returns should bear any particular relationship with private attitudes towards risk with respect to private goods. On the other hand, if the good supplied by the government is a close substitute to the private good, there is a presumption that the government and private attitudes towards risk should bear a close relationship to each other, and this is discussed in detail below.

2. The second question that remains to be answered is, is there any simple modification of the tax system which would allow the economy to attain an "optimum"? The answer is yes. If, in addition to our proportional tax, we allow the government to levy a lump sum tax on each industry (a franchise tax), then the constrained pareto optimum is, of course, unaffected. On the other hand, we can, as illustrated in Figure 9, obtain the allocation identical to the centralized allocation in our competitive economy in the following manner. Denote by t_R^* and t_S^* the optimal "tax" rates on gross output in the controlled economy; let z be the optimal proportion of total investment allocated to the safe industry, $\hat{t}_R$ and $\hat{t}_S$, the tax rates on gross output in the competitive economy, and $\bar{t}_R$ the tax rate per unit of investment in the risky industry (independent of the realized level of profits in the industry). Then we require that (a) the after tax ("adjusted") coefficient of variation equal the before tax ("adjusted") coefficient of variation [so (3.15) may be satisfied],

$$\frac{(1 - \hat{t}_R)\mu_R - \bar{t}_R - r(1 - \hat{t}_S)}{\sigma_R(1 - \hat{t}_R)} = \frac{\mu_R - r}{\sigma_R}, \tag{3.18}$$

and (b) the tax revenue in each state of nature be equal to the allocation to public goods in the constrained pareto optimal problem, which implies that

$$\hat{t}_R = t_R^* \tag{3.19}$$

and

$$r\hat{t}_S z + (1-z)\bar{t}_R = zt_S^* r . \tag{3.20}$$

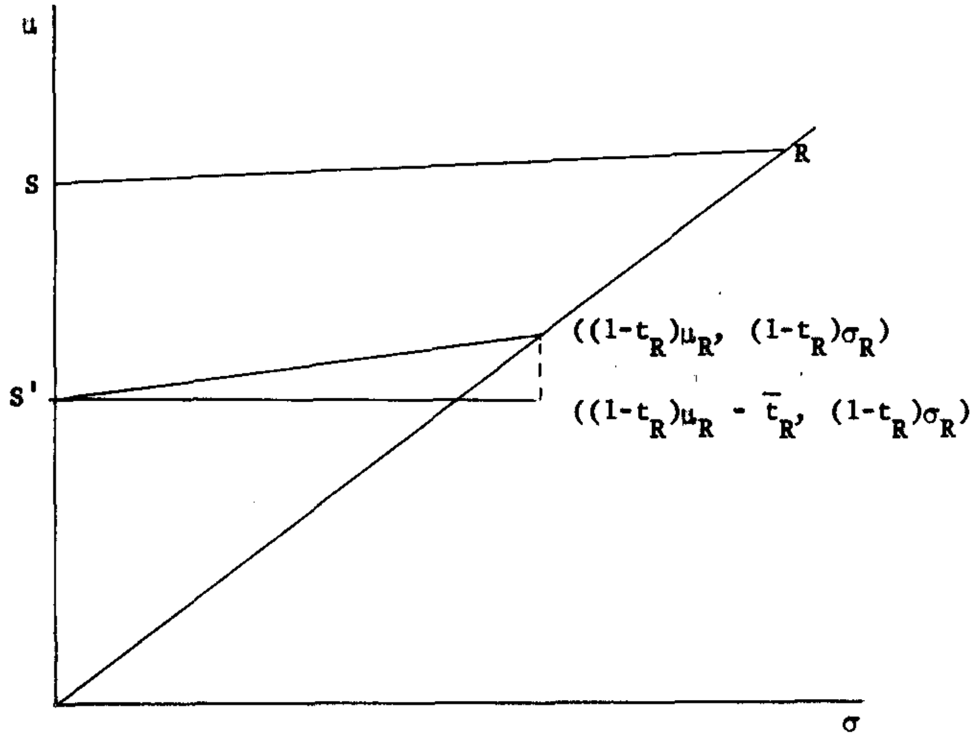

FIGURE 9

Proportional Production and Franchise Taxes

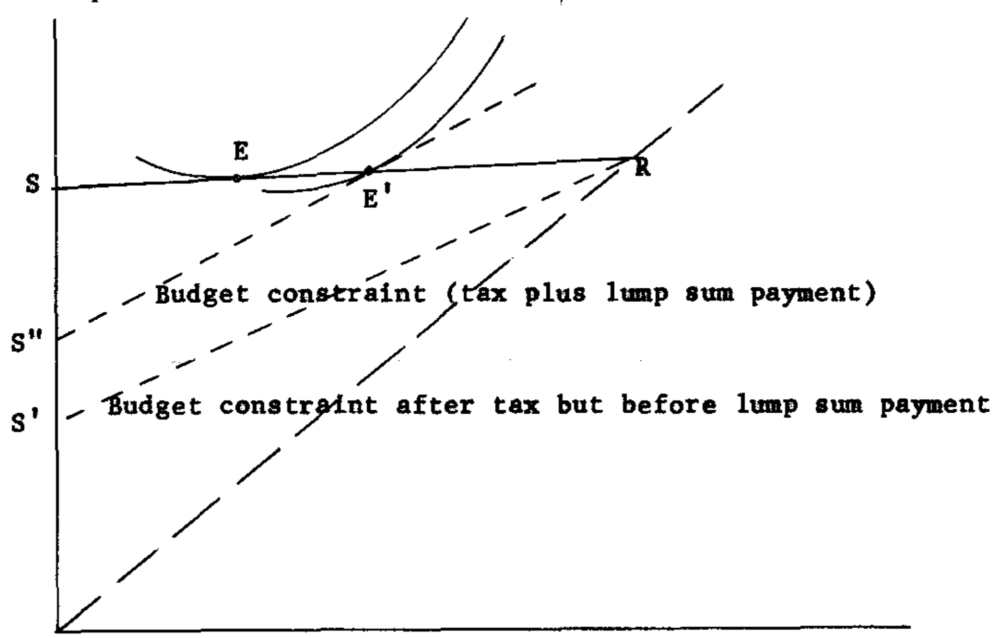

FIGURE 10

The solution to (3.18)-(3.20) requires that

(3.21a) $$\hat{t}_S = zt_S^* + (1-z)t_R^*$$

and

(3.21b) $$\bar{t}_R = zr(t_S^* - t_R^*)$$

The proportional tax rate in the risky sector is the same as in the controlled economy; in the safe industry, the profits tax is a weighted average between the rates of taxation in the controlled economy in the two sectors, with the weights being the proportion of total investment in the given sector, and the franchise tax is proportional to the difference between the tax rates in the two sectors in the controlled economy. Thus, of course, if the controlled solution involved proportional taxation, at the same rate in both sectors, in our competitive economy no franchise tax need be levied. Note that if $t_R^* > t_S^*$, then there is a negative "franchise tax." But in all cases, the average tax rate paid by the risky industry is greater or smaller than that paid by the safe industry as $t_S^* \lessgtr t_R^*$, as straightforward calculation will verify.

(c) Taxes when Proceeds Redistributed

If the government uses the revenues from the profits taxes to make lump sum payments (in the different states of nature) or to provide a public good which acts as a perfect substitute in consumption for the private good (i.e. enters additively into the utility function with the private good), then it is easy to show that if consumers correctly perceive the riskiness of the government subsidy, the <u>allocative neutral tax requires taxation of both industries at the same rate</u>. We wish to maximize

$$M(rI_S(1 - t_S) + \mu_R(1 - t_R)I_R + rI_S t_S + \mu_R t_R I_R,\ \sigma_R(1 - t_R)I_R + \sigma_R t_R I_R)$$

where I_S and I_R depend on the tax rates t_S and t_R :

$$-\frac{M_2}{M_1} = \frac{\mu_R(1 - t_R) - r(1 - t_S)}{\sigma_R(1 - t_R)}$$

It is clear that if $t_R = t_S$, the allocation is identical to the no-tax allocation, which can be shown in our model to be Pareto optimal. (In Figure 10 we illustrate the consequences of taxing the safe industry at a higher rate than the risky.) This result should not be surprising, for in this case a tax at the same rate on both industries is identical to a general tax on investment, for we have already noted that when the two industries are taxed at the same rate, the slope of the opportunity locus remains unchanged, and since the proceeds of the tax are redistributed, there is no income effect from the tax. Hence, such a tax is completely non-distortionary when investment is inelastically supplied. (See below for a discussion of the optimal tax structure when investment is not inelastically supplied.)

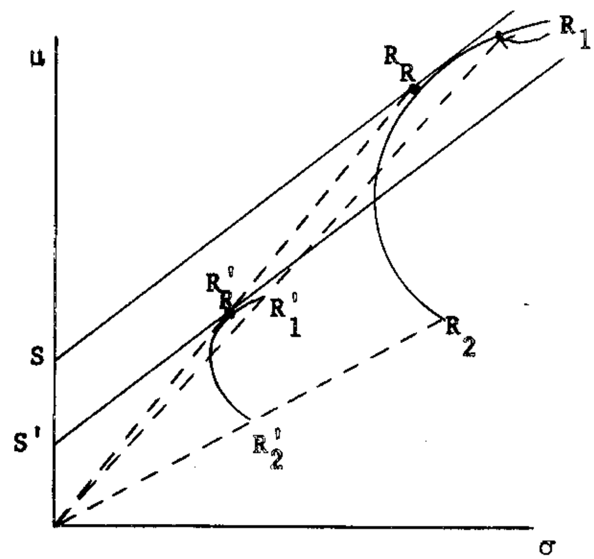

FIGURE 11a

Proportional Tax on Production. No Change in Allocation of Investment Among the Risky Industries

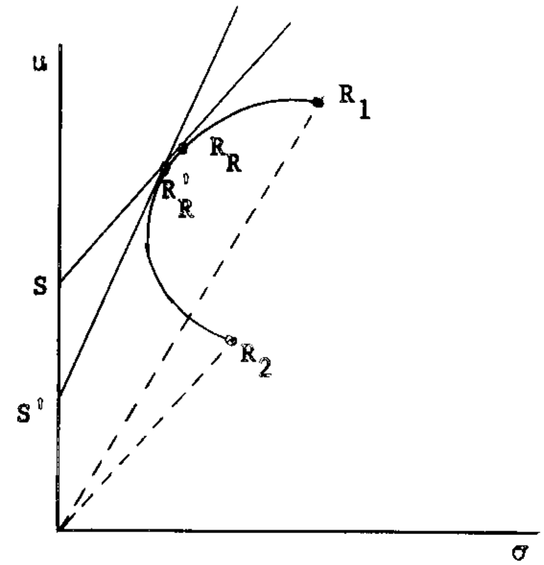

FIGURE 11b

Tax on Safe Industry Only. Increased Allocation to Risky Industry with Lower Mean

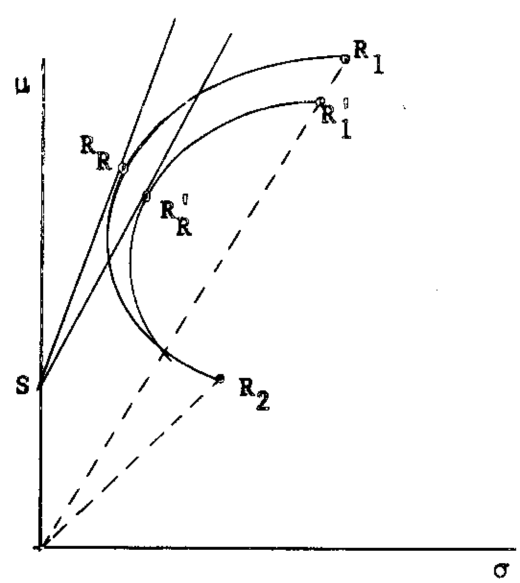

FIGURE 11c

Tax on Risky Industry with Higher Mean: Increased Proportion of Risky Assets in First Industry

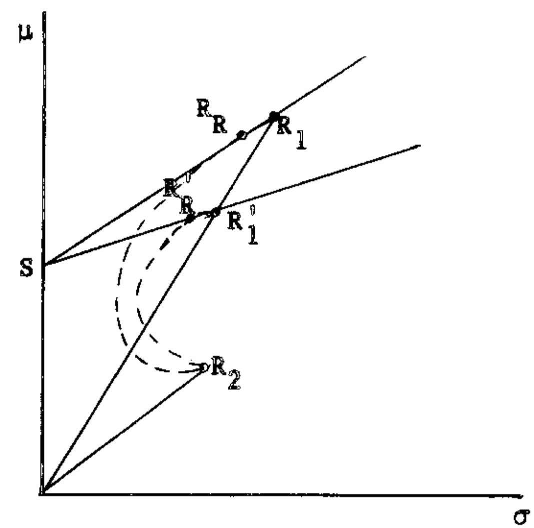

FIGURE 11d

Tax on Risky Industry with Higher Mean: Decreased Proportion of Risky Assets in First Industry

APPENDIX TO SECTION 3

Two Risky Industries

In this appendix, we extend the analysis for the mean-variance model to the case of two or more risky industries.[37] (Recall from the preceding section that all firms in a given industry have the same production function, and industries differ from one another only in the pattern of returns across the states of nature.)

In the mean variance model, we can divide the process of portfolio allocation into two steps: what proportion of one's wealth one allocates to the safe asset (and thus what proportion to the risky assets), and what proportion of the amount invested in the risky assets is to be allocated to each risky security. It is easy to show, for the mean-variance model, that individuals will demand risky assets in the proportions which minimize the coefficient of variation (adjusted to take the return on the safe asset as the origin), i.e. if there are two risky industries, and $\hat{\lambda}$ is the ratio of investment in the first to the total investment in both risky industries,[38] competitive equilibrium requires that $\hat{\lambda}$ be chosen to minimize

$$(3.22) \qquad \frac{\left(\hat{\lambda}^2\sigma_1^2 + 2\hat{\lambda}(1-\hat{\lambda})C + (1-\hat{\lambda})^2\sigma_2^2\right)^{1/2}}{\hat{\lambda}\mu_1 + (1-\hat{\lambda})\mu_2 - r} \equiv \frac{\sigma_R}{\mu_R - r}$$

where μ_i and σ_i are the mean return and standard deviation of return per dollar invested in the i^{th} risky industry, $C/\sigma_1\sigma_2$, the correlation coefficient and μ_R and σ_R are the mean and standard deviation of return per dollar invested in the risky industries, when divided among the risky industries in the proportions given by $\hat{\lambda}$.[39] This implies that

$$\frac{\mu_1 - \mu_2}{(\mu_R - r)} = \frac{\hat{\lambda}\sigma_1^2 - (1-\hat{\lambda})\sigma_2^2 + (1 - 2\hat{\lambda})C}{\sigma_R^2} \tag{3.23}$$

As in the preceding sections, consumers will allocate their portfolio between the safe industry and the risky industries so that

$$-\frac{M_1}{M_2} = \frac{\sigma}{\mu_R - r} \tag{3.24}$$

From (3.23) it is clear that if we consider proportional production or profits taxes, if we tax both risky industries at the same rate, and if we are interested in allocative neutrality with respect to the proportion of our risky assets allocated to each of the two risky industries, we must tax the safe and risky industries at the same rate.[40] This is clear from Figure 11a. Point R_1 represents the mean and standard deviation of terminal wealth if we invest all our resources in the first risky industry, R_2 is the corresponding point for the second, the curved line joining them gives the mean-standard deviation combinations resulting from different proportions invested in the two risky industries. S represents the return (terminal wealth) from investing in only the safe industry. The opportunity locus is the line SR_R from S tangent to R_1R_2 .

If we impose a production tax on all three industries at the same rate, we simply move all points on the opportunity locus towards the origin, i.e. it is equivalent to a simple change in scale. Hence the optimal allocation among the alternative risky assets is unaffected.

But although a proportional tax at the same rate in all industries is allocatively neutral in its relative effects on different risky industries, as before it is not allocatively neutral in its effect on the allocation between risky industries as a whole and the safe industry (except in the special case of unitary wealth (income) elasticities).

If we do tax the safe industry at a different rate from the risky, we must tax the risky industries at different rates to preserve allocative neutrality among the risky industries, e.g. as we decrease the tax on the safe industry, we increase the allocation to the industry with the higher mean. (See Figure 11b.) To offset this we must change the relative tax rates on the two risky industries. It is important to observe that whether we increase or decrease the tax on, say, the industry with the higher mean does not depend at all on the properties of the indifference curves, since the choice of the proportions in which risky assets are to be purchased depends only on the properties of the assets themselves. Unfortunately whether an increase in the tax on the industry with higher mean results in an increased or decreased relative allocation to it is ambiguous; consider, for simplicity, the case where the two risky industries are independent, i.e. $C = 0$. Then

$$\frac{\hat{\lambda}}{1-\hat{\lambda}} = \frac{[\mu_1(1 - t_1) - r(1 - t_S)]/\sigma_1^2(1 - t_1)^2}{[\mu_2(1 - t_2) - r(1 - t_S)]/\sigma_2^2(1 - t_2)^2}$$

so that

$$-\frac{d\hat{\lambda}}{dt_1} \sim \frac{\mu_1(1 - t_1)}{\mu_1(1 - t_1) - r(1 - t_S)} - 2 = \frac{2r(1 - t_S) - \mu_1(1 - t_1)}{\mu_1(1 - t_1) - r(1 - t_S)}$$

i.e. depends on the extent to which the after tax mean return from the first industry exceeds that from the safe industry. If μ_1 is not large relative to r, then a tax on the first industry reduces the demand for it (relative to the second) as one might expect, but otherwise a tax on the first industry may actually increase the demand for it. Similar results obtain when $C \neq 0$.

4. Optimal Taxes on Investment in Safe and Risky Industries

In this section, we consider the structure of optimal taxes on investment in the safe and risky industries. The government wants to raise sufficient revenue to purchase a given amount of the factor, I_g. The factor will be used to purchase a commodity which does not affect the individual's allocation among alternative risky assets or his supply of the factor, although the taxes used to raise the revenue will. The government is not allowed to levy other (non-distortionary) taxes.[41]

We begin our analysis with the simplest case, where there are only two industries, a safe industry and a risky industry, both of which have constant returns to scale, reserving our analysis of the case of two risky industries to an appendix. The return per dollar invested in the safe and risky industries before tax are given by r and $e(\theta) = g(\theta)$ respectively. Hence if a tax is imposed on the purchase of securities of the safe industry at the rate t_S so the price of the safe security is $p_S = 1 + t_S$, and on the purchase of securities of the risky industry at the rate t_R, so its price is $1 + t_R = p_R$, then the return per dollar invested (after tax) is simply $r/(1 + t_S)$ and $e/(1 + t_R)$ respectively.[42] If the initial wealth of the individual (i.e. that not derived from selling his factor) is w_0,[43] we can express the allocation problem of the representative consumer as follows:

$$\text{(4.1)} \qquad \text{maximize } EU(r'Z_S + e'Z_R) + L(w_0 - Z_S p_S - Z_R p_R)$$

where, it will be recalled, Z_S is the demand for securities of the safe industry, Z_R that for the risky industry, e' and r' are the after tax returns per dollar invested in the risky and safe securities, respectively, and $-L'$ is the marginal disutility of labor (supplying an extra unit of the factor).

Thus, total welfare attained may be expressed simply as a function of the prices of the securities (i.e. of the taxes imposed), and of initial wealth, w_0:

$$\Psi(p_S, p_R, w_0)$$

Similarly, his supply of labor and his demand for securities of the safe and risky firms may be written simply as a function of the tax rates.

The objective of the government is to choose that set of tax rates which maximizes Ψ subject to all markets clearing, i.e.

(4.2) $$\max_{\{p_S,\ p_R\}} \Psi$$

(4.3) $$\text{subject to } I = I_g + I_S + I_R$$

where I_S and I_R are the derived demands for the labor factor in the safe and risky industry, and are therefore simply given by

(4.4) $$I_S = Z_S$$

and

(4.5) $$I_R = Z_R$$

In Appendix A we show that the solution to this problem requires[44]

(4.6) $$\frac{t_S}{Z_S}\left(\frac{\partial Z_S}{\partial p_S}\right)_{\bar{\Psi}} + \frac{t_R}{Z_S}\left(\frac{\partial Z_S}{\partial p_R}\right)_{\bar{\Psi}} = \frac{t_S}{Z_R}\left(\frac{\partial Z_R}{\partial p_S}\right)_{\bar{\Psi}} + \frac{t_R}{Z_R}\left(\frac{\partial Z_R}{\partial p_R}\right)_{\bar{\Psi}}$$

The percentage change (along the compensated demand curve) in the demand for the two securities must be the same. For small taxes, if L' were constant (constant marginal disutility of supplying I), so there were no income effect,

(4.7) $$\frac{t_S}{Z_i}\left(\frac{\partial Z_i}{\partial p_S}\right)_{\bar{\Psi}} + \frac{t_R}{Z_i}\left(\frac{\partial Z_i}{\partial p_R}\right)_{\bar{\Psi}}$$

is the total percentage change in the demand for the i^{th} security induced by the tax system. (4.6) asserts then that the percentage reduction in demand of each security be the same (and indeed equals the percentage increase in the supply of the factor). This is close to what one might mean by allocative neutrality in this case. Thus, under these assumptions the optimal tax is allocatively neutral in this special sense (same percentage change in investment). On the other hand, when there are income effects or when the tax is not small, (4.6) does not imply that there be the same percentage change in the investment in the two industries when comparing the new equilibrium to the pre-tax equilibrium.

It may be useful to give a simple geometric interpretation to the above result. The optimal tax must have the property that it is impossible to raise additional revenue without lowering utility. As we noted earlier, we can write the level of expected utility simply as a function of t_S and t_R (p_S and p_R). The duality relationships between the direct and indirect utility function guarantee that the indifference curves between t_S and t_R are downward sloping and convex, as depicted in Figure 12. An individual is indifferent then between any pair of tax rates lying on the same indifference curve. The slope of the indifference curve is given by

$$\frac{dt_R}{dt_S} = -\frac{\Psi_{p_S}}{\Psi_{p_R}} = -\frac{Z_S}{Z_R}$$

The revenue from the tax (in I numeraire) is just

$$T = t_S Z_S + t_R Z_R$$

so along an indifference curve

$$\frac{dT}{dt_S} = Z_S + t_S\left(\frac{\partial Z_S}{\partial t_S}\right)_{\bar{\Psi}} + t_R\left(\frac{\partial Z_R}{\partial t_S}\right)_{\bar{\Psi}} - \left\{Z_R + t_S\left(\frac{\partial Z_S}{\partial t_R}\right)_{\bar{\Psi}} + t_R\left(\frac{\partial Z_R}{\partial t_R}\right)_{\bar{\Psi}}\right\}\frac{Z_S}{Z_R}$$

When $dT/dt_S = 0$, using the symmetry of the compensated price derivatives,

$$\frac{t_S}{Z_S}\left(\frac{\partial Z_S}{\partial t_S}\right)_{\bar{\Psi}} + \frac{t_R}{Z_S}\left(\frac{\partial Z_S}{\partial t_R}\right)_{\bar{\Psi}} = \frac{t_S}{Z_R}\left(\frac{\partial Z_R}{\partial t_S}\right)_{\bar{\Psi}} + \frac{t_R}{Z_R}\left(\frac{\partial Z_R}{\partial t_R}\right)_{\bar{\Psi}}$$

which is exactly the result derived above for the optimal tax.

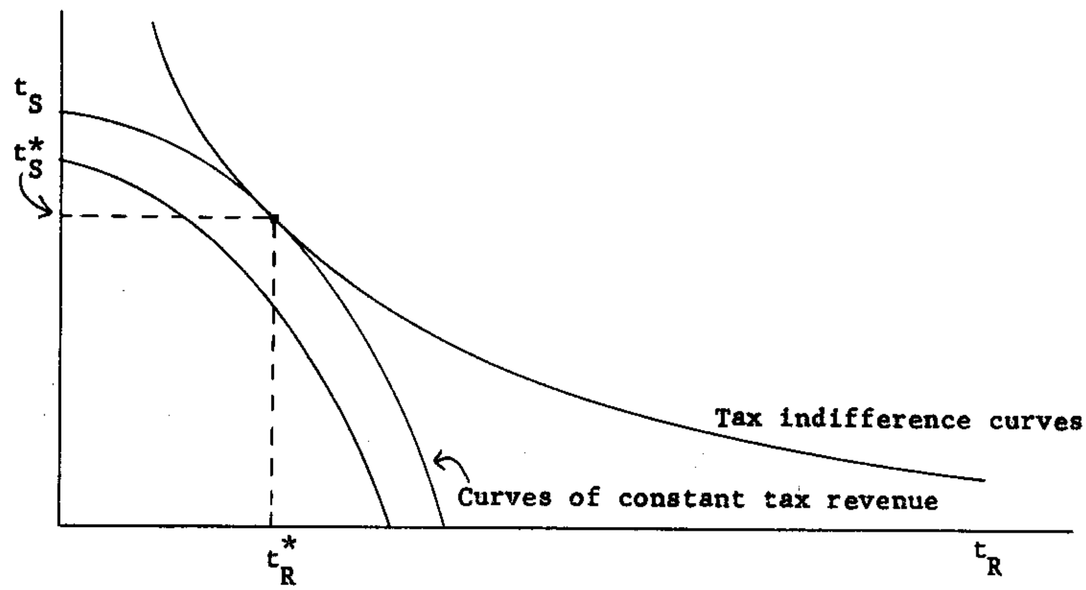

FIGURE 12

Determination of Optimal Tax Structure

Our problem now is to see exactly what (4.6) implies for the relative tax rates for the different securities, i.e. we require a calculation of the compensated demand schedules for our particular problem. This we do in Appendix B. Substituting the calculated values of the compensated price derivatives into (4.6) we obtain (after some manipulation)[45]

(4.8) $$t_R - t_S = -\frac{t_S EU''Y(e-r)}{-L''I - EU''rY}$$

The denominator is unambiguously positive. On the other hand, we observe that we can rewrite the numerator,

(4.9) $$-EU''Y(e-r) = E\left(\frac{-U''}{U'}Y\right)U'(e-r) = E\rho U'(e-r) ,$$

using the Arrow-Pratt measure of relative risk aversion, $-U''Y/U' \equiv \rho$. Then, defining ρ^* to be the value of ρ when $e = r$, (4.9) may be written

(4.10) $$E\rho U'(e-r) = E(\rho - \rho^*)U'(e-r) + \rho^* EU'(e-r)$$

Observe that the second term is zero (making use of the first order conditions); if ρ is constant, the first term is zero; if ρ increases with Y , when $e > r$, $\rho > \rho^*$, and conversely when $e < r$. Hence, the first term is positive. Similarly, if ρ decreases with Y , the first term is negative. Thus, we have established that <u>the tax rate on the risky industry should be greater than, equal to, or less than that on the safe industry, as relative risky aversion is an increasing, constant, or decreasing function of income</u>.

But it is easy to show that the wealth elasticity of demand for the safe asset is greater or less than unity as the individual has increasing constant or decreasing relative risk aversion [2, 19]. Thus, the above result may be restated in the following way: <u>The risky industry should be taxed at a greater (smaller) rate than the safe industry if the wealth-elasticity of demand for the risky asset is less than (greater than) unity, i.e. the industry with the higher wealth-elasticity should be taxed at the lower rate</u>.[46]

In the analysis so far, we have assumed (a) constant returns to scale in both industries, (b) the proceeds of the tax are not distributed to individuals (or do not affect their portfolio allocation), and (c) all individuals are identical. The first two of these points will now be discussed; the third is the major subject of the next section.

(a) If there is decreasing returns to scale in one or both industries, the initial ownership claims of the individuals will have a positive value (instead of, as in the constant returns to scale case, a value of zero). But these can be taxed at up to 100% without creating any distortionary effects; and hence the optimal tax structure will in fact tax them fully.[47] If, on the other hand, as seems likely in practice, a tax at less than 100% is applied on such initial ownership claims, then the effect of the tax in reducing producer surplus must also be taken into account. (With constant returns to scale, there is no producer surplus, only consumer surplus. The above calculations may be thought of as simply a detailed calculation of the tax structure which minimizes the loss of consumer surplus from the imposition of the tax.) It is easy to show that the smaller the elasticity of supply (the closer to constant returns to scale) the higher the tax rate (other things being equal), as suggested by the conventional partial equilibrium analysis.[48] (b) If the government spends the proceeds of the tax to provide a good which is a close substitute for the private good, the result that the sign of $t_R - t_S$ depends on whether the expenditure elasticity of the demand for the risky security is less than or greater than unity still obtains, but this will not depend in general on simply whether there is increasing or decreasing relative risk aversion, as in the earlier analysis. Note that it will, in the special case where the government invests in the two kinds of investments in the same proportions as the individuals do. But if, as another polar case, the government were to invest only in a safe project, then $t_R - t_S$ will be of the sign of

$$-EU''(Y-F)(e-r)$$

where

$$Y = Z_R e + rZ_S + F$$

and F is the (fixed) level of income provided by the government. This will be positive if there is increasing relative risk aversion and decreasing absolute risk aversion. But in other cases, the sign is indeterminate.

APPENDIX A

Derivation of Optimal Tax Formula

The solution to the constrained maximization problem (4.2) may be derived as follows: if ν is the Lagrange multiplier on the constraint (4.3), necessary conditions for the solution to (4.2) are given by

$$\Psi_{P_S} + \nu\left(\frac{\partial Z_S}{\partial P_S} + \frac{\partial Z_R}{\partial P_S} - \frac{\partial I}{\partial P_S}\right) = 0$$

(A.1)

$$\Psi_{P_R} + \nu\left(\frac{\partial Z_S}{\partial P_R} + \frac{\partial Z_R}{\partial P_R} - \frac{\partial I}{\partial P_R}\right) = 0$$

From the budget constraint

$$p_S Z_S + p_R Z_R = I + w_0 \tag{A.2}$$

we obtain by differentiation

$$p_S \frac{\partial Z_S}{\partial p_i} + p_R \frac{\partial Z_R}{\partial p_i} + Z_i = \frac{\partial I}{\partial p_i} \tag{A.3}$$

Substituting into (A.1) we obtain

$$\begin{aligned} \Psi_{p_S} - \nu\left(t_S \frac{\partial Z_S}{\partial p_S} + t_R \frac{\partial Z_R}{\partial p_S} - Z_S\right) &= 0 \\ \Psi_{p_R} - \nu\left(t_S \frac{\partial Z_S}{\partial p_R} + t_R \frac{\partial Z_R}{\partial p_R} - Z_R\right) &= 0 \end{aligned} \tag{A.4}$$

The securities of the two kinds of firms are, in this problem, simply like any ordinary commodities, and so must satisfy the basic Slutsky relationships:

$$\frac{\partial Z_i}{\partial p_j} = \left(\frac{\partial Z_i}{\partial p_j}\right)_{\overline{\Psi}} - Z_j\left(\frac{\partial Z_i}{\partial w_0}\right) \tag{A.5}$$

The change in the demand for the i^{th} security can be broken down into two parts: the substitution effect, the change that would have resulted from the tax if, simultaneously, we had compensated him by an increase in his wealth to make him as well off as before, and the income effect. Moreover

(A.6)
$$\left(\frac{\partial Z_i}{\partial p_j}\right)_{\bar{\Psi}} = \left(\frac{\partial Z_j}{\partial p_i}\right)_{\bar{\Psi}}$$

the compensated change in the demand in the demand for the i^{th} security from a change in the price of the j^{th} security (tax on the j^{th} security) is equal to the compensated change in the demand for the j^{th} security from a change in the price of the i^{th} security (tax on the i^{th} security). Finally, we recall the well-known fact that the increase in wealth required to compensate an individual for the increase in the price (tax) on the i^{th} commodity (security) is just equal to the individual's consumption (purchases) of that commodity (security):

(A.7)
$$\left(\frac{dw_0}{dp_i}\right)_{\bar{\Psi}} = -\frac{\partial \Psi/\partial p_i}{\partial \Psi/\partial w_0} = Z_i$$

Substituting these into (A.4), we obtain

(A.8)
$$Z_S(\nu - \Psi_{w_0}) - \nu\left(t_S\left(\frac{\partial Z_S}{\partial p_S}\right)_{\bar{\Psi}} + t_R\left(\frac{\partial Z_S}{\partial p_R}\right)_{\bar{\Psi}} - t_S Z_S \frac{\partial Z_S}{\partial w_0} - t_R Z_S \frac{\partial Z_R}{\partial w_0}\right) = 0$$
$$Z_R(\nu - \Psi_{w_0}) - \nu\left(t_S\left(\frac{\partial Z_R}{\partial p_S}\right)_{\bar{\Psi}} + t_R\left(\frac{\partial Z_R}{\partial p_R}\right)_{\bar{\Psi}} - t_S Z_R \frac{\partial Z_S}{\partial w_0} - t_R Z_R \frac{\partial Z_R}{\partial w_0}\right) = 0$$

or rewriting

(A.9)
$$\frac{t_S}{Z_S}\left(\frac{\partial Z_S}{\partial p_S}\right)_{\bar{\Psi}} + \frac{t_R}{Z_S}\left(\frac{\partial Z_S}{\partial p_R}\right)_{\bar{\Psi}} = \frac{t_S}{Z_R}\left(\frac{\partial Z_R}{\partial p_S}\right)_{\bar{\Psi}} + \frac{t_R}{Z_R}\left(\frac{\partial Z_R}{\partial p_S}\right)_{\bar{\Psi}}$$
$$= \left(1 - \frac{\Psi w_0}{\nu}\right) + t_S \frac{\partial Z_S}{\partial w_0} + t_R \frac{\partial Z_R}{\partial w_0}$$

APPENDIX B

Calculation of Compensated Price Derivatives

From the first order conditions of utility maximization

(B.1)

$$EU'(Z_R e + Z_S r)e - p_R L'(-(p_R Z_R + p_S Z_S)) = 0$$

$$EU'(Z_R e + Z_S r)r - p_S L'(-(p_R Z_R + p_S Z_S)) = 0$$

we obtain by total differentiation

(B.2)

$$\begin{bmatrix} EU''e^2 + L''p_R^2 & EU''er + L''p_R p_S \\ EU''er + L''p_R p_S & EU''r^2 + L''p_S^2 \end{bmatrix} \begin{bmatrix} dZ_R \\ dZ_S \end{bmatrix}$$

$$\begin{bmatrix} L' - p_R Z_R L'' & - L''p_R Z_S \\ - p_S Z_R L'' & L' - L''p_S Z_S \end{bmatrix} \begin{bmatrix} dp_R \\ dp_S \end{bmatrix}$$

If we call the determinant of the LHS of the coefficients of the above equation D, we can obtain by direct calculation

(B.3)

$$\frac{1}{Z_R}\frac{dZ_R}{dp_R} = \left\{ \frac{L'}{Z_R}(EU''r^2 + L''p_S^2) - L''(EU''p_R r^2 - p_S EU''er) \right\} / D$$

$$\frac{1}{Z_S}\frac{dZ_S}{dp_R} = \left\{ - \frac{L'}{Z_S}(EU''er + L''p_R p_S) + L''\frac{Z_R}{Z_S}(p_R EU''er - p_S EU''e^2) \right\} / D$$

$$\frac{1}{Z_R}\frac{dZ_R}{dp_S} = \left\{ - \frac{L'}{Z_R}(EU''er + L''p_R p_S) + L''\frac{Z_S}{Z_R}(p_S EU''er - p_R EU''r^2) \right\} / D$$

$$\frac{1}{Z_S}\frac{dZ_S}{dp_S} = \left\{ \frac{L'}{Z_S}(EU''e^2 + L''p_R^2) - L''p_S(p_S EU''e^2 - p_R EU''er) \right\} / D$$

To calculate the compensated price changes, we must calculate the income effects,

(B.4)

$$\frac{dZ_R}{dw_0} = \frac{L''(EU''r^2 p_R - EU''erp_S)}{D}$$

$$\frac{dZ_S}{dw_0} = \frac{L''(EU''r^2 p_S - EU''erp_R)}{D}$$

thus arriving at the results,

(B.5)

$$\frac{1}{Z_R}\left(\frac{\partial Z_R}{\partial p_R}\right)_{\overline{\Psi}} = \frac{L'}{Z_R}\,(EU''r^2 + L''p_S^2)/D$$

$$\frac{1}{Z_S}\left(\frac{\partial Z_S}{\partial p_R}\right)_{\overline{\Psi}} = -\frac{L'}{Z_S}\,(EU''er + L''p_R p_S)/D$$

$$\frac{1}{Z_R}\left(\frac{\partial Z_R}{\partial p_S}\right)_{\overline{\Psi}} = -\frac{L'}{Z_R}\,(EU''er + L''p_R p_S)/D$$

$$\frac{1}{Z_S}\left(\frac{\partial Z_S}{\partial p_S}\right)_{\overline{\Psi}} = \frac{L'}{Z_S}\,(EU''e^2 + L''p_R^2)/D$$

APPENDIX C

Optimal Tax Structure with Two Risky Assets

The introduction of more than one risky asset not only complicates the calculations, but actually may change the analysis in more fundamental ways. In particular, the simple relationships used in the previous section, between the properties of the utility functions, i.e. whether relative risk aversion was decreasing, constant, or increasing, and the structure of the demand functions no longer necessarily hold. Somewhat heuristically, the reason for this may be put as follows: As the individual becomes wealthier, if he has increasing relative risk aversion, he wants a "safer" portfolio. If there are only two assets, one safe and one risky, the only way by which this can be done is to invest relatively more in the safe asset. But if there are more than two assets, he can, as an alternative, increase the proportions in which he holds either of the two risky assets.[49] The only set of circumstances in which the results cited earlier can be shown to always go through are those in which the portfolio separation theorem obtains, i.e. the proportions in which the risky assets are held are independent of wealth, so a single mutual fund of risky assets may be formed. The portfolio allocation problem is then divided into two steps: finding the optimal proportion in which the risky assets are held, and finding the optimal proportions of dividing the entire portfolio between safe and risky assets. The precise conditions under which the portfolio separation theorem obtain have been explored elsewhere [5]. Here, we shall simply assume as we did earlier in Section 3 that the preference of the individual (attitudes towards risk) may be completely represented in terms of the mean and standard deviation of terminal wealth. As before, our strategy will be to derive the optimal tax structure in terms of certain properties of the utility function, and then to interpret these properties for the specific problem at hand. To simplify the calculations, we shall assume that the marginal utility of leisure (work) is constant, and that there are only two risky assets, but the results may be extended in a straightforward way to the more general case.

Expected utility may be written simply as a function of the quantity of the alternative securities owned by the individual:

$$\text{(C.1)} \qquad EU(Y) = \phi(Z_1, Z_2, Z_3)$$

We can derive, in exactly analogous manner to the preceding case, the result that the optimal tax structure implies that (where all price derivatives are along the compensated demand schedules)

$$\text{(C.2)} \quad \frac{t_1}{Z_1}\left(\frac{\partial Z_1}{\partial P_1}\right)_{\bar{\phi}} + \frac{t_2}{Z_1}\left(\frac{\partial Z_1}{\partial P_2}\right)_{\bar{\phi}} + \frac{t_S}{Z_1}\left(\frac{\partial Z_1}{\partial P_S}\right)_{\bar{\phi}} = \frac{t_1}{Z_2}\left(\frac{\partial Z_2}{\partial P_1}\right)_{\bar{\phi}} + \frac{t_2}{Z_2}\left(\frac{\partial Z_2}{\partial P_2}\right)_{\bar{\phi}} + \frac{t_S}{Z_2}\left(\frac{\partial Z_2}{\partial P_S}\right)_{\bar{\phi}}$$

$$= \frac{t_1}{Z_S}\left(\frac{\partial Z_S}{\partial P_1}\right)_{\bar{\phi}} + \frac{t_2}{Z_S}\left(\frac{\partial Z_S}{\partial P_2}\right)_{\bar{\phi}} + \frac{t_S}{Z_S}\left(\frac{\partial Z_S}{\partial P_S}\right)_{\bar{\phi}}$$

which may be written in matrix notation, letting $A_{ij} = (\partial Z_i/\partial P_j)_{\bar{\phi}}$ and t be the tax vector

$$\text{(C.3)} \qquad At = -\alpha Z$$

or

(C.4) $$t = -\alpha A^{-1}Z$$

where α is simply a proportionality factor. The first order conditions of utility maximization may be written

(C.5) $$\phi_i = p_i$$

where without loss of generality we have let the marginal utility of leisure equal unity. Totally differentiating (C.5) leads to

(C.6) $$\{\phi_{ij}\}[dZ_i] = [dp_i]$$

so

(C.7) $$\frac{dZ_i}{dp_i} = \left(\frac{\partial Z_i}{\partial p_i}\right)_{\bar{\phi}} = \frac{\hat{\phi}_{ij}}{|\phi|}$$

where $\hat{\phi}_{ij}$ is the cofactor of the ij^{th} element of the matrix $\{\phi_{ij}\}$, and $|\phi|$ is the determinant of the matrix $\{\phi_{ij}\}$. But since the matrix of cofactors (divided by the determinant) is just the inverse matrix, its inverse is the matrix ϕ_{ij}, i.e.

(C.8) $$t_i = -\alpha\sum_j \phi_{ij}Z_j$$

By assumption, we can write expected utility as a function of simply mean and standard deviation, i.e.

(C.9) $$\phi(Z_1, Z_2, Z_S) \equiv EU(Z_1e_1 + Z_2e_2 + Z_Sr) \equiv M(\mu, \sigma)$$

where

$$\mu \equiv Z_1\mu_1 + Z_2\mu_2 + Z_S r$$

$$\sigma^2 = Z_1^2\sigma_1^2 + 2Z_1Z_2C + Z_2^2\sigma_2^2$$

The calculations will be somewhat simplified if we write

(C.9') $$M \equiv M(Z_1\mu_1 + Z_2\mu_2 + Z_S r,\ Z_1 h(Z))$$

where

$$h^2 \equiv \sigma_1^2 + 2ZC + Z^2\sigma_2^2$$

and

$$Z = \frac{Z_2}{Z_1}$$

Thus the first order conditions are

(C.10)
$$\phi_1 = M_1\mu_1 + (h - h'Z)M_2 = p_1$$
$$\phi_2 = M_1\mu_2 + h'M_2 = p_2$$
$$\phi_3 = M_1 r = p_S$$

or

(C.11a) $$\frac{\mu_1}{h - h'Z} - \frac{\mu_2}{h'} = \left[\frac{p_1}{h - h'Z} - \frac{p_2}{h'}\right]\frac{r}{p_S}$$

which can be solved for $Z = Z^*$ as a function of μ_1, μ_2, r, p_1/p_S

and p_2/p_S, and

(C.11b)
$$-\frac{\mu_R' - r}{\sigma_R'} = \frac{M_2}{M_1}$$

where

$$\mu_R' = \frac{\mu_1 Z_1 + \mu_2 Z_2}{p_1 Z_1 + p_2 Z_2} = \frac{\mu_1 + \mu_2 Z^*}{p_1 + p_2 Z^*}$$

(C.12)

$$\sigma_R' = \frac{Z_1 h(Z^*)}{p_1 Z_1 + p_2 Z_2} = \frac{h(Z^*)}{p_1 + p_2 Z^*}$$

Differentiating (C.10), we obtain

$$\phi_{11} = M_{11}\mu_1^2 + 2M_{12}\mu_1(h - h'Z) + M_{22}(h - h'Z)^2 + M_2 \frac{Z^2}{Z_1} h''$$

(C.13)
$$\phi_{12} = M_{11}\mu_1\mu_2 + M_{12}[\mu_2(h - h'Z) + \mu_1 h'] + M_{22}(h - h'Z)h' - M_2 h'' \frac{Z}{Z_1}$$

$$\phi_{13} = M_{11}\mu_1 r + M_{12} r(h - h'Z)$$

etc.

Substituting into (C.8), we obtain

(C.14)
$$t_i = -\alpha\{\mu_i M_{11}\mu + M_{12}[(h - h'Z)\mu + \mu_i Z_1 h] + M_{22}(h - h'Z)hZ_1\}$$

which, upon substituting (C.10) becomes

(C.15)
$$\frac{t_i}{p_i} = -\alpha\left\{\frac{\mu_i}{p_i}\left[\left(M_{11} - M_{12}\frac{M_1}{M_2}\right)\mu + \sigma\left(M_{12} - M_{22}\frac{M_1}{M_2}\right)\right] + \frac{1}{M_2}[\mu M_{12} + \sigma M_{22}]\right\}$$

We observe that [from (C.11b)]

$$\text{(C.16)} \qquad \frac{d(1-a)}{d \ln w_0} \sim \frac{M_{21}\mu + M_{22}\sigma}{M_2} - \frac{M_{11}\mu + M_{12}\sigma}{M_1}$$

where, as before (1-a) is the proportion of total assets held in the form of money and

$$\frac{d \ln Z_S}{d \ln w_0} = 1 + \frac{d \ln (1-a)}{d \ln w_0}$$

(C.15) becomes

$$\text{(C.16)} \qquad \frac{t_i}{p_i} = \alpha'\left(\frac{\mu_i}{p_i}\left(\frac{d \ln Z_S}{d \ln w_0} - 1\right)\right) + \alpha''$$

where α' and α'' are constants. (C.16) has some very important implications:

(i) The structure of taxes among the different risky industries depends simply on differences in mean returns; it does not depend at all on either variances or covariances, although there may, of course, be some relationship between means and variances.

(ii) Although the allocation among alternative risky assets depends simply on characteristics of the assets themselves and not on preferences, the tax structure does depend on preferences.[50]

(iii) For small taxes, (C.16) can be approximated by

$$\text{(C.17)} \qquad t_i = \frac{\alpha'}{1-\alpha'}\left(\frac{d \ln Z_S}{d \ln w_0} - 1\right)\mu_i \approx \alpha'\left(\frac{d \ln Z_S}{d \ln w_0} - 1\right)\mu_i$$

Thus, whether industries with higher mean are taxed at a higher or lower rate depends completely on whether the elasticity of demand for the safe asset is greater or less than unity. Thus, if there is unitary elasticity of demand, all assets are taxed at the same rate. If $\mu_1 > \mu_2 > r$, then if $d \ln Z_S/d \ln w_0 > 1$

$$t_1 > t_2 > t_S$$

while if $d \ln Z_S/d \ln w_0 < 1$

$$t_1 < t_2 < t_S$$

5. <u>Taxation and Equity</u>

The discussion so far has focused on economies with identical consumers (the "representative individual"). Although this is convenient for the analysis for the distortionary effects of alternative tax structures, it ignores, of course, the important distributional questions. There are two separate issues here:

(i) <u>Horizontal Equity</u>. We should like to treat individuals who are in "all essential respects" identical, identically. Difficulties arise, however, in defining what is to be meant by "all essential respects." What is clear is that if two individuals have identical consumption patterns except for the kind of ice cream they consume, one eating only chocolate, the other only vanilla, a tax system which only taxed ice cream, and taxed chocolate and vanilla ice cream at differential rates would violate the principle of horizontal equity.

As the careful reader may note, there is in general an ambiguity in the meaning of horizontal equity; there will be few cases as clear cut as that just presented. This does not necessarily deprive the concept of all interest. We can attempt to make the concept more precise. We define horizontal equity in the following manner: Consider an optimal lump sum taxation scheme; consider two individuals which would have paid the same lump sum tax. Then a tax structure is horizontally equitable if the amount by which each must be compensated to restore him to his original level of utility is the same. Society may take horizontal equity as a value in itself; and it is not unlikely that this value will conflict with other social objectives, e.g. vertical equity (redistribution); that is, it may be true that commodities which on average have a very high income elasticity (and therefore are a good basis for vertical redistribution)[51] have a high variance in expenditure within any income (utility) group and therefore taxing them results in horizontal inequity. Proportional taxes on perfumes and automobiles may be a case in point. The optimal tax structure will depend then on the relative social weight placed on horizontal equity and on the other objectives of social policy. (Indeed by increasing sufficiently the relative weight on horizontal equity, our optimal tax problem is simply one of finding that tax structure which comes as close as possible to horizontal equity.)

In any case, it should be emphasized that some notion of horizontal equity--in an undoubtedly very ambiguous form--is an important part in all Western concepts of justice, and most governments seem to pay some attention to horizontal equity in the design of their tax structure.

Now no tax system is going to be horizontally equitable if individuals who would have been taxed the same amount under a lump sum tax system have different preferences. If, for instance, the supply curve of labor of two individuals who would have paid the same lump sum tax are different then a uniform consumption tax would be "horizontally inequitable." For small taxes, the amount which an individual needs to be compensated to attain the pre-tax level of utility is $\Sigma C_i dp_i$.[52] If, as in our model, the change in price is just equal to the tax, this is equivalent to $\Sigma t_i C_i$. It is clear that any tax system which differentiates in the rates levied on different kinds of commodities is liable to be "horizontally inequitable." For horizontal equity we want to tax those commodities for which the consumption patterns are most alike among individuals who we would tax equally if lump sum taxes could be imposed. I would suggest that this is likely to be true for broader categories of commodities than for narrow categories, e.g. for "cheese" rather than for "Blue cheese" or "Cheddar cheese" and for "food" rather than for "cheese." If it could be shown that individuals who would have paid the same lump sum tax supply the same amount of labor, a tax on aggregate consumption (or on labor income) would be horizontally equitable.[53]

In the particular case in point, the relevant empirical question is how significant are the disparities in investment patterns among different individuals in the same wealth-income class. This is essentially equivalent to the question of how different are different individual's attitudes towards risk (in the same wealth-income group). I suspect that such differences are significantly larger than, say, that in the proportion of income spent on housing or food, and thus, from the viewpoint of horizontal equity alone, differentiation between safe and risky assets would not be desirable.

(ii) Vertical Equity. A second general maxim of taxation is that we should like to tax the "rich" at least as high a percentage of their income (expenditure) and probably at a higher percentage than the "poor." Here even if individuals differed only with respect to initial wealth, if the only taxes that were available were proportional taxes on safe and risky assets (so that the only means of redistributing income would be the imposition of a higher tax rate on the security which is demanded relatively more by wealthier individuals) the objective of redistribution and that of "effeciency" would conflict with one another: Assume for instance that all individuals have increasing relative risk aversion; if we were to consider only efficiency (minimizing loss of consumer surplus) both the rich and the poor would prefer that the proportional tax rate on the risky asset be at a higher rate than that on the safe asset (for given revenue). On the other hand, because of increasing relative risk aversion, for redistributive purposes, we should like to tax the safe asset at a higher rate than the risky.

To see these results somewhat more precisely, consider an economy in which there are only two groups, the rich and the poor, differing with respect to their endowment of labor.

Our objective is to maximize social welfare

$$\ell U^1 + (1-\ell)U^2 \equiv \ell \Psi^1(p_S,\ p_R,\ w_0^1) + (1-\ell)\Psi^2(p_S,\ p_R,\ w_0^2) \tag{5.1}$$

where ℓ is the relative weight placed on utility of the rich and U^1 and U^2 are the utility levels of the rich and the poor respectively, subject, of course, to the market clearing constraints. The first order conditions may be written

$$v_S = \frac{t_S}{p_S}(\gamma_S \eta^1_{SS} + (1-\gamma_S)\eta^2_{SS}) + \frac{t_R}{p_R}(\gamma_S \eta^1_{SR} + (1-\gamma_S)\eta^2_{SR}) \tag{5.2}$$

$$v_R = \frac{t_S}{p_S}(\gamma_R \eta^1_{RS} + (1-\gamma_R)\eta^2_{RS}) + \frac{t_R}{p_R}(\gamma_R \eta^1_{RR} + (1-\gamma_R)\eta^2_{RR}) \tag{5.3}$$

where

$$v_i \equiv \gamma_i\left(\frac{\ell}{\nu}\Psi^1_{w_0} + t_S\frac{\partial Z^1_S}{\partial w_0} + t_R\frac{\partial Z^1_R}{\partial w_0}\right) + (1-\gamma_i)\left(\frac{(1-\ell)}{\nu}\Psi^2_{w_0} + t_S\frac{\partial Z^2_S}{\partial w_0} + t_R\frac{\partial Z^2_R}{\partial w_0}\right) - 1$$

(5.4)

$$\eta^k_{ij} = \left(\frac{\partial \ln Z^k_i}{\partial \ln p_j}\right)_{\bar{\Psi}^k}$$

$$\gamma_i = Z^1_i/Z_i\ , \quad (1-\gamma_i) = Z^2_i/Z_i$$

and ν is the Lagrange multiplier associated with the market clearing constraint.

The interpretation of (5.2) and (5.3) should be clear. We noted in the previous section that optimal taxation required an equal percentage reduction (along the compensated demand schedule) in all commodities.[54] That will be the case here if and only if $v_S = v_R$. v_S will equal v_R if $\gamma_S = \gamma_R$, i.e. the percentage of the safe industry owned by the rich is the same as the percentage of the risky industry which they own. This is equivalent to the rich and the poor allocating their assets between the two industries in the same proportions. More generally, (5.2) and (5.3) state that the percentage reduction in output will be greater in the industry which is in relatively greater demand by the rich, just as one might have expected. This, of course, will imply that the structure of taxes may be very different from that proposed in the previous section.

To see this, we derive from (5.3)

$$(5.5) \qquad \frac{t_S/p_S}{t_R/p_R} = \frac{v_S(\gamma_R\eta^1_{RR} + (1-\gamma_R)\eta^2_{RR}) - v_R(\gamma_S\eta^1_{SR} + (1-\gamma_S)\eta^2_{SR})}{v_R(\gamma_S\eta^1_{SS} + (1-\gamma_S)\eta^2_{SS}) - v_S(\gamma_R\eta^1_{RS} + (1-\gamma_R)\eta^2_{RS})}$$

Attention needs to be drawn to two cases.

(a) If $\gamma_S = \gamma_R$ then it is impossible to achieve any redistribution objectives by taxing the two assets differentially; i.e. no matter what the tax structure, the rate of taxes paid to expenditure is the same for the rich and poor. This does not necessarily mean that distributional considerations are not important in the design of the tax structure. For the distribution of the loss of consumer surplus may differ among alternative tax structures.

When $\gamma = \gamma_S = \gamma_R$, (5.5) becomes

$$(5.5') \qquad \frac{t_S/p_S}{t_R/p_R} = \frac{(\gamma\eta^1_{RR} + (1-\gamma)\eta^2_{RR}) - (\gamma\eta^1_{SR} + (1-\gamma)\eta^2_{SR})}{(\gamma\eta^1_{SS} + (1-\gamma)\eta^2_{SS}) - (\gamma\eta^1_{RS} + (1-\gamma)\eta^2_{RS})} .$$

When $\gamma = 1$, (5.5') may be shown to be equivalent to the results established in the preceding section. When cross elasticities are zero (which is very unlikely) (5.5') gives the familiar result that tax rates are inversely proportional to demand elasticities. More generally, (5.5') states that the ratio of tax rates should be inversely proportional to the difference between the own price elasticity and the cross elasticities, where each of the elasticities is taken to be a weighted average of the elasticities of the two groups. Note, however, that the weights to be attached to the elasticities of the rich and the poor are in proportion to their contribution to the purchases of the commodities; since the rich purchase more than the poor, the optimal tax structure is closer to that which would prevail if there were only rich individuals than if there were only poor. This is a somewhat surprising result; one might have thought that even if the rich and poor spent the same proportion of their income on different assets if price elasticities differed distributional considerations would require that the optimal tax structure use a weighted average of the price elasticities, with a greater weight on the poor than simply their relative contribution to demand. But this does not seem to be the case.

(b) On the other hand, if $\gamma_S \neq \gamma_R$, the result obtained earlier about whether $t_R \gtrless t_S$ may actually be reversed. To see this, consider the case where the compensated price elasticities of the two groups are the same. Then (5.5) takes on the form

$$(5.6) \qquad \frac{t_S/p_S}{t_R/p_R} = \frac{v_S\eta_{RR} - v_R\eta_{SR}}{v_R\eta_{SS} - v_S\eta_{RS}} = \frac{\frac{v_S}{v_R}\eta_{RR} - \eta_{SR}}{\eta_{SS} - \frac{v_S}{v_R}\eta_{RS}} = \frac{\eta_{RR} - \frac{v_R}{v_S}\eta_{SR}}{\frac{v_R}{v_S}\eta_{SS} - \eta_{RS}}$$

We first observe that, for fixed η_{ij}, the ratio of the relative tax on the safe to the risky industry is an increasing function of v_S/v_R.[55]

This has the immediate implication that the industry which is in relative demand by the rich is taxed at a higher rate than it would have been had we ignored distributional considerations.[56] Indeed, not only is it possible that we tax the industry with the lower price elasticity at a lower rate, than the other industry, but we may actually subsidize it (i.e. impose a negative tax).

Consider the polar case where $v_R/v_S = 0$. Then

$$\frac{t_S/p_S}{t_R/p_R} = -\frac{\eta_{RR}}{\eta_{RS}}$$

The relative tax structure depends simply on the ratio of the own elasticity to the cross elasticity of demand for the risky asset. This will be greater or less than unity as the expenditure elasticity of the demand for the risky asset is positive or negative, or, what is equivalent, as the Arrow-Pratt measure of Absolute Risk Aversion $(-U''/U')$ is decreasing or increasing with wealth.[57] In the normal case $t_S/p_S > t_R/p_R$. This should be contrasted with the earlier result, in which the relative tax rates depended on whether the expenditure elasticity was greater or less than unity. Thus, in those cases where the expenditure elasticity on the risky asset is between zero and one, efficiency considerations alone dictate that the risky industry be taxed at a higher rate than the safe, while distributional considerations suggest that (for the polar case examined here), the safe industry be taxed at a higher rate.[58] The tax structure depends crucially on the relative emphasis to be placed on these two considerations.[59]

6. Price Uncertainty: The Many Commodity Model

The sole source of uncertainty in the preceding analysis was technological; i.e. arose from uncertainty about the physical output of given production processes. In fact, firms often seem more concerned with price uncertainty than with such technological uncertainty: the uncertainty of what the price at which they can sell their output next period will be. There are many reasons for price uncertainty, the most important of which may in fact be associated with oligopolistic markets--the uncertainty associated with the behavior of the other producers in the given market. However, even in competitive markets, there may be price uncertainty arising from variations in the relative quantities of different commodities produced. This has been particularly characteristic of primary commodity markets. This is not the place for an extensive discussion of uncertainty in such models. We do wish, however, to make note of how much of what we have said earlier is dependent on the one commodity assumption (no price uncertainty).

First, we should note that in general relative prices will move in opposite directions to the changes in relative quantities, and so the fluctuations in profits are to some extent ameliorated. Indeed, if the elasticities of demand are small, the fluctuations in profits may move in the opposite directions to fluctuations in output.

Secondly, it should be observed that when there is more than one commodity, it is not unambiguous which is the "safe" asset, which is the risky. For the returns to different assets will be correlated with movements in relative prices, and the marginal utility of income will also depend on relative prices. For instance, the return to housing is likely to be highly correlated with the price of housing, that in agriculture with the price of food. For an individual who consumes only the services of houses, an investment in housing would, from a consumption point of view, be a safe asset, while an investment in agriculture would be risky. On the other hand, for an individual who consumed only food, an investment in housing would be risky, and one in agriculture would be safe.[60] In particular, it should be observed that even if the real output from an investment were perfectly certain, fluctuations in outputs of other commodities would still make the given investment risky, since its price relative to other prices would vary and the marginal utility of income would vary. Moreover, attitudes towards risk will in general depend on relative prices.[61]

To illustrate how these points affect the analysis, we consider the following special case. Assume the indifference map is homothetic and can be represented (in one ordinal numbering) by the parametrization

$$(6.1) \qquad U = \frac{\delta x_1^A + (1-\delta)x_2^A}{A/2} \qquad 0 < A < 1\ , \quad \text{or} \quad A < 0$$

where x_1 and x_2 are the two commodities. For simplicity, and without loss of generality, we shoose the units in which x_1 and x_2 are measured so that $\delta = (1-\delta)$. The indirect utility function, giving the maximum level of utility attainable at a given set of prices/q_i and a given income, corresponding to (6.1) is, letting $q_2 = 1$ (x_2 be the numeraire),

$$(6.2) \qquad \hat{U} = \frac{Y^A}{A}\,[1 + q_1^{A/A-1}]^{1-A}$$

where Y is the individual's income. If he has an initial wealth, I, invests a proportion z in the first industry and 1-z in the second, his income, in x_2 numeraire, is

$$(6.3) \qquad Y = I\{ze_1 + (1-z)e_2\}$$

where e_1 and e_2 are the returns per dollar invested in each of the two industries.

For simplicity, we shall assume the production in both industries is described by a stochastic constant returns to scale production function

$$X_i = g_i(\theta)I_i$$

so

$$e_i = \frac{x_i q_i}{V_i}$$

where V_i is the value of the firm. Because of constant returns to scale, competitive equilibrium requires $V_i = I_i$,

$$e_i = q_i g_i \tag{6.4}$$

If, as we have assumed earlier, all individuals are identical, then the relative prices in each state of nature will be determined by the relative marginal utilities in the given states, i.e.,

$$\frac{q_1}{q_2} = \left(\frac{g_1 I_1}{g_2 I_2}\right)^{A-1} \tag{6.5}$$

Assume the representative individual has a strictly concave cardinal utility function of the form

$$\tilde{U} = H(U) \tag{6.6}$$

$$H' > 0 , \quad H'' < 0$$

The individual's optimal portfolio allocation is given by

$$EH'\{Y^{A-1}(1 + q_1^{A/A-1})^{1-A}(e_1 - e_2)\} = 0 \tag{6.7}$$

where the individual takes the probability distribution of q_i and e_i as given. The competitive equilibrium allocation is then that (unique) allocation for which [substituting (6.3)-(6.5) into (6.7)]

(6.8)

$$EH'\left\{\frac{(zg_1)^A}{((1-z)g_2)^{A-1}} + (1-z)g_2\right\}^{A-1}\left\{1 + \left(\frac{zg_1}{(1-z)g_2}\right)^{A/A-1}\right\}^{1-A}\left(g_1^A\left(\frac{z}{(1-z)g_2}\right)^{A-1} - g_2\right) = 0$$

To ascertain the effect of a proportional investment tax at the same rate in the two industries on the allocation of investment resources, we observe, as previously, that such a tax is equivalent to a reduction in I. Hence, since it is easy to establish that (6.8) is concave in z, the change in z is of the sign of

$$(6.9) \qquad E\,\frac{(-H''U)}{H'}\,H'Y^{A-1}\left(1+q_1^{A/A-1}\right)^{1-A}(e_1-e_2)$$

For reasons analogous to those in the one commodity case, it is natural to call

$$\frac{-H''U}{H'}$$

the measure of relative risk aversion. Hence, we observe that if there is constant relative risk aversion the allocation of investment is unaffected by a proportional investment tax, regardless of the pattern of risk or consumption.

On the other hand, when H is not of constant elasticity the effect will depend both on taxes and the pattern of risk. Consider, for instance, the case where only the first industry has a variable output: $g_2 = 1$ in all states of nature. Then, for any given allocation, z, the higher g_1, the better off is the individual; on the other hand, whether g_1^A increases or decreases with g_1 depends on whether A is greater or less than zero. For this, it immediately follows that a proportional tax on investment in both industries results in an increased allocation to the industry with no technological uncertainty if the elasticity of substitution[62] between the commodities is less than unity $(A < 0)$ and risk aversion increases with utility or if the elasticity of substitution between the commodities is greater than unity $(A > 0)$ and risk aversion decreases with utility. Conversely, investment in the industry with no technological uncertainty is reduced if there is an elasticity of substitution between commodities less than unity and risk aversion decreases with utility, or if the elasticity of substitution between commodities is greater than unity and risk aversion increases with utility.

These results suggest that if the elasticity of substitution were equal to unity, then the pattern of allocation would be unaffected by the tax, and that in fact is the case. The utility function may be written in the form

$$U = x_1^A x_2^{1-A} \tag{6.1'}$$

and the indirect utility function is of the form

$$\hat{U} = A^A (1-A)^{1-A} Y (q_1^{-A} q_2^{-A}) \tag{6.2'}$$

Relative prices are just inversely proportional to relative quantities:

$$\frac{q_1}{q_2} = \frac{x_2}{x_1} \frac{A}{1-A} = \frac{g_2 I_2 A}{g_1 I_1 (1-A)} \tag{6.5'}$$

The individual's optimal portfolio allocation is given by (again letting x_2 be the numeraire)

$$EH'Yq_1^{-A}(e_1 - e_2) = 0 \tag{6.7'}$$

so that, if all individuals are identical, the competitive equilibrium allocation would be given by the value of z for which

$$EH'\left\{\left(\frac{(1-z)g_2A}{1-A} + g_2(1-z)\right)\left(\frac{g_2(1-z)A}{g_1z(1-A)}\right)^{-A}\left(\frac{(1-z)g_2A}{z(1-A)} - g_2\right)\right\}$$

(6.8')

$$= \left((1-z)A - z(1-A)\right)\frac{A^{-A}}{(1-A)^{2-A}}\left(\frac{1-z}{z}\right)^{1-A} EH' g_2^2\left(\frac{g_2}{g_1}\right)^{-A} = 0$$

which, for an interior solution, requires

$$1-z = A$$

independent of the pattern of uncertainty or attitudes towards risk (i.e. of the function H). A proportional tax on investment in both industries, or indeed, a tax at a differential rate in the two industries, cannot affect the pattern of allocation of investment; it can only affect the relative prices which prevail in the different states of nature.

These results are in accord with what we would expect. With unitary elasticity, price changes exactly offset quantity changes. If the elasticity of substitution is less than unity, the price changes more than offset the quantity fluctuations.

Hence the industry with no fluctuations in output experiences, in effect, greater uncertainty than that experiencing the technological uncertainty. Accordingly, it "behaves" like the relatively risky industry.[63] Conversely when the elasticity of substitution is greater than unity, in which case price variations are not sufficient to compensate for the quantity variations.

7. Taxation and Financial Policy

The taxes considered in the previous sections have differentiated among investment (profits) in different industries, but not in the manner in which the funds for investment are raised (and hence the form in which the profits are distributed). As a result the Modigliani-Miller theorem, asserting that the value of the firm is independent of the financial policy pursued by the firm (provided that it does not borrow a sufficient amount to go bankrupt in any state of nature) remained valid throughout the previous analysis.

We now consider what happens when the government does treat from a tax point of view different financial instruments differently. In particular, we shall consider the case, where, as in the United States, a distinction is made between debt and equity financing. Assume there is a corporate profits tax at the rate t_p. Interest payments on debt are tax deductible. On the other hand, individuals must pay a tax of t_r on interest income receipts, and a tax at the rate t_e on receipts from equities. Hence the total tax payments to the government in state θ if the profits of the firm are $\pi(\theta)$, if the firm issues B bonds, and the rate of interest is $\hat{r} \equiv r-1$, are

$$t_p(\pi - \hat{r}B) \qquad \text{corporate profits tax}$$

$$+ t_r\hat{r}B \qquad \text{tax on interest payments by individuals}$$

$$+ t_e(1 - t_p)(\pi - \hat{r}B) \qquad \text{tax on income from equities by individuals}^{64}$$

$$= [t_p + t_e(1 - t_p)]\pi - \hat{r}B(t_p + t_e(1 - t_p) - t_r)$$

Observe that in each state of nature, tax payments are increased or decreased with B as

$$(7.1) \qquad t_p + t_e(1 - t_p) - t_r = t_e - t_r + t_p(1 - t_e) \lessgtr 0$$

If $t_r = t_e$, as Modigliani and Miller [13] and Baumol and Malkiel [3] implicitly assume, this is unambiguously positive, so tax payments in every state of nature are reduced by issuing bonds. Hence, it will be in the interest of all individuals for firms to finance as much of their investment as they can by issuing bonds. The effect of this kind of tax on the investment decisions can easily be seen as in Figure 13, where we have again for simplicity assumed constant returns to scale in the two industries. Assume the maximum proportion of investment in the risky industry which can be financed by bonds is $K_R < 1$. Then the mean return after tax is just

$$\mu' = \mu - (\mu - 1 - \hat{r}K_R)(t_p + t_r(1 - t_p)) - t_r\hat{r}K_R$$

$$= \mu - (\mu - 1)(t_p + t_r(1 - t_p)) + \hat{r}K_R t_p(1 - t_r)$$

$$\sigma' = \sigma(1 - t_p)(1 - t_r)$$

i.e. it is equivalent to a simple income tax at the rate $t_p + t_r(1 - t_p)$ plus a "lump sum" subsidy (independent of the state of nature) of $K_R(1 - t_r)t_p\hat{r}$ per unit of investment. One would normally expect that the safe industry, can, however, finance a larger proportion of its investment through bonds, since there is no risk of default i.e. $K_S > K_R$. Hence, the after tax rate of return on safe investments is given by

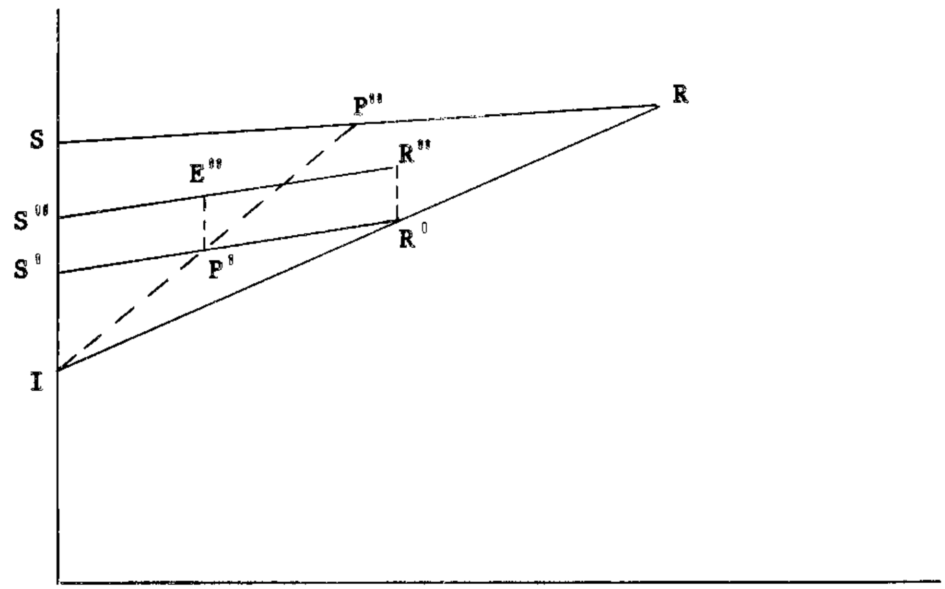

FIGURE 13a

Effects of Profits Taxes: $t_e = t_r$, $K_R = K_S$

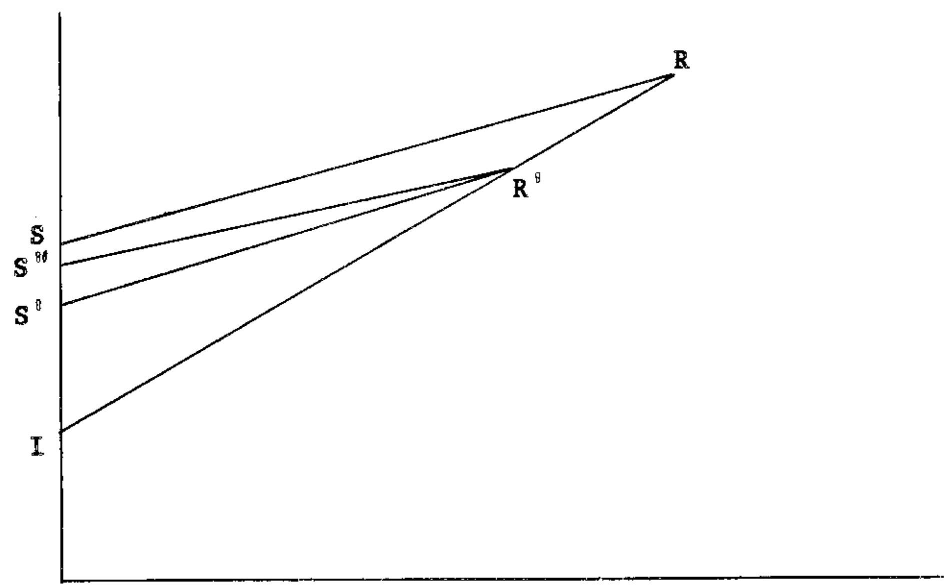

FIGURE 13b

Effects of Profits Taxes: $t_e = t_r$, $K_R = 0 < K_S$

$$\hat{r}' = \hat{r}[1 - (t_p + t_r(1 - t_p)) + K_S(1 - t_r)t_p]$$

In Figure 13a, we have depicted the effects of the tax on real investment. If $K_R = K_S = 0$, this is just a proportional profits tax, at the rate $t_p + t_r(1 - t_p)$; the points R and S move towards point I in the same proportion (to $R'S'$); and so the opportunity set moves down in parallel, as discussed earlier (Section 3). If $K_R = K_S > 0$, the opportunity set moves up; the point R' moves up to R'', S' to S''; the amount of the upward shift is identical, and so the new opportunity set is still parallel to the original. If E'' is the new equilibrium, the pre-tax mean-variance corresponding to E'' is given by P'', which is found as follows. The mean and standard deviation, pre-lump sum subsidy but after the income tax, corresponding to E'' is P', i.e. the same standard deviation but smaller mean, and the pre-tax point corresponding to P' is just P'', as in the earlier analysis. Whether the tax structure increases or decreases risk taking depends on whether P'' is to the left or right of E, the initial equilibrium. A sufficient condition in the mean variance model that the tax increase risk taking is that there is increasing or constant relative risk aversion (income elasticity of the demand for safe assets greater than unity). One would generally expect, however, that $K_R < K_S$, i.e. risky firms can finance a smaller proportion of their investment through debt. In that case, the "lump sum" subsidy to the risky industry is smaller than that to the safe, so that the new opportunity locus is flatter than the pre-tax locus (Figure 13b). The effect of this is normally (if the price elasticity of the demand for safe asset is negative) to reduce risk taking from what it would have been had $K_R = K_S$. Whether risk taking is increased or decreased relative to the no-tax situation depends on the extent to which K_R and K_S differ, and the income and price elasticities of the demand for the safe asset.

It is important to distinguish these real effetts from the tax from the superficially more dramatic but economically less significant changes in the financial structure of firms. Now it no longer is true that firms are indifferent as to the debt-equity ratio; in this case each firm will attempt to have as high a debt equity ratio as it can, subject to the constraints of bankruptcy.

On the other hand, just the opposite conclusions obtain if t_r is sufficiently greater than t_e, which seems to be the case for at least some wealthy Americans. The returns from the firm which are not distributed to bondholders may be redistributed to the stockholders by several devices which take advantage of the special treatment of capital gains. The firm may buy back their shares (although few companies seem to avail themselves of this opportunity) or they may reinvest the funds. The latter is possible even if the given stockholder does not wish to invest his profits (e.g., he wishes to consume them), for provided that there exists some other individual who would have been willing to invest the given amount in the firm, as there will normally be in a growing dynamic economy, the original stockholder simply sells that proportion of his shares which represent the increment in value resulting from the investment of the firm to the new shareholder. If capital gains are taxed at one half the rate of normal income, $t_c = \frac{1}{2} t_r$ then the critical rate for t_r, at which the individual is indifferent as to the financial policy of the firm, is given by

$$t_r = \frac{2t_p}{1 + t_p}$$

or, if $t_p = .5$, $t_r = .67$. Individuals with a personal tax rate in excess of 67% would prefer the firm to issue no debt.

If capital gains are taxed at either one-half of the normal rate or 25%, whichever is lower, then the critical value of t_r (with $t_p = .5$) is .625.

In fact, however, because the tax on capital gains is only paid when the gains are realized, and if they are not realized upon death, they are not paid at all, the effective rate on capital gains may in fact be considerably less than one half that of the personal tax on interest income. At the extreme, if $t_e = 0$, the critical value of $t_r = t_p$. All individuals whose personal tax rate exceeds the corporate tax rate desire that the firm have a zero debt equity ratio. In this case, the interest deductibility provisions have no effect, and the real effects are identical to those analyzed earlier in Section 3.

Note that the effect of the interest deductibility provision is to polarize firms into two groups: those which want a very low (zero) debt equity ratio, catering to individuals in very high income brackets, and those which want a very high debt equity ratio. On the other hand, a large percentage of the wealth of the United States is owned by individuals whose personal tax bracket is in the range where the savings resulting from the firms changing its financial policy is relatively small. Consider, for instance a firm which had pursued a policy of a zero debt equity ratio, and decided it would immediately refinance itself, resulting in a debt equity ratio of 1/2 (greater than most firms in fact have). Assume $t_p = .5$ and the ratio of the mean return on the risky asset to that on the bond (before tax) is 2. An individual in the 50% bracket who pays half the rate on capital gains as on interest income would have his taxes reduced by only 5%. Under similar conditions, an individual who was in the 60% bracket and who planned to die before realizing his capital gains would make a savings of only 5% if the firm changed its financial policy from a debt equity ratio of 1/2 back to zero. Given the variations in the tax structure in the last 30 years, the costs associated with dramatic changes in the financial structure and the normal lags of response, it is not surprising that the tax system has apparently not had a major effect on the financial structure of firms. If, however, the tax system were to remain unchanged for a long period, it would seem not unlikely that gradually firms would take advantage of even these small gains.

The important point to observe is that these two provisions of the tax structure--the tax deductibility of interest payments with special treatment of capital gains--is, in terms of its effects on the pattern of allocation of investment, much closer to being equivalent to a uniform proportional tax system than has previously been thought. This is not to say that the effects of the tax system on the financial structure of particular firms might not be quite significant. If such a polarization as suggested above should occur, one might expect firms whose maximum debt equity ratio was small, e.g. risky firms, to be more likely to pursue an all equity policy, while firms with little variance to pursue a high debt equity policy. Since the major method of taking advantage of the special provisions of capital gains is through investment, again one would expect firms with high investment rates to have a low debt equity ratio.[65]

One might argue that it would be far simpler to eliminate the interest deductibility provision of the tax system, and tax all income uniformly. Although in the model presented here, this would clearly seem advantageous over the present system, in more general models the answer seems less clear. For a number of reasons, one may want to tax pure profits and rents at a higher rate than income from capital. Unfortunately, it is often difficult except for payments on bonds to differentiate the two; but where it is possible, we do treat them differently.

8. Summary and Concluding Remarks

The object of this study has been to describe the effects of alternative tax policies on the allocation of resources between safe and risky industries and to determine, under alternative objectives, the design of the optimal tax structure. The major results of the study may be summarized as follows:

1. If the proceeds of the tax are not redistributed as lump sum payments (and do not in any other way affect individual's allocation decisions), then the partial equilibrium results derived earlier [19] obtain in our competitive general equilibrium model; for instance, a proportional tax on production at the same rate in all industries, decreases, leaves unchanged, or increases investment in the risky industries relative to the safe industry

as individuals have an expenditure elasticity of demand for risky assets greater, equal to, or less than unity.

2. If a uniform proportional production (or profits) tax is levied on all risky industries, the tax which results in the proportion of assets allocated to the safe industry remaining unchanged will not in general be neutral in its effects on the allocation among risky assets.

3. If the proceeds of the tax are redistributed as lump sum payments, allocative neutrality requires that safe and risky industries be taxed at the same rate.

4. If the government has a utility function over the mean and variance of the proceeds of the tax (e.g. the proceeds are used to finance a public good) and the government is restricted to imposing proportional (production or profit) taxes, the level of social welfare obtainable under a centralized system of production (allocation of investment) is greater than that obtainable under a decentralized (competitive) system of production. If the government is risk neutral, and the marginal utility of (mean) income to the government is the same as that to the private sector, then the proportional production tax rates should be inversely related to the mean returns, so the safe industry is taxed at a higher rate than the risky. If, on the other hand, the marginal rate of substitution of the government between risk and mean return is the same as in the private sector, and the marginal utility of (mean) income is the same, then the optimal tax structure imposes the same tax rate on both sectors.

5. If the supply of the factor input is not inelastic, if all individuals

are identical, and if there are only two industries, both having constant returns to scale, then the optimal tax structure requires that investment in the industry with an income elasticity greater than unity be taxed at a lower rate, i.e. the safe industry if relative risk aversion is increasing, the risky industry if relative risk aversion is decreasing. The relative tax rates among different risky industries depend only on the mean returns, and in general, the tax rate is an increasing or decreasing function of mean return as the income elasticity of risky assets is greater or less than unity (relative risk aversion is decreasing or increasing).

6. There is a conflict in the design of the tax structure between the objectives of minimizing loss of consumer surplus and horizontal and vertical equity (distribution).

7. When there are two commodities, variations in relative outputs will result in variations in relative prices. If there is constant relative risk aversion or if the elasticity of substitution between the two commodities is unity, allocative neutrality requires that they be taxed at the same rate. If only one of the industries experience technological uncertainty, it acts like the risky industry if the elasticity of substitution between commodities is greater than unity, but like the safe industry if the elasticity of substitution between commodities is less than unity.

8. The important effects of the differential tax treatment of debt and equity are not the change in financial policy which it causes, but the distortionary effects on real production. If there were no special treatment of capital gains, the interest deductibility provisions would be equivalent to imposing

a higher tax rate on riskier industries (industries for which the maximum debt financing is relatively small). The special treatment of capital gains means that some individuals (in high tax bracket) would prefer firms to have an all equity policy. The present system of favorable treatment of capital gains--cum interest deductibility is probably reasonably close, for a large percentage of wealthholders, to being neutral in its effect on financial policy (and hence is essentially equivalent to a uniform tax rate on safe and risky industries). But if financial neutrality is a desirable objective, then it would be far simpler to accomplish this goal by eliminating both the provisions for interest deductibility and the special treatment of capital gains.

From these results, no clear answer emerges to the important policy question of whether the government should tax riskier industries at lower or higher rates than safe industries. The answer depends on detailed knowledge not only on the kind of tax (e.g. production versus profit) and empirical information about income and price elasticities of demand, but also on the objectives of the government, e.g. the importance placed on distributional objectives. Indeed we have noted that the objectives of "efficiency" and "redistribution" are likely to be in conflict. What is clear is that those who advocate government subsidies to risk taking, in the form of special tax treatment (e.g. oil industry, special treatment of capital gains), special regulations (drug industry), insurance schemes, etc. have a difficult case in establishing the desirability of these policies on the grounds either that a proportionate tax system discriminates against them unfairly or that it would be desirable to increase resources to such risk taking activities.

My own judgment is that unless a far stronger case can be made for such special treatment of risky industries than has heretofore been made, a uniform tax rate is the wisest course to pursue: not only does it have the advantages of administrative simplicity and horizontal equity, but it avoids the political conflicts and opportunities for the exertion of influence of special interests groups which inevitably accompany the differentiation in taxation between different sorts of expenditure.

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TABLE OF SYMBOLS

Individuals' Utility

$U^j(Y)$	Utility of income (terminal wealth)
$Y^j(\theta)$	j^{th} individual's terminal wealth (income) in state θ
$A = -U''/U'$	Arrow-Pratt measure of absolute risk aversion
$\rho = -U''Y/U'$	Arrow-Pratt measure of relative risk aversion
$L^j(-I^j)$	j^{th} individual's utility function for "leisure" (disutility for supplying factor)
I^j	j^{th} individual's supply of factor
$U^j = U^j(Y) + L^j(-I^j)$	Total utility of j^{th} individual (from income and leisure)
$M^j(\mu^j, \sigma^j)$	Expected utility of income, expressed as a function of mean and standard deviation
μ^j	Mean of individual's income
σ^j	Standard deviation of individual's income
$\phi(z_1, z_2, \ldots, z_S)$	Expected utility of income as a function of purchases of different securities
$\Psi(p_1, p_2, p_3, w_0)$	The indirect-expected utility of income function

Individual's Portfolio Allocation

β_i^j	j^{th} individual's initial ownership share of i^{th} firm
λ_i^j	Proportion of wealth invested in i^{th} firm by j^{th} investor
λ	Ratio of investment in first risky industry to that in second
a	Fraction of resources invested in risky industry

w_0	Initial wealth of individual (not derived for selling factor, I)
z_i	Individual's purchases of i^{th} risky asset (if there is only one risky asset, it is denoted by z_R)
z_S	Individual's purchases of safe asset

Firms

X_i	Output of i^{th} firm
I_i	Investment of i^{th} firm
θ	State of nature
$g_i(\theta)$	Output of i^{th} firm in state θ at unit investment level
$f_i(I_i)$	Ratio of output at investment level I_i to output at unit investment level
B_i	Number of bonds issued by i^{th} firm
$r-1$	Rate of interest on a perfectly safe asset
V_i	Total market value of i^{th} firm
E_i	Market value of i^{th} firm's shares

Securities

$e_i(\theta)$	(Gross) return per dollar invested in i^{th} firm's shares
$p_i = V_i/I_i$	Ratio of value of i^{th} firm to its investment
μ_R	Mean return per dollar invested in risky asset(s)
σ_R	Standard deviation of return per dollar invested in risky asset(s)
μ_i	Mean return per dollar invested in i^{th} risky asset

σ_i	Standard deviation of return per dollar invested in i^{th} risky asset
C	Covariance (per unit of investment)

<u>Government</u>

$\Omega(Y, T)$	Utility of income and public goods
T	Proceeds of tax (expenditure on public goods)
$G(\bar{T}, [E(T-\bar{T})^2]^{1/2})$	Social evaluation of tax revenue (expressed as function of mean and standard deviation)
t_S, t_R	Tax rates on safe and risky industries (nature of tax defined in relevant sections)
'	Denotes after-tax value of given variables
I_g	Government purchases of factor
ν	Shadow price on market clearing constraint
ℓ	Relative weight placed on utility of the rich

FOOTNOTES

[1]These are both sufficient conditions. A further sufficient condition is that the rate of return on the safe asset is zero. These results assume that there are only two assets, a perfectly safe asset and a risky asset. If there is more than one risky asset, these results will only be true in general if the conditions for the portfolio separation theorem obtain [5]. Condition (a) is equivalent to the assumption of increasing absolute risk aversion, i.e. if $U(Y)$ is the utility function, $-U''/U' = A$ is the measure of absolute risk aversion, [2, 15]; then $dA/dY > 0$. Condition (b) is equivalent to the assumption of decreasing absolute and increasing relative risk aversion, i.e. if $\rho = -U''(Y)Y/U'$ is the measure of relative risk aversion, $d\rho/dY > 0$, $dA/dY < 0$.

[2]The only exception to my knowledge is the recent study by Shibata [17]. But her analysis involves a number of very restrictive assumptions and is primarily concerned with price uncertainty rather than technological uncertainty.

[3]This result holds only if there is no special treatment of capital gains. See below.

[4]See sections 3 and 7.

[5]Indeed, one way of viewing a profit tax is as an investment tax with the proceeds of the tax used to purchase shares in the given firm.

[6]We present the model in somewhat greater generality than in fact we will employ in the subsequent analysis. The various cases we consider will be alternative modifications and specializations of this general model.

[7]Because inputs precede outputs in time, this model is sometime referred to as a two period model. For a discussion of the limitations of the one period model, see [8].

[8]It should be emphasized that the assumption that increases in investment lead to equiproportionate increases in output in each state of nature is crucial for the analysis. For without it, except under the rather unrealistic assumption that there are as many securities as states of nature, it is not clear what meaning can be given to competitive price-taking behavior. Diamond has shown that the competitive allocation for the market presented here is a Constrained Pareto Optimum. When the technology is not of the form (2.1), or when firms have a choice as to the pattern of output across

the states of nature, then there are strong reasons to believe that the market allocation will not even be a constrained Pareto Optimum, in which case tax policy will be concerned not only with raising revenue, but also with alleviating the distortions in the market allocation [11, 23].

[9]Without loss of generality, we may take $f(1) = 1$.

[10]This assumption, in conjunction with the assumption that there are fewer firms than states of nature, implies that there will be fewer securities in the stock market than states of nature. We also assume that individuals cannot trade directly with one another income in one state of nature for income in another state of nature. Thus the stock-market is the only mechanism in this model for handling risks. These assumptions are not completely realistic; some firms do issue preferred stocks and other debt instruments which are less risky than common stocks but riskier than ordinary bonds, and insurance and betting do allow individuals to take or avoid some risks that they could not by means of securities in the stock market alone; but introducing these considerations would complicate the analysis without seriously altering the basic qualitative propositions.

[11]We assume that there is a perfect capital market and that firms never issue so many securities as to go bankrupt in any state of nature. Thus r is the same for all firms and constant across the states of nature. If we assume that there are sufficiently high "transaction" costs involved in bankruptcy, it can be shown that firms will never in fact go bankrupt. In the simple two period model presented here, if a firm is not able to meet its bond obligations, i.e., if for any θ, $X_i(\theta) < rB_i$ it is said to go bankrupt. (In practice, a far more common occurrence is the deferral of interest or principal payments, but this can only be incorporated into a multi-period model.) For a detailed discussion of the role of bankruptcy in the absence of taxation in a simple model of the stock market, see Stiglitz [21].

[12]See below, Section 7. It is clear that provided the tax does not discriminate between interest and non-interest income, the MM proposition will remain valid. (This requires, of course, provisions for full loss off set.)

[13]Without loss of generality, we assume firms have no outstanding bonds at the beginning of the period.

[14]By the arguments above, the value of the firm, in the absence of taxes which discriminate between alternative forms of income will depend simply on its level of investment, not on its financial policy. Later, in Section 7, we shall modify an analysis to take account of taxes which do discriminate between alternative forms of income.

[15] If there are non-constant returns to scale, the firm assumes that its market value increases in proportion to $f_i(I_i)$, i.e.

(2.5) $$\frac{dV_i}{V_i dI_i} = \frac{f_i'(I_i)}{f_i(I_i)}$$

Since we shall explicitly assume that all firms and individuals are price takers, by the usual Fisherian arguments, it can be shown that it will be in the interest of all stockholders to have the firm maximize (2.4), i.e. the market value of the original shareholders:

(2.6) $$\frac{dV_i}{dI_i} = 1 = \frac{V_i f_i'}{f_i}$$

Since $V_i \geq I_i$, if the solution to (2.6) yields a value of $I_i > V_i$, I_i is set equal to zero. (Rather than operating at a loss the firm always has the option of shutting down.)

[16] When all firms are price takers, it can be shown that we can describe the behavior of the industry by aggregating all the firms in the same risk class together into a single (price taking) firm, even when there are not constant returns to scale.

Assume there were a firm whose output was given by

(2.7) $$X_i(I_i, \theta) = \max_k(\Sigma f_{i_k}(I_{i_k}))g_i(\theta)$$

$$\text{s.t. } \Sigma I_{i_k} = I_i$$

If it maximizes the market value of the original shareholders

(2.8) $$1 = \frac{f_{i_k}' V_i}{\Sigma f_{i_k}} \qquad \text{all } k$$

This should be compared with the disaggregate equilibrium. By definition all firms in the same risk class have the same value of V_{i_k}/f_{i_k}. Moreover,

(2.9) $$f_{i_k}' \cdot V_{i_k} = f_{i_k}$$

(from (2.6)) from which it follows that f_{i_k}' has the same value for all k.

The total value of securities in the industry is just $\Sigma V_{i_k} = V_i$. Summing (2.9) over k, we then obtain

$$f'_{i_k} \cdot V_i = \sum_k f_{i_k} \qquad (2.10)$$

which is identical to (2.8).

[17] Because of constant returns to scale, the value in equilibium of the individual's initial ownership claims on firms is zero. More generally, if he initially owns β_i^j of the i^{th} firm, his total wealth, which he can allocate among alternative securities and bonds, is given by

$$I^j = \sum_i \beta_i^j (V_i - I_i) \qquad (2.12)$$

[Since $V_i - I_i$ represents the total value of initial ownership claims in the i^{th} firm, and the individual initially owns β_i^j of the i^{th} firm, the value of his initial ownership claims in the i^{th} firm is just $\beta_i^j(V_i - I_i)$.]

[18] This formulation assumes that the level of utility attained from a terminal wealth of Y^j depends only on the level of Y^j, not on the state of nature. This would not be the case if individuals traded this commodity internationally for other commodities which were to be consumed, and the international price ratios depended on θ, or if the level of utility attainable from any given income depended on such events as the weather, whether there is a war, etc. Although the results on the constrained Pareto Optimality [8] of the competitive equilibrium do not depend on this assumption, those results relating to the structure of taxes do. The reason for this should be clear: If U depends on θ (except through $Y(\theta)$), a security may (from a utility point of view) be "safer" than a bond (e.g. it is often asserted that because of inflation, in real purchasing power a security is "safer" than a bond); thus the demand function for the security has the properties usually associated with the safe asset and conversely for the bond. Since, as will be apparent shortly, the structure of optimal taxes depends on properties of these demand functions, the conclusions of Section 4 on which assets should be taxed at a higher rate then have to be reversed.

[19] Equivalently [using (2.8)], we have

$$\frac{EU'X_i}{rEU'} = V_i \qquad (2.15)$$

[20]If there are two or more firms which have perfectly correlated returns, i.e. belong to the same risk class, then there is an inessential indeterminacy in the demand for the securities of a given firm. But as we have noted above, we can aggregate all firms of the same risk class together.

[21]From now on, we shall, when there is no ambiguity, drop the superscript j.

[22] $-L^{j'}$ is the disutility of supplying an extra unit of the factor; throughout we assume an additive utility function.

[23]If there is not constant returns to scale, it will also depend on the vector of initial ownership claims.

[24]In the case of non-constant returns to scale, from (2.6) we derive the demand for factors and the supply of securities by the i^{th} firm:

$$1 = p_i\left(\frac{d \ln f_i}{d \ln I_i}\right) \tag{2.26}$$

which may be solved for I_i as a function of p_i:

$$I_i = I_i(p_i) \tag{2.27}$$

Market equilibrium requires again $I_i = Z_i$. A general equilibrium is a set of prices, $p_1, \ldots, p_n$, which generates demands for securities [given by (2.22)] equal to supply [given by (2.26)] and demands for factors equal to supply [given by (2.21)]. The usual fixed point arguments can be used to ensure the existence of at least one such price vector.

[25]With the exception of those relating to two or more risky industries.

[26]Throughout this and the remaining sections of the paper (except Sections 5 and 7) we will conduct the analysis as if all individuals were identical (the "representative man").

[27]As we have drawn the tangency in Figure 1, it lies between S and R. It is not possible, of course, for it to lie to the right of R; since society as a whole clearly cannot "borrow" (sell the safe security short) in that case the economy simply specializes in the risky investment.

[28] Elsewhere, it has been established that whether the income elasticity of the safe asset is greater or less than unity depends simply on whether the utility function, $U(Y)$, from which $M(\mu,\sigma)$ is derived, has increasing, or decreasing relative risk aversion. See [2].

[29] The proofs of these propositions are straightforward. In the general case of an individual maximizing expected utility, we have

$$EU' \cdot (e-r) = 0$$

The sign of the change in the proportion of resources invested in the safe asset when r is increased is found by implicit differentiation with respect to r:

$$-EU' + EU''(1-a)(e-r)I = EU'(-1 - U''Y/U') + EU''eI$$

The rest of the analysis is straightforward. (Cf. [19])

[30] These results are essentially unaffected by presence of non-constant returns to scale. In Figure 6, we have drawn the production possibilities schedule for the economy in which there is decreasing returns to scale in the safe industry. The competitive equilibrium can be shown, as in the usual analysis, to be at the point E. The after production (gross output) tax curve, when the same proportionate tax rate is imposed in both industries is depicted in the figure by the dotted curve. As before, at any allocation of real resources between the two investments, the mean and standard deviation are both reduced proportionately, so that the after-tax return corresponding to E is A. Whether the proportional tax is allocatively neutral, increases or decreases risk taking depends solely on whether the income consumption curve is a straight line through the origin (elasticity of demand for the safe asset of unity)bends upwards (greater than unity elasticity of demand for the safe asset) or bends downward.

[31] More generally, sufficient conditions that an increase with tax increase risk taking is that there be (a) increasing absolute risk aversion or (b) decreasing absolute risk aversion and increasing or constant relative risk aversion or (c) $r-1 = 0$.

[32] Note that this implies that if $r = 1$, the demand curves for risky securities is upward sloping.

[33] Risk neutrality is equivalent to zero relative risk aversion (a utility function which is linear in income).

[34]This criterion is not meant to yield a complete ordering of tax structures, but allocative neutrality is alleged to be a "desirable characteristic" in any tax system.

[35]Although this may turn out to be the case; we are suggesting that allocative neutrality is a property which one ought to prove is desirable for a tax system, rather than assume ab initio.

[36]The analysis for profits (income taxes) is similar and hence is omitted.

[37]For a discussion of the difficulties in extending the general model to two or more risky assets, see below, Section 6.

[38]In terms of our previous notation,

$$\hat{\lambda} = \lambda_1/\lambda_1 + \lambda_2$$

where λ_i is the proportion of total assets allocated to the i^{th} risky industry.

[39]We have already noted that competitive equilibrium requires that the level of investment in each industry be equal to the demand for its securities when the ratio of the value of the firm to its level of investment is unity for all firms.

[40]In the special case where $\mu_1 = \mu_2$, $\hat{\lambda}$ will be chosen simply to minimize variance:

$$\hat{\lambda}\sigma_1^2 + (1 - 2\hat{\lambda})C - (1-\hat{\lambda})\sigma_2^2 = 0$$

Then $\hat{\lambda}$ does not depend at all on r ; hence the tax rate on the safe industry cannot affect the relative allocation among risky industries.

[41]In practice, governments do levy some taxes that appear to be very close to lump sum or non-distortionary taxes, e.g. poll taxes and land taxes, but such taxes are relatively insignificant and clearly do not raise enough revenue; the government must then resort to distortionary taxes. Even taxes like land taxes are often distortionary; e.g. the value of land on which the tax is likely to be based will in general depend on improvements.

[42]We could have alternatively formulated the problem as if the tax were imposed at the level of the firm rather than at the level of the individual i.e. on the firms investment rather than on the individual's purchase of

securities of the firm. In this particular model, the two are perfectly equivalent, that is, if a tax is imposed on the investment of the safe industry at the rate $\tilde{t}_S$, the value of the industry will be (in competitive equilibrium) $V_S = I_S(1 + \tilde{t}_S)$; similarly, $V_R = I_R(1 + \tilde{t}_R)$. Then the after tax returns per dollar invested in the safe and risky industry will be given by

$$r' = \frac{r}{1 + \tilde{t}_S}$$

and

$$e' = \frac{g(\theta)}{1 + \tilde{t}_R}$$

It is clear that if $\tilde{t}_i = t_i$ these are identical to the expressions derived above for after-tax returns.

[43]Because of constant returns to scale, unless the government makes transfer payments, $w_0 = 0$.

[44]The formal solution to this problem is identical to that presented in the more general context of optimal commodity taxation, i.e., for instance [9, 25, 4, or 16].

[45]Alternatively, direct substitution of (B.5) into (4.6) yields

$$\frac{t_R}{Z_R} L'[EU''r^2 + L''p_S^2] - \frac{t_S}{Z_R} L'[EU''er + L''p_R p_S]$$

$$= -\frac{t_R}{Z_S} L'[EU''er + L''p_R p_S] + \frac{t_S}{Z_S} L'[EU''e^2 + L''p_R^2]$$

Multiplying by $Z_S Z_R / L'$ we obtain

$$EU''Y(rt_R - et_S) + (t_R p_S - t_S p_R)L''I = 0$$

or

$$t_S EU''Y(r-e) + rEU''Y(t_R - t_S) + (t_R - t_S)L''I = 0$$

[46] This latter result can be shown to be a general property following from the additive structure of the utility functions.

[47] These are like pure windfall gains; in our model, there is no difficulty in identifying these pure windfall gains from gains that were anticipated (with a finite probability),
a tax on which would have distortionary effects. In a multiperiod model, however, the ownership claims at the beginning of one period are largely the proceeds from investments in previous periods. All of this suggests that extreme caution be used in applying the results derived from the simple one period models.

[48] Since uncertainty plays no special role in this result, the proof is omitted; the reader is referred to [25] for a detailed discussion of these problems.

[49] Cass and I have explored these questions in much greater detail elsewhere, see [6].

[50] In view of the results in the appendix to the preceding section, this result should not be too surprising.

[51] Clearly, by introducing progressive tax rates, we can increase the progressivity of the tax structure even if there is a unitary elasticity to demand. But the high marginal tax rates may result in a loss of "efficiency," the objective of tax policy with which this paper has until now been concerned.

[52] In this general discussion, C_i is the consumption of the i^{th} commodity or (minus) the supply of the i^{th} factor.

[53] Again, it is important to note the trade-offs: If there is a fair degree of homogeneity in consumption patterns among the members of a given population, we may be willing to sacrifice the loss in horizontial equity for the gain in efficiency resulting from the introduction of differential tax rates on different kinds of expenditure (investment in different kinds of securities).

[54] More formally, the percentage reduction in demand (along the compensated demand schedules) for small taxes is

$$\sum_{k=1}^{2} \left(t_S \left(\frac{\partial Z_i^k}{\partial P_S} \right)_{\bar{u}^k} + t_R \left(\frac{\partial Z_i^k}{\partial P_R} \right)_{\bar{u}^k} \right) \qquad i = S,\ R$$

(which is identical to the right hand side of (5.2) and (5.3)); $-v_S \gtrless -v_R$ as $\gamma_S \gtrless \gamma_R$ provided

$$\frac{\ell}{\nu}\Psi^1_{w_0} - \frac{(1-\ell)}{\nu}\Psi^2_{w_0} + t_S\left(\frac{\partial z^1_S}{\partial w_0} - \frac{\partial z^2_S}{\partial w_0}\right) + t_R\left(\frac{\partial z^1_R}{\partial w_0} - \frac{\partial z^2_R}{\partial w_0}\right) < 0$$

The first term represents the marginal social evaluation of income to the two different groups; one would normally expect

$$\ell\Psi^1_{w_0} << (1-\ell)\Psi^2_{w_0}$$

i.e. that the marginal dollar given to the poor raises social welfare considerably more than one given to the rich. The second two terms represent the tax gain that would result from giving an extra dollar to the two different groups. If the marginal spending propensities were the same, these terms would be the same for the two groups. Alternatively, if the rich took a larger percentage of the increased income in the form of leisure and had a marginal propensity to consume (for each unit of expenditure) on the more heavily taxed security (commodity), that was less than that on the less heavily taxed security (commodity), then

$$t_S\left(\frac{\partial z^1_S}{\partial w_0} - \frac{\partial z^2_S}{\partial w_0}\right) + t_R\left(\frac{\partial z^1_R}{\partial w_0} - \frac{\partial z^2_R}{\partial w_0}\right) < 0$$

It is, of course, possible that the above expression be positive. Although it seems very likely that even when it is positive, the differences in marginal social evaluations $(\ell\Psi^1_{w_0} - (1-\ell)\Psi^2_{w_0})/\nu$ are much greater than the differences in marginal tax revenues, it is not possible on a priori grounds to rule out the possibility that just the opposite is true. In that case, the percentage reduction in the commodity (security) which is in relative demand by the rich is smaller than that of the other commodity (security). In the ensueing discussion we shall, however, ignore that possibility.

[55]The derivative of (5.6) with respect to $v \equiv v_S/v_R$ is

$$\frac{\eta_{RR}(\eta_{SS} - v\eta_{RS}) + (v\eta_{RR} - \eta_{SR})\eta_{RS}}{(\eta_{SS} - v\eta_{RS})^2}$$

which is of the sign of

$$\eta_{RR}\eta_{SS} - \eta_{SR}\eta_{RS}$$

[56] Assuming, as we have argued will normally be the case, that

$$\frac{\ell}{\nu}\Psi^1_{w_0} - \frac{(1-\ell)}{\nu}\Psi^2_{w_0} + t_S\left(\frac{dZ^1_S}{dw_0} - \frac{dZ^2_S}{dw_0}\right) + t_R\left(\frac{dZ^1_R}{dw_0} - \frac{dZ^2_R}{dw_0}\right) > 0$$

(See Ftn. 54, above.)

[57] Using the results of Appendix B of Section 4, we obtain

$$\frac{\eta_{RS}}{\eta_{RR}} - 1 = \frac{EU''erp_S - EU''r^2p_R}{EU''r^2p_S + L''p_Rp_S^2} \approx E(-U'')\left(\frac{e}{p_R} - \frac{r}{p_S}\right)$$

$$= E\ \frac{-U''}{U'}\ U'\left(\frac{e}{p_R} - \frac{r}{p_S}\right) \gtreqless 0 \quad \text{as} \quad \frac{d(-U''/U')}{dW} \gtreqless 0$$

(Cf. [19].)

[57] The empirical evidence is ambiguous. The figures cited in [1] do suggest that the expenditure elasticity for risky assets is between zero and unity, but studies such as those cited by [19] indicate that the expenditure elasticity is greater than unity. In that case, as we have already argued, $-v_R > -v_S$. In the case of $v_S/v_R = 0$ by exactly parallel arguments,

$$\frac{t_S/p_S}{t_R/p_R} = -\frac{\eta_{SR}}{\eta_{SS}}$$

The right hand side of the above equation can be shown to be less than unity if there is increasing absolute risk aversion (negative expenditure elasticity of demand for risky assets) or decreasing absolute and increasing relative risky aversion (expenditure elasticity between zero and unity). With decreasing relative risk aversion (which we would expect under the given assumption that $\gamma_R > \gamma_S$, since it implies that the expenditure elasticity on risky assets is greater than unity) t_S may be either greater or less than t_R.

[59] Not only may v_R/v_S take on values between zero and one, but it may actually become negative. This implies that t_R/t_S will be even smaller than indicated above, and indeed may become negative, i.e. the risky industries are subsidized and the safe taxed. To see this, consider the case where the revenue

to be raised by the government is small. That implies that either t_S and t_R are of opposite sign or they are both small. The latter case implies that v_S and v_R are both small. But we may write

$$v_R = \frac{(1 - \gamma_R)(1 + v_S)}{1 - \gamma_S} - 1 = \frac{(\gamma_S - \gamma_R) + (1 - \gamma_R)v_S}{1 - \gamma_S}$$

But as v_S goes to zero, v_R does not (and conversely). Hence t_S and t_R must be of opposite sign, i.e. the industry which is in relative demand by the poor is subsidized.

[60]The implications of this point for the term structure of interests rates are developed in [22].

[61]For an extensive discussion of this see [20].

[62]The elasticity of substitution gives the percentage change in relative consumption of the two commodities from a percentage change in relative prices:
$$- \frac{d \ln X_1/X_2}{d \ln p_1/p_2} = \frac{1}{1-A} \gtrless 1 \quad \text{as} \quad A \gtrless 0 .$$

[63]In the sense that increases in wealth result in increased proportion of wealth allocated to the industry with technological uncertainty when there is increasing risk aversion and a decreased proportion when there is decreasing risk aversion.

[64]In the two period model used throughout this paper, all profits must be distributed at the end of the period either to bond holders or shareholders. The tax rate paid by shareholders depends, of course, on the form in which these returns are paid, i.e. as capital gains or dividends. The results of the two period model may easily be extended to the more general case.

[65]Certain non-financial considerations are, for some firms at least, probably fairly significant. The larger the debt equity ratio, the less financial resources are required for a take-over bid, etc. These complications make clear cut empirical results unlikely. For a more extensive discussion of these points, see J.E. Stiglitz, "Some Aspects of the Pure Theory of Corporate Finance," [25]. For a discussion of the effects of taxation on financial policy explicitly taking into account the effects of capital gains taxation, see also D.E. Farrar and L.L. Selwyn, "Taxes, Corporate Policy and Return to Investors," National Tax Journal, December, 1967, pp. 444-454.