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AT YALE UNIVERSITY

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COWLES FOUNDATION DISCUSSION PAPER NO. 296

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RECENT DEVELOPMENTS IN PRICE DYNAMICS

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August 13, 1970

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by

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I. Introduction

The present paper surveys recent developments in price dynamics.¹ The intention is to sort those theories which show promise for understanding the actual movement of prices and for leading to the formulation of models suitable for econometric testing and forecasting.

A casual glance at the empirical literature on price and wage movements indicates that much work remains for econometricians. Existing models have made serious prediction errors in recent years. Moreover, for the most part, these models lack a solid grounding in theoretical reasoning. The basic theoretical problem, and one which has not been studied to a significant extent, is that price theories are traditionally equilibrium theories. Macroeconomic inflation theory, on the other hand, is usually thought to be a disequilibrium phenomenon.² As we will note below, some

*This research was supported by the National Science Foundation, the National Bureau of Economic Research, and the Federal Reserve Board. The views are, however, the responsibility of the author.

¹As used in this paper, "prices" include factor prices as well as product prices, although the stress is on the latter.

²The notions of equilibrium and disequilibrium are used as in the survey by Hahn and Matthews [1964]. Price theory is equilibrium in the sense that it discusses prices toward which a stable system without disturbances will tend as time passes. Macroeconomic inflation theory is disequilibrium in that it usually assumes price (or inflation) expectations and certain other variables are fixed.

first steps have been made in the linking up of macroeconomic inflation theory to microeconomic equilibrium theory; much remains to be done.

Section II below discusses classical static and dynamic theories. Section III then turns to the "new" microeconomics of inflation, with special emphasis on the role of uncertainty in price dynamics. Section IV discusses some recent econometric theories of inflation and attempts to give them a firm theoretical footing. In Section V, some problems in macroeconomic theories of inflation are discussed. Finally, we survey recent econometric studies of inflation in Section VI and apply the theoretical remarks to the specifications.

II. Microeconomic Foundations: Classical Analysis

In the present section we will review the classical microeconomic foundations of price dynamics. The objective is to ascertain what of this literature, if anything, is relevant for econometric studies of price and wage dynamics.

A. Classical Statics

There is a long and rich literature on price theory in market economies. Economic science has developed an elegant and closely reasoned theory of the determination of equilibrium relative prices in competitive markets, as well as some discussion of the kinds of distortions that would be found in markets with certain kinds of imperfections.

The basic proposition in classical statics is that the real magnitudes of the system are unaffected by a change in the absolute price level which

leaves relative prices unchanged.¹ This proposition is easily shown for competitive market structures, but some reflection indicates that it must hold for any structure with a unique equilibrium.² The basic proposition has the corollary that in a full equilibrium, inflation of the nominal price level (with relative prices unchanged) does not affect any real magnitudes. As recent developments make clear, however, this condition is quite restrictive for it assumes that real interest rates (including the real interest rate on money) must also be unchanged by inflation. It is important to note that there is an underlying assumption that money prices do not affect tastes or technology. It is clear that in any realistic inflation these conditions will not be met.

Classical price theory does not give much attention to the determination of the absolute price level. The main reason for this lack of attention is that absolute prices are, for the most part, irrelevant to the real magnitudes of the system. When something has to be said about the level of absolute prices, often a crude quantity theory of prices is invoked, in which the level of absolute prices is determined by "the money supply."

B. Classical Dynamics: Tatonnement

The most important system of price dynamics discussed in classical

¹This proposition should be strictly interpreted. First, all prices must change proportionally, including factor prices and the price of money. Second, expectations about future price movements must be unaffected. Third, the distribution of income and wealth must also be unaffected in the transition. It should be noted that this proposition is not sufficient for the "monetary dichotomy" often discussed (see Modigliani [1962]).

²This is also shown formally in Bailey [1962], Chapter V.

price theory was the tatonnement process. Although other mechanisms are discussed, only the tatonnement has survived.^{1,2} The earliest attempt to formalize an adjustment mechanism was Walras' tatonnement hypothesis. This hypothesis is simply that the price of a good rises (or falls) when its excess demand is positive (or negative). This hypothesis turned out to be extremely fruitful for analytical results. It is not clear whether this hypothesis was meant to be a realistic description of the actual method used by markets to equilibrate.³ Walras' description of the adjustment process is quite interesting: He argues that the "practical solution" is an extremely efficient algorithm for finding the "theoretical and mathematical solution."⁴

Walras' hypothesis has been widely criticized for its unreality. In his review of Walras' Elements, Edgeworth argued that "Walras' laboured lessons indicate a way, not the way of descent to equilibrium."⁵ Patinkin,

¹See Samuelson [1947], pp. 263 ff. for a review of some dynamic mechanisms which had been discussed in early writers.

²It is puzzling that the problem of price dynamics received virtually no treatment even in modern microeconomics. An examination of 9 popular books on price theory indicate 6 have no discussion of inflation, price dynamics, and disequilibrium price adjustment (Mansfield [1970], Baumol [1965], Leftwich [1966], Bain [1967], Malanos [1962], Ferguson [1969]). 3 have a few formal notes on "tatonnement" or cobweb theorems" (Cohen and Cyert [1965], Lancaster [1969], and Stigler [1966]). Apparently the leading writers feel that economic science has nothing to contribute to price dynamics.

³Goodwin argues that "Walras disavowed [tatonnement] as a practical device." (Goodwin [1953], p. 59). Patinkin disagrees and holds that Walras describes the actual tatonnement used by the market. (Patinkin [1965], pp. 531-532). A third interpretation is that the problem that the theoretical and practical adjustment mechanisms solve is the same, but the method of solution is different.

⁴Walras [1881], p. 106.

⁵See Edgeworth [1889].

on the other hand, criticizes Edgeworth and claims that the tatonnement "is the way which represents the workings of economic phenomena."¹ Patinkin's argument is certainly a major puzzle, for the theoretical tatonnement can hardly be considered a realistic description of the way markets work; nor is it the only mechanism which is stable, as modern work on non-tatonnement stability theory indicated.²

The appeal of the tatonnement for modern economists hypothesis is probably the strength of the stability results which can be proved. This is clearly a weak foundation. First, it presumes a competitive market and casual observation indicates that this obtains for only a small part of a modern economy. Second, even in competitive markets, there is no good evidence that tatonnement is a realistic description. There are certainly other mechanisms which are possible, and the other proposed adjustment mechanisms have just as good a claim to realism as the tatonnement.³

¹Patinkin [1965], p. 538.

²See Negishi [1962] and Hahn [1970]. In a recent discussion of "The Computer and the Market" (Lange [1967]), Lange stresses that the "market process with its cumbersome tatonnements appears old-fashioned. Indeed, it may be considered as a computing device of the pre-electronic age." According to Lange, the tatonnement is responsible for "unpleasant fluctuations" (indeed, it may be unstable); it operates slowly; and it cannot make proper decisions about growth and development problems because of the lack of accurate information on futures pricing. He concludes that replacing the tatonnement with mathematical programming aided by the electronic computer is indispensable for optimal long-term planning.

It is interesting to compare Lange's view of the efficiency of the tatonnement with that of Walras just cited in text.

³See Section III below.

A particularly useful exposition of the tatonnement price-adjustment mechanism was given by Arrow [1960].¹ His treatment extends the static stability analysis to markets with increasing demands. Arrow considers a competitive economy with n goods. Excess demand functions (the difference between supply and demand) are linearized to give:²

$$(1) \quad X = Mp + n$$

where X , p , and n are $(n \times 1)$ vectors of excess demands, prices and constants and M is a $(n \times n)$ matrix of coefficients. Assuming a continuous adjustment process, we have

$$(2) \quad \dot{p} = KX$$

¹Also see Negishi [1962] for an excellent summary of the problem of stability.

²Equation (1) can be derived as follows. Assume that households have demand functions $Y^d = D(p)$ and firms have supply functions $Y^s = S(p)$, Y^d , Y^s , and p being output demanded, output supplied, and prices. Excess demand is defined as $X = Y^d - Y^s = D(p) - S(p)$. We assume an equilibrium exists, i.e. that there is a set of prices, \bar{p} , such that $D(\bar{p}) = S(\bar{p})$. Taking the first order Taylor expansion around $p = \bar{p}$ yields

$$\begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix} = \begin{pmatrix} D_1(\bar{p}) - S_1(\bar{p}) \\ \vdots \\ D_n(\bar{p}) - S_n(\bar{p}) \end{pmatrix} + \begin{pmatrix} D_1^1 - S_1^1 & D_2^1 - S_2^1 \dots D_n^1 - S_n^1 \\ \vdots \\ D_1^n - S_1^n & D_2^n - S_2^n \dots D_n^n - S_n^n \end{pmatrix} \begin{pmatrix} p_1 - \bar{p}_1 \\ \vdots \\ p_n - \bar{p}_n \end{pmatrix} + 0^2(p - \bar{p})$$

$$= (D(\bar{p}) - S(\bar{p})) + [D'(\bar{p}) - S'(\bar{p})](p - \bar{p}) + 0^2(p - \bar{p})$$

where D_j^i (or S_j^i) is the derivative of the i^{th} demand (supply) function with respect to the j^{th} price evaluated when excess demands are zero and $0^2(p)$ is a second-order term in prices. Recall $D(\bar{p}) - S(\bar{p}) = 0$, so the first order term is $X = [D'(\bar{p}) - S'(\bar{p})]p - [D'(\bar{p}) - S'(\bar{p})]\bar{p}$. Thus the matrix M in the text is $[D'(\bar{p}) - S'(\bar{p})]$ and the matrix n is $-[D'(\bar{p}) - S'(\bar{p})]\bar{p}$.

where \dot{p} is a $(n \times 1)$ vector of time derivatives, dp_i/dt , and K is an $(n \times n)$ positive, diagonal matrix of adjustment coefficients. Combining (1) and (2) we have

$$(3) \quad \dot{p} = KMp + Kn$$

Under some strict conditions, (3) will be stable.¹ The solution is $\dot{p} = 0$, $X = 0$, and therefore $p = -M^{-1}n$.

Thus the first proposition is the following: a tatonnement system is stable if goods are gross substitutes. Arrow next introduces the problem of adjustment when some demand or supply functions are constantly changing over time. If a demand function is moving out, we can rewrite our excess demand function as follows:

$$(4) \quad X = Mp + n + et$$

where e is a $(n \times 1)$ vector of coefficients and t is time. But now we find a surprising result: with constantly changing demands, there will be shortages or surpluses which will not vanish. To see this, simply differentiate (4) with respect to time and substitute for \dot{p} from (2), obtaining:

$$(5) \quad \dot{X} = MKX + e$$

Arrow shows that \dot{X} tends to 0, so excess demands do not vanish but rather tend toward:

¹The usual set of sufficient conditions is to assume all goods are gross substitutes. See Negishi [1962]. This implies that excess demand for good j rises if the price of good i ($i \neq j$) falls. In terms of the present model, this would imply that the matrix $(I-M)$ is positive. As is well known, this condition is sufficient for stability of the system. The problem is slightly more complicated than the usual analysis because of the inclusion of supply in the present model.

$$(6) \quad \lim_{t \rightarrow \infty} X(t) = -K^{-1} M^{-1} e$$

Moreover, by substituting (6) into (2) we see that

$$(7) \quad \lim_{t \rightarrow \infty} \dot{p}(t) = -M^{-1} e$$

Since $-M^{-1}$ is nonnegative, we will have rising prices if all demands are rising.

Our final proposition is thus that prices will be changing on all markets, that this change will tend toward some constant rate of change, and that the final rate depends only on conditions of supply and demand, but not on reaction speeds. Arrow has also shown that the propositions stated above also hold when some restricted forms of price expectations are added to the model.¹

The importance of the Arrow results seems to be generally overlooked by much of the recent discussion of price dynamics, especially in the Phillips curve literature. In a model with secular change in the economic variables, persistent inflation or deflation may be consistent with long-run equilibrium. In fact, if the demands are generally rising, inflation is the lubricant necessary for assuring that the demands are met by increasing, if not completely adequate supplies. Without generally rising prices, producers have no signals to read to show them that certain sectors will need expanding production in the future.

¹Let p^* be expected prices, and let price expectations be determined by

$$\dot{p}^* = K^*(p - p^*) + n^0.$$

In this model expected prices rise, asymptotically, at the same rate as actual prices, but future prices do not necessarily equal present prices times the rate of change of prices. If this latter condition is imposed, the system becomes $[M + M^1(1+\dot{p})]\dot{p} + e = 0$, which does not have an easy solution.

A particularly simple numerical example is where all goods have own-elasticities of -1 and zero cross-elasticities. Interpret each variable as the logarithm of the original variable. Asymptotically, the price of each good is rising at the rate of increase of demand for that good. In an economy growing at four percent, the asymptotic rate of inflation is four percent.

One serious shortcoming in this analysis is that expectations of inflation do not explicitly enter the price dynamics (see equation (2) above). At this point we have not investigated dynamics sufficiently to say whether or not this is justified.

C. The Phillips Curve Controversy Reviewed: Is the Economic System Neutral to Inflation?

Classical price dynamics presents certain suggestive models for further exploration. Its main weakness is inability to provide a normative or profit-maximizing basis for the tatonnement, or other hypotheses. This shortcoming has recently come to the fore because of certain controversies surrounding the trade-off between inflation and unemployment. The controversy, briefly stated, is whether there is a long-run trade off between resource unemployment and inflation. In terms of the Arrow model of tatonnement presented above, the debate turns around the form of the price dynamics function contained in equation (2):

$$(2) \quad \dot{p} = KX$$

Some economists, notably Phelps and Friedman,¹ have argued that the function

¹Friedman [1968], Phelps [1970b].

should have a term incorporating the expected rate of change of prices:

$$(2F) \quad \dot{p} = KX + \lambda \dot{p}^e$$

λ a $(n \times n)$ matrix. According to this school of thought (call it "accelerationist"), two conditions are met: (1) $\dot{p}^e \rightarrow \dot{p}$ as $t \rightarrow \infty$ and (2) $\lambda = I$. The first condition is quite straightforward: it simply assumes that expectations are, in the long-run, correct. The second condition is more difficult to interpret. It says that the price-adjustment mechanism responds fully to fully anticipated inflation. Both of these assumptions are part of the accelerationist hypothesis.¹ Using the system of the previous section, assuming prices are correctly anticipated (so $\dot{p}^e \rightarrow \dot{p}$) and recalling that $\dot{X} = MKX + e$, we obtain $X = -K^{-1}M^{-1}e$. Putting this in (2F) yields:

$$(8) \quad \dot{p} = -(I - \lambda)^{-1}M^{-1}e$$

Clearly as $\lambda \rightarrow I$, $\dot{p} \rightarrow \infty$; and since $\dot{X} = M\dot{p} + e$, $X \rightarrow 0$ as $\dot{p} \rightarrow \infty$. If $\lambda = I$, $KX = 0$, which implies that $X = 0$. As long as $(I - \lambda)$ is nonsingular, (8) will hold.

We can thus summarize the argument conveniently with the present model. When price dynamics can be summarized by a Walrasian market-clearing equation, the question of whether there is a natural rate of unemployment, corresponding to clearing of all markets and no excess demands in any market in the long-run

¹Phelps ([1970b] and earlier works) recognizes the necessity of both conditions for the accelerationist position. Friedman [1968], on the other hand, appears to assume the first condition ($\dot{p}^e \rightarrow \dot{p}$) is sufficient. For a fuller discussion and a counterexample to Friedman's view, see Nordhaus [1970].

($X = 0$ in the system here), revolves around the price-clearing equations. If the adjustments in each market respond completely to the expected rate of change of prices (and $[I - \lambda]$ is thus singular), there will be only one level of output consistent with bounded rates of inflation, that where there are no excess demands or supplies.¹

Thus the question of whether there is a "natural rate of unemployment" (or whether it is "inflation neutral") revolves around whether the λ matrix in equation (2F) is such that $(I - \lambda)$ is singular. There are four points to be made about this. First, it is not a sufficient condition for inflation neutrality that price- or inflation-expectations be correct.² As is easily seen in the system above, expectations can be perfect, yet if the adjustment mechanism is not singular the system can have excess demands or supplies. Second, it is often supposed that complete adjustment in the labor market (i.e. the labor market's $\lambda = 1$) is sufficient for inflation neutrality. This is not generally true, but rather assumes that the rest of the economy is also inflation-neutral. Third, the theoretical argument that markets should display inflation-neutrality rests mainly on the proposition that rational individuals should display inflation neutrality. This proposition does not generally follow, for markets do not generally follow any collective rationality. We simply do not know enough about market behavior to know how they behave with respect to fully anticipated inflation.³ Fourth, there has by now accumulated a great deal of empirical evidence about the actual response of markets, especially labor markets, in periods of rising prices. The evidence is overwhelmingly against the hypothesis that markets are inflation-neutral.⁴

¹Incidentally, there will be an equilibrium for the model where $X = 0$, or $\dot{p} = -M^{-1}(n + et)$, which implies that $\dot{p} = -M^{-1}e$. In this case prices adjust immediately to wipe out excess demands.

²See Footnote 1, p. 10.

³While the argument cannot be generally made that markets are inflation-neutral, the analysis of the "new microeconomics" discussed in the next section does lead to that conclusion. The reason for this is that prices, and price adjustments, are set by individual firms who are, by assumption, inflation-neutral.

⁴Much of the evidence is reviewed by Bodkin et al. [1966]. See also Gordon [1970] for an explicit test of the accelerationist position. To my knowledge the only writer who finds evidence of inflation neutrality is Cagan [1968b] in a National Bureau research note, but this does not seem to have been followed up. Some evidence is reviewed in Section VI below.

III. The "New" Microeconomics of Inflation

In the last couple of years a new wave of theoretical analysis of the microeconomic foundations of inflation theory has appeared. One of the leading writers in this new wave, E.S. Phelps, has given the following explanation for the outpouring:¹

It is notorious that the conventional neoclassical theory of the supply decisions of the household and of the firm are inconsistent with Keynesian employment models and with the post-Keynesian economics of inflation....It seems clear that macroeconomics needs a microeconomic foundation.

The stated purpose of the analysis can hardly be faulted.

The new microeconomics is based on two fundamental assumptions:

(1) Economic agents are rational and--within the economic constraints of their environments--households behave as to maximize "lifetime expected utility" and firms "doggedly" maximize "net worth."² (2) The most important environmental constraint considered is "that collating information about potential exchange opportunities is costly and can be performed in various ways."³ Put differently, the "new" theory accepts the neoclassical analysis except the assumption of perfect and costless information structure. In one way or another, almost all recent developments put these two assumptions to work on the age-old problems of economic analysis: What are optimal prices, wages, employment, output, etc.? Taken together, these contributions clear up many of the inconsistencies in current macroeconomic thinking.

We will first give a brief description of the effect of uncertainty on decisions, then turn to the implications of this for the microeconomics of inflation.

¹Phelps [1969], p. 147. The "non-Walrasian" economics examined here are really "non-general equilibrium," for they do not always deny the kinds of "Walrasian" adjustment mechanism discussed in Section II above.

²These quotations are from Phelps [1970], p. 3. Also see Phelps [1969].

³Alchian [1970], p. 28.

A. Uncertainty and Prices

The Walrasian paradigm which is usually the starting point for most microeconomic reasoning assumes that products are homogeneous and that all economic agents have full information about both the quality and the price of relevant goods. Further work of Arrow and Debreu¹ has extended this analysis to markets characterized by uncertainty. It is extremely important to note, however, that in the Arrow-Debreu world there are (as in the Walrasian paradigm) no costs of obtaining information about characteristics of goods or about states of the world. As Stigler pointed out several years ago² these assumptions are crucial for the validity of the general equilibrium model. Under conditions of uncertainty, where markets are imperfect, reliable information is costly and the economic agent has quite a different task. It is now possible that he will spend a good deal of his time engaged searching for goods that suit his tastes, determining the cost and characteristics of products, and hedging himself against risks which he cannot market.

It is instructive to consider "information" as an additional commodity.³ Especially where goods are heterogeneous, indivisible, infrequently purchased, or immovable, it costs real resources to gather information about the different dimensions of a good. This good is an intermediate good since it does not yield satisfaction directly. There are several consequences of this added complication. Often, workers will decide to spend time searching for a better job ("unemployment"), goods will wait for potential customers (they are "inventories"), and in certain cases specialists in information ("brokers") will help facilitate transactions.

Some progress has been made recently on the problem of information. The simplest kind of model for integrating uncertainty into price theory starts with buyers (or sellers) uncertain about the price, p , of a commodity,

¹Arrow [1964] and Debreu [1959].

²Stigler [1961].

³This is suggested by Alchian [1970]. He suggested that this inclusion re-introduces Say's law, but this reasoning is unclear.

x . They take a sample from a group of n sellers, and get offers (p_1, \dots, p_n) . If the cost of an offer is c , then they will continue to sample until the expected decrease in price on the next offer is less than c . For example, if prices are normally distributed with mean, μ , and variance, σ^2 , then the minimum offer, $p(n)$, at the n^{th} observation is approximately¹

$$p(n) = \mu - \sigma/\sqrt{2 \log n}$$

If the seller is looking for one item, then the minimum net cost, $p(n) + cn$, comes at approximately²

$$(1) \quad n = \sqrt{2 \sigma/c}$$

Clearly, the amount of optimal search depends directly on the dispersion of prices reigning in the market and inversely with the cost of search. As the numerical examples in footnote 2 of this page indicate, a considerable amount of search may be consistent with the uncertainty presently found in markets. A number of important phenomena can be explained by this simple search model. First, it is clear that under the usual conditions markets need not be perfect in the sense that there is perfect information and all prices are the same. If the costs of information gathering are significant, a certain amount of product differentiation may be a long-run phenomenon.³ Secondly, one of the interpretations of the phenomenon of persistent unemployment in market economies is that this is simply a result of workers shopping around for better wages, or of employers refusing to lower wages in periods of declining demand because of the probability that they will lose workers and be unable to rehire them in more affluent periods without substantial

¹Gumbel shows that the modal largest value for a sample of n from a normal variable $(N(0, 1))$ is approximately $\sqrt{2 \ln(0.4n)} \approx \sqrt{-1+2 \ln n}$. (See Gumbel [1958], section 4.23, pp. 136-140.) The approximation in the text is found in Alchian [1970].

²Net price is $p^* = \mu - \sigma/\sqrt{2 \log n} + cn$, so $\frac{dp^*}{dn} = -\sigma/\sqrt{2} (\log n)^{-1/2} n^{-1} + c = 0$. Thus the sample which minimizes cost is:

$$n^2 \log n = 2\sigma^2/c^2$$

For n around 2 this is approximately the number in the text. Just to get a rough check, the optimal size of sample for different items (assuming \$5 per sample) is something like: washing machines: 2; automobiles: 12; houses: 80. The coefficients of variation are from Stigler [1966], p. 4 for washing machines and automobiles. The figure for houses is from unpublished regressions of Peter Mieszkowski.

³See Stigler [1961] and Phelps and Winter [1970].

search costs.¹ Third, since price information is a scarce resource (in that it is costly to obtain and to store it) there will be real costs to rapid changes in prices since this makes past price quotations obsolete and, therefore worthless.² Fourth, in economies which are in a constant state of flux (with movement in and out of the labor force, with fluctuations in aggregate demand, and with firms taking these parameters into account when they make their decisions) there will generally be unemployment of all factors and outputs for a significant amount of time.³

Before we turn to the significance of these findings for price dynamics, let us note some of the unexplored or unexplained problems in these theories. The most important shortcoming is that, without exception, the theories have not succeeded in marrying supply and demand. Glancing back at equation (1), p. 14, above, we see that it gives optimal behavior when dispersion of prices and costs of search are given. But search costs, and certainly the amount of price dispersion, are crucial endogenous variables of the system. To be complete, the new microeconomies must explain these magnitudes as well.⁴

It is perhaps in order to offer some suggestion for how the two sides of the market can be combined. Assuming there are large number of potential buyers and sellers for a good, it should be noted that the optimal search rule (1) is applicable for both sides of the market. That is, both buyers and sellers will generally have incentives to do some sampling. For reasons which are not always clear, however, it is customary in most markets for one of the two sides to post a price. If institutions are well arranged, one would suspect that the price setter is the one that does not do the

¹See Holt [1970], Alchian [1970].

²See Alchian [1970].

³See Alchian [1970], Gordon and Hynes [1970], and Nordhaus [1970].

⁴As concrete examples, consider the following. Both Stigler [1961] and Alchian [1970] consider the optimal search and stopping rules when price dispersion is given. They fail to discuss the sources of price dispersion, and why it is a permanent, as opposed to a transient phenomenon. Similarly Stigler [1962] and Rees [1966] apply the theory to the labor market and do not explain the dispersion in wage offers on the part of the firm. Holt, in many places, analyzes the optimal behavior of the individual worker in detail, and in the aggregate, without looking into the firm's demand for labor (see Holt [1970]).

Several authors look at the decisions from the point of view of the firm alone, for example, Phelps and Winters [1970], Mortenson [1970], and Phelps [1970a].

searching,¹ and that this arrangement might minimize aggregate search costs (search costs are cnx , where x is now the total number of transactions and c and n are the same as above).

The link between buyer and seller, in fact, would seem to be very close to the mechanism specified by Phelps and Winter [1970].² What happens in markets with search is that the low-price sellers attract more customers than the high price customers. This can be shown in Figure 1. Let $f(p)$ be the density function of existing prices and $f(p,n)$ the minimum of a sample of n .³ The density function of sales, $f(p,n)$ is closely related to $f(p)$ by equation (1): if $n = 1$, then the density function of sales will be exactly the density function of prices. But as the size of the sample increases, first to 5, then to 10, the distribution becomes more and more concentrated toward lower-priced sales. If sales are repetitive, and customers have memories, there will be cumulative movement toward low priced firms, and high-priced firms will be driven out of business. Once the dispersion of price is known to be low (around $2/3$ of the search cost for a price quotation),⁴ the system becomes stable.

It may also become stable with higher dispersion if customers have low memories, or are infrequent purchasers, or if there is a steady stream of uninformed buyers which comes into the market. A good example of where high dispersion of prices can persist for long period is the perennial "tourist trap," where the terrain is unfamiliar, purchases are infrequent and no substitutes are available. In the case of a stable system of prices with high dispersion, firms will be faced with a clearly defined, negatively sloped demand curve between price and flow of purchases. It is easily seen

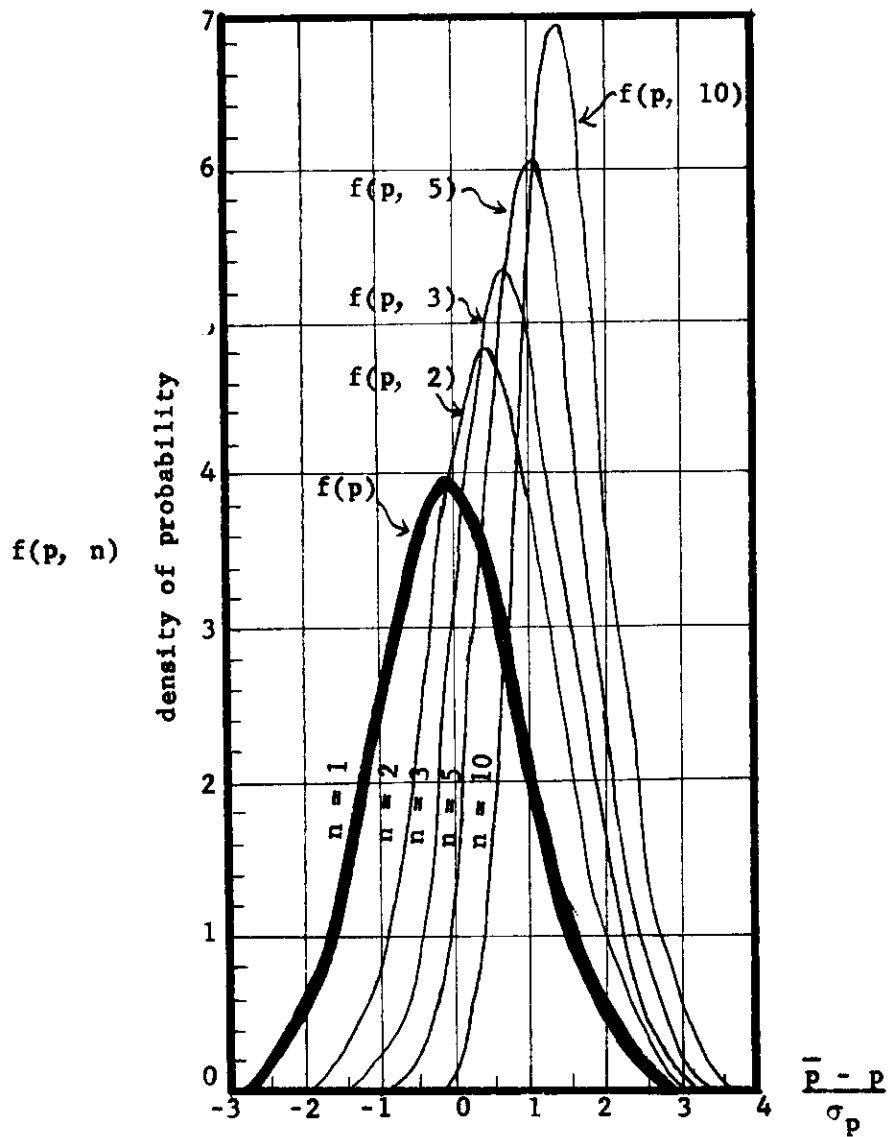
¹Advertising is in some ways an exception to this rule, for it is an attempt to inform (or, more likely, to try to misinform) the broad mass of consumers about the terms of a given deal.

²See Section III-B below.

³We have adapted Gumbel's figure by graphing $(\bar{p} - p)$, thus looking at the maximum rather than the minimum. By viewing Figure 1 in a mirror, the more intuitive distribution $(p - \bar{p})$ can be seen.

⁴See equation (1), p. 14 above.

FIGURE 1. Density Function of Prices for Samples of Size 1, 2, 3, 5, 10 from a Normal Distribution



Source: Gumbel [1958], p. 132.

that sales prices will be approximately distributed with mean $\{m - \sigma/2 \log n\}$ and standard deviation $\{\sigma(.40 + .6n^{-1/2})\}$, where m and σ are the moments of the price distribution.¹ These distributions define the static demand curve in the stable limit.²

In the next section we will apply some of the concepts developed to price dynamics.

B. Uncertainty and Price Dynamics

Several papers have applied, more or less directly, some of the ideas discussed above to derive theories of disequilibrium wage and price behavior. The new features of these dynamic theories are the following: (1) even though firms or households behave rationally, they do not learn about possibilities instantaneously. As a result, parties have temporary monopoly power in markets. (2) Given the dynamics of adjustment, firms will have an optimal path over time for prices and wages, the path depending on initial conditions, prices, costs, and the usual factors. (3) In general, the optimal paths and comparative dynamics for price, output, etc. behave very much according to the usual textbook examples when modifications for existence of monopoly power are made.

One of the most elegant treatments is that of Phelps and Winters [1970]. They utilize the basic competitive model with one major modification: they assume that although the elasticity of demand for an individual firm is infinite in the long run, it is finite in the short run. Their economy is therefore a kind of Chamberlinean model, where each firm's dd curve is like the industry's DD curve (in fact it has the same elasticity instantaneously in the Phelps-Winter model) but in the long run, the firm's dd

¹The mean is from footnote (1), p. 14 above. The standard deviation of the mean is an approximation from a figure in Gumbel [1958], p. 135.

²After the present paper was written, an excellent application of some of the concepts to consumer theory was published (Nelson [1970]).

curve becomes perfectly horizontal at average industry price. The major simplifying assumption is that the fraction of the market possessed by a single firm is a first-order differential equation in the firm's price and the industry price:

$$(1) \quad \dot{x}_i = H(p_i, \bar{p})x_i$$

where x_i is the i^{th} firm's share of the industry, p_i is the i^{th} firm's price, and \bar{p} is the average price in the industry. (As is customary, dots over variables represent time rates of change of those variables.) Moreover, $H(\bar{p}, \bar{p}) = 0$.¹

It is not clear how Phelps and Winter derive this basic behavior hypothesis.² From our discussion in Section III-A, it is clear that the qualitative properties of equation (1) could be derived from the model of search under conditions of uncertainty discussed there. On the other hand, the quantitative properties of (1) would depend on the number of searches. Thus, referring back to Figure 1 on p. 17, we know that while the bold line is the density function of prices, the thin lines are the density functions of sales. For example, if the number of searches is $n = 5$, then the "breakeven price" (\tilde{p}) (at which the firm just has its proportional share of customers) comes at about $\sigma/2$ below the mean. Below \tilde{p} , the firms have more than their share of the market, while above \tilde{p} , firms have less. Thus the Phelps-Winter assumption of $H(\bar{p}, \bar{p}) = 0$ --meaning that a firm which sets price at the industry average will lose no customers--does not square with the underlying search model except in the case where no search occurs. In fact, as we have indicated in Section III-A above, the Phelps-Winter assumption will hold asymptotically as (or rather if) the high price firms

¹This is not stated explicitly, but it follows immediately from that assumption that $H(p, p^1) = -H(p^1, p)$. (Phelps-Winter [1970], p. 312.)

²They give the following explanation: "Over time, customers gradually shift from the firms charging higher prices to those charging lower prices. The formulation of this dynamic process offered here is suggested by the thought that the process by which information about prices is transmitted is essentially one of 'comparing notes' in the course of random encounters among customers." (Phelps-Winter [1970], p. 311). It is puzzling why this explanation, rather than formal sampling, is chosen by the authors. Perhaps they feel that search costs are much lower by their method. We chose to interpret the process as a formal search process and use the results of the previous section.

get selected out of the population and price dispersion decreases. A second problem with the model is that the assumption of a first-order differential equation is not persuasive. By itself, the search model does not generate the first-order equation directly, for if customers have no memory, the firm's share with a constant price differential is unchanging. To get a constant erosion of customers, it must be assumed that individuals have memory and do some sampling every period and go to the lowest period firm. Thus if all customers sample and have memory, it is as if the size of sample increases over time, and the density function of sales moves further and further to the left. This would indeed drive out the high-period firms, as Phelps-Winter assume.

Assuming that the firm produces at constant marginal and average cost, and that $x\eta(p)$ is the demand function for the i^{th} firm's product,¹ the firm wishes to maximize its net worth in (2) subject to (1).

$$(2) \quad V = \int_0^{\infty} e^{-rt} [p-c]x\eta(p)dt$$

Let us further simplify by setting $H(p_i, p) = k(\bar{p} - p_i)$, $k > 0$. Using standard techniques in optimal control, we can derive the following set of conditions for describing the firm's optimal price behavior (omitting subscripts):

$$(3a) \quad \dot{x} = k(\bar{p} - p)$$

$$(3b) \quad \dot{q} = q[r - k(\bar{p} - p)] - (p - c)\eta$$

$$(3c) \quad \eta'x[p + \eta/\eta' - c] + qkx = 0$$

q is the conjugate variable of x , and is interpreted as the "shadow price of patronage." It is easily seen that if k is zero (the firm is a monopolist) the condition is simply that the bracketed term in (3c) holds; this is the

¹For purposes of exposition, we are simplifying the Phelps-Winter model.

standard monopoly condition that the ratio of marginal cost to price be equal to one plus the inverse of the elasticity of demand, $c/p = 1 + 1/\epsilon$, where $\epsilon = \eta'x/\eta =$ price elasticity of demand.

In final equilibrium, where the firms share is constant, $\dot{x} = \dot{q} = 0$, so we know from (3):

$$(4a) \quad p = \bar{p}$$

$$(4b) \quad q = \eta(p - c)/r$$

$$(4c) \quad (p - c)/p = \left[\frac{kp}{\epsilon\eta} - \epsilon \right]^{-1}.$$

These conditions tell us that the firm's price tends toward the average industry price,¹ and that at that price the firm's "monopoly power," $(p-c)/p$, is less than it would be for a monopolist according to reaction speeds (k) and discount rates (r).

Moreover, Phelps and Winter derive a rather complicated rule for disequilibrium price behavior. From their description it appears that price moves toward the equilibrium (\bar{p}, \bar{x}) in (4) in a way similar to the Walrasian mechanism [see their equations (42)]:

$$(5) \quad \begin{aligned} \dot{p} &= \alpha_1(p - \bar{p}) + \alpha_2(x - \bar{x}) \\ \dot{x} &= \beta_1(p - \bar{p}) \end{aligned}$$

The most important question for Phelps and Winter, insofar as they are attempting to propose non-Walrasian adjustment mechanisms, is to show that firms adjust output rather than price when demand shifts. For this to be the case, β_1 in equation (5) must be large relative to α_1 and α_2 .

In this case, increases in demand would be met mainly by increases in output rather than increases in price.

¹This is a major criticism of the Phelps-Winter model. As we noted above this result comes from a misspecification of the customer-flow dynamics in a search model. The correct specification would lead to a non-zero dispersion, the dispersion dependent on search costs and customer memories.

A careful examination of the relevant sections of the Phelps-Winter paper (pp. 325-335) indicate that (aside from the usual static variables) the non-Walrasian case where output rather than price responds is important when interest rates are high, or when reaction speeds (k) are low. This result, although not completely spelled out, it is of interest. It indicates that one can expect "non-tatonnement" behavior where conditions approximating monopoly conditions obtain, including the condition that price is significantly above marginal cost. Similarly, if profit margins are low, the Phelps-Winter model predicts the tatonnement model will give relatively good productions. As we noted above in discussion of equation set (4), this case corresponds to conditions where competitive behavior obtains.

Another set of papers¹ examines the problem of wage dynamics in heterogenous labor markets. Using the basic framework set forth in the last section, this theory demonstrates that a worker will have a declining "acceptance wage" which depends basically on the worker's perceptions of the market opportunities. The authors show that (for given expectations about the general trend of wages) there will be a trade-off between the unemployment rate and the rate of wage inflation like that postulated in the Phillips curve. In other words, the new microeconomics leads one to predict that for labor markets there will be a non-Walrasian adjustment mechanism, but a mechanism in which prices respond to excess demands or supplies in the general manner predicted by the Walrasian adjustment mechanism. The major difference is that one has to pay close attention to the movement of quantities, as well as prices, in order to understand the market trends. Moreover, because firms and workers are quantity-adjusters as well as (perhaps more than) price-adjusters, there may be sizeable unemployed resources as economic agents adjust to changing conditions.

This brief discussion of the "new microeconomics" of inflation indicates that some interesting breakthroughs have been made in rationalizing some of the difficult details of modern macroeconomics. In particular, we now have a better understanding of the relation between unutilized resources and price

¹Holt, [1970], Mortensen [1970], Phelps [1970b].

movements. The authors stress that in equilibrium (that is, after all expectations have adapted to the new environment) the rate of inflation is completely immaterial to the level of real magnitudes, and this certainly rates as one of the most important results of the new microeconomics. On the other hand, there is very little which is truly dynamic, in the sense of explaining the actual rate of inflation insofar as this is a disequilibrium phenomenon.

IV. Optimal Pricing in the Long and Short Run

In recent years, significant progress has been made in understanding the empirical behavior of prices. It is slightly troubling, however, that these empirical studies have proceeded independently of price theory. The authors rely more heavily on questionnaires or behavioral studies of business behavior¹ than on the traditional body of economic thought. The main reason for this has been discussed above: namely, while there is an elegant theory of equilibrium price behavior, there is no well-developed theory of price dynamics. A second problem with existing empirical studies is that they lean heavily on markup theories of pricing. These theories have been attacked by some as antithetical to profit-maximizing behavior,² while others have argued that mark-up pricing is not at variance with optimizing behavior.³

This section uses a different approach to derive models of price behavior. The approach is conventional in its assumptions, using neoclassical pricing and production theory. These conventional assumptions lead to powerful restrictions on the optimal price (or markup) equations customarily used and may help to shed light on certain ancient controversies in the area of price theory. The general conclusion is that the theoretical basis behind

¹Two behavioral studies are widely cited: Hitch and Hall [1939] and Kaplan et al. [1958]. Other studies are by D.C. Hague, I.F. Pierce, and A.C. Cook.

²Machlup ([1952], [1968]).

³See Simon ([1957], [1959]), Eckstein [1964], and Williamson [1964].

existing pricing practice, as well as current econometric estimation, may be more sound than has sometimes been thought.

A. Long-Run Pricing

The neoclassical theory of competitive economies has developed a unified, consistent theory of individual and aggregate price behavior. For the most part there has been little or no application of this theory to empirical estimates of price behavior, probably because neoclassical theory is mainly equilibrium theory. The most common tool used by economists investigating price behavior is mark-up pricing theory. Some economists have found that firms use mark-up pricing as a rule of thumb in short-run pricing decisions, but no relationship between this and profit maximization has been shown. In fact, as we show there is a fairly definite relationship under the usual assumptions of neoclassical production and demand theory.¹

The present section derives a model of long-run or equilibrium pricing. The results resemble closely some of the earlier mark-up ideas.

We assume that a firm has a technologically given production relation in the long run given by a production function. It is assumed that there are no lags in the production process, so that orders, production, and demand are all known immediately and can respond to changes in the independent variables instantaneously. To simplify the analysis, assume the firm uses capital (K), labor (L), and materials (M) as inputs. Homogeneous physical output (X) is then given by

$$(1) \quad X = F(K, L, M) .$$

L is measured in manhours and M in physical units. K is the services of past investments. To start with we examine a representative firm producing the output. The firm faces a well-defined demand curve, where output sold is a function of the price that the firm sets for its product:²

¹The relation between cost functions and profit-maximizing prices has been discussed by de Menil [1969]. The idea that markup pricing is consistent with profit maximization is treated briefly in Bodkin et al. [1966].

²Equation (2) should be interpreted as the demand curve perceived by the firm. If there are no interdependencies, then (2) would be an unbiased estimate of the true demand curve. If interdependencies do exist, then (2) is the true demand curve modified by the conjectural variations of other firms.

$$(2) \quad X = G(p, Y)$$

where Y is money income or output.

The firm's criterion in setting price is that the expected value of discounted profits be maximized, where θ is a discount rate. Using hats (^) to represent expected values, this criterion is:

$$(3) \quad \max_{\{p\}} U = \int_0^T \{\hat{p}_t \hat{X}_t - \hat{C}_t\} e^{-\theta t} dt .$$

where C_t is cost. At the present stage of the analysis we assume there are no temporal dependencies,¹ so (3) reduces to instantaneous maximization of the profit flow, Π , at every point of time. The criterion is thus:

$$(4) \quad \max_{\{X\}} \Pi = p_t X_t - C_t .$$

Later in the analysis it will be useful to recognize the existence of lags and frictions, which makes a solution to the pricing decision much more complicated.

We will now give an explicit solution under different assumptions about production.

Case 1. Fixed Proportions. We assume that demand is log linear, so that

$$(5) \quad X = B p^{-b_1} Y^{b_2} .$$

If the production function is constant returns, then

$$(6) \quad X = \min \left[\frac{K}{a_1}, \frac{L}{a_2}, \frac{M}{a_3} \right] .$$

A cost minimizing firm² will thus set

¹This will be the case where a perfect used capital goods market exists.

²Recall that we assume perfect factor markets. This implies that factors can be adjusted immediately and the cost minimizing input level given in (7) can be attained.

$$(7) \quad X = \frac{K}{a_1} = \frac{L}{a_2} = \frac{M}{a_3}$$

The instantaneous rate of profit is¹

$$(8) \quad \begin{aligned} \Pi &= pX - C \\ &= (X^{-1/b_1} b_1^{1/b_1} b_2^{b_2/b_1})X - wL - qK - vM, \end{aligned}$$

where w is the wage per man-hour, q is the service price of capital,² and v is materials price.

Putting (7) into (8) and maximizing³ gives us the price equation:

$$(9) \quad p = \frac{b_1}{b_1 - 1} (a_1 w + a_2 q + a_3 v),$$

which is the simplest form of the markup equation. We thus see that where production is by fixed coefficients and demand is constant elastic, then we have a precise "full-cost pricing" or "markup" price determination. Furthermore shifts in demand only influence price if they change elasticity.

¹Note from (5) that $p = X^{-1/b_1} b_1^{1/b_1} b_2^{b_2/b_1}$.

²Since a perfect capital market exists, q is the "rental" price of a unit of capital.

³Putting (7) into (8) gives

$$(i) \quad \Pi = AX^{1-1/b_1} - X(a_1 w + a_2 q + a_3 v)$$

where $AX^{-1/b_1} = p$, and $-1/b_1$ is the price elasticity of demand

$$\frac{\partial \Pi}{\partial X} = AX^{-1/b_1} (1 - 1/b_1) - (a_1 w + a_2 q + a_3 v)$$

Since $AX^{-1/b_1} = p$,

$$(ii) \quad p = (1 - 1/b_1)^{-1} (a_1 w + a_2 q + a_3 v)$$

$$= \left(\frac{b_1}{b_1 - 1} \right) (a_1 w + a_2 q + a_3 v)$$

(When there is not constant returns we get quite different results.) Note that the coefficient on the linear regression

$$(10) \quad p = \alpha_0 + \alpha_1 w + \alpha_2 q + \alpha_3 v$$

has the restriction that $\alpha_0 = 0$ and $\alpha_j/\alpha_1 = a_j/a_1$, which is a very strong restriction.

If Hicks-neutral technological change is proceeding at rate h , then a term e^{-ht} should be introduced in (9). If there are not constant returns the results are much less nice, with price given by¹

$$(11) \quad p = \left(\frac{b_1 B^{\frac{1}{m} - 1}}{(b_1 - 1)^m} \right)^\sigma (a_1 w + a_2 q + a_3 v)^{\sigma_Y} \sigma^{b_2 \left(\frac{1}{m} - 1 \right)}$$

where m is the degree of homogeneity of the production function and $\sigma = \{1 + b_1(1/m - 1)\}^{-1}$. We thus have the important restriction that if there are constant returns (or close to it) then price will be (almost) unaffected by changes in demand.²

We have used a constant-elasticity demand curve, although this is certainly too restrictive. We can, however, use a general demand curve linearized at the equilibrium price. This gives:

$$(12) \quad p = c_0 - c_1 X + c_2 Y, \quad c_1 > 0$$

¹In this case $X = \min[K/a_1, L/a_2, M/a_3]^m$. Then we have $X^{1/m} = K/a_1 = L/a_2 = M/a_3$ and (i) in footnote 3, page 26 becomes:

$$(i') \quad \Pi = AX^{1 - 1/b_1} - X^{1/m}(a_1 w + a_2 q + a_3 m).$$

The derivative of (i') is

$$\frac{\partial \Pi}{\partial X} = 0 = p(1 - 1/b_1) - \frac{1}{m} X^{1/m-1} (a_1 w + a_2 q + a_3 m)$$

$$\text{using (5)} \quad = p(1 - 1/b_1) - \frac{1}{m} B^{1/1-m} p^{-b_1(1/m-1)} Y^{b_2(1/m-1)} (a_1 w + a_2 q + a_3 m)$$

Thus if $\{\sigma = 1 + b_1(1/m-1)\}^{-1}$

$$p = \left(\frac{b_1}{b_1 - 1} \right)^\sigma \frac{1}{m^\sigma} B^{(1/m-1)\sigma} Y^{\sigma b_2(1/m-1)} (a_1 w + a_2 q + a_3 m)^\sigma$$

²This result was also shown by Kalecki [1939].

Putting (12) and (7) into (4) we have

$$(13) \quad \begin{aligned} \Pi &= X[c_0 - c_1X + c_2Y] - wL - qK - vM \\ &= X[c_0 - c_1X + c_2Y - (a_1w + a_2q + a_3v)] \end{aligned}$$

Maximizing we have¹

$$(14) \quad p = \frac{c_0 + c_2Y}{2} + \frac{wa_1 + qa_2 + va_3}{2}$$

In this formulation, demand does enter the pricing equation since demand is no longer constant elasticity.

Case 2. Cobb-Douglas. A second common form of production function, which is usually thought to be more applicable to the long run than the fixed-coefficients model, is the Cobb-Douglas or log-linear model of production (h the rate of Hicks-neutral technological change):

$$(15) \quad X = c_0 K^{a_1} L^{a_2} M^{a_3} e^{ht}$$

with demand

$$(16) \quad X = Bp^{-b_1} Y^{b_2}$$

We can form our profit function:

$$(17) \quad \Pi = AX^{1 - 1/b_1} - wL - qK - vM$$

where $C = c_0 B^{1/b_1} Y^{b_2/b_1}$. Maximizing with respect to all inputs we have

$$\begin{aligned} \frac{\partial \Pi}{\partial X} &= c_0 - c_1X + c_2Y - (wa_1 + qa_2 + va_3) - c_1X \\ &= p - c_1X - (wa_1 + qa_2 + va_3) \\ &= p - (c_0 + c_2Y - p) - (wa_1 + qa_2 + va_3) \end{aligned}$$

Thus
$$p = \frac{c_0 + c_2Y}{2} + \frac{wa_1 + qa_2 + va_3}{2}$$

$$(18) \quad \begin{cases} \frac{\partial \Pi}{\partial K} = a_1 \left(1 - \frac{1}{b_1}\right) p \frac{X}{K} - q = 0 \\ \frac{\partial \Pi}{\partial L} = a_2 \left(1 - \frac{1}{b_1}\right) p \frac{X}{L} - w = 0 \\ \frac{\partial \Pi}{\partial M} = a_3 \left(1 - \frac{1}{b_1}\right) p \frac{X}{M} - m = 0 \end{cases}$$

Thus

$$(19) \quad X = C e^{ht} \left(\frac{ka_1}{q}\right)^{a_1} \left(\frac{ka_2}{w}\right)^{a_2} \left(\frac{ka_3}{v}\right)^{a_3}$$

where $k = p^{1-b_1} B \left(1 - \frac{1}{b_1}\right) Y^{b_2}$.

Putting (16) into (19),

$$B p^{-b_1} Y^{b_2} = C e^{ht} \left(\frac{a_1}{q}\right)^{a_1} \left(\frac{a_2}{w}\right)^{a_2} \left(\frac{a_3}{v}\right)^{a_3} k^{a_1+a_2+a_3}$$

Setting $m = a_1 + a_2 + a_3$, the degree of homogeneity of the production function,

$$p^{-[b_1+(1-b_1)m]^{-1}} = C e^{ht} \left(\frac{a_1}{q}\right)^{a_1} \left(\frac{a_2}{w}\right)^{a_2} \left(\frac{a_3}{v}\right)^{a_3} B^{m-1} Y^{b_2(m-1)}$$

Finally set $d = -[b_1 + (1 - b_1)m]^{-1}$, so

$$(20) \quad p = (B^{m-1} C)^d e^{hdt} \left(\frac{a_1}{q}\right)^{a_1 d} \left(\frac{a_2}{w}\right)^{a_2 d} \left(\frac{a_3}{v}\right)^{a_3 d} Y^{b_2(m-1)d}$$

Equation (20) gives the basic, long-run rule for the profit-maximizing price. We can simplify greatly for purposes of discussion by assuming constant returns to scale ($m = 1$). In this case, we get

$$(20a) \quad p = C^d e^{-ht} \frac{a_1^{a_1} a_2^{a_2} a_3^{a_3}}{q^{a_1} w^{a_2} v^{a_3}}$$

where $C' = C(1 - 1/b_1)$. The optimal price is closely related to average and marginal cost, in fact it equals these when $b_1 = \infty$, the competitive case. Under non-competitive conditions, the ratio of price to marginal cost equals $(1 - 1/b_1)$, the usual rule under static conditions. Three important points should be made about the optimal price. First is the effect of factor prices on price. As equation (20a) shows, the logarithm price is a log-linear function of factor prices. Thus a rise of one percent in the wage rate leads to a rise of a_2 percent in the optimal price. The second point is that the cost of capital is an important component of the optimal price. It is slightly surprising that cost of capital has been omitted from statistical estimates of price behavior. The third point is that productivity does not explicitly appear in the equations (more precisely, it appears through the time trend). The omission of productivity is a result of the assumption that technological change proceeds smoothly at an exponential rate and does not vary over the business cycle.

One interesting application of the model is to examine the relation of the optimal price to the well-known "mark-up pricing" model. According to Eckstein, the target-return pricing formula is the following:¹

$$(21) \quad p = \frac{\hat{\pi}K}{\bar{X}} + \frac{\bar{L}}{\bar{X}} w + \frac{\bar{M}}{\bar{X}} v$$

where $\hat{\pi}$ is the target rate of return and bars over variables standard levels of operation. We can compare (21) to (20) by linearizing (20):

$$(22) \quad p = c_0 + c_1 q + c_2 w + c_3 v$$

¹This formula is from Eckstein ([1964], p. 269). Eckstein views (21) as the most defensible member of the markup family. Crude full-cost (or cost-plus) pricing differs from (21) in that the variables relate to actual rather than standard cost; Eckstein points out that, given cyclical productivity behavior, the crude versions could lead to the absurd result of price declines in expansions.

If the target-return pricing scheme is to be an approximation to the optimal pricing rule, then we must find that the coefficients in the two equations are equal (i.e. we must find $c_0 = 0$, $c_1 = \frac{\hat{\pi}K}{\bar{X}q}$, $c_2 = \bar{L}/\bar{X}$, $c_3 = \bar{M}/\bar{X}$).

To determine the c_i coefficients in (22) from the optimal equation (20) take derivatives

$$(23) \quad \frac{dp}{dq} = \frac{p}{q} \frac{a_1}{b_1 + m(1 - b_1)} = c_1$$

$$(24) \quad \frac{dp}{dw} = \frac{p}{w} \frac{a_2}{b_1 + m(1 - b_1)} = c_2$$

$$(25) \quad \frac{dp}{dv} = \frac{p}{v} \frac{a_3}{b_1 + m(1 - b_1)} = c_3$$

Thus the "target-return pricing" corresponds to the optimum when:

$$(26) \quad c_1 = \frac{\hat{\pi}K}{\bar{X}q} = \frac{pa_1}{q[b_1 + m(1 - b_1)]}$$

$$(27) \quad c_2 = \frac{\bar{L}}{\bar{X}} = \frac{p}{w} \frac{pa_2}{[b_1 + m(1 - b_1)]}$$

$$(28) \quad c_3 = \frac{\bar{M}}{\bar{X}} = \frac{p}{v} \frac{pa_3}{[b_1 + m(1 - b_1)]}$$

Taking the coefficient on capital (c_1) first, recall from (18) that $\frac{qK}{pK} = a_1(1 - 1/b_1)$. Thus

$$\frac{a_1}{b_1 + m(1 - b_1)} = \frac{q\hat{\pi}K}{pq\bar{X}} = \frac{\hat{\pi}K}{p\bar{X}}$$

or

$$\hat{\pi} = \frac{p\bar{X}}{qK} \frac{qa_1}{b_1 + m(1 - b_1)} = \frac{q}{[b_1 + m(1 - b_1)](1 - 1/b_1)}$$

If $m = 1$, the constant returns case; we have

$$(29) \quad \hat{\pi} = \frac{q}{\left(1 - \frac{1}{b_1}\right)}$$

A competitive market with constant returns implies $b_1 = \infty$ and $\hat{\pi} = q$.

With higher elasticities the rate of profit (as customarily measured) is above the cost of capital. We thus see that the interpretation of the "target rate of return" is the cost of capital over 1 minus the inverse of the elasticity of demand.

Next we wish to determine the conditions under which the target-return model gives the correct rule for labor costs. That is, when does $c_2 = \bar{L}/\bar{X}$? Using (27) and recalling that $wL/pX = a_1(1 - 1/b_1)$, we have

$$(30) \quad c_2 = \frac{\bar{L}}{\bar{X}} \frac{1}{\left(1 - \frac{1}{b_1}\right)[b_1(1 - m) + m]}$$

This equals \bar{L}/\bar{X} (as predicted by the target-return model) only when $m = 1$ and $b_1 = \infty$; that is, only under perfect competition and constant returns to scale. In general, as in the case of the capital coefficient, c_2 will be greater than \bar{L}/\bar{X} under noncompetitive conditions.

Finally it is easily seen that for materials

$$(31) \quad c_3 = \frac{\bar{M}}{\bar{X}} \frac{1}{\left(1 - \frac{1}{b_1}\right)[b_1(1 - m) + m]}$$

which is exactly analogous to the labor coefficient.

In summary, the optimal pricing rule does not coincide with the target-return pricing rule except under competitive conditions. To the extent that the model used here is an accurate reflection of the underlying conditions of demand and production, this result implies that target-pricing is not a good rule of thumb except under competitive conditions. A better rule of thumb would be to follow the optimal price rule in equation (20) or the linearized rule in equation (22) with proper coefficients.

Case 3. Constant elasticity of substitution (CES). A final production function we examine is the constant elasticity of substitution, or CES.¹ This is somewhat more general than either case 1 or case 2, and includes them as limiting special cases.

The constant returns to scale CES production function is written

$$(32) \quad X = e^{ht} [\delta_1 K^{-\rho} + \delta_2 L^{-\rho} + (1 - \delta_1 - \delta_2) M^{-\rho}]^{-\frac{1}{\rho}}$$

$-1 < \rho < \infty$, and where the partial elasticities of factor substitution are $(1+\rho)^{-1}$. It can be shown that the profit maximizing price for the CES in (32) is given by

$$(33) \quad p = A \left[B_m \frac{1}{(1+\rho)^\rho} + C_w \frac{1}{(1+\rho)^\rho} + D_v \frac{1}{(1+\rho)^\rho} \right]^{\frac{\rho}{b_1[\rho^2 + 1 - \frac{1}{b_1(1+\rho)}]}}$$

where A, B, C, and D are constants.

C. Tax Shifting

We have up to now investigated some of the properties of optimal pricing decisions under specified kinds of demand and production functions. One important finding is that the optimal price looks suspiciously like the mark-up equation often used in econometric estimates. There is, however, one important difference from the standard interpretation which concerns tax shifting.

Some of the "full-cost mark-up" theories of price include profit taxes in the cost of producing output and, therefore, in the full cost. Unlike the other elements of cost, the inclusion of profit's tax is an irrational element for a monopolist. If the mark-up equation is considered to be a "rule of thumb," a first approximation of an optimal pricing rule such as the ones we derived above, then profits taxes have no place in the cost if the rule of thumb is rational.

¹ See Arrow, et al. [1961].

D. Short-Run Pricing

The model developed in Section B is a long-run theory of price determination. How can this be translated into a short-run model? On the one hand, it might be argued that a distributed lag adjustment model would be appropriate to take into account the fact that adjustment to long-run desired levels are impeded by uncertainty, administrative delays, and long-run contracts. On the other hand, market structure and price administration might lead to short-run pricing decisions which are inconsistent with long run profit maximization. In this section we note some of the arguments which have been made concerning the deviation of short-run from long-run behavior. The remarks are organized around the three problems of uncertainty, market structure, and costs of adjustments.

1. It is sometimes argued that uncertainty about the form of demand or cost functions causes stickiness in the price mechanism.¹

Actually, although uncertainty may aggravate the problem of stickiness, but it cannot be the cause of it.²

To gauge the effect of uncertainty, we introduce uncertainty in the demand and production functions by recognizing the existence of random disturbances. For simplicity we will examine only log-linear production and demand functions. It is assumed that there are multiplicative disturbances in both output and demand:

$$X = CK^{\frac{a_1}{L}} L^{\frac{a_2}{M}} M^{\frac{a_3}{ht}} U_1 \quad E(U_1) = 1$$

$$X = Bp^{-b_1} Y^{b_2} U_2 \quad E(U_2) = 1$$

Carrying out the maximization it can be shown that the optimal price (that is, the price which maximizes ex post profits) is:

$$(34) \quad p = ke^{\frac{hdt}{q}} \frac{-a_1^d}{w} \frac{-a_2^d}{v} \frac{-a_3^d}{Y} \frac{b_2^{(m-1)d}}{U_1^d U_2^{(m-1)d}}$$

¹This argument is made by Cagan [1968]. Also see Stigler [1961].

²This point is also made by Gordon and Hymes [1970], p. 375 ff.

where k is a constant. (This equation is related in an obvious way to equation (20) in Section IV-B.) There are two sorts of difficulties which uncertainty raises. First, if U_1 and U_2 are not known, then the profit-maximizing price must be replaced by the price which maximizes expected profits, which implies replacing $(U_1^d U_2^{(m-1)d})$ by its expected value, 1 .¹ Since the ex post optimal price will differ by the factor $(U_1^d U_2^{(m-1)d})$, this gives the first deviation of the short-run optimal price from actual price. Second, we must recognize that all the decision parameters in (33) (a_1 , a_2 , d , and so forth) must be replaced by their expected values. To the extent that the firm's estimates of the decision parameters deviate from the actual parameters, there is further a difference between the short-run and the optimal price.

Nevertheless, there is nothing in the introduction of uncertainty per se which leads to stickiness of price. Since (33) holds at every point of time, the flexibility of price (measured, say, by its coefficient of variation) will be magnified by uncertainty unless the disturbances are negatively correlated with other independent variables.

2. The second qualification on short-run, instantaneous price adjustment is that firms do not adjust price because of oligopolistic interdependencies. Most writers feel that the stickiness of price is chiefly due to fear of "rocking the boat."² One model of the process is the "kinky demand curve," showing how short-run price movements may have a certain inflexibility.³ Yet if prices are stable, the problem of how they get to long run equilibrium in the first place is now the troublesome issue.

¹We are assuming the firm is risk-neutral. This is inessential to the conclusion.

²Cf. Eckstein [1964].

³The original kinky demand curve was in Sweezy [1939], with criticism by Stigler [1947]. Also see Cyert and Cohen [1965], Chapter 12.

Recent evidence on price flexibility by Stigler and Kindahl [1970] casts serious doubts on the standard notions about price inflexibility. Recall that the "stickiness" usually works downward, according to this theory, while prices should retain their upward flexibility. In contrast to this theory, Stigler and Kindahl find that by examining transactions prices (rather than list prices) there is substantial downward flexibility of price even during contractions. On the other hand, there does seem to be evidence that competitive industries are more likely to behave as predicted by standard price theory than noncompetitive industries.¹ The evidence, however, is that the inflexibility in prices is associated with all forms of noncompetitive markets, rather than only markets with oligopoly of small numbers.²

The test of the effect of different forms of noncompetitive market structure on price inflexibility given in Stigler and Kindahl relies on cross-sectional analysis. As an attempt to determine whether market structures affect price flexibility directly, observations on the price of primary aluminum were obtained for the period 1909-1966. The interest in looking at the primary aluminum industry over this period is that there was an exogenous change in the market structure of aluminum after World War II when the Federal government sold aluminum plants to two new companies, thus changing a monopoly into tripoly. If the inflexibility of price is due to oligopolists' fear of "rocking the boat," there should have been a noticeable increase in price inflexibility after World War II. Table 1 gives estimates of the average and standard deviation of price change for periods within the whole period. The following conclusions seem to be apparent. For all samples, the frequency and dispersion of price change fell sizeably over the sixty year period. The

¹The following table gives the percentage of prices behaving according to theory. "According to theory" is interpreted as prices which rise more than 5% in booms and fall more than 5% in recessions.

	Concentration Ratio			
	0 to 25	25 to 50	50 to 75	75 to 100
According to theory	90%	60%	45%	57%
Not according to theory	10%	40%	55%	43%

Source: Stigler and Kindahl, p. 61, fn. 5 and p. 63, fn. 8.

²It is clear that the "rocking the boat" or "kinky-demand curve" reasoning holds for industries with more than one seller.

TABLE 1

Average, Standard Deviation, and Frequency of Price Changes: Primary Aluminum

<u>Period</u>	<u>Mean Price Change Per Cent</u>	<u>Standard Deviation, Price Change, Per Cent</u>	<u>Frequency of Price Change Per Cent of Periods (changes per month)</u>
<u>All Years</u>			
1909-29	-.000017	.0464	71
1929-41	-.0032	.0146	9
1948-66	.0021	.0140	20
<u>Deflationary Years, Only</u>			
1909-29	-.022	.0379	74
1929-41	-.011	.0266	25
1948-66	-.0059	.0176	13
<u>Stable Years, Only</u>			
1909-29	-.0006	.0238	52
1929-41	-.00046	.00454	4
1948-66	-.000145	.01136	12
<u>Inflationary Years, Only</u>			
1909-29	.032	.0656	90
1929-41	none	none	--
1948-66	.0073	.0147	35

Note: Deflationary (inflationary) years are those where the December to December price fell (rose) by more than 5 percent. I am indebted to Robert Neugebauer for performing these calculations.

most important change in flexibility came between the first and second periods (divided by 1929), rather than between the second and third periods (divided by the Second World War). The conclusion would seem to be that events surrounding the Great Depression were more important in leading to price inflexibility than those surrounding the change in market structure. This conclusion reinforces the conclusion from Stigler and Kindahl mentioned above.

3. The final source of stickiness in price is cost of price adjustment. There are several possible sources of stickiness. First, there may be some physical costs of price changes: this would include certain costs of drawing up prices, printing new books, informing personnel, etc. This might be important for goods, especially low-priced items, where changing prices often would be annoying, but it could hardly be significant for larger goods where the

ratio of price to sales cost is low. Second, there might be a reaction from the demand side to changes in price: customers might change product from pique. Finally, since knowledge about prices is scarce, there is an incentive not to change price because of the obsolescence of knowledge which would occur. In any of these cases, price would change only when actual and desired price differ by some finite amount. Eckstein uses this as an argument for stickiness: "Under non-competitive conditions, deviation from list price occurs only when the market conditions differ from normal by more than some threshold amount."¹

As we noted above a more significant source of inflexibility in the short-run is the fact that buyers like to have prices stable as a service. Cagan writes:²

Most buyers would like to have assured supplies at stable prices to ease the problems of financing and scheduling production over time. One of the valued services of a supplier is his ability to take care of regular customers and maintain a "fair" price at all times....It is well known that commercial banks ration credit during period of "tight money" to accomodate loyal customers who patronize the bank during other periods as well.

In light of our estimates of the search costs in Section II above, this explanation seems to be reasonable. It implies that suppliers bear the risk of fluctuations by keeping their prices stable over business cycles. There is one puzzle in the institutional arrangement, however, for to the extent that firms hold prices constant in anticipation of future price declines, it is speculating and assuming the risks of price changes. One would expect that (except for labor markets) households, for whom the goods are a small part of their "portfolio," would prefer to bear the risk rather than compensate the firm to bear it.

¹Eckstein [1964], p. 270.

²Cagan [1968], p. 10. In light of results of Stigler and Kindahl [1970] it might be questioned whether the stability of list prices (when transaction prices are changing) is really a significant service. Moreover Kane and Malkiel [1965] argue that for banks price inflexibility is a peculiarity of the banking product, so this example should be used with caution.

V. Problems in the Macroeconomic Theory of Inflation

Up to now we have considered the problems of price determination and price inflation in the firm. Ultimately, however, we are interested in predicting aggregate as well as individual price movements. There is an important logical step from micro to macro, and the present section considers some of the aspects of this problem.

A. Aggregation

Many of the important propositions about price and wage dynamics rely heavily on aggregation phenomena. For example, the Schultze hypothesis about the existence of inflation with no aggregate excess demand relies on nonlinearities of response.¹ In a very similar way, Lipsey's theory of the Phillips curve assumes nonlinearities in the response mechanism of wages.²

For a simple model of aggregation it is helpful to start with the optimal pricing rules derived in the previous section.

Assume that all firms in an industry behave symmetrically. The production function is n-sectors, with gross output given by the production function

$$X_i = C_i K_i^{\alpha_i} X_{1i}^{\beta_{1i}} \dots X_{ni}^{\beta_{ni}} L_i^{\gamma_i} e^{-h_i t}, \quad \alpha_i, \beta_{ij}, \gamma_i \geq 0$$

From (20), in Section IV-A the prices of output is then (assuming W and Q are the price of labor and capital):

$$p_i = e^{-(c_i + h_i t)d_i} [Q_i^{\alpha_i} p_1^{\beta_{1i}} \dots p_n^{\beta_{ni}}] \frac{b_{2i}^{d_i} (1-m_i)^{d_i} \gamma_i^{d_i}}{Y}$$

where c_i is a constant, h_i the rate of technological change in the i^{th} industry, and b_{1i} and b_{2i} , the price income elasticities of demand,

¹Schultze [1959].

²Lipsey [1961].

and $m_i = \alpha_i + \gamma_i + \sum_j \beta_{ij}$ and $d_i = [b_{1i} + (1 - b_{1i})m_i]$. Taking logarithms and letting lower-case letters represent logs:

$$p_i = -(c_i + h_i t)d_i + d_i \alpha_i q + d_i \beta_{1i} p_1 + \dots + d_i \beta_{ni} p_n + d_i \gamma_i w + d_i (1 - m_i) b_{2i} y$$

Set $a_i = d_i \alpha_i$, $f_{ij} = d_i \beta_{ij}$, $g_i + d_i \gamma_i$, $l_i = d_i (1 - m_i) b_{2i}$, and let p , c , h , a , g , f , and l be the matrices corresponding to the subscripted variables. Further let $c' = (c + ht)d$, and the logarithms of the price, we have:

$$p = c' + wg + pf + qa + lY$$

$$p(I-f) = c' + wg + qa + lY$$

It can be shown that $(I-f)^{-1}$ exists and is positive, so

$$p = (c' + wg + qa + ly)(I-f)^{-1},$$

or

$$p = c'^* + wg^* + qa^* + y l^*$$

where a star behind a matrix indicates that it has been post-multiplied by $(I-f)^{-1}$. Using a fixed weight price index:

$$\theta = (\theta_1 \dots \theta_n)$$

where

$$\sum \theta_i = 1,$$

we get an aggregate price index, \bar{p} , by averaging prices:

$$(1) \quad \bar{p} = p \cdot \theta = c'^* \theta + wg^* \theta + qa^* \theta + y l^*$$

This (in logarithmic form) gives us our aggregate price equation.

Equation (1) assumes that reaction to changes in costs of demand is instantaneous. As we have noted in earlier sections, this assumption is doubtful.

If reaction is accounting to a first-order lag, and \hat{p} is the desired price in (1) we have

$$(2) \quad \Delta \bar{p}_t = \lambda[\hat{p}_t - \bar{p}_{t-1}] = \lambda[c^* + w_t g^* + q_t q^* + y \ell^* - \bar{p}_{t-1}]$$

Applying the fixed weight index, we have

$$\Delta \bar{p}_t = \lambda[c^* \cdot \theta + w_t g^* \cdot \theta + q_t a^* \cdot \theta + y \ell^* - \bar{p}_{t-1} \cdot \theta]$$

Solving, we get the standard form of a distributed lag price equation:

$$(3) \quad \bar{p}_t = \sum_{i=1}^{\infty} (1-\lambda)\lambda^i [c^* \cdot \theta + w_{t-i} d^* \theta + q_{t-i} a^* \theta + g_{t-i} \ell^* \theta]$$

or

$$(4) \quad \bar{p} = c^* \theta + g^* \theta \left[\sum_{i=1}^{\infty} (1-\lambda)\lambda^{-i} w_{t-i} \right] + a^* \theta \left[\sum_{i=1}^{\infty} (1-\lambda)\lambda^{-i} q_{t-i} \right] + \ell^* \theta \left[\sum_{i=1}^{\infty} (1-\lambda)\lambda^{-i} y_{t-i} \right]$$

Recalling the definitions of variables, we find that the aggregate price index is a function of past behavior of wages and capital costs. As in the usual model, the lag structures on the past factor prices (the terms in brackets) are determined by the form of the distributed lag. The coefficients on the prices (the terms in front of the brackets) are a very complicated function of the production structure of the economy. For example the coefficient on wages is $g^* \theta$ ($= g(I-F)^{-1} \theta$). This is a weighted average of the direct and indirect elasticities of the outputs with respect to labor. Other coefficients are similarly determined.

2. Institutional Forces

Given certain important results in the new microeconomics of inflation theory, especially the theorem that in a world with rational individuals the system is homogeneous of degree zero in the rate of inflation, many writers turn to institutional factors to explain observed rates of inflation. One of the most important of these is the non-synchronization hypothesis first advanced by Koopmans and later developed by Akerlof and by Phelps.¹ This is closely related to the more developed literature on income determination in open inflation and inflation in Keynesian systems which preceded the development of the Phillips curve literature.

¹Koopmans [1945], Phelps [1970b], Akerlof [1969].

The basic idea behind both types of models is that at the current level of absolute or relative prices there is no equilibrium in the economy. The agents in the economy, whether they are firms (as in the model of Phelps and Akerlof) or labor unions (as in the model of Koopmans and some of the Keynesian models), have inconsistent demands on the national output or on the level of relative prices. This is perhaps most familiar in the Keynesian models of inflation.¹ Assume money national income is divided between three income sources, wages (W_t), profits (Q_t), and a residual of rents and interest income (R_t). It is assumed that laborers and firms try to recapture their share of income, a and b respectively, while rentiers must be content with their money fixed shares. For simplicity, assume that both laborers and firms adjust prices and wages completely, but with a lag of one period. Holzman shows that a shock of demand, when the economy is in a full employment situation, will lead to an inflationary spiral until shares of the different recipients are back to their desired level. The rise of price, in this simple example, is $x/[1-a(1-b)]$, where x is the shock as a proportion of national income. If either firms or labor are slow to adjust, the height of the inflation is smaller.

The non-synchronization models of Koopmans, Akerlof, and Phelps are quite similar in that now several agents in the economy (firms, workers, or whatever) have inconsistent ideas about the desired relative prices. In the case where part of the economy can absorb the loss in income (as the rentier sector does above) or where real balance or other effects lead to a decline in demand, the inflation will eventually damp out. On the other hand, if the firms are trying to maintain constant differentials, and if they learn correctly the general trend of prices or wages, then (as Phelps has argued) the markup will be augmented by the expected rate of inflation. If people are perfectly rational, and computing time is free, then the process is unstable and will accelerate. We thus must look to some other sector of the economy to accept lower relative prices or lower incomes in order to stop the inflationary acceleration.

¹The following discussion is based on the model of Holzman [1950].

V. Recent Empirical Studies of Price Behavior*

Price behavior has only recently been the object of detailed econometric investigation. Unfortunately, it is not clear that the studies have proved fruitful. As Fromm and Taubman write about their simulations of the Brookings model, "An examination of the complete model solutions for 1961-62 reveals that the wages and prices sector is one of the larger contributors of errors in the aggregate results."¹

The present section reviews the important recent studies of the behavior of prices, with attention confined to studies of United States prices. In addition, when several sectors are studied, attention is confined to manufacturing.

The following studies are reviewed:²

<u>Year</u>	<u>Authors and Sources</u>	<u>Sectors Studied</u> ³
1959	Edwin Kuh [1959]	Corporate sector*
1965	Charles Schultze and Joseph Tryon [1965]	Durable and Nondurable Manufacturing;* Wholesale and Retail Trade; Regulated Industries; Contract Construction; Other
1968	Gary Fromm and Paul Taubman [1968]	Durable and Nondurable Manufacturing;* Wholesale and Retail Trade; Regulated Industries; Contract Construction; Other
1966	George Perry [1966]	Manufacturing*
1967	Lawrence Klein [1967] and Michael Evans [1969]	Manufacturing*
1968	Robert Solow [1968]	Private Nonfarm Deflator*
1968	Otto Eckstein and Gary Fromm [1968]	All Manufacturing,* also Durable and Nondurable Manufacturing
1970	Robert J. Gordon [1970]	Private, Nonfarm Economy*
1970	Leonall C. Anderson and Keith M. Carlson [1970]	GNP Deflator*

*I am indebted to Peter Reuter for assistance in preparation of the present section.

¹Fromm and Taubman [1968], p. 11.

²It was decided to restrict the survey to the published literature. For this reason the excellent material by George de Menil, prepared for the MIT-FRB model, has been omitted (see, for example, de Menil [1969]). de Menil's work is of particular significance because he uses a theoretical approach to deriving optimal price.

³The stars refer to the results discussed in the present paper.

A. General Form of Equations¹

There is quite general agreement about the structure of price equation to be used, although the exact reasoning and variables differ. The theoretical structure behind these equations is best laid out in Schultze and Tryon. There are three important assumptions:²

1. Prices are set as a markup over "standard" or "normal" costs. (Most studies take this assumption without discussion.) In general, this implies that price changes occur with speed and certainty when long-run costs change, but more slowly when short-run costs change.³ Most authors assume that changes in wages are considered by producers to be permanent, and therefore these enter immediately into "normal" costs. Short-run fluctuations in productivity, on the other hand, do not enter fully since these are usually transient phenomena. To smooth out short-run fluctuations in productivity, most authors take 12 quarter moving averages.

For statistical purposes most authors use unit average labor cost (total compensation/total real output) to measure labor cost. The long debate between the full-cost pricing proponents and the marginalists--who would insert marginal labor cost--is seldom mentioned. Sometimes there is a discussion of unit material cost, and, rarely, unit capital cost.

2. The second general hypothesis concerns the effect of deviations of actual from normal unit costs. Most authors postulate that these temporary changes in cost will affect prices less than permanent changes. For the most part temporary changes prove statistically insignificant.

There is, however, a serious problem which arises in the use of short-run productivity. As is well-known, short-run productivity movements tend to be primarily (and positively) related to short-run movements in output. As a result, these equations predict a decline in price when output is rising relative to trend. This prediction runs counter to prior reasoning. If the cyclical movement of productivity is known to the producers, it may be that

²The assumptions laid out here are not fully subscribed to by all authors. Deviations will be noted below.

¹The discussion in Sections V-A through V-C omits the "monetarist" version of Anderson and Carlson. The monetarist model is so different that it is discussed separately in Section V-D.

³There is, to my knowledge, no formal proof that this is optimal behavior. In general, this kind of adaptive expectations assumes that there are costs to adjusting price. As we noted above, the presence of adjustment costs has never been convincingly demonstrated.

it is the demand effect which is showing up, rather than the cost effect. (We return to this below.)

3. Finally, it is generally felt that the markup over cost is influenced by the level of output relative to capacity. This leads the authors to introduce additively several variables representing demand pressure. One might suppose that there would be an interaction effect, so that demand pressures would affect the markup over cost through the coefficients on labor, capital, and materials costs.

To get a general picture of the kind of equations customarily run, examine Table 2. As Table 2 shows, every equation in one form or another uses labor costs, either cost per manhour or cost per unit product. And almost invariably, labor costs are highly significant. Moreover, about half the equations contain a significant term in either materials costs or costs of farm products. The second striking feature of Table 2 is that, on the demand side, there is very little uniformity, with equations using capacity utilization, inventory-sales ratios, and ratios of new orders or unfilled orders to sales. This seeming inability to find a significant (and consistent!) impact of demand is quite surprising.¹

As can be seen, with the exception of the Brookings model and the Klein-Evans equation, there are no entries from the well-known, large-scale econometric models. Many models do not explicitly include price equations, but rather let prices be determined residually. This category includes Klein's early models, the Klein-Goldberger model and the Suits-Michigan model. The Wharton model uses the "Klein" price equation listed in Table 2. To date, published versions of the MIT-FRB model have not included price equations. Clearly, this sector has been troublesome.

B. Specific Variables Cost

Next let us turn to specific cost variables used in the price equations.

¹The problem of finding a significant impact of demand on prices contrasts sharply with labor markets, where excess supply makes the most important contribution.

TABLE 2. Overview of Price Equations

Author	Dependent Variable	Cost		Materials Costs	Demand			
		Short-Run	Long-Run		Capacity Utilization	Inventory Output	Unfilled-Order Sales	New Orders Sales
Kuh, 1959	p^{corp}	$.305w_{-1}^c$ (7.1)	$-.227(X/MH)^c$ (4.3)					
Schultze-Tryon (Brookings), 1965	$p^{m,d}$ $p^{m,n}$	$.371(ULC-ULC^N)$ (2.5)	$+1.845ULC^N$ (43.9) $+1.490ULC^N$ (20.7)	$+ .279p^f$		$-.059 \left[\frac{I}{X} - \frac{I}{X} \right]_{-1}$ $-.098 \left[\frac{I}{X} - \frac{I}{X} \right]_{-1}$		
Fromm-Taubman (Brookings), 1968	$p^{m,d}$ $p^{m,n}$	$.8155(ULC-ULC^N)$	$+1.1069ULC^N$ $+1.3109ULC^N$	$+ .1033p^f$		$-.2427 \left[\frac{I}{X} - \frac{I}{X} \right]_{-1}$ $-.2945 \left[\frac{I}{X} - \frac{I}{X} \right]_{-1}$		
Perry, 1966	$\Delta P^m / p^m$	$.466\Delta w^m / w^m$ (2.39)		$.344 \frac{\Delta P^f}{p^f}$ (14.9)	$-.610\Delta CU^P$ (5.0)	$+ .149CU^P$ (3.0)		
Klein-Evans, (1967)	p^m	$.542ULC^m$ (6.1)			$.246CU^w$ (6.8)			
Solow, 1968	$\Delta P^{nf} / p^{nf}$	$.2145 \frac{\Delta ULC}{ULC}$ (5.6)*		$.0371 \frac{\Delta P^f}{p^f}$ (5.0)*	$.0005CU^w$ (2.1)*	-1		
Eckstein-Fromm, 1968	p^m	$.179ULC$ (3.8)		$.079p^{ra}$ (4.9)	$.001CU^w$		$.162 \frac{\Delta O_u}{S}$ (2.7)	-1
Gordon, 1970	$\Delta P^{nf} / p^{nf}$	$.1939 \frac{\Delta ULC}{ULC}$ (5.0)	$.7531 \frac{\Delta ULC^N}{ULC}$ (15.1)				$.1518 \frac{\Delta(O/S)}{O/S}$ (4.0)	
Anderson- Carlson, 1970	$(\Delta P^{gnp})GNP_{-1}$				$\Sigma \theta_1 CU_{t-1}^{ac}$	$\Sigma \theta_1 = .09$ (9.18)		

TABLE 2 (continued)

Author	Other	Estimate Period	Durbin Watson	Standard Error of Estimate	R ²⁺
Kuh, 1959	"Demand Ratchets"	49.I-58.I	n	n	.900
Schultze-Tryon (Brookings), 1965		48.II-60.IV 48.II-60.IV	n n	n n	.974 .900
Fromm-Taubman (Brookings), 1968		48.I-60.IV 48.I-60.IV	n	n	n
Perry, 1966		47.I-60.IV	n	n	.921
Klein-Evans, 1967	.043 Korean War Dummy (7.1) + .606 $\sum_{i=1}^m p_{t-i}^m$ (8.65)	48.I-60.IV	0.60	.0098	.982
Solow, 1968	+ .4271 π_s^e (10.0) - .0047 (2.4)* Guidepost Dummy	47.I-66.IV	n	n	.9088
Eckstein-Fromm, 1968	.819 p_{-1}^m (19.16)	54.II-65.IV	1.64	.0022	.995
Gordon, 1970	.0881 $\frac{\Delta e^g}{e^g}$ (7.1)	51.I-60.IV	.93	.0053	.899
Anderson-Carlson, 1970	.86 Δp_{ac}^e (8.55)	55.I-69.IV	1.41	1.07	.87

t-statistics in parentheses.

* = t-statistics conditional on price expectations parameter.

+ = sources do not generally note whether R² is corrected for degrees of freedom.

n = not given.

Key to Table 2. The regression selected in each case is the one which has the lowest standard error of estimate. The variables used are as follows:

Dependent variables:

- p^{corp} = Price deflator for corporate output
 $p^{\text{m,d}}$ = Deflator or wholesale price index for durable manufacturing
 $p^{\text{m,n}}$ = Deflator or wholesale price index for nondurable manufacturing
 p^{m} = Deflator or wholesale price index for manufacturing
 p^{nf} = Private nonfarm deflator
 p^{gnp} = GNP deflator

Labor costs:

- w^{c} (Kuh) = Average hourly earnings in the corporate sector
 $(X/MH)^{\text{c}}$ (Kuh) = Output per manhour in the corporate sector
 ULC (Schultze-Tryon) = Compensation of employees/gross product originating
 ULC^{N} (Schultze-Tryon) = Compensation per manhour / $\sum_{-11}^0 \frac{1}{12} \left[\frac{\text{output}}{\text{manhour}} \right]_{-i}$
 ULC (Fromm-Taubman) = Compensation per manhour/real gross product originating
 ULC^{N} (Fromm-Taubman) = Compensation per manhour / $\sum_{-11}^0 \frac{1}{12} \left[\frac{\text{real output}}{\text{manhour}} \right]_{-i}$
 w^{m} (Perry) = Wage rate of straight time hourly earnings
 ULC^{m} (Klein) = "Unit labor cost"
 ULC (Solow) = Private wage bill/real private nonfarm output
 ULC^{N} (Eckstein-Fromm) = $a e^{bt}$, a , b regression coefficients
 ULC (Gordon) = Compensation in private nonfarm economy/private nonfarm output (adjusted by Gordon)
 ULC^{N} (Gordon) = $\sum \theta_i ULC_{t-i}$, θ_i Almon weights

Material costs:

- p^{f} = Farm deflator (several different versions used)
 p^{r} = Raw materials price index
 p^{ra} = Faith Halfter Ando's raw materials price index

Capacity utilization:

- CU^{P} = (Perry) FRB index of manufacturing production/Commerce Dept. Real Net Value of Structures and Equipment
 CU^{W} = Wharton index of capacity utilization
 CU^{ac} = $[GNP_t p_t^{\text{gnp}} - GNP_{t-1} p_{t-1}^{\text{gnp}}] - [GNP_t^{\text{F}} - GNP_{t-1}^{\text{F}}]$

Other variables:

I/X = Ratio of inventories to output originating

(I/X) = Trend or 12 quarter moving average of (I/X)

(Ou/S) = Unfilled orders/sales

(O/S) = New orders/sales

π_s^e = (Solow) $\theta \sum_{i=0}^T (1-\theta)^i \pi_{t-i}^{nf}$, where π^{nf} is annual rate of change of p^{nf} . Series starts in 1929.

e^g = (Gordon) Total employment rate

GNP = Gross National Product, constant dollars

GNP^F = Potential Output (Council of Economic Advisers), constant dollars

$\Delta p_{ac}^e = GNP_{t-2} p_{t-2}^{gnp} \left\{ \left[\left(\sum_{i=1}^{17} \theta_i \frac{\pi_{t-i}^{gnp}}{u_{t-i}} \cdot 4 \right) \cdot 0.01 + 1 \right] \frac{1}{4} - 1 \right\}$, where $\pi^{gnp} = \Delta p^{gnp} / p^{gnp}$, u is unemployment rate, θ_i are Almon weights.

1. Labor costs.

As we noted above, every equation (except the monetarist) includes some form of labor costs. All but Perry use unit labor costs, but the lag structure varies widely among different equations. It is customary to define unit labor cost as $ULC = (Comp/X) = \text{Total compensation of labor/Real Output}$.¹ This can be further defined as $ULC = (Comp/MH)(MH/X)$, where MH = manhours. Great care is not always taken to make sure that the labor cost refers to the same sector as the price index, but this can be rationalized as a short-coming of the data. Many authors do not include productivity explicitly (i.e., MH/X is omitted) and this can perhaps be understood as implicitly assuming productivity is growing smoothly.

In any case, we can calculate the estimated effect of ULC on prices explicitly for most of the regressions, and these are presented in Table 3.² Given that the unit labor cost is the only uniformly present variable, it is hard to rationalize the range of long-run elasticities (.3744 to 1.845). The most likely culprit is omission of correlated variables, such as raw materials prices or capital costs. It is comforting to note that the two lowest elasticities (Solow, Perry) all include a measure of raw materials prices. Table 6 gives the lag structures on unit labor costs in different equations, while Chart 1 shows graphically the lag structures on the compensation term. It is clear that very little consensus on lag structures has been reached by the surveys. It is slightly puzzling to note that several authors assume that reactions to wage changes are instantaneous. A general argument has been made for at least some delay in reaction to costs. Moreover, the evidence is reasonably strong that some delay in reaction to changes in unit labor costs exist (see Gordon's estimates) but it is not clear that this delay holds for wages as well as productivity. It would therefore seem better to include some lag in compensation.

¹The Schultze-Tryon price equations list ULC as compensation per unit of current dollar output (see p. 286). I presume this should read constant dollars.

²It should be warned that, due to the hesitance of authors (or editors?) to report all the summary statistics, it is exceedingly difficult to compare different equations. As can be seen in Table 2, standard errors, means of variables, and elasticities are virtually never mentioned. Other embarrassing statistics, like Durbin-Watson Statistics, are also often ignored.

TABLE 3
Elasticities of Price with Respect to Unit Labor Costs

	<u>Immediate</u>	<u>Long-Run</u>	<u>Equation Number</u>
Kuh (p^{corp})	[.227 - .305] ^{b, c}	[.227 - .305] ^{b, c}	
Schultze-Tryon			
($p^{\text{m, d}}$)	[0.492 - 1.845] ^{a, c}	1.845 ^a	(9.7)
($p^{\text{m, n}}$)	1.490 ^a	1.490 ^a	(9.8)
Fromm-Taubman			
($p^{\text{m, d}}$)	[.839 - 1.1068] ^{a, c}	1.1068 ^a	(A.57)
($p^{\text{m, n}}$)	1.3109 ^a	1.3109 ^a	(A.58)
Perry (p^{m})	0.466	0.466	(5.8)
Klein (p^{m})	[0.542] ^b	[1.375] ^b	p. 92
Solow (p^{nf})	.2145	.3744	
Eckstein-Fromm (p^{m})	.179	.989	2-(4)
Gordon (p^{nf})	.1939	.9470	(1)
Anderson-Carlson (p^{gdp})	0.0 ^d	0.0 ^d	Table II

^a Since $p = 1.0$ for 1954, the coefficient is approximately the elasticity. No mean of dependent variable is given.

^b Number is coefficient. Since no mean of dependent variable is given, elasticity cannot be calculated.

^c Higher figure related to increase in wages; lower figure is for decrease in productivity.

^d A priori coefficient.

The lag structure on average productivity hardly seems better. Some authors use a 12-quarter average to determine "normal" productivity.¹ There is no reason to expect a rectangular distribution and, again, Gordon's results would tend to argue against it.

¹ Schultze-Tryon and Fromm-Taubman.

TABLE 4

Lag Distribution of Variables in Unit Labor Costs
(Total Sums to Unity)

		Period (quarters)											
		t	t-1	t-2	t-3	t-4	t-5	t-6	t-7	t-8	t-9	t-10	t-11
Kuh													
Schultze-Tryon:													
<u>Comp</u>	Durable	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
<u>MH</u>	Nondurable	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
<u>X</u>	Durable	0.267	0.067	0.067	0.067	0.067	0.067	0.067	0.067	0.067	0.067	0.067	0.067
<u>MH</u>	Nondurable	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083
Fromm-Taubman:													
<u>Comp</u>	Durable	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
<u>MH</u>	Nondurable	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
<u>X</u>	Durable	0.730	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
<u>MH</u>	Nondurable	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083
Perry,	Comp/MH	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Klein,	Comp/X	.394	.15	.15	.15	.15	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Solow,	Comp/X	.573	.03	.026	.024	.021	.019	.017	.015	.014	.013	.011	.011
Eckstein/Fromm													
	Comp/MH	.181	.148	.121	.100	.081	.067	.054	.044	.036	.030	.025	.020
	X/MH	.181	.148	.121	.100	.081	.067	.054	.044	.036	.030	.025	.020
Gordon,	Comp/X	.548	.254	.143	.053	-.006	-.033	-.034	-.018	.002	.016	.017	.008

One might ask, finally, how the results surveyed here fit in with the theoretical results discussed in earlier sections. For purposes of concreteness, we will assume that production is by a Cobb-Douglas production function with constant returns to scale. In this case, for individual firms, the coefficient on unit labor costs should be approximately equal to (wage bill/total sales). For a sector (such as manufacturing) the "total sales" should refer to sales outside of the sector (see Section V-A). It is not easy to get an estimate of the relevant total sales figure to go in the denominator. Under highly simplified assumptions, however, it is approximated that the elasticities of labor, capital, and materials are as follows:¹

	<u>Labor Share</u>	<u>Capital Share</u>	<u>Material Share</u>
Nondurable manufacturing	.24	.08	.68
Durable manufacturing	.40	.13	.47
Total manufacturing	.32	.10	.59
Private nonfarm	.61	.19	.20
GNP	.73	.22	.05

The estimates are not firm, but they do give a rough idea of what one should expect from econometric estimates of long-run elasticities. The comparison of the a priori figures with Table 5 indicates that the estimates are not only widely dispersed, but also far from the a priori estimate on average. In general, the estimated elasticities are not far from unity. This upward bias is what would be expected from estimation which omits variables (such as materials and capital costs) which are positively correlated with unit labor costs.

¹The figures for GNP are from the national income accounts, while for manufacturing (including mining) is from Input-Output Tables (see Liebling [1955], p. 312). The other estimates are made as follows. The share of labor, capital, and imports as a fraction of national income are the numbers given for GNP. To get the figures for other sectors, it is assumed that the proportion of sales of sector *i* going to sector *j* is proportional to total sales of sector *j*. The prediction by this method is .48 for manufacturing, which compares with the total of .59 for manufacturing from Input-Output Tables.

2. Materials Costs

Although raw materials prices are clearly a component of costs, most authors chose to exclude them. This omission is partly due to the fact that (at least up until recently) good indexes of prices for inputs into different sectors have not been available.¹ Some authors use farm prices (or the farm deflator) under the presumption that this is a good proxy for the correct raw materials price index; it would probably be more suitable to use the farm deflator as an instrumental variable in the formal sense.

Table 5 shows the estimated elasticities of price with respect to raw materials. As can be seen, the estimates range from a low of .0647 for Solow to a high of .436 for Eckstein and Fromm. It is difficult to rationalize the use of the farm deflator for a serious estimate of the effect of raw materials. It may well be that omitting the Korean war period also gives lower estimates of the elasticity.

TABLE 5
Elasticities of Prices with Respect to Raw Materials Prices

		<u>Long Run Elasticity</u>	<u>Variable</u>
Schultze-Tryon	$(p^{m,n})$.279 ^a	Farm deflator.
	$(p^{m,d})$	[0.0] ^b	
Fromm-Taubman	$(p^{m,n})$.1033 ^a	Implicit deflator for value of cash receipts from farm markets, CCC loans, and income in kind.
	$(p^{m,d})$	[0.0] ^b	
Perry	(p^m)	.344	Index of crude nonfood materials for manufacturing, except fuel.
Eckstein-Fromm	(p^{ra})	.436	F.H. Ando index of raw material prices.
Solow	(p^{nf})	.0647	Farm deflator.

^aSee note a, Table 3.

^bThis variable was apparently excluded when it proved insignificant.

^cEquation (2)-(2).

¹Eckstein and Fromm use "special indexes prepared by Dr. Faith Halfter Ando. These indexes are composed only of crude raw materials. The published materials price indexes are not suitable for price equations because they include a considerable share of semi-finished goods which are actually value added of manufacturing." Eckstein and Fromm [1968], p. 1167.

How do these elasticities square with theoretical results of Section IV? Recall that (in the Cobb-Douglas case) the estimates elasticity should be approximately the share of raw materials. For manufacturing, the share of raw materials is approximately 50 percent. It would appear, therefore, that (except for the Eckstein-Fromm estimates) the estimates in Table 7 for manufacturing are inconsistent with a Cobb-Douglas production function. The observed elasticities imply the elasticity of substitution between raw materials and other inputs is significantly larger than unity. Solow's estimate, on the other hand, is probably not far off for non-farm product.

3. Capital Costs.

As we noted above, none of the studies surveyed here introduce capital costs; most, indeed, do not even discuss the question.¹ This lapse is explained as follows by Eckstein and Fromm:²

The traditional version of the classical theory of the firm calls for no direct influence of the size of the capital stock on short-run, profit maximizing, price-output decisions; the capital stock makes itself felt through the short-run cost curve....Alternatively, short-run cost can be defined to include the quasi-rent on capital, with the quasi-rent varying with the rate of utilization. However the traditional exposition does not solve explicitly for the quasi rents and hence leaves the influence of the utilization of capital vague.

While the omission of capital costs can be rationalized in "the classical theory of the firm" as determining the level of the short-run cost curve, but not changes in the short-run cost curve, most authors rely on markup or target-return pricing as the theoretical underpinning of the empirical work. In the target-return pricing, changes in unit capital costs are as legitimate a part of costs as unit labor and unit materials costs. Although they recognize capital should be included, Eckstein and Fromm argue: "However,

¹This refers to the equations discussed above. Schultze and Tryon introduce unit capital consumption allowance into regulated industries with good results. Eckstein and Fromm introduce an after tax rate of return in the durables equation, but the negative sign indicates that they are getting the result which comes from fitting the profit identity.

²Eckstein-Fromm [1968], p. 1163.

the available time series on capital are not sufficiently precise to reveal the inevitably small quarterly changes in [the capital-normal output ratio], and so this approach could not be used."¹

In light of recent work in investment theory, it is surprising that no attempt has been made to incorporate a "unit capital cost" variable into the analysis of price behavior. While it is true that movements in the ratio of book-value or replacement cost of capital to normal output would move quite slowly, the same cannot be said about unit capital cost. In the familiar neoclassical analysis, unit capital cost is

$$UCC = \frac{qK}{X}$$

where $q = (r + \delta)p_K + T(t)$, where r is the relevant interest rate, δ is the depreciation rate, and $T(t)$ are functions of tax rates (especially investment tax credit and depreciation allowances). Part of q will reflect "quasi-rents," and thus will be irrelevant to the classical competitive firm; part will reflect user-cost (in Keynes' sense) and thus will be relevant to all firms. But for the true target-return firm, the total unit cost of capital (UCC) should enter in pricing decisions.

Once this is realized, the claim that UCC does not move much from quarter to quarter is easily perceived to be incorrect. The q term moves mightily when changes in tax legislation occur. Thus, for example, using the theoretical model in section IV, it would be expected that the investment tax credit of 1962 would lower long-run price by about 15 percent ($= .07 \times .25$), and that its repeal in 1969 would raise long-run price by the same percentage.² It is difficult to resist remarking that this a priori estimate is very close to Solow's estimate using the guidepost dummy. The result in this case would not be a result of the guideposts, but rather a result of the investment tax credit which was invoked (and revoked) about the same time.

¹Ibid., p. 1198.

²For this example we note that the investment tax credit applied to the entire economy.

C. Specific Variables: Demand

As we noted above, demand variables do not show up consistently and significantly. The following successes have been reported.

1. Capacity utilization.

The only variable which has been tried and proved in several equations is an index of capacity utilization. Eckstein and Fromm, Solow, and Klein report success using the Wharton index of capacity utilization, while Perry uses an index of the output-capital ratio. The only elasticities reported or easily derived are given in Table 6.

TABLE 6
Elasticity of Price with Respect to Capacity Utilization

<u>Author</u>	<u>Long Run Elasticity</u>
Perry (p^m)	.15
Solow (p^{nf}) ^a	.166
Eckstein-Fromm (p^m)	.477

^aSolow gives no mean of dependent variable, but it seems clear that he uses the series as published which has a mean of about 87.

It is not clear from the estimates how responsive price is to demand changes. Since a "normal" recession involves perhaps a ten percent change in the Wharton capacity utilization index, expected decline (relative to what would otherwise occur) is between 1.5 and 4.8 percent in prices. There will also be effects through decline in unit labor costs. This effect is not insubstantial.

2. Inventories.

The Brookings model has consistently used the inventory-sales ratio (more precisely the deviation of the ratio from a 12-quarter moving average) as an explanatory demand variable.¹

¹Eckstein and Fromm appear to use the same variable but report no results. They state the capacity utilization is superior statistically to other demand variables which they tried.

3. Orders.

Orders variables have also been used as a demand variable. Gordon uses the rate of change of the (new orders/sales) ratio and reports favorable results. Eckstein and Fromm use the lagged change in the (unfilled orders-sales) ratio and get barely significant results.

4. Actual Unit Labor Costs: A Hidden Demand Variable?

As we have noted above, some authors include actual unit labor costs (or current compensation per manhour) as part of the cost side of full-cost pricing.

Actual unit labor cost is $w_t MH_t / X_t$. It is quite clear that both output (X_t) and productivity (X_t / MH_t) are variables which are cyclically related. Moreover, it is probably more correct to identify changes in output with changes in demand than with productivity. Therefore, it seems useful to inquire how much of the power of actual unit labor costs is demand related and how much cost related. Since the two have opposite signs, estimates of the effect of unit labor cost should be biased down unless the separate and correctly specified demand effect is excluded. The short-run bias may explain the fact that running price equations in first differences gives substantially lower coefficients on actual unit labor costs than running in levels.

D. Specific Variable: Miscellaneous

A few other variables have been thrown into the kettle from time to time.

1. Guidepost Dummy.

Solow introduces a "guidepost dummy" to represent that "the wage-price guidepost effort might have had some effect on restraining the rise in manufacturing prices."¹ He reports favorable t-statistics for the dummy variable, and these suggest that guideposts might have restrained prices by 1/2 of one percent.²

¹Solow, p. 18.

²This result compares with the completely independent estimate by Brainard and Lovell [1966] that preventing the rise in steel prices kept prices 0.1 to 0.3 percent lower.

The dummy variable approach to the guideposts lacks elegance and conviction. Moreover, Gordon reports that in his equation the guidepost dummy is insignificant.¹

2. Expected Rate of Inflation.

As an attempt to separate out changing price expectations from the effect of guideposts, Solow also introduces the expected rate of inflation. He uses the model of adaptive expectations introduced by Cagan, and (apparently) calculates the "expected rate of inflation," $(\dot{p}/p)^e$, as

$$\left(\frac{\dot{p}}{p}\right)^e = \theta \sum_{i=0}^T \left(\frac{\dot{p}}{p}\right)_{t-i} (1 - \theta)^i + \left(\frac{\dot{p}}{p}\right)^e_{t-T}.$$

He then estimates his price equation for θ running from 0.0 to 0.9. He finds that the maximum likelihood is at $\theta = 0.9$, which implies a very short lag. On the other hand, the standard error of θ at $\theta = 0.9$ is apparently fairly large.

It is difficult to know what to make of Solow's result because he gives no theoretical reason for including price expectations in the price equation. As has been suggested by several authors,² it might be appropriate to include them in the wage equation, as Gordon [1970] does. Inclusion in the price equation is more difficult to understand. The most plausible reason is that Solow is running a "reduced-form" price equation, having solved the price and wage equations.

¹Gordon [1970], p. 17. Note that because of the particular method Solow used (searching over a grid), his t-statistic on the guidepost dummy overestimates the correct t-statistic. His estimate is conditional on θ (the rate of adaptation of price expectations) while the correct t-statistic is unconditional on θ . The correct t-statistic would be obtained by calculating the sum squared residuals when searching over θ for D included and D excluded and then calculating the appropriate F-statistic.

²Especially E.S. Phelps.

3. Rate of Growth of Employment Rate.

Gordon includes the rate of change of "the employment rate" in his price equation. His reasoning is as follows:¹

In a classical view of the labor market, firms equate the marginal product of labor to the real wage of workers. And the marginal product declines at high rates of employment as more workers are added to a relatively fixed stock of capital. Thus an increase in employment can be achieved only if the real wage is reduced, which, for a given nominal wage rate, requires a price increase. In short, the ratio of prices to wages at standard capacity becomes an increasing function of the employment rate, and the employment rate therefore becomes a variable in the price equation.

It is difficult to see exactly what Gordon is getting at. It might be assumed that equilibrium real wage would be lower at high rates of employment, but why should firms raise price in response to higher employment? If price is raised because wages are higher, then this would be reflected by higher unit labor costs; if because productivity is lower, then this should show up in unit labor costs through lower productivity.

Whatever the theory, my casual guess about why the employment rates shows up is that it is (through Okun's law) a proxy for changes in capacity utilization. Since capacity utilization has proved quite successful, the employment rate should also be a good variable.

E. A "Monetarist" Price Equation

Although the so-called "monetarist" school has been extremely active on many fronts, only recently has a formal entry been put into the competition for best price equation. The study by Anderson and Carlson [1970], which is primarily designed as a model for explaining short run movements in GNP, also contains an explicit and novel price equation. Their equation contains two variables, a "demand" variable and a price "anticipations" variable (see Table 2 and the key to Table 2 for the exact definitions).

The general assumption behind the price equation is that price is determined by the intersection of an aggregate demand and supply curve.² It

¹ Gordon [1970], p. 13.

² The description is contained in Anderson and Carlson [1970], Appendix A.

is assumed that "the observed values fall on the supply line." From this it is easily seen that change in price is a function of change in demand and change in supply. It does not seem possible, however, to reconcile the description of the assumptions with the conclusion the authors' reach.¹

A second assumption is that the price anticipations term moves up the supply curve. Here, again, there seems to be a minor misspecification for they add in price anticipations directly, whereas the price response should be a function of the slopes of the curves.²

We have noted that the "demand" variable actually used by Anderson and Carlson does not square with the theoretical specification. The price anticipations variable is also difficult to understand. Recall that anticipated price change is

¹An algebraic version runs as follows. The supply function is $p = S(X, X^F)$, while demand is $pX = \bar{Y}$, where \bar{Y} is predetermined by the total spending equation. They assume that the slope is a linear function of $(X^F - X)$; therefore $\frac{dp}{dX} = \alpha_0 + \alpha_1(X^F - X)$. Integrating this yields $p = \alpha_0 X + \alpha_1 X X^F - \frac{1}{2} \alpha_1 X^2 + \alpha_2$. Inserting the demand relation into the supply relation yields (i):

$$(i) \quad p = \alpha_0 \bar{Y}/p + \alpha_1 \frac{\bar{Y} X^F}{p} - \frac{1}{2} \alpha_1 \left(\frac{\bar{Y}}{p} \right)^2 + \alpha_2,$$

which is of third degree in p . The form of the equation "derived" by Anderson and Carlson is

$$(ii) \quad \Delta p = \beta_0 + \beta_1 \{ \Delta \bar{Y} + (X - X^F) \}.$$

How can this be derived from (i) above is an interesting puzzle. In fact, (ii) does not appear integrable. Moreover, the figure on p. 22 of their article appears to assume price is a linear function of $1/(X - X^F)$ (much in the spirit of Phillips curves).

²Thus if a and b are the price-elasticities of demand and supply, and c is the percentage rise in the supply curve for each percentage rise in anticipated price, then the coefficient on anticipated price in the reduced form should be $-\frac{bc}{a+b}$, not simply c . Since by assumption the price elasticity of supply (b) is a function of $(X - X^F)$, so is the term $(-bc/a+b)$.

$$\Delta P_{ac}^e = GNP_{t-1} \left\{ \left[\left(\sum_{i=1}^{17} \theta_i \frac{\pi_{t-i}^{gnp}}{u_{t-i}/4} \right) \right] .01 + 1^{\frac{1}{4}} - 1 \right\} .$$

In the first place the weights in the anticipations (the θ_i) are derived from the long-run interest rate equation. There is no explanation for this; one could, perhaps, argue that the same price expectations should hold for firms setting prices as for owners of securities. The second unusual feature is that Anderson and Carlson divide the rate of inflation by the ratio ($u_{t-i}/4$), where u_{t-i} is the percentage unemployment rate. Their explanation is as follows:¹

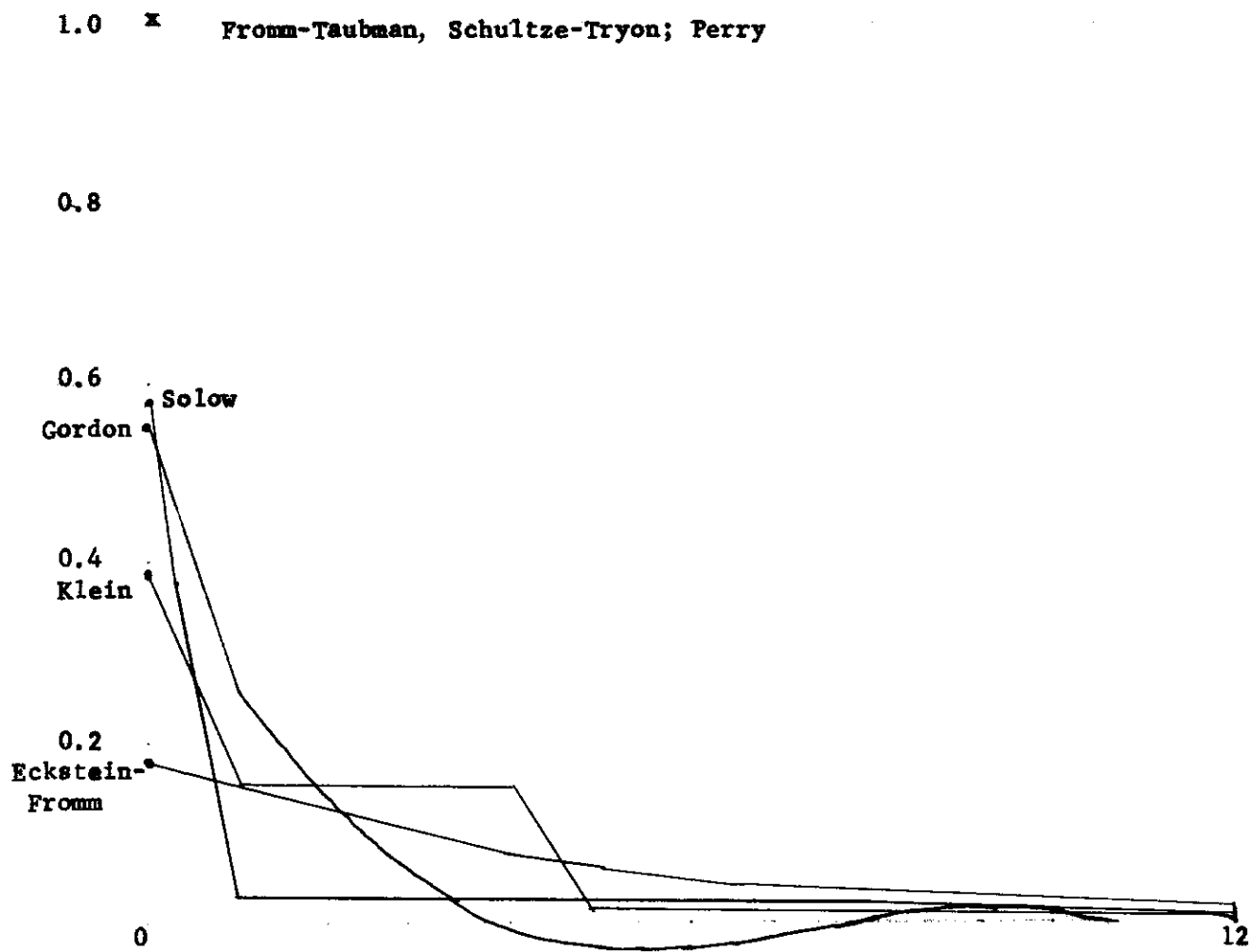
In the process of constructing a measure of anticipated price change, past changes in prices are adjusted by a summary measure of current economic conditions. Since price changes tend to lag changes in total spending, the degree of resource utilization as measured by the unemployment rate is used as a leading indicator of future price movements. For example, if unemployment is rising relative to the labor force, decision-making economic units would tend to discount current inflation in forming anticipations about future price movements. Reflecting this consideration, the price change in each quarter is divided by an index of the unemployment rate applicable to that quarter. Thus the measure of price anticipations would be less for a given inflation rate accompanied by high or rising unemployment than when unemployment is low or falling.

This is an interesting addition to the literature on price expectation.² Unfortunately, it is fallacious as it stands. Surely, if they are trying to take into account the change in conditions, the change in the unemployment rate (ΔU) should be used. In fact, U and ΔU have only a $R^2 = .28$ for the post war period. This error casts serious doubt on this price equation, for the term ($4\pi^{gnp}/u$) is highly correlated with the general tightness of the economy, in much the same way the ($1/u$) is so correlated in the Phillips curve analysis. It would appear that the monetarists have let the Keynesians in through the back door.

¹Ibid., p. 13.

²The same argument was used by Lipsey [1961] in his discussion of Phillips curves.

FIGURE 2. LAG STRUCTURES ON COMPENSATION



Source: Table 4.

Since the proof of the pudding is in the eating, the final question about the monetarist price equation is how well it fits the data. Because of the peculiar form of the equation,¹ it is impossible to compare on the basis of the data supplied in the article. On the other hand, both an inspection of the chart (on p. 17), a comparison with the Wharton model (p. 16) and the root mean error for the 1968-1969 period reveals an unimpressive fit within the period.² The root mean squared error of the price level for 1963-1965 (a period of the authors' choice) is 0.60 for the monetarist model and 0.33 for the Wharton model.³

In summary, the major innovation in the monetarist price equation is an attempt to explain price behavior on the basis of a demand variable and price anticipations without any cost variables. In the current form, the results are largely unsatisfactory.

General Observations

From this survey of recent econometric equations of price behavior I would conclude with the following observations.

1. Most of the specifications and interpretations have proceeded without the benefit of formal theory. As a result, the implicit elasticities of price with respect to different costs are difficult to understand. The simple theoretical apparatus outlined in Section IV above indicates that (under conditions where production is roughly Cobb-Douglas) the elasticities should be approximately equal to the relative shares. In practice, the estimated elasticities range from .3744 to 1.845 for labor costs and from .0647 to .436 for materials costs. Moreover, capital costs are usually omitted from the analysis, even in the long-run analysis. One

¹The dependent variable is actually $(\Delta p)GNP$, rather than Δp . This form is due to the authors' desire to explain the change in GNP due to price changes. The authors could circumvent many of the problems by using logarithms of variables.

²The estimates are "ex post dynamic simulations," according to Anderson and Carlson.

³In recent forecasts, the model predicted 4.9 percent for 1970-I (versus the actual 6.8) and 4.7 percent for 1970-II (versus 4.0).

would hope that by putting a heavier weight on theoretical specification, and a smaller weight on goodness of fit, that the results might coincide more closely with what theoretical specification would indicate.

2. A second problem revealed by this survey is the wide disparity in approaches to the lag structures. As can be seen in Table 4 and Chart 1 above, most authors assume that compensation affects price with no lag, while productivity is smoothed over as much as 12 quarters. The only author who allows flexible lags (Gordon) does not, however, allow for different lags on compensation and productivity, so the hypothesis of different lags is not really tested.

3. The equations reviewed above make it plain that very little is known about the structure of the impact of demand on prices (apart from the effect through the unemployment rate on wages). The only variable which appeared in several studies was capacity utilization, which had an elasticity between 15 and 45 percent. It might be suspected, however, that the capacity utilization effect could be confounded with the productivity effect, as well as simultaneous equation bias, since in most equations capacity utilization entered in an unlagged form. In any case, given the wide disparity of results, forecasters should probably be wary of inclusion of most demand variables (except capacity utilization) because of the high probability of spurious correlation. The inclusion may perhaps account for some of the high unreliability of wage and price forecasts noted at the beginning of this section.¹

¹One important piece of evidence which may shed light on the problem of demand is a recent study by George Hay [1970]. Hay investigated a model of pricing in which the firm simultaneously set prices, production, and inventories. He shows that, for plausible parameter values, most of the response to changes in demand will be met by increases in production and buildup of unfilled orders. His contention seems to be borne out by regression results, although he has not given the complete solution in which the long-run impact of demand on prices can be determined.

4. Finally, there are some disturbing details of econometric technique. In most of the studies, very little attention has been given to the structure of errors. It has apparently become customary to use 4-quarter differences for variables in price and wage equations. This custom follows the work of Dow and Dicks-Mireaux. The following explanation is given by Perry,¹

If wage negotiations are spread evenly throughout the year, one fourth of all wages will be negotiated in each quarter. In a given quarter, this will result in a change in an aggregate wage index about one fourth as great as the change in those wages that were actually negotiated that quarter. The change in an aggregate wage index over a year will span four such quarters....All the preceding variables are used as four quarter averages.

Solow, Perry, and Eckstein-Fromm are quite explicit about using this form for price behavior, while Gordon is ambiguous on this count. However reasonable using four-quarter moving averages for wage equations may be,² the same rationale does not exist for prices since there is no general pattern of setting prices for contractual periods of one year.

It is also noted that authors are quite casual about whether they use prices in levels or in first logarithmic differences. Given the upward trend of prices, it might appear more logical to run regression in the logarithms, or the first differences of logarithms, rather than in natural units.

Moreover, given the evident serial correlation in the residual pattern in the level equations, it would appear that much more attention should be given to the structure of errors. As Eckstein and Fromm's results indicate, taking the first difference of an equation can have drastic results on the coefficients of an equation.³

¹Perry [1966], pp. 31-32.

²In a private communication, George de Menil has indicated that explicit tests of the structure of errors casts serious doubts on the validity of Perry's technique.

³The single most important coefficient is on unit labor costs. It is disturbing to note that taking first difference of their standard equation [equation (2)-1] lowers the sum of coefficients on ULC^N from 0.810 to 0.390.

There is also a persistent habit of omitting outlying observations (Korean War, early postwar, periods of high demand).

Finally, none of the authors except Perry¹ give any attention to the simultaneous equation bias. This is especially disturbing, given the inclusion of current output and current wages in almost all equations.

¹Perry gives estimates of a system of equations both by ordinary least squares and two-stage least squares. The estimates are extremely close for the price equation. The problem is that raw materials prices and capacity utilization are taken as exogenous, so that the test is hardly conclusive.

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