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AN ECONOMIC THEORY OF TECHNOLOGICAL CHANGE

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I. Introduction

It is commonplace to assert that most of the growth of output is due to technological change. It is more accurate to state that the growth of cannot explain most of the growth of output. At the present time there is no compelling empirical evidence pointing toward technological change rather than associating increases in productivity with economies of scale, learning by doing, errors of measurement, or even sunspots. The failure to explain the residual may be due to the absence of an adequate theory -- especially microtheory -- of the generation and transmission of new knowledge. Once a reasonable model of the inventive process has been developed and tested, it should be easier to sort out the sources of productivity change among the different claimants. The present paper is a crude attempt to accomplish this task. The first 5 sections of the paper present an economic model of the inventive process and discuss its properties. Section 6 then applies the model to two industries in order to test its predictions.

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II. The Setting of the Problem

Although technology has long been recognized as an important part of the economic scene, invention has not been fully integrated into economic analysis. (Invention will be used as a general term for activities which expand the level of technical knowledge.) In fact, the "Schumpeter tradition", -- treating invention as an exogenous force acting on the economic system -- led to widespread neglect of fundamental relations between invention and economic activity. Following the lead of the late Jacob Schmookler, important connections have been uncovered, indicating that over half of all patented inventions are assigned to profit-oriented corporations and that time series on patents show striking cum-cyclic correlations. This evidence has led most analysts to abandon the Schumpeter tradition.

Recent work integrating invention into conventional economic analysis (such as [1], [4], [11]) has given us a deeper understanding of the important problems in the economics of invention. These studies have highlighted: (1) the high degree of uncertainty residing in the outcome of inventive activity; (2) the public good character -- or inappropriability -- of inventions, except under extreme legal arrangements like a patent system; and (3) the indivisibility of invention, meaning that once a new process has been discovered it can be spread to all firms at (virtually) zero marginal cost.

It is essential that these distinctive properties of information be included in a serious model of the inventive process. In general, models investigating productivity (for example, Lucas [5] or Griliches and Jorgenson [2]) treat inventions like any old capital good, with no problems of indivisibility or inappropriability. If the work mentioned in the last paragraph is taken seriously, then the capital-theoretic models of invention do not capture the important characteristics of invention.

The model analyzed and tested here attempts to cope with the problems of invention in a way quite different from the capital-theoretic view. The scenario is roughly as follows. The economy is a neoclassical in all regards except in the production of knowledge. There are two conventional factors of production, capital and labor, which are combined to produce output by an aggregate production function. Labor, capital, and output markets all behave competitively.

There are several new twists which are introduced to generate the model of invention: (1) Inventions are produced within the system. An invention is viewed as a new process of production, or as a new vector of input-output coefficients.

(2) Any invention is potentially a public good in the sense that it is indivisible, or that it can in theory be used universally at zero marginal cost. In practice the spread of inventions can be impeded for some time by inventor secrecy or by legal arrangements such as exclusive rights to trade secrets or to patented inventions. In the model we assume that the inventor has exclusive rights to use and/or license the invention for T years, after which the invention enters the public domain

as public knowledge. T can be interpreted indifferent ways, according to context as the life of a patent the average lead-time, or the length for which secrets can be held.

(3) The most important point is that from an economic point of view the inventor has a monopoly over the invention for T years. He can either produce output or license the invention to other producers. After T years, his monopoly vanishes, and anyone can use the invention.

It should be stressed that monopoly over information is essential for a sensible treatment of invention, when invention is a public good. There is no more sense to having a "competitive" market for a blueprint for the xerography process than a competitive market for nation defense.

In sum, the model used here is a traditional neoclassical model except for the introduction of a multitude of temporary little monopolies on information. It resembles in many ways the Schumpeterian view of the process of capitalist growth, with inventors taking the role of entrepreneurs and getting all the surplus. Instead of entrepreneurs getting the profits from introducing "new combinations" or innovations (as in the Schumpeterian scheme) inventors get profits from inventions. As will become apparent, this changes the shape of the capitalist economy in some interesting ways.

III. The Model

In setting down the model we will first give the conventional framework to be retained -- equations (1) and (2) -- and then give the technology and market for inventions equations (3) through (10). Important variables are as follows:

A = Level of productivity	R = Total inventive inputs
C = Consumption	T = Life of monopoly on invention
D = Cost per invention	V = Value of invention
I = Net investment	w = Wage rate
K = Capital stock	z = Aggregate savings rate
L = Labor	α = Output elasticity of capital
N = Number of inventions	β = Elasticity of $\hat{A}$ with respect to N
p = Price level	$-\gamma$ = Elasticity of $\hat{A}$ with respect to A
q = Price of capital service	μ = Share of inventors
Q = Output	π = Rate of population growth
r = Interest rate	ϕ = Discount factor
s_i = Per unit royalty on i^{th} invention	$\sigma = 1/(1 - \beta)$
S = Total royalties per unit of output	

Output is produced by a Cobb-Douglas production function:

$$Q = AK^{\alpha}L^{1-\alpha} \tag{1}$$

where A is the level of Hicks-neutral technological change.[2] Output is devoted to three uses: consumption (C), net physical investment (I), and research or inventive inputs (R). We assume that a constant fraction of income, $(1-z)$, is consumed. Labor grows exponentially at rate π. Since all conventional markets are competitive, factor rewards are equal to marginal physical product times net price, i.e. market price less royalty payments.

Since the net rental on capital is $q = rp$, the equilibrium level of capital is given by

$$K = \frac{\alpha Q}{r}\left(\frac{p-S}{p}\right) . \tag{2}$$

So much for the conventional markets. We now examine the market for invention. First, we find the level of royalties for a given profile of past inventions. This is quite simple under the assumption that <u>inventions are technically independent</u>. More precisely, we assume that the absolute contribution per invention to total productivity, A, is independent of whether other inventions are used. This implies that the i^{th} (of m inventions) contributes ΔA_i to productivity independently of whether inventions $j \neq i$ are used.[3]

Under this condition it is easily shown that the royalty on the i^{th} invention as a proportion of market price equals the ratio of the change in productivity due to the i^{th} invention to level of productivity:[4]

$$\frac{s_i}{p} = \frac{\Delta A_i}{A} \tag{3}$$

Using (3) we can derive an extremely interesting result. Add together the per-unit royalties for all unexpired inventions, i.e. all inventions less than T years old (call them inventions $1, \ldots, m$), and let S be the sum of royalties on these inventions. Using (3) we have:

$$\frac{S}{p} = \sum_{i=1}^{m} \frac{s_i}{p} = \frac{\sum_{i=1}^{m} \Delta A_i}{A(t)} = \frac{A(t) - A(t - T)}{A(t)} .$$

The share of inventors, μ, is given by:

$$(4) \qquad \mu = \frac{SQ}{pQ} = \frac{S}{p} = \frac{A(t) - A(t - T)}{A(t)} = 1 - \frac{A(t - T)}{A(t)} .$$

The distributional theorem in equation (4) is remarkably general. It holds for any production function and any past profile of inventions. It tells us that the share of inventors is the ratio of the increase in technology over the last T years to the actual level of technology. If a is the annual rate of technology then the share of inventors is $1-e^{-aT}$, or somewhat less than aT. Equation (4) looks almost like the statement that the compensation of inventors is equal to the marginal product of invention, but that is not correct. It is rather that inventors capture all the surplus for the life of the invention. In fact, non-inventors get at time t exactly what could have been produced with the technology of period $(t - T)$;[5] inventors get all the extra output due to the increase in technology over the past T years.[6]

Two further assumptions are necessary. First, the supply of inventors is taken to be perfectly elastic at the going wage rate. This is relatively harmless, and an extension to scarce, competitive inventors would be simple. The crucial assumption for all this analysis -- and an assumption which cannot be swept under the rug -- is that there is a relation between the amount of inventive input and the rate of technological change. Put differently, there is a reasonably well-defined production function for technology. This is a questionable assumption. On the one hand, it is difficult to point to any laws of production of technology. On the other hand, the fact that profit-oriented firms are becoming more involved in research would indicate that they feel there is a positive relation between inventive inputs and technology. Moreover, there is some scattered empirical evidence of such a relation between research inputs and productivity (see [7]).

Formalizing these assumptions, we can write the rate of technological change as a function of the number of inventions We use the simple log-linear function:[7]

(5) $$\dot{A}/A = N^{\beta} A^{-\gamma}$$

Equation (5) states first that the rate of technological advance is an increasing function of the number of inventions produced at a given time, N . We assume, following arguments of Machlup [6] and others, that for a given initial stock of knowledge, A , inventions add less and less to the level of technology -- thus diminishing returns to N . The other feature of equation (5) is that the rate of advance may be a function of the <u>level</u> of knowledge. Thus, if knowledge has advanced very far, then (for a given N) the rate of advance might be lower. The opposite might also be the case. In the dynamic analysis it can be shown that for stability A must be a retarding force on technological advance, thus $\gamma > 0$.

We can now calculate the profit of inventors. The N^{th} invention gives an increase in productivity, $A'(N)$, given by

$$A'(N) = \beta N^{\beta-1} A^{1-\gamma} \tag{6}$$

Since from (3) the royalty, $s_N(t)$, equals the rate of productivity change and setting D equal to the cost per period per invention, we have the profit of the marginal invention, $V(N)$, equal to

$$V(N) = \int_0^T s_N(t) Q(t) e^{-rt} dt - D \tag{7}$$

The first term is the per unit royalty for the N^{th} invention times the number of units discounted back by a constant interest rate for the life of the monopoly. The second term is the cost. Putting (3) and (6) into (7), and normalizing by setting $p = 1$ (thus converting r into a real interest rate), we have:

$$V(N) = \int_0^T \beta N^{\beta-1} A^{-\gamma} Q e^{-rt} dt - D \ .$$

If inventors assume that output and technology will grow in the future at their present rates of growth, $\hat{Q}$ and $\hat{A}$, the equilibrium becomes:

(8) $$V(N) = \beta N^{\beta-1} A^{-\gamma} Q \phi - D$$

where ϕ is a capitalization factor given by:

(9) $$\phi = \frac{1 - \exp[(\hat{Q} - \gamma\hat{A} - r)T]}{r + \gamma\hat{A} - \hat{Q}}$$

Inventor equilibrium is attained when $V(N) = 0$, i.e. when the marginal inventor makes zero profits.[8] Therefore equilibrium in the invention market implies:

(10) $$N = \left[\frac{\beta Q A^{-\gamma}\phi}{D}\right]^{\frac{1}{1-\beta}}$$

This structural equation for the demand for inventions bears remarkable similarity to Schmookler's intuitive description of the determinants of inventive activity. A larger industry will induce a larger flow of inventions -- this is a consequence of the indivisibilities associated with knowledge. Schmookler concurs, writing in a chapter entitled "The Amount of Invention is Governed by the Extent of the Market...":[9]

> The most important relation for our purpose however, is the fact that given the expected market share and the expected cost of invention for each possible invention, the number of machines it will pay to invent will vary directly with S , the expected size of the market.

Schmookler did not investigate the other variables, although cost (D) and prior knowledge (A) did enter into his discussion.

IV. Instantaneous Properties of the System

We now have enough information to describe completely the economy. We will first discuss the instantaneous equilibrium. We can reduce the system to five variables, Q , A , N , r , and K , defined by the behavior equations (1), (2), (5), and (10), and by the savings assumption:

(11.a) $$Q = AK^{\alpha}L_o e^{\pi(1-\alpha)t}$$

(11.b) $$ND + I = zQ$$

(11.c) $$K = \frac{\alpha Q}{r}(1 - \mu)$$

(11.d) $$\dot{A}/A = N^{\beta}A^{-\gamma}$$

(11.e) $$N = \left(\frac{\beta Q A^{-\gamma}\phi}{D}\right)^{\frac{1}{1-\beta}}$$

Differentiating all but the (11.b) logarithmically with respect to time, and letting hats over variables represent proportional rates of change, (thus $\hat{x} = \dot{x}/x$):

(12.a) $$\hat{Q} = \hat{A} + \alpha\hat{K} + \pi(1 - \alpha)$$

(12.b) $$NC + I = zQ$$

(12.c) $$\hat{r} = \hat{Q} - \frac{\hat{\mu}\mu}{1-\mu} - \hat{K}$$

(12.d) $$\hat{\hat{A}} = \beta\hat{N} - \gamma\hat{A}$$

(12.e) $$\hat{N} = \sigma\hat{Q} - \sigma\gamma\hat{A} + \sigma\hat{\phi}, \quad \text{where} \quad \sigma = 1/(1 - \beta) .$$

The important features can be seen by examining the (12.a) and the combination of (12.d) and (12.e):

(13) $$\hat{Q} = \hat{A} + \alpha\hat{K} + \pi(1 - \alpha)$$

(14) $$\hat{\hat{A}} = \beta\sigma\hat{Q} - \gamma(1 + \beta\sigma)\hat{A} + \beta\sigma\hat{\phi}$$

These equations clarify the interdependence between the growth of output and the growth of knowledge. Knowledge growth affects output by the conventional additive term, $\hat{A}$, in equation (13). The new twist is that this feeds back through the knowledge market, increasing the acceleration of knowledge by the factor $\beta\sigma$.

A second important interaction can be seen by examining equation (11.c). Recalling from (4) that $(1-\mu) = A(t-T)/A(t)$ -- which equals $\exp[-\hat{A}T]$ in equilibrium -- we see that a rise in the rate of technological change depresses the capital stock by cutting into the post-royalty marginal product of capital. This is a result which does not seem to have been noted before. Rewriting (11.c) we have the equilibrium capital-output ratio:

$$K/Q = \alpha \exp[-\hat{A}T]/r$$

If, as empirical evidence indicates, interest rates remain roughly constant and technological change speeds up, then the equilibrium capital-output ratio should be declining.

V. Asymptotic Properties of the System

The economic system described in equation set (12) is rather different from the standard neoclassical model. The chief differences in structure are that technology can be interpreted in the growth equation as a capital good with unitary exponent, and that the technology-formation function is quite strange compared to the capital-formation equation. This strangeness is due to the fact that technology is subject to monopoly pricing rather than competitive pricing.

A long-run balanced growth path can be found as follows. Recalling from (11) that $\mu\hat{\mu}/(1-\mu) = \hat{A}(t-T) - \hat{A}(t)$, substituting this and (12.c) into (12.a),we have (15). (16) is from (14):

$$\hat{Q} = \hat{A}(t) + \pi + \frac{\alpha\hat{A}(t - T)}{1 - \alpha} - \frac{\alpha\hat{r}}{(1 - \alpha)} \tag{15}$$

$$\hat{\hat{A}} = \beta\sigma\hat{Q} - \gamma(1 + \beta\sigma)\hat{A} + \beta\sigma\hat{\phi} \tag{16}$$

Consider paths with constant interest rates, $\hat{r} = 0$. It is easily verified for such paths that the long-run equilibrium is reached when $\hat{\hat{A}} = \hat{\phi} = 0$, so from (16) and recalling $\beta\sigma = \beta/(1 - \beta)$:

$$\hat{Q} = \frac{\gamma}{\beta}\hat{A} \tag{17}$$

Putting this into (15) for $\hat{\hat{A}} = 0$ implies

$$\hat{A} = \frac{\pi\beta(1 - \alpha)}{\gamma(1 - \alpha) - \beta} \tag{18}$$

It can be shown that a necessary condition for (17) and (18) to be a stable equilibrium is that the denominator is positive, so $\hat{A}$ will always be positive.[10]

Equation (18) has a few remarkable features. First, it is linear in population growth. If population is stagnant then eventually technology will stagnate. The reason for this peculiar, non-neo-classical, feature is that since $\gamma < 0$ the annual number of inventions must be growing to keep up with the diminishing returns in the knowledge function. But the number of inventions can grow only if income is growing faster than $\hat{A}$. Thus unless population growth gives a stimulus to growth of income, both income and technology will eventually stagnate.

This peculiar result is one of several due to the more general feature of the model, that it displays aggregate economies of scale. As is obvious from the production function, doubling all "factors" -- A , K , and L -- quadruples the level of output. One of the nice features of the present model is that it allows this realistic feature without introducing inconsistencies into the market structure. Such large increasing returns are consistent with profit-maximization because information is temporarily monopolized.

Other magnitudes of the system are easily found. From (12) and (17) it is easily seen that $\hat{N} = \hat{Q}$. Therefore the shares of output devoted to investment and invention are constant. We can find the interest rate by combining (11.d) and (11.e), obtaining

$$\frac{DN}{Q} = \beta\phi\hat{A} \qquad (19)$$

So the interest rate is given by the solution of (19), (11.b), and (11.c):

(20) $$z = \frac{DN}{Q} + \frac{I}{Q} = \beta\phi\hat{A} + \frac{\alpha\hat{Q}e^{-\hat{A}T}}{r}$$

This does not seem to be tractable. Since $\frac{dz}{dr} < 0$, it is easily seen that there exists a single value of r satisfying (33). For T large and β close to unity we know from (33) that $r = \hat{A}(1 + \gamma\alpha)/z$.[11]

VI. Empirical Tests of the Model

The model described above has two important and independent features which are readily testable. These two are the technology production function -- equation (21) -- and the inventor equilibrium -- equation (22):

(21) $$\hat{A} = k_o + k_1 N^{\beta} A^{-\gamma}$$

(22) $$\log N = k_2 + (\frac{1}{1-\beta})\log Q - (\frac{\gamma}{1-\beta})\log A + (\frac{1}{1-\beta})\log \phi$$

The first equation gives the rate of technological change as a function of the number of inventions and the level of technology. The second gives the flow of inventions as a function of the level of output, technology, and the discount factor, ϕ.

Estimation. It is clear that the theoretical specification gives us a great deal of overidentification since -- ignoring disturbances -- there are eight coefficients from which only five parameters must be recovered. In fact, we will not be able to use these restrictions in estimation.

In testing the theoretical model we have made two additions. Since the data are from industries, and it is natural to expect that some technological change will come from outside the industries, we have added a constant term, k_o, to equation (21) representing the exogenous rate of technological change. Second, the structure of the random elements must be discussed. It seems unlikely that disturbances in the invention equation (22) will feed back through either Q or A in a period of time sufficiently short as to cause a significant bias. This implies that ordinary least squares estimates of equation (22) will give unbiased estimates. The same cannot be said of equation (21) because there is a lagged dependent variable. We must therefore make the stronger assumption that the errors in (21) are serially independent. Once this assumption has been made ordinary least squares estimation will give consistent estimates of the coefficients.[12] Finally, ϕ was dropped from equation (22) since it was statistically insignificant.

Equation (22) was estimated by ordinary linear least squares. Estimation of (21) was somewhat more complicated. First round estimates were derived by estimating the equation linearized at its mean. To find the maximum likelihood estimator a grid in the neighborhood of the first round estimates was then examined.

<u>Data</u>. We use the only two industries for which long, continuous time series are available: railroads and agriculture. The most drastic assumption needed is that the annual number of patents issued is a good proxy for the number of inventions.

It seems a good bet that the inventions and patents move roughly together. On the other hand, there may well have been a long-run tendency for the propensity to patent to decline. It is, however, hard to see why there would be systematic variations in the propensity to patent over ten or twenty year periods. We have used Schmookler's data on patents [4], while output, labor, and capital data are from Kendrick [12]. The rate of total factor productivity was recalculated using a Cobb-Douglas production function with $\alpha = .25$, rather than Kendrick's method. (The two correlate with $\bar{R}^2 = .9989$ and $\bar{R}^2 = .9997$ for railroads and agriculture, respectively.) The series of ϕ were calculated from series on railroad bond yields from [13] and rates of growth of output.

Results. The empirical results for the two industries are mixed. Roughly speaking, the "technology production function" hypothesis is decisively rejected, while the "inventor equilibrium" hypothesis seems to be consistent with the historical experience.

First examine the production of knowledge hypothesis. There are three different versions. The strictest version of the hypothesis is the non-linear hypothesis examined in the theoretical analysis above. This equation was estimated by non-linear methods, as described above, and is reported as regressions 1 and 2 in Table I. The results are unfavorable to the hypothesis that there is a relationship as specified in the model. Looking at the likelihood ratio of the maximum likelihood estimate compared with the hypothesis that $\beta = \gamma = 0$, we find that the hypothesis of a significant difference is rejected at the 50% confidence

The predictions Table III shows the prediction. While they are not too near the actual levels, they at least have the right order of magnitude.

In summary, we have found that the flow of patented inventions behaves according to the predictions of the economic model of the inventive process outlined here. This means, roughly, that patents are positively correlated with increases in output but negatively correlated with the level of productivity. On the other hand, the hypothesis of a relation between the flow of patented inventions and the growth of productivity is not borne out by the two histories examined.

These two results thus imply a significant paradox. While it is easy to reject the technology production function hypothesis because of unfavorable empirical evidence, this leaves the positive results in the other equation -- the inventor equilibrium -- unexplained. Put differently, if there is no relationship between the flow of new inventions and productivity advance, how can the strong correlations between economic activity and inventions be explained?

Our possibility,of course, is that the historical experience represents a highly improbable occurence. Thus, if the true β is zero and if the other assumptions (including normality) are correct, the probability that a $\hat{\beta}$ as large as that in either equation is observed is less than five in ten-thousand. Clearly an alternative hypothesis would be more congenial.

TABLE II. Estimates for Inventor Equilibrium

Regression Number	Industry	Regression Coefficients and t-statistics	R^2	SEE	D-W	$\hat{\gamma}$	$\hat{\beta}$
7	Railroad	$\log N = -1.853 + 1.2867 \log Q - 2.262 \log A$ (-2.72) (12.70) (-19.50)	.8997	.1878	.632	1.76	.223
8	Agriculture	$\log N = 6.582 + .01288 \log Q - 1.135 \log A$ (1.96) (.03) (-2.58)	.3164	.3395	.185	(a)	(a)
9	Railroad	$\log N = -2.964 + 1.273 \log Q - 3.026 \log A + .0187T$ (-4.07) (13.95) (-11.42) (3.16)	.9140	.1745	.745	2.20	.272
10	Agriculture	$\log N = -5.678 + 1.896 \log Q - .3245 \log A - .0217T$ (-1.55) (3.47) (-.81) (-5.25)	.5311	.2834	.344	0.17	.473

(a) $\hat{\beta}$ is outside of the admissible region.

level. Aside from the problem of the statistical significance of the results, at least one of the three coefficients about which one has prior information is wrong in both equations.

Equation 3 through 6 in Table I test linear approximations to the technology production hypothesis. In none of these are the results in strong conformity to prior restrictions. Sign patterns are different between the two industries and none of the eight coefficients takes the expected sign at a significant level.

The second part of the hypothesis is the inventor equilibrium. Regressions 7 and 8 in Table II give estimates of equation (22), while regressions 9 and 10 introduce a time trend to offset long-run structural changes not encompassed by the model.[13] Examining the equations where a time trend is introduced, the model seems to be appropriate and predicts the signs and magnitudes of the coefficients extremely well. The estimates for β are positive, between zero and one, and not far apart. The coefficients on $\log A$ also have the right sign, although that for agriculture is insignificant. We thus have found that the part of the hypothesis predicting inventor behavior performs remarkably well.

As a final test we can use the coefficients estimated and displayed in Table II to predict the equilibrium rate of technological change. Using equation (17), we recall that $\hat{A} = \beta\hat{Q}/\gamma$. Although it may involve certain illegitimate assumptions, it is at least a further check on the results to see how well it predicts.

TABLE I Estimates for Knowledge Production Function, 1890-1953*

Regression Number	Industry	Regression Coefficients and t-statistics	$\bar{R}^2$	SEE	D-W
1	Railroad[a, b]	$\hat{A} = -.0440 + .2809N^{-.2}A^{0.}$ (-.70) (1.12)	.0199	.0594	1.40
2	Agriculture[a, c]	$\hat{A} = .221 - .2099N^{0.}A^{-.5}$ (3.10) (-2.94)	.1227	.0543	2.92
3	Railroad	$\hat{A} = .1719 - .0212 \log N - .0105 \log A$ (1.18) (-.99) (-.43)	.0216	.0598	1.41
4	Agriculture	$\hat{A} = -.0592 + .0105 \log N + .1105 \log A$ (-.43) (.51) (2.7)	.1238	.0547	2.90
5	Railroad	$\hat{A} = .439 - .000012N - .00201A$ (.99) (-.65) (-.08)	.0134	.0601	1.40
6	Agriculture	$\hat{A} = -.0808 + .000004N + .0868A$ (-.43) (.16) (2.46)	.1141	.0551	2.94

*Numbers in parentheses are t-coefficients. SEE is the standard error of estimate. D-W is the Durbin-Watson coefficient.

[a] The minimum was at a corner of the grid, so the maximum occured outside the a priori limit given by the theoretical model.

[b] The approximate standard errors of β and γ were 600. and 50.

[c] The approximate standard errors of β and γ were 60. and 600.

TABLE III.

Predicted and Actual Rates of Productivity Change
(Percent)

INDUSTRY	PRODUCTIVITY CHANGE		
	ACTUAL	PREDICTION I[a]	PREDICTION II[b]
Railroad	2.59	.35	.45
Agriculture	1.24	(c)	2.20

[a]Using regression 7.

[b]Using regressions 9 and 10.

[c]Not applicable since estimate was inadmissable.

Rather than throw our lot in with coincidence, the following explanation seems worth consideration. The structure of the random elements may well be such as to lead to the observed results. Several observers have noted that the returns from inventions are highly disperse. (Indeed, this has led many observers to conclude that patent data are worthless as a measure of inventive output.) Scherer [10] reports a rough test showing the value of patents to be distributed according to the arc-sine distribution.

To follow through this line of argument, assume each invention is unpredictable, so that the marginal contribution of the i^{th} invention has a random term ε_i which is quite disperse. We can thus rewrite structural equation (21) as

$$\dot{A}/A = N^{\beta}A^{-\gamma} + \sum_{i=1}^{N} \varepsilon_i(t) \tag{21'}$$

Under normal circumstances we would expect that $\Sigma\varepsilon_i(t)$ would collapse around zero by the law of large numbers. On the other hand, very disperse distributions do not obey the law of large numbers. But distributions which do not obey the law of large numbers have infinite variances. Thus if the random elements are so disperse or so unstable that the law of large numbers does not hold for $\Sigma\varepsilon_i(t)$ it will generally follow that (21') does not obey the usual properties needed for consistent statistical estimation.

Furthermore, if inventors are more or less risk neutral or do not perceive the massive uncertainties or research, then inventor equilibrium in equation (22) will be unchanged. We thus conclude that if the relation between invention and technological change is characterized by high degrees of randomness, then changes in technology might be white noise while the inventor equilibrium would be very well behaved. Both these would be true for any stable Paretian distribution with skewness parameter less than one such as the arc-sine distribution mentioned above. Both these fit descriptions the empirical results reported here.

FOOTNOTES

1. One of the illustrative shortcomings of the capital-theoretic model is that the competitive solution is Pareto-optimal as long as institutions are taken as given. This is seriously at variance with the conventional wisdom.

2. Some cheap generalizations can be made as follows. If production is not Cobb-Douglas, the asymptotic results hold if technological change is Harrod-neutral and the elasticity of substitution is less than one. The instantaneous equilibrium in this case becomes much more complex. If the elasticity of substitution is greater than one the system explodes for any technological change.

3. This assumption of no joint-production is particularly bothersome. If inventions were not independent but inventors acted independently the problem would disappear. The problem is that with interdependence a game-theoretic problem of royalties arises. When inventions are independent the Nash equilibrium -- the solution used here -- is composed of dominant strategies and is thus unobjectionable. When inventions are not independent the Nash equilibrium is no longer dominant.

4. This proposition was first shown by Arrow [1], and has been expanded in [9]. A demonstration runs roughly as follows: $\frac{\Delta A_i}{A}$ is the percentage cost-reduction which can be attained by a firm using the i^{th} invention. The total cost reduction including royalty, is thus $\left(-\frac{\Delta A_i}{A} + \frac{s_i}{p}\right)p$. Putting (3) into this we see that s_i is the maximum royalty that an inventor can charge. It can be shown rigorously that as long as A_i/A is small (10) will also be the profit-maximizing royalty for the inventor.

5. This is trivial: Let $K^{\alpha}L^{1-\alpha} = F$. Then $(1-\mu)A(t)F = [A(t-T)/A(t)]A(t)F = A(t-T)F$.

6. Equation (4) can be applied to show (i) that there is generally a bias in estimating output elasticities from relative shares. Thus if all inventor's royalties accrue to "capital" then the observed share overestimates capital's share by $(1-\alpha)[1 - \exp(-\hat{A}T)]$; (ii) in this case using observed weight underestimates $\hat{A}$ by the factor $\exp(-\hat{A}T)$; (iii) as invention moves from the personal to the corporate sphere, capital's observed share will rise significantly.

7. We have shifted to continuous analysis and will do so for the next two sections. This is inessential for the argument. Although continuous analysis causes some problems of understanding, the interpretation of discrete numbers of inventions and discrete changes in A gives the usual view.

8. This equilibrium assumption implies some kind of priority in discovering new techniques. An alternative assumption might argue for all inventors averaging zero profits. In this case the equilibrium condition is (10) with the β inside the bracket replaced by 1.

9. [11], p. 115.

10. Stability analysis of the differential-difference system in (12) is quite difficult. So far we have only succeeded in showing stability for the system which has a constant interest rate at the equilibrium value.

11. One of the stock questions asked of such a model is whether it is efficient. The immediate response here would be that it must be inefficient due to the monopolistic restrictions on invention. What then is the optimal legal environment?

    The optimal kind of legal environment for promoting economic welfare concerns such things as subsidizing or taxing inventive activity and changing the economic or legal life of the invention (say by changing the legal life of the patent). The answer seems to be that in the present model the competitive allocation of resources to invention will be less than is optimal if $T < \infty$. The system can be made efficient either by setting T at infinity or by subsidizing invention. To take a simple numerical example, if $r = .10$, $T = 10$, and $(\hat{Q} - \gamma\hat{A})$ is small, then $\phi = 6.5$. The social rate of return 0.15. By subsidizing 35 percent of research costs, the system could be made efficient. This conclusion would be modified if labor were not inelastically applied or if there were more than one good.

12. For proof of these statements, see any standard econometrics text. We are in serious trouble if the movements in $\hat{A}$ are due to the business cycle -- as they are in the short run. We must hope that any correlation between the business cycle and independent variables in (21) cancel out. On this score we cannot be too optimistic.

13. The time trend is introduced to pick up systematic changes in the propensity to patent, in the resources needed per invention, and in the cost of inputs.

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