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**THE TIME ALLOCATION OF CONSUMPTION UNDER
DEBT LIMITATIONS**

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THE TIME ALLOCATION OF CONSUMPTION UNDER DEBT LIMITATIONS*

by

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I. Introduction

One of the more important areas of consumer theory is that of the time allocation of consumption. In the models of consumer behavior over time, the optimal consumption program is selected subject only to the lifetime wealth constraint and the income stream is important only to the extent that it determines lifetime wealth; consumption at any age bears no direct relationship to income at that age. However, in order for current consumption to be independent of current income, it is necessary in these models to permit the incurring of unlimited debt by the consumer over certain time intervals. The subject of this paper is the time allocation of consumption when there are specific limits on the amount of debt that the individual can incur at any age. The income stream has a more direct connection with consumption when such debt limitations are included as a constraint on consumer behavior.

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It must be emphasized that the subject is the time allocation of consumption rather than the utilization of credit facilities. In the formulation of the model, most of the factors associated with the use of credit have been simplified and eliminated from the analysis. In particular, it will be assumed that the interest rate is independent of the level of an individual's assets, i.e. the borrowing and lending rates are equal. Also, no special consideration will be given to the problem of consumer durables purchases. Thus, the results of the analysis would only apply in a world of rented durables or equal sale and purchase prices for used durables.

The notion of debt applicable to this study is that of unsecured loans. Thus, the availability of such credit is quite limited, for it is one thing to borrow money to buy a car and quite another to borrow money to rent a car. A little reflection indicates that this latter notion of credit or debt is the appropriate one for long-term or lifetime allocation of consumption. The more common consumer durable credit is relevant only to short-run time allocation. One may buy a car on time only if one might pay cash for it several years hence anyway. If the resale value of the durable is always greater than the unpaid balance, then no debt of the type under consideration, to which we shall refer as unsecured debt, is involved. The fact that such unsecured debt is limited makes the analysis all the more relevant because the case of no credit at all is a special (and simple) case.

The other major assumptions involved in the analysis are that the individual has firm expectations concerning future prices, income, and his life span, and that his preferences are unchanging over time. Compared to the above noted assumptions, the technical assumptions made in the analysis are very minor. The above assumptions which are common to most economic analysis, explicitly or implicitly, preclude any realistic detailed description of behavior. All that can be hoped for is that the model captures the general characteristics of economic behavior. Maintaining a generality in the technical details does not appreciably enhance the realism of the model, but it encumbers the analysis greatly. The prime concern of this paper is with constructing a workable model of the time allocation of consumption under credit restrictions by judicious selection of simplifying assumptions such that the major factors are included and the various effects are easily visualized.

2. Technical Description of the Model

It is assumed that an individual's preferences are defined over consumption programs and that those preferences can be represented by a utility functional of the type

$$(2.1) \quad \int_0^T u(c(t))g(t)dt$$

In this functional $u(c(t))$ is the instantaneous utility

function and $g(t)$ is the subjective discount function. The time interval is from the time of planning, time 0 , to the time of death, T .

In order to keep the analysis as simple as possible, it is necessary to make certain assumptions about the instantaneous utility function. The required assumptions are that the marginal utility function, $u'(c)$, is continuous and strictly decreasing and that

$$u'(0) = \infty$$

$$u'(\infty) = 0$$

The assumption that $u'(c)$ is continuous and strictly decreasing is required in order that its inverse function should exist. This is equivalent to assuming strict concavity of $u(c)$. The other assumption eliminates consideration of the non-negativity of consumption and the possibility of satiation. These strong assumptions are not really necessary in the analysis, but they simplify the results considerably.

3. The Credit Constraint

In the standard model of the time allocation of consumption there is only one constraint on consumption, i.e., that net assets of death must be non-negative. In the present analysis the individual must plan his consumption in such a way that his debts at any age never exceed the amount which the credit institutions are willing to grant. If $A(t)$ is the level of net assets at age " t ", then that credit constraint

is of the form

$$(3.1) \quad A(t) \leq D(t)$$

where $D(t)$ is the limit imposed by the credit institutions. It is to be emphasized that $A(t)$ is the net assets. The debt limit $D(t)$ is, therefore, the amount of unsecured credit available to the individual at age t . The debt limit may be zero at all ages, but we need not assume this for the analysis. It is reasonable to assume that the debt limit is near zero for the very young and very old. The constraint in the conventional models is simply $D(T) = 0$.

This formulation of debt restriction abstracts from all the institutional arrangements for consumer loans such as time payments. The debtor pays off his loans only as his increasing age induces the loan agencies to diminish the amount of debt he will be allowed to hold.

The credit constraint (3.1) can be written in a more convenient form by solving the differential equation,

$$(3.2) \quad \frac{dA}{dt} = y(t) + i(t)A(t) - p(t)c(t)$$

where $y(t)$ is non-interest income, $p(t)$ is the price level, and $i(t)$ is the money rate of interest. Since we are dealing with only a single consumption good, changes in the price level and the money rate of interest could be reduced to a single real rate of interest. However, we choose to keep all quantities in money terms, because the results then suggest the form of the results for the many-commodities case. The solution of

the differential equation (3.2) and substitution into (3.1) gives the credit constraint as

$$(3.3) \quad \int_0^t j(\tau) p(\tau) c(\tau) d\tau \leq \int_0^t j(\tau) y(\tau) d\tau + j(t) D(t)$$

where

$$j(t) = e^{-\int_0^t i(\tau) d\tau}$$

For the convenience we will write the credit constraint as

$$(3.4) \quad C(t) \leq Y(t) \quad \text{For } 0 \leq t \leq T .$$

The variable $C(t)$, which appears on the left-hand side of (3.3), is the cumulative discounted consumption expenditures at age t . For many reasons, it will be convenient to deal with the function $C(t)$, rather than the consumption program $c(t)$. Therefore, the term expenditure path will be used for the function $C(t)$, $0 \leq t \leq T$. The variable $Y(t)$ includes the cumulative discounted autonomous income as well as gifts, inheritances, and a term involving the debt limit.

The income variable implicitly involved is very close in spirit to "discretionary income" as defined by the National Industrial Conference Board [10].

If the debt limit $D(t)$ is equal to the discounted income which the individual will earn in the next δ time periods, then the influence of credit on the constraint has a particularly simple form, i.e.,

$$(3.5) \quad \text{if } D(t) = \int_t^{t+\delta} j(\tau) y(\tau) d\tau$$

$$(3.6) \quad \text{then } C(t) \leq \int_0^{t+\delta} j(\tau) y(\tau) d\tau$$

The debt limit $D(t)$ would depend in some way upon the income of the individual. Figure 3.1 illustrates the credit constraints for various sorts of debt limits.

4. The Time Allocation of Consumption

There are two results from the conventional analysis that are required for the problem at hand. The rigorous analysis of the time allocation of consumption has been given by Yaari [14]. One may consider the conventional analysis as having to do with the following constrained maximization problem:

$$\text{Max } U = \int_{t_1}^{t_2} u(c(t))g(t)dt$$

$$\text{Subject to } \int_{t_1}^{t_2} j(t)p(t)c(t)dt \leq W$$

$$\text{and } c(t) \geq 0$$

The first condition for an optimal consumption program $c^*(t)$ is that it must use up all available resources, i.e.,

$$\int_{t_1}^{t_2} j(t)p(t)c^*(t)dt = W. \text{ This, of course, only applies to the}$$

the case in which there is no bequest motive. The other result is the usual condition of equating at the margin, i.e.,

$$u'(c^*(t))g(t) = \lambda j(t)p(t) \text{ where } \lambda \text{ is a positive constant,}$$

[Yaari, 13, p. 307]. Roughly speaking, the marginal utility of an additional discounted dollar of wealth spent on consumption at time t must be equal to the marginal utility of an additional

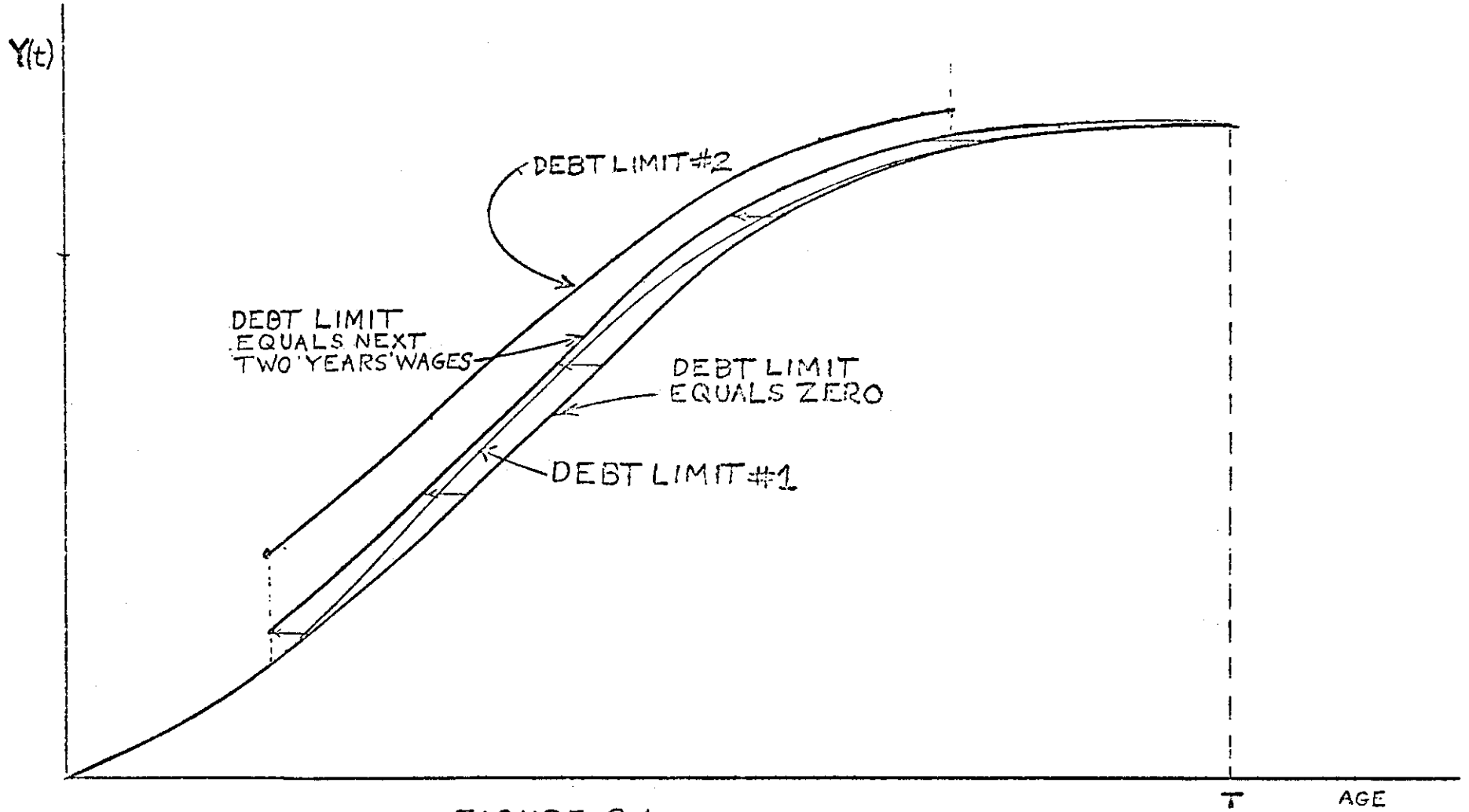
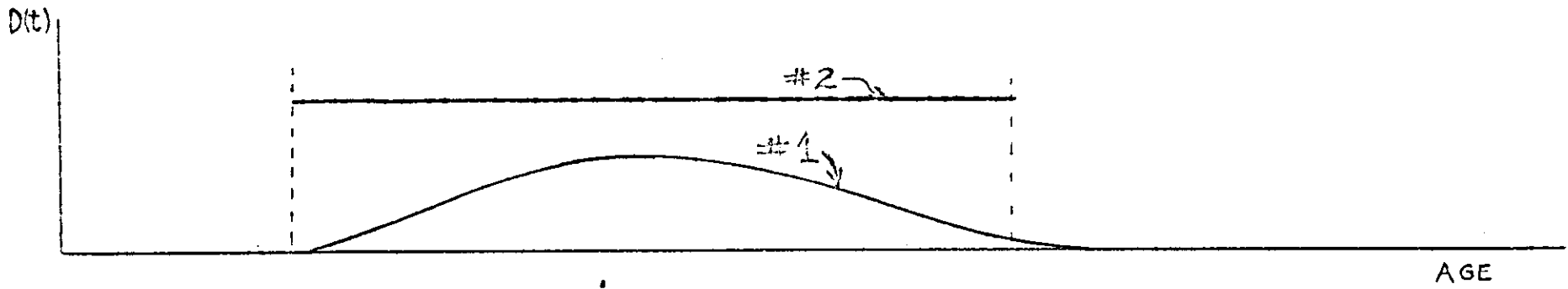


FIGURE 3.1

discounted dollar spent on consumption at any other time. An alternative statement is that of the programs costing the same amount of wealth on an interval the one associated with a constant multiplier function gives the maximum utility.

5. The Time allocation of Consumption Under Debt Limitations

In the conventional analysis, it was found that for the optimal consumption program $c^*(t)$

$$(5.1) \quad u'(c^*(t))g(t) = \lambda p(t)j(t)$$

Thus, if one specifies a value of λ , then one can solve

$$(5.1) \text{ for } C^*(t) .$$

For the present analysis, it is convenient to deal with a special function that may be associated with a given program. For lack of a better name, we will call this function "the multiplier function" because of its relation to the Lagrangian multiplier. We define the multiplier function

program to be $\lambda(t) = \frac{u'(c(t))g(t)}{j(t)p(t)}$. Under the assumptions

made on $u'(c)$, there will be a unique multiplier function associated with each consumption program, and a unique consumption program corresponding to any non-negative $\lambda(t)$, $0 \leq t \leq T$. The assumptions concerning $u'(c)$ were made so as to guarantee this condition.

The optimal consumption program under debt limitations can be determined from necessary conditions for the multiplier

function associated with the optimal program. These necessary conditions will be given here with a sketch of the proofs.

Lemma 1:

A necessary condition for an optimal solution $c^*(t)$ to solve the maximization problem,

$$\text{Max } U = \int_0^T u(c(t))g(t)dt$$

$$\text{Subject to } C(t) = \int_0^t j(\tau) p(\tau)c(\tau)d\tau \leq Y(t) ,$$

is that the multiplier function $\lambda(t)$ be non-increasing where

$$\lambda(t) \text{ is defined as } \lambda(t) = \frac{u'(c^*(t))g(t)}{j(t)p(t)} .$$

If the multiplier function is increasing on some interval (t_1, t_2) , we can replace that part of the consumption program with one associated with a constant multiplier function, say λ^* . This will increase utility on this interval and thus the original could not have been optimal. By choosing the λ^* greater than $\lambda(t_1)$, we insure that the credit constraint would **not** be violated. (See Figure 5.1.)

Lemma 2:

The multiplier function $\lambda(t)$ associated with an optimal program is non-decreasing except on the constraint, i.e., except when $C(t) = Y(t)$. The proof is essentially the same as that of lemma 1. If the multiplier function is decreasing and $C(t) < Y(t)$ on some interval, then we may improve

upon the original program by "splicing in" a consumption program associated with a constant multiplier function. In order to visualize this operation, see Figure 5.2.

These two lemmas provide the basis of a theorem which enables us to find the optimal consumption program. The theorem will be stated in a somewhat imprecise form in the interest of clarity. (See Figure 5.3.) theorem 1:

Let $C_{\lambda}(t, t_1)$ be the expenditure path associated with a constant multiplier function for $t > t_1$. At each value of t_1 the optimal multiplier function is the maximum value of λ such that $C_{\lambda}(t, t_1) = Y(t)$ for some t between t_1 and T .

Suppose that this maximum value is λ^* and that the value of the multiplier function for the optimal program is less than λ^* . This means that the optimal expenditure path is above $C_{\lambda^*}(t, t_1)$. But $C_{\lambda^*}(t, t_1)$ intersects $Y(t)$ at some point. The optimal expenditure path would have to intersect $C_{\lambda^*}(t, t_1)$ at some point and hence there would be an interval on which the expenditure path is non-optimal, contrary to assumption.

If the value of the multiplier function at t_1 were greater than λ^* , then the expenditure path would be below $C_{\lambda^*}(t, t_1)$. But an optimal multiplier function cannot change values except on the constraint. However, λ^* is the maximum value of the multiplier such that the associated expenditure

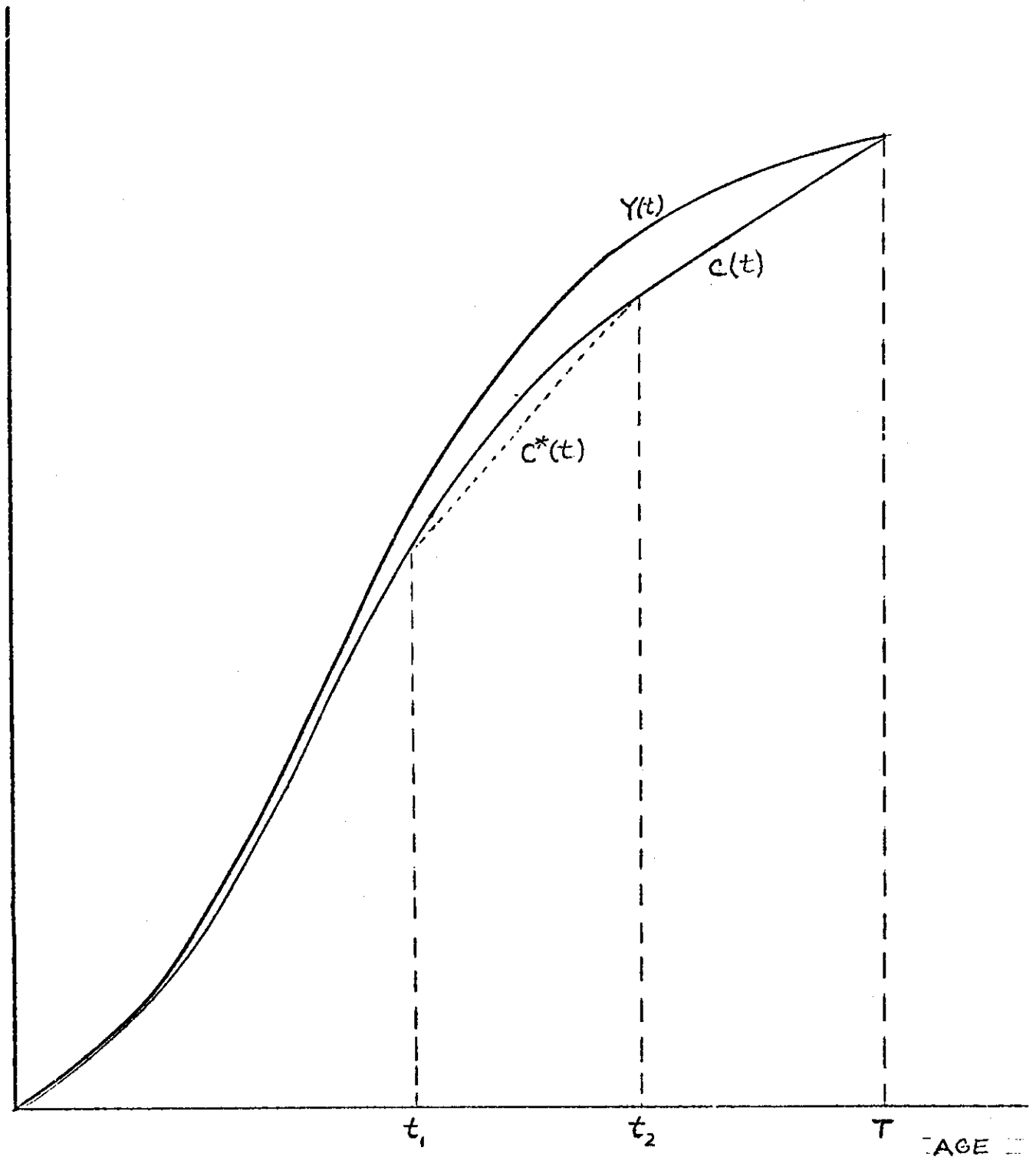
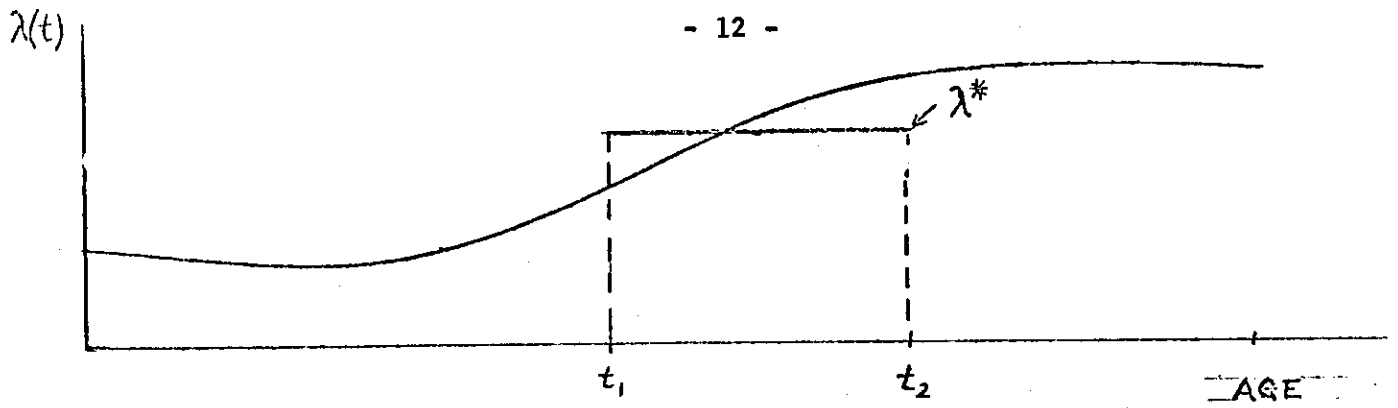


FIGURE 5.1

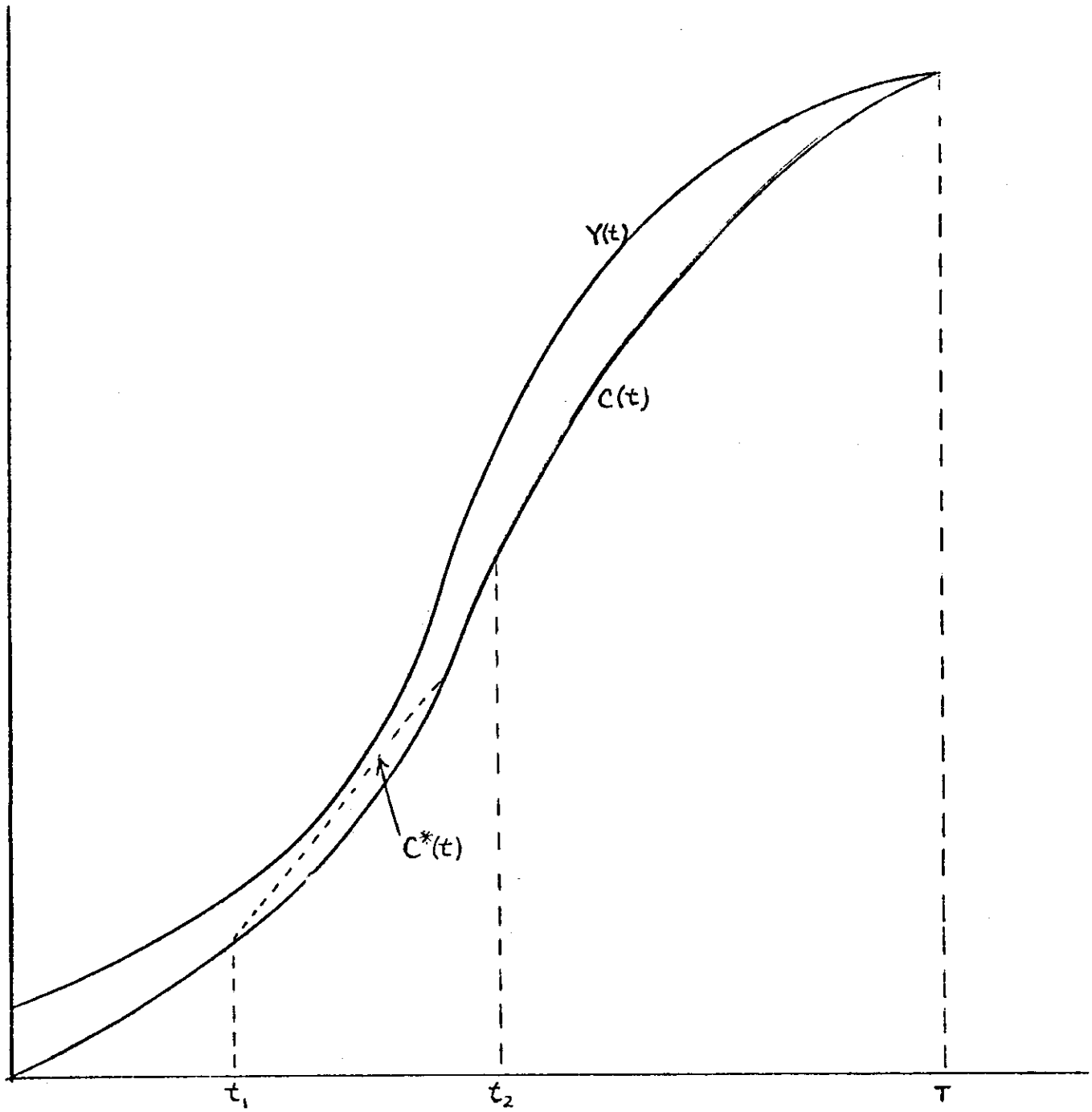
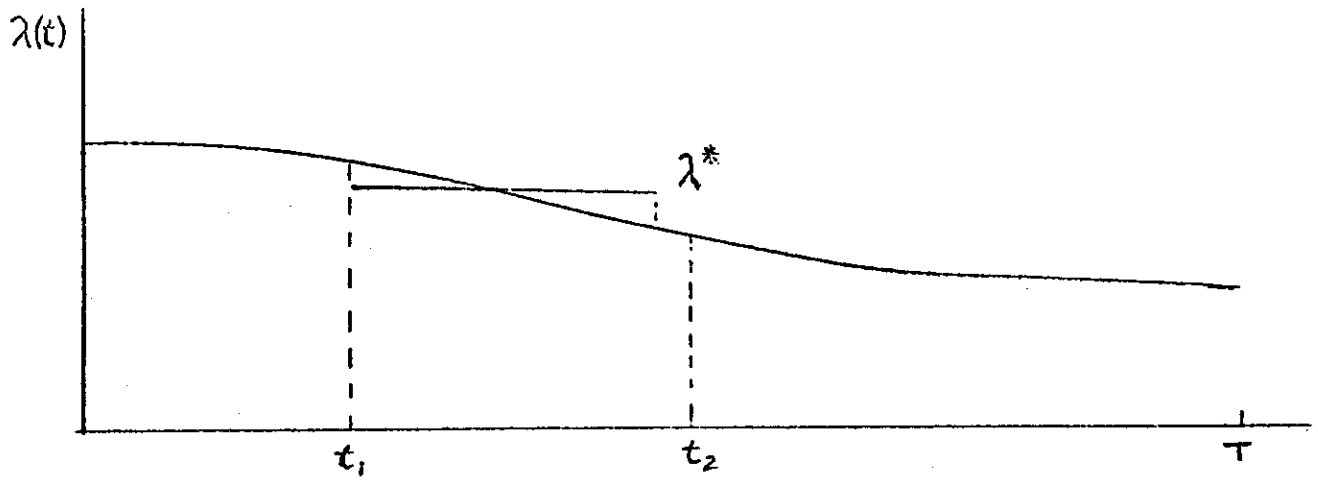


FIGURE 5.2

path intersects the constraint. If the assumed optimal expenditure does not intersect the constraint before the end of the life span, there will be unconsumed resources, and this clearly could not be optimal.

Since the optimal expenditure path may be on the constraint over some interval, some modification of the theorem must be made. The correct version is given in terms of those values of λ such that $C_\lambda(t, t_1) \geq Y(t)$ for some t such that $t_1 \leq t \leq T$.

Figure 5.3 illustrates the meaning of theorem 1 geometrically. At any time t_1 there will be a one-parameter family of expenditure paths diverging from point $(t_1, C(t_1))$. Each path corresponds to a different non-negative value of λ . Two paths will not cross because of the concavity assumption on $u(c)$. Small values of λ generate more vertical expenditure paths (higher rates of consumption expenditure) and high values of λ generate more horizontal expenditure paths (lower rates of consumption expenditure). The set of values of λ is partitioned into those values such that the expenditure paths intersect the constraint in the interval (t_1, T) and those which do not. The optimal value of λ separates the two sets. In Figure 5.3 the expenditure path is the one that is tangent to the constraint. When the consumer is on the constraint the same

principle applies but may involve moving along the constraint. This may mean spending current income and any other funds the consumer can obtain.

As can be seen from Figure 5.3, theorem 1 gives a condition for the determination of the time horizon and the principle is the same as that involved in the determination of the visual horizon. In determining the consumption program one also finds the time horizon and the level of normal income. Here normal income is taken to be the consumption-determining income variable. As it turns out, there is no direct connection between normal income and normalcy or permanence.

The results can be intuitively explained and justified in a much simpler form than has been presented. In this model, expenditure can always be transferred forward, but not always backward. If one considers a very short period of time, the influence of the interest rate and of subjective time preference can be ignored and the optimality requires equal rates of consumption at the beginning and end of the time interval if such equality can be achieved. Consider the time just before the consumer reaches his debt limit and just after he does. Once the debt limit is reached consumption is constraint determined so the consumer must allocate the expenditure of his income and assets such that the spending of his last cent does ^{not} involve any abrupt decrease in his consumption. There may be (because of the abrupt availability of funds) situations in which the

consumer chooses a **pattern** of consumption involving abrupt increases in consumption but not decreases. This, of course, would not necessarily hold true for models involving uncertainty.

The rest of the paper consists of a cursory treatment of the application of theorem 1 to examples and aspects of consumer behavior.

6. Logarithmic Utility

In order to explain and interpret the preceding results, it is expedient to use a very special case, i.e., $u(c) = \ln c$ and $g(t) = 1$. The advantage of this case is that the expenditure path $C(t)$ is a straight line for consumption programs associated with a constant value of the multiplier. It is convenient to assume that income becomes available in a continuous stream. Also, for purposes of illustration it is desirable to consider an income stream which is more variable than would be found in practice. An example of the time allocation of consumption under debt limitations is given in Figure 6.1. The optimal expenditure path for this problem involves two types of segments; the first is such that $C(t) = Y(t)$ over some interval and the second is such that $C(t) < Y(t)$ over an interval. The second type is a straight line. The junction of the two types comes where the transition between the two types can take place smoothly; i.e., where the straight line is tangent to the credit constraint.

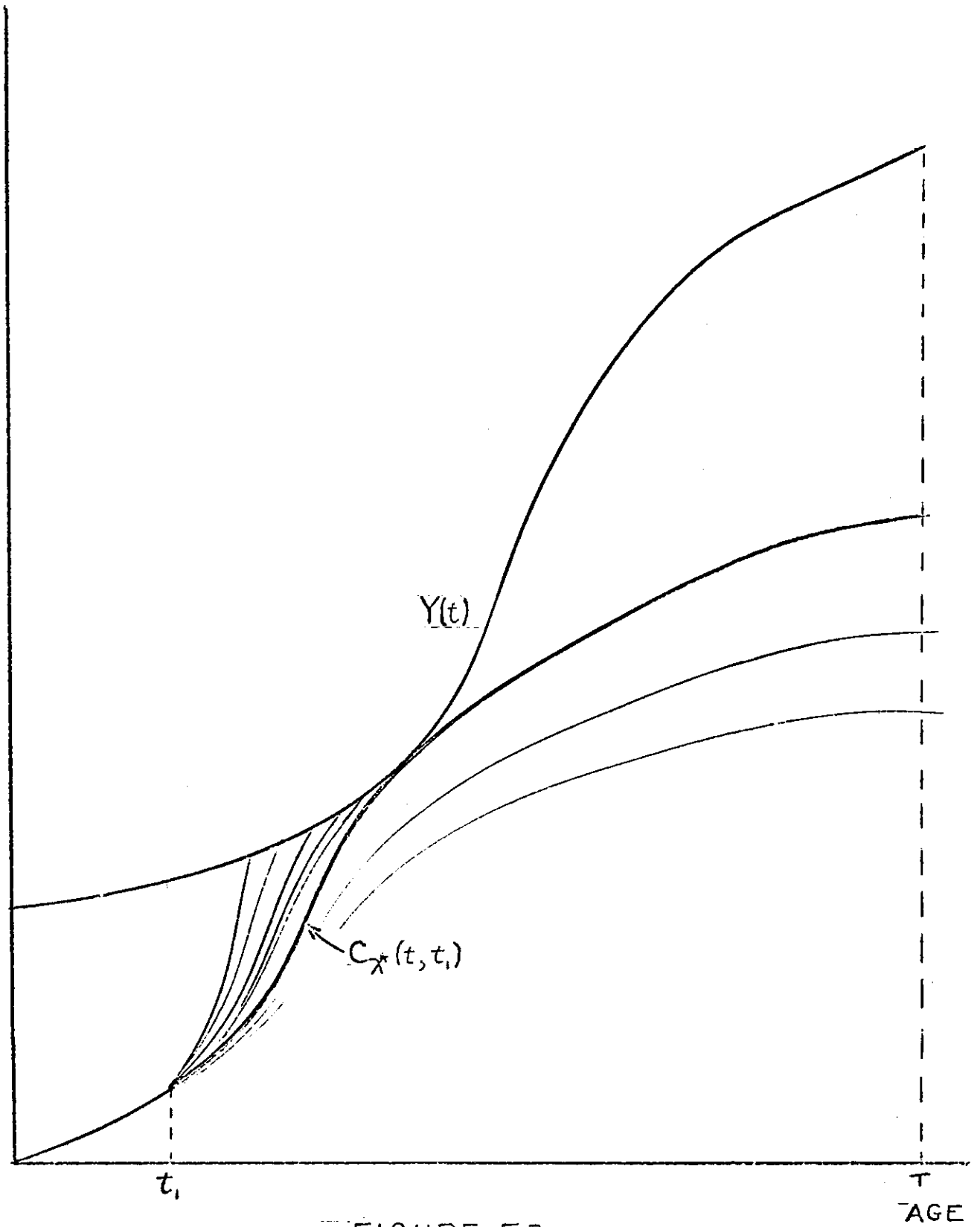


FIGURE 5.3

It is interesting to note that the reciprocal of the multiplier function can be expressed as

$$\frac{1}{\lambda(t_1)} = \min_{t < t_1} \frac{Y(t) - Y(t_1)}{t - t_1}$$

The case of logarithmic utility with arbitrary subjective discount function is almost as simple as the first case. By defining a new variable of integration,

$S = \int_0^t g(\tau) d\tau$, a new "time" variable, as it were, one can convert the maximization problem

$$\max \int_0^T u(c(t))g(t)dt$$

$$\text{subject to } C(t) \leq Y(t)$$

to

$$\max \int_0^S u(c(s))ds$$

$$\text{subject to } C(s) \leq Y(s) .$$

The constraint for the new problem is found by plotting Y versus s rather than t . If $u(c) = \ln c$, then the optimal expenditure paths in terms of the variable "s" will be straight lines when $C(s) < Y(s)$. Thus, the optimal expenditure path $C(s)$ is easily determined graphically. However, we are interested in $C(t)$ rather than $C(s)$, and this is found simply by plotting "C" versus "t" rather than "s". Figure 6.2 is

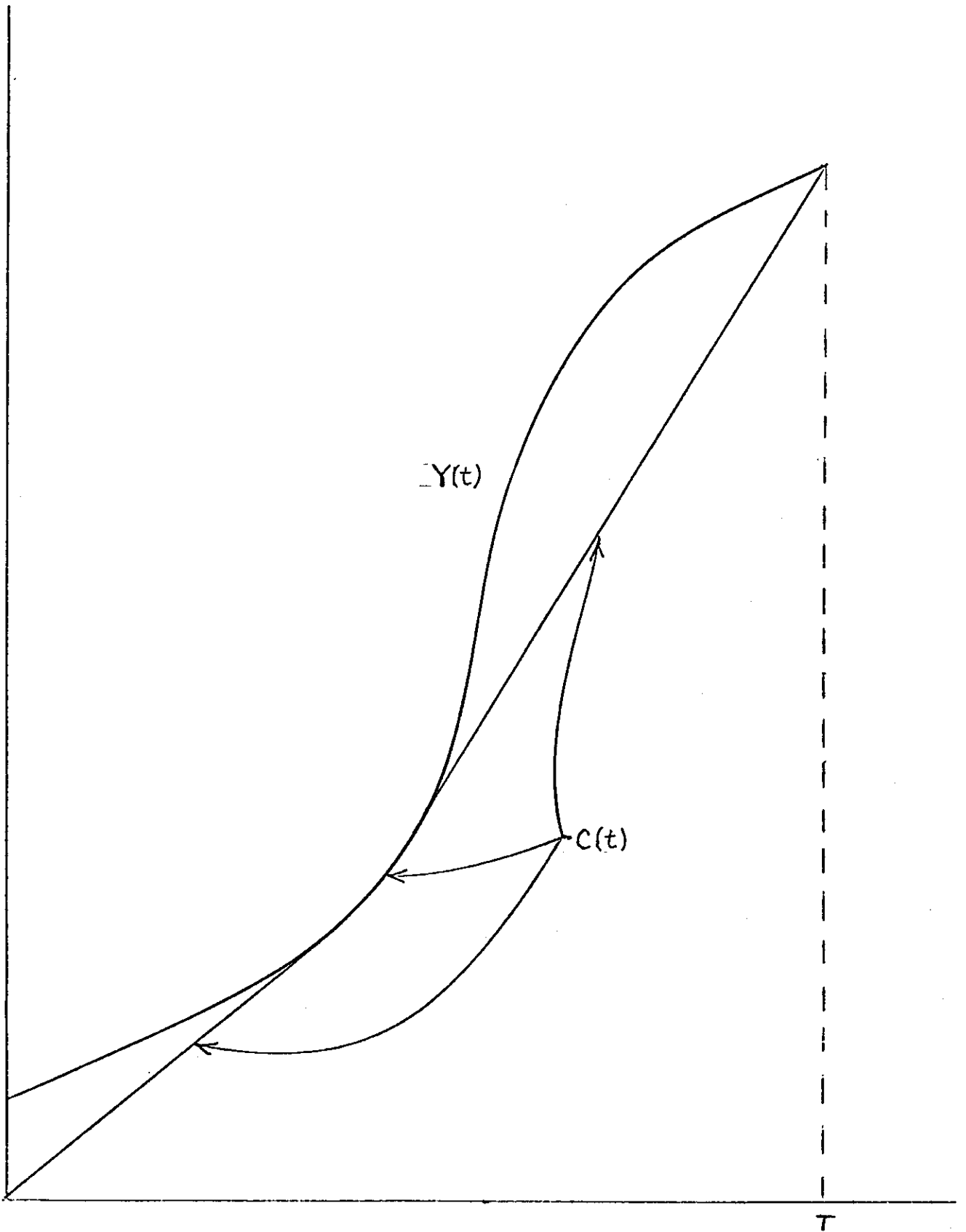


FIGURE 6.1

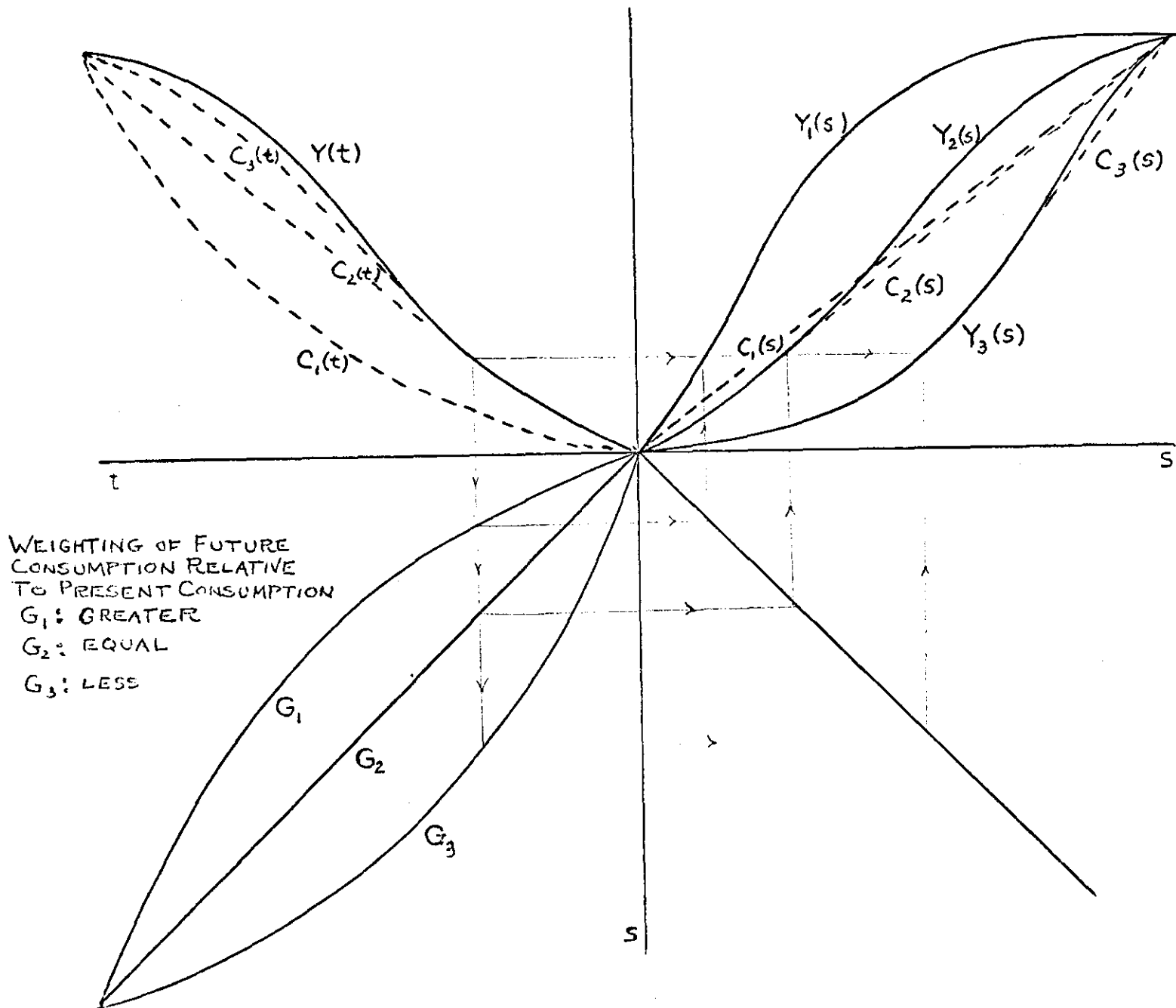


FIGURE 6.2

a graphical presentation of the above operation. The usual nomographic devices are used to keep the presentation simple. This variable has the dimensions of income and one could take $1/\lambda(t)$ to be normal income. For this case normal income would be

$$\frac{1}{\lambda(t_1)} = \min_{t < t_1} \frac{Y(t) - Y(t_1)}{\int_{t_1}^t g(\tau) dt}$$

where $g(\tau)$ is the subjective discount function. We find that normal income is always non-decreasing.

7. Effect of Uncertain Life Span

In a study of a production-smoothing problem, Modigliani and Hohn [9., p. 46] state that, "One significant aspect of the solution is that under certain conditions...the optimal plan consists of a sequence of plans covering successive intervals of the entire horizon and having the following property: the production schedule is identical with the optimum plan for the corresponding interval considered separately; it depends, in other words, on requirements for earlier or later periods."

They found, in effect, a mechanism for determining the planning horizon for a firm. One is not able to say that a decision-maker does not require information about developments which take place beyond the horizon because such information

was necessary in determining the horizon. However, it appears that less precise information is needed in determining the horizon than would be required to determine the optimal plan over the entire future.

The same type of analysis can be performed for individuals differing only with respect to the length of their lives. It is convenient to have income become available only at discrete instants. This is shown in Figure 7.1, where it is assumed that the individual is receiving an annuity which terminates only when he dies. This assumption is not critical for the result, but is included because of the importance of Social Security payments for lifetime-allocation problems. The fact that an individual's consumption in his early years should not depend critically upon his estimate of his life expectancy is a desirable feature of the model. Consumption in the last allocation period does depend upon life span expectancy, but this is not too unreasonable.

8. Involuntary Pension Programs

One type of problem which is easily considered in this analysis is that of the effect of involuntary pension programs. The purpose of such programs is to rearrange the income stream of an individual by making deduction from his wages and providing income when he retires. These programs are not intended to have a neutral effect on individual savings. Let us suppose that the rearrangement of the income stream

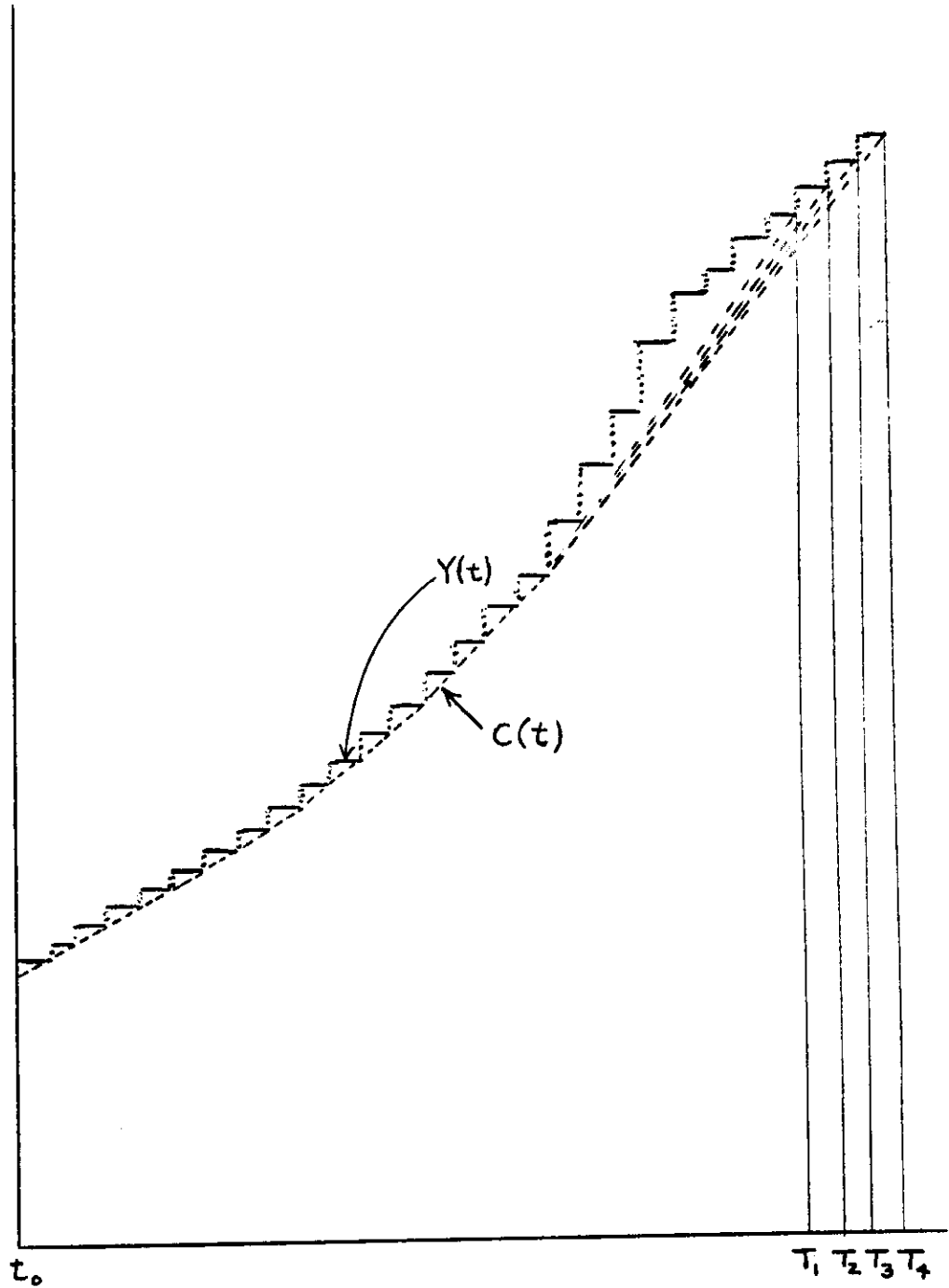


FIGURE 7.1

leaves the lifetime wealth unaffected. Figure 8.1 shows an exaggerated example for the case of logarithmic utility and no systematic time preference.

The most immediate result is that with the contractual saving plan the individual commences his discretionary saving at a later age. Of significance, however, is the fact that his retirement consumption is higher with the pension plan than without it and that his total savings are greater at each age than they would be without the pension plan. In other words, contractual saving programs should have a positive effect on total savings rather than simply displacing other types of savings. Interestingly enough, this effect has been found in survey studies by Cagan [1].

9. Windfalls

One implication of the permanent income hypothesis is that the disposition of a windfall differs from that of regular income. The empirical studies dealing with this implication of the permanent income hypothesis have had difficulties in the matter of whether or not a particular payment was anticipated or not. In the model considered here, it is assumed that the individual makes decisions on the basis of firm expectations. The expectations need not be fulfilled and the individual may revise his plans as time goes on, but he does not explicitly acknowledge the uncertainty of future

prices and income. However, the model provides interesting information about the disposition of a cash gift which we will take as equivalent to a windfall. For simplicity, the regular income will be considered as a continuous stream. Figures 9.1 and 9.2 show the disposition of a cash gift at two different ages. In the first case the "windfall" comes during a period of credit shortage and is expended over a short interval. In the figure, it is assumed that the individual has no time preference and the instantaneous utility function is logarithmic. If the individual has a systematic time preference, then the gift would be consumed over an even shorter interval. In the second case, the gift arrives during a period of positive accumulated savings and raises consumption over the entire allocation period. In the first case, it would not matter whether or not the individual anticipated the gift. The only thing that would affect the consumer's decision would be a change in the current debt limit of the individual on the basis of the expected receipt of the gift. If the gift is a temporary tax cut for next year, one would not expect this to influence the policies of the credit institutions, so that an announced tax cut for next year would probably have no influence on current consumption of those for which debt limits are operative.

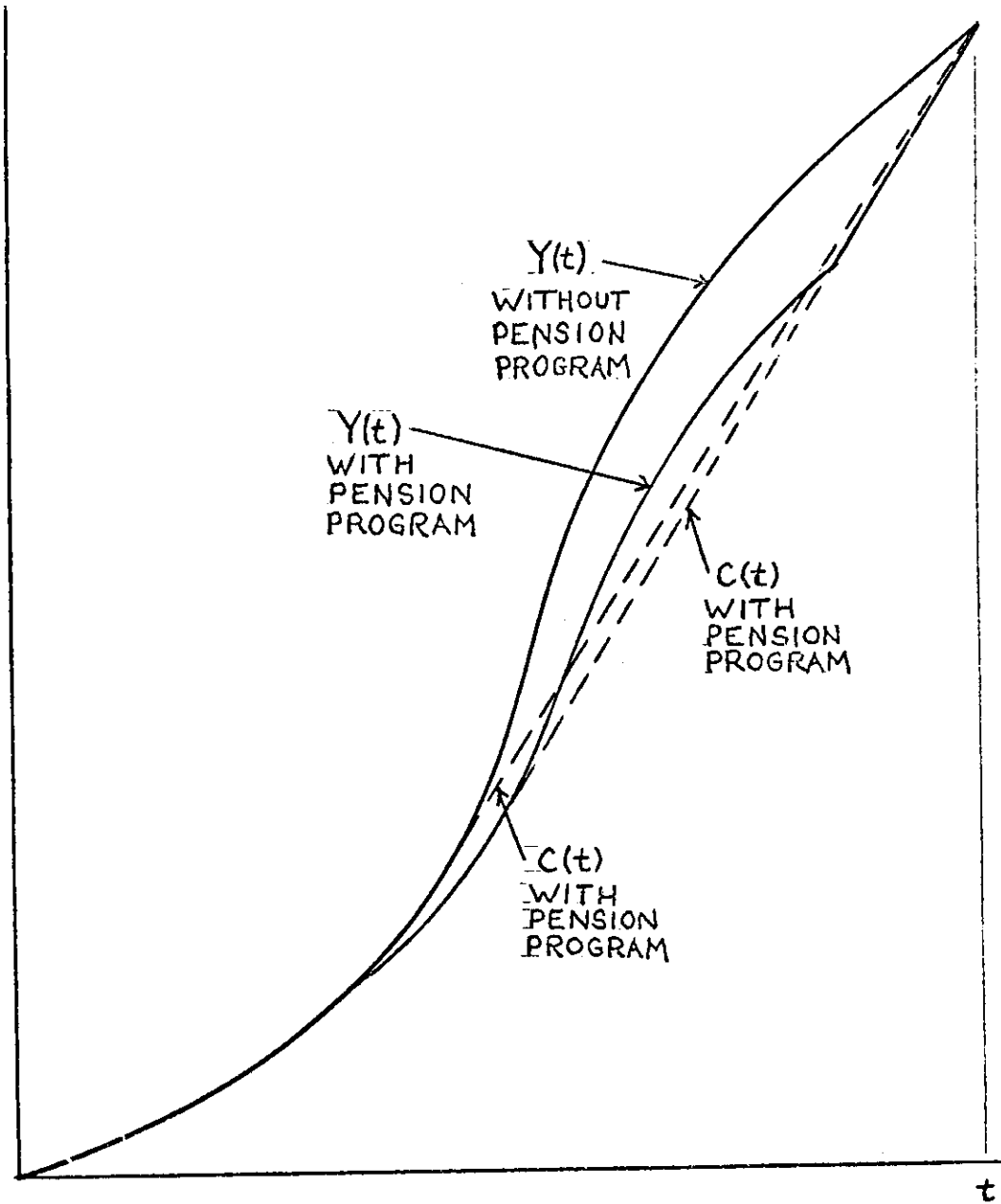


FIGURE 8.1

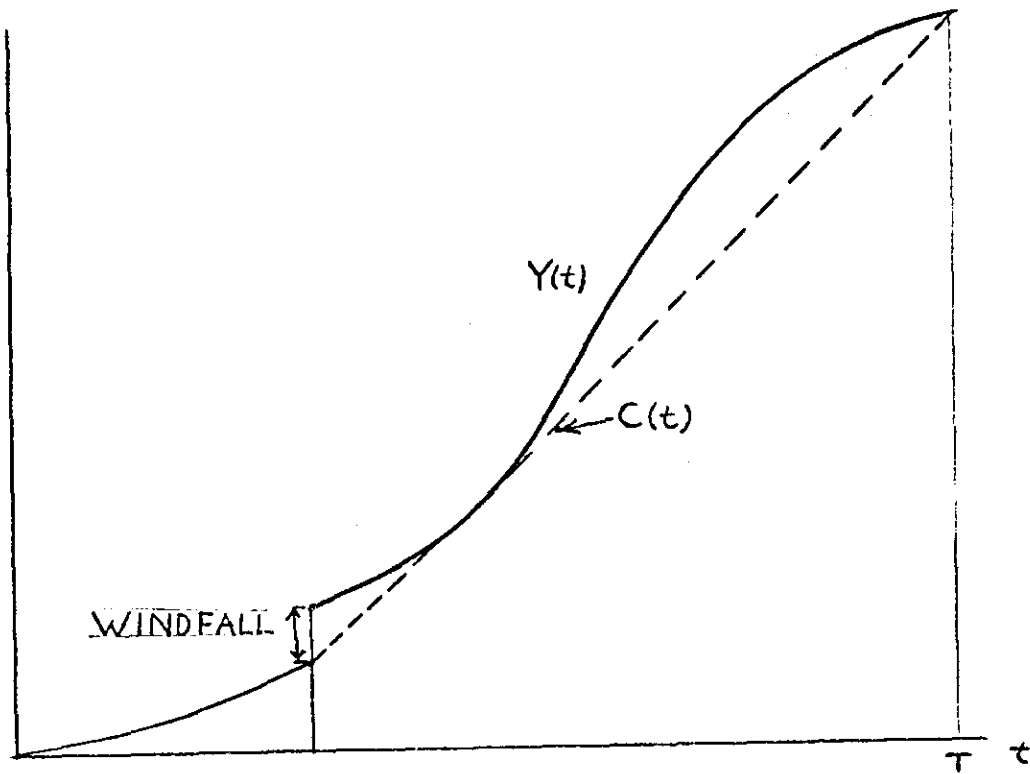


FIGURE 9.1

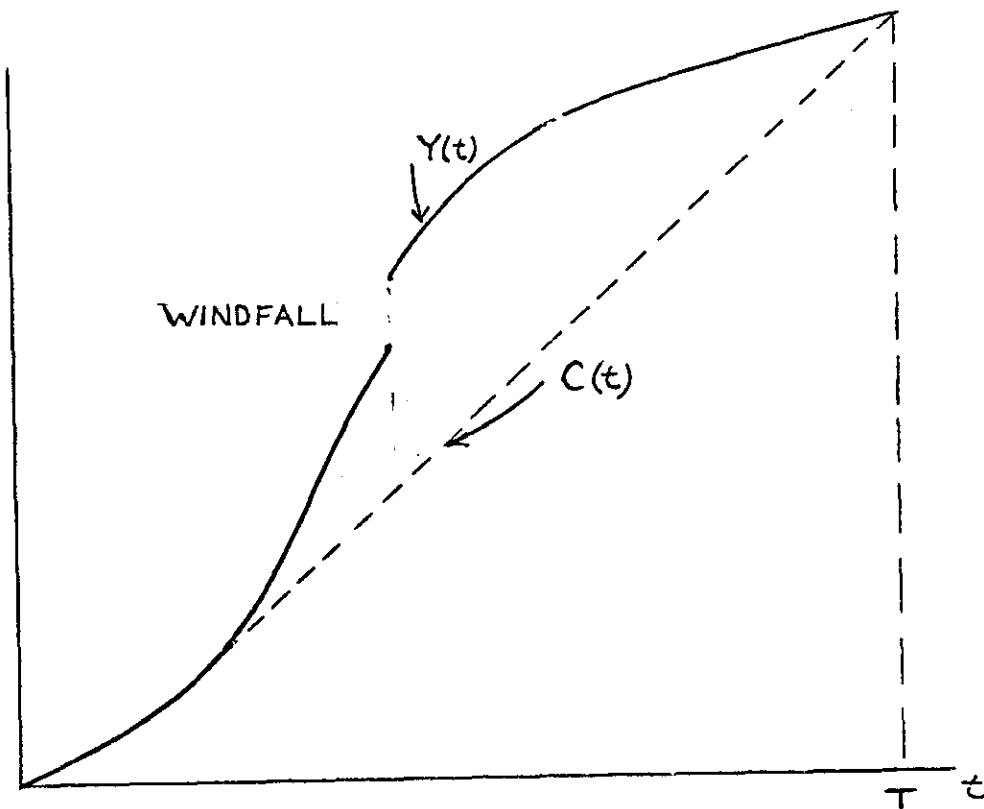


FIGURE 9.2

If the gift came during a period of positive net worth, then the anticipation of the gift would make a difference. Figure 9.2 gives the case of a fully anticipated gift, whereas Figure 9.3 gives the case in which the gift arrived entirely unexpectedly. At time t_1 the individual finds that his credit constraint is #2 rather than #1, and he revises his consumption accordingly.

It is worthwhile to note that when the credit constraint is given as a step function, a small windfall may be entirely consumed in the allocation period within which it falls as is shown in Figure 9.4.

10. Cross-Section Consumption Functions

It is of interest to plot consumption expenditure versus income for an individual with a parabolic stream assuming $i(t) = 0$ and $D(t) = 0$. This is shown in Figure 10.1. If the economy consisted of identical individuals with identical income streams, the cross-section consumption function would consist of two portions, in one of which consumption expenditure is a direct function of current income and another in which consumption is independent of current income.

If we limit consideration to constant interest rate and constant rate of time discount, then only the difference between the two parameters is significant. The plots of consumption versus income for the two relevant cases are given in Figures 10.2 and 10.3.

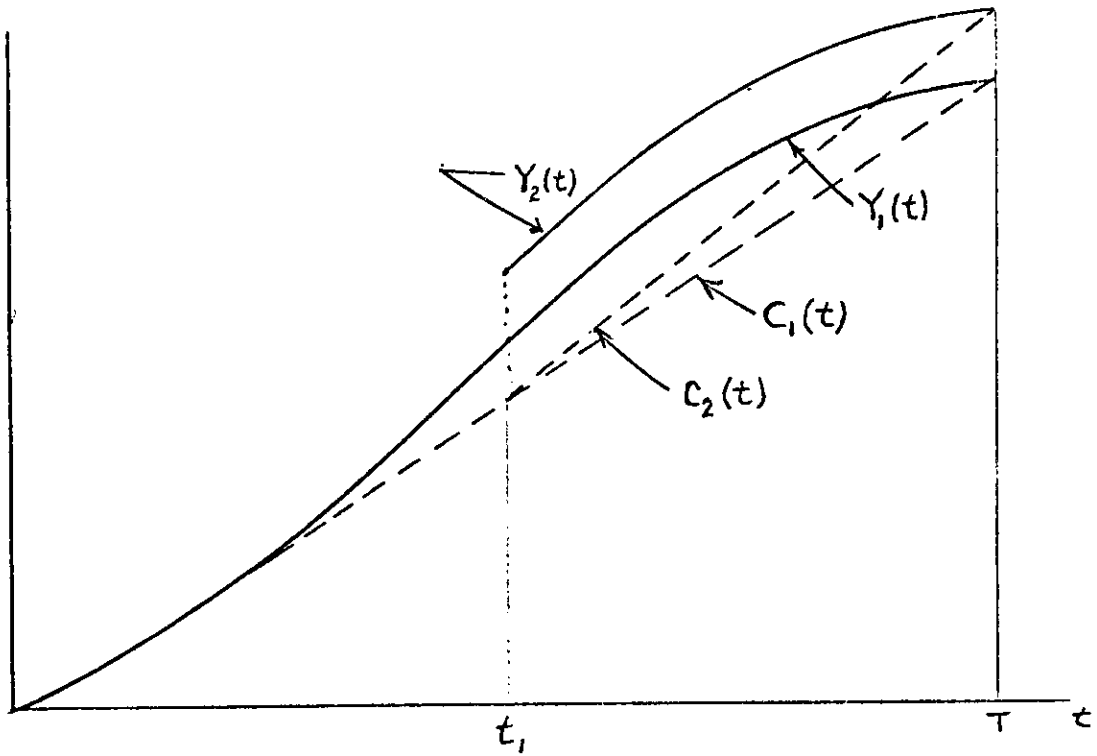


FIGURE 9.3

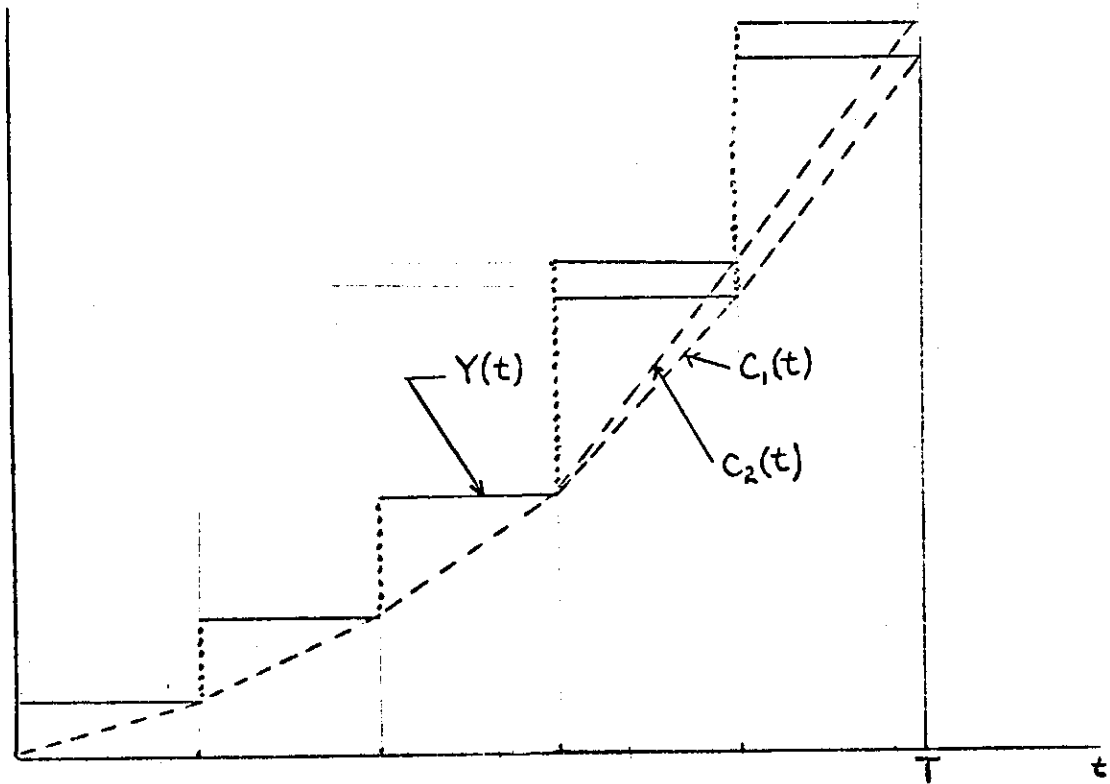


FIGURE 9.4

11. The Keynesian Consumption Schedule

In order for the main features of the Keynesian income determination model to hold, it is only necessary that there exist a relationship between consumption expenditures and aggregate disposable income in the current accounting period when all other factors determining consumption are held fixed. Some theorists might argue from a pure theory of consumption that changes in the level of disposable income in the current period would affect consumption expenditures over such a long period that the effect on current consumption expenditures would be negligible. As a consequence of their analysis, these critics feel that the multiplier effects would be insignificant. In order to construct the Keynesian consumption schedule for an individual, we assume a given credit constraint for the individual and determine the consumption expenditures which would occur during the accounting period. We then alter the credit constraint curve by assuming an increase in disposable income in the accounting period. One can then construct a schedule of the level of consumption in the accounting period that would prevail for various levels of disposable income for the accounting period. The role of anticipations of income changes comes in through the individual being able to adjust consumption expenditures between now

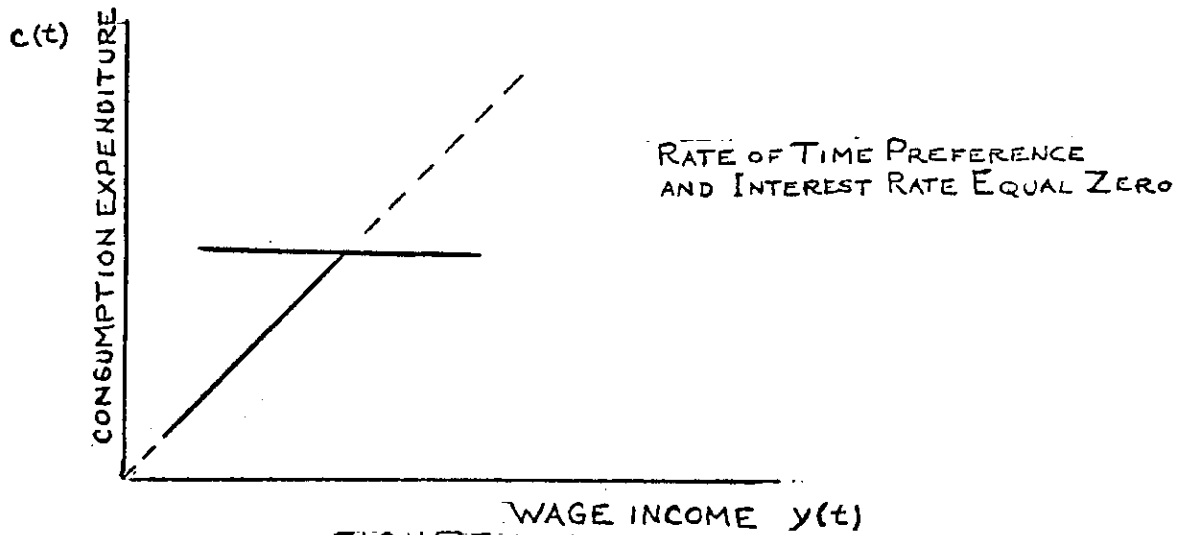


FIGURE 10.1

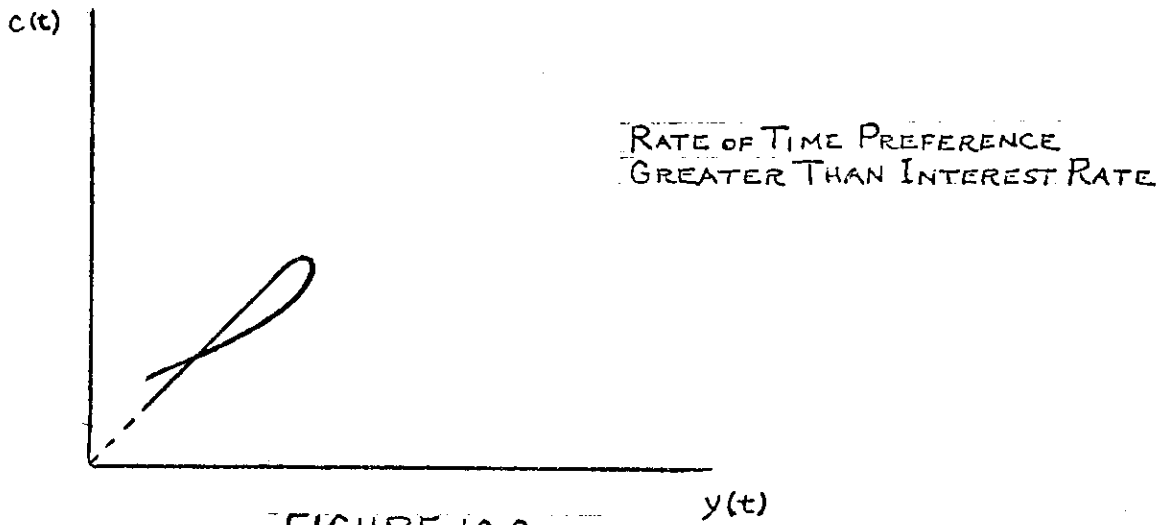


FIGURE 10.2

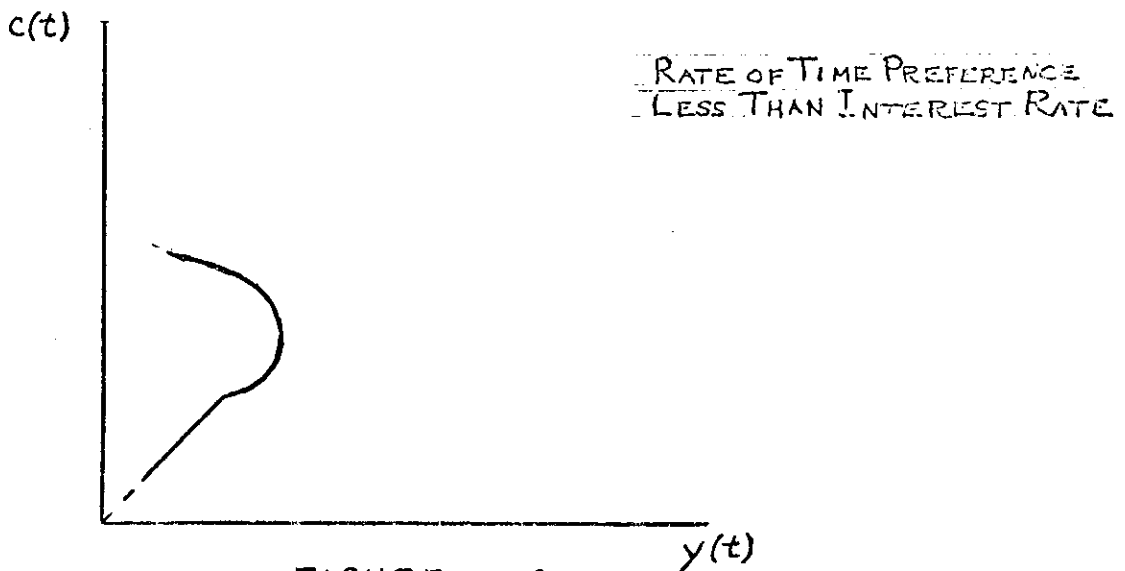


FIGURE 10.3

and the time at which the income change occurs. However, in the credit constraint model, the individual's ability to make such adjustments may be severely restricted. For example, consider the situation of someone who is living from paycheck to paycheck. If a tax cut is announced two years in advance, it would not affect his consumption in the intervening period unless credit institutions saw fit to increase the debt limit for the individual on the strength of the tax cut. The effect of a temporary change in income does depend upon the relative size of the accounting period to the allocation period, but there is no reason to take the accounting period as the remaining life span. For those individuals whose allocation period is less than the accounting period, the effects are simple. Their consumption changes dollar for dollar with the change in disposable income up to some definite level. At the threshold level, the allocation period changes and there is an accompanying reduction in their MPC out of current disposable income. Figure 11.1 illustrates the responses to the temporary changes in income. The magnitude of the change that is required to alter the allocation period from a week or a month to a lifetime, of course, depends upon the particular income stream and the rate of time preference of the individual, but one can compute the value for a hypothetical individual with a logarithmic utility function. Suppose an individual has a steady job where his wages increase 3 per cent per year

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CONSUMPTION
EXPENDITURE

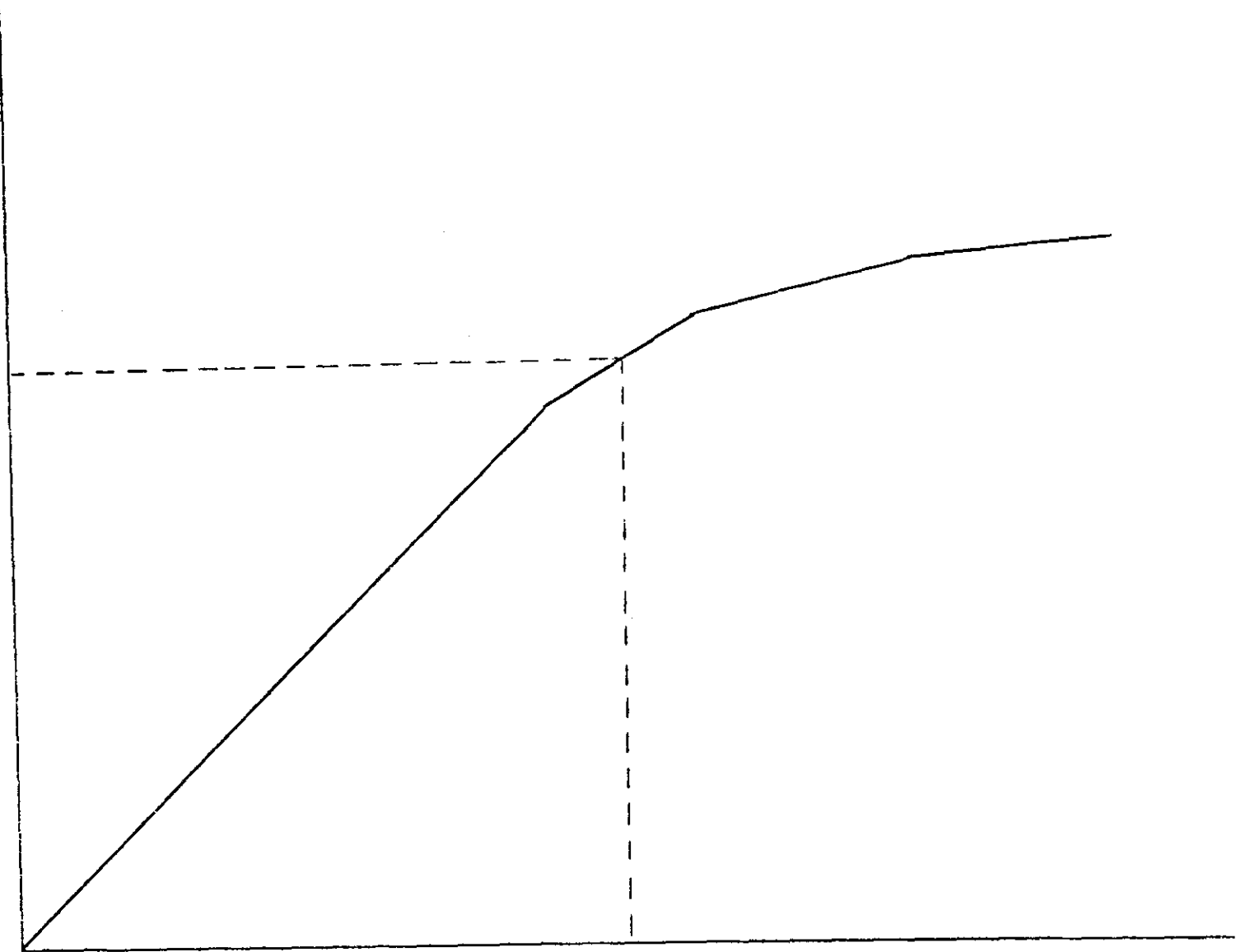


FIGURE III

EXPECTED
INCOME

CURRENT
DISPOSABLE
INCOME

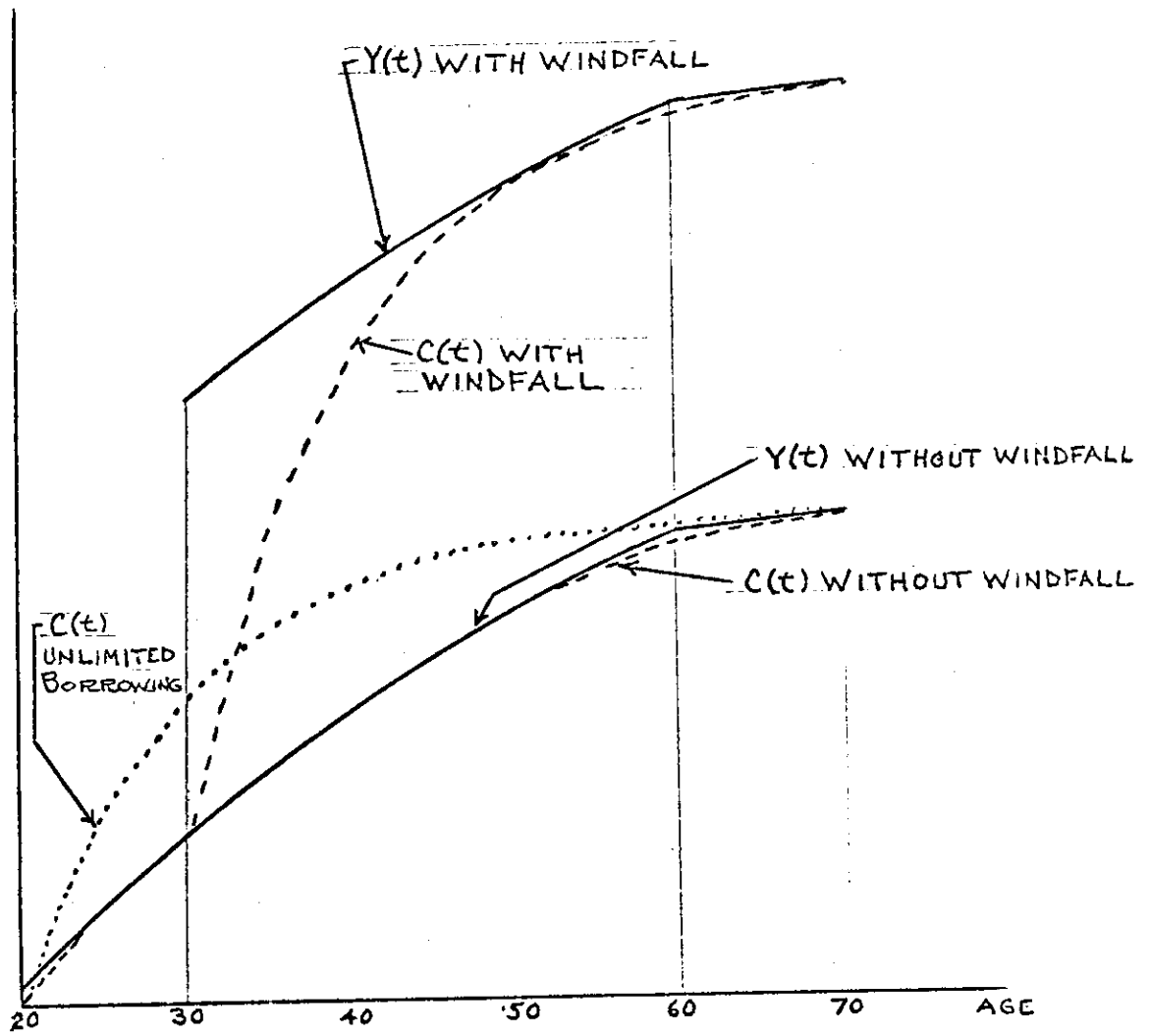


FIGURE 11.2

and that he retires at age sixty on a pension equal to one quarter of his rate of pay at retirement and lives to age seventy. Suppose that his rate of time preference is 10 per cent, while rate of interest is a constant 5 per cent throughout his lifetime. Also let us assume that credit institutions will carry an amount of this individual's debt equal to his annual income. This is quite a generous estimate of the amount of unsecured credit available to an individual. Within three years after gaining access to this credit, it will be fully used. Thereafter, the individual's allocation periods are the periods between wage payments until a point six years before retirement. At that time, he begins to pay off his debt and save to supplement his retirement income. Now suppose that at age thirty, he is to receive a gift such that he never has to worry about the credit constraint again. That is to say, he will be able to choose a consumption program consistent with his lifetime wealth and the credit constraint will not be violated for this program. How large must this gift or windfall be? Calculations show that if his yearly income is \$10,000 per year, then the magnitude of the windfall would have to be over \$200,000. Figure 11.2 gives a graphical determination of magnitude. One may conceive of this quantity being determined in the following way. A person with a subjective discount rate of ten per cent will spend 94 per cent of his lifetime wealth between age 30 and 54 if he is subject to no credit restrictions. However,

only 85 per cent of his lifetime wealth from wage earnings is available in that period. By how much must his lifetime wealth be increased through a gift or windfall at age 30 in order that 94 per cent of his total lifetime wealth be available by age 54? The answer as was given above is 1.4 times the total discounted value of his wage earnings.

12. Conclusions

This paper deals with the influence of a particular type of credit restriction, debt limitations, on the time allocation of consumption. It is felt that credit restriction is an important factor in the consumption decision and that the analysis of formal models involving credit restrictions provide useful insights into the problem. This paper analyzes a model of consumer behavior, with problems of consumer durable purchases and uncertainty eliminated, in which the net indebtedness of a consumer cannot exceed an institutionally imposed limit. The result is a method for determining the time horizon and normal income for an individual. The concept of normal or consumption-determining income which arises in the model includes credit, but excludes interest income.

The apparatus of the analysis can be applied to certain topics in consumer behavior, such as the disposition of windfalls. The methods provide a link between the pure theory of consumer behavior and the consumption function of

Keynesian analysis. There is reason to expect minor windfalls to be entirely consumed over intervals shorter than one year.

The model does not include uncertainty, but the results indicate that some elements of uncertainty may not be important for individuals subjected to debt limitations.

The model considered in this paper is not sufficiently complex to be applied without qualifications to empirical situations, but it does indicate an approach to consumer behavior over time which is of use in clarifying the role of current income in determining consumption and may aid in defining empirically the concept of normal income.

REFERENCES

1. Cagan, P., The Effect of Pension Plans on Aggregate Savings, NBER Occasional Paper No. 95, 1965.
2. Dreyfus, S.E., Dynamic Programming and the Calculus of Variations, New York: Academic Press, 1965.
3. Fisher, I., The Theory of Interest, New York: Kelly and Millman, 1954.
4. Friedman, M., A Theory of the Consumption Function, Princeton, N.J.: Princeton University Press, 1957.
5. _____, "Windfalls, the 'Horizon,' and Related Concepts in the Permanent-Income Hypothesis," in C. Christ, et al. Measurement in Economics, Stanford: Stanford University Press, 1964, pp. 3-28.
6. Howard, R. Dynamic Programming and Markov Processes, New York: John Wiley, 1960.
7. Koopmans, T. C., "Water Storage Policy in a Simplified Hydroelectric System," Proceedings of the First International Conference on Operations Research, Oxford, 1957.
8. Modigliani, F. and R. Brumberg., "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data," in K. Kurihara (ed.), Post Keynesian Economics, New Brunswick, N.J.: Rutgers University Press, 1954, pp. 388-436.
9. _____, and F. E. Hohn. "Production Planning Over Time and the Nature of the Expectation and Planning Horizon," Econometrica, Vol. 23 (1955), pp. 46-66.

10. National Industrial Conference Board, "Discretionary Income," Technical Paper No. 6, 1958.
11. Pontryagin, L. S., V. G. Bol'tanskii, R.S. Gamkrelidze and E. F. Mischenko. The Mathematical Theory of Optimal Processes, New York: Pergamon Press, 1964.
12. Tobin, J. "The Consumption Function," mimeographed copy of an extended version of an article prepared for The International Encyclopedia of the Social Sciences, 1966.
13. Yaari, M. "On the Consumer's Lifetime Allocation Process," International Economic Review, Vol. 5 (1964), pp. 304-317.
14. _____, "On the Existence of an Optimal Plan in a Continuous-Time Allocation Process," Econometrica, Vol. 32 (1964), pp. 576-590.
15. _____, "Uncertain Lifetime, Life Insurance and the Theory of the Consumer," Review of Economic Studies, Vol. 32 (1965), pp. 137-150.