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A LONG-RUN COST FUNCTION FOR THE LOCAL SERVICE

AIRLINE INDUSTRY: AN EXPERIMENT ON NON-LINEAR

ESTIMATION

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Footnote *

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A LONG-RUN COST FUNCTION FOR THE LOCAL
SERVICE AIRLINE INDUSTRY:
AN EXPERIMENT ON NON-LINEAR ESTIMATION

by

George Eads, Marc Nerlove, and William Raduchel*

... a hoarse voice spoke next.
"change engines ---" it said, and
there it choked and was obliged
to leave off.

Through the Looking Glass

In this study we formulate and estimate a cost function for the U.S. local service airline industry. Sections 1 and 2 discuss certain characteristics of the industry and of the regulation of the industry by the Civil Aeronautics Board (CAB) which influence the form of the cost function and the method of estimation chosen. The model is outlined in Section 3. The data available and the method of estimation are discussed in Section 4.

1. The Local Service Airline Industry

The U.S. local service airline industry was established in the period immediately following World War II when the

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Civil Aeronautics Board decided to extend air service to communities not then receiving service. Instead of utilizing the existing, or trunk, carriers a new class of specialist carriers was created.¹ Although the nature of the routes flown and the aircraft used by the local service carriers have changed somewhat over the last two decades, Table 1 shows that a meaningful distinction can still be made between these carriers and the trunk carriers: In 1966 the thirteen local service carriers together flew only about thirty percent as many revenue passenger miles as did United, the largest trunk carrier, while originating nearly as many passengers and making almost three times as many departures as United. The aircraft flown by the local service carriers in 1966 were on the average only one-half the size of those flown by the trunks, and the average local service flight flew only one-fourth as far as did the average trunk flight. It is also apparent that a significant difference exists in average aircraft size and average flight stage length even between the largest local service carrier and the smallest trunk carrier. These data, as well as other aspects of the operations of the local service and trunk carriers, are convincing evidence that the local service airline industry constitutes a meaningful unit for separate analysis.

TABLE 1
 SELECTED COMPARISONS OF TRUNK AND LOCAL SERVICE CARRIERS,
 DOMESTIC OPERATIONS, 1966^a

	All Trunks	United ^b	Continental	Northeast	All Locals	Mohawk	Lake Central
Revenue Passenger Miles (000)	56,802,788	11,893,275	1,870,625	933,018	3,467,510	423,247	116,296
Revenue Passenger Originations (000)	79,382	15,679	2,376	2,035	15,540	1,921	693
Aircraft Departures Performed	2,290,949	504,273	73,033	70,121	1,479,063	121,312	91,542
Revenue Passengers per Aircraft (avg.)	58.7	56.0	53.1	48.0	21.2	27.7	15.1
Available Seats per Aircraft (avg.)	100.4	96.9	96.0	82.6	42.3	51.5	33.3
Overall Flight Stage Length (miles)	434.6	445.2	482.1	276.9	111.7	125.9	85.5
On-Line Passenger Trip Length (miles)	715.6	758.5	787.4	458.4	223.1	220.3	167.9

a. Source: Civil Aeronautics Board, Monthly Report of Air Carrier Traffic Statistics (December 1966).

b. On the basis of revenue passenger miles, United is the largest trunk carrier; Northeast is the smallest; and Continental, the second smallest. Continental is included here because it, rather than Northeast, is considered to be representative of the smaller trunk carriers. Although Northeast has many aspects of a local service carrier, it can be seen that a substantial break exists between Northeast and Mohawk, the largest of the local service carriers. Lake Central is the smallest local service carrier.

For the most part the original aircraft used by the local service carriers were either war-surplus training aircraft or slightly-modified versions of civilian pleasure aircraft. Following the war, however, large numbers of war-surplus Douglas DC-3's were available at bargain prices, and a substantial pool of pilots and mechanics familiar with the aircraft existed. Consequently, the industry soon settled on the 21-28 seat DC-3 as its standard aircraft, not because it was considered ideal for the short-haul, low-density operations of the industry, but because it was readily available.²

Almost immediately the local service carriers considered replacing the DC-3 with an aircraft of equal or smaller size which would be more economical to operate because it would be specifically designed for local service operations. Larger aircraft were considered, however, as average passenger loads began to increase and as the local service carriers began to acquire longer and denser routes from the trunk carriers. A few Fairchild F-27's, a twin-turboprop aircraft of 36-40 seat capacity, were purchased by the local service carriers; most carriers, however, added larger piston engine aircraft - Convair 240's, 340's, and 440's and Martin 202's and 404's seating 36-52 passengers —

both to replace many of their DC-3's and to increase total capacity. These larger piston engine aircraft also were not designed for local-service operations but to specifications of the trunklines. They were purchased primarily because of their low initial price.

Recently the local service carriers have been disposing of their remaining DC-3 aircraft and have been either buying new turboprop aircraft such as the Fairchild FH-227 (successor to the F-27) and the Japanese Nishon YS-11 or converting their larger piston engine aircraft to turboprop power.³ They have also been purchasing large numbers of "small" jets: the BAC 111, DC-9, Boeing 727, and Boeing 737. These can seat up to 170 passengers (the Boeing 727-200), but most seat from 70 passengers (BAC 111) to 100 passengers (DC-9-30).⁴

The acquisition of these larger aircraft and the phasing out of the DC-3's have been justified on the basis of traffic growth: between 1952 and 1966 total aircraft miles rose 400 percent and revenue passengers, 900 percent. Although there are many local service routes on which traffic is so great that even aircraft twice the size of the DC-3 might be too small and although the average passenger load for local service carriers rose from 8.3 in 1952 to 21.2 in 1966, there remain a substantial number of routes on which the DC-3 is a much larger aircraft than traffic warrants.⁵

Since certain of the equipment changes now being undertaken by the local service carriers may be unwarranted given both current and projected traffic levels, a subject of obvious interest is the impact of these equipment changes on local service airline industry costs. Our cost function is formulated so as to shed light on this issue.

Since the end of 1966 three mergers involving seven of the thirteen local service carriers have been consummated: Frontier-Central (early 1967), Bonanza-West Coast-Pacific (mid-1968), and Allegheny-Lake Central (mid-1968). These mergers have been encouraged by the CAB. A cost function such as the one we are proposing may illuminate the extent of returns to scale and thus the possible consequences of mergers for efficiency.

2. The Nature of CAB Regulation of the Local Service Airline Industry.

The local service airline industry is regulated by the CAB under authority granted by the Civil Aeronautics Act of 1938 as amended by the Federal Aviation Act of 1958. The CAB regulates entry through its authority to grant Certificates of Public Convenience and Necessity to provide air service. Prior to 1955 such certificates required periodic renewal, but in 1955 Congress, over strong CAB objections,

required the Board to grant permanent certificates.⁶ A carrier must receive CAB permission to initiate or terminate service on a route, and such permission usually is not granted without a full hearing concerning whether the service is needed and which carrier is to be allowed to provide it. Local service carriers generally are not allowed to fly routes in competition with other local service carriers. Quite recently, however, they have been allowed to enter into direct competition with trunk carriers. Once a route has been authorized it must receive service amounting to at least two flights a day each way provided that traffic stays above the level of five originating passengers per station per day and that the average passenger load on the segment is at least seven passengers. Stations and routes not meeting these standards are candidates for elimination under the CAB's "Use-It-Or-Lose-It" policy, first enunciated in 1958. But such elimination is not automatic. Forty-one stations not meeting the standard were still being served in 1966.⁷

The CAB cannot control directly the maximum number of flights a carrier offers over a route, but exercises indirect control through its method of subsidy payment. Since about 1960 the CAB's method of subsidy compensation has included a limitation on the number of flights per station per day which are subsidized, though the maximum number has not

remained constant. Any excess flights must have their costs covered by revenues generated.

The CAB generally has been unsuccessful in its attempts to control the size of aircraft used by the local service carriers. The subsidy formula is established so as to be "neutral" as between aircraft types, though as Swaine has observed [23, pp. 157-62], for a short time the subsidy formula actually made it profitable to use larger aircraft than necessary.

It may be concluded that the CAB has effective control over the number of stations served by the local service carriers and substantial, but not complete, control over the total number of aircraft miles flown. However, it has little effective control over the aircraft mix used by the carriers to fly these aircraft miles. To the extent that the measure of output adopted below depends on the aircraft mix, it cannot be considered fully exogenous.

3. A Cost Model for the U.S. Local Service Airline Industry

In his study of the costs of thermal electric power generation, Nerlove [19] estimated a cost function which was a reduced form of a Cobb-Douglas production function assuming that output and factor prices are exogenous and that

the firms in the industry minimize costs. We shall employ a version of this approach but modify it for the following reasons:⁸

One major problem with the use of the approach just outlined is that, except by chance, one never observes a firm on the long-run total cost function but instead observes it on a short-run total cost function. The firm is unable in the short run to adjust all of its factors of production to the optimum level for the output level it is given to produce during the period of observation.⁹ Furthermore, it is not possible to predict a priori the direction of the bias that might result from the estimation of a long-run cost function from a scatter of points generated by observations on a family of short-run cost functions. Meyer and Kraft [17, p. 324] suggest that such bias may be eliminated by averaging observations over several time periods and performing a regression on the averaged data. This technique appears to be highly arbitrary. In an industry such as the one we are studying, moreover, it is not feasible because of the small number of firms. We estimate a family of short-run cost functions in a way that enables us to derive the parameters of the underlying long-run function.

The relationship between the long-run cost function and the associated family of short-run cost functions is well-known and available in a number of locations.¹⁰ Consider a production function,

$$(1) \quad y = f(x_1, \dots, x_n),$$

where y represents output and x_1, \dots, x_n represent inputs, x_n being the quantity of the fixed factor. The short-run cost function obtained from (1), assuming prices exogenous and cost minimization by the firm, can be written,

$$(2) \quad c_s = \varphi(y, p_1, \dots, p_{n-1}, x_n) + p_n x_n,$$

where p_1, \dots, p_n are prices. This equation states nothing more than that short-run total costs are the sum of short-run variable costs and fixed costs. Writing (2) as an implicit function we obtain

$$(3) \quad c_s - \varphi(y, p_1, \dots, p_{n-1}, x_n) - p_n x_n =$$

$$G(y, p_1, \dots, p_n, x_n) = 0.$$

The condition existing at each point of tangency between a short-run cost function and the long run cost function is

$$(4) \quad \frac{\delta G}{\delta x_n} = 0.$$

Solving (4) for x_n and substituting the result into (3), we obtain the long-run cost function,

$$(5) \quad H(y, p_1, \dots, p_n) = 0.$$

which is a function only of output and the input prices.

This suggests an indirect method of estimating the parameters of the unobservable long-run cost function: one could estimate the parameters of a family of short-run cost functions and use the above relationship to determine the parameters of the long-run cost function.

A second problem in applying the methods of the electric power study [19] is defining output for the local service airline industry. A single dimension does not suffice, but using a multidimensional output definition raises problems of a theoretical nature. Klein [14, pp. 226-236] faced a similar problem in connection with his rail cost study: He defined output for the railroads as consisting of freight ton miles and passenger miles. However, when two or more such variables are incorporated directly in a log-linear production function, the implied product transformation loci are convex rather than concave; thus the second-order conditions for profit maximization cannot be satisfied. Klein argued that this was irrelevant, since the regulated nature of the industry precluded substitution among different outputs, i.e., railroads were not free to vary the mix of passenger and freight services offered. Unfortunately, the choice of a particular relationship among the output levels may affect the statistical relationship

of the input levels both among themselves and to the output levels. To avoid the possibility of such misspecification, it seems preferable to adopt a theoretically more defensible form.

Consider the family of functions given by

$$(6) \quad y^n = \tau_1 a_1^n + \tau_2 a_2^n + \dots + \tau_m a_m^n,$$

where $1 < n < \infty$. Interpret the a_i 's as various measures of airline output such as the number of stations served or the number of seat miles flown. The τ_i are weights to be estimated. Defining output, y , as the n^{th} root of the right-hand side of (6) produces product transformation curves of the proper shape. If $n = 2$ and $\tau_1 = \tau_2 = \dots = \tau_m$, the product transformation locus is an m -dimensional hypersphere. If $\tau_1 \neq \tau_2 \neq \dots \neq \tau_m$, it is an m -dimensional hyperellipsoid.¹¹

In view of the questions to which the cost function is relevant, it is desirable that the output measure adopted incorporate some measure of aircraft choice. Let us assume that there are k aircraft types. Let a_i be the i^{th} output component defined in (6), and let

$$(7) \quad a_i = \epsilon_1 a_{i1} + \epsilon_2 a_{i2} + \dots + \epsilon_k a_{ik},$$

where a_{ij} represents the output measure i flown by aircraft type j and ϵ_j represents a weight to be determined.¹²

A final reason for modification of the methodology is that one factor of production, fuel, appears to be used in fixed proportions; that is, fuel costs appear to be determined only by the price of fuel and by output and not by the prices of other factors of production. The use of a cost function derived from a Cobb-Douglas production function implies that the elasticities of substitution between factors of production are equal and equal to one. The use of a CES production function removes the restriction of unitary elasticity of substitution but still implies equality. It is possible, however, to derive a cost function having an underlying production function which permits different elasticities of substitution between different pairs of factors. Irrespective of the underlying production functions, the relationship between the long- and short-run cost functions examined above holds.

Our model takes the following form:

$$(8) \quad c_f = p_f^f \text{ (fuel cost equation)}$$

$$(9) \quad c_{ao(s)}^* = (c_{ao(s)} - p_2 x_2) = k p_1^{1/\alpha_1} x_2^{-\alpha_2/\alpha_1}$$

(short run variable cost equation)

$$(10) \quad c_{ao(l)} = k' y^{1/r} p_1^{\alpha_1/r} p_2^{\alpha_2/r} \text{ (all other costs, long run)}$$

(11) $c_t = c_{ao(\ell)} + c_f$ (long run total costs); where

(12) $y = \text{output} = [\tau_1 (\epsilon_1 a_{11} + \epsilon_2 a_{12} + \dots + \epsilon_k a_{1k})^2 + \tau_2 a_2^2]^{\frac{1}{2}}$, and

where: $r = \alpha_1 + \alpha_2$,

$k = \alpha_0^{-1/\alpha_1}$,

$k' = r [\alpha_0^{\alpha_1} \alpha_1^{\alpha_2}]^{-1/r} = r [(\frac{\alpha_1}{k})^{\alpha_1} \alpha_2]^{-1/r}$,

$\alpha_0, \alpha_1, \alpha_2$ = parameters to be estimated,

c_f = fuel costs,

p_f = price of fuel, piston (PFP) and turbine (PFT),

f = gallons of fuel used (GFP (piston fuel) and GFT (turbine fuel)),

$c_{ao(s)}$ = total costs except fuel, short run,

$c_{ao(\ell)}$ = total cost except fuel, long run,

$c_{ao(s)}^*$ = short run variable costs (COST),

p_1 = price of labor less pilots and copilots (PLNP),

p_2 = price of pilots and copilots (PPILOT)

x_2 = stock of pilots and copilots (PCP),

c_t = total long-run costs,

a_{11}, \dots, a_{1k} = number of aircraft miles flown by aircraft type (or group) a, ..., k (ACMA, ..., ACMK),

a_2 = number of stations served, (STA), and

$\tau_1, \tau_2, \epsilon_1^m, \dots, \epsilon_k$ = parameters to be estimated,

normalized so that $\tau_1 + \tau_2 = 1$ and $\epsilon_1 + \dots$

$+ \epsilon_k = 1$.¹³

The reader will note that this cost function is derived in part from a Cobb-Douglas production function. The short-run elasticity of substitution between all factors of production is zero while the long-run elasticity of substitution is zero between fuel and the two other factors of production, (pilots and copilots and labor other than pilots and copilots), while it is equal to one between the latter two factors of production. The stock of pilots and copilots is assumed to be the factor whose input is fixed in the short run.

The Choice of the Fixed Factor. To assume a labor factor as fixed in the short-run is somewhat unconventional and requires explanation. To contend that the stock of pilots and copilots is a proper measure of the flow of fixed factor services, it is necessary to argue that short-run changes in the stock of pilots and copilots are at least

as difficult to make as short-run changes in the stock of aircraft. We also argue that short-run changes in the utilization of the stock of pilots and copilots are not significant, thus eliminating the problem that arises from using a stock to measure a flow. The argument requires a brief description of the markets for aircraft and pilots and the ways in which an airline might be able to vary its input of each of these services in the short run.

Local service airlines purchase aircraft in two markets: the new aircraft market and the used aircraft market. Until the advent of the pure jets, the only aircraft purchased in the new aircraft market was the F-27; all the large piston and DC-3 aircraft purchased were used aircraft. New aircraft are seldom available for immediate delivery. At the time of his order the purchaser is usually assigned a position in a delivery queue, and actual delivery does not take place for several months or, in some cases, for several years.

The used aircraft market works differently: There usually exists a small pool of used aircraft of various types which are available for purchase on relatively short notice. Such purchases are arranged through aircraft brokers, and the carrier may acquire the aircraft from almost any point in the world. Because of safety regulations, the buyer of used

aircraft is aware of the condition of any aircraft in which he may be interested. The market is highly competitive, and the price reflects the condition of the aircraft. Thus, if an airline is willing to acquire used aircraft, it is possible for it to obtain aircraft on relatively short notice without paying a substantial premium for early delivery.

If purchases of new or used aircraft were the only two ways in which a carrier might increase the supply of aircraft services, it would be necessary to consider the stock of aircraft as fixed to the industry as a whole, at least in a period as short as a quarter. This would introduce some complication in the treatment of aircraft services as variable for the individual air carriers.

Fortunately, two other sources exist: First, there is a world market for the leasing of aircraft by one carrier from another. A carrier with temporary over-capacity can reduce it and a carrier needing additional capacity on a short-run basis can obtain it rapidly. Unlike the used aircraft market, the market for leased aircraft is not limited to older types of aircraft, but short-term leasing may be relatively expensive.

Second, the utilization rates of existing fleets of aircraft may be varied in the short run. Though limited by maintenance requirements set by law, significant short-run changes in fleet utilization are possible and are, in fact,

widely observed. The cost of maintaining a small number of fully depreciated aircraft in order to be able to provide short-run increases in capacity is not large and airlines are known to maintain such hedging stocks. (Note that such stocks result in difficulties in using the stock of aircraft to measure services from the fixed factor.)

In contrast, the stock of pilots and copilots can not be varied so easily as the stock of aircraft. To be sure, there usually exists a pool of trained and licensed pilots who may be hired by an airline, but in many cases newly-hired or promoted pilots and copilots must be trained and certified to fly the particular types of aircraft the airline owns. This is time consuming, and the problem is exacerbated by the large variety of aircraft flown by the local service carriers.

Furthermore, even if pilots certified to fly the particular types of aircraft possessed by the airline are available, they cannot be put to work immediately upon being hired, because they must be thoroughly familiarized with the routes over which they will be flying before they can fly with paying passengers. Route familiarization requires the use of flight time of senior pilots, which is obviously in short supply when an airline is faced with an unplanned increase in its output. Differences in operational procedures among

airlines and pilot seniority rules also serve to hamper carrier flexibility in the acquisition and termination of pilots in response to short-run fluctuations in demand.

Utilization of the stock of pilots and copilots also cannot be varied easily. Airlines are discouraged from maintaining a stock of pilots in excess of normal requirements by union agreements which require that a large portion of a pilot's salary be paid regardless of whether or not he flies. The maximum amount of time a pilot can fly during a month and at one continuous stretch is regulated by the Federal Aviation Administration. Collective bargaining agreements concerning pilot and copilot scheduling restrict actual utilization considerably below these limits. [1, especially pp. 57-156].

The airline strike of 1966 provides perhaps the best example illustrating that the stock of pilots and copilots, rather than the stock of aircraft, is the fixed factor in the short run. The local service carriers which served areas also served by the struck trunk carriers were faced with an unexpected and massive increase in demand and were able to take advantage of this increase only to a limited extent. This was partly because of a limited stock of aircraft and pressure on ground facilities, but the major difficulty

mentioned in reports during the strike was the problem of obtaining sufficient pilot time to fly the aircraft the local airlines had at the rates of aircraft utilization they were able to achieve.¹⁴

The choice of the number of pilots and copilots as a measure of services from the fixed factor appears justified.¹⁵

4. Data and Estimation

The Data. The data used in this study consist of quarterly observations on twelve of the thirteen local service carriers over a nine-year period from the first quarter of 1958 through the fourth quarter of 1966, virtually the entire period of transition from use of the DC-3 to use of larger aircraft. At the end of 1957, ten of the thirteen local service carriers were operating all DC-3 fleets. The first of the turboprop aircraft, the F-27, was introduced in 1958. Extension of the period of observation prior to 1958 in order to cover the entire equipment transition was precluded by a major change in the CAB accounting system which took effect in 1957 as well as difficulty of access to earlier data.

Aircraft Classes. The local service airlines have used several aircraft types. Table 2 lists all of the aircraft

TABLE 2

SELECTED SPECIFICATIONS OF AIRCRAFT USED BY THE LOCAL SERVICE AIRLINES^g

Aircraft Group	Aircraft Type	Crew	Passengers	Gross Weight ^a	Speed ^b	Engine Characteristics
A	DC-3	2	21-28	25,000	167	2, piston, 1275 hp. ^c
	Nord 262	2	27-29	22,930	224	2, turboprop, 1065 eshp. ^d
B	Convair 240	2	40	41,790	265	2, piston, 1800 hp.
	Convair 340	2	44	47,000	284	2, piston, 1800 hp.
	Convair 440	2	44-52	49,100	289	2, piston, 1800 hp.
	Martin 202	2	36-40	39,900	286	2, piston, 1800 hp.
	Martin 404	2	40	43,000	280	2, piston, 1800 hp.
C	F-27A	2	36-48	42,000	293	2, turboprop, 2020 eshp.
	FH-227	2	44-52	43,500	293	2, turboprop, 2230 eshp.
	Convair 540	2	48-52	50,670 ^e	322	2, turboprop, 3500 eshp.
	Convair 580	2	52	50,670 ^e	342	2, turboprop, 3750 eshp.
	Convair 600	2	46	45,000	312	2, turboprop, 3025 eshp.
D	DC-9-10	2	90	77,700	559	2, turbofan, 14000 lbs. ^f
	BAC 111-200	2	79	78,500	550	2, turbofan, 10410 lbs.
	Boeing 727	3	70-131	161,000	520	3, turbofan, 14000 lbs.

a. Normal gross weight in pounds.

b. Best cruise speed in miles per hour.

c. Brake horsepower.

d. Equivalent shaft horsepower

e. Maximum landing weight.

f. Pounds thrust.

g. Sources: [20], [33], [25, pp. 21-22], and [26, pp. 176-77].

types that were operated up to the end of 1966 with the exception of small single engine and twin engine aircraft, most of which had been replaced by DC-3's by the early 1950's. Table 2 also illustrates the similarity among a number of these aircraft types and suggests that a suitable grouping of similar types would not result in the loss of a great deal of information.

Another reason for grouping is that a costing method such as the one proposed here should not find significant differences in cost behavior among closely similar aircraft types. Since each carrier tends to operate only one of the aircraft from each of the groups (e.g., either Martin 202's or Convair 240's, but not both), if differences in cost behavior between two similar aircraft were found by the model, it might show the existence of interfirm differences in the method of operation rather than true significant differences in aircraft cost characteristics. Conversely, by adjusting the sample to remove all firm effects, some of the variance due to the different types of aircraft operated by the firms might be simultaneously removed.

Group A consists of the DC-3 and the Nord 262. Although the Nord 262 is turbine powered and might be conceivably placed in Group C with the other turboprop aircraft, its

similarity in size and power to the DC-3 plus the limited experience in its operation dictate the chosen grouping. Only one airline, Lake Central, has purchased the Nord, and it began Nord operations only late in 1965. Furthermore, the Nord was grounded for a substantial period of time due to in-flight engine failures which have since been corrected.

The case for grouping aircraft types is most obvious among the large piston aircraft which are referred to as Aircraft B: the Convair 240, 340, and 440 and the Martin 202 and 404. They are all of virtually the same size and seating capacity. Their cruising speeds are similar, and they are powered by modifications of the same Pratt & Whitney engine, the R-2800. The major difference among the aircraft in this group is that the Convair 240 and the Martin 202 are unpressurized, while the other aircraft are.

Aircraft Group C consists of all turboprop aircraft excluding the Nord 262. On the basis of power alone, it might appear that the turboprop aircraft should be divided into two classes: (1) the F-27 and FH-227, and (2) the turboprop Convair conversions. However, since the turboprop Convairs did not begin significant operations until the very end of our sample period, there are insufficient observations on their operations to justify grouping them separately.

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Group D consists of pure jet aircraft. Local service airline experience with these aircraft is also quite limited, the first having been put into service by Mohawk in 1965. Towards the end of the sample period, however, their use was becoming quite widespread. The major lack of homogeneity in this group involves the Boeing 727. This aircraft has three engines, as compared with two for the BAC 111 and the DC-9, and it carries a flight crew of three, instead of the usual two. Only two local service airlines, Pacific and Frontier, use this aircraft, and they each acquired it in mid-1966.

Estimation of the Fuel Equations. Separate equations were estimated for turbine and piston fuel. Estimation of a single equation requires a single fuel price, but a single price index is not meaningful, since it depends solely upon the relative weights given to the prices of piston and turbine fuel. Furthermore, the use of a single equation implies the possibility of substitution between turbine and piston fuel, and this is not possible once a choice of aircraft has been made.

The entire sample was used to estimate the piston fuel equation. A smaller, nine-carrier, seven-quarter sample was used to estimate the turbine fuel equation. The shorter period

encompassed all significant turbine operations for most carriers with the exception of those few which bought F-27's in 1958 and 1959. The equations were estimated in natural units, not logarithms, and were forced through the origin on the assumption that if no aircraft miles were flown and no aircraft departures were made, no fuel was used.

The method of estimation used was the two-round procedure developed by Balestra and Nerlove [2, pp. 593-599], and the reader is referred to [2] for details. In their procedure it is assumed that the residual disturbance term can be divided into two stochastically independent parts, a time-invariant firm effect and a remainder. The ratio of the contribution to total variance of the time-invariant effect to the total variance is denoted by ρ . It is also possible to include a firm-invariant time effect. This was not done both for reasons of computational simplicity and because the need for it in this case was not readily apparent. An estimate of the disturbance variance-covariance matrix was obtained in the first round and used in the second round to obtain the coefficient estimates by means of generalized least squares.

The estimates are shown in Table 3.¹⁶ The relative magnitudes and signs of the coefficients are as expected. The multicollinearity tests suggested by Farrar and Glauber [7]

TABLE 4
FUEL EQUATIONS

I.	GFP = 0.595 ACMA + 1.093 ACMB + 8.639 ACDA	
	(0.041) (0.033) (3.408)	
	+ 17.283 ACDB	R ² = 0.9930 ^a
	(3.675)	ρ = 0.2793
II.	GFP = 0.679 ACMA + 1.241 ACMB	
	(0.009) (0.006)	
		R ² = 0.9915
		ρ = 0.3758
III.	GFT = 1.222 ACMC + 2.842 ACMD + 40.825 ACDC	
	(0.201) (0.374) (27.245)	
	+ 94.723 ACDD	R ² = 0.9874
	(69.432)	ρ = 0.3487
IV.	GFT = 1.517 ACMC + 3.369 ACMD	R ² = 0.9847
	(0.027) (0.090)	ρ = 0.3855

^aThe R²'s shown in Table 3 and Table 4 are the ratio of the explained variance of the original dependent variable (denoted u'u) to the total variance of the original dependent variable (denoted y'y),

$$\text{i.e. } R^2 = \frac{y'y - u'u}{y'y}$$

The explained variance calculated from the second round estimates of the two-round Balestra-Nerlove procedures is the explained variance of the transformed dependent variable (denoted u*'u*). The following equation gives the relationship between u'u and u*'u*,

$$u'u = \frac{1 - \rho + T\rho}{1 - T\rho} u*'u^*,$$

where ρ is as defined in footnote 18 and T is the number of time periods in the sample.

indicate that there is significant collinearity present. As would be expected, the diagnostic section of the tests indicates that important collinearity exists between the aircraft mile and aircraft departure variables for each aircraft type. Consequently, a second equation for each fuel type was estimated using only the aircraft mile variables in each case. The results are also shown in Table 3. Note that the estimated values of ρ are much closer for regressions II and IV than for I and III.

Estimation of the "All Other Costs, Short Run" Equation.

This equation was estimated in two parts. It was not found possible in earlier estimation attempts [6, pp. 81-85] to obtain simultaneous estimates of the fixed factor coefficient, α_2/α_1 and ρ , the ratio of the variance of the firm "effect" to the total residual variance. A possible reason for this difficulty is that the factor does vary, albeit slowly, over time and that its level reflects interfirm differences to some extent. The method adopted was to obtain an independent estimate of the fixed factor coefficient from a derived demand equation for pilots and copilots. The equation estimated was of the form:

$$x_2 = k'y^{1/r} (p_2/p_1)^{-\alpha_1/r} v,$$

where $k'' = \alpha_2 (\alpha_0 \alpha_1^{\alpha_1} \alpha_2^{\alpha_2})^{-1/r}$,

x_2 = number of pilots and copilots (PCP),

y = output (ACMT),

p_1 = price of labor net of pilots and copilots (PLNP),

p_2 = price of pilots and copilots (PPILOT), and

v = an error term whose logarithm is assumed to have the properties outlined in Balestra and Nerlove [2, pp. 594-595].

Total aircraft miles (unweighted) was used as a simple proxy for output in this equation.¹⁷

This equation is a long-run derived demand equation for pilots and copilots; therefore, a way must be found to deal with the disturbing influences of short-run demand fluctuations. Ideally, we should formulate a distributed lag structure and estimate it as part of the equation. For simplicity, however, we took eight-quarter moving averages of the price and output variables, centered on the fourth quarter. In doing so, a specific lag distribution of exceptionally simple form was imposed. The simple output definition and the arbitrary distribution assumed are perhaps the weakest links in our estimation procedure and more work is clearly needed in this connection. For this reason we use this equation only to determine the necessary adjustment to costs in the short-run cost equation.

TABLE 4
DERIVED DEMAND EQUATION FOR PILOTS

$$\begin{aligned} \log \text{ PCP} &= 0.2712 + 0.7840 \log \text{ ACMT} \\ &\quad (0.2265) \quad (0.0207) \\ &\quad - 0.3624 \log (\text{PPILOT/PLNP}) \quad R^2 = 0.882^a \\ &\quad (0.1458) \quad \rho = 0.397 \end{aligned}$$

^aSee Note a, Table 3.

The results of the estimation of the derived demand equation for pilots and copilots are shown in Table 4 above. The estimates were obtained using the two-round Balestra-Nerlove procedure previously described. The signs of the coefficients are all as expected and the level of significance is high.

From Table 4 we see that $\frac{-\alpha_1}{r} = -0.3624$ and $1/r = 0.7840$. From these we obtain $r = 1.2755$, $\alpha_1 = 0.4622$, and, by utilizing the definition of r , $\alpha_2 = 0.8133$. Finally we obtain the estimate we are seeking, $\alpha_2/\alpha_1 = 1.7596$. This value multiplied by the logarithm of the number of pilots and copilots was added to the logarithm of the dependent variable for each observation. This "adjusted" variable was then used as the dependent variable in the equation explaining short-run costs less fuel costs.

In estimating the equation for short-run costs less fuel costs, we employed a maximum-likelihood approach rather than the two-round procedure used for the previous five equations. The two-round procedure was ruled out by the highly nonlinear definition of output (see equation (12)). As will appear, our results are reasonable economically, which was not the case in the earlier study of Balestra and Nerlove [2, pp. 599-60] when the maximum-likelihood approach was attempted in a similar context.

The likelihood function to be maximized is of the form,

$$(14) \quad L(\psi, \rho, \sigma^2) = -\frac{NT}{2} \log 2\pi - \frac{1}{2} \log |\Omega| - \frac{1}{2} u' \Omega^{-1} u,$$

where N is the number of firms in the sample; T is the number of time periods; Ω , ρ , and σ^2 are defined exactly as in Balestra and Nerlove [2, pp.594-5],¹⁸ ψ stands for the parameters of u, and where u is defined as follows:

$$(15) \quad u = c' - \log k - \frac{\kappa}{2} \log [\tau_1 (\epsilon_1^{ACMA} + \epsilon_2^{ACMB} + \epsilon_3^{ACMC} + \epsilon_4^{ACMD})^2 + \tau_2 (STA)^2],$$

where κ is the short-run elasticity of all costs excluding fuel costs with respect to changes in output, $c' = \log \text{COST} + \frac{\alpha_2}{\alpha_1} \log \text{PCP}$, k is the constant term of equation (9), and $\frac{\alpha_2}{\alpha_1}$ is the estimate obtained from the derived demand equation presented in Table 4.

Balestra and Nerlove [2, pp. 608-12] show that (14)

can be rewritten as:

$$(16) \quad L(\psi, \xi, \eta) = -\frac{NT}{2} \log 2\pi - \frac{N}{2} \log \xi - \frac{N(T-1)}{2} \log \eta \\ - \frac{1}{2} \left\{ \frac{M_1(\psi)}{\xi} + \frac{M_2(\psi)}{\eta} \right\},$$

where: $\xi = \sigma^2 [(1-\rho) + T\rho]$;

$$\eta = \sigma^2 (1-\rho),$$

$$M_1(\psi) = \frac{1}{T} \sum_{n=1}^N \sum_{t=1}^T \sum_{t'=1}^T u_{nt} u_{nt'}; \text{ and}$$

$$M_2(\psi) = \sum_{n=1}^N \left[\sum_{t=1}^T u_{nt}^2 - \frac{\sum_{t=1}^T u_{nt}^2}{T} \right]^2.$$

The calculations may be simplified by partially maximizing (16) with respect to certain of the parameters and then using a computer algorithm to maximize the partially-maximized, or concentrated, likelihood function with respect to the remaining parameters.^{19,20} The maximum-likelihood estimates for ξ and η can also be written as: [2, p. 612]:

$$\hat{\xi} = \frac{M_1(\psi)}{N}, \text{ and } \hat{\eta} = \frac{M_2(\psi)}{N(T-1)}$$

Substituting these definitions for ξ and η into (15) and simplifying, we obtain:

$$(18) \quad \hat{L}(\psi) = -\frac{NT}{2} (\log 2\pi + 1) - \frac{N}{2} \log \frac{M_1(\psi)}{N} \\ - \frac{N(T-1)}{2} \log \frac{M_2(\psi)}{N(T-1)}.$$

This expression is a function only of ψ . Once the values of the parameters which maximize $\hat{L}(\psi)$ have been found, the corresponding values for ρ and σ^2 can be calculated [2, p. 612]:

$$(19) \quad \rho = \frac{(T-1)M_1(\psi) - M_2(\psi)}{(T-1)[M_1(\psi) + M_2(\psi)]}, \text{ and}$$

$$(20) \quad \sigma^2 = \frac{M_1(\psi) + M_2(\psi)}{NT}$$

The u which was used in maximizing (18) was not the one defined in equation (15), but was

$$(21) \quad u = C' - \frac{\kappa}{2} \log [(\tau_1 ACMA + \tau_2 ACMB + \tau_3 ACMC + \tau_4 ACMD)^2 + \chi(STA)^2].$$

A maximum for the likelihood function was found within the theoretically admissible range for ρ ($\hat{\rho} = 0.619$). Instead of estimating τ_i and χ directly, we estimated their square roots in order to ensure that the estimates themselves would be positive. The resulting constrained estimates were then normalized to obtain estimates of the cost function parameters:

$$(22) \quad e_i = \frac{\tau_i}{\sum \tau_i},$$

$$(23) \quad \tau_1 = \frac{(\sum \tau_i)^2}{(\sum \tau_i)^2 + \chi},$$

$$(24) \quad \tau_2 = \frac{\chi}{(\sum \tau_i)^2 + \chi}, \quad \text{and}$$

$$(25) \quad \log k = \frac{\kappa}{2} \log [(\sum \tau_i)^2 + \chi]$$

Using the normalized parameter estimates, the equation for short-run variable costs (less fuel costs), $c^*_{ao(s)}$, can be written as:

$$(26) \quad c^*_{ao(s)} = 279.81 \left[0.102 (0.197ACMA + 0.242 ACMB + 0.202 ACMC + 0.359 ACMD)^2 + 0.898 (STA)^{2.125} \right]$$

PCP ^{-1.759} $\hat{\sigma}^2 = 0.0205$

No attempt was made to estimate the asymptotic variance-covariance matrix for the parameter estimates, because of the complexity of the calculations required.²¹ This does not mean that nothing can be said about the reliability of the estimates obtained. In fact, much may be learned by observing the paths of the parameter estimates, their derivatives, and the value of the likelihood function during the maximization procedure. The ratio of the "explained" variance of costs to the total variance was extremely high ($R^2 = 0.93$).

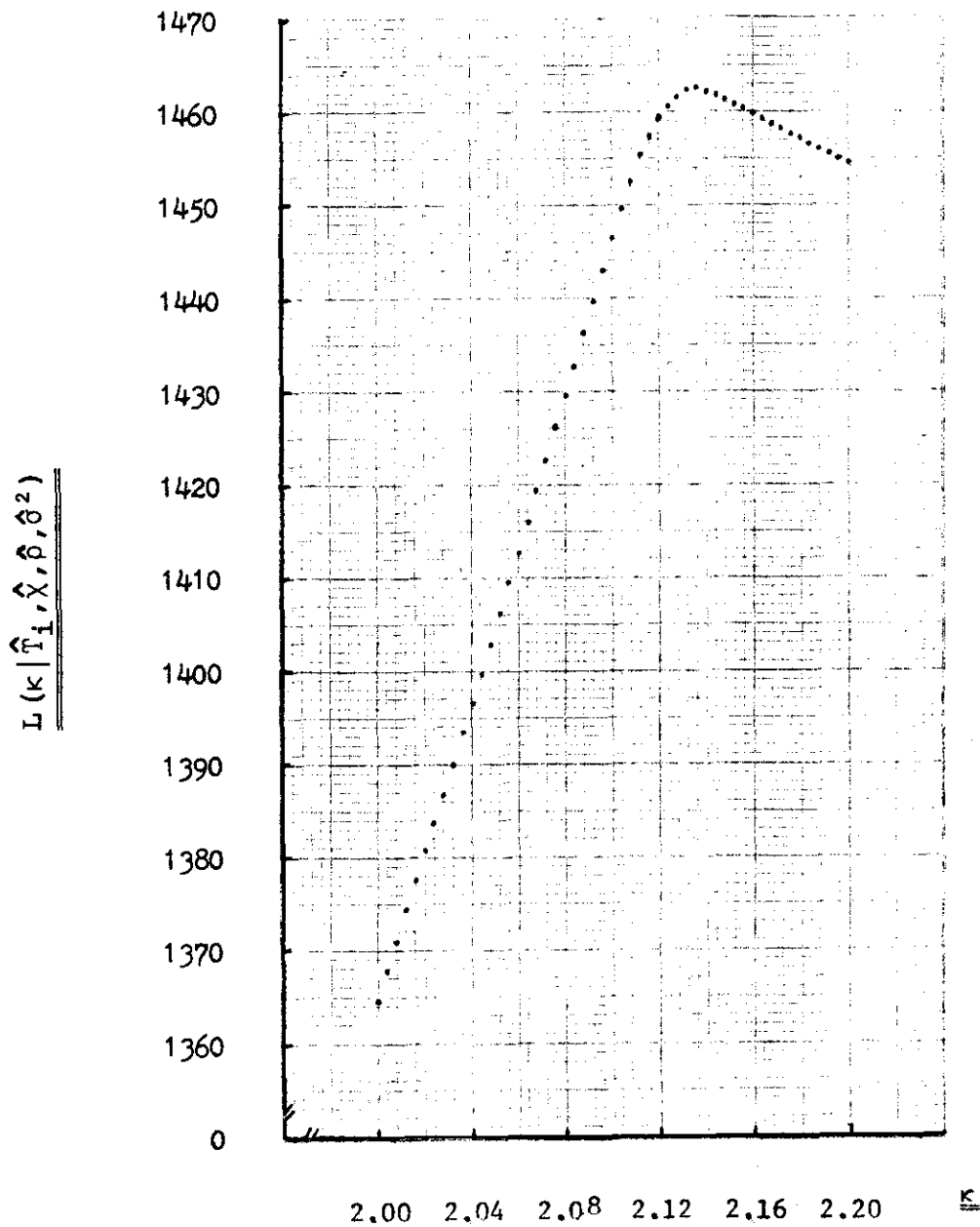
Since the algorithm obtains only local maxima, several starting points were used. In all trials the same aircraft-mile weights, the ε_i , were obtained upon normalization. This means that the relative sizes of the T_i 's were constant regardless of their absolute levels.

The value of $\frac{\partial L}{\partial \kappa}$ alternated between large negative and large positive values for very small changes in κ near the indicated maximum of the likelihood function. Regardless of how much the step size of the search procedure was reduced, $\frac{\partial L}{\partial \kappa}$ could not be made close to zero. This does not necessarily indicate that the estimate of κ was poor; indeed the behavior of the gradient may indicate that the likelihood function is sharply peaked at $\hat{\kappa}$ and that the estimate of κ is, in fact, a good one.

To test this hypothesis, the estimates of all the parameters except κ were inserted into the likelihood function, and the function was evaluated for changes in κ . This evaluation was performed for $2.000 \leq \kappa \leq 2.200$ with a step size of 0.002. The results, shown in Figure 1, confirm the hypothesis advanced. The likelihood function is sharply peaked, with the indicated maximum at $\kappa = 2.136$, only 0.012 away from the maximum indicated by the algorithm. The difference

FIGURE 1

A TEST OF THE RELIABILITY OF THE ESTIMATE OF κ



in the value of the likelihood function between the two points is small [$L(\kappa = 2.124) = 1460.8$ and $L(\kappa = 2.136 = 1462.3$]. Furthermore, the value of $\frac{\partial L}{\partial \kappa}$ changes rapidly around the maximum. It is 1053.0 at $\kappa = 2.124$, 117.6 at $\kappa = 2.134$, and -148.6 at $\kappa = 2.138$. Consequently, it is not likely that $\frac{\partial L}{\partial \kappa}$ could be reduced to zero unless the search procedure were extremely efficient.

Less confidence may be attached to the estimates of the τ_i 's, the relative weights for the aircraft mile synthetic variable and the non-synthetic variable. The smallest value that could be obtained for the first derivatives of the τ 's and χ , the parameters which were normalized to yield τ_1 and τ_2 , ranged from 0.9 to 4.8. Reducing the step size did not reduce the size of the derivatives, though the algorithm continued to indicate that a maximum had been found. The sign of the derivatives did not change. This matter has yet to be further explored.

The Long-Run Total Cost Function for the Local Service Airline

Industry. The result developed in Section 3 may now be used to derive the long-run total cost function from the short-run variable cost function. From equation (26), $\frac{1}{\alpha_1} = 2.125$ and $\alpha_1 = 0.471$. Note that this estimate of α_1 is remarkably close to the corresponding estimate from the derived demand equation

for pilots, thus giving increased confidence in the reliability of the estimate. From the derived demand equation for pilots and copilots (Table 4), we have already obtained $\frac{\alpha_2}{\alpha_1} = 1.759$, which implies that $\alpha_2 = 0.770$.²² From these results, we can obtain the long-run cost elasticity with respect to changes in the price of pilots, $\frac{\alpha_2}{r} = 0.638$, and the long-run cost elasticity with respect to changes in the price of labor net of pilots, $\frac{\alpha_1}{r} = 0.362$. The constant term for the long-run cost equation, k' , is obtained as

$$(27) \quad k' = r \left[\left(\frac{\alpha_1}{k} \right)^{\alpha_1} \alpha_2^{\alpha_2} \right]^{-1/r} = 14.79$$

Combining these results with the fuel equations shown in Table 3, we obtain the long-run total cost equation:

$$(28) \quad c_\ell = 14.79 [0.102 (0.197 ACMA + 0.242 ACMB + 0.202 ACMC + 0.359 ACMD)^2 + 0.898 (STA)^2]^{0.770} \\ PLNP^{0.362} \quad PPILOT^{0.638} \\ + PFP (0.679 ACMA + 1.241 ACMB) + PFT (1.517 ACMC + 3.369 ACMD).$$

It is this equation which may be used to answer questions such as those raised at the end of the first section of the paper.

APPENDIX

Construction of Variables

This Appendix describes the construction of the variables used in the equations whose estimation is described in the body of the paper. The source of the data is the CAB's Report of Financial and Operating Statistics for Certificated Air Carriers [31], otherwise known as "Form 41." The portions of Form 41 that were used in variable construction are filed quarterly by each carrier, with the exception of Schedule P-41, "Taxes," which is filed annually. Copies of the complete Form 41 are available only in the Public Records Room of the CAB in Washington, but certain data from Form 41 are published by the CAB in summary form, for example, in [27], [28], [29], and [30] and in a number of trade publications, including Aviation Week and Flight Magazine.

Dependent Variable

The dependent variable (COST)²³ is "Total Operating Expenses" (P-1.2, 7199)²⁴ less "Depreciation and Amortization Expenses" (P-1.2, 7000). "Depreciation and Amortization Expenses" includes as major categories depreciation expenses for flight equipment, for maintenance equipment and hangers,

and for general ground property (see Schedule P-3 for a breakdown by categories). The latter two items were considered to be legitimate items of fixed cost and were subtracted from "Total Operating Expenses" for that reason. There is general agreement, however, that the depreciation expenses for flight equipment reported by the carriers do not reflect the flow of services provided by aircraft in any meaningful sense. For this reason, they, too, were subtracted from "Total Operating Expenses."²⁵ Interest expenses were not subtracted from "Total Operating Expenses" since they are not included in this account, but are found in Accounts 8187.1-8187.3 (Schedule P-3), which appear on Schedule P-1.2 consolidated into the general account "Non-Operating Income and Expense, Net" (8100). Since the stock of pilots and copilots was used as a measure of the flow of services of the fixed factor, wages of pilots and copilots (P-10) were also subtracted as a fixed cost. Fuel expenses (P-5.2, 5145.1) (including an estimate of fuel taxes) were subtracted from "Total Operating Expenses" since separate fuel expense equations were estimated.

Independent Variables

Price of Fuel [Piston (PFP) and Turbine (PFT)]: These variables were constructed by dividing "Fuel Expenses"

(P-5.2, 5145.1) by "Gallons of Fuel Issued" (T-3, 9992) and adding an estimate of the fuel tax per gallon of fuel. According to this definition, a fuel price was undefined for a carrier unless that carrier actually used the particular type of fuel during the accounting period.

The estimate of the fuel tax per gallon was obtained by allocating "Fuel Taxes" (P-41, filed annually) between piston and turbine fuel and dividing the result by the number of gallons of each type of fuel issued during the year. This assumes that any change in the fuel tax occurred on the first day of the year, but the assumption was unavoidable.

There is a two cent per gallon Federal tax on piston fuel but no tax on turbine fuel. The Federal tax rates have not changed since 1951. The states differ in their policies as to whether turbine fuel is taxed and also in the rates of tax they apply to aviation fuels. Their rates have changed more frequently than the Federal tax rate.

Since tax rates differ between states, a carrier changing its pattern of operations becomes subject to a different effective tax rate. State fuel taxes were allocated on the basis of the areas served by each carrier using data from the table "State and Federal Aviation Gasoline and Jet Fuel Taxes (cents per gallon): 1919-1964" ([28], 1965 ed., pp.518-19). Certain somewhat arbitrary assumptions

were unavoidable in the course of this allocation of state fuel taxes, but we feel that no serious biases in the fuel price variable were introduced as a result.

Price of Labor Net of Pilots and Copilots (PLNP): Payroll data by class of employee are reported on Schedule P-10; but, although Schedule P-10 is filed quarterly, the wage bill figures on it are stated in annual rates, therefore they were divided by four. "Payroll taxes" (P-41) are reported only annually, and therefore must be prorated. This was done by weighting them by the quarterly wage bill. The variable PLNP was obtained by dividing the net quarterly wage bill including prorated payroll taxes (the quarterly wage bill for pilots and copilots was subtracted) by the net number of employees for the quarter (P-10) (i.e., total employees less pilots and copilots).

Price of Pilots and Copilots (PPILOT): The quarterly payroll for pilots and copilots (P-10) was divided by the number of pilots and copilots (P-10). It was not possible to include payroll taxes in the quarterly pilot and copilot payroll since tax payments by employee class are not recorded.

Fuel Usage [Piston (GFP) and Turbine (GFT)]: The data for these variables were obtained from Schedule T-3 (999.1)

which lists gallons of fuel issued by aircraft type. GFP is the total gallons of fuel used by piston-engine aircraft and GFT is the corresponding total for turbine aircraft. It was assumed that all fuel issued during a quarter was used during that quarter.

Aircraft Miles [ACM(i), where i Represents the Aircraft Group, and ACMT]: ACM(i) consists of "Total Aircraft Miles" (T-3, 9597), reported by aircraft type and aggregated into aircraft groups according to the scheme outlined in the text. It includes miles flown in both scheduled and unscheduled service. Total Aircraft Miles (ACMT) is the unweighted sum of ACMA, ACMB, APMC, and ACMO.

Aircraft Departures [ACD(i)]: This consists of total departures performed by aircraft type aggregated by aircraft group. It is the sum of "Total Departures Performed in Scheduled Service" (T-4) and "Total Departures Performed in Non-scheduled Revenue Flights" (T-3, 8684).

Stations Served (STA): This variable was obtained from a count of the stations served listed on Schedule T-4.

Stock of Pilots and Copilots (PCP): The data for this variable was obtained from Schedule P-10, "Payroll".

FOOTNOTES

1. The following official Civil Aeronautics Board carrier designations are the ones which we shall use:

Local Service Carriers

Trunk Carriers

Allegheny	Ozark	American	Northeast
Bonanza	Pacific	Braniff	Northwest
Central	Piedmont	Continental	TWA
Frontier	Southern	Delta	United
Lake Central	Trans Texas	Eastern	Western
Mohawk	West Coast	National	
North Central			

2. In 1953, 150 of the 152 aircraft in the local service airlines' fleets were DC-3s. [16]
3. Conversion kits are manufactured either by the Allison Division of General Motors or by Rolls Royce/Convair.
4. It is interesting to note that the largest aircraft operated by the trunk carriers in the late 1940's and the early 1950's when the local service carriers began operations were Constellations and DC-6's seating about 60 passengers.
5. In 1963 the Systems Analysis and Research Corporation (SARC) performed a study of the potential market for smaller transport aircraft for the Federal Aviation Agency [24] which concluded:

... in 1975 [projecting 1963 traffic growth rates] there should be more widespread use of smaller aircraft than is presently made of the DC-3. Current [1963] DC-3 operations of the local service carriers account for some 45.5% of total routings operated by the local service carriers, whereas in 1975, even with the greater traffic volumes which are forecast, 61% of the local industry routings will most economically be operated with aircraft of less than 40-seat capacity." [24, p. 29]

6. For a discussion of this controversy see Swaine [23, pp. 66-77].
7. [34]; this source also reported that "in eleven years of Use-It-Or-Lose-It conditions, 101 points have suffered a suspension of service."
8. A synthetic cost function approach similar to that employed by Caves [5, pp. 65-78], Straszheim [21, Chapter 4 and Appendix B] and the Systems Analysis and Research Corporation [24, pp. 31-56] was rejected for reasons outlined in Eads [6, pp. 31-36].
9. Borts [3, pp. 110-115] contends that the firm will not even be on its short-run cost function but on some hybrid function. Reasons are advanced in Eads [6, pp. 42-45] for believing that this is not the case for firms in the local service airline industry.
10. See, for example, Henderson and Quandt [11, pp. 58-60].
Two works in which the relationship between the short-run and the long-run cost functions has been used are Bressler's 1952 study of city milk distribution [4, pp. 228-53] and the study of the costs of electricity generation reported in Johnston [13, pp. 64-73]. In both of these studies, plant size was included as a variable in the cost function. The short-run cost function was obtained by holding plant size constant and varying output.

The long-run cost function was obtained by allowing plant size to vary and tracing out a number of short-run cost curves. The long-run cost function was then constructed graphically as the envelope of the short-run functions. We are aware of no instance prior to Eads [6] in which the short-run, long-run relationship has been utilized as it is here. For possible reasons, see Eads [6, pp.52-54].
11. If $2 < n < \infty$, the transformation locus becomes an m -dimensional supersphere on superhyperellipse. For a discussion of the properties of such surfaces see Gardner [10].

12. Since all carriers do not fly all aircraft types, zero observations will appear. This raises a serious problem in a log-linear output formulation, since the logarithm of zero is undefined. The problem has sometimes been treated by adding a constant to each observation of each output measure, thus assuring that no zeros appear. However, this introduces a bias, since a constant added to a small number results in a larger percentage increase than does the same constant added to a larger number.
- Zeros cause no problems of this sort when an output formulation such as (7) is adopted.
13. For details concerning the construction of the variables see the Appendix. The acronyms used for the variables to follow are defined either in the appendix or in the text as they appear.
14. For example, Mohawk Airlines reported near the end of July that it would have to suspend operations for the last three days of the month unless the pilot's union allowed it to waive portions of the union contract dealing with pilot scheduling. Other carriers saved pilot time by skipping low-density steps and concentrating on the larger markets. Aircraft capacity was seldom mentioned as a major constraint. See [15] and [22].
15. Our confidence in this choice was increased by the highly unsatisfactory results obtained when measures of aircraft services were used. See Eads [6, pp. 79-80 and Appendix D].
16. Each coefficient can be interpreted as the number of gallons of fuel required for flying one aircraft mile or for performing one takeoff and landing using aircraft of the appropriate group, everything else being held constant.
17. The correlation between the price ratio and total aircraft miles was not significantly different from zero.
18. That is,
- let σ_{μ}^2 = the variance of the time-invariant firm effect,
- σ_{ν}^2 = the variance of the remaining disturbance,

$$\sigma^2 = \sigma_{\mu}^2 + \sigma_v^2$$

$$\rho = \sigma_{\mu}^2 / \sigma^2$$

and

$$\Omega = \sigma^2 \begin{bmatrix} A & 0 & \dots & 0 \\ 0 & A & \dots & 0 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 0 & 0 & & A \end{bmatrix}, \quad (NT \times NT),$$

and

$$A = \begin{bmatrix} 1 & \rho & \rho & \dots & \rho \\ \rho & 1 & \rho & \dots & \rho \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ \rho & \rho & \rho & \dots & 1 \end{bmatrix}, \quad (T \times T).$$

19. The computer algorithm used was developed in Fletcher and Powell [8] and [9] and modified by William Raduchel.
20. The important principle of stepwise maximization is discussed in Hood and Koopmans, [12, pp. 156-58].
21. It is known that the maximum-likelihood estimates are, under general circumstances, distributed with asymptotic variance - covariance matrix given by the inverse of the matrix of second partial derivatives evaluated at the estimated parameter values. See, for example, Mood and Graybill.
22. To the extent that output is in fact endogenous the estimate obtained for r is biased. For example, if a positive correlation between output and the distance of the production function existed, then there would be a negative correlation between output and the disturbance in the cost function (since v , the disturbance in the cost function, equals $1/u$, where u is the disturbance in the production function. This means that the estimate of $1/r$ may be

biased downward, that is, there may be a bias in the direction of increasing returns to scale. If one calculates the degree of returns to scale from equation (28) below (which includes the fuel cost equations), the estimate is less than one indicating decreasing returns to scale. The potential bias noted above cannot affect this conclusion.

23. The capital letters in parentheses following a variable name are the acronym that is used to designate the variable in the text except where explicitly stated otherwise.
24. The first number in the parentheses is the schedule of Form 41 on which the data appear. The second number is the relevant account on that schedule. For the official CAB definition of what each account is to include see [32].
25. This decision was made only after extensive study of the CAB accounts and after conversations with a number of CAB staff members. We are unsure just how much the fact that our results may differ from those of others may be due to our decision to exclude aircraft depreciation expenses as either a fixed or a variable cost. Investigation of this point is currently under way.

BIBLIOGRAPHY

- [1] Baitsell, John M. Airline Industrial Relations: Pilots and Copilots. Boston: Division of Research, Graduate School of Business Administration, Harvard University, 1966.
- [2] Balestra, Pietro and Marc Nerlove, "Pooling Cross Section and Time Series Data in the Estimation of a Dynamic Model: The Demand for Natural Gas." Econometrica, 34 (July 1966), pp. 585-612.
- [3] Borts, George H. "The Estimation of Rail Cost Functions," Econometrica, 28, (January 1958), pp. 108-131.
- [4] Bressler, R.G., Jr. City Milk Distribution. Cambridge, Massachusetts: Harvard University Press, 1952.
- [5] Caves, Richard E. Air Transport and Its Regulators. Cambridge, Massachusetts: Harvard University Press, 1962.
- [6] Eads, George C. "A Cost Function for the Local Service Airlines." Unpublished dissertation, Yale University, 1968.
- [7] Farrar, Donald E. and Robert R. Glauber, "Multicollinearity in Regression Analysis: The Problem Revisited." The Review of Economics and Statistics, 44 (February 1967), pp.92-107.
- [8] Fletcher, R. "Certification of Algorithm 251[E4], Function Minimization." Collected Algorithms from CACM. (looseleaf, pp. 251-52).
- [9] Fletcher, R., and M.J.D. Powell, "A Rapidly Convergent Descent Method for Minimization." The Computer Journal, 6 (1963), pp. 163-68.
- [10] Gardner, Martin. "Mathematical Games." Scientific American, 213 (September 1963), pp. 222-232.

- [11] Henderson, James M., and Richard E. Quandt. Micro-economic Theory. New York: McGraw-Hill, 1958.
- [12] Hood, W.C., and T. C. Koopmans, eds., Studies in Econometric Method. Cowles Commission Monograph No. 14. New York: John Wiley & Sons, Inc., 1953.
- [13] Johnston, J. Statistical Cost Analysis. New York: McGraw-Hill, 1960.
- [14] Klein, Lawrence R. A Textbook of Econometrics. Evanston, Illinois: Row, Peterson and Company, 1953.
- [15] "Local Revenues Rise as Strike Continues." Aviation Week, 85 (August 15, 1966), pp. 40-43.
- [16] "Local Service Airlines: Number of Scheduled Aircraft, 4th Quarter, 1953." Flight Magazine, 41 (June 1954), p. 53.
- [17] Meyer, John R., and Gerald Kraft. "The Evaluation of Statistical Costing Techniques as Applied in the Transportation Industry." The American Economic Review, 51, (May 1961), pp. 313-34.
- [18] Mood, Alexander M., and Franklin A. Graybill, Introduction to the Theory of Statistics. 2nd ed. New York: McGraw-Hill, 1963.
- [19] Nerlove, Marc. "Return to Scale in Electricity Supply." In C. Christ, et. al., Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yehuda Gronfeld. Stanford: Stanford University Press, 1963.
- [20] "Specifications." Aviation Week, 88 (March 18, 1968), pp. 179-224.
- [21] Straszheim, Mahlon R. International Air Transport and Economic Development. Washington: The Brookings Institution (preliminary version).
- [22] "Strike Clogs Airline Reservations Systems." Aviation Week, 85 (July 25, 1966), pp. 41-43.

- [23] Swaine, Howard R. "Subsidization of Local Air Services." Unpublished dissertation, University of California at Los Angeles, 1965.
- [24] Systems Analysis and Research Corporation. Economic Analysis of the Short-Haul Transport. Boston: Systems Analysis and Research Corporation, 1964. (mimeographed)
- [25] Taylor, John W.R., ed. Jones' All the World's Aircraft. 1961 - 2nd ed. New York: McGraw Hill, 1961.
- [26] Taylor, John W.R., ed. Jones' All the World's Aircraft. 1965-66 ed. New York: McGraw-Hill, 1965.
- [27] U.S. Civil Aeronautics Board, Costs and Statistics Division, Bureau of Accounts and Statistics. Aircraft Operating Cost and Performance Report for Calendar Years 1965 and 1966. Washington, 1967.
- [28] _____, _____, _____. Handbook of Airline Statistics. (Various editions.) Washington.
- [29] _____, _____, _____. Monthly Report of Air Carrier Traffic Statistics. (Various issues) Washington.
- [30] _____, _____, _____. Quarterly Report of Air Carrier Financial Statistics. (Various issues). Washington.
- [31] _____, Report of Financial and Operating Statistics for Certificated Air Carriers, Form 41. (First quarter of 1968 through fourth quarter of 1966). Washington.
- [32] _____, Uniform System of Accounts and Reports for Certificated Air Carriers in Accordance with Section 407 of the Federal Aviation Act. Washington, 1961. (looseleaf)
- [33] "U.S. Commercial Transports." Aviation Week, 84 (March 7, 1966), p. 199.
- [34] "'Use-It-Or-Lose-It' Cities Drop to 41." Flight Magazine, 56, (June 1967), p. 67.