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**LAGS IN FISCAL AND MONETARY IMPACTS ON INVESTMENT  
IN PRODUCERS' DURABLE EQUIPMENT**

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LAGS IN FISCAL AND MONETARY IMPACTS ON INVESTMENT  
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by

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I. Introduction

Both fiscal and monetary policy-makers share the desire to influence the flow of business fixed investment. The overall purposes to which such influence would contribute may be short-run (for example, to counteract a business cycle) or long-run (to affect the rate of growth of potential output). In the first case, it is desirable to apply a policy tool whose effects are highly concentrated in time, for if there is a long lag the cyclical situation may have drastically changed by the time the policy takes hold. In the second case, it is desirable to affect the flow of investment more gradually, or else the policy itself may create short-run instability. In either case, as Griliches and Wallace have emphasized, "whether or not a particular stabilization or growth policy will actually do more harm than good depends crucially on the form of the lag function."<sup>1</sup>

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<sup>1</sup>[16], p. 328.

Through tax changes in 1954, 1962, and 1964 the federal government has sought to encourage investment spending, presumably with long-run goals in mind. On the other hand, monetary changes since the Treasury-Federal Reserve Accord have presumably aimed at rapid and reversible influence. Without doubt, the off-setting changes in the tax treatment of investment passed in 1966 and 1967 were explicitly designed to have short-run effects on investment.

This paper seeks to evaluate the direct<sup>1</sup> effects of fiscal and monetary instruments, as well as other determinants, on expenditures for producers' durable equipment, the largest component of business fixed investment. In particular, I am concerned not only with how much a particular policy affects investment, but also when the effect occurs. Jorgenson, in a series of papers with several colleagues, has drawn the striking conclusion that "any measures which result in a once-over change in demand for capital will result in a relatively short and sharp boom in investment demand followed by a lengthy period of steadily worsening stagnation induced by a decline in total investment expenditures relative to their peak levels."<sup>2</sup> This conclusion, if true, has important policy implications. However, a controversial feature of the model from which it is derived is the assumption that all of the determinants

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<sup>1</sup>By the direct effect of a change in a policy parameter I mean the effect of the parameter change with all other determinants of investment unchanged. A tax credit stimulates investment indirectly through feedbacks on the other determinants of investment, but the magnitude of these feedbacks can be discussed only in the context of a complete model.

<sup>2</sup>[26], pp. 85-86. See also Jorgenson [25], Jorgenson and Stephenson [27][28] and Hall and Jorgenson [18][19][20].

of investment act with the same distributed lag. I have attempted to relax this restrictive assumption.

The model discussed in this paper is heavily influenced by Jorgenson's neo-classical investment model, in the extended version developed by Hall and Jorgenson to study the effects of tax policy on investment. All fiscal and monetary parameters in their model affect investment via changes in the imputed rent on the price of the services of capital goods. By assumption, the elasticity of investment demand with respect to this rent is unity, and thus the elasticity of investment demand with respect to each of the determinants of the rent is an assumed, rather than estimated, value. In removing this second restrictive assumption, I provide estimates of the direct effects of various policy changes which are less dependent on the particular way the model is put together.

In the next section, a rationale is developed to justify the estimation of separate lag distributions for different determinants of investment. The rationale is stated in terms of a neo-classical model in which factor proportions are variable only up to the point when new machines are installed.<sup>1</sup> With the addition of assumptions about the way in which machines wear out, and the way in which expectations about future prices are formed, this model implies that the short-run elasticity

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<sup>1</sup>See Johansen [24], Solow [38], Massell [33], Phelps [35], and Kemp and Thanh [29] for theoretical developments of this model, mostly in the context of long-run growth models.

of investment demand with respect to changes in the rent will never exceed the long-run elasticity. It should be emphasized however that this rationale, while perhaps plausible, is not the only way in which a divergence in lag distributions could be justified.

The model is used to explain quarterly data for aggregate expenditures on producers' durable equipment. The long run price elasticity of demand for equipment is estimated to be very close to unity, with the short run elasticity being considerably smaller. At the same time, the short run elasticity of equipment demand with respect to output substantially exceeds the long run elasticity. Tax parameters as well as the interest rate and the yield on equities all appear to be important determinants of the level of expenditures, although the quantitative impact of the Investment Tax Credit, adopted in 1962, appears to be greater than that of any of the other policy measures studied.

## II. Specification of a Distributed Lag Model

### A. Introduction to the Model

The importance to policy-makers of knowledge about the speed with which monetary and fiscal policies can affect investment was emphasized in the introduction to this study. In order to acquire this knowledge, it is necessary to estimate a model of investment demand in which explicit attention is given to monetary and fiscal parameters. In this section, I discuss the specification of such a model, in which a number of terms reflecting relative prices, capital costs, and fiscal institu-

tions are combined into a single expression for the imputed rent on a piece of equipment. In the long run, it is assumed that the demand for equipment is a log linear function of this rent, which is identified with the price of the services provided by a unit of equipment. In addition, the demand for equipment is assumed to be proportional to the sum of (a) desired changes in capacity and (b) replacement of capacity.

A crucial feature of this model is the assumption that changes in the determinants of the rent may well affect investment expenditures with a different lag distribution than exists for changes in the determinants of desired capacity. I develop the rationale for this assumption in terms of a neo-classical model in which factor proportions are variable only up to the point when new machines are installed. Given the current state of knowledge (or ignorance) about real world possibilities for changes in factor proportions, it cannot be assumed a priori that all of the determinants of investment affect expenditures with the same time pattern.

The stylized world on which my model of investment behavior is based is one in which a firm, at time  $t$ , must make investment decisions  $\eta$  periods ahead. For ease of exposition, I will call the periods "days," and assume that any equipment which is ordered on day  $t$  is delivered on day  $t+\eta$  and can be immediately put into production.

From the vantage point of day  $t$ , the firm knows (with certainty) that even if it orders no new equipment, on day  $t+\eta$  it will have a certain amount of equipment on hand. This equipment includes the machinery available on day  $t$ , plus the equipment previously ordered which will be delivered between  $t$  and  $t+\eta$ , less the equipment which will be retired during this same period. The factor proportions on all

of this equipment are assumed to be fixed, although in general they will differ from machine to machine. Each machine, if operated for a "day" by the appropriate amounts of the cooperating factors, can produce a certain amount of output,<sup>1</sup> and this amount is defined as the "capacity of the machine."<sup>2</sup> The aggregate capacity of all the machines which will be in existence the morning of  $t+\eta$ , with no new investment, is  $Z_0$ .

The firm must make decisions about (a) how much new capacity to order for delivery  $\eta$  days later, and (b) what "blueprint" should be used to specify the factor proportions on the new capacity. I assume that the two decisions are separable, and consider first the question of decision (b), given the amount of new capacity to be ordered. I assume that the firm's objective in deciding about factor proportions is minimization of cost.

#### B. Choice of Factor Proportions

In a situation in which relative prices will change over the lifetime of the capacity, it will be impossible, in general, to choose proportions which use the optimal amount of all factors at all times. In order to simplify the problem of choice, I assume that starting from

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<sup>1</sup>For the present I assume there is only a single homogeneous output.

<sup>2</sup>In fact, unless fixed capital is used on a 24-hour basis, there is still no unique measure of capacity. However, it does not seem terribly unreasonable to assume that institutional patterns set a "normal" degree of utilization which can be altered only by incurring high costs.

an initial rent  $c(0)$ , earned by a new machine, the flow of rent is expected to decline exponentially, so that  $c(t) = c(0)e^{-\delta t}$ . This decline may result from physical deterioration of the capacity so that the flow of output it can produce becomes less over time, from rises in the wages of rents on non-fixed factors, from changes in the price of the output produced, or from other causes. This assumption is a very crude approximation, and is by no means innocuous.<sup>1</sup> If it is granted, however, the present value of the stream of rents earned (after deduction of income taxes) by a unit of new capacity over its lifetime may be written:

$$\int_0^{\infty} e^{-rt}(1-u)c_i(0)e^{-\delta t}dt + kq_i + q_i(1-k')u \int_0^{\tau} e^{-rs}D(s)ds$$

where  $q_i$  is the price of the  $i^{\text{th}}$  capital good (when new),

$k$  is the rate of tax credit on investment in the  $i^{\text{th}}$  good,

$k'$  is the rate of tax credit which must be deducted from the depreciation base,

$r$  is the appropriate discount rate (including adjustment for risk),

$u$  is the rate of direct taxation of business income,

$\tau$  is the lifetime of the  $i^{\text{th}}$  good prescribed for tax purposes,

$D(s)$  is the proportion of the depreciation base for an asset of age  $s$  which may be deducted from taxable income,

$\delta$  is the exponential rate of decline in the value of services provided by a unit of the  $i^{\text{th}}$  good.

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<sup>1</sup>This assumption, in particular, implies that the quasi-rent will approach zero only asymptotically. However, if the reason for the expected decline in quasi-rent is, for example, an expected rise in all wages at a constant rate  $w$ , then  $c(t) = c(0)(1 - \beta e^{wt})$  (where  $\beta$  is the initial wage share), and this function declines at an increasing rate and reaches zero in finite time.



The first term in this expression is the discounted stream of quasi-rents, after deduction of direct taxes. The second term allows for tax credits of a certain proportion of the cost of the machine. The third term is the discounted value of the stream of taxes saved as a result of a deduction of depreciation expenses from taxable income.

Equating this present value to  $q_i$ , the price of a new machine, it is possible to solve for  $c_i(0)^*$ , the quasi-rent which must be earned by a new machine to make its purchase worthwhile. Using  $z$  to denote

$(1 - k') \int_0^{\tau} e^{-rs} D(s) ds$ , the present value of the depreciation deduction,

the equality is:

$$q_i(1 - k - uz) = \int_0^{\infty} e^{-(\delta+r)t} c_i(0)^*(1 - u) dt,$$

and integrating the right hand side leads to<sup>1</sup>

$$(1) \quad c_i(0)^* = \frac{q_i(r + \delta)(1 - k - uz)}{1 - u}$$

The book of "blueprints" tells how much of the malleable output must be molded into each of the models of machinery which are available. In order to proceed further, I assume that at any particular point of time, the available blueprints correspond to a neo-classical production function

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<sup>1</sup>This derivation closely follows that of Hall and Jorgenson [18] and the formula arrived at differs from theirs only because I have included  $k'$  in  $z$ .

of the special constant elasticity of substitution class<sup>1</sup> (in the first degree homogeneous version as originally developed by Arrow, Chenery, Minhas, and Solow [2]). Assuming that there are  $m$  factors, this function, with suitable normalization,<sup>2</sup> may be written

$$(2) \quad Q = \left[ \sum_{i=1}^m \alpha_i X_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where  $Q$  is output in physical terms,

$X_i$  is the input of the  $i^{\text{th}}$  factor,<sup>3</sup>

$\sigma$  is the elasticity of substitution, and

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<sup>1</sup>This  $m$ -factor CES function, though itself a special case, is still relatively general compared to the two most popular alternatives, the Cobb-Douglas and Leontief production functions, both of which are special cases of the class of CES functions.

<sup>2</sup>As originally written by ACMS, the right hand side was multiplied by an "efficiency" parameter,  $\gamma$ , but  $\sum_{i=1}^m \alpha_i$  was normalized to equal one. With no loss of generality the "efficiency" parameter may be set equal to one. If the alternative normalization is adopted, the sum of all the "distribution" parameters (the  $\alpha$ 's) is then equal to  $\gamma^{\frac{\sigma-1}{\sigma}}$ .

<sup>3</sup>This input may be thought of as a flow of services per unit of time, or, without loss of generality, as a stock of fixed factor which provides a flow of services proportional to the stock, with the proportionality factor normalized to 1 by appropriate choice of units. Changes in units will of course change the distribution parameter for the factor in question, but as long as  $\sigma$  does not change over time this will not cause any difficulties. The input of the factor "machinery," in what follows, is the number of units of the homogeneous output which are frozen into the machine of the particular blueprint chosen.

$\alpha_i$  is the distribution parameter for the  $i^{\text{th}}$  factor (as noted below, these parameters may well change over time).

Suppose, for the moment, that all factor prices will not vary over the lifetime of the capacity to be installed. In this case, the assumed decline in rent could come about if, for example, a certain portion of the capacity "dissolved" (was struck by lightning) each period, thus reducing both the output and the costs by exactly the same proportion. For this special case, a single set of factor proportions will minimize costs at all times. Given knowledge of the factor prices (the  $c$ 's) conditions for cost minimization subject to the production constraint (2) may be derived by forming the Lagrangian expression:

$$C = \sum_{i=1}^m c_i X_i + \lambda \left[ Q - \sum_{i=1}^m \alpha_i X_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

and, as necessary conditions, setting the  $m$  first derivatives with respect to the  $X_i$  equal to zero. This leads to the equations

$$(3) \quad c_i - \lambda \alpha_i \left( \frac{Q}{X_i} \right)^{\frac{1}{\sigma}} = 0, \quad i = 1, \dots, m.$$

Now, from (3) it would be possible to derive  $m-1$  equations expressing the cost-minimizing ratios of each of the other  $m-1$  factors to  $X_i$ , e.g. (reordering so that the  $i^{\text{th}}$  factor is the first)

$$\frac{X_j}{X_1} = \frac{\alpha_j^\sigma c_1^\sigma}{\alpha_1^\sigma c_j^\sigma}, \quad j = 2, \dots, m.$$

Substituting these ratios into the production function leads to the expression

$$(4) \quad \frac{X_1}{Q} = \left[ \alpha_1 + \sum_{j=2}^m \frac{\alpha_j^\sigma c_1^{\sigma-1}}{\alpha_1^{\sigma-1} c_j^{\sigma-1}} \right]^{\frac{\sigma}{\sigma-1}}$$

for the amount of the first factor to be embodied in (or needed to cooperate with) each unit of new capacity.

Not only is (4) very awkward, especially if the number of factors is greater than two, but it is also necessary to make assumptions about the time paths of each of the  $\alpha_j$  in order to use this expression to derive the demand for a factor. Unless it is assumed that each of the distribution parameters is constant over time, or changes according to some exponential time pattern, the use of (4) leads to hopeless difficulties. In addition, there are difficulties with regard to the choice of units in which to measure each of the other factors. In computing a wage rate, should labor be measured in man-hours, or education adjusted man-hours, or what? Although it will indeed prove necessary to make arbitrary smoothness assumptions about technical changes in the distribution parameter for equipment, it seems desirable to make as few of these assumptions as possible.

Fortunately, it is possible to decrease the required number of such assumptions by using the economic interpretation of the Lagrange multiplier,  $\lambda$ , and adding one relatively plausible behavioral assumption. Note first that  $\lambda$  represents the minimum average total cost of output produced with the newest technology. If the firm is investing in new equipment, this average cost must represent its marginal cost as well.<sup>1</sup> The interpretation of  $\lambda$  as a minimum average cost may be verified by rewriting equations (3) in the form

$$c_i = \lambda \alpha_i X_i^{-\frac{1}{\sigma}} \left[ \sum_{i=1}^m \alpha_i X_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}, \quad i = 1, \dots, m,$$

multiplying both sides of each equation by  $X_i$ , and summing all  $m$  equations, to get

$$\begin{aligned} \sum_{i=1}^m c_i X_i &= \lambda \left[ \sum_{i=1}^m \alpha_i X_i^{\frac{\sigma-1}{\sigma}} \right] \left[ \sum_{i=1}^m \alpha_i X_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} \\ &= \lambda \left[ \sum_{i=1}^m \alpha_i X_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ &= \lambda Q. \end{aligned}$$

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<sup>1</sup>When the firm is investing in new machinery, the average variable costs on the oldest machinery in use cannot exceed the appropriately calculated average total cost using new machinery. The "trick" comes in calculating average total cost, which must include allowance for expected decreases in value of new machinery. Without knowledge about expectations of future prices it is not possible to make such calculations. The simplified rules of thumb I have assumed make such calculation easy, but they may not be at all appropriate.

Thus

$$\lambda = \frac{\sum_{i=1}^m c_i X_i}{Q},$$

the average cost with the latest technology, assuming the static cost-minimizing factor proportions are used.

Now with the addition of the behavioral assumption that price is set as a markup on marginal (or minimum average) cost, so that

$$(5) \quad p = M\lambda, \quad M \geq 1,$$

and that this markup is constant over time, then it is possible to use observed output price as a proxy for minimum average cost. Substituting (5) into the first equation of (3), only this one equation is required to write the demand for the first factor per unit of new capacity as a function of its rental, its distribution parameter, the price of output, and  $M$ , i.e.

$$(6) \quad \frac{X_1}{Q} = \frac{\alpha_1 p}{c_1 M}^\sigma = \alpha_1^\sigma v_1^\sigma,$$

where  $v_1$  is substituted for the ratio of  $p/c_1$  (the inverse of the product rent for equipment) to  $M$  (the constant markup factor).

The derivation of (6) has proceeded under the assumption of a static world, with no changes in prices or technology. In a time series analysis, it is unrealistic to assume that the underlying production function will not change over time. As long as technical change is factor-

augmenting only with respect to factors other than machinery, the marginal product of machinery will not, however, be changed.

Factor augmentation, in the sense in which I am using it, may be defined as follows. A change in one of the distribution parameters  $\alpha_j$  cannot be distinguished from a change in the units in which the factor  $X_j$  is measured. Thus, the production function, (2), governing additions to capacity at time  $t$ , may be written with unchanged distribution parameters, but with scale factors which multiply the quantities of each factor when these quantities are measured in time-invariant units. These scale factors are functions of time, and thus the production function may be written

$$(2a) \quad Q_t = \left[ \alpha_1 (a_{1t} X_{1t})^{\frac{\sigma-1}{\sigma}} + \sum_{i=2}^m \alpha_i (a_{it} X_{it})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where the first factor is equipment. In (2a) technical change which increases  $a_{it}$  is factor-augmenting with respect to the  $i^{\text{th}}$  factor; a single unit of that factor goes further. Each of the derivatives of (2a) with respect to any factor involves only the scaling of that particular factor, so that

$$\frac{\partial Q_t}{\partial X_{it}} = \alpha_i a_{it} \left( \frac{Q_t}{a_{it} X_{it}} \right)^{\frac{1}{\sigma}}$$

Thus, for equipment, a change in  $a_{it}$  ( $i = 2, n$ ) is Harrod-neutral.

If, however,  $a_{1t}$  changes (which includes the case of Hicks-neutral technical change, in which all the  $a_{it}$  rise by the same proportion), the marginal product of a unit of equipment will change over time. Without loss of generality, all equipment-augmenting technical progress may be treated as embodied in new equipment, as Hall has shown that "given rates of embodied and disembodied change and a given deterioration function cannot be distinguished from a lower rate of embodied technical change, a higher rate of disembodied change, and a higher rate of deterioration."<sup>1</sup> This means that the rate of disembodied equipment-augmenting change can be arbitrarily normalized to zero; if ways are found to make old equipment more productive it will simply show up as slower deterioration of the capacity embodied in that equipment.

As soon as the possibility of technical change is admitted, the assumption that relative prices and factor costs do not change is untenable. In addition, if the model is to be applied empirically, there must be some recognition that prices are, in fact, constantly changing. The factor proportions which would minimize costs if the production function of time  $t$ , and the costs of time  $t$ , were to persist forever, namely

$$(6a) \quad \frac{X_1}{Q} = \frac{1}{a_{1t}} \left( \frac{a_{1t} \alpha_1 p_1}{c_{1t} M} \right)^\sigma = a_{1t}^{\sigma-1} \alpha_1^\sigma v_{1t}^\sigma = V_t ,$$

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<sup>1</sup>[7], p. 98.



will generally be the wrong proportions in a world of changing prices.

For the purposes of estimation, I am going to assume that entrepreneurs choose their factor proportions by a simple sub-optimal rule of thumb. This rule of thumb involves using factor proportions  $V_t^*$  which are calculated as a distributed lag function of past values of  $V$ , the static optimum amount of equipment per unit of capacity. That is:

$$(7) \quad V_t^* = \sum_{k=0}^{\infty} \gamma_k V_{t-k} .$$

There is no particular reason why the weights  $\gamma$  should add up to one.

This rule of thumb is undoubtedly a vast oversimplification. It would be desirable to explicitly consider the ways in which expectations about interest rates, equipment prices, and prices of other factors are formed, and the choice of optimum proportions in the face of such expectations. However, such an explicit model would require many more specific assumptions than I have been willing to make.

### C. Gross Additions to Capacity

The decision as to how much capacity to order at time  $t$  for delivery  $\eta$  days later depends on the capacity needed for replacement, and on desired net additions to capacity. Temporarily ignoring replacement, desired net additions to capacity should equal the difference between the capacity the firm "desired" to have in period  $t+\eta-1$  (and placed orders to achieve) and the capacity desired in period  $t+\eta$ . I assume

"desired" capacity ( $Z^*$ ) is a roughly constant<sup>1</sup> multiple ( $z^* \geq 1$ ) of output planned for period  $t+\eta(Q_{t+\eta}^*)$ , that is

$$Z_{t+\eta}^* = z^* Q_{t+\eta}^* .$$

Furthermore, I assume that  $Q^*$  may be approximated by a function of past outputs

$$(8) \quad Q_{t+\eta}^* = \sum_{i=0}^{\infty} \xi_i Q_{t-i} .$$

Then net additions to desired capacity are

$$(9) \quad Z_{t+\eta}^* - Z_{t+\eta-1}^* = z^* \left( \sum_{i=0}^{\infty} \xi_i (Q_{t-i} - Q_{t-i-1}) \right) .$$

For different firms, and for different types of equipment within a firm, the lead time  $\eta$  will vary. In deriving an aggregate model, then, it is desirable to specify that actual net additions to capacity are a distributed lag function of past desired net additions, with the weights adding up to one and each particular weight ( $\psi_j$ ) indicating the proportion

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<sup>1</sup>With fixed proportions prevailing ex post, there will be an optimal degree of excess capacity which depends on the certainty with which demand expectations are held, the disutility of not having enough capacity, and the cost of holding excess capacity. Thus, in general,  $z^*$ , the desired capacity-output ratio, will be a function of relative prices among other things, but for simplicity I assume it is approximately constant. See Manne [31] and Smith [37] for approaches to the problem of the optimal degree of excess capacity for relatively special cases.

of the capacity ordered in period  $t$  which is installed in period  $t+j$ .<sup>1</sup>  
 Thus, an aggregate equation for actual net additions to capacity would be:

$$Z_t - Z_{t-1} = z^* \left\{ \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} \psi_j \xi_i (Q_{t-i-j} - Q_{t-i-j-1}) \right\} ,$$

This equation can be written in general form as

$$Z_t - Z_{t-1} = z^* \left\{ \sum_{k=0}^{\infty} \phi_k (Q_{t-k} - Q_{t-k-1}) \right\} ,$$

But this is just a first order difference equation with a very simple solution,

$$(10) \quad Z_t = z^* \sum_{k=0}^{\infty} \phi_k Q_{t-k} + C ,$$

where  $C$  is an arbitrary constant which must be zero if actual capacity is equal to desired capacity in a steady state.

If net additions to capacity actually follow such a pattern (either without error, or with very small, non-serially correlated errors) then current capacity can be adequately approximated as a function of past output levels. This approximation is not likely to be very good in the case of an individual firm or industry, or even in the aggregate in the face of a substantial downswing, for the lag between desired and actual

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<sup>1</sup>In other words,  $\psi_j$  represents the proportion of the equipment aggregate for which the waiting period  $\eta$  is equal to  $j$ .

net decreases in capacity cannot be expected to resemble the lag for net increases.<sup>1</sup> But in the absence of any direct measures of capacity or capacity utilization for the economy as a whole, an approximation of this sort is perhaps the best available measure.

The synthetic stock of capacity implicitly derived is the analogue in this model to the stock of capital which ordinarily appears in investment functions. The crucial difference in this model is that the amount of investment necessary to replace a unit of capacity which wears out (or becomes obsolete) will depend on recent relative prices rather than the amount of investment which originally took place.

Turning to replacement demand, by far the simplest assumption is that the flow of services from a unit of capacity declines exponentially from the time the capacity is installed. With the proper adjustment for changes in the rate of discount, such a deterioration pattern would imply exponential decline in the prices of used machinery. Since existing evidence suggests that exponential decline may be an adequate approximation to the true pattern,<sup>2</sup> this assumption can be rationalized, and leads to

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<sup>1</sup>The problem is the well-known one of the asymmetry in the operation of the acceleration principle. Aggregation, of course, does not solve the problem, except to the extent that some firms can sell their excess capacity to others. Nevertheless, the declines in aggregate net new orders for machinery in the post-Korean period, even in the worst recessions, have been moderate enough to encourage the feeling that the bias from this source will not be serious.

<sup>2</sup>See, for example, Terborgh [39], Chapters 4 and 5, Griliches [15], pp. 121-123, and Chow [5].

the conclusion that the proportion of existing capacity to be replaced in a given time period is a constant  $(\delta)$ .<sup>1</sup> Then gross additions to capacity, or gross investment in capacity units ( $I^Z$ ) may be written as

$$I_t^Z = z^* \sum_{k=0}^{\infty} \phi_k (Q_{t-k} - Q_{t-k-1}) + \delta z^* \sum_{k=0}^{\infty} \phi_k Q_{t-k-1},$$

using (10) to express the net stock of capacity at the end of period  $t-1$  (the beginning of period  $t$ ).

#### D. Equipment Expenditures

To derive an investment function, gross additions to capacity must be scaled by a factor which represents the incremental ratio of equipment to capacity considered optimal at the time the plans are made final. The rule of thumb for choosing this marginal ratio is given in (7).  $V_t^*$  as given in (7) must be multiplied by net additions to desired capacity in period  $t+\eta$ , as given by (8), and also by the amount of

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<sup>1</sup>A simplification of this sort may not be tenable, however, in a model with ex post fixed proportions, to the extent that rising variable costs eliminate the quasi-rents on old equipment even before it is physically worn out. For, in such circumstances, the proportion of capacity retired in any given period would be a function of, among other things, relative prices. If there is some possibility of choosing the durability of machinery, and more durable machines cost more, one would expect, however, that market forces would lead to the manufacture of capital goods in which the physical life is normally less than the economic life. See Bardhan and Britto [3] for the development of a model in which capital goods are always scrapped before they are economically obsolete.

replacement to take place in period  $t+\eta$ , which will be  $\delta z^*$  times the output planned for  $t+\eta$ . Thus, "desired gross additions to capacity" ( $I_{t+\eta}^{*Z}$ ) are given by

$$\begin{aligned} I_{t+\eta}^{*Z} &= z^* \left( \sum_{i=0}^{\infty} \xi_i (Q_{t-i} - Q_{t-i-1}) \right) + \delta z^* \left( \sum_{i=0}^{\infty} \xi_i Q_{t-i-1} \right) \\ &= z^* \sum_{i=0}^{\infty} \xi_i (Q_{t-i} - (1 - \delta) Q_{t-i-1}), \end{aligned}$$

and "planned expenditure" ( $E_{t+\eta}^*$ ) is

$$E_{t+\eta}^* = z^* \left\{ \sum_{k=0}^{\infty} \gamma_k V_{t-k} \right\} \left\{ \sum_{i=0}^{\infty} \xi_i (Q_{t-i} - (1 - \delta) Q_{t-i-1}) \right\}.$$

Adding a lag between plans and expenditures, as before, aggregate equipment expenditures in period  $t$  are expressed in terms of past output levels and relative price ratios as

$$E_t = z^* \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{i=0}^{\infty} \psi_j \gamma_k \xi_i (V_{t-k-j}) (Q_{t-i-j} - (1 - \delta) Q_{t-i-j-1}).$$

This expression is a special case of the general form

$$(11) \quad E_t = \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} \beta_{ij} V_{t-i} Q_{t-j},$$

an equation which involves a large (in principle infinite) matrix of coefficients ( $\beta_{ij}$ ) of relative price and output terms. Unrestricted estimation of the matrix, however, is not possible or desirable.

The common sense of (11) is that equipment spending is a complex weighted sum of the effects of lagged relative prices and output levels, interacting multiplicatively. But if quarterly changes in  $V$  and  $Q$  are small relative to their levels, (11) is approximately proportional to

$$(12) \quad E_t \approx \sum_{i=0}^n \lambda_i V_{t-i} \tilde{Q}_t + \sum_{j=0}^n \mu_j Q_{t-j} \tilde{V}_t$$

where  $\tilde{Q}_t$  and  $\tilde{V}_t$  are approximations to the levels of  $Q$  and  $V$  over the period  $t-n, t$ . On this interpretation

$$\lambda_i \approx \sum_j \beta_{ij} = \sum_{k=0}^i z^* \delta \psi_k \gamma_{i-k}$$

and

$$\mu_j \approx \sum_i \beta_{ij} = \sum_{k=0}^j z^* \psi_k (\xi_{j-k} - (1 - \delta) \xi_{j-k-1}) .$$

That is, the  $\lambda$ 's represent row sums of the  $\beta_{ij}$  coefficient matrix and the  $\mu$ 's represent column sums. Since all of the  $\psi$ 's and  $\gamma$ 's are assumed positive the  $\lambda$ 's should all be positive.<sup>1</sup> With respect to the  $\mu$ 's, only the first non-zero  $\mu_j$  weight can be specified a priori

---

<sup>1</sup>This statement requires amplification. The  $\gamma$  weights, which partly represent expectations about future relative prices, might conceivably contain negative terms; the assumption that they should all be positive implies that  $V_t^*$  is a positively weighted sum of past values of  $V_t$ , which may not be true.

to be positive, although it seems likely that the first few  $\mu_j$  would be positive with the weights applied to more distant output effects negative. In other words, both recent and distant relative prices are expected to enter the equation with the same sign, but only recent output levels are expected to have a positive effect on investment while past levels should have a negative effect. This simply reflects the familiar acceleration principle; for a given level of current output a higher level of past output means a smaller output rise and hence less investment. The distinction between the expected pattern of  $\lambda$ 's and the  $\mu$ 's arises because without ex post substitution investment does not react to a change in relative prices but only to the level of the ratio.<sup>1</sup> In this model there is no acceleration effect with respect to relative prices.

In Table 1, I have worked out a numerical example of what the full coefficient matrix  $\beta$  would be, given the arbitrarily chosen sets of weights<sup>2</sup>

---

<sup>1</sup>To the extent that more capacity is replaced when  $V$  is rising, due to the fact that old capacity (put in place when  $V$  was low) becomes uneconomical,  $E$  might react to changes in  $V$  even when ex post substitution does not exist. As noted earlier, the assumption of exponential retirement patterns excludes this possibility, but the assumption may not be valid.

<sup>2</sup>These weights imply (a) that output planned at  $t+\eta$  is one-half of output at  $t$ , plus 40% of output at  $t-1$ , plus 15% of output at  $t-2$ , (b) that  $V_t^*$  is  $.4V_t + .3V_{t-1} + .2V_{t-2} + .1V_{t-3}$ , and (c) that 25% of orders placed in period  $t$  are filled in that period, 50% in period  $t+1$ , and 25% in period  $t+2$ .



$$[\xi_0 \ \xi_1 \ \xi_2] = [.5 \ .4 \ .15]$$

$$[\gamma_0 \ \gamma_1 \ \gamma_2 \ \gamma_3] = [.4 \ .3 \ .2 \ .1]$$

$$[\psi_0 \ \psi_1 \ \psi_2] = [.25 \ .5 \ .25] .$$

TABLE 1

EXAMPLE OF  $\beta$  MATRIX WITH FIXED PROPORTIONS EX POST

		Coefficients of Q lagged j periods						Row sums*
		0	1	2	3	4	5	
i =	j =							
Coefficients of V lagged i periods	0	.050	-.005	-.021	-.014	.000	.000	.010
	1	.038	.096	-.026	-.052	-.027	.000	.029
	2	.025	.072	.032	-.043	-.041	-.014	.032
	3	.012	.049	.027	-.028	-.029	-.010	.021
	4	.000	.025	.022	-.013	-.017	-.007	.010
	5	.000	.000	.012	-.001	-.006	-.003	.003
column sums*		.125	.237	.048	-.151	-.120	-.034	.105

\*Totals may not check because of rounding

In the example, the row sums are all positive, while the column sums are first positive and then negative. Reading across each row the same pattern is evident; the coefficients are first positive and then negative.

#### E. The "Standard" Neo-Classical Model

The functional forms (11) and (12) are particularly appropriate for testing the hypothesis that a more general functional form is needed than is provided by the "standard" neo-classical model, because

the standard model too can be fitted into these forms. I identify the "standard" model with the path-breaking work of Jorgenson, mentioned in the introduction to this study; the most important particular in which it differs from the model I have specified is that factor proportions are assumed to be freely variable at all times. In this case, (6a) would give the appropriate conditional operating rule for the cost-minimizing ratio of capital stock<sup>1</sup> to output.<sup>2</sup>

Jorgenson recognizes, however, that there exists a lag -- or series of lags -- between the making of plans and the delivery of equipment. In the most rigid form of this model, plans are made without regard for this lag; one interpretation of this assumption would be that expectations are supposed to static,<sup>3</sup> i.e.

$$Q_{t+\eta}^* = Q_t$$

and

$$V_t^* = V_t .$$

---

<sup>1</sup>I am also assuming that the "standard" model includes enough assumptions to assure the existence of an aggregate capital stock. On this see Fisher [11].

<sup>2</sup>The operating rule does not, however, specify how the optimum output is chosen.

<sup>3</sup>Nothing at all would be changed if expectations embodied some sort of constant trend so that, e.g., "expected" output ( $Q^*$ ) was a constant multiple of current output. The constant trend factor could not, in the estimation, be untangled from the other coefficients.

A desired stock of equipment,  $K^*$ , is computed as

$$K_t^* = V_t^* Q_{t+\eta}^* = V_t Q_t .$$

If equipment is ordered to cover any changes in desired capital stock, and a proportion  $\psi_j$  of the aggregate total of equipment ordered in period  $t$  is delivered in period  $t+j$ , then

$$(13) \quad K_{t+1} - K_t = \sum_{j=0}^{\infty} \psi_j (V_{t-j} Q_{t-j} - V_{t-j-1} Q_{t-j-1})$$

and

$$(14) \quad K_{t+1} = \sum_{j=0}^{\infty} \psi_j V_{t-j} Q_{t-j} ,$$

where  $K_{t+1}$  is the aggregate stock at the beginning of period  $t+1$ .

Then, since (13) provides an expression for net investment in period  $t$ , and replacement in that same period will be  $\delta K_t$  if exponential retirement is assumed, this means

$$E_t = \sum_{j=0}^{\infty} \psi_j (V_{t-j} Q_{t-j} - V_{t-j-1} Q_{t-j-1}) + \delta K_t .$$

Substituting for  $K_t$  using (14) leads to

$$(15) \quad \begin{aligned} E_t &= \sum_{j=0}^{\infty} \psi_j (V_{t-j} Q_{t-j} - V_{t-j-1} Q_{t-j-1}) + \delta \sum_{j=0}^{\infty} \psi_j V_{t-j} Q_{t-j} \\ &= \sum_{j=0}^{\infty} \psi_j (V_{t-j} Q_{t-j} - [1 - \delta] V_{t-j-1} Q_{t-j-1}) . \end{aligned}$$

This corresponds to the general form

$$E_t = \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} \beta_{ij} V_{t-i} Q_{t-j},$$

with the matrix  $\beta$  being simply:

$$\begin{bmatrix} \psi_0 & 0 & 0 & \dots \\ 0 & \psi_1 - (1-\delta)\psi_0 & 0 & \dots \\ 0 & 0 & \psi_2 - (1-\delta)\psi_1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

The row and column sums are obviously identical. The expected sign pattern of the weights on the main diagonal would be  $\beta_{11} > 0$ ,  $\beta_{22} = ?$ , depending on whether or not  $\psi_1 > (1-\delta)\psi_0$ . All the weights could be positive, if  $\psi_{i+1} \geq (1-\delta)\psi_i$  for all  $i$ . But this would appear unlikely, for this restriction implies that the mean lag between orders and deliveries of equipment would have to be at least  $1/\delta$  periods (i.e., 6 to 12 years, for plausible values of  $\delta$ ).

Clearly, however, there is nothing about the assumption that factor proportions can vary ex post which requires static expectations. If price and output expectations are not static; but are instead given by

$$Q_{t+\eta}^* = \sum_{i=0}^{\infty} \xi_i Q_{t-i},$$

and

$$V_{t+\eta}^* = \sum_{k=0}^{\infty} \gamma_k V_{t-k},$$

then an argument precisely analogous to the one just given leads to the expression

$$(16) \quad E_t = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{i=0}^{\infty} \psi_j \gamma_k \xi_i (V_{t-k-j} Q_{t-i-j} - [1 - \delta] V_{t-k-j-1} Q_{t-i-j-1}).$$

In this case the row and column sums of the  $\beta$  matrix will not be identical, unless the coefficients  $\gamma_k$  are identically equal to the coefficients  $\xi_i$ . A condition which is sufficient, though not necessary, to guarantee that all the row sums in the  $\beta$  matrix corresponding to (16) will be positive is that

$$\gamma_{i+1} \geq (1 - \delta) \gamma_i \quad \text{for all } i.$$

In Table 2, I give a numerical example of a  $\beta$  matrix derived from equation (16) using the same sets of  $\psi$ ,  $\gamma$ , and  $\xi$  weights which were used in deriving Table 1. In this example, both the row sums and the column sums are first positive and then negative, which would be the "normal" case.

To summarize, I have shown that in the most extreme form of the "standard" model (with the implicit assumption of static expectations) the  $j^{\text{th}}$  row sum of the coefficient matrix of equation (11) would be

TABLE 2

EXAMPLE OF  $\beta$  MATRIX FOR STANDARD NEO-CLASSICAL  
MODEL WITH NON-STATIC EXPECTATIONS

		Coefficients of Q lagged j periods						Row sums*
		0	1	2	3	4	5	
Coefficients of V lagged i periods	0	.050	.040	.015	.000	.000	.000	.105
	1	.038	.085	.055	.016	.000	.000	.194
	2	.025	.061	.000	-.020	-.012	.000	.055
	3	.012	.038	-.004	-.061	-.045	-.014	-.074
	4	.000	.014	-.009	-.045	-.033	-.010	-.083
	5	.000	.000	-.010	-.030	-.021	-.007	-.068
	6	.000	.000	.000	-.011	-.009	-.003	-.024
Column sums*		.125	.237	.048	-.151	-.120	-.034	.105

\*Totals may not check because of rounding

identical to the  $j^{\text{th}}$  column sum. In the less restrictive case in which expectations about both V and Q are formed as weighted averages of past observations on V and Q, with all the weights having positive signs, the presumption is that both the row sums and the column sums would be positive for the first few rows (or columns) and then negative. This contrasts to the hypothesized sign pattern of row sums derived from a theoretical model in which ex post substitution is not possible. In that case, all the row sums would be positive.<sup>1</sup>

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<sup>1</sup>At least given the assumptions that  $V_t^*$  is a positively weighted average of past values of V, and that the amount of capacity desired is not a function of V.

As noted in the introduction, it should made a big difference to policy-makers whether or not a policy which affects the implicit rent on equipment sets off a short-lived investment boom. Thus, for the purposes of short-run policy, the most important thing is to be able to estimate equation (11) or equation (12), and to derive a qualitative (and quantitative) description of the  $\lambda$  weights. However, it would also be desirable to be able to interpret the estimates, and to be able to draw conclusions about the sort of capital-theoretic model that generated the data. I have noted that an unambiguous interpretation will not generally be possible. All that can be said is that, if all the  $\lambda$  weights turn out to have the same sign, or if only a few of them are negative, if they differ very significantly from the distribution of the  $\mu$  weights, then these results are not inconsistent with a model in which ex post substitution is not possible. Also, this pattern would seem improbable in a model in which factor proportions could be easily altered even after fixed equipment was in place.

### III. Estimation of the Parameters of the Model

#### A. Choice of Variables and Parameters

This section describes an attempt to use the model specified in equations (1), (6a), and (11) to explain the quarterly time series of expenditures for Producers' Durable Equipment in the United States. This aggregate is the most comprehensive series available on equipment expenditures by American firms; it is also the only quarterly series available which separates equipment expenditures from expenditures for

new construction. In a sense, the estimation of the lag distributions corresponding to the model provides a test of the "putty-clay" hypothesis -- that choice of technique is possible only up to the point that new machinery is put into place.<sup>1</sup>

In the development of the model in Section II, it was assumed that there was only one homogeneous output. For empirical applications, however, it may be more "realistic" to think of the production function, (2), as a relationship which is simply an aggregate approximation, entailing a summation over many products for each of which there exists at any moment several "blueprints" defining the feasible methods of production.<sup>2</sup> The approximation will be useful if the demand for investment goods, as a function of relative prices and gross additions to capacity, behaves as predicted by (6a). In particular, the separate estimation of  $\sigma$ , which represents the long-run elasticity of equipment demand with respect to the inverse of product rent, will provide an important test of the degree to which the demand for equipment is sensitive to the price of a unit of equipment services, and thus to fiscal and monetary parameters which affect the rent.

The central hypothesis of this paper is contained in the fol-

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<sup>1</sup>Models of this sort have become known as "putty-clay" models because, in one stylized version, machinery is assumed to be made of "putty," which can be shaped into any given form until it is put into place; then it sets and becomes hard-baked "clay."

<sup>2</sup>Of course, all of the usual index number problems arise.



lowing equation:

$$(17) \quad E_t = \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} \beta_{ij} V_{t-i} Q_{t-j} + \epsilon_t .$$

For the estimation, the variables in equation (17) are defined as follows:

$E_t$  is deflated expenditures for Producers' Durable Equipment (in 1958 dollars, seasonally adjusted quarterly totals at annual rates),<sup>1</sup>

$Q_t$  is Business Gross Product (GNP less the gross product of Government, Households and Institutions, and Rest of World, in 1958 dollars, seasonally adjusted at annual rates),

$V_t$  is a variable which is proportional to the equilibrium ratio of equipment to output, given the prices and technology of period  $t$ ,

and  $\epsilon_t$  is an independently distributed random error.

As specified in equation (6a),

$$V_t = a_{1t}^{\sigma-1} (\alpha_1 p_t)^{\sigma} (c_{1t} M)^{-\sigma} .$$

Two of the parameters,  $\alpha_1$  and  $M$ , are constants independent of time;  $\alpha_1$  is the distribution parameter for equipment in the production function, and  $M$  is the markup proportion over minimum average cost.

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<sup>1</sup>Unless otherwise stated, all seasonally adjusted variables are used as provided by the Commerce Department.

In the estimation these parameters are absorbed into the coefficients  $\beta$ , and it is not possible to identify them separately. Thus, for the purposes of estimation,  $V$  is defined as

$$V_t = a_{1t}^{\sigma-1} P_t^{\sigma} c_{1t}^{-\sigma}$$

In this expression

- $\sigma$  is the price elasticity of demand for equipment,
- $a_{1t}$  is the technical change parameter for equipment,
- $P_t$  is the implicit price deflator for Business Gross Product (seasonally adjusted),
- $c_{1t}$  is the imputed rent per unit of new equipment.

The technical change parameter  $a_{1t}$  is assumed to follow a smooth trend.<sup>1</sup> A parameter  $h$  is defined such that

$$a_{1t}^{\sigma-1} = e^{ht}$$

with  $t$  equal to zero at the midpoint of 1958 and incremented by one each quarter. The rate (per year) of equipment-augmenting technical change,  $h'$ , is then

$$h' = 4h/(\sigma-1),$$

since

$$a_{1t} = e^{ht/(\sigma-1)}.$$

The expression for the imputed rent on new equipment is derived from (1),

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<sup>1</sup>Unless some such assumption about the smoothness of technical change is made, there is no possible way to distinguish between the effects of technical change and those of relative prices.

$$c_{1t} = \frac{q_t(r_t + \delta)(1 - k_t - u_t z_t)}{1 - u_t},$$

In this expression

- $\delta$  is the rate of decline of value of the services provided by a unit of fixed equipment (per year),
- $q_t$  is the implicit price deflator for Producers' Durable Equipment (seasonally adjusted),
- $r_t$  is the discount rate (per year),
- $u_t$  is the general rate of income taxation for corporations,
- $z_t$  is the present value of the depreciation deduction,
- $k_t$  is the effective rate of tax credit against equipment purchases.

Although it was initially my intention to estimate  $\delta$ , this did not prove feasible, and it was specified to be .16 (see below).

The choice of appropriate measures for  $r_t$ ,  $z_t$ , and  $k_t$  requires more extensive comment.

The choice of an appropriate empirical measure of the discount rate is a problem which has led to a great deal of disagreement, but from the point of view of policy it is crucial. The discount rate implicit in investment decisions made in an uncertain world might not be highly correlated with any observed market yields. It might even be beyond the influence of monetary authorities, or their influence on it might be weak.

In a riskless world, the market rate of interest would provide an adequate approximation to the unobserved discount rate appropriate to investment decisions.<sup>1</sup> Yields on equities might differ from the interest rate, but only in so far as growth in earnings (either in real terms or because of inflation) was anticipated. The risk factor, however, cannot be ignored, and in so far as the discount rate fluctuates cyclically due to changing market interpretations of the risk involved, a yield on equity may be more closely correlated with the discount rate. As a very rough approximation, the discount rate might be represented by a weighted sum of Moody's Industrial Bond Yield (R), Moody's Industrial Dividend-Price Ratio (S),<sup>2</sup> the corporate tax rate (u), and a time trend (equal to zero at the midpoint of 1958):

$$r_t = (c_0 + c_1 R_t + c_2 S_t + c_3 t)(1 - c_4 u_t) .$$

The general formula for  $z_t$ , the present value of the depreciation deduction, given in Section III, is

$$z_t = (1 - k') \int_0^{\tau} d^{-rs} D(s) ds ,$$

---

<sup>1</sup>In their most recent study of the electric utility industry, Modigliani and Miller [34] found that, for the three years studied, the long term interest rate seemed to be the variable most closely related to their measure of the cost of capital.

<sup>2</sup>If dividend payouts are an approximately constant proportion of earnings (when earnings have been adjusted for overstatement of depreciation) but only adjust slowly to earnings fluctuations, the dividend-price ratio will be a more accurate representation of actual market discounting of expected earnings than the more volatile ratio of actual earnings to price.

In this formula,  $k'$  represents any tax credit which must be deducted from the depreciation base,  $r$  represents the discount rate,  $D(s)$  represents the proportion of the cost basis for an asset of age  $s$  that may be deducted from income for tax purposes, and  $\tau$  gives the life-time of the asset for tax purposes.

Until the end of 1953, the dominant method of depreciation in the United States was the straight-line method, for which

$$D(s) = \begin{cases} \frac{1}{\tau} & \text{for } 0 \leq s \leq \tau, \\ 0 & \text{otherwise.} \end{cases}$$

Following Hall and Jorgenson,<sup>1</sup> the present value of the deduction for straight-line depreciation is:

$$zsl_t = (1 - k'_t) \frac{1}{r_t \tau_t} (1 - e^{-r_t \tau_t}),$$

if the current discount rate is expected to persist.

Starting in 1954, two "accelerated" depreciation methods were permitted by law, the sum of the years' digits method, and the double declining balance method. The SYD method, as Hall and Jorgenson show, "dominates the double declining balance and straight-line formulas in the range of discount rates and life times with which we are concerned."<sup>2</sup>

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<sup>1</sup>[18], p. 394.

<sup>2</sup>[18], p. 395.

The advantage over the double declining balance method, however, is small, especially for assets with relatively short lives, and this factor, along with the computational ease of the declining balance method, may account for the fact that double declining balance depreciation has actually been preferred. Because the present values for the two accelerated methods are close together for the relevant range of lifetimes and discount rates, with the SYD method being dominant, I have chosen to use the formula for SYD depreciation to represent all depreciation taken by accelerated methods. The formula for the deduction is:

$$D(s) = \begin{cases} \frac{2(\tau - s)}{\tau^2} & \text{for } 0 \leq s \leq \tau, \\ 0 & \text{otherwise,} \end{cases}$$

and the present value of the deduction is:

$$zsyd_t = (1 - k_t^d) \frac{2}{r_t \tau_t} \left[ 1 - \frac{1}{r_t \tau_t} (1 - e^{-r_t \tau_t}) \right].$$

As calculated by Hall and Jorgenson,<sup>1</sup> the present value of the depreciation deduction for the two methods and selected lifetimes and discount rates is given in Table 3.

Information about the extent to which accelerated methods have actually been adopted is given in Appendix Table A-12 in Ture [41]. The

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<sup>1</sup>[18], p. 395.

TABLE 3

PRESENT VALUES FOR DEPRECIATION DEDUCTION

Lifetime	Discount Rate	zsl	zsyd
5	.06	.864	.907
5	.12	.752	.827
10	.06	.752	.827
10	.12	.582	.696
25	.06	.518	.643
25	.12	.317	.456

table indicates that according to the Treasury Department's Life of Depreciable Assets Survey in 1959, 29.7 percent of the Production Machinery, Transportation Vehicles and Equipment, and Furniture and Office Machinery, which had been acquired since 1953, was being depreciated by the double declining balance method; and 22.7 percent was being depreciated by the sum of the years' digits method. On the basis of this information I have chosen to represent the present value of the depreciation deduction (per dollar of equipment purchased) as:

$$z_t = w_t \cdot zsyd_t + (1 - w_t) \cdot zsl_t$$

with  $w_t = .524$  starting in the first quarter of 1954. As Figure 1 shows, there is apparently no trend (or learning curve) associated with the adoption of accelerated depreciation by corporations. Thus, the assumption that  $w$  was zero before 1954, and then rose immediately to some constant value, does not seem completely unreasonable, at least from

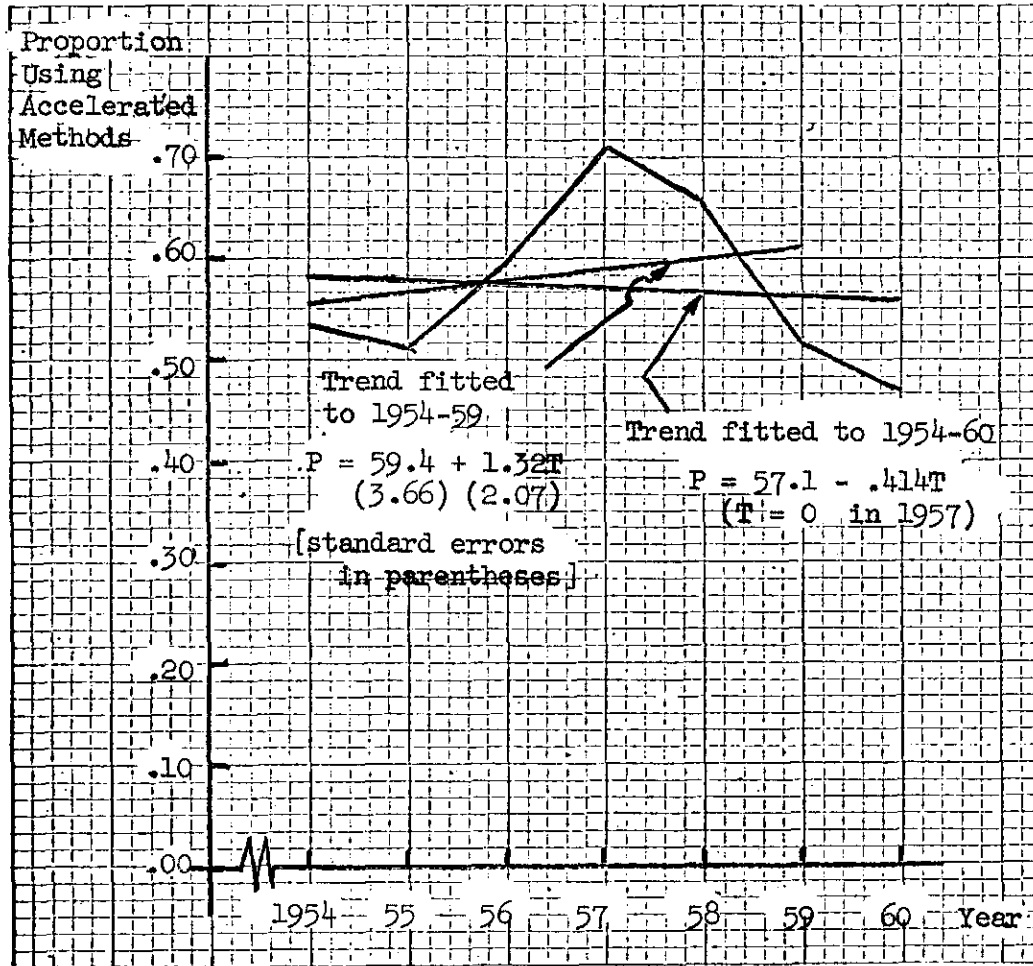


FIGURE 1

PROPORTIONS OF DEPRECIABLE ASSETS OF CORPORATIONS DEPRECIATED  
 BY ACCELERATED METHODS, BY YEAR OF PURCHASE

Source: Ture [41], Table A-5, p. 133.



this evidence. However, it should be noted that the sample for this part of the LDA survey was rather small, and also that this conclusion is quite the opposite of the one reached by Wales [45], using quite different data and methods, in his 1966 Ph.D. dissertation at MIT. Further work is needed to clear up the inconsistency.

Equipment life-times, represented by  $\tau$ , have been adjusted to take account of changes in guideline lives and actual practice, in so far as evidence exists. The adjustments follow those of Hall and Jorgenson<sup>1</sup> with the introduction of new depreciation guidelines assumed to shorten average tax lifetimes from 15.1 to 13.1 years, starting in 1962/3.

When a tax credit for investment was first adopted in 1962, a portion of the law known as the Long Amendment provided that any credit claimed have to be deducted from the depreciation base. The Long Amendment was repealed by the Revenue Act of 1964; starting in 1964/1,  $k'$  is assumed to be zero.

The tax credit was first requested in the President's tax message of April 20, 1961, but it was not passed by the House of Representatives until March 29, 1962; passage by the Senate came on September 6th, and the bill was signed into law on October 16th. Although business confidence that a credit would be passed must have built up after House passage, substantial assurance about the effective date and terms of the

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<sup>1</sup>[18], p. 400.

credit could not have existed before the bill was reported by the Senate Finance Committee in August.<sup>1</sup> The parameter  $k$ , representing the effective rate of tax credit, is somewhat arbitrarily assumed to be zero until the third quarter of 1962, and to be equal to 5% for later periods.

The estimated effective rate is lower than the maximum statutory rate, 7%, for several reasons. There are restrictions on the applicability of the credit to short-lived equipment. In addition, the maximum credit allowed for public utility companies was only 3%. Finally, there are restrictions on the amount of the credit which could be taken in any one year. These restrictions have now been amended, but the most important restriction in the original law provided that the amount of the credit taken in any one year could not exceed the first \$25,000 of tax liability plus one-fourth of any remaining tax liability.

My estimate of the actual effective rate is based on information pertaining to 1963, the only year for which the tax credit was fully in effect and for which relatively complete data have been published. Current dollar FDE expenditures in 1963 amounted to 34.8 billion dollars. Adding up all the figures for "cost of eligible property" which can be found in various places in the 1963 Statistics of Income, U.S. Business Tax Returns, [42], and making allowance for some double counting of partnerships and small corporations, the total is about 32.98 billion. Considering that the Statistics of Income figures are not quite complete,

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<sup>1</sup>This summary of events is based on [44].

and more important, are based on the bookkeeping years of the taxpaying units, the two numbers are remarkable close. For practical purposes it seems unlikely that identifying PDE and "cost of eligible property" as one and the same thing will lead to serious error.

Adding up all the "tentative credits" for corporations, sole proprietors and partnerships (again adjusting for double counting), I have arrived at the figure 1.917 billion.<sup>1</sup> This is 5.5% of 34.8 billion, and I would call it the "effective rate of tentative credit." It is lower than 7% for two reasons. About 20% of the equipment was purchased by public utilities, who only could claim 3% credit. This lowers the effective rate by 12%. Furthermore, for corporations (the only unit for which the two figures are available) "qualified investment" is only about 89% of "cost of eligible property," because much of the eligible property has tax lives of less than 8 years.<sup>2</sup> These two adjustments account for virtually all of the difference between .07 and .055.

However, because of the restrictions on the maximum credit which could be claimed, actual credits taken amounted to only about 1.39 billion, or 4.0% of PDE spending. The difference, however, was not lost. Because of carryback and carryforward provisions, it seems likely that most of this credit could eventually be claimed, although the delay would

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<sup>1</sup>The "tentative credit" is the amount which could be claimed if there were no restrictions on amounts claimed.

<sup>2</sup>"Qualified investment" differs from "cost of eligible property" because assets with short lives receive weights of less than one in counting up "qualified investment."

make the present value of a dollar's credit amount to less than a dollar. Furthermore, when planning their investments, corporations must be assumed to expect to make profits in the relatively near future. It is very hard to derive an expression for just how the portion of the tentative credit not immediately claimed should enter in. But it will not do to ignore it completely -- surely on a priori grounds the true "effective rate of tax credit" should be at least as close to the rate of tentative credit as it is to the rate of actual credit. I have chosen 5% as a compromise between 4% and 5.5%.

#### B. Restrictions on Lag Distributions

At the close of Section II, I argued that the interesting parameters in equation (17) are the row and column sums of the  $\beta$  matrix -- the  $\lambda$  and  $\mu$  weights. One feasible method of approximation involves estimating two coefficients out of each row and column of the coefficient matrix  $\beta$ , with these two coefficients acting as proxies for all the rest of the coefficients in the row or column. Some experimentation with various patterns reveals that the results are quite insensitive to the choice of which coefficients are to be estimated, as long as at least two coefficients in each row are estimated.<sup>1</sup> The practice of estimating

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<sup>1</sup>Estimating more than two coefficients in each row and column produced negligible improvements in the unadjusted coefficient of determination and reduced the adjusted coefficient of determination in all cases in which it was tried.

two diagonal sets of coefficients  $\beta_{i,i}$  and  $\beta_{i,i-1}$ , for  $i = 2, \dots, n$  has been adopted for all regressions which are reported here. The maximum lag,  $n$ , has been chosen equal to 12, implying that the estimated value of equipment spending is based on values of  $V$  and  $Q$  for the preceding three years.<sup>1</sup> Since  $\mu_j = \beta_{jj} + \beta_{j+1,j}$  and  $\lambda_k = \beta_{kk} + \beta_{k,k-1}$ , the coefficients to be estimated determine  $\mu_j$  for  $j = 1, 12$  and  $\lambda_k$  for  $k = 2, n$ .<sup>2</sup> Thus, the equation to be estimated, which I will refer to as the Expenditures Equation, is

$$(18) \quad E_t = \sum_{i=2}^{12} C_i V_{t-i} Q_{t-i+1} + \sum_{i=2}^{12} D_i V_{t-i} Q_{t-i} + \epsilon_t \cdot$$

---

<sup>1</sup>In the theoretical development, all lag weights were specified to extend over the infinite past. Following the development of Chapter 1, it would have been possible to obtain consistent estimates of infinite lag distributions using non-linear methods. In this case, finite lags are more convenient, especially because there are several different lag distributions to be estimated. On a priori grounds it is at least as plausible to assume that the effects of a change in a particular variable will become negligible after a finite period as it is to assume that the effects will continue forever. In addition, even though only a finite number of lagged values of  $Q$  enter equation (18), it is shown in Section III-D that this does not generally imply that the complete adjustment of capacity to a change in output takes place within a finite period (see Table 12).

<sup>2</sup>This estimation procedure requires the first row sum,  $\lambda_1$ , to be zero. Alternative regressions in which this requirement was removed produced no evidence that  $\lambda_1$ , the row sum representing the effects of  $V$  lagged only one quarter, was significantly different from zero. The regressions reported below show estimated values of  $\lambda_2$  as well which are essentially zero; no significant effects of variations in relative prices are found until the third quarter after the change.

The coefficients  $C_i$  correspond to  $\beta_{i,i-1}$ , while the coefficients  $D_i$  correspond to  $\beta_{ii}$ .

Multicollinearity would not permit the estimation of all 22 of the C and D coefficients, and to overcome this difficulty, I have used the technique developed by Shirley Almon [1].

The C coefficients are constrained to be values of a third degree polynomial in  $i$ , with the additional constraint that the value of the polynomial for  $i = 13$  should be zero. This last constraint has the effect of forcing the C coefficients to approach zero as the index approaches 13. In mathematical form the constraints applied amount to:

$$C_m = A_3(m^3 - 2197) + A_2(m^2 - 169) + A_1(m - 13) \quad \text{for } 2 \leq m \leq 13.$$

A similar constraint has been applied to the estimated D coefficients:

$$D_n = B_3(n^3 - 2197) + B_2(n^2 - 169) + B_1(n - 13) \quad \text{for } 2 \leq n \leq 13.$$

With the insertion of these constraints into equation (18) all but 6 of the linear coefficients may be eliminated. The coefficients  $A_1$ ,  $A_2$ ,  $A_3$  and  $B_1$ ,  $B_2$ ,  $B_3$  could be estimated, or else the constraints could be applied by making use of Lagrangian interpolation weights.<sup>1</sup> I have

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<sup>1</sup>See [1] for a more extensive discussion of the use of these weights. Except for rounding error in computation; the alternative methods for eliminating all but six coefficients will produce identical results.

chosen to use the Lagrangian weights; the six coefficients actually estimated,  $C_1^*$ ,  $C_2^*$ ,  $C_3^*$ , and  $D_1^*$ ,  $D_2^*$ ,  $D_3^*$  are linear functions of, respectively,  $A_1$ ,  $A_2$ ,  $A_3$  and  $B_1$ ,  $B_2$ ,  $B_3$ .

### C. Preliminary Exploration of the Parameter Space

Even with these simplifications, estimation of the parameters of equation (18) is a complicated non-linear problem. The variable  $V$  is a nonlinear function of  $\sigma$ ,  $\delta$ ,  $c_0$ ,  $c_1$ ,  $c_2$ ,  $c_3$ ,  $c_4$ , and  $h$ . If it is assumed that the error term  $\epsilon$  in equation (18) is distributed as  $N(0, \sigma_\epsilon^2)$ , that  $E\epsilon\epsilon' = \sigma_\epsilon^2 I$ , and that  $\epsilon$  is independent of all the right hand variables, then maximum likelihood estimates of the parameters of (18) may be obtained by minimizing the sum of squared residuals with respect to all eight nonlinear parameters and the six linear parameters ( $C^*$  and  $D^*$ ). Section III-D describes the simultaneous nonlinear estimation of all these parameters (with the exception of  $c_3$ ,  $c_4$  and  $\delta$ ).

It is a considerably simpler task to obtain maximum likelihood estimates of the lag parameters alone conditional on some assumed values of the nonlinear parameters. This estimation involves only linear methods. Because there is no guarantee that the nonlinear maximization technique will produce a global maximum of the likelihood function, it is generally a good idea to conduct some preliminary exploration of the parameter space. With so many nonlinear parameters, a systematic search of the parameter

space is not feasible; a discussion of the nonsystematic preliminary search which has actually been carried out is included below. This discussion is useful because it provides some sort of "feel" for the sensitivity of the estimated error variance to the various parameters.

It is possible that more can be learned about the role and interactions of each of the variables than can be learned from reference only to a final set of estimates (complete with asymptotic standard errors whose interpretation is not at all clear).

Twelve preliminary trials have been carried out. For each trial, particular values of all the nonlinear parameters except  $\sigma$  have been guessed, and then an exhaustive search has been made in the interval (0, 2) for the value of  $\sigma$  which maximized the unadjusted coefficient of determination. The parameter values assumed in the various preliminary trials are indicated in Table 4. The assumption that  $h$  is equal to zero implies that all technical change is other-factor augmenting. It should be noted that there is no particular reason why the weights  $c_1$  and  $c_2$  should sum to one, or any other specific value. Trials 1 and 2 represent constant after tax discount rates. Trial 3 corresponds to the assumption contained in the original Hall and Jorgenson paper [18] that the before tax rate of return was constant at .14 throughout the period. In the other trials the assumed value of  $c_4$  is an approximation to the "desired" proportion of debt in capital structure, included in the cost of capital in the manner suggested by the Modigliani-Miller theory. Trials 7 and 12 use a discount rate based only on Moody's industrial



bond yield, but the constants  $c_0$  (the intercept) and  $c_3$  (the time trend in the discount rate) are adjusted so that the discount rate actually used in the calculations has no trend, and has a mean value of 6%.<sup>1</sup> Similar adjustments have been made in the constants for trial 8, in which the discount rate is based only on the dividend-price ratio.

In all of the preliminary trials, the parameter  $w$  is assumed to be equal to .4, while the effective rate of tax credit ( $k$ ) is assumed to be 7%. As several experiments discussed in Section F below make clear, the model is not at all sensitive to the assumed value of  $w$ . Only the estimated value of  $\sigma$  is sensitive to the assumption about  $k$ ; the experiments suggest that if  $k$  had been assumed equal to .05 in the preliminary trials, instead of .07, the estimates values of  $\sigma$  would have been about 20% higher.

For all of the trials, the sample period includes quarterly data from the third quarter of 1951 through the fourth quarter of 1965 (a total of 57 observations, with 1952/3 eliminated due to abnormal effects of the steel strike and seizure).<sup>2</sup> For the same sample period, two of the most popular alternative models of investment behavior have

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<sup>1</sup>This formulation might be interpreted as implying that the trend in  $R$  does not affect the discount rate, but is instead offset by an opposite trend in the risk adjustment, and thus that only deviations about the trend are of significance.

<sup>2</sup>Observations on all the variables were available from 1947, but the equation could be fitted only for periods for which  $n$  lagged values were available. The time period starting in 1951 was chosen because some preliminary experiments were carried out with  $n$  assuming values as high as 18, but the right hand variables with very long lags turned out to have insignificant coefficients.

TABLE 4

TRIAL VALUES OF PARAMETERS FOR PRELIMINARY ESTIMATES

Trial Number	$\delta$	h	$c_0$	$c_1$	$c_2$	$c_3$	$c_4$
1	.16	0	.07	--	--	--	--
2	.16	0	.04	--	--	--	--
3	.16	0	.14	--	--	--	1.
4	.16	0	0	1.0	.5	0	.2
5	.16	0	0	.75	.75	0	.2
6	.16	0	0	.5	1.0	0	.2
7	.16	0	.01907	1.	0	-.0003178	0
8	.16	0	.03202	0	1.	+.0004180	0
9	.16	0	0	.667	.333	0	.2
10	.16	0	.03	.667	.333	0	.2
11	.16	0	-.03	1.5	.5	0	.2
12	.16	0	.01907	1.	0	-.0003178	.2

also been used to explain equipment spending. The first of these models is a straightforward, though not rigid, version of the acceleration principle, with investment specified to be a function of changes in output and of net capital stock (K). The model estimated is:

$$(19) \quad E_t = \alpha \sum_{i=1}^{17} \beta_i \Delta Q_{t-i} + \delta K_{t-1} + \epsilon_t .$$

The second alternative model corresponds to the standard neo-classical model with static expectations (see Section II-E). In this model desired capital stock is assumed to be the product of  $V_t$  and  $Q_t$ , and investment expenditures are assumed to be the sum (a) a distributed lag function of changes in "desired" capital stock, and (b) replacement. The relationship is specified to be:

$$(20) \quad E_t = \alpha \sum_{i=1}^{17} \beta_i (V_{t-i} Q_{t-i} - V_{t-i-1} Q_{t-i-1}) + \delta K_{t-1} + \epsilon_t .$$

In calculating  $V$  for use in equation (20), nonlinear parameter values have been chosen to make it correspond as closely as possible to the model of Hall and Jorgenson. These parameters are the same as those used in Trial 3, except that  $w$  is set equal to 1 after 1954, in accordance with the H-J assumption that the best proxy for the present value of depreciation deductions taken after 1954 is the present value for sum of the years' digits pattern. The assumed value of  $\delta$ , .16, is between the value Hall and Jorgenson used for manufacturing equipment (.1471) and the value they

used for non-farm non-manufacturing (1923).<sup>1</sup>

Both (19) and (20) are simply special cases of the more general model I have presented (except for the addition of the net capital stock as a right hand variable).<sup>2</sup> In (19),  $\sigma$  is zero, and all of the terms involving relative prices drop out of the equation. In (20),  $\sigma$  is 1.0, and all of the elements of the  $\beta$  matrix not on the main diagonal are assumed to be zero. The more general form of the standard neo-classical model (with non-static expectations) can best be represented by estimating the lag distributions in equation (18). It is possible for the sign pattern of the estimated row sums to conform to either the predictions of the "putty-clay" model (with all of the row sums positive, as in Table 1) or the general form of the standard neo-classical model (with the row sums positive for short lags and negative for long lags,

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<sup>1</sup>Despite these similarities, the results are not comparable to the original results of Hall and Jorgenson for several reasons; they used the consumer durables deflator and estimated the lag by a considerably different method, as well as carrying out the estimation on net rather than gross investment. In a more recent paper, Hall and Jorgenson [20] present more comparable results, both in terms of the deflator used, and the way in which the lag distributions are estimated.

<sup>2</sup>The argument in Section III-E suggests that if either (19) or (20) were the correct specification, and if the errors were not autocorrelated, the net capital stock term would not add anything to the equation. In fact, it proved highly significant in both (19) and (20) and the errors were still autocorrelated. Several different net capital stocks, developed from historical data going back to 1909, were used; those calculated on the basis of an assumed  $\delta$  equal to .16 provided the best fits. This same capital stock, when added to the better fitting versions of (18), added virtually nothing to the explanatory power of the equation.

as in Table 2).

In estimating both (19) and (20) the Almon polynomial technique has been used to restrict the lag weights; the restriction involves a fourth degree polynomial, according to the equation:

$$\beta_n = A_4(n^4 - 104976) + A_3(n^3 - 5832) + A_2(n^2 - 324) + A_1(n - 18)$$

for  $1 \leq n \leq 18$  .

Examination of Table 5 reveals that it is possible to "explain" equipment spending quite well under a number of different discount rate assumptions, including the assumption that nothing can affect the discount rate used in investment decisions. However, the best fitting trials are those (trials 4, 7, 9, 10, and 12) which emphasize the bond yield as the most important indicator of the prevailing discount rate. The best of these (trial 9) explains 35% of the residual variance left unexplained in the best constant rate equation. It is also the best trial from the point of view of serial correlation, which is very serious for most of the cases. The improvement in fit is not trivial; given the crudeness of the linear formulation and the fact that no systematic method of choosing the  $c$  weights has been used, it is surprisingly large. Inclusion of  $S$  in the cost of capital seems to result in a slight improvement over formulations based solely on trend-adjusted  $R$  (compare trials 4 and 9 to trials 7 and 12) but the results are hardly conclusive. When the cost of capital is based heavily on stock market yields, however, not only does the statistical fit worsen markedly, but also the estimates

of the price elasticity of demand are much lower. This implies that many of the fluctuations in the stock market (which are reflected in the denominator of  $S$ ) produce no corresponding change in investment; the only way the model permits less weight to be given to these fluctuations is through a lower estimate of  $\sigma$ . The possibility that the role of the stock market has been fundamentally misspecified in this equation cannot, however, be ruled out. It could be that movements in  $S$  are primarily reflections of expectations (confidence) in which case it is a mistake to include them as part of the user cost of capital. Such an expectational variable might well have a large short-run effect but little long-run effect on investment, but if it is included with relative price variables which may have greater effects in the long run than in the short run, the various lag distributions will be muddled. Some support for such a hypothesis may be found in the residuals for trial 9. There is, for example, a tailing off of equipment spending in late 1962 and early 1963; because  $S$  is constrained to act with the same lag as all the other user cost variables, the sharp (20%) rise in  $S$  as a result of the market break in 1962 does not help much in explaining this dip. At this time, allowing for more than two lag distributions does not seem fruitful, but it might be explored at some later date.

The improvement of trial 9 over the two alternative models is due to a multitude of factors and cannot be attributed to any single change. However, the difference between trial 3 and the standard neo-classical model is almost completely to be accounted for by the additional flexibility in the lag structure allowed by the estimation of more elements

TABLE 5

GOODNESS OF FIT STATISTICS AND ESTIMATES OF  $\sigma$  ; PRELIMINARY TRIALS

Trial	Estimated Value of $\sigma$	Coefficient of Multiple Determination <sup>1</sup>	Adjusted Standard Error of Estimate <sup>2</sup> (in millions of 1958 dollars)	Coefficient of Variation <sup>3</sup>	Durbin-Watson Statistic
1	.87	.97936	804.9	.027	.97
2	.94	.98286	732.7	.024	1.17
3	.98	.98370	714.4	.024	1.24
4	.85	.98859	597.7	.020	1.74
5	.65 <sup>4</sup>	.97356	910.0	.030	.79
6	.35 <sup>4</sup>	.95406	1199.4	.040	.50
7	.74	.98737	628.8	.021	1.58
8	.65 <sup>4</sup>	.96359	1067.7	.036	.56
9	.90	.98936	577.1	.019	1.88
10	.89	.98739	628.4	.021	1.57
11	.60 <sup>4</sup>	.98039	783.5	.026	1.10
12	.77	.98793	614.8	.020	1.66
Accelerator Model					
	(.00) <sup>5</sup>	.95233	1218.	.041	.46
Standard Neo-Classical Model (Static Expectations)					
	(1.00) <sup>5</sup>	.92269	1541.	.051	.43

<sup>1</sup>Unadjusted.

<sup>2</sup>Sum of Squared Residuals/(Number of Observations - k), where k = 7 for trials 1-12 and k = 6 for trials 4.3 and 4.4.

<sup>3</sup>Adjusted SEE/Mean of Dependent Variable.

<sup>4</sup>Price elasticity of demand estimated only to nearest .05; other estimates are to nearest .01.

<sup>5</sup>By assumption.

in the  $\beta$  matrix, for the nonlinear parameters in both trials are virtually identical.<sup>1</sup> Allowing the more general lag structure leads to the "explanation" of nearly 80% of the residual variance left unexplained by the standard neo-classical model.

Comparison of trial 3, in which the only relative price variables which appear are tax parameters and the price of equipment relative to that of output, and the accelerator model, in which the equipment-output ratio is held constant, shows that allowing for substitution, even without any consideration of monetary variables, significantly improves the explanation. But this is true only if the relative price variable  $V$  is allowed a lag structure distinct from the lag structure for output  $Q$ . The standard neo-classical model as stated in equation (20), which does not allow this freedom, explains less of the variance of  $E$  than does the accelerator model.<sup>2</sup>

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<sup>1</sup>The only difference is in  $w$ , which hardly affects the fit at all. The addition of the net capital stock to trial 3 does not improve its explanatory power. The superior fit in trial 3 cannot be attributed to the free estimation of  $\sigma$ ; the estimated value (.98) is very close to 1, with an asymptotic standard error of about .07.

<sup>2</sup>This striking result does not hold, however, when the two alternative models are reestimated using the consumer durable deflator to deflate investment expenditures (and as the price index for equipment in calculating the rent). The coefficient of variation for the accelerator model was virtually unchanged when this alternate deflator was used, but the fit for the standard neo-classical model improved markedly, so that it provided a slightly better explanation than the accelerator model. The sum of squared residuals, however, was still nearly four times as big as the residual sum of squares for a version of trial 9 using the same deflator. These suggestive results were not explored further, but one interpretation is that the rigid form of the standard neoclassical model is much more sensitive to the precise specification than the other equations.

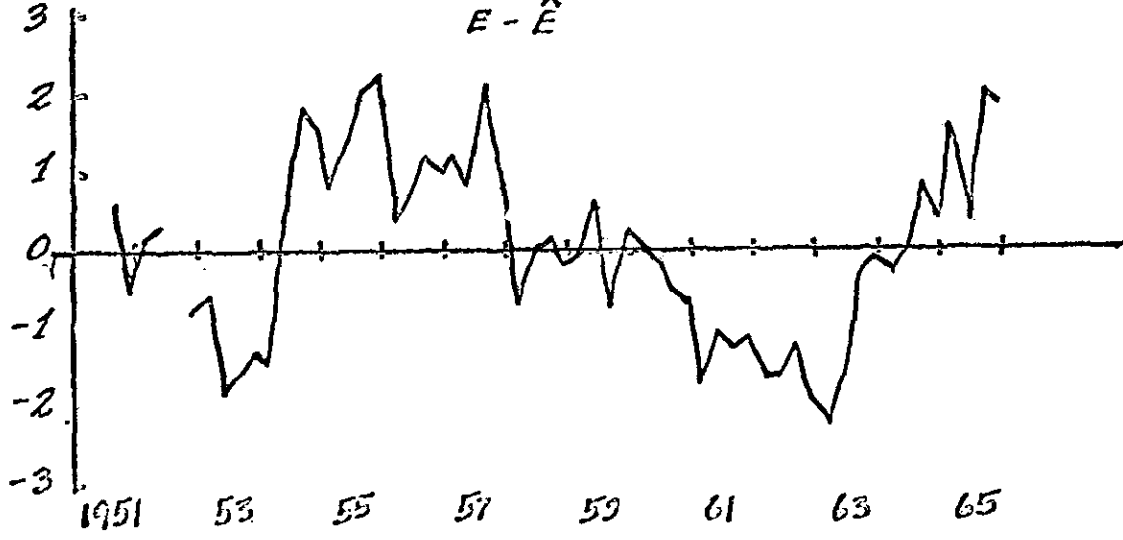
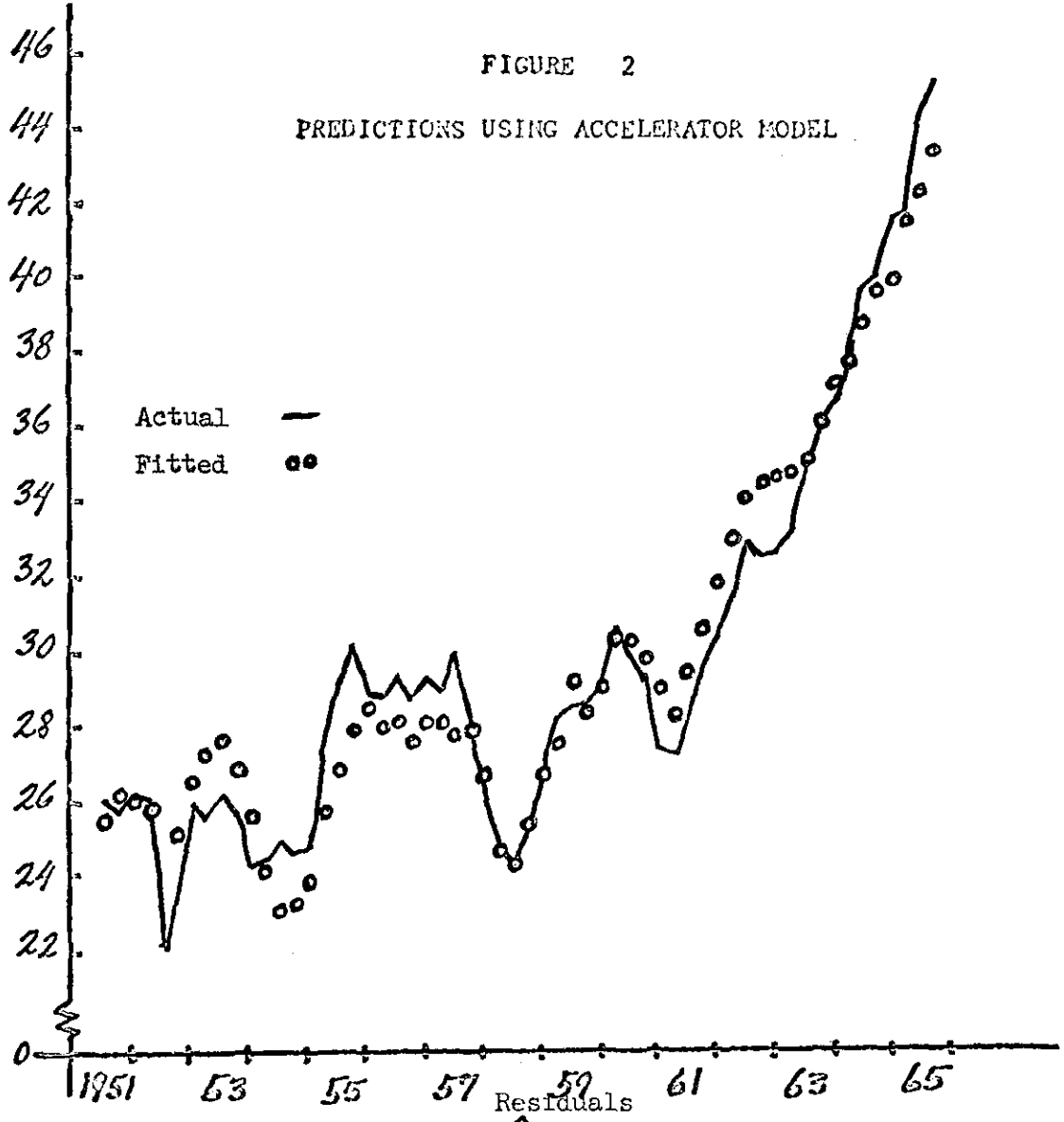


Fitted and actual values of expenditures, as predicted by the accelerator model and the standard neo-classical model, are given in Figures 2 and 3. As the Durbin-Watson ratios confirm, the errors are highly serially correlated. Examination of Figure 2 shows that although the accelerator model based on output alone explains the major qualitative movements of the investment series, without relative price effects the investment booms of the mid-fifties and mid-sixties are underpredicted, and the investment slowdown of the early sixties is not sufficiently reflected. In Figure 3, it can be seen that the introduction of relative price effects without a separate lag smoothes out the peaks and troughs in the series of estimated values even more. An explanation for this pattern can be derived from a comparison of the estimated lag structures for equations (19) and (20). When relative price effects are added without a separate lag structure, the apparent misspecification seems to bias the estimated accelerator coefficients downward in the first few periods. The mean lag is increased as the lag structure "lengthens," and the sensitivity of predicted investment to changes in output is considerable reduced.

The lag distributions estimated for the various trials with the general lag specification are of particular interest. In every case they conform to the qualitative pattern represented in Table 2 and suggested by the "putty-clay" model, as opposed to the pattern represented in Table 2 and suggested by the standard neo-classical model with non-static expectations. For all trials, the row sums ( $\lambda$  weights) are all either positive or insignificantly different from zero, while the column

FIGURE 2

PREDICTIONS USING ACCELERATOR MODEL



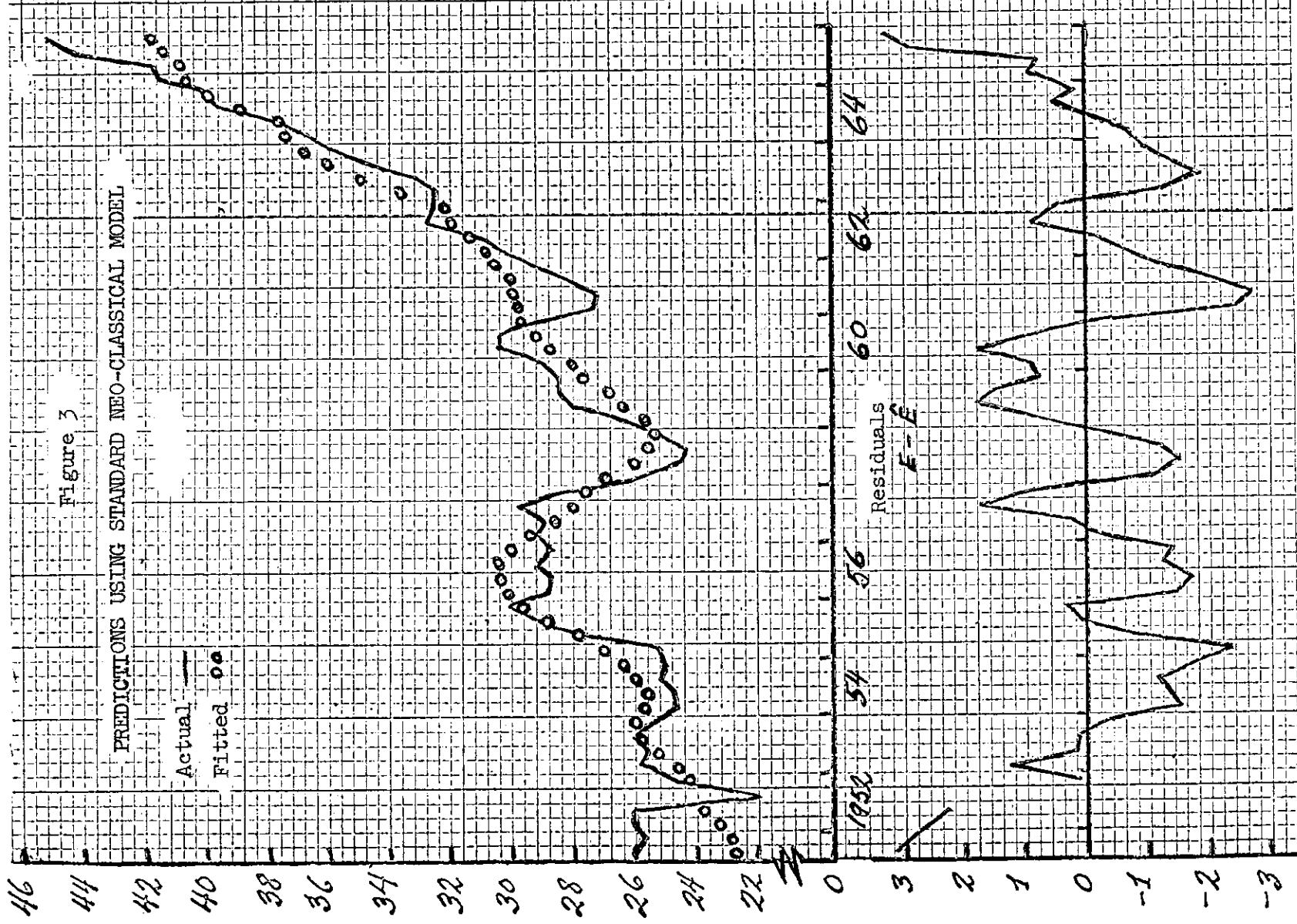


TABLE 6

ESTIMATES OF LAG COEFFICIENTS AND STANDARD ERRORS FOR TRIAL 9

	<u>Estimated Coefficient</u>	<u>Estimated Standard Error</u>
C <sub>2</sub>	.0363	(.0032)
C <sub>3</sub>	.0334	(.0024)
C <sub>4</sub>	.0300	(.0023)
C <sub>5</sub>	.0262	(.0024)
C <sub>6</sub>	.0223	(.0025)
C <sub>7</sub>	.0182	(.0025)
C <sub>8</sub>	.0142	(.0025)
C <sub>9</sub>	.0104	(.0025)
C <sub>10</sub>	.0069	(.0024)
C <sub>11</sub>	.0040	(.0020)
C <sub>12</sub>	.0016	(.0013)
D <sub>2</sub>	-.0356	(.0039)
D <sub>3</sub>	-.0315	(.0027)
D <sub>4</sub>	-.0274	(.0025)
D <sub>5</sub>	-.0234	(.0027)
D <sub>6</sub>	-.0195	(.0028)
D <sub>7</sub>	-.0158	(.0028)
D <sub>8</sub>	-.0123	(.0026)
D <sub>9</sub>	-.0091	(.0024)
D <sub>10</sub>	-.0062	(.0021)
D <sub>11</sub>	-.0037	(.0017)
D <sub>12</sub>	-.0016	(.0010)

TABLE 7

ESTIMATED COEFFICIENTS OF  $V_{t-1}Q_{t-j}$  FOR TRIAL 9

1/j	1	2	3	4	5	6	7	8	9	10	11	12	$\lambda_i = \sum_j \beta_{ij}$
1	---												.0000
2	.0363	-.0356											.0007
3		.0334	-.0315										.0019
4			.0300	-.0274									.0026
5				.0262	-.0234								.0028
6					.0223	-.0195							.0032
7						.0182	-.0158						.0024
8							.0142	-.0123					.0019
9								.0104	-.0091				.0013
10									.0069	-.0062			.0007
11										.0040	-.0037		.0003
12											.0016	-.0016	.0000
Column Sums													
$\mu_j = \sum_i \beta_{ij}$	.0363	-.0022	-.0015	-.0012	-.0011	-.0013	-.0016	-.0019	-.0022	-.0022	-.0021	-.0016	$\sum \lambda_i = \sum \mu_j$ = .0177

sums ( $\mu$  weights) are first positive and then negative. For trials 1, 2, 3, 5, and 7, the first  $\lambda$  weight has the "wrong" sign; otherwise the estimated row sums are all positive. The lag distributions are discussed at greater length in Section III-D; it seems clear, however, that the results of the preliminary estimation strongly support the hypothesis that, however the discount rate is specified, the lag structure between changes in  $V$  and equipment spending is significantly different from the lag structure leading from changes in  $Q$ .

The reason for the insensitivity of the estimated lag structures to the way in which the discount rate is specified seems to be that changes in  $V$  are dominated by changes in tax policy (see Figure 5 below) which affect all of the trials in the same way. The lag distributions estimated for trial 9 may be taken as typical. Table 6 gives the C and D coefficients and standard errors estimated for trial 9; in Table 7 the estimated weights are arranged in the form of a  $\beta$  matrix (but it should be recalled that the coefficients which are estimated are really only proxies for the complete set of coefficients in each row and column).

#### D. Non-Linear Estimation

An iterative technique has been used to obtain estimates of the parameters of the nonlinear version of the Expenditures Equation which maximize the likelihood function, at least locally. Because there is no guarantee that there are not several local maxima, and because the iterative technique used cannot guarantee a global maximum, the iter-

ative process has been started from a number of initial sets of estimates, including all of the sets of parameters used in the preliminary trials. All of the sets of initial parameters tried have led to the same local maximum.

The technique used is called the "maximum neighborhood" method by its originator, Donald Marquardt. It combines the favorable features of two better known techniques for solving nonlinear equations, Gauss' method and the method of steepest ascent. Gauss' method involves linearization of the model by expanding it in a Taylor series about the initial guesses of the parameters (and truncating after the first order terms). The normal equations for the linearized model can be solved, and they provide a new set of parameter estimates which will usually give a larger value for the likelihood function when they are inserted into the original (not the linearized) model. However, the method can break down if the linear approximation is not sufficiently good in the neighborhood of the "corrected" parameter estimates -- the new estimates may actually give a smaller value of the likelihood function.<sup>1</sup>

Using the method of steepest ascent, however, it is always possible to improve the parameter estimates in such a way as to increase the likelihood function (except at a local maximum or a saddle point). The difficulty is that convergence may be extremely slow. Marquardt's method

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<sup>1</sup>In this case, Hartley [22] recommends correcting the parameter estimates by only a fraction of the vector of corrections provided by the linearized model.

in effect, performs an optimum interpolation between the Taylor series method and the gradient method, the interpolation being based upon the maximum neighborhood in which the truncated Taylor series gives adequate representation of the nonlinear model.<sup>1</sup>

Convergence to a local maximum has been considered complete only when every one of the corrections estimated from the linearized model has passed the following test:

$$\frac{|b_i^{q+} - b_i^q|}{.001 + |b_i^q|} < .00005$$

where  $b_i^q$  is the value of the  $i^{\text{th}}$  parameter on the  $q^{\text{th}}$  iteration, and  $b_i^{q+}$  is the "corrected" value of the parameter. Depending on the initial guesses, convergence for the model has involved as few as three iterations or as many as sixty.

In practice, it appears that convergence cannot be achieved if both  $h$  and  $c_3$  are allowed to vary; since both represent trend terms, the linearized model is too nearly singular to get meaningful results. Thus,  $c_3$  has been arbitrarily set at zero. Similarly, it has seemed necessary arbitrarily to normalize either  $\delta$ ,  $c_1$ , or  $c_2$ , and to estimate, in effect, only the ratios  $c_1/\delta$ ,  $c_2/\delta$ , and  $c_1/c_2$ .

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<sup>1</sup>[32], p. 431. The program I used embodies the Marquardt algorithm and is a slight revision of IBM Share Program No. SDA-3094-01.



Thus, the parameter  $\delta$  has been set at .16 in all cases (some rough tests indicate the results are not at all sensitive to the absolute value of  $\delta$ , within the range .10 - .20). No attempt has been made to estimate  $c_4$ , which plays a very small role in the model in any case. Instead, this parameter has been set at .2 on the basis of rather casual examination of movements in debt-equity ratios at market value and book value.

Thus, nonlinear solution of the model has involved obtaining least squares estimates of the parameters  $c_0$ ,  $c_1$ ,  $c_2$ ,  $\sigma$ ,  $h$ , and the  $C^*$  and  $D^*$  parameters. Asymptotic standard errors have been computed, but they are in fact only the standard errors of the parameter estimates as computed from the linearized model. If the Taylor series model is written

$$\hat{E} = f(X; \hat{b}) + P(b - \hat{b})$$

where  $f(X; \hat{b})$  is the  $n \times 1$  vector of predicted values of  $E$  as a nonlinear function of the matrix of right hand variables  $X$ , and the  $k \times 1$  vector of final parameter estimates  $\hat{b}$ , while  $P$  is the  $n \times k$  matrix of partial derivatives of the estimated values of  $E$  with respect to each of the parameters, evaluated at  $\hat{b}$ , i.e.,

$$P_{ij} = \frac{\partial f_i}{\partial b_j} \Big|_{\hat{b}},$$

then

$$\frac{\sum_{t=1}^n (E_t - \hat{E}_t)^2}{n - k} (P'P)^{-1}$$

is the asymptotic variance-covariance matrix of the parameter estimates from which I have derived asymptotic standard errors.

The fruits of the nonlinear estimation are given in Table 8, which gives the estimated parameters and various summary statistics, Table 9, which shows the estimated long-run elasticities of equipment spending with respect to all of the important variables and policy parameters in the model, and Figure 4, in which the fitted and actual values and the residuals are plotted. The unadjusted coefficient of variation is hardly improved at all over trial 9, and the adjusted standard error is actually slightly larger. The Durbin-Watson statistic, however, is slightly closer to 2.0.

The estimated values of both  $c_1$  and  $c_2$  are large compared to their asymptotic standard errors, confirming the apparent significance of the bond yield and the dividend-price ratio in the explanation of investment. The estimate of  $c_0$  is negative, and if  $\hat{c}_0 + \hat{c}_1 R + \hat{c}_2 S$  is taken as an estimate of the discount rate, it seems unreasonably low. But examination of the underlying model indicates that, of the two places in which the discount rate enters, the only place where its absolute level makes any difference is when it is used to discount depreciation patterns in computing the present value of the depreciation deduction. In this role, a low discount rate may act primarily as an ad hoc adjustment to decrease<sup>1</sup> the importance of changes in depreciation rules as a determinant

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<sup>1</sup>At a discount rate of zero, all depreciation patterns have the same present value.

TABLE 8  
 NONLINEAR ESTIMATES OF PARAMETERS AND ASYMPTOTIC STANDARD  
 ERRORS FOR EXPENDITURES EQUATION

<u>Parameter</u>	<u>Estimated Value</u>	<u>Asymptotic Standard Error</u>
$c_0$	-0.008	(0.002)
$c_1$ (bond yield)	-0.535	(0.096)
$c_2$ (stock yeild)	0.098	(0.028)
$\sigma$	1.022	(0.069)
$h$	0.00182	(0.00030)
$C_1^*$	0.0254	(0.0053)
$C_2^*$	0.0184	(0.0032)
$C_3^*$	0.0069	(0.0022)
$D_1^*$	-0.0263	(0.0061)
$D_2^*$	-0.0161	(0.0030)
$D_3^*$	-0.0058	(0.0020)

Summary Statistics

Coefficient of Multiple Determination <sup>1</sup>	.98954
Adjusted Standard Error of Estimate <sup>2</sup>	.597 x 10 <sup>9</sup>
Coefficient of Variation <sup>3</sup>	2.00%
Durbin-Watson Statistic	1.95
Sum of Squared Residuals	16.37 x 10 <sup>18</sup>
Mean of Dependent Variable	29.89 x 10 <sup>9</sup>
Number of Observations	57

<sup>1</sup>Unadjusted.

<sup>2</sup>Sum of Squared Residuals/(Number of Observations - 11).

<sup>3</sup>Adjusted SEE/Mean of Dependent Variable.

TABLE 9  
ESTIMATED LONG-RUN ELASTICITIES OF  
EQUIPMENT SPENDING WITH RESPECT TO VARIOUS DETERMINANTS

Determinants	Elasticities Estimated from Nonlinear Version of Expenditures Equation			
	Evaluated at Relative Prices of			
	1953/1	58/1	63/1	65/4
Output	1.00 <sup>a</sup>	1.00 <sup>a</sup>	1.00 <sup>a</sup>	1.00 <sup>a</sup>
Price of Output	1.02	1.02	1.02	1.02
Price of Equipment	-1.02	-1.02	-1.02	-1.02
Bond Yield	-.20	-.21	-.22	-.23
Dividend-Price Ratio	-.07	-.05	-.04	-.02
Corporate Tax Rate	-.20	-.18	-.17	-.06
Proportion of Depreciation by Accelerated Methods	n.a.	.02	.02	.02
Service Lifetime for Tax Purposes	-.10	-.09	-.09	-.09
Rate of Tax Credit	n.a.	n.a.	.05	.10
Time	.01 <sup>b</sup>	.01 <sup>b</sup>	.01 <sup>b</sup>	.01 <sup>b</sup>

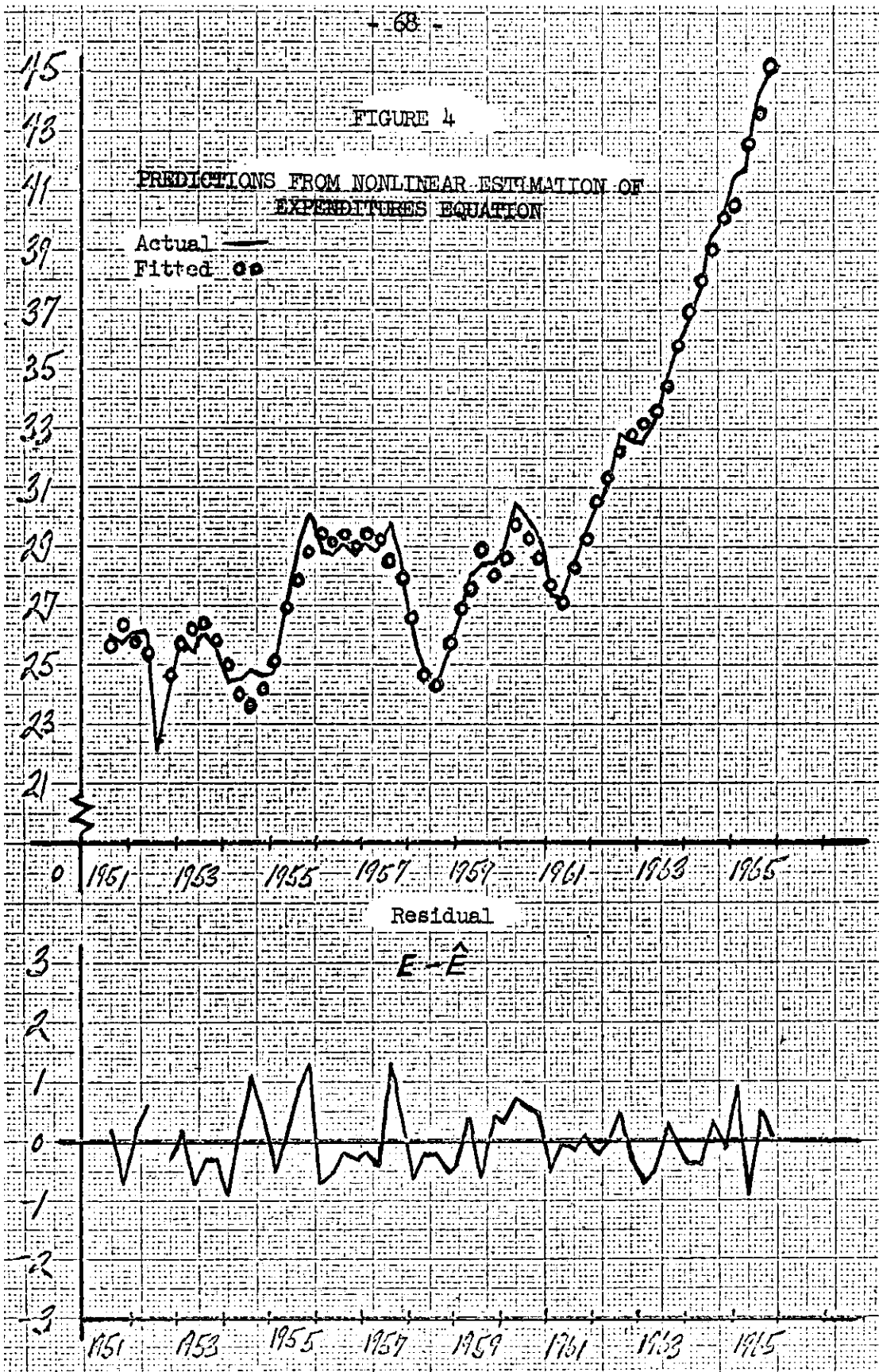
<sup>a</sup>By assumption

<sup>b</sup>Since the origin of variable "time" is completely arbitrary, these elasticities have been calculated as  $\frac{\partial E}{\partial t} \frac{1}{E}$  instead of  $\frac{\partial E}{\partial t} \frac{t}{E}$ . For the purposes of these calculations, time is measured in years, although for the other calculations it is measured in quarters.

FIGURE 4

PREDICTIONS FROM NONLINEAR ESTIMATION OF EXPENDITURES EQUATION

Actual —  
Fitted ○○



of investment. Thus, it decreases the elasticity of investment demand with respect to, for example, a change in guideline lives.

The estimated trend term,  $h$ , is positive, and indicates that will all other variables held constant, the ratio of equipment spending to output is estimated to rise at a rate of about .73% per year. Although  $h$  was included in the equation to allow for technical change, it would not be proper to interpret this parameter as a reliable measure of the degree to which technical change is "capital-augmenting" or "capital-using".

Instead, the estimate of  $h$  seems to change around primarily to offset the trends in other variables. For example, the higher the estimate of  $c_1$  the greater is the weight given to the bond yield, which has a very definite uptrend, in the determination of the discount rate. Other things equal, then, the rent will be higher, and the equilibrium equipment-output ratio lower. But if the equilibrium equipment-output ratio is not to fall over time (and Figure 5 indicates that it was about as high in 1965 as it was in 1948) then the estimate of  $h$  must be higher whenever, ceteris paribus,  $c_1$  is higher. If, then a high estimate of  $c_2$  results from a high cyclical partial correlation between orders and the bond yield, the equilibrium effects are offset by an algebraically larger estimate of  $h$ . The relationship of all this to "technical change" seems rather remote. Despite the fact that it cannot be interpreted, it still seems useful to allow for a trend in order to minimize the danger of falsely accepting one of the other variables as important when it is really acting as a proxy for the trend.

The estimated price elasticity is close to one, and also close to the preliminary estimates. The estimated effect of the tax credit is relatively large; with repeal of the Long Amendment the elasticities in Table 9 indicate permanent elimination of the investment tax credit would lead to an eventual permanent reduction of about ten percent in equipment spending. Accelerated depreciation is estimated to have a considerably smaller effect; the impact of variations in guideline lives within the range which has been contemplated is also relatively small.<sup>1</sup> The very large change over time in the elasticity with respect to the corporate tax rate requires some comment. This elasticity is proportional to the derivative of the rent with respect to  $u$ . From the expression for the rent (equation (1)) the relevant derivative is:

$$\frac{q(r + \delta)(1 - k - z)}{(1 - u)^2} .$$

Thus, the elasticity is proportional to  $1 - k - z$ , and when  $z$ , the present value of the depreciation deduction, is close to 1, the elasticity is quite sensitive to the value of  $k$ , the rate of tax credit. For a piece of equipment on which the whole credit of 7% can be claimed, if  $z$  is greater than .93 (which is only possible after repeal of the Long Amendment) an increase in the tax rate should increase investment, for

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<sup>1</sup>The simulations contained in Section III-F of this paper give considerably more information about the estimated effects of tax policies.

the discounted depreciation deductions exceed the cost of the machine (net of credit), and the higher the investment, the greater the savings on the excess deductions.

The time path of  $V$ , which summarizes all of the relative price and trend effects, is plotted in Figure 5. Apart from the general downtrend through most of the period, attributable to rising equipment prices (relative to the rest of output), and rising interest rates, there are three major movements in the series, all primarily the results of changes in tax laws. In 1954-1955, the adoption of accelerated depreciation provided a significant additional investment incentive. In the last half of 1962, adoption of the tax credit, along with liberalization of depreciation guidelines, provided another significant offset to rising costs, and the repeal of the Long Amendment in 1964, making it unnecessary to deduct the tax credit from the depreciation base, added to the value of this incentive. Finally, temporary repeal of the tax credit in late 1966 led to a situation in which the indicated equilibrium ratio of equipment per unit of capacity was only slightly above its low levels of the late fifties and early sixties (with the improvement due to the relative stability of equipment prices since 1958).

Figure 5 is of particular interest in light of the conclusion drawn by Hickman [23], based on investment functions fitted for the period 1949-60, that the capital-output ratio in the United States was declining. The importance of this conclusion is that, if true, it might mean that private investment demand would be insufficient to sustain full employment. Figure 5 indicates that the marginal capital-output ratio might



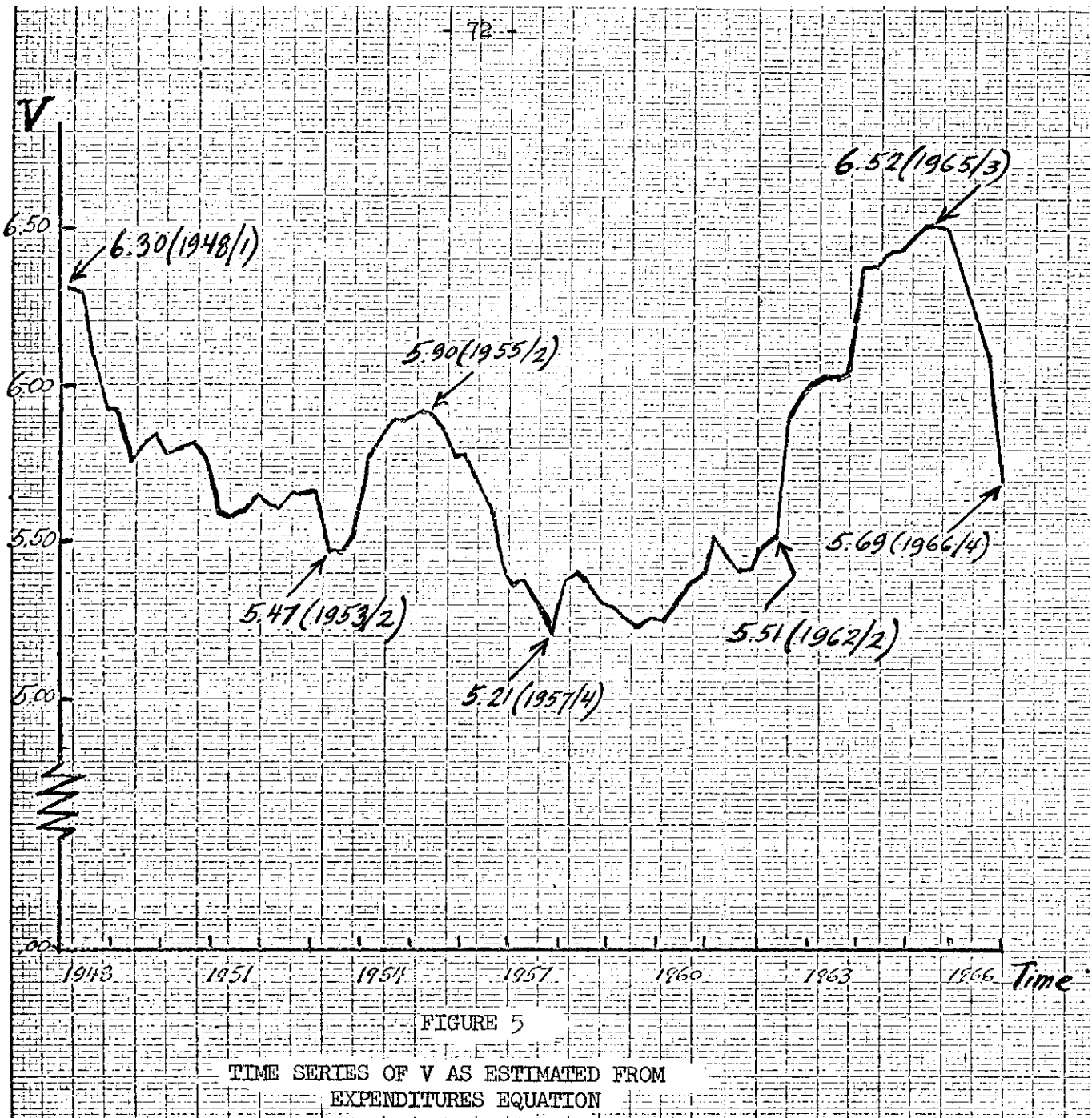


TABLE 10

LAG DISTRIBUTIONS OBTAINED FROM  
NONLINEAR ESTIMATION OF EXPENDITURES EQUATION

Estimated Values of

	Coefficients of $V_{t-i} Q_{t-i+1}$	Coefficients of $V_{t-i} Q_{t-i}$	Column Sums	Row Sums
i	$\hat{C}_i$	$\hat{D}_i$	$\hat{\mu}_i$	$\hat{\lambda}_i$
0	---	---	---	---
1	---	---	.0254	---
2	.0254	-.0263	-.0019	-.0009
3	.0245	-.0238	-.0011	.0007
4	.0227	-.0210	-.0007	.0017
5	.0203	-.0181	-.0007	.0022
6	.0174	-.0151	-.0009	.0023
7	.0143	-.0121	-.0011	.0021
8	.0110	-.0093	-.0014	.0017
9	.0079	-.0067	-.0016	.0012
10	.0050	-.0043	-.0017	.0007
11	.0026	-.0024	-.0015	.0003
12	.0009	-.0009	-.0009	.0000

$$\Sigma \hat{\beta}_{i,j} = \Sigma \hat{C}_i + \Sigma \hat{D}_i = \Sigma \hat{\mu}_i = \Sigma \hat{\lambda}_i = .01197$$

well have declined during the period studied by Hickman, although not necessarily for technological reasons. But it also indicates that at least the desired ratio of equipment to output has been substantially affected by government policy since that time. Hickman did not allow for this effect (which was less important for his sample period), but it has played a large role in the revival of investment demand in the last four years.

Table 10 gives estimates of the lag parameters derived from the non-linear estimates of  $C^*$  and  $D^*$  parameters. Perhaps the easiest way to make intuitive sense out of Table 10 is to speak in terms of percentages of the steady state response to a change in one of the determinants of investment which will occur in each time period. Table 11 interprets the lag distributions for changes in  $Q$  or  $V$  in terms of these percentages.

TABLE 11  
EFFECTS OF PERMANENT CHANGES IN  $Q$  OR  $V$  ON FLOW OF  
EXPENDITURES FOR EQUIPMENT,  $n$  QUARTERS AFTER THE CHANGE  
(Changes are expressed as percentages of steady state response.)

Quarters after Change												
0	1	2	3	4	5	6	7	8	9	10	11	12
Effects of Change in $Q$ <sup>1</sup>												
---	212	197	188	182	176	169	159	148	134	120	108	100
Effects of Change in $V$ <sup>2</sup>												
---	---	-5	2	17	35	53	70	83	92	98	100	100

<sup>1</sup>Assuming no change in  $V$ .

<sup>2</sup>Assuming constant growth in  $Q$ , at 4%.

The sum of all the coefficients shows that a rise in output of one billion (1958) dollars will eventually increase the flow of PDE expenditure by  $Vx .01197$  billion, or at the 1965/4 level  $V (6.494)$  by roughly \$71 million. Due to the accelerator effect this response will be exceeded as capacity is initially adjusted upward, and only after three years will the response die down to the steady state effect, which is replacement on the equipment needed to produce \$1 billion of output. Table 11 gives the increments in PDE spending stemming from a sustained rise in output as percentages of the steady state increment.

For any given set of relative prices, it is possible to compute the equilibrium increment to an imaginary net stock of equipment<sup>1</sup> which will be brought about by a sustained unit change in output. The lag weights, when combined with an a priori value of  $\delta$ , can be used to derive an

TABLE 12

PROPORTION OF ADJUSTMENT OF STOCK OF EQUIPMENT  
TO CHANGE IN Q COMPLETED AFTER N QUARTERS

Estimated from Expenditures Equation

n	$\delta = .04$	$\delta = .08$	$\delta = .12$	$\delta = .16$	$\delta = .20$
4	8%	15%	22%	29%	36%
8	13%	25%	36%	46%	55%
12	17%	32%	45%	56%	66%
16	20%	37%	51%	63%	72%
20	23%	42%	57%	68%	77%
40	37%	61%	76%	86%	92%

<sup>1</sup>If all relative prices are held constant, and so is technical change, it is possible to speak of a stock of equipment, since all machines are of the same model.

expression for the proportion of the adjustment from one equilibrium stock to another which will be completed within  $n$  quarters after the change in output. For the estimated weights the results, as a function of the assumed  $\delta$ , are shown in Table 12. For "reasonable" values of  $\delta$ , in the range .08 to .16, the adjustment seems relatively slow; only 42-68% of the adjustment takes place within the first five years, and it is more than ten years before the adjustment even approaches 90% completeness. Long lags in capital-stock adjustment models, of course, are nothing new, but it is these long lags which have led to criticism of many of the simpler versions of such models.

The speed of response of equipment spending to a change in  $V$  varies with the growth rate of output; the faster output is growing the faster substitution will take place. With investment running at a level of around \$50 billion and 4% growth in real output, a one percent change in the interest rate (from the 1965/4 level of 4.72%) would eventually change the flow of PDE spending by about 5% (or 2.5 billion dollars), but a year after the interest rate change less than 20% of the eventual effect would be felt. If the change were then reversed<sup>1</sup> the effects of

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<sup>1</sup>In case the "reversal" only restored interest rates to the original level (as in a sequence, 4%, 5%, 4% as opposed to 4%, 5%, 3%) it would take over a year for the second change simply to offset the effects of the first change enough to bring spending back to the level that existed when the second change took place. Such lag effects may not be reasonable, but they could result from the lag of shipments behind orders and also from a process in which steps once taken to adopt a new technique result in the "forgetting" of the old technique. Note must also be taken of the lag between the application of policy instruments and the effect on those factors directly determining investment; in the case of tax changes the lag may be negligible but in the case of interest rates it may be

the original change would continue to build up and it would take more than a year for an equal deviation of interest rates in the opposite direction (from the presumed "normal" level) to restore spending on equipment to the level which would have ensued had no changes taken place.<sup>1</sup> Table 13 gives the time pattern of effects for the sequence of interest rate changes assumed in this particular case; similar patterns can be computed from Table 11 for any desired sequence of changes.

TABLE 13

EFFECT OF A CHANGE IN  $V$  FOLLOWED AFTER ONE YEAR BY A CHANGE OF TWICE THE ORIGINAL MAGNITUDE IN THE OPPOSITE DIRECTION

(Effects are given in percentages of the steady state effect of the original change, assuming that output is growing at a 4% rate)

Quarters after first change:	1	2	3	4	5	6	7	8	9	10
Effects on $E$ :	---	-5%	+2%	17%	35%	63%	66%	49%	22%	-8%

Figure 6 compares the estimated short and long-run responses of equipment spending to changes in  $V$  and  $Q$ , based on the non-linear

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substantial (despite Tucker's very appealing case [40] for a short lag, which unfortunately fails to distinguish between short and long rates, with the latter more likely to directly affect fixed investment). In so far as monetary policy can rapidly influence the stock market, this may increase its effectiveness, but little evidence is available on this point.

<sup>1</sup>Because this research contributes to a larger project on monetary policy (as noted in Appendix 1), I have also been concerned with the possibility that rationing or "availability" effects might lead to more rapid monetary influences on investment. A variety of variables which have been suggested as likely to reflect such effects have been experimentally introduced into the model with very short lags, but none has come close to making a significant contribution to the explanation.

estimates, to some previous estimates of lags in the investment process. The results are not really comparable, especially in so far as the Jorgenson [25] and Griliches-Wallace [16] models included structures as well as equipment.<sup>1</sup> The most recent Hall-Jorgenson estimates (from [20]), based on yearly data for equipment spending only conform relatively well to my estimates of the lag for changes in output. But their model excludes any possibility of interest rate effects; only changes in tax policy and the relative price of new equipment influence  $V$  in their model. Their results are not inconsistent with the view that the relatively larger and more frequent fluctuations in output exerted a dominant influence in the estimation of the lag pattern. Granting this possibility, because no separate lag pattern was allowed for tax changes it would seem dangerous to base policy conclusions on the apparently large and rapid effects of tax policy found in their study.

The most disturbing lag coefficient is the relatively large response of investment in the first quarter following a change in output. Some investment functions have specified a priori that no stimulus lagged less than two quarters can have any effect, in view of the supposed accuracy of investment anticipations. But Eisner [7] and Evans and Green [10] have found that unexpected rises in sales, or simply changes in sales occurring after anticipations have been reported, can add something

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<sup>1</sup>However, in view of the small proportion of construction in the total investment of manufacturers, the short run response of construction to relative price changes would have to exceed the long run response by a factor of ten or more to produce lag patterns for total investment like those implicit in the models cited, and this seems unlikely.

Short Run Response  
 as Percentage  
 of Long Run Response

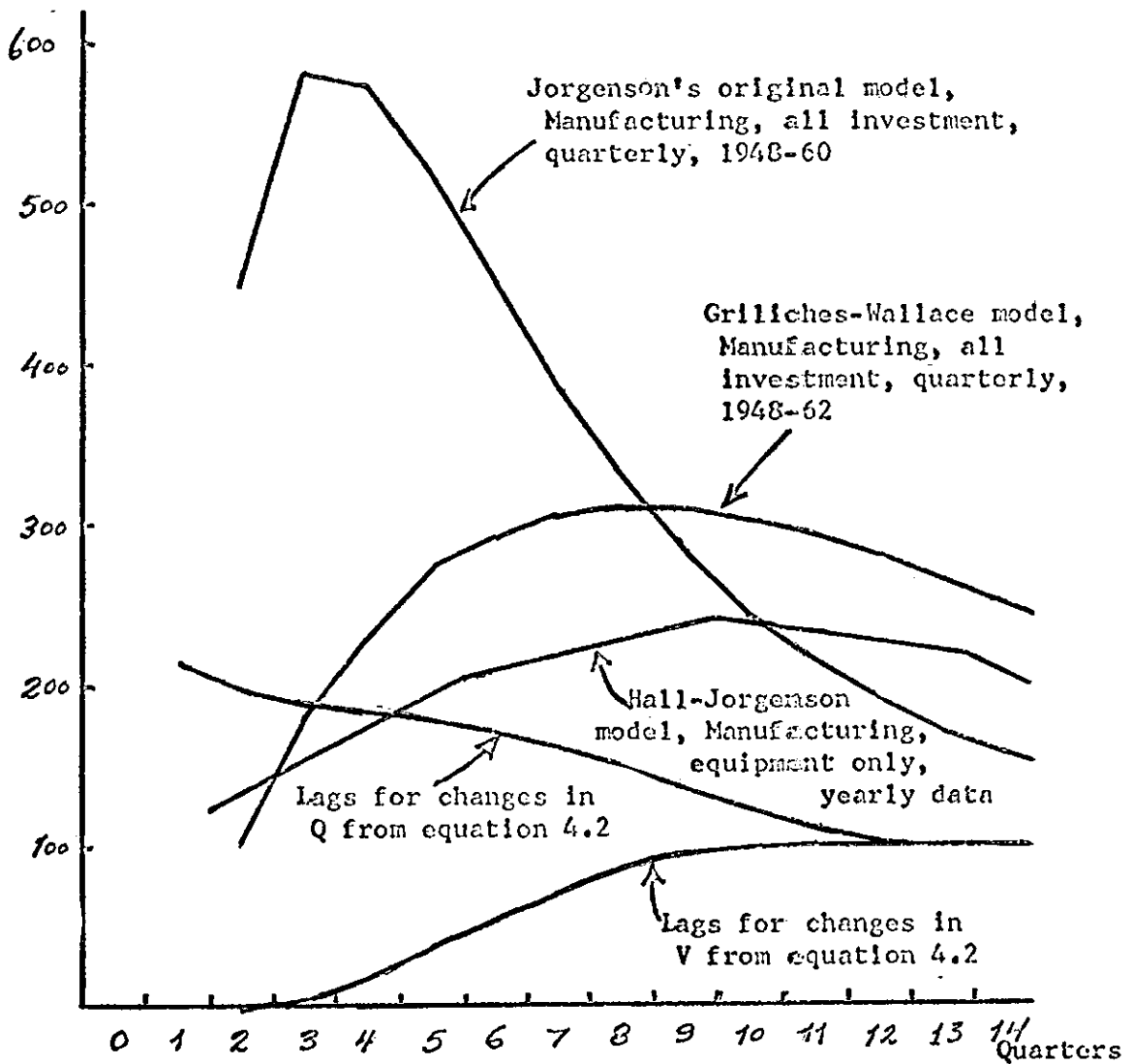


FIGURE 6

COMPARISON OF ESTIMATED LAG DISTRIBUTIONS

Sources: Jorgenson [25], Table 4, Griliches and Wallace [16],  
 Table 4, and Hall and Jorgenson [20] Table 2.



to explanations based on anticipations alone.<sup>1</sup> In principle, changes in output might well be reflected very rapidly in changes in orders (especially cancellations) and at least some of these orders could be rapidly translated in expenditures. Although the elasticity of  $E_t$  with respect to  $Q_{t-1}$  seems too large to reconcile with Eisner's estimates of the elasticity of  $I_t$  with respect to  $Sales_{t-1}$ , the principal feasible alternative, constraining this lag coefficient to be zero, does not seem theoretically justifiable. In fact, even the requirement that changes in output cannot affect investment in the same quarter is somewhat hard to defend; this specification was adopted largely to minimize statistical problems resulting from simultaneous determination.

The distinction between output and sales should not, however, be overlooked. It is possible that changes in output (which, unlike sales, are under the control of the producer) are in fact correctly anticipated, and that the high correlation of output changes and nearly simultaneous investment is a result of this correct anticipation. Viewed in a slightly different way, changes in output may result from some external cause (say changes in orders) which also stimulates investment demand. If this is the case, orders themselves should be studied (though this can only be done on an industry level), but as a first approximation the linkage from aggregate demand (via orders) to output and then to investment may not be seriously misspecified if the intermediate orders stage is suppressed.

---

<sup>1</sup>Eisner's conclusions were tempered by the relatively unsuccessful results of tests based on extrapolation beyond the sample period, but extrapolations with the functions estimated by Evans and Green generally produced better predictions than the anticipations did.

A much less optimistic interpretation would point to the fact that if serial correlation is present, lagging the endogenous output variable will not be enough to remove problems brought about by failure to consider all of the simultaneous equations in the underlying economic system. Despite the nice-looking Durbin-Watson statistics, the presence of serial correlation must still be suspected. The difficulties may be compounded by the fact that the lagged output variable includes the dependent variable, equipment spending, as one of its components. Although estimation of a more complete system would not be feasible in this study, a cautious interpretation of the lag distributions is certainly advisable.

Another way to judge the "reasonableness" of the estimates is to examine the equilibrium ratios of equipment "stock" (under idealized circumstances where such stocks make sense) to output, and also to compare the equilibrium shares of equipment spending in output. Both of these pairs of statistics are functions of the estimated values of  $V$ , and thus change over time.

The formula for the equilibrium ratio of equipment to capacity output, given the relative prices and technology of period  $t$ , is

$$V_t(\sum_{ji} \beta_{ij})/\delta .$$

The estimate of  $\sum_{ij} \beta_{ij}$  is .01197. Neither the absolute values of this sum nor the absolute value of the estimate of  $V$  has any independent importance, but the product of  $V$  and  $\sum_{ij} \beta_{ij}$  represents the steady-state stream of replacements on the capacity needed to produce a unit of output.

Thus, for any assumed value of the rate of replacement  $\delta$  this implies a certain ratio of the stock of equipment to capacity.

The implied ratios for  $\delta = .16$  are given in Table 14.<sup>1</sup> In

TABLE 14  
EQUILIBRIUM EQUIPMENT-OUTPUT RATIOS

	Estimated in Chapter 4	Estimated in Chapter 5
Prices of:		
1955/1	.4406	.4465
1960/1	.3961	.3504
1965/1	.4837	.4542

a stationary state with no growth in output, the proportion of equipment spending in business GNP would range between 5.7% and 7.0% as estimated from the Expenditures Equation.<sup>2</sup> With output growing steadily at 4% the range would be about 7.5%-9.0%. By comparison, the actual share of EPD in Business GNP was 9.6% in the first quarter of 1948, fell as low as 6.5% as a result of the steel crisis in 1952, and rose as high as 8.4% in the capital goods boom of 1955. At the low point in 1961 the ratio was only 6.0%, but it had risen to 8.0% by the end of 1965.

---

<sup>1</sup>The "actual" ratio of the constant dollar net stock of equipment, consistent with exponential depreciation at 16% per year, was in the range of 35% of output throughout this period. But this ratio is not really comparable to the "equilibrium" ratios, which assume the existence of a steady state.

<sup>2</sup>For the range of values of  $V$  observed during the sample period.

#### IV. Policy Simulations and Sensitivity Analysis of Tax Parameters

##### A. Policy Simulations

In order to more fully evaluate the effect of tax policy changes, monetary policies, and relative price changes, I have used the model, with the parameter estimates reported in Table 8, to simulate the time path of equipment expenditures under a number of alternative assumptions about tax parameters and other variables which affect  $V$ . The simulated time paths of predicted equipment spending may be compared to the predicted values given the actual policy parameters (or independent variables), in order to compute a measure of the direct effects of a policy such as accelerated depreciation or the investment tax credit. Without a complete model of national income determination these computations are of limited meaning, for the path of output, as well as the other determinants of investment in equipment, is surely altered by induced changes in investment. Nevertheless, these computations might be thought of as the outcomes of controlled experiments in which the government sought to provide the same level of aggregate demand in two ways: (1) by direct government purchases of equipment, or (2) by tax incentives to encourage private equipment purchases, interest rate manipulations, etc.

Seven sets of simulations have been carried out which represent the direct effects of various combinations of tax policies which have been adopted since 1954. The results are tabulated in terms of constant dollar effects in Table 15, and as proportions of actual expenditures in Table 16. The policies include the adoption of accelerated depreciation methods in 1954, the promulgation of new depreciation guidelines

in 1962, adoption of an investment tax credit in 1962, the repeal of the Long Amendment in 1964, and the reduction of general income tax rates for corporations in 1964 and 1965. There are interactions among the various policies -- the depreciation guidelines would have induced slightly more investment if accelerated depreciation had not already been in effect, for example -- and thus the effect of all the policies together is not the sum of the effects of each policy alone.

All of the policies together are estimated to have directly induced over 17 billion (1958) dollars of gross expenditures between 1954 and 1966, with the largest effects coming in 1964-1966. The most important single policy is the tax credit--even without repeal of the Long Amendment it would have directly induced over 6 billion dollars of gross investment by the end of 1966; with the Long Amendment repealed the effects add up to more than 9 billion (constant) dollars. The directly induced investment is smaller than the tax reductions for 1962 and 1963, approximately equal to the reductions in 1964, and surely considerably in excess of the reductions for 1965 and 1966.<sup>1</sup> Accelerated depreciation policies are estimated to have had disappointingly small effects (amounting to roughly a 2% increase above what expenditures would otherwise have been for each year after 1956). The depreciation guidelines apparently increased expenditures by about 1% in each year after 1964.

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<sup>1</sup>Recall that the estimated effective rate of tax credit for 1963 is 4.0%; for later years it might be higher, but probably not much more than 5.0%.

TABLE 15

ESTIMATED DIRECT EFFECTS OF TAX POLICIES

(In billions of 1958 dollars)

	A	B	C	D	E	F	G
1954	-.01						-.01
1955	.22						.22
1956	.43						.43
1957	.44						.44
1958	.43						.43
1959	.51						.51
1960	.55						.55
1961	.57						.57
1962	.64						.64
1963	.67	.05	.22		.22		.95
1964	.70	.36	1.51	*	1.51	*	2.57
1965	.77	.54	2.14	.90	3.04	.05	4.41
1966	.84	.59	2.35	1.96	4.31	.16	6.06
1954-66	6.76	1.55	6.22	2.85	9.07	.21	17.77

\*Indicates less than \$50 million.

- A: Accelerated Depreciation.
- B: Depreciation Guidelines.
- C: Investment Tax Credit (with Long Amendment).
- D: Repeal of Long Amendment.
- E: Investment Tax Credit (without Long Amendment).
- F: General Corporate Tax Reductions (of 1964 and 1965).
- G: All Policies Combined.

TABLE 16

ESTIMATED DIRECT EFFECTS OF TAX POLICIES

(As percentages of actual expenditures)

	A	B	C	D	E	F	G
1954	*						*
1955	0.8%						0.8%
1956	1.5%						1.5%
1957	1.5%						1.5%
1958	1.7%						1.7%
1959	1.8%						1.8%
1960	1.8%						1.8%
1961	2.0%						2.0%
1962	2.0%						2.0%
1963	2.0%	0.2%	0.6%		0.6%		2.8%
1964	1.8%	1.0%	3.9%	*	3.9%	*	6.7%
1965	1.8%	1.2%	5.0%	2.1%	7.0%	0.1%	10.2%
1966	1.7%	1.2%	4.8%	4.0%	8.8%	0.3%	12.4%

\* Indicates less than 0.05%

In view of the large magnitude of the effects attributed to the Tax Credit, and the brief period during which this provision has been effective, it might be useful to give quarterly breakdowns of the estimated effects. This is done in Table 17.

TABLE 17

ESTIMATED INCREMENTS TO EQUIPMENT SPENDING  
 ATTRIBUTED DIRECTLY TO TAX CREDIT  
 (INCLUDING REPEAL OF LONG AMENDMENT)

	From Expenditures Equation
1962/3	----
1962/4	----
1963/1	-.09
1963/2	.02
1963/3	.29
1963/4	.66
1964/1	1.04
1964/2	1.39
1964/3	1.62
1964/4	1.97
1965/1	2.33
1965/2	2.86
1965/3	3.27
1965/4	3.69
1962 total	.00
1963 total	.22
1964 total	1.51
1965 total	3.04

Note: Units are billions of 1958 dollars, seasonally adjusted at annual rates.



The assumptions built into the model make it inappropriate for use in evaluating a temporary change in tax law, if the change is widely publicized as temporary, and expectations are formed on the assumption that the announcements are true. Thus, no attempt has been made to simulate the effects of temporary suspension of the investment tax credit in late 1966 and early 1967.

Two additional simulations have been made to predict what equipment expenditures might have been if (a) the bond yield had remained constant at 3.75% and the dividend-price ratio had remained constant at 4.39% throughout the postwar period, and (b) if  $V$  had remained constant at its mean value throughout the sample period. These simulations are reported in Tables 18 and 19. In both cases, of course, the estimated effects are only "partial" -- total output is assumed to follow its actual time path.

The most striking characteristic of the interest rate simulation is the large negative stimulus arising between 1956 and 1961, which is apparently associated with the movement of the bond yield from a relatively low level before 1955 to a relatively high plateau after 1960.<sup>1</sup> If an attempt is made to match up turning points in the interest rate series and turning points in the estimated effect on equipment spending,

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<sup>1</sup>For the 38 quarters between 1947 and 1956/2, quarterly averages of Moody's industrial bond yield all fell in the range 2.60%-3.39%. The yield rose sharply between 1956/2 and 1959/3; for the 26 quarters starting with 1959/3 the quarterly averages remained in the range 4.38%-4.72%.

TABLE 18  
NET DIRECT EFFECTS OF INTEREST RATES  
(As percentages of actual expenditures)

Year	Quarters				Average for Year
	1	2	3	4	
1950	2.6%	2.2%	2.0%	2.3%	2.3%
1951	2.6%	2.8%	2.8%	3.1%	2.8%
1952	2.9%	2.8%	3.0%	2.6%	2.8%
1953	2.5%	2.6%	2.6%	2.7%	2.6%
1954	2.8%	2.3%	1.7%	1.4%	2.1%
1955	1.4%	1.6%	1.9%	2.2%	1.8%
1956	2.7%	3.0%	3.0%	3.1%	3.0%
1957	3.0%	2.9%	2.4%	1.8%	2.5%
1958	1.4%	0.8%	-0.4%	-1.2%	0.2%
1959	-1.6%	-1.7%	-1.8%	-1.7%	-1.7%
1960	-1.7%	-2.0%	-2.4%	-2.9%	-2.3%
1961	-3.6%	-4.0%	-4.2%	-4.2%	-4.0%
1962	-4.1%	-4.0%	-3.8%	-3.8%	-3.9%
1963	-3.8%	-3.8%	-3.6%	-3.6%	-3.7%
1964	-3.4%	-3.3%	-3.1%	-3.1%	-3.3%
1965	-3.0%	-3.2%	-3.2%	-3.2%	-3.2%
1966	-3.3%	-3.3%	-3.2%	-3.4%	-3.3%

TABLE 19

NET DIRECT EFFECTS OF ALL RELATIVE PRICES COMBINED

(As percentages of actual expenditures)

Year	Quarters				Average for Year
	1	2	3	4	
1950	12.2%	10.2%	8.2%	7.7%	9.4%
1951	7.4%	6.8%	6.4%	6.1%	6.6%
1952	5.2%	4.4%	4.2%	3.1%	4.2%
1953	2.4%	2.1%	1.8%	2.1%	2.1%
1954	2.0%	1.4%	0.5%	0.1%	1.0%
1955	0.3%	1.0%	1.7%	2.5%	1.4%
1956	3.4%	4.0%	4.0%	3.9%	3.8%
1957	3.3%	2.6%	1.4%	-0.1%	1.8%
1958	-1.4%	-2.5%	-4.6%	-6.2%	-3.6%
1959	-7.1%	-7.3%	-8.0%	-7.8%	-7.6%
1960	-8.2%	-8.6%	-9.1%	-9.7%	-8.9%
1961	-10.7%	-10.9%	-10.0%	-10.3%	-10.7%
1962	-9.8%	-9.2%	-8.8%	-8.8%	-9.2%
1963	-9.2%	-8.6%	-7.2%	-5.6%	-7.6%
1964	-3.9%	-2.4%	-1.3%	-0.1%	-1.9%
1965	1.0%	2.4%	3.3%	4.2%	2.8%
1966	5.1%	5.6%	5.8%	5.9%	5.6%

the lags vary from 2 to 8 quarters, with the average lag being about 6 quarters -- 1 1/2 years. When the lag between monetary changes and movements in long term interest rates is added in the result is certainly not encouraging toward the use of monetary policy to achieve countercyclical influences on investment.

B. Sensitivity Analysis of Tax Parameters

Because tax policy -- especially the tax credit -- plays such an important role in the explanation of equipment spending, I have attempted to assess the sensitivity of the results to the assumed values of the parameters  $k$  and  $w$ . The model has been completely reestimated with assumed values of  $k$  -- the effective rate of the tax credit -- varying all the way from zero to .20. The somewhat surprising result, recorded in Table 20, is that the estimated price elasticity adjusts to largely offset even very extreme assumed values of  $k$ . The best explanation apparently is achieved with  $k$  assumed to be .11, but the improvement in fit is quite small.

Because of the offsetting variations in the estimate of  $\sigma$ , the estimated direct stimulation of equipment spending due to the credit varies much less than the variations in  $k$ . Thus even if  $k$  is assumed to be .10 rather than .05, a 100% increase, the estimated direct impact on equipment spending in 1965 increases by only 41%, from \$3.04 billion to 4.30 billion.

It would be very nice if the statistics in Table 20 could be used to construct a confidence interval for  $k$  (and for direct effects

TABLE 20

SUMMARY TABLE FOR ALTERNATIVE ASSUMPTIONS

ABOUT EFFECTIVE RATE OF TAX CREDIT

Assumed value of k	R <sup>2</sup>	$\overline{SEE}$	$\Sigma \hat{\epsilon}^2$	D.W. Statistic	Estimate of $\sigma$	Estimated direct Effect of tax credit (billions of 1958 dollars)		
						1963	1964	1965
.00	.98682	669.71	20.632	1.55	1.112	.00	.00	.00
.01	.98772	646.48	19.225	1.67	1.319	-.03	.29	.68
.02	.98848	626.13	18.034	1.78	1.305	-.04	.60	1.39
.03	.98898	612.37	17.250	1.85	1.224	.035	.94	2.05
.04	.98932	602.96	16.723	1.91	1.125	.131	1.25	2.61
.05	.98954	599.63	16.375	1.95	1.022	.219	1.50	3.04
.06	.98969	592.49	16.148	1.97	.959	.334	1.77	3.50
.07	.98978	589.89	16.007	1.98	.879	.409	1.95	3.80
.08	.98983	588.37	15.925	1.99	.808	.477	2.11	4.05
.09	.98986	587.57	15.881	2.00	.740	.519	2.21	4.20
.10	.98987	587.22	15.862	2.00	.676	.541	2.29	4.30
.11	.98987	587.16	15.859	2.00	.619	.554	2.34	4.36
.12	.98987	587.28	15.865	2.00	.573	.570	2.39	4.43
.13	.98986	587.52	15.878	2.00	.533	.582	2.44	4.47
.14	.98985	587.83	15.895	2.00	.498	.596	2.48	4.53
.15	.98984	588.18	15.914	2.00	.470	.614	2.53	4.59
.16	.98982	588.54	15.934	2.00	.440	.612	2.55	4.61
.17	.98981	588.93	15.954	2.00	.415	.618	2.57	4.62
.18	.98980	589.31	15.975	1.99	.394	.625	2.59	4.65
.19	.98978	589.70	15.996	1.99	.374	.633	2.62	4.68
.20	.98977	590.08	16.017	1.99	.356	.637	2.64	4.70

of the credit). But because of the nonlinearities in the model, it simply cannot be assumed that the ratio of the estimated error variance to the true variance will have anything like a chi-square distribution, and thus none of the standard tests is appropriate. Nevertheless, to give an intuitive feel for the size of the reduction in error variance which comes as a result of relaxing the assumption  $k = k^*$  (where  $k^*$  is an a priori value specified for  $k$ ), I have tabulated values of the ratio

$$F' = \frac{Q_1/1}{Q_0/45}$$

where  $Q_0$  is the sum of squared residuals when  $k$  is estimated along with the other parameters,  $Q_2$  is the sum of squared residuals conditional on  $k = k^*$ , and  $Q_1 = Q_2 - Q_0$ . I have labelled this ratio  $F'$  because it is computed in the same way as a statistic which would have an  $F$  distribution under the null hypothesis that  $k = k^*$ , in the general linear model.<sup>1</sup> The ratio  $F'$  for various values of  $k^*$ , is found in Table 21. Since the improvement in "fit" from relaxing my prior assumption, that  $k$  is approximately .05, does not seem terribly large, I have adopted the prior assumption. At the same time, both tables 20 and 21 show quite clearly that the time series examined provide little support for the hypothesis that  $k^* = 0$  (meaning that the tax credit

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<sup>1</sup>If the model were completely linear, this is exactly the ratio which would be computed, with 57 observations, to test the hypothesis that 1 of 12 coefficients took a specified value, while the other 11 were estimated without restriction. See Graybill [14], p. 133-140.

TABLE 21

F' STATISTICS FOR ALTERNATIVE ASSUMED VALUES OF k

<u>k*</u>		<u>F'</u>
.00	. . . . .	13.54
.01	. . . . .	9.55
.02	. . . . .	6.17
.03	. . . . .	3.95
.04	. . . . .	2.45
.05	. . . . .	1.46
.055	. . . . .	1.03
.06	. . . . .	0.82
.07	. . . . .	0.42
.08	. . . . .	0.19
.09	. . . . .	0.06
.10	. . . . .	0.01
.11	. . . . .	0.00
.12	. . . . .	0.02
.13	. . . . .	0.05
.14	. . . . .	0.10
.15	. . . . .	0.16
.16	. . . . .	0.21
.17	. . . . .	0.27
.18	. . . . .	0.33
.19	. . . . .	0.39
.20	. . . . .	0.45

has no effect at all). A substantial reduction in error variance can be achieved by adopting almost any alternative hypothesis which implies that  $k > 0$  .

The results suggest that the estimated price elasticity is not very robust to assumptions about k . But it should be noted that the regressions fitted to samples which did not include post 1962 data all produced estimated price elasticities which were close to 1.0. Still, on the basis of this evidence the possibility that highly visible policies such as the tax credit may have larger effects than would be produced

by an equivalent reduction in equipment prices cannot be ruled out. The key factor is the effect on expectations about eventual factor price changes, and this effect could occur in many ways.

Similar experiments with the accelerated depreciation parameter  $w$  have revealed that the model is almost totally insensitive to this parameter within the entire admissible range (0 to 1). With  $k$  set at .05, varying  $w$  over this entire range the maximum variation in the sum of squared residuals is only 1.3%. The best fit is for  $w = 0$ , while the worst is for  $w = 1$ . None of the estimates of the other parameters change by more than a few percent as  $w$  is varied.

The sensitivity of the likelihood function to  $w$  is so small that no firm conclusions about even the direct effects of accelerated depreciation can be derived from the data. Trying to estimate  $w$  empirically would be futile; no "significant" results could be obtained. But this also means that the data give no support (in this model) to the hypothesis that accelerated depreciation has any effect on investment at all! Of course, the data do not deny the hypothesis either -- they simply shed no light. Since my prior value for  $w$ , derived from analysis of Ture's data, is about .5, I conclude that I cannot reject this hypothesis, and it remains a part of the model. Some other researcher, whose null hypothesis was  $w = 0$ , could not reject that either. This result on the (statistically) insignificant effects of accelerated depreciation in a neo-classical aggregative model is essentially the same result Eisner [4] reports in his critique of Hall and Jorgenson. But the results on the tax credit are by no means so ambiguous. Given the evidence that



some tax incentives matter, this only strengthens my view that, with respect to other cases where the evidence is ambiguous, the presumption still should be that tax incentives make a difference. But it should be made clear that in the case of accelerated depreciation the conclusion is drawn on the basis of presumptions, not empirical evidence.

## V. Conclusions

The main conclusions of the empirical portion of this study include the following:

- (a) Relative prices affect equipment spending with a much longer lag than changes in output;
- (b) The long-run price elasticity of equipment spending is apparently close to one, but cannot be estimated with any great precision;
- (c) The investment tax credit of 1962 has probably directly stimulated more investment spending than the policy has cost the **government** in taxes, while the results of other fiscal policies are more ambiguous;
- (d) Variations in measures of the cost of capital seem to influence equipment spending in accordance with their theoretical role, but stable estimates of the effects of these variables have not been obtained;
- (e) The marginal ratio of equipment spending to output is sensitive to direct fiscal and monetary policy measures, and this ratio need not decline if government policies are not directed toward this goal.

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