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THE OPTIMAL LIFE OF A PATENT

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by

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One of the important peculiarities of information is that it is expensive to produce but cheap to reproduce. How does a price system react to this phenomenon? One reaction will be to avoid the production of information altogether. Alternatively, firms may decide to keep their information to themselves. In other cases the price system breaks down into imperfect competition. All three of these laissez-faire reactions are inefficient and it may be argued that it is desirable to impose social controls like a patent system or government control on the market for information. In the present paper we discuss some economic aspects of a patent system.

1. A Model of the Patent System: Run-of-the-Mill Inventions.

Patents are licenses for a monopoly over information. They are used, presumably, when mechanisms for producing information efficiently are not available. In other words, they are a kind of second

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best, or optimal feasible, solution to the problem of inducing the optimal rate of technological change.

Assuming that the patent system has been chosen as the proper instrument for promoting technological change, the question arises, what kind of patent system should be organized? What should be the life of a patent?[1/] Should there be compulsory licensing? Should there be fees?

We propose to examine the question of the optimal life of a patent in a simple model of patenting proposed by Arrow [1962]. It is assumed that the product market is competitive, and that potential investors can invest scarce resources to produce inventions. The inventor sets an arbitrary royalty for licenses on the invention. (Alternatively, the inventor may be a monopolist of the final product since the solution is exactly the same.)

We assume that there are constant costs of production both before and after the invention. The invention is used in the industry forever. There are enough industries or potential products so that any one invention does not disturb the demand or supply for inventions in the future.

Assume that the demand for all goods is described by the

1/ The problem of the optimal life seems to have escaped discussion in the literature, with the exception of Machlup ([1958], pp. 66 ff.). All arguments about existence of the patent system are really arguments about the length of life of patents since the abolition of a patent system means imposing a zero lifetime.

linear demand curve:

(1) $$Y = a - dp \;.$$

The cost of production (and thus the price) before the invention is c_0. A certain amount of resources (R) is invested in inventive inputs, giving a new process with cost $c_1 = c_0 - B(R)$. The conventional cost and demand curves are pictured in Figure 1.[2/]

Arrow has shown[3/] that for the linear demand curve the royalty will equal the area ABCD when GO/HO is greater than one-half, that is for inventions which are not drastic.

Assuming that the inventor sets the profit-maximizing royalty, he will set the per unit royalty (r) equal to the differency AB in Figure 1, that is, he will collect the entire cost reduction on the output OG. Therefore, <u>for the life of the patent</u> output will be reduced by GH from the level which would obtain if the invention were freely available. Under certain simplifying assumptions we can measure society's deadweight loss as CDE.[4/]

2/ If arc elasticity is $-d$, then $\Delta Q/Q = -d\Delta p/p$, so the requirement is that $d(c_1 - c_0)/c_0 < 1$. For elasticities in the neighborhood of unity this implies that cost be reduced more than 50 percent, certainly a drastic invention. Product inventions will be "drastic" under this definition. The theory is modified for drastic inventions in section 2.

3/ Arrow [1962], pp. 619-622.

4/ This result is well-known and is discussed in Samuelson [1948], Chapter 7.

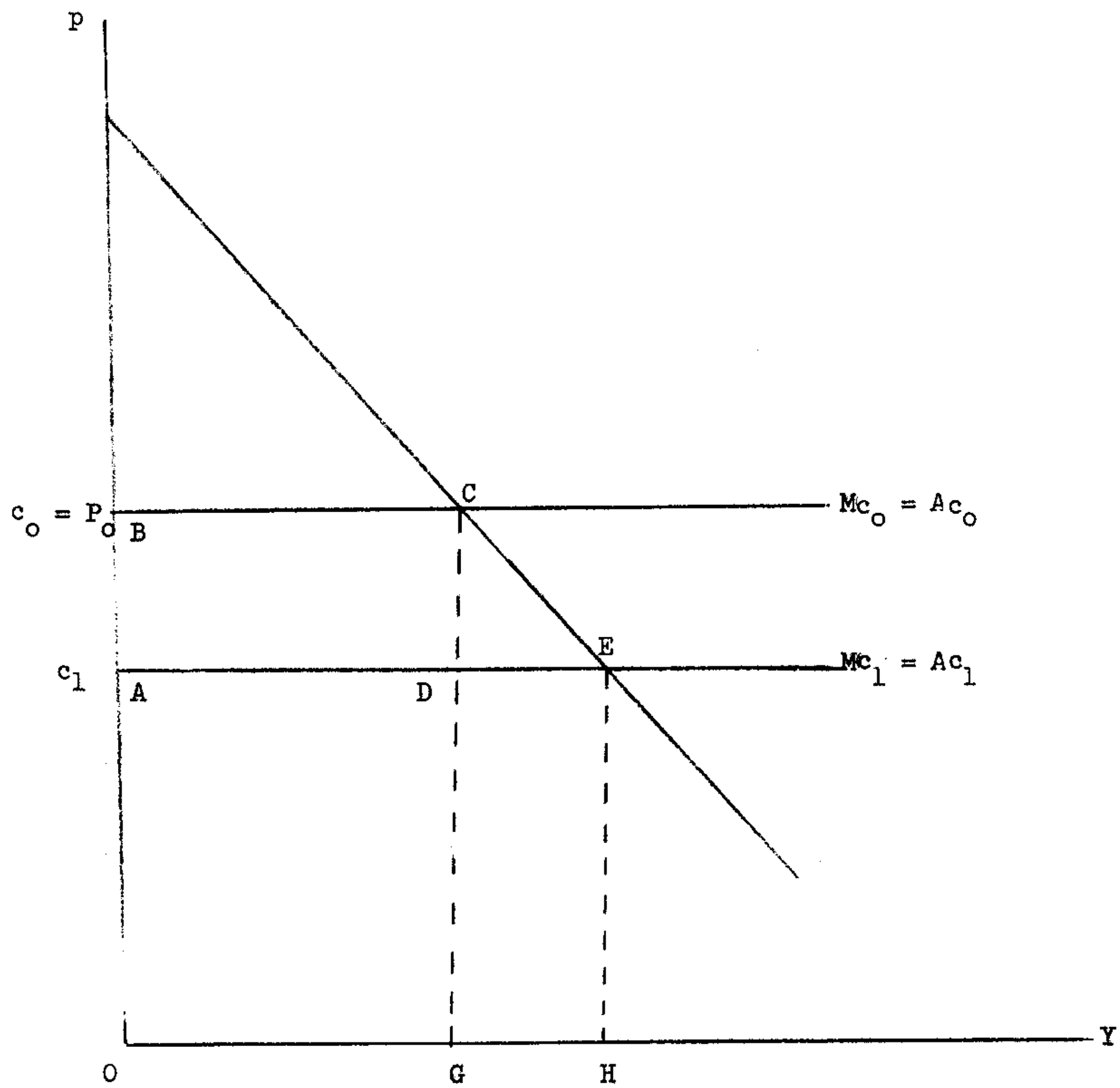

Figure 1

EQUILIBRIUM IN PRODUCT MARKET BEFORE AND AFTER INVENTION
(RUN-OF-THE-MILL PRODUCT INVENTION)

After the patent expires, the process will be used universally, price will fall to OA , and there will be no further deadweight loss.

We now want to examine the optimal life of the patent, T . In the absence of the patent it is assumed that no research will be performed since it is impossible to appropriate the invention. Therefore the gain to society is ABCD for all time and DCE from T on. Society's cost is the resources invested in research.

We can formalize the model as follows. The profit of the inventor is:

$$\pi(R) = \int_0^T Y_0 r e^{-\rho v} dv - sR \tag{2}$$

The royalty for run-of-the-mill inventions is given by

$$r = p_0 - p_1 = c_0 - c_1 = B(R) \tag{3}$$

where Y_0 is output before the invention, r is the unit royalty, R is the amount of research, s is the cost of research, and ρ is the discount rate. From now on s and ρ are to be interpreted as shadow prices. $B(R)$ is the "invention possibility function."[5]

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[5] $B(R)$ is a novel feature of the model, not present in Arrow [1962] or McGee [1966]. It is absolutely essential for determining the absolute life. We assume $B'(R) > 0$, $B''(R) < 0$ at least after a point.

Since inventors are profit-maximizing, they maximize (2) given the invention possibility function in (3) and the life of the patent, T . This implies

$$\pi'(R) = 0 = \int_{0}^{T} Y_o B'(R) e^{-\rho v} dv - s$$

$$= \frac{Y_o B'(R)[1 - e^{\rho T}]}{\rho} - s$$

Define $\varphi(T) = 1 - e^{\rho T}$. We thus have

(4) $$B'(R)\varphi Y_o = s\rho$$

Similarly the benefit to society from the invention is:[6]

(5) $$W = \int_{0}^{\infty} rY_o e^{-\rho v} dv + \int_{T}^{\infty} \frac{1}{2}(Y_1 - Y_o) r e^{-\rho v} dv - sR$$

From (1) we know

$$Y_1 - Y_o = -d(p_1 - p_o) = rd$$

So

(6) $$W = \frac{rY_o}{\rho} + \frac{r^2 d[1 - \varphi(T)]}{2\rho} - sR$$

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[6] We take welfare to be the sum of profits and consumer surplus. This assumption will give correct (Pareto-optimal) results when the marginal utility of income is strictly constant.

We now maximize W in (6) subject to the constraint in (4).[7] The Lagrangean is

$$(7) \qquad L = \frac{B(R)Y_o}{\rho} + \frac{dB(R)^2[1 - \varphi]}{2\rho} - sR + \lambda[B'(R)\varphi Y_6 - s\rho]$$

Differentiate (7) with respect to both φ and R :

$$(8) \qquad \frac{\partial L}{\partial \varphi} = - \frac{dB^2}{2\rho} + \lambda B'Y_o = 0$$

$$(9) \qquad \frac{\partial L}{\partial R} = \frac{B'Y_o}{\rho} + \frac{dB'B(1 - \varphi)}{\rho} - s + \lambda B''\varphi Y_o = 0$$

Solving for λ in (8) and (9) and equating:

$$(10) \qquad \frac{dB^2}{2\rho B'Y_o} = - \frac{B'Y_o + dB'B(1 - \varphi) - s\rho}{\rho B''\varphi Y_o}$$

Cancelling and recalling from (4) that $s\rho = B'(R)\varphi Y_o$:

$$-B''\varphi dB^2 = 2B'^2[Y_o + dB(1 - \varphi) - \varphi Y_o]$$

$$(11) \qquad \varphi[-B''B^2d + 2B'^2(dB + Y_o)] = 2B'^2(Y_o + dB)$$

7/ We maximize welfare subject to the constraint that inventors follow the rule in (4). This is equivalent to setting

$$\frac{dW}{d\varphi} = \frac{\partial W}{\partial \varphi} + \frac{\partial W}{\partial R}\frac{\partial R}{\partial \varphi} \qquad \text{equal to zero.}$$

We ignore the horrible second-order conditions.

Thus

$$\varphi = \frac{2B'^2(dB + Y_o)}{2B'^2(dB + Y_o) - B''B^2 d} \tag{12}$$

We convert d into an elasticity by setting $Y_o = p_o = 1$.

Then we have the maximizing T defined by:

$$\varphi(T) = \frac{dB + 1}{dB(1 + \frac{\sigma}{2}) + 1} \tag{13}$$

where

$$\sigma = -\frac{B''B}{B'^2}$$

and

$$T = -\frac{1}{\rho}\log(1 - \varphi) , \qquad 0 \leqq \varphi \leqq 1 , \qquad 0 \leqq T \leqq \infty \tag{14}$$

Thus the optimal life of the patent depends on the elasticity of demand (d) , the importance of the invention (B) , and the curvature of the invention possibility function (σ) .8/

8/ One of Alfred Marshall's rare slips seems to concern the problem of optimal life of a patent: "If it were possible to adapt the duration of each patent grant to its peculiar conditions, the public interest would call for a specially long period for patents relating to processes to which the law of Increasing Return applied strongly, but in which its effects are slowly developed." (Marshall [1919], p. 407). Exactly the opposite conclusion seems to apply, as can be seen by drawing a declining cost curve for the invention. Marshall's confusion probably came from the same error which led him to propose bounties for increasing returns industries.

The formula in (13) coincides roughly with what our intuition would tell us. Since σ is positive, we know that the optimal life is a finite, positive period. As the function $B(R)$ tends toward fixed proportions $(B'' \to -\infty)$ the optimal life tends to zero, while if there are no diminishing returns $(B'' \to 0)$ the optimal life becomes very large.

Other results can be shown which are perhaps less obvious. Note that for more important inventions the optimal length of life is shorter, since the importance of the second order dead-weight loss increases.[9/] Similarly, as the elasticity of demand for the good in question rises, the optimal life decreases. This last result is obvious in the limit, since for completely inelastic demand there is no deadweight loss.

One result of this model is that compulsory licensing can <u>never</u> be more efficient than a simple patent system. If there is a fixed maximum on fees, this will discourage any invention which has a greater reduction of cost, without any counteracting positive incentive. If there is a decision as to the "fair fee," this will act as a tax if the fair fee is less than the monopoly royalty and will

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9/ It is almost universally argued that more important inventions should have longer lives. (Unimportant inventions are, by statute, exempted from patent coverage. See the discussion in Machlup [1958], section 4-D.) To the extent that more important inventions are riskier or require longer periods of development, a longer life may be justified. But in general more important inventions involve larger second order effects and thus should have shorter lives.

This result holds for run-of-the-mill inventions examined here. The conventional wisdom is, however, correct for drastic process inventions as is shown in section 2 below.

have no effect otherwise. As will be shown below, a tax always leads to a less efficient solution. Therefore, compulsory licensing cannot increase efficiency.

By a similar argument it can be seen that instituting annual fees to keep the patent alive, as is done in many European countries, decreases the efficiency of the system.

2. Drastic Inventions

The second kind of invention is what we will call "drastic" inventions. A drastic invention is one where the royalty is determined on the downward-sloping part of the derived-demand-for-the-invention curve. This will happen with linear demand functions whenever output is more than doubled. By convention we say that the output of a previously unproduced good is more than doubled, so product inventions are drastic. Figure 2 shows the product market and derived demand for a drastic invention. JCE is the product demand, with constant average cost curves BC and AE for before and after the invention, respectively. DEF is the derived demand for the patented invention, with DEGH its marginal revenue. Since the inventor behaves like a monopolist, with no marginal cost, he will set $MR = 0$, which occurs at H. This gives equilibrium royalty of OR and equilibrium price OK.

It is easily verified that OR is exactly one-half of JA

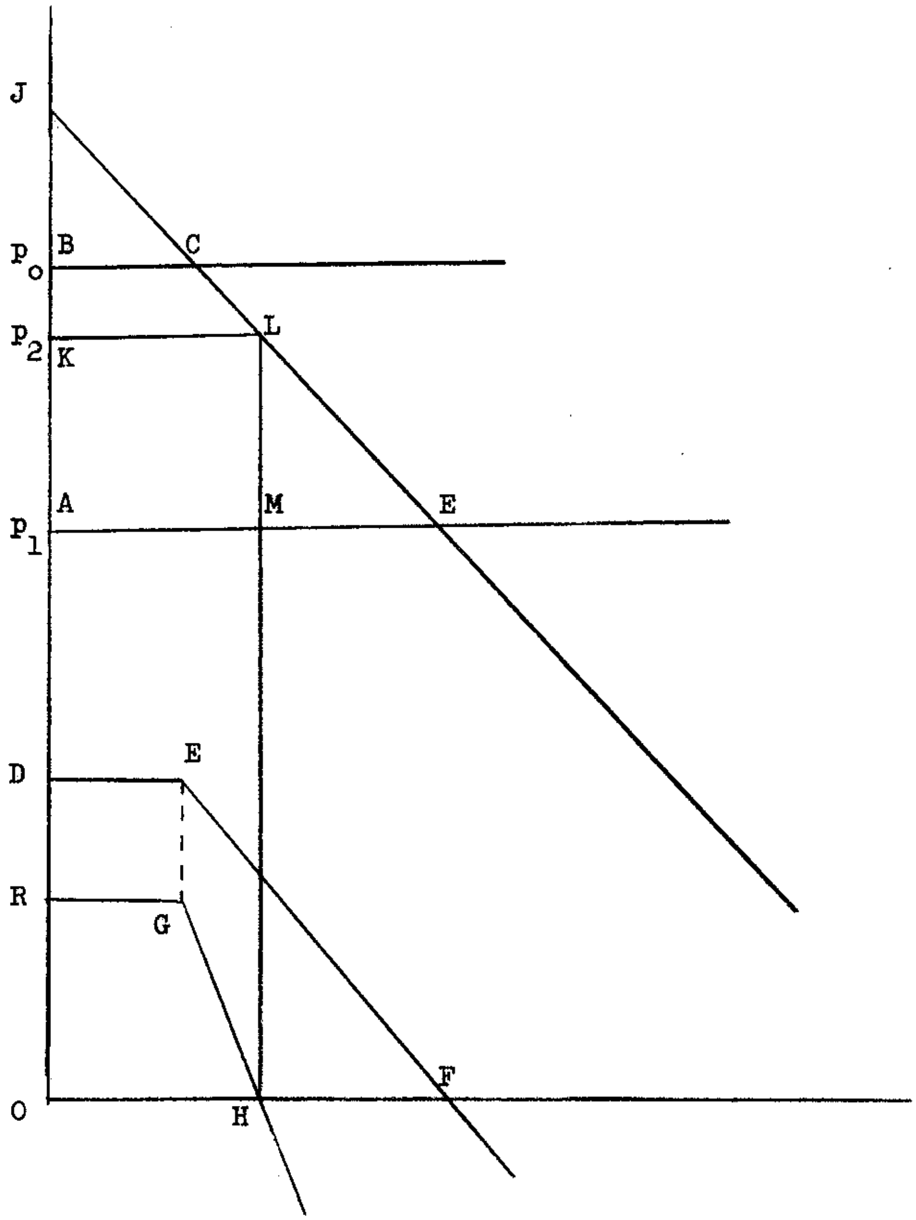

Figure 2

EQUILIBRIUM IN THE PRODUCT MARKET BEFORE AND AFTER INVENTION,

DRASTIC INVENTION

(which shows why the definition of drastic invention given above is the correct one).

The profit of the inventor is:

$$\pi(R) = \int_0^{T_1} \frac{1}{2}(a - p_1)[a - d(a - \frac{1}{2}\{a - p_1\})]e^{-\rho v}dv - sR .$$

Since $p_1 = p_o - B(R)$,

$$\pi(R) = \frac{1}{2}\int_0^{T} (a - p_o + B)(a - \frac{1}{2}da + \frac{1}{2}dB - \frac{1}{2}dp_o)e^{-\rho v}dv - sR .$$

Thus the necessary condition for the inventor's maximum is:

$$\pi'(R) = 0 = \frac{\varphi B'}{4\rho}(2a - da + dB - dp_o + da - dp_o + Bd) - s .$$

or

$$(15) \qquad 2\rho s = \varphi B'(a - dp_o + dB)$$

The benefit to society is given by

$$W = \int_0^{\infty} AJE - \int_0^{\infty} BJC - \int_0^{T} MEL - sR$$

$$= \int_0^{\infty} \frac{1}{2}(a - p_o + B)[a - d(p_o - B)]e^{-\rho v}dv$$

$$- \int_0^{\infty} \frac{1}{2}(a - p_o)(a - dp_o)e^{-\rho v}dv$$

$$- \int_0^{T_1} \frac{1}{8}(a - p_o + B)[a - d(p_o - B)]e^{-\rho v}dv - sR .$$

Since the second term is a constant we can ignore it and write the Lagrangean:

$$L(R, \varphi) = (a - p_o + B)(a - dp_o + dB)\left[\frac{1}{2\rho} - \frac{\varphi}{8\rho}\right] - sR + \lambda[\varphi B'(a - dp_o + dB) - 2\rho s]$$

To simplify, define

$$u = a - p_o + B$$

$$w = a - dp_o + dB$$

which gives

$$L = \frac{(4 - \varphi)}{8\rho} uw - sR + \lambda[\varphi B'w - 2\rho s] .$$

Maximizing with respect to R and φ :

$$\frac{\partial L}{\partial R} = \frac{(4 - \varphi)}{8\rho} B'(ud + w) - s + \lambda\varphi(B'^2 d + B''w)$$

$$\frac{\partial L}{\partial \varphi} = - \frac{uw}{8\rho} + \lambda B'w$$

Eliminating λ :

$$- \frac{(4 - \varphi)B'(ud + w) - 8s\rho}{8\rho\varphi(B'^2 d + B''w)} = \frac{u}{8\rho B'}$$

Solving for φ and using (15):

$$\varphi = \frac{4B'^2(ud + w)}{w(5B'^2 - B''u)}$$

$$= \frac{4(ud + w)}{w\left(5 - \frac{B''u}{B'^2}\right)}$$

We see that $-\frac{B''u}{B'^2} = \frac{\sigma u}{B}$, so

$$(16) \qquad \varphi = \frac{4B\left(1 + \frac{du}{w}\right)}{5B + \sigma u} .$$

or, using the convention $p_o = Y_o = 1$, note that for process inventions $u = d + B$ and $w = 1 + dB$

$$(17) \qquad \varphi = \frac{4B(1 + dB + d^2)}{(1 + dB)[5B + \sigma(d + B)]}$$

This holds only for process inventions. For product inventions $Y_o = 0$, and we get the following expression for the optimal life:

$$(18) \qquad \varphi = 4B \frac{2 + \frac{a(d - 1)}{a - dc_o + dB}}{\sigma(a - c_o + B) + 5B} .$$

where c_o is the original cost of production.

How does the optimal life of drastic inventions compare with the optimal life for run-of-the-mill inventions? The general form of (17) and (18) makes interpretation rather difficult. The following comments come from the numerical analysis described in section 4.

Equation (17) gives the length of life for drastic process inventions. One rather surprising result is that the optimal life generally rises with the importance of the invention. This is puzzling in light of the opposite relation for run-of-the-mill inventions.

The rationale seems to be that for drastic inventions half the decrease in cost is passed on in lower prices during the life of the patent, while for run-of-the-mill inventions decreases in cost do not affect price at all. It is, thus, not necessary to wait until the patent expires to reap the benefits of technological change for drastic inventions.

A second difference from the result for run-of-the-mill inventions is that in certain circumstances the optimal life may be infinite. This can be seen by substituting $d = B = 1$ and very small σ into (17). The reason for this is that when inventions become very important the incentive effect of lengthening the life is more beneficial than the deadweight loss.[10/]

Equation (18) gives the length of life for product inventions. The pattern of optimal life for this type is not at all regular and discussion will be deferred to section 4.

3. Problems with the Analysis.

The theory we have been outlining has innumerable problems, which may perhaps vitiate the results. On the other hand, it is fair to say that even the problems of the model illustrate some of the

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10/ It is in this case that the static character of the model produces the most unacceptable results. With a progressive technology, giving perpetual patents is a dubious policy.

more fascinating questions in the economics of knowledge.[11]

1. We have assumed that the total supply of inventions is unaffected by the rate and importance of invention. This assumption violates one's intuition on the importance of prior knowledge in producing more knowledge.

This particular problem has both a "local" and a "global" aspect. We will first discuss global interdependencies of knowledge. The problem is that as the rate of invention or knowledge production increases, more (or possibly less) inventions can be produced at less (or possibly greater) cost. The usual reasons for this dependency is that production of knowledge as an output depends on the existing stock of knowledge as an input. As the stock of knowledge increases this affects the productivity in making new knowledge in two ways. First, there is a technical complementarity, which we will call the imitation effect, which increases the productivity of other resources. Second, there is a technical substitution, or rivalry, we will call the depletion effect, which reduces productivity; this depletes the

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[11] We will not include in the list standard sorts of economic questions, as follows: (i) We have assumed that the marginal utility of income is constant; (ii) Do inventors (or firms) maximize profits; (iii) How do individuals behave in the presence of massive uncertainty (although see footnote 30); etc.

number of potential theorems-to-be-proved by proving one.[12]

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[12] We can formalize this by writing the following cost function for research for an inventor:

$$C_1 = G(R_1; R_2, .., R_n; R_{n+1}, .., R_m) .$$

The function G gives the cost of production to the first inventor of an invention. This is a function of his research (R_1). In addition, people working on "similar" inventions affect his productivity $(R_2, \ldots, R_n)$. Then there is the group working on research in general $(R_{n+1}, \ldots, R_m)$. The second through m^{th} terms reflect external economies.

Now we usually assume that research is always complementary, so that $G_i > 0$, $i = 1, \ldots, m$. Raising the level of research globally (i.e., for R_2 through R_m) lowers the cost of a given invention; this is the imitation effect and most generally be favorable.

On the other hand, inventors are not merely interested in producing knowledge. They want to produce increases or advances in knowledge. Since other research may increase what is known prior to an invention, the advance may be slimmer than originally anticipated. It is in this sense that other research is rival to our research; it may deplete the number of theorems to be proved.

Let us make this more concrete. Before two inventors get to work, assume that pencils can be produced at 5 cents a piece. The first inventor can invent a process for producing pencils at 3 cents by investing $1,000 in research. A second inventor, however, invests $500 and gets a process for 4 cents per pencil. The first inventor now finds it would only cost him $800 to invent his process. The reduction of cost of $200 is the imitation effect, but the fact that the invention saves only 1 cent (instead of 2 cents) is the depletion effect.

Some of the pathologies associated with the economics of information are due to very strong imitation effects. The fact than an invention, once produced, is economically a free good means that the cost of producing (i.e., reproducing) it is negligible.

The depletion effect is in reality a pecuniary (but not technological) external diseconomy, whereas the imitation effect is both a technological and a pecuniary external economy.

The long footnote is necessary for an understanding of interdependence. We can readily see that the imitation effect of general (or basic) research may be strong for both research which is closely related and that which is unrelated. On the other hand, the depletion effect is usually negligible except for research which is closely related.

Global interdependence (we will call this "global spillover") formally means that the costs of performing any invention depend on the general level of research. We would expect that globally the imitation effect would outweigh the depletion effect; thus there would be technological external economies to increasing to general level of inventive activity.[13]

The presence of externalities from global spillover will generally mean that the optimal life as calculated above is understated. (It is obviously not correct to say that if patented research activity is small then the understatement is small.) The conceptual framework for handling externalities of this nature has been analyzed elsewhere,[14] so it will not concern us here. Merely note that in some sense the spillover is closely related to how perfectly the patent system allows appropriation. If an invention can really be appropriated, in the sense that the ideas can be completely controlled, then global spillover is curtailed. Thus, if a patent controls not only the mousetrap, but the concept of spring traps, the possible spillover

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[13] See Nelson, et.al. [1967], p. 77.

[14] See Nordhaus [1967], Chapter 3.

will be much smaller. This is the important issue of the optimal "breadth" of a patent, to which we return in point 4 below.

2. So much for global spillovers. Local spillovers are much more complicated in their effect; these are interactions between inventors working on technically similar problems. Whereas with global spillovers there is a presumption that imitation effect outweighs depletion effect, the opposite may be true for local spillovers. How unfortunate for economists that there is but one general equilibrium to discover, which has been "depleted" since 1881.

The real questions about patent systems are involved with questions of local spillovers: these are the issues of competing patents, inventing around patents, suppressing patents, and overlap of patents. We will take these up in turn.

3. Competing patents are the center of many questions involving a patent system. If several inventors are working on the same project, are producing the same ideas, but the earliest gets the patent, we can say two things: there is a duplication of effort and society got the invention. Somehow it would be desirable to avoid the duplication, especially if it was caused by a patent system and if the invention would have been forthcoming anyway.

This is unfortunately vague, but the process of competing patents is not easily described. There seem to be three kinds of interactions between competing inventions or inventors, all of which are peculiar to inventions which are "close" to each other. The first interaction is the pecuniary external diseconomy discussed above, the depletion effect. If a process is invented and provided free of charge, lowering costs of production by X dollars, then (for non-drastic inventions) the royalty rate on any new invention is reduced by X dollars.[14a]

What is the implication of this bizarre form of pecuniary external diseconomy for the optimal life? The answer is unclear. If existing prices reflect true social costs and inventors' foresight is accurate, then this pecuniary diseconomy would not appear to bias the optimal life; this is the standard argument about the efficiency of a price system. On the other hand, the essence of the economy under examination is that prices do not represent social costs due to the inappropriability of information. It must merely be an assertion, therefore, to claim that the depletion effect does not bias the optimal life.

A second interaction is the race to the patent office, the "horserace effect." People working on an invention are racing against others with the same idea. One obvious result is to have time enter strongly in the decisions, but it is not clear how this affects the research. A second result is that the expected value of the invention

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14a/ What if there are two patents (n patents) which are perfect substitutes in production, but each is legally valid, competing for the derived demand for technology? We have duopoly (n - opoly) with all its indeterminancies. We thus assume a competitive patent market, or that an old patent has expired.

(the private value) is less than its discounted royalties. If on the average a company finds that the proportion $q < 1$ of its patents are first in line at the Patent Office the first term in (2) should be multiplied by q. This obviously lowers the optimal R. On the other hand, since there is duplication of effort the social cost is greater. If there are n competing inventors, then the social cost in (5) is nsR rather than sR. The net effect of this is very surprising. If all companies are equally successful and each arrives at the Patent Office $1/n^{th}$ of the time, then we have the truly remarkable result that the horserace effect completely washes out.[15/] The horserace is about the purest kind of depletion effect one can imagine, but it does not change the optimal life at all.[16/] This strengthens the presumption that the depletion effect does not distort the optimal life.

A third kind of local interaction is where a higher rate of invention shortens the economic life of patents. In this case, royalty rates go to zero before the patent expires. This is really

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15/ This can be demonstrated as follows: (4) becomes $qB'(R)Y_o = s\rho$, while the third term in (7) is nsR. Working through the equilibrium gives:

$$\text{(i)} \quad \varphi = \frac{bB + 1}{bB(1 + \frac{\sigma}{2}) + nq}$$

If the leadtimes are distributed evenly, then $q = 1/n$ so (i) becomes precisely (13).

16/ Note that the horserace effect differs from the first effect since only one of the inventions of the former kind is valid while all the latter are.

a special subcase of another kind of model of the optimal life.[17]

4. The next difficulty with the model is part of one of the most fascinating questions in the economics of information: the optimal "breadth" of a patent. The breadth of the patent is the amount of the information in an invention which the patent allows the inventor to appropriate. This leads to questions like the following: Should the inventer reap the reward from imitations in other fields, or from ideas inspired by the invention? Should there be protection

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[17] More specifically, we have assumed that no technological progress occurred during the life of the patent. A more realistic assumption is that the competitive costs of production decline continuously, say according to $p_o(t) = p_o e^{-gt}$. If initial royalty is r_o , then $r(t) = r_o + p_o(e^{-gt} - 1)$. Although the complete solution has proved intractable, it appears that the life of the patent will always be less than the economic life (i.e., $T < \frac{1}{g} \log \frac{B(R)}{p_o}$) .

What then is the effect when a global change in the patent laws increases the amount of resources devoted to research and shortens the economic lifetime of inventions (this seems intuitively very plausible). The answer seems to be that it shortens the optimal life. The depletion effect in this case means that there is an upward bias in our formulas.

In certain industries the rate of invention is so high that the economic life of an invention is shorter than the legal life of the patent. Comanor [1964] presents data indicating that in the ethical drug industry the maximum impact of a patent on sales is in the second year after its introduction; after about five years the impact has declined sharply. (Comanor [1964], Table 2, p. 376.) In such an industry the length of life would have to be drastically shortened to affect the research effort. The considerations used here would indicate that the optimal life in the industry should be much smaller than at present.

against "patenting around" inventions? Since the breadth of a patent determines how much of the new knowledge is appropriable, it is appealing to assert that the breadth of patents on all inventions or discoveries should be approximately the same. Yet by statute discovery of laws of nature are specifically excluded from patents (zero breadth), while many mechanical inventions would appear to be almost completely appropriable (large breadth).

This question is closely related to the question of the "optimal externality."[18/] Since internalization has costs (in this case a monopoly on information) it is not always the optimal feasible policy to completely internalize the externality since this would lead to more deadweight loss than necessary.[19/]

A simple example of the breadth question can be built into the model used earlier. Let us assume that patent authorities control the amount of spillover from the patent, given by the breadth parameter, θ . If the invention lowers cost from c_o to c_1 we assume that after the invention the proportion θ spills out from the invention so that the freely available technology has cost $c = c_1 + \theta(c_o - c_1)$. If $\theta = 0$ (no breadth) then all the invention spills over, and conversely.

18/ The literature on "optimal externality" refers to the distribution of the gains or losses from internalizing externalities. See for example Dolbear [1966]. AER March 1967

19/ This is obvious in the discussion of the optimal life. To internalize completely means to make the life of the patent infinite.

In Figure 3 if cost falls from c_o to c_1, then competitive cost falls by $(1 - \theta)(c_o - c_1)$.

What is the optimal breadth of a patent? Curiously, there is no determinate answer, except that $\varphi\theta$ equals a constant.[20/] The addition of a second policy tool does not lead to an increase in welfare because the two maximum conditions are linearly dependent. Therefore there is no unique optimal breadth of patents.[21/]

A final note is in order. It is probably true that if the inventor can discriminate perfectly, then the solution is for infinite life. The perfectly discriminating solution is Pareto-optimal. With this consideration in mind, it might be asserted that price discrimination in patents is not a bad idea, as is commonly assumed.

[20/] We show this for run-of-the-mill inventions only. Take (2) and note that $r = \theta B(R)$, so that (4) becomes (4*)

$$\varphi\theta B'Y_o = s\rho \quad (4^*)$$

The Lagrangean, (5*), includes a term which takes into account the gain in welfare in period (0, T) due to lower price:

$$L = \frac{BY_o}{\rho} + \frac{dB^2(1 - \varphi)}{2\rho} + \frac{\varphi(1 - \theta)dB^2}{2\rho} - sR + \lambda[B'\varphi Y_o\theta - s\rho] \quad (5^*)$$

It can be shown that the equilibrium reduces to the single equation; $\varphi\theta = (bB + 1)[bB(1 + \sigma/2) + 1]^{-1}$, which is closely related to (13). It is interesting to note that the addition of another policy variable does not allow any increase in welfare.

[21/] We can also raise the question of the "third best," that is, what the optimal breadth should be when the life cannot be changed. It can be shown from the equilibrium that if the life is set for average inventions, important inventions should have a smaller breadth than average and small inventions should have larger breadth. The latter is clearly not the case in the present patent laws.

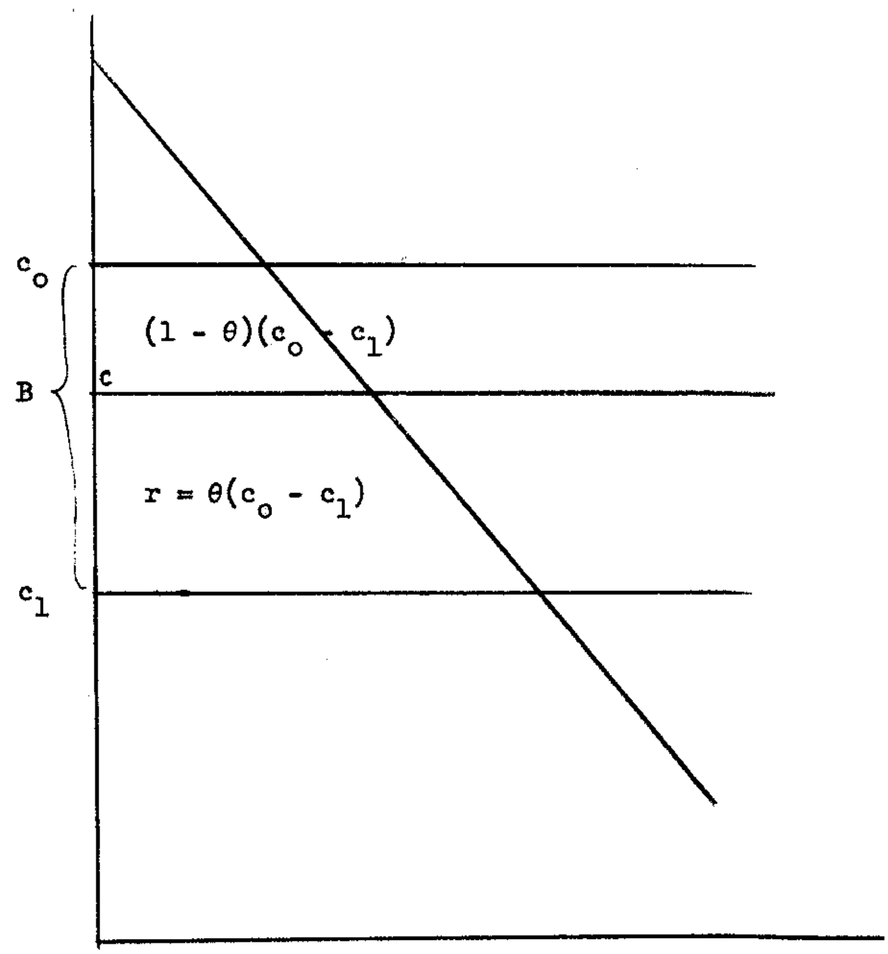

Figure 3

EXTERNAL ECONOMICS DETERMINED BY BREADTH OF PATENTS

4. Empirical "Estimates" of the Optimal Life.

Although the model developed in this paper can stand on its own, we are ultimately interested in applying the model to estimate the optimal life in the real world. Given the knowledge of the parameters of the system, it is more than modesty to note that the results are fanciful. One may therefore interpret the empirical results as somewhere between firm estimates and numerical examples.

The variables which determine the optimal life are b, ρ, B, and σ. The first three of these pose no great difficulties in estimation: elasticity of demand (b), discount rate (ρ), and reduction in per unit cost for the invention (B). The parameter σ, representing the curvature of the invention function, is probably impossible to estimate with much confidence.

It is perilous to guess a plausible value for σ. Many authors have used the functional form $B(R) = \beta R^{\alpha}$ to show the relationship between inventive inputs and productivity growth.[23] This seems to be a reasonable kind of function both from the relationship of research of size of firm and the research-output ratio.[24] The value of σ can be calculated as follows:

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[23] The use of a log-linear function has been canonized by Griliches [1964], Mansfield [1965], Minasian [1962], and others. Suffice it to lament that the real world has not read these works.

[24] See Nordhaus [1967], section 3-C.

$$\sigma = - \frac{BB''}{(B')^2}$$

(19) $$= - \frac{\beta\alpha(\alpha - 1)R^{\alpha-2}\beta R^{\alpha}}{\beta^2\alpha^2 R^{2\alpha-2}}$$

$$= \frac{1 - \alpha}{\alpha}$$

Thus σ represents the ratio $(1 - \alpha)/\alpha$ where α is the elasticity of output with respect to research. Empirical work mentioned in footnote 24 gives a value of $\alpha = .10$ as most likely, making $\sigma = 9$.

There has been no examination of the average value of B (the percentage cost reduction of the invention), although this task would pose no serious analytical difficulties. As a guess, it would appear that since there are a large number of inventions issued annually, the average reduction in cost would be small. We take a range around $B = .05$ as plausible. Note that this makes most inventions except product inventions "run-of-the-mill" inventions.[25/]

We take the value $b = 1.0$ as an average value for the elasticity of demand.

The final parameter is the discount rate, ρ . The appro-

25/ For an excellent discussion of patent statistics, see Schmookler [1966].

priate level for this would seem to be about 0.20 (that is, 20%).[26]

We have listed the optimal life for inventions under the specified assumptions in Tables 1 through 3. Table 1 gives the optimal life (T) for run-of-the-mill, process inventions. We see that the optimal lives for run-of-the-mill inventions are not too far from the life under United States patent laws. If we are to take 1.0 for the usual elasticity of demand, the optimal life ranges from 27 years for really trivial inventions $(B = .001)$ to 1.5 years for the most important $(B = 1.0)$. If we think $B = .05$ is an average invention, the optimal life is about nine years. This value is moderately sensitive to changes in the size of the invention or the elasticity of demand. Numerical evaluation shows that for $B = .05$ and $d = 1.0$, the optimal life is moderately sensitive to changes in σ, with $T = 5.6$ when $\sigma = 20$ and $T = 12.2$ when $\sigma = 4$.

One might conclude from the present analysis that the actual life for run-of-the-mill inventions is longer than is optimal. It must be emphasized that this is a highly uncertain estimate.

The results are much more satisfactory for drastic process inventions. Surprisingly, the optimal life turns out to be quite insensitive to changes in any of the parameters.

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[26] Most estimates of the (real) social rate of return are in the vicinity of 20% (see Eckaus and LeFeber [1962], Solow [1965], Chapter 3, or Phelps and Phelps [1966]). If the inventor has a required rate of return $\rho^* \neq \rho$, then the present analysis must be modified slightly, as is shown in footnote 30 below.

TABLE 1

OPTIMAL LIFE OF PATENT FOR RUN-OF-THE-MILL PROCESS INVENTIONS

Value of d \ Value of B	.001	.01	.02	.05	.10	.20	.50	.75	.90
.25	34.0	22.5	19.1	14.7	11.6	8.7	n	n	n
.50	30.5	19.1	15.8	11.6	8.6	6.2	3.7	n	n
.75	28.5	17.1	13.9	9.8	7.2	5.0	2.9	2.4	n
1.0	27.0	15.8	12.6	8.7	6.2	4.2	2.5	2.0	1.9
1.5	25.0	13.9	10.8	7.2	5.0	3.4	2.1	1.7	1.6
2.0	23.6	12.6	9.6	6.2	4.2	2.9	1.8	1.5	1.4
4.0	20.2	9.6	6.9	4.2	2.9	2.0	1.4	1.2	1.2
10.0	15.8	6.2	4.2	2.6	1.8	1.4	1.2	1.1	1.1

n = Not applicable

Notes to Table 1. We have assumed the rate of discount $(\rho) = .20$, $\sigma = 9.0$, and that fiscal policy is neutral (in the sense of section 5). The cells labeled n refer to the fact that these are drastic inventions, for which the optimal life is given in Table 2 . The optimal life is calculated according to equations (13) and (14) in the text.

TABLE 2

OPTIMAL LIFE OF PATENT FOR DRASTIC PROCESS INVENTIONS

Value of d \ Value of B	.25	.50	.80	.90	1.0
.10	1.9	1.4	1.4	1.5	1.5
.25	1.0	1.3	1.4	1.5	1.5
.50	n	1.2	1.4	1.4	1.4
.75	n	n	1.4	1.4	1.4
1.00	n	n	n	n	1.4
2.00	n	n	n	n	n

n = not applicable

Notes to Table 2. Assumed values for other parameters are that $\sigma = 9$ and $\rho = .20$ and that fiscal policy is neutral. The optimal life is calculated according to equation (18) and (14). Drastic inventions imply that $B > d$, so for elasticities of demand around 1.0, process inventions will almost never be drastic.

TABLE 3

OPTIMAL LIFE OF PRODUCT INVENTIONS

For $a = 1.0$

Value of d \ Value of B	.005	.01	..10	.50	.80	1.0
.01	1.7	1.7	1.7	1.7	1.7	1.7
.10	1.7	1.7	1.7	1.8	1.8	1.8
.50	1.7	1.7	1.9	2.4	2.6	2.8
1.0	4.2	4.2	4.2	4.2	4.2	4.2
2.0	1.7	1.6	1.2	4.2	∞	9.7
10.0	1.7	1.7	1.4	0	∞	∞

For $a = 0.80$

Value of d \ Value of B	.50	.80	.95	1.0
.01	1.9	2.1	2.0	2.0
.10	2.1	2.2	2.2	2.2
.50	2.8	3.3	3.3	3.4
1.0	5.3	5.7	5.4	5.3
2.0	0.0	∞	∞	20.5
10.0	0.5	0	∞	∞

Notes to Table 3. For this table it is assumed that $\rho = .20$, $\sigma = 9.0$, and that fiscal policy is neutral. The values given above represent the solution to equation (18) in the text. Recall that for a product invention to be economically feasible, $B > c_o - a$ (which explains why we only examine $B > .2$ in the second half of the table). We have normalized by setting $c_o = 1$.

Even more surprising is the fact shown in Table 2 that the optimal life for drastic process inventions seems to be very small, in the order of one-tenth of the actual life of patents. The reason for the very small life seems to be that drastic inventions are very important inventions and thus have a great deal of potential deadweight loss if they have long life.

A final pattern of optimal lives appears for product inventions, as is shown in Table 3. Recall that the parameter a gives the intercept of the demand function (after the cost of production before the invention is normalized at 1). The pattern of lives does not make a great deal of sense. It is easily seen that for relatively inelastic demand the optimal life is very short; on the other hand, when the elasticity of demand is greater than unity weird patterns appear and no general rule seems to apply.[27/]

5. Fiscal Incentives for Invention.[28/]

There are several ways to influence inventions in a patent system. We have so far discussed but two, the life and breadth of the patent. It is also possible to increase the return of inventors (and thus the level of invention) by giving invention tax credits or by subsidizing inputs. We discuss this problem briefly in the present section. Fiscal measures in the present model can be intro-

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[27/] A third kind of life of interest for product inventions is that where the product had very high cost to begin with. Taking the limit in (18) as c_o tends to infinity shows the optimal lifes to be zero.

Although we do not have an explanation for the strange results in Table 3, it may be that second order conditions are not assured for $d > 1$. It is easily verified that the second order conditions for a social optimum -- that is, the Lagrangean, not the profit function -- are very messy. This could account for the bizarre results in Table 3.

[28/] This section deals with fiscal incentives for run-of-the-mill, process duced by rewriting (2):

$$(2^*) \qquad \pi(t) = \int_0^T (1 + \delta)Y_o re^{-\rho t}dt - (1 + \gamma)sR \qquad \delta \geq -1$$

$$\gamma \geqq -1$$

Here $-\gamma$ is the subsidy on research inputs and $-\delta$ the tax on revenue. If $\gamma = \delta$ then $-\gamma$ is the profits tax. By going through the same procedure we see that the Lagrangean in (7) becomes:

$$(19)\ L = \frac{B(R)Y_o}{\rho} + \frac{dB(R)^2(1 - \varphi)}{2\rho} - sR + \lambda[B'(R)\varphi Y_o - s\rho\frac{(1 + \gamma)}{1 + \delta}]$$

By differentiating (19) with respect to the fiscal parameters we get:

$$(20) \qquad \frac{\partial L}{\partial \gamma} = - \frac{\lambda s\rho}{1 + \delta} < 0$$

$$(21) \qquad \frac{\partial L}{\partial \delta} = \frac{\lambda s\rho(1 + \gamma)}{(1 + \delta)^2} > 0$$

and the optimal life is given by

$$(22) \qquad \varphi(T) = \frac{bB + 1}{bB(1 + \frac{g}{2}) + k}, \qquad k = \frac{1 + \delta}{1 + \gamma}$$

Clearly the optimum is unaffected by a profits tax (assuming, of course, that ρ does not change).

The present fiscal policy toward research has been described elsewhere.[29] It was there shown that for corporations $\delta = -.48$ and $\gamma = -.58$. Therefore, for corporations, $k = 1.20$. Table 4 shows the optimal life for different fiscal parameters (with $\rho = .20$ and $\sigma = 9$). The optimal life under the present tax structure appears to be considerably less than 17 years.

By examining (20) and (21) we can determine the optimal tax policy. The maximum of L occurs when $\lambda = -1$ and $\delta = \infty$, or for $k = \infty$ -- which gives the optimal life to be zero. We thus know that when there are no constraints on fiscal incentives the optimal patent policy is to have a very high invention tax credit and subsidy on inputs and a very short life. In the limit, the deadweight loss is zero and Pareto efficiency is reached. Since a royalty is charged only for an infinitesimal period of time this solution is in accord with the conventional theorem that the efficiency price of information may be zero.

It is obviously impossible to obtain the optimal policy. A more realistic policy might be to give tax shelter to inventors. Thus if royalties from patents are tax-free, but costs continue to be deductible, then this is equivalent to $\delta = 0$ and $-\gamma =$ the tax rate. For corporations with $-\gamma = .48$, this implies a value of k of about 2. In this case the optimal life is about three

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[29] See Nordhaus [1967], section 3-C.

TABLE 4

OPTIMAL LIFE OF PATENT, IN YEARS,

WITH DIFFERENT FISCAL INCENTIVES

Elasticity of demand (b)	Fiscal Parameter (k) 1.0	1.2	2.0
0.25	14.7	8.0	3.3
0.5	11.6	7.3	3.2
1.0	8.7	6.2	3.1
2.0	6.2	5.0	2.8
10.0	2.6	2.4	1.9

Notes to Table 4. We have assumed that $\rho = .20$, $B = .05$, and $\sigma = 9$. The optimal lives are calculated according to equations (22) and (14). The fiscal parameter k is to be interpreted as follows: $k = 1.0$ is fiscal neutrality; $k = 1.2$ gives the fiscal parameter under the present current write-off for R and D ; $k = 2.0$ gives the value for corporations when returns from inventions are tax-free.

years. For individual inventors, $-\gamma$ would probably be less than .48 for unimportant inventions and higher for important inventions, and the optimal life would be longer or shorter, respectively.[30]

A final policy for increasing efficiency would be for the government to purchase the rights to inventions after patents have been issued. This would in theory, completely eliminate inefficiency. The policy is especially useful for important inventions, where the deadweight loss is significant.

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[30] A few final comments on other problems are in order. Suppose that inventors are risk averse and require a risk premium on invention. Let ρ^* be the required rate of return on invention and ρ the social rate of return. From Mansfield's work [1964] and [1965] it would appear that ρ^*/ρ might be as high as 2. It is easily verified that this is equivalent to a $k = 1/2$. Thus even under favorable fiscal conditions, say where there was a tax shelter, the optimal life would be in the range of thirty years for $B = .05$. If there are discrepancies between private and social marginal product for other reasons they can be handled in the same way.

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