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A NOTE ON TAXATION, INCREASING RETURNS DUE TO
SET-UP COSTS AND COMPETITIVE EQUILIBRIUM

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A NOTE ON TAXATION, INCREASING RETURNS DUE TO
SET-UP COSTS AND COMPETITIVE EQUILIBRIUM*

by
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1. The Problem

Suppose that a closed economy has all of the properties of convexity of preference sets and production sets except that in order to use certain production processes set-up costs must be paid. The set up costs cause increasing returns and destroy the competitive equilibrium; is there a tax policy which restores the equilibrium and would it always be approved unanimously or by a majority vote?

An example of increasing returns is provided by:

$$y = \max [\sqrt{x} - \sqrt{a}, 0]$$

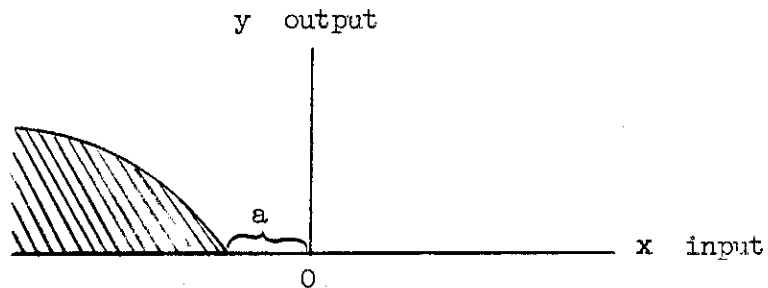


Figure 1

* Research undertaken by the Cowles Commission for Research in Economics under Contract Nonr-3055(01) with the Office of Naval Research.

The effect of the set-up costs is to move the production possibility set away from the origin.

First the simple problem involving only one set-up cost is investigated. In order to answer our questions concerning taxation we may consider and compare three cases as follows:

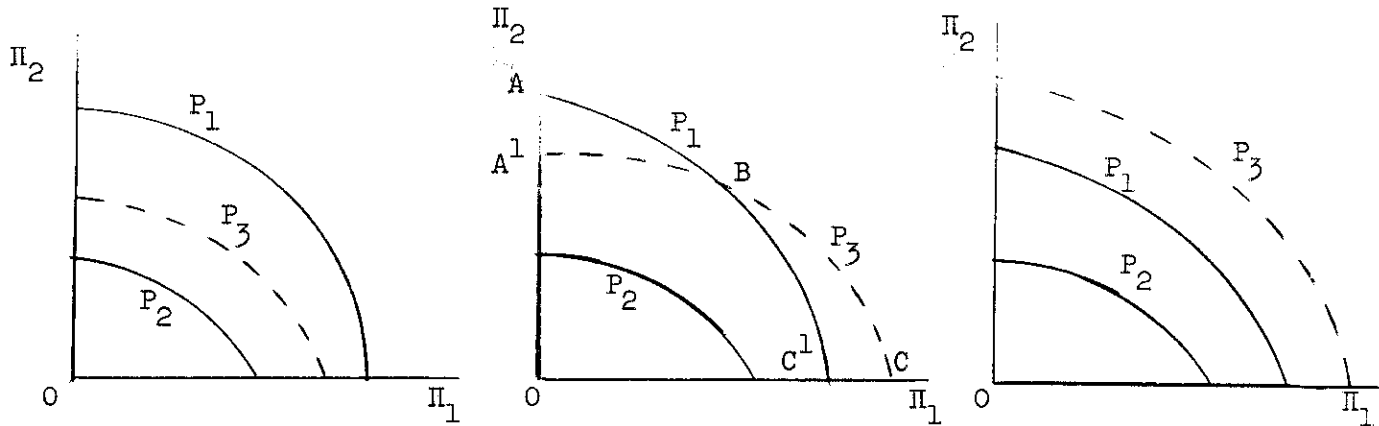
(1) Consider the closed economy with the initial distribution of resources given. Suppose that by government or by some other fiat the process involving set-up costs is not available to the members of the economy. Let production and trade take place in the economy without using the forbidden process. We may calculate the Pareto optimal surface for this economy as is shown in Figure 2.

(2) Consider the same state as above in (1) with one change. The government taxes the members of the economy (in resources) precisely up to the level necessary to pay for all of the set-up costs (whether they are indivisible or not makes no difference). We calculate the new Pareto optimal surface for the economy not using production process involving set-up costs after the government has removed resources via taxation.

(3) We calculate a third Pareto optimal surface which results from trade and production as above in (2) with one change. The government uses the taxes to subsidize the set-up costs, hence the new trade and production takes place with the members of the economy (with their diminished resources) in a position to use all production

processes without paying set-up costs and without increasing returns.

The three steps noted above give rise to three cases as indicated in Figures 2a, b and c. In these figures we demonstrate



the two-person case where Π_1 and Π_2 represent the gains to the first and second individuals respectively. In each figure P_1 , P_2 and P_3 represent the Pareto optimal surfaces derived from steps 1, 2 and 3.

In Figure 2a P_3 is always worse than P_1 . This means that set-up costs cannot be recouped by production. There is no incentive to use processes involving set up costs and no need for taxation in order to have a decentralized pricing system.

In Figure 2c P_3 is always better than P_1 . This means that set-up costs can always be recouped by production. Any taxation scheme which results in the domination of P_1 by P_3 and hence pays for the set-up cost subsidy will enable the economy to function using a price system.

Figure 2b shows two cases caused by the nonconvexity of the Pareto optimal surface. The nonconvex surface ABC shows the Pareto optimal possibilities. If the "solution" had been in the range AB without production, at best the possibilities with production would be on the segment A^1B which means that taxation would cause the second individual to always be worse off as compared with the no production outcome. Had the previous solution been in the range BC^1 then a taxation scheme could be found resulting in a more optimal outcome on BC.

2. The General Case

The above demonstration has been given for two individuals and a set-up cost. The general case is the same except that there is a finite combinatoric problem which must be explored when there are n individuals and k set-up costs. In particular 2^k Pareto optimal surfaces must be considered. The outer boundary of the union of all of these Pareto optimal surfaces describes the optimal possibilities as is shown in Figure 3, where the number 1 stands for

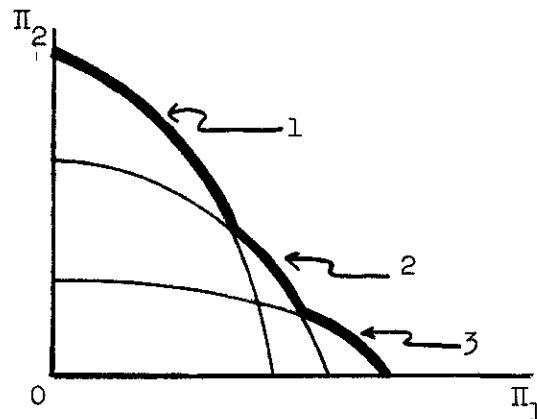
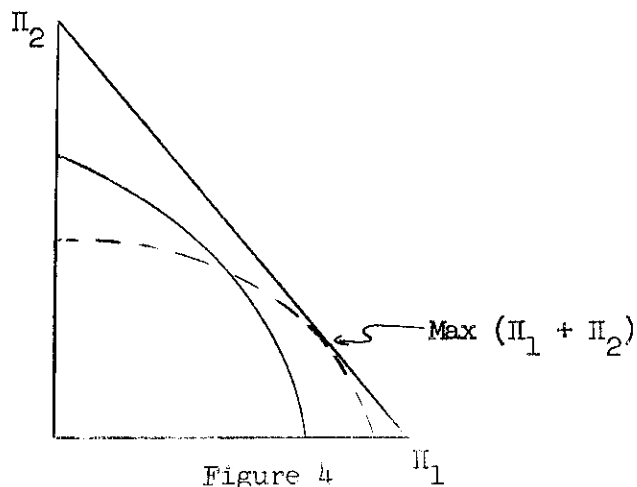


Figure 3

the Pareto optimal surface without production; 2 with one production process active and 3 with two production processes active.

3. Transferable Utility and Taxation

It is evident that with a transferable "u-money" (or with the use of mixed strategies) the Pareto optimal surface obtained from the union of the production possibilities will be flat as is shown in Figure 4 (which is an extension of Figure 2b). As the joint maximum is defined this is computed and the (side-payment)



optimal surface is the line with slope of -1 through this point.

For n persons and k set-up costs the process involves the maximization of the objective function $\Pi_1 + \Pi_2 + \dots + \Pi_n$ over the 2^k production possibilities.

4. The Competitive Equilibrium and Taxation without Transferable Utility

In general the Pareto optimal surface will be nonconvex

and consist of a set of "scalloped sections" each corresponding to a different array of activities with set-up costs being used. The question: "Is there a taxation which would permit all to behave as though they were in a normal competitive market?" is, if unmodified, trivial. The answer is yes; one has to pick a taxation to correspond to the payment for the set up costs for any one of the subsets of the 2^k possibilities which forms part of the boundary of the Pareto optimal surface. A more important question is: "Would the members of society vote approval for such a tax scheme?" This is a game theoretic problem and can only be answered by specifying the choices and powers of all individuals. In particular the status quo must be defined.

First a simple case (as illustrated in Figure 2c) can be disposed of. If the Pareto optimal surface is the boundary of a convex set there is only one level of taxation that must be considered and all will be able to agree on what processes should be active.

When there are say, m "scallops" in the surface there will be m different efficient levels of taxation to be considered. What happens if no taxation is agreed upon? This forces us to state what we mean by a "solution." In the next section we deal with several cooperative solutions; here however the game and its solution are as follows: Each individual may be assumed to have as part of his value system a liking for democratic voting processes and a market economy with prices. Thus we take as given the proposition

that there exists a government and that taxation will be determined by vote and that individuals will honor the outcome.

The pure technical problem of reducing 2^k alternatives to m efficient alternatives having been overcome, we are still left with the voting problem. Each of the m alternatives is for a level of taxation and the burden of this could be distributed in many ways. It is easy to observe that all the problems of the voting paradox exist. Binary choice between tax schemes may lead to non-transitivity of group preferences. However at least from a weak welfare point of view this is not important. The (possibly highly limited) goal was to show that it is always possible to find a tax scheme which brings about a competitive market and results in a Pareto optimal outcome. If we impose upon the society a voting mechanism for the selection of the taxes then depending upon the specifics of the mechanism different outcomes may be expected. Binary choice will lose transitivity, but keep Pareto optimality and simple majority decision. If all m alternatives are presented simultaneously then transitivity is no problem but the first choice may be decided upon only by plurality.

In the above we imposed as a desideratum taxation determined by vote and an economy run by the price system. If everyone agrees to abide by the rules of the game specified below then a Pareto optimal outcome can result.

Move 1: is made by "Nature," optimal tax and subsidy schemes are calculated and a device selects a set of propositions upon which

a vote is to be held.

Move 2: all individuals vote and the voting mechanism is such that a tax and subsidy scheme is selected.

Move 3: the taxes are used to pay set-up costs decided upon in the vote. The processes with these set-up costs (now paid) are available to anyone who wants to use them. Everyone now trades and produces in a competitive market.

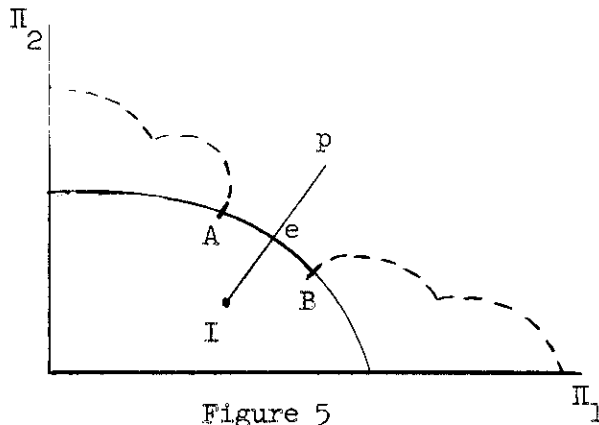


Figure 5

The limit on the distribution of taxes is shown in Figure 5. Once the level has been selected its distribution must be such that from the after-tax initial resource point I the resultant price system p intersects the segment AB as is indicated at point e. Such tax schemes always exist.

5. Other Solutions and Measures of Solution

The solution given in Section 4 was based upon the assumptions that all members of the society were willing to vote upon a taxation scheme to pay for the set up costs. We may view the pro-

blem in a completely different manner and consider the economy as a general cooperative game and examine the value^{1/} and core^{2/} solutions. There is little difficulty in formalizing the problem as a game, as a "C-game"^{3/} is obtained immediately. A C-game is one in which the amount obtained by any coalition is independent of the acts of the other members of society.

The Shapley and Harsanyi values always exist but would be difficult to compute in general. Each one of the 2^n coalitions must be evaluated as in Sections 2 or 3.

Does a core exist? In order to answer this question care must be taken to specify the nature of the game. For example if a simple majority of the voters can impose any tax scheme they want then no game has a core. If there is no vote but merely open bargaining among all coalitions then some games will have a core and some will not. For example consider the following three person game:

There are three goods, the second can be manufactured from the first if a set-up cost of one unit of the third is paid. There are three individuals in the society with resources $(1, 0, 1)$ $(1, 0, 1)$ and $(0, 1, 0)$. They all have the same utility function $\min [x, y, z]$. The production condition can be expressed $y = \delta x$ where $\delta = 1$ if $z = 1$ $\delta = 0$ if $z = 0$.

The characteristic function of this game is:

$$V(1) = V(2) = V(3) = 0$$

$$V(1,2) = V(1,3) = V(2,3) = 1$$

$$V(1,2,3) = 1$$

It has no core and it corresponds to the case where the Pareto optimal surface has segments or "scallop" corresponding to different optimal production and exchange arrangements. If the Pareto surface after set-up costs are paid remains convex (as in Figure 2c) then the game will have a core.

It is conjectured that if there are a fixed number of set-up costs to be paid then the limit game formed by considering successively larger games with more players of each type will have a strong ϵ -core in the sense of Shapley and Shubik^{4/} even if the Pareto optimal surface is nonconvex.

6. Some Further Difficulties and Summary

If the set-up costs are of variable size or if they occur in "steps" as would be the case where one or more of an indivisible machine is needed, then the above analysis in principle remains the same but becomes relatively useless without a specific algorithm.

It is conjectured that a weak ϵ -core will exist in limit games with "step" indivisibilities.

Although a practical algorithm for finding the various "solutions" to the economic problems of set-up costs is highly desirable the goal of this paper is somewhat less. It is to show that as soon as taxation is considered great care must be taken in defining the game. Different solutions will be highly sensitive to slight changes in the rules (the core and voting for example).

Given that the taxation mechanism is defined then there always is a way of taxing and paying subsidies so that the market can function via a price system to achieve a Pareto optimal outcome.

Many of the major difficulties in taxation, voting, subsidies and welfare economics lie not so much in the analysis of a model but in deciding what is the concept of solution to be used and what is a legitimate and relevant model of the politico-economic process.

Footnotes

- 1/ Shapley, L. S., "A Value for N-Person Games," in H. W. Kuhn and A. W. Tucker (Eds.), Contributions to the Theory of Games, Vol. II, Princeton University Press, 1953, pp. 307-317.
- 2/ See: Luce R. D. and H. Raiffa, Games and Decisions, John Wiley and Sons, 1957.
- 3/ Shapley, L. S. and M. Shubik, "Competition, Welfare and the Theory of Games," (unpublished manuscript RAND) Chapter III.
- 4/ Shapley, L. S. and M. Shubik, "Quasi-bore in a Monetary Economy with Nonconvex Preferences," RAND RM-3518-1-PR, October 1965.