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PRICE STRATEGY OLIGOPOLY WITH PRODUCT VARIATION

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by

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1. The Market With Undifferentiated Products

Suppose that two firms each with a constant average cost c face a market for an undifferentiated product. For simplicity we consider a model with a linear demand, constant average costs and given capacities for the firms; however our remarks are more general.

In the classical writings on duopoly the various authors[1] observed it is possible to formulate a duopoly model where the firms use either quantity or price as their strategy. The use of quantity as a strategy leads to the well known Cournot model[2]. It has been suggested that price is a far more natural variable for the firm to use in its strategy. Both Bertrand and Edgeworth[3] investigated the simple price model with the sellers selling an undifferentiated product. They obtained different results and the difference can be explained in terms of different assumptions concern-

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ing capacity restrictions (or equivalently rising costs).

A great amount of the difficulty in dealing with oligopolistic price strategy models comes in describing the demand conditions that exist when various prices are quoted. This is especially striking when the products are totally undifferentiated; however even with differentiation, oligopoly contingent demand curves[4] may have kinks and bends which make them difficult to analyze.

Chamberlain suggested in his theory of monopolistic competition[5] that one should treat each competitor as though he were a monopolist in the sense that each individual provides a slightly differentiated product or service when compared with any other even though the other may purportedly be selling the same good. In this paper we mathematically formulate an example of firms in monopolistic competition and investigate noncooperative behavior as the number of firms increases and as the level of product variation is decreased.

2. The Bertrand-Edgeworth Examples

We consider two firms each selling the same product to a market whose aggregate demand can be represented by $q = a - bq$. We wish to describe the demand if more than one price exists. This can be done by considering an aggregate consumer with a quadratic utility function:

$$(1) \qquad u(q) = aq - \frac{b}{2}q^2 \qquad \text{where} \quad q = q_1 + q_2$$

It is easy to see that the consumer in this instance will always buy from the firm with the lower price, as long as it has sufficient supplies. If it runs out of supplies then the constrained consumer maximization is such that it might make purchases from the other firm as well. It follows immediately that if each firm has sufficient capacity to supply the whole market at any price (at or above costs), then the firm with the lower price will take all. In this case the noncooperative equilibrium, will, as was observed by Bertrand, be the competitive equilibrium where each firm sets price equal to marginal cost (in this case marginal cost equals average cost).

Edgeworth noted that if capacities were limited the consumers might also buy from the higher priced firm and instead of settling down immediately to the competitive price, price in the market would be indeterminate and would tend to fluctuate over a range. This has been described as the Edgeworth cycle[6].

At what capacity level does the nature of the solution change, and how is the solution affected by the presence of more competitors? A diagramatic and algebraic investigation of these questions is presented.

Let each firm have an average cost c and a capacity k. Figures 1 and 2 represent conditions where respectively each firm has a capacity of $k = \frac{a - c}{b}$ and $k = \frac{a - c}{2b}$.

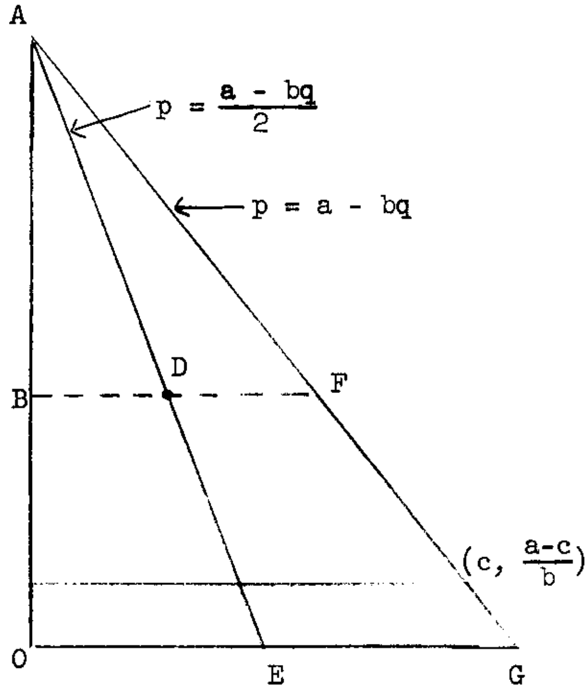

Figure 1

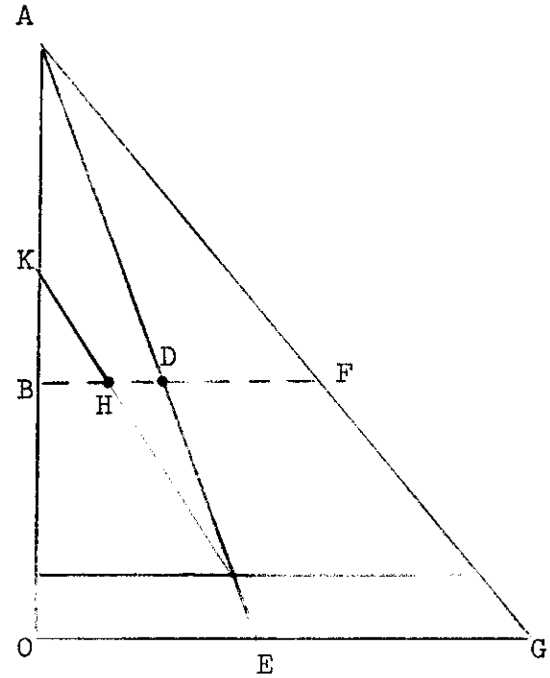

Figure 2

In Figure 1 the line AG describes the overall market demand when the firms charge the same price. The line AE is the demand on the individual firm when both charge the same price. Suppose that one firm charges a price represented by OB . We may draw the contingent demand curve faced by the other firm as it varies its price. First consider what it will sell if it charges a price higher than OB . It will sell nothing as is indicated by the segment AB . The reason is evident, because at any price above c , the other firm has enough capacity to satisfy the whole market. If both firms name the same price, the market will be split at D . If the second firm undercuts the first then its demand will be given by FG . Hence, given the price of the first firm, the contingent demand for the second as it varies its price from p = a to p = 0 is given by the broken curve AB D FG . For every price of the first firm there will be a contingent demand defined for the second firm. Furthermore as long as the first firm prices at or above cost, if the second firm names a higher price than the first there will be no market left.

In Figure 2 we consider the effect of a capacity limitation. We note that together they have enough capacity to satisfy the market at price equals cost, hence the efficient solution is not effected by capacity restrictions. This is not so for the structure of contingent demand. This change is critical for the change in noncooperative behavior.

Suppose that the first firm charges the price given by OB . At that price the consumer are willing to buy BF , however the firm has only enough capacity to supply HF leaving an unsatisfied demand of BH . Suppose that the second firm is charging a higher price than the first, its demand may not be completely wiped out. In this instance there will be some demand left up to the price OK as is indicated by the segment of the contingent demand curve given by HK . As the price of the second firm varies across the whole range the total contingent demand curve is given by AK KH D and FG . This possibility that there may be some demand left for the higher priced firm destroys the stability of the efficient point as a noncooperative equilibrium. This can be seen from Figure 3. Suppose that the first firm sets its price equal to cost. If it had enough capacity to saturate the market then there

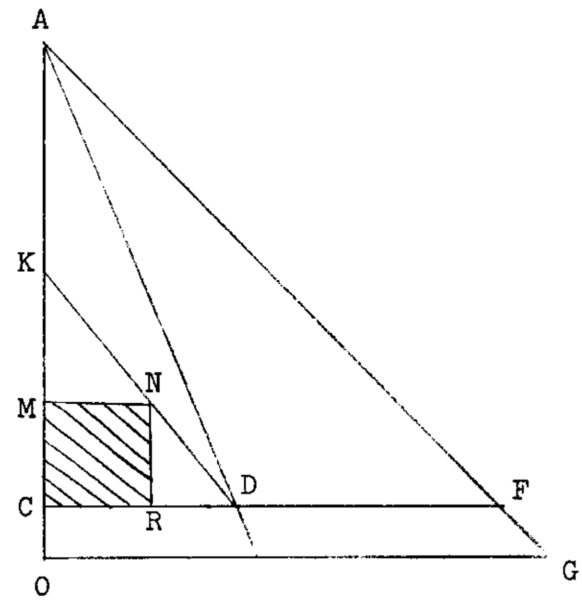

Figure 3

would be nothing left for the other at any higher price. The argument holds symmetrically hence the efficient point is also a non-cooperative equilibrium point inasmuch as given that either knows that the other is charging $p_i = c$ neither is motivated to change his price.

When we limit capacity to $k = (a - c)/2b$ (or the amount given by DF in Figure 3) we may not only show why this equilibrium is destroyed but can even demonstrate how far the second firm would raise its price given the information that the first was charging c . The contingent demand curve, given that the first firm charges c is shown by AKD and FG . The best move for the second firm is to maximize monopolistically by charging a price OM . It obtains a profit indicated by the rectangle MNCR .

Any capacity less than $k = (a - c)/b$ causes the destruction of the equilibrium point at $p = c$.

3. Contingent Demand and Product Differentiation

The discontinuity in the contingent demand curves was a natural result of the assumptions of the absolute identity of products and rational consumer behavior. This should disappear if we introduce product differentiation. A way to do so which yields us a model sufficiently simple to permit a diagramatic and algebraic treatment is by introducing an extra term into the previously used utility function. This term reflects a degree of differentiation among the goods. In spite of its relative simplicity the model is rich enough to illustrate the interesting features of oligopolistic demand and to serve as the basis for the study of the change in oligopolistic behavior as the number of competitors is increased.

The aggregate utility function used is given by:

$$(2) \quad u = aq - \frac{b}{n}q^2 - \epsilon\left[\frac{\Sigma q_i^2}{n} - \frac{q^2}{n^2}\right] - \sum_{i=1}^{n} p_i q_i \quad \text{where} \quad q = \sum_{i=1}^{n} q_i$$

This contains the parameter n in various terms. The reason for its inclusion in the utility function will be explained when we investigate oligopolistic behavior involving markets with increasing numbers of firms and customers. For our discussion of contingent demand we limit ourselves to $n = 2$. This gives us:

$$U = aq - \frac{b}{2}q^2 - \frac{\epsilon}{4}(q_1 - q_2)^2 - p_1q_1 - p_2q_2 , \tag{3}$$

Where ϵ is the parameter which controls the degree of product differentiation. If it is zero, then the products are perfect substitutes. The higher it is, the more complementary they become and the more the third term dominates the utility function. As the model we are investigating is an open partial model of the economy we ignore income effects as a first approximation and hence may take into account the budget constraint of the consumers by subtracting the terms p_1q_1 and p_2q_2 directly in the utility function.

Using equation (3) and the condition for consumer optimization we may solve for consumer demand in terms of prices. This gives us:

$$\frac{\partial u}{\partial q_1} = a - bq - \frac{\epsilon}{2}(q_1 - q_2) - p_1 = 0 \tag{4}$$

$$\frac{\partial u}{\partial q_1} = a - bq - \frac{\epsilon}{2}(q_2 - q_1) - p_2 = 0 . \tag{5}$$

From (4) and (5) by addition and subtraction (6) and (7) are obtained:

$$2a - 2bq = p_1 + p_2 \tag{6}$$

(7) $$-\epsilon(q_1 - q_2) = p_1 - p_2 ;$$

these yield

(8) $$q_1 = \frac{2a - p_1 - p_2}{4b} - \frac{p_1 - p_2}{2\epsilon}$$

or $$\boxed{q_1 = \frac{2a - (1 + \frac{b}{\epsilon})p_1 - (1 - \frac{b}{\epsilon})p_2}{4b}}$$

and similarly for q_2 . These solutions will hold only if

(9) $$\epsilon \geq \frac{2b(p_1 - p_2)}{2a - p_1 - p_2} .$$

Condition (9) is obtained directly from (8) by setting $q_1 = 0$.

If $p_1 > p_2$ and (10) $0 \leq \epsilon \leq \frac{2b(p_1 - p_2)}{2a - p_1 - p_2}$ then $q_1 = 0$

and from (5) we obtain:

(11) $$\boxed{q_2 = \frac{a - p_2}{b + \frac{\epsilon}{2}}} ,$$

thus we have a continuous contingent demand with two "kinks" as is shown in Figure 4. The labels are for the case where p_1 is fixed and p_2 varies.

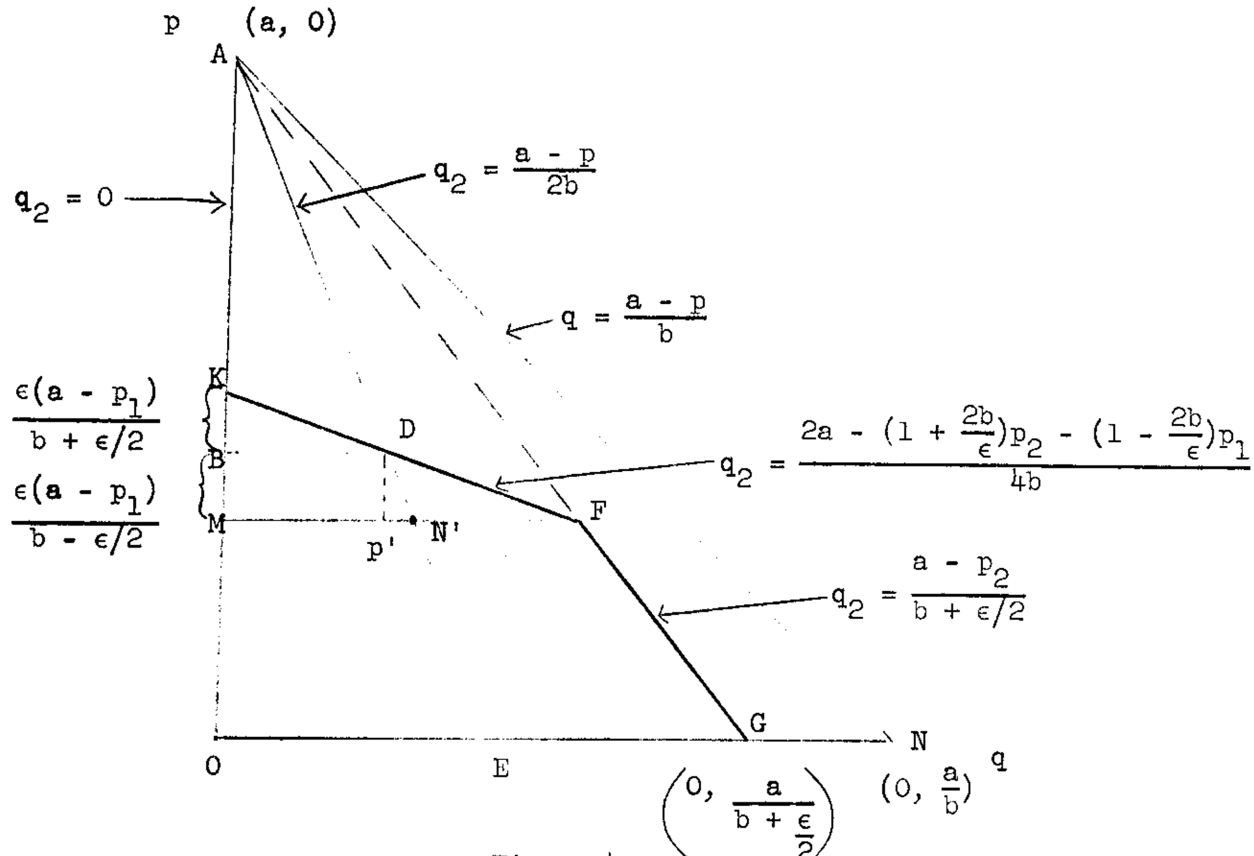

Figure 4

The contingent demand is given by AKDFG . It is of interest to note that because the goods are not perfect substitutes the low priced firm will never obtain "all of the market" in the sense of the amount ON which would be sold if both charged the same low price. The gap GN is a measure of the lack of substitutability. The line AN is an "apples and oranges" addition of the amounts sold by both when they charge the same price. The line AE gives the amount sold by one firm on the assumption that the other is charging the same price. The equations for the three line segments AK , KF and FG which make up the contingent demand are given. At the point F the second firm has completely priced the first out of the market.

4. The Noncooperative Price Duopoly Without Capacity Constraints

When the firms are equal and capacity constraints are not tight we are able to solve for the noncooperative equilibrium in the market by observing that a symmetric equilibrium will exist which enables us to write down payoff or revenue functions involving demands as given in equation (8). Thus

$$(12) \qquad P_i = (p_i - c)\left(\frac{2a - (1 + \frac{b}{\epsilon})p_i - (1 - \frac{b}{\epsilon})p_j}{4b}\right)$$

can be written and solved analytically giving:

$$(13) \qquad \boxed{p_i = \frac{2a + c(1 + \frac{2b}{\epsilon})}{3 + \frac{2b}{\epsilon}}}$$

A straightforward check that this is an equilibrium point is obtained by setting the price of one firm at that given by (13) and checking for the maximum for the other using all three segments of his contingent demand.

When $\epsilon = 2b$ the point G is moved to E and (13) simplifies to

$$(14) \qquad p_i = \frac{a + c}{2} ,$$

each may charge his monopoly price.

As ϵ approaches zero p_i approaches c which is the efficient point solution for undifferentiated goods.

The solution given by equation (13) is not the efficient point solution for this monopolistic market. If the economy were being run for the benefit of the consumer the condition that price equals cost would still prevail. It is of interest to note however that as differentiation is removed even with only two competitors the noncooperative equilibrium (without capacity constraints) approaches the efficient solution as indicated by the Bertrand model.

The noncooperative equilibrium can be illustrated by means of a diagram as is shown in Figure 5 below. N is the equilibrium

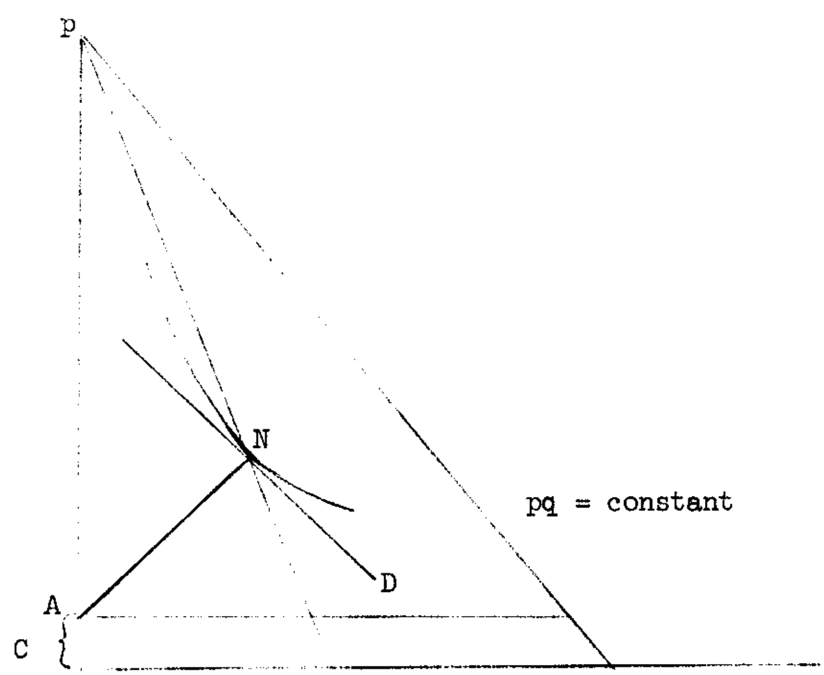

Figure 5

point. The curve "pq = constant" is the highest isoprofit curve attainable when on the contingent demand curve ND . The slope of AN is the negative reciprocal of ND .

5. The Noncooperative Price Oligopoly Without Capacity Constraints

In order to appreciate the effect of increasing the number of competitors in such a way that we can study the change in economic power as numbers increase we introduce the parameter n into the utility function. This has the effect that the size of the market faced by any individual firm, if all are charging the same price will be the same regardless of the number of competitors. The absolute economic size of all firms remains constant but their size relative to the sum of all markets decreases.

Rewriting the utility function for the n person market

$$U = aq - \frac{b}{n}q^2 - \epsilon \left[\frac{\Sigma q_i^2}{n} - \frac{q^2}{n^2} \right] - \sum_{i=1}^{n} p_i q_i \tag{15}$$

we differentiate to obtain demand conditions and solve for demands in terms of all prices.

$$\frac{\partial U}{\partial q_i} = a - \frac{2bq}{n} - \frac{2\epsilon}{n}q_i + \frac{2\epsilon}{n^2}q - p_i = 0 \quad \text{or} \tag{16}$$

$$p_i = a - \frac{2b}{n}q - \frac{2\epsilon}{n}q_i + \frac{2\epsilon}{n^2}q \, . \qquad i = 1, \ldots, n \tag{17}$$

Summing all of the equations (17) and dividing by n we obtain the average price:

$$\bar{p} = \frac{1}{n}\Sigma p_i = a - \frac{2b}{n}q - \frac{2\epsilon}{n^2}q + \frac{2\epsilon}{n^2}q = a - \frac{2b}{n}q \ . \tag{18}$$

Hence

$$p_i - \bar{p} = -\frac{2\epsilon}{n}(q_i - \bar{q}) \qquad \text{where} \quad \bar{q} = q/n \ . \tag{19}$$

Using (18) we may write $\bar{q} = \frac{n(a - \bar{p})}{2nb} = \frac{a - \bar{p}}{2b}$.

From (19) $q_i - \bar{q} = \frac{n}{2\epsilon}(p_i - \bar{p})$

hence (20) $\boxed{q_i = \frac{a - \bar{p}}{2b} - \frac{n}{2\epsilon}(p_i - \bar{p})}$.

The revenue for the i^{th} firm may be expressed as:

$$P_i = (p_i - c)q_i \tag{21}$$

taking derivations, setting them equal to zero and then setting all $p_i = p$ we may solve for the symmetric noncooperative equilibrium.

$$\frac{\partial}{\partial p_i}((p_i - c)q_i) = (p_i - c)\left[-\frac{1}{2nb} - \frac{n}{2\epsilon} + \frac{1}{2\epsilon}\right] + \frac{a - \bar{p}}{2b} - \frac{n}{2\epsilon}(p_i - \bar{p}) = 0 \tag{22}$$

or $$(p - c)\left[\frac{-\epsilon - n\,(n-1)b}{2\epsilon nb}\right] + \frac{a - p}{2b} = 0$$

giving (23) $$p = \frac{n(n-1)bc + \epsilon(na + c)}{n(n-1)b + \epsilon(n + 1)}$$

As ϵ approaches zero p approaches c as we have already noted in the case of duopoly. Here however we have a further convergence result. As $n \to \infty$ we observe that $p \to c$. In other words as the number of competitors increases even though they are selling differentiated products the noncooperative equilibrium approaches the efficient point solution where price equals costs.

The results will be qualitatively the same for firms with increasing marginal costs provided that they are not so steep as to have the effect of capacity limitation.

6. Price Oligopoly With Capacity Constraints

It was possible to demonstrate in Section 2 that in price competition capacity is of critical importance in preserving the stability of equilibrium. If the capacity of the competitor with the lowest price is limited this will increase the market to the others. It is evident that this effect will be present for differentiated as well as undifferentiated competitors.

The condition needed to preserve the equilibrium in the market for an undifferentiated good is that each firm has enough capacity to supply the whole market at price equals cost. In this section we investigate the conditions needed when the firms sell products differentiated as in the demand structure given in equation (20) as derived from the utility conditions given in (15).

Let each firm have a capacity k , suppose that all except the first is selling at capacity, from (17) we may write:

$$(24) \qquad p_1 = a - \frac{2(n-1)}{n}bk + (n-1)\frac{2\epsilon}{n^2}k - \frac{2}{n}\left[b + \epsilon - \frac{\epsilon}{n}\right]q_1$$

The optimal price and production for the first firm are determined by setting $\dfrac{d[(p_1 - c)q_1]}{dp_1} = 0$, which gives:

$$(25) \qquad q_1 = \frac{a - c + \frac{2(n-1)}{n}(\frac{\epsilon}{n} - b)k}{\frac{4}{n}\left(b + \epsilon(\frac{n-1}{n})\right)} .$$

The profit at (p_1, q_1) is:

$$(26) \qquad (p_1 - c)q_1 = \frac{\left[a - c + 2\frac{(n-1)}{n}(\frac{\epsilon}{n} - b)k\right]^2}{\frac{8}{n}\left(b + \epsilon(\frac{n-1}{n})\right)}$$

From (25) we may write the profit at the noncooperative equilibrium as:

$$(27) \qquad P = \frac{\epsilon(a - c)^2(n(n-1)b + \epsilon)}{2b(\epsilon(n+1) + n(n-1)b)^2}$$

By setting (26) = (27) and solving for k we can determine the critical capacity below which the equilibrium is destroyed. We call the critical capacity k_n to stress its dependence upon n.

$$(28) \quad k_n = \frac{n^2(a - c)}{2(n-1)(nb - \epsilon)} \left\{ 1 - \frac{1}{n} \frac{\sqrt{\epsilon(bn + \epsilon(n-1)(n(n-1)b + \epsilon)}}{(\epsilon(n+1) + n(n-1)b)} \right\}$$

For $\epsilon = 0$ we obtain $k_n = \dfrac{n^2(a - c)}{2(n-1)nb}$.

This gives $k_2 = \dfrac{(a - c)}{b}$ and $k_\infty = \dfrac{(a - c)}{2b}$; this shows that with undifferentiated products the duopolists need an excess capacity so large that each could individually saturate the whole market, if a pure strategy equilibrium is to be preserved. As the numbers grow each needs only in the limit his efficient production capacity.

For any fixed ϵ as $n \to \infty$ we have:

$$k \to \frac{a - c}{2b} + 0\left(\frac{1}{n^{3/2}}\right) \to \frac{a - c}{2b} .$$

For duopoly in general (28) gives:

$$k_2 = \frac{2(a - c)}{(2b - \epsilon)} \left\{ 1 + \frac{1}{2}\left(\frac{2b + \epsilon}{2b + 3\epsilon}\right)\sqrt{\epsilon} \right\} .$$

We can show the destruction of the pure strategy noncooperative equilibrium by a diagram as in shown in Figure 6. In

Figure 4 the contingent demand with no capacity constraint was AKDFG this has a kink at F . When capacity constraints come into

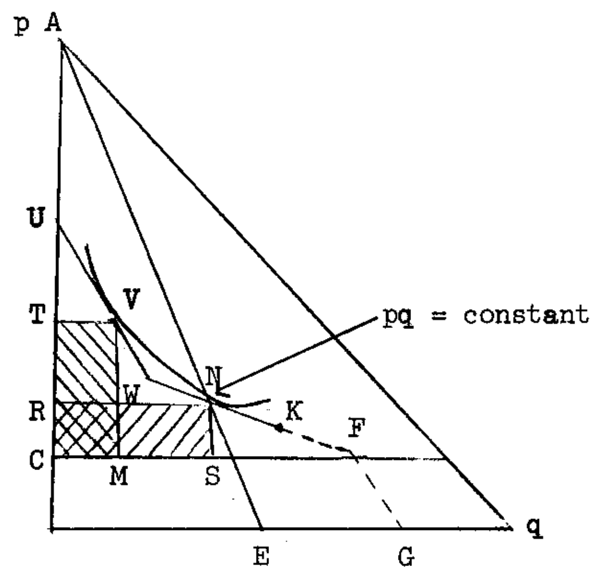

Figure 6

effect an added "kink" is introduced as is shown in Figure 6 where the contingent demand is AUWNFG . The point K is the capacity limit of the lower priced firm which cause the kink at W and the branch of demand UW .

When the isoprofit curve becomes tangent to the contingent demand at two points the pure strategy equilibrium is destroyed. The profit from raising price will be as high as maintaining the same price as the competitor. The profits are shown in Figure 6 by areas TVMC and RNSC . N is the pure strategy noncooperative equilibrium that would prevail if capacity were plentiful and V is the point at which profit possibilities destroy the equilibrium. It can only exist if there is the added kink at W which in turn depends upon capacity limitations.

The interpretation of these results is straightforward. In a price oligopoly there will be price instability unless there is sufficient overcapacity. For any level of capacity of $k > (a - c)/2b$ for a fixed ϵ the introduction of more competitors will eventually remove the price instability, i.e. there will be a specific $n = n(k, \epsilon)$ beyond which there will be a pure strategy equilibrium in the market, and as n increases this equilibrium approaches the efficient solution.

In many oligopolistic markets with few firms we expect some overcapacity however not necessarily enough to prevent price instability. In larger markets we may expect that the overcapacity is sufficient to lead to stability. Although we have presented our analysis in terms of a specific example, our results appear to be general for any economically reasonable symmetric model.

FOOTNOTES

1. Cournot, A. A., Researches into the Mathematical Principles of the Theory of Wealth, New York, Macmillan, 1897.

   Edgeworth, F. Y., Papers Relating to Political Economy, London: Macmillan.

2. Cournot, A. A., op.cit.

3. See Shubik, M., Strategy and Market Structure, New York, Wiley, 1959, Chs. 4 and 5.

4. Ibid., Ch. 5.

5. Chamberlain, E. H., The Theory of Monopolistic Competition, Cambridge, Harvard University Press, 6. ed., 1950.

6. Shubik, M., op.cit. Ch. 5.